Transportation
Library

TA
1
.A526

TRANSACTIONS

OF THE

AMERICAN SOCIETY

OF

CIVIL ENGINEERS

(INSTITUTED 1852)

VOLUME 96

1932

Edited by the Secretary, under the direction of the Committee on Publications.

Reprints from this publication, which is copyrighted, may be made on condition that the full title of Paper, name of Author, and page reference are given.

TRANSPORTATION LIBRARY

NEW YORK
PUBLISHED BY THE SOCIETY

1932

Entered according to Act of Congress, in the year 1932, by the AMERCAN SOCIETY OF CIVIL ENGINEERS, in the office of the Librarian of Congress, at Washington.

NOTE.—This Society is not responsible for any statement made or opinion expressed in its publications.

CONTENTS

PAPERS

No.		PAGE
1793	**ANALYSIS OF CONTINUOUS FRAMES BY DISTRIBUTING FIXED-END MOMENTS.**	
	By Hardy Cross	1
	Discussion:	
	By C. P. Vetter	11
	L. E. Grinter	11
	S. S. Gorman	20
	A. A. Eremin	21
	Elmer F. Bruhn	22
	A. H. Finlay	23
	R. F. Lyman	27
	R. A. Caughey	28
	Orrin H. Pilkey	29
	I. Oesterblom	32
	Robert A. Black	35
	H. E. Wessman	40
	Edward J. Bednarski	52
	S. N. Mitra	55
	Jens Egede Nielsen	55
	F. E. Richart	56
	William A. Oliver	60
	R. R. Martel	60
	Clyde T. Morris	66
	Francis P. Witmer	69
	T. F. Hickerson	74
	F. H. Constant	79
	W. N. Downey	80
	E. C. Hartmann	93
	Thomas C. Shedd	94
	David M. Wilson	95
	Marshall G. Findley	101
	George E. Large	101
	Sophus Thompson and R. W. Cutler	108
	Alfred Gordon	111
	A. W. Earl	112
	A. Floris	116
	I. M. Nelidov	117
	E. A. MacLean	123
	George M. Dillingham	125
	Donald E. Larson	127
	J. A. Van den Broek	133
	Hardy Cross	138

No. PAGE

1794 SOME ASPECTS OF WATER CONSERVATION.

By **R. A. Sutherland** 157

Discussion:

By E. C. Eaton 198
A. W. Simonds 206
W. H. R. Nimmo 207
C. R. Olberg 212
John W. Pritchett 215
A. J. Gilardi 220
R. A. Sutherland 225

1795 CONSTRUCTION OF LA OLA PIPE LINE.

By **W. B. Saunders** 230

Discussion:

By Ernest B. Nelson 266
H. L. Thackwell 267
William E. Rudolph 268
W. B. Saunders 270

1796 THEORY OF SIMILARITY AND MODELS.

By **Benjamin F. Groat** 273

Discussion:

By H. Bateman 304
W. P. Roop 305
A. V. Karpov 308
Jasper O. Draffin 326
Ed S. Smith, Jr. 327
F. J. Eisner 329
Benjamin F. Groat 335

1797 TESTS OF BROAD-CRESTED WEIRS.

By **James G. Woodburn** 387

Discussion:

By H. W. King 417
Sherman M. Woodward 418
David L. Yarnell 419
Boris A. Bakhmeteff 423
D. D. Curtis 434
Erik G. W. Lindquist 435
Ralph L. Parshall 440
James G. Woodburn 447

1798 HIGHWAY LOCATION: GENERAL CONSIDERATIONS.

By **Gibb Gilchrist** 454

Discussion:

By C. S. Mullen 462
H. L. Thackwell 462
B. P. Harrison 464
E. P. Arneson 465
Gibb Gilchrist 466

No. PAGE

1799 HIGHWAY LOCATION: PRACTICAL CONSIDERATIONS.

By **A. R. Losh** 468

Discussion:

By H. K. Bishop 477
J. C. Carpenter 480
H. J. Spelman 482
H. S. Kerr 483
H. W. Griffin 485
A. R. Losh 487

1800 STRESSES IN GRAVITY DAMS BY PRINCIPLE OF LEAST WORK.

By **B. F. Jakobsen** 489

Discussion:

By William Cain 514
William Gore 520
L. J. Mensch 524
Lars R. Jorgensen 528
C. A. P. Turner 530
Donald P. Barnes 536
Anthony Hoadley 539
Fred W. Ely 542
M. H. Gerry, Jr. 547
Fredrik Vogt 548
Eugene Kálmán 556
Fernand Campus 567
A. Floris 571
Robert E. Glover 578
P. Wilhelm Werner 582
B. F. Jakobsen 584

1801 FORMULAS FOR RAINFALL INTENSITIES OF LONG DURATION.

By **Merrill M. Bernard** 592

With Discussion:

By W. B. Gregory 607
W. W. Horner 607
Adolph F. Meyer 608
R. W. Powell 609
C. E. Grunsky 612
Charles W. Sherman 616
Merrill M. Bernard 617

1802 ECONOMICAL DESIGN OF HYDRAULIC PIPE-LINE DREDGE.

By **John Francis Cushing** 625

Discussion:

By R. L. Vaughn 651
Arthur L. Collins 658
John Francis Cushing 661

No. PAGE

1803 THE BUTTRESSED DAM OF UNIFORM STRENGTH.
By Herman Schorer 666

Discussion:
By James Girand 689
Fred A. Noetzli 692
B. F. Jakobsen 696
Calvin V. Davis 697
A. A. Eremin 701
Eugene Kalman 703
A. Floris 710
Fredrik Vogt 713
A. C. Schwager 714
Paul Bauman 715
I. M. Nelidov 716
Herman Schorer 720

1804 RELOCATING LAND SURVEYS IN TEXAS.
By B. F. Williams 735

Discussion:
By J. M. Howe 744
Arthur D. Kidder 745
W. J. Powell 745

1805 FLOW OF WATER IN TIDAL CANALS.
By Earl I. Brown 749

Discussion:
By R. L. Faris 814
G. T. Rude 815
Eugene E. Halmos 819
W. M. Black 821
H. F. Flynn 822
Ralph G. Davis 827
Earl I. Brown 831

1806 THE DON MARTIN PROJECT.
By Andrew Weiss 835

Discussion:
By C. H. Howell 855
Robert S. Stockton 857
Andrew Weiss 859

1807 A DISCHARGE DIAGRAM FOR UNIFORM FLOW IN OPEN CHANNELS.
By Murray Blanchard 865

Discussion:
By Lynn Perry 871
J. B. Macphail 873
G. B. Pillsbury 873
Lynn Crandall 875
Benjamin E. Jones 876
A. J. Matheson 878
L. Standish Hall 879
Ray E. Mackenzie 880
Murray Blanchard 882

No. PAGE

1808 LEGAL PROBLEMS INVOLVED IN ESTABLISHING SET-BACK LINES.

By Clifton Williams 887

Discussion :

By J. M. ALBERS 896
J. E. WILLOUGHBY 898
HORACE H. SEARS 899
GEORGE H. HERROLD 901
CLIFFORD C. MUHS 902
H. B. COOLEY 903
WALTER C. SADLER 905
H. J. MCFARLAN 907
DONALD M. BAKER 910
E. G. WALKER 912

1809 THE EIGHT-MILE CASCADE TUNNEL, GREAT NORTHERN RAILWAY: A SYMPOSIUM 915

PART I — PRELIMINARY STUDIES AND RESULTS OF IMPROVING CASCADE CROSSING.

By D. J. Kerr 917

PART II — SURVEYS, CONSTRUCTION METHODS, AND A COMPARISON OF ROUTES.

By Frederick Mears 926

PART III — CONSTRUCTION PLANS AND METHODS.

By J. C. Baxter 950

Discussion :

By R. H. KEAYS 1001
J. G. SULLIVAN 1003
FREDERICK MEARS 1004

1810 MEASURING IRRIGATION DELIVERIES IN THE PANJAB.

By E. S. Lindley 1005

1811 SURVEYS FOR SWIFT RIVER RESERVOIR OF THE BOSTON METROPOLITAN WATER SUPPLY.

By N. LeRoy Hammond 1020

Discussion :

By C. B. BREED 1029
C. L. HALL 1033
FRANK C. SELLNOW 1033
N. LEROY HAMMOND 1036

No. PAGE

1812 **RUN-OFF — RATIONAL RUN-OFF FORMULAS.**

By **R. L. Gregory and C. E. Arnold** 1038

Discussion :

By LE ROY K. SHERMAN 1100
FRANCIS BATES 1100
JOHN W. RAYMOND, JR 1115
REGINALD A. RYVES 1118
G. S. TAPLEY 1122
W. I. HICKS 1124
CARL H. REEVES 1126
JOHN M. KEMMERER 1127
LEONARD L. LONGACRE 1128
G. H. HICKOX 1134
DONALD M. BAKER 1136
ALBERT R. ARLEDGE 1139
CLARENCE S. JARVIS 1140
C. E. GRUNSKY 1142
RALPH W. POWELL 1149
MERRILL M. BERNARD 1150
CHARLES H. LEE 1161
R. L. GREGORY AND C. E. ARNOLD 1163

1813 **GRAVITY DAMS ARCHED DOWN STREAM.**

By **B. J. Lambert** 1178

Discussion :

By EUGENE KALMAN 1184
WILLIAM P. CREAGER 1187
T. L. E. HAUG 1188
B. F. JAKOBSEN 1189
A. FLORIS 1190
FLOYD A. NAGLER 1191
B. J. LAMBERT 1192

1814 **FLOW OF GROUND-WATER AS APPLIED TO DRAINAGE WELLS.**

By **M. R. Lewis** 1194

Discussion :

By L. STANDISH HALL 1207
O. W. ISRAELSEN 1208
M. R. LEWIS 1210

1815 **DESIGN OF A REINFORCED CONCRETE SKEW ARCH.**

By **Bernard L. Weiner** 1212

Discussion :

By G. D. HOUTMAN 1321
J. CHARLES RATHBUN 1323
A. A. EREMIN 1327
BERNARD L. WEINER 1330

No. PAGE

1816 **MANUFACTURING CONCRETE OF UNIFORM QUALITY.**

By **William M. Hall** 1336

Discussion:

By Henry B. Seaman 1359
Theodore Belzner 1360
John Sanford Peck 1360
J. W. Kelly 1361
Stanton Walker 1362
Edward E. Bauer 1363
O. Bonney and D. T. Mitchell 1365
C. E. Arnold 1367
Morris Mason 1371
John H. Griffith 1372
W. B. Wendt 1374
William M. Hall 1376

1817 **SOME REFLECTIONS REGARDING ENGINEERING PRACTICE: ADDRESS AT THE ANNUAL CONVENTION, YELLOWSTONE NATIONAL PARK, WYOMING, JULY 6, 1932.**

By **Herbert S. Crocker** 1380

MEMOIRS OF DECEASED MEMBERS

Howard Adams Carson, Hon. M. Am. Soc. C. E. 1386
Raymond Edmond Adams, M. Am. Soc. C. E. 1388
William George Addis, M. Am. Soc. C. E. 1389
William Henry Adey, M. Am. Soc. C. E. 1390
Andrews Allen, M. Am. Soc. C. E. 1391
Louis Russell Ash, M. Am. Soc. C. E. 1393
Peter Christian Asserson, M. Am. Soc. C. E. 1397
Frank Harrison Bailey, M. Am. Soc. C. E. 1403
Samuel Pond Baird, M. Am. Soc. C. E. 1406
Bernard Arthur Behrend, M. Am. Soc. C. E. 1407
James Wartelle Billingsley, M. Am. Soc. C. E. 1411
Fred Milton Bisbee, M. Am. Soc. C. E. 1413
Howard Edward Boardman, M. Am. Soc. C. E. 1415
William James Boucher, M. Am. Soc. C. E. 1417
William Algernon Brackenridge, M. Am. Soc. C. E. 1419
Gustave Maurice Braune, M. Am. Soc. C. E. 1422
Hugh Brodie, M. Am. Soc. C. E. 1426
Paul King Bunn, M. Am. Soc. C. E. 1428
Hector Robins Burroughs, M. Am. Soc. C. E. 1429
George Raymond Campbell, M. Am. Soc. C. E. 1431
Edward Carlos Carter, M. Am. Soc. C. E. 1433
Willis Chipman, M. Am. Soc. C. E. 1435
John Christian Valdemar Christensen, M. Am. Soc. C. E. 1438
St. John Clarke, M. Am. Soc. C. E. 1439
James Retzer Comly, M. Am. Soc. C. E. 1442
Henry St. Leger Coppee, M. Am. Soc. C. E. 1444
Charles James Craigmile, M. Am. Soc. C. E. 1445
William Warren Cummings, M. Am. Soc. C. E. 1447
Sverre Dahm, M. Am. Soc. C. E. 1448
Frank Eugene Davidson, M. Am. Soc. C. E. 1451
Joseph Phineas Davis, M. Am. Soc. C. E. 1453
John Stanley Dawson, M. Am. Soc. C. E. 1457
Floyd Gossett Dessery, M. Am. Soc. C. E. 1458

PAGE
Harry Espenship Detwiler, M. Am. Soc. C. E. 1460
Frederic Clark Dunlap, M. Am. Soc. C. E. 1461
James Harvey Edwards, M. Am. Soc. C. E. 1463
George Ezra Ellis, M. Am. Soc. C. E. 1466
Charles Clement Elwell, M. Am. Soc. C. E. 1467
Frank Collins Emerson, M. Am. Soc. C. E. 1470
Lloyd Tilghman Emory, M. Am. Soc. C. E. 1474
Howard Richards Farnsworth, M. Am. Soc. C. E. 1476
Frank Foster, M. Am. Soc. C. E. 1478
Herman Fougner, M. Am. Soc. C. E. 1480
James Hillhouse Fuertes, M. Am. Soc. C. E. 1482
Edward Gagel, M. Am. Soc. C. E. 1486
Benjamin Hurt Hardaway, M. Am. Soc. C. E. 1489
James Clark Harding, M. Am. Soc. C. E. 1494
Edward Willard Howe, M. Am. Soc. C. E. 1495
Andrew Murray Hunt, M. Am. Soc. C. E. 1497
Edgar Jadwin, M. Am. Soc. C. E. 1498
Andrew Martin Jensen, M. Am. Soc. C. E. 1504
William Henry Jones, M. Am. Soc. C. E. 1505
Alfred Lawrence Kehoe, M. Am. Soc. C. E. 1506
Sentaro Kondo, M. Am. Soc. C. E. 1507
William Henry Lawton, M. Am. Soc. C. E. 1508
George Herbert Leland, M. Am. Soc. C. E. 1510
Milo Stuart MacDiarmid, M. Am. Soc. C. E. 1511
Ernest McCullough, M. Am. Soc. C. E. 1512
Horace Miller Marshall, M. Am. Soc. C. E. 1514
Montgomery Meigs, M. Am. Soc. C. E. 1516
Joseph Davis Metcalfe, M. Am. Soc. C. E. 1518
Eusebius Joseph Molera, M. Am. Soc. C. E. 1520
Richard Montfort, M. Am. Soc. C. E. 1521
Stephen Henley Noyes, M. Am. Soc. C. E. 1521
Frederick Sylvester Odell, M. Am. Soc. C. E. 1524
Arthur Onderdonk, M. Am. Soc. C. E. 1525
William Collins Phelps, M. Am. Soc. C. E. 1527
Henry Tegmeyer Porter, M. Am. Soc. C. E. 1528
Harry Longyear Preston, M. Am. Soc. C. E. 1531
Albert Reesor Raymer, M. Am. Soc. C. E. 1534
Abraham Moreau Reynolds, M. Am. Soc. C. E. 1536
Knud Sophus Riser, M. Am. Soc. C. E. 1537
Earl Ivan Roberts, M. Am. Soc. C. E. 1539
William Lawrence Saunders, M. Am. Soc. C. E. 1540
Anton Schneider, M. Am. Soc. C. E. 1542
Philip Schuyler, M. Am. Soc. C. E. 1547
Mitsugu Sengoku, M. Am. Soc. C. E. 1549
Robert Ames Shailer, M. Am. Soc. C. E. 1553
Willis Appleford Slater, M. Am. Soc. C. E. 1555
Jay L. Stannard, M. Am. Soc. C. E. 1558
Eugene Washington Stern, M. Am. Soc. C. E. 1560
Frank Stoddard Stevens, M. Am. Soc. C. E. 1562
Lewis Thornburg, M. Am. Soc. C. E. 1564
Lester Waldo Tucker, M. Am. Soc. C. E. 1566
William Francis Tye, M. Am. Soc. C. E. 1568
George William Volckman, M. Am. Soc. C. E. 1570
Fred J. Wagner, M. Am. Soc. C. E. 1572
Samuel Tobias Wagner, M. Am. Soc. C. E. 1574
John Simpson Walker, M. Am. Soc. C. E. 1576
Andrew Jackson Wiley, M. Am. Soc. C. E. 1577
Henry Dickinson Woods, M. Am. Soc. C. E. 1584
Charles Worthington, M. Am. Soc. C. E. 1586
Adolph Yappen, M. Am. Soc. C. E. 1588
Elmer Zarbell, M. Am. Soc. C. E. 1589
Julian Wilmot Alger, Assoc. M. Am. Soc. C. E. 1590
Albert Ross Bailey, Assoc. M. Am. Soc. C. E. 1592
George Castleman Breckinridge, Assoc. M. Am. Soc. C. E. 1594

PAGE
Morton Burden, Assoc. M. Am. Soc. C. E. 1595
Robert Morris Drake, Assoc. M. Am. Soc. C. E. 1597
Preston Burt Easterbrooks, Assoc. M. Am. Soc. C. E. 1597
James Henry Gallivan, Assoc. M. Am. Soc. C. E. 1600
James Hendricks Hallett, Assoc. M. Am. Soc. C. E. 1601
Lewis Stirling Hereford, Assoc. M. Am. Soc. C. E. 1603
William Chaffin Howe, Assoc. M. Am. Soc. C. E. 1603
Frank Burdette Kiel, Assoc. M. Am. Soc. C. E. 1604
John Bruce Mailey, Assoc. M. Am. Soc. C. E. 1607
Alfred Stowe Mirick, Assoc. M. Am. Soc. C. E. 1608
Joseph Lincoln Murphy, Assoc. M. Am. Soc. C. E. 1609
Robert McKnight Pardee, Assoc. M. Am. Soc. C. E. 1614
William Couts Randolph, Jr., Assoc. M. Am. Soc. C. E. 1615
Homer Austin Reid, Assoc. M. Am. Soc. C. E. 1616
Herbert Jones Sowden, Assoc. M. Am. Soc. C. E. 1618
Paul Bertram Spencer, Assoc. M. Am. Soc. C. E. 1619
Samuel Houston Ware, Assoc. M. Am. Soc. C. E. 1621
Charles Edward Willacker, Assoc. M. Am. Soc. C. E. 1622
Ferd Clark Wintermute, Assoc. M. Am. Soc. C. E. 1624
Thomas Rogers Hughes, Jun. Am. Soc. C. E. 1625
Elmer Barthold Lawson, Jun. Am. Soc. C. E. 1626
Bernard Joseph Merickel, Jun. Am. Soc. C. E. 1627
John Harry Miller, Jun. Am. Soc. C. E. 1628
William Josiah Karner, Affiliate, Am. Soc. C. E. 1630
Edward Alfred Simmons, Affiliate, Am. Soc. C. E. 1632
Edward Dean Adams, F. Am. Soc. C. E. 1636

ERRATA

Page 95, Line 19, the word, "employed," should read "employ."
Page 1010, Line 29, for "$\surd^2$," read V^2.

PAPERS AND REPORTS OF SPECIAL COMMITTEES PUBLISHED IN PROCEEDINGS, AM. SOC. C. E. APRIL, 1931, TO MAY, 1932, INCLUSIVE, NOT PUBLISHED IN TRANSACTIONS VOL. 96.

	Number of *Proceedings.*
AN ANALYSIS OF MULTIPLE-SKEW ARCHES ON ELASTIC PIERS. By **J. Charles Rathbun**	April, 1931
SPECIFICATIONS FOR BOUNDARY SURVEYS: PREPARED BY THE COMMITTEE OF THE SURVEYING AND MAPPING DIVISION ON BOUNDARY SURVEYS	May, 1931*
FINANCING STREET AND HIGHWAY IMPROVEMENTS. By **R. W. Crum**	August, 1931
PROGRESS REPORT OF THE COMMITTEE OF THE SANITARY ENGINEERING DIVISION ON NEW JERSEY SEWAGE EXPERIMENTS	August, 1931*

---

* Published in *Proceedings* only.

	Number of *Proceedings.*
BIBLIOGRAPHY OF PHYSICAL PROPERTIES AND BEARING VALUE OF SOILS. Compiled by **Morris Schrero**	August, 1931*
DESIGN OF LARGE PIPE LINES. By **Herman Schorer**	September, 1931
HIGHWAY CONSTRUCTION MANAGEMENT. By **T. Warren Allen**	September, 1931
CONSTRUCTION MANAGEMENT. By **Richard Hopkins**	September, 1931
SOILS MECHANICS RESEARCH. By **Glennon Gilboy**	October, 1931
ENGINEERING FEATURES OF THE ILLINOIS WATERWAY. By **Walter M. Smith**	October, 1931
CONSTRUCTION WORK ON A FEDERAL RECLAMATION PROJECT. By **Morris Mason**	October, 1931
WATER PRESSURES ON DAMS DURING EARTHQUAKES. By **H. M. Westergaard**	November, 1931
WESTERN HIGHWAY PRACTICE, WITH SPECIAL REFERENCE TO CALIFORNIA. By **C. S. Pope**	November, 1931
LOW-COST BITUMINOUS ROADS. By **Walter N. Frickstad**	November, 1931
EFFECTS OF BENDING WIRE ROPE. By **Frederick C. Carstarphen**	December, 1931
LAKE CHAMPLAIN BRIDGE. By **Charles M. Spofford**	December, 1931
CONSTRUCTION PLANT AND METHODS OF ERECTING STEEL BRIDGES. By **A. F. Reichmann**	December, 1931
FULTON STREET, EAST RIVER TUNNELS, NEW YORK, N. Y. By **Miles I. Killmer**	December, 1931
WIND-BRACING CONNECTION EFFICIENCY. By **U. T. Berg**	January, 1932
PUBLIC SUPERVISION OF DAMS: A SYMPOSIUM	January, 1932
STEREO-TOPOGRAPHIC MAPPING. By **C. H. Birdseye**	January, 1932
DETERMINATION OF PRINCIPAL STRESSES IN BUTTRESSES AND GRAVITY DAMS. By **W. H. Holmes**	January, 1932
ECONOMIC PROPORTIONS AND WEIGHTS OF MODERN HIGHWAY CANTILEVER BRIDGES. By **J. A. L. Waddell**	February, 1932

---

* Published in *Proceedings* only.

Number of *Proceedings.*

DESIGN CHARACTERISTICS OF THE READING OVERBUILD TRANSMISSION LINE.
By **Frederick W. Deck** .... February, 1932

STRESSES IN REINFORCED CONCRETE DUE TO VOLUME CHANGES.
By **C. P. Vetter** .... February, 1932

SECOND PROGRESS REPORT OF SUB-COMMITTEE NO. 31, COMMITTEE ON STEEL OF THE STRUCTURAL DIVISION, ON WIND-BRACING IN STEEL BUILDINGS. .... February, 1932*

SUSPENSION BRIDGES UNDER THE ACTION OF LATERAL FORCES.
By **Leon S. Moisseiff and Frederick Lienhard** .... March, 1932

CONSTRUCTION METHODS ON THE WELLAND SHIP CANAL.
By **E. G. Cameron** .... March, 1932

STRESSES IN INCLINED ARCHES OF MULTIPLE-ARCH DAMS.
By **George E. Goodall and Ivan M. Nelidov** .... March, 1932

THE COMPENSATED ARCH DAM.
By **A. V. Karpov** .... April, 1932

DEVELOPMENT OF HYDRO-ELECTRIC POWER AS AN AID TO IRRIGATION.
By **Charles C. Cragin** .... April, 1932

IMPROVEMENT OF HUDSON RIVER BY NARROWING THE NAVIGABLE FAIRWAY.
By **Lynne J. Bevan** .... May, 1932, Pt. 1

APPLICATION OF DURATION CURVES TO HYDRO-ELECTRIC STUDIES.
By **G. H. Hickox and G. O. Wessenauer** .... May, 1932, Pt. 1

STANDARD SYMBOLS AND GLOSSARY FOR HYDRAULICS AND IRRIGATION: COMPILED BY SPECIAL COMMITTEE ON IRRIGATION HYDRAULICS .... May, 1932, Pt. 1*

ENGINEERING SEISMOLOGY: NOTES ON AMERICAN LECTURES.
By **Dr. Kyoji Suyehiro** .... May, 1932, Pt. 2*

---

* Published in *Proceedings* only.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

TRANSACTIONS

Paper No. 1793

ANALYSIS OF CONTINUOUS FRAMES BY DISTRIBUTING FIXED-END MOMENTS[1]

By Hardy Cross,[2] M. Am. Soc. C. E.

With Discussion by Messrs. C. P. Vetter, L. E. Grinter, S. S. Gorman, A. A. Eremin, Elmer F. Bruhn, A. H. Finlay, R. F. Lyman, R. A. Caughey, Orrin H. Pilkey, I. Oesterblom, Robert A. Black, H. E. Wessman, Edward J. Bednarski, S. N. Mitra, Jens Egede Nielsen, F. E. Richart, William A. Oliver, R. R. Martel, Clyde T. Morris, Francis P. Witmer, T. F. Hickerson, F. H. Constant, W. N. Downey, E. C. Hartmann, Thomas C. Shedd, David M. Wilson, Marshall G. Findley, George E. Large, Sophus Thompson and R. W. Cutler, Alfred Gordon, A. W. Earl, A. Floris, I. M. Nelidov, E. A. MacLean, George M. Dillingham, Donald E. Larson, J. A. Van den Broek, and Hardy Cross.

Synopsis

The purpose of this paper is to explain briefly a method which has been found useful in analyzing frames which are statically indeterminate. The essential idea which the writer wishes to present involves no mathematical relations except the simplest arithmetic. It is true that in order to apply the method it is necessary to determine certain constants mathematically, but the means to be used in determining these constants are not discussed in the paper, nor are they a part of the method. These constants have been derived by so many writers and in so many slightly different ways that there is little occasion to repeat here the whole procedure.

The reactions in beams, bents, and arches which are immovably fixed at their ends have been extensively discussed. They can be found comparatively readily by methods which are more or less standard. The method of analysis herein presented enables one to derive from these the moments, shears, and thrusts required in the design of complicated continuous frames.

---

[1] Published in May, 1930, *Proceedings*.

[2] Prof., Structural Eng., Univ. of Illinois, Urbana, Ill.

DEFINITIONS

For convenience of reference, definitions of three terms will be introduced at once. These terms are "fixed-end moment", "stiffness", and "carry-over factor".

By "fixed-end moment" in a member is meant the moment which would exist at the ends of the member if its ends were fixed against rotation.

"Stiffness", as herein used, is the moment at one end of a member (which is on unyielding supports at both ends) necessary to produce unit rotation of that end when the other end is fixed.

If one end of a member which is on unyielding supports at both ends is rotated while the other end is held fixed the ratio of the moment at the fixed end to the moment producing rotation at the rotating end is herein called the "carry-over factor."

EFFECT OF JOINT ROTATION

Imagine any joint in a structure, the members of which are being deformed by loads, or in some other way, to be first held against rotation and then released. Call the algebraic sum of the fixed-end moments at the joint the "unbalanced fixed-end moment". Before the joint is released this unbalanced fixed-end moment will not usually be zero; after the joint is released, the sum of the end moments at the joint must be zero. The total change in end moments, then, must equal the unbalanced fixed-end moment. This may be stated in another way by saying that the unbalanced fixed-end moment has been "distributed to" the connecting members in some ratio.

When the joint is released all connecting members rotate through the same angle and this rotation at the end is accompanied by a change in end moment. The change in end moments is proportional to the "stiffness" of the members.

Hence, it may be said that when the joint is released the unbalanced fixed-end moment is distributed among the connecting members in proportion to their stiffness.

The rotation of the joint to produce equilibrium induces moments at the other ends of the connecting members. These are equal in each member to the moments distributed at the rotating joint multiplied by the carry-over factor at the rotating end of the member. This follows from the definition of "carry-over factor".

MOMENT DISTRIBUTION

The method of moment distribution is this: (*a*) Imagine all joints in the structure held so that they cannot rotate and compute the moments at the ends of the members for this condition; (*b*) at each joint distribute the unbalanced fixed-end moment among the connecting members in proportion to the constant for each member defined as "stiffness"; (*c*) multiply the moment distributed to each member at a joint by the carry-over factor at that end of the member and set this product at the other end of the member; (*d*) distribute these moments just "carried over"; (*e*) repeat the process until the moments to be carried over are small enough to be neglected; and (*f*) add all moments

—fixed-end moments, distributed moments, moments carried over—at each end of each member to obtain the true moment at the end.

To the mathematically inclined the method will appear as one of solving a series of normal simultaneous equations by successive approximation. From an engineering viewpoint it seems simpler and more useful to think of the solution as if it were a physical occurrence. The beams are loaded or otherwise distorted while the joints are held against rotation; one joint is then allowed to rotate with accompanying distribution of the unbalanced moment at that joint and the resulting moments are carried over to the adjacent joints; then another joint is allowed to rotate while the others are held against rotation; and the process is repeated until all the joints are "eased down" into equilibrium.

BEAM CONSTANTS

This method of analysis is dependent on the solution of three problems in the mechanics of materials. These are the determination of the fixed-end moments, of the stiffness at each end, and of the carry-over factor at each end for each member of the frame under consideration. The determination of these values is not a part of the method of moment distribution and is not discussed in this paper.

The stiffness of a beam of constant section is proportional to the moment of inertia divided by the span length, and the carry-over factor is — ½.

The proof or derivation of these two statements and the derivation of formulas for fixed-end moments is left to the reader. They can be deduced by the use of the calculus; by the theorems of area-moments; from relations stated in *Bulletin 108* of the Engineering Experiment Station of the University of Illinois (the Slope-Deflection *Bulletin*); from the theorem of three moments; by what is known to some as the column analogy method;[3] or by any of the other corollaries of geometry as applied to a bent member. Formulas for fixed-end moments in beams of uniform section may be found in any structural handbook.

SIGNS OF THE BENDING MOMENTS

It has seemed to the writer very important to maintain the usual and familiar conventions for signs of bending moments, since these are the conventions used in design.

For girders the usual convention is used, positive moment being such as sags the beam. For vertical members the same convention is applicable as for girders if the sheet is turned to read from the right as vertical members on a drawing are usually read. The usual conventions for bending moments are, then, applicable to both girders and columns if they are looked at as a drawing is usually lettered and read.

Moments at the top of a column, as the column stands in the structure, should be written above the column and those at the bottom of the column, as the column stands in the structure, should be written below the column when the sheet is in position to read the columns. This is necessary because positive moment at the right end of a beam and at the top of a column both represent tendencies to rotate the connected joint in the counter-clockwise direction.

---

[3] "The Column Analogy," by Hardy Cross, M. Am. Soc. C. E., *Bulletin 215*, Eng. Experiment Station, Univ. of Illinois, Urbana, Ill.

It makes no difference whether girder moments are written above or below the girder. Either arrangement may be convenient. Confusion will be avoided by writing column moments parallel to the column and girder moments parallel to the girders.

When any joint is balanced the total moment to the right and to the left of the support is the same, both in absolute value and in sign. The unbalanced moment is the algebraic difference of the moments on the two sides of the joint.

LIMITATION OF METHOD

From the fact that the terms, "stiffness" and "carry-over factor", have been defined for beams resting on unyielding supports, it follows that direct application of the method is restricted to those cases where the joints do not move during the process of moment distribution. The method, however, can be applied in an indirect way to cases in which the joints are displaced during the moment distribution, as indicated later.

As the method has been stated, it is restricted only by this condition that the joints are not displaced. If this condition is satisfied it makes no difference whether the members are of constant or of varying section, curved or straight, provided the constants (*a*) fixed-end moments at each end, (*b*) stiffness at each end, and (*c*) carry-over factor at each end, are known or can be determined. Such values can be derived by standard methods and may be tabulated for different types of members and conditions of loading.

It will be found that in most cases accuracy is needed only in the fixed-end moments. It does not ordinarily make very much difference how, within reason, the unbalanced moments are distributed, nor, within reason, how much of the distributed moments are carried over.

In the "Illustration" which follows it has been assumed that the members are straight and of uniform section. The stiffnesses, then, are proportional to the moments of inertia, (I), divided by the lengths, (L), but the relative values given for $\frac{I}{L}$ in this problem might quite as well be the relative stiffness of a series of beams of varying section. In this latter case, however, the carry-over factors for the beams would not be $-\frac{1}{2}$.

ILLUSTRATION

The illustration given (Fig. 1) is entirely academic. It is not intended to represent any particular type of structure nor any probable condition of loading. It has the advantage for the purpose of this paper that it involves all the conditions that can occur in a frame which is made up of straight members and in which the joints are not displaced.

The loads on the frame are supposed to be as indicated. The relative values of $\frac{I}{L}$ for the different members are indicated in circles.

The fixed-end moments in all members are first written. In this problem they are arbitrarily assumed to be as shown, as follows: at *A*, 0; at *B*, in *BA*, 0, and in *BC*, — 100; at *C*, in *CB*, — 100, in *CF*, + 80, in *CD*, — 200, and in *CG*, — 50; at *F*, + 60; at *G*, — 50; at *D*, in *DC*, — 100, and in *DE*, 0; at *E*, in *ED*, 0, and in the cantilever, — 10.

Before proceeding to a solution of the problem, attention may be called to the arrangement of the computations. The moments in the girders are written parallel to the girders; those in the columns, parallel to the columns.

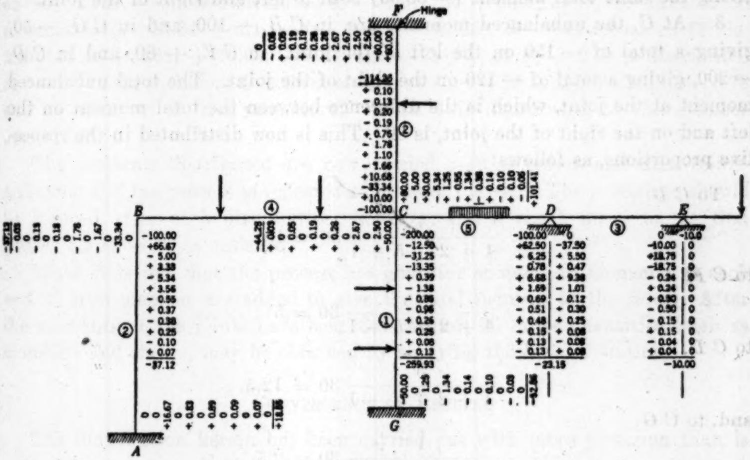

F
Hinge
+ 2.50
− 60.00
+ 60.00
−114.26
+ 0.10
− 0.13
+ 0.20
− 0.19
+ 0.75
− 1.78
+ 1.10
− 1.66
+ 10.68
− 33.34
+ 10.00
− 100.00
②
+ 80.00
− 5.00
+ 30.00
− 5.34
+ 1.25
− 0.55
+ 1.34
− 0.38
+ 0.14
− 0.10
+ 0.10
− 0.08
+ 101.41
B
④
C
⑤
D
③
E
− 37.12
+ 0.05
− 0.13
− 0.18
− 1.78
− 1.67
− 33.34
− 100.00
+ 66.67
− 5.00
+ 3.33
− 5.34
+ 3.56
− 0.55
+ 0.37
− 0.38
+ 0.25
− 0.10
+ 0.07
− 87.12
②
− 44.28
+ 0.05
+ 0.05
+ 0.19
+ 0.28
+ 2.67
+ 2.90
− 50.00
+ 2.50
− 50.00
− 200.00
− 12.50
− 31.25
− 13.35
+ 0.39
− 1.38
− 0.88
− 0.96
+ 0.26
− 0.25
+ 0.08
− 0.13
− 259.93
①
− 100.00
+ 62.50
+ 6.25
− 0.78
+ 6.68
+ 1.69
+ 0.69
− 0.51
+ 0.48
− 0.15
+ 0.13
− 0.13
0
− 37.50
+ 5.50
+ 0.47
+ 9.38
− 1.01
− 0.12
+ 0.30
+ 0.25
+ 0.08
− 0.08
+ 0.08
− 23.15
0
− 10.00
+ 18.75
− 18.75
− 0.24
+ 0.24
+ 0.50
− 0.50
− 0.15
+ 0.15
− 0.04
+ 0.04
− 10.0
− 10.00
+ 16.67
+ 0.83
+ 0.89
+ 0.09
+ 0.07
+ 18.55
− 50.00
− 1.25
− 1.34
+ 0.14
+ 0.10
− 0.08
− 52.58
A
G

Fig. 1.

The original fixed-end moments are written next to the members in which they occur, the successive moments distributed or carried over being written above or below these, but farther from the member.

The arrangement of the moments in the columns in positions above the columns, when the paper is turned into a position to write these moments, for the top of the columns (at *B*, *F*, and *C*), and in positions below the columns for the bottom of the columns (at *A*, *C*, and *G*), is an essential part of the sign convention adopted.

The moment at *C* in the girder, *BC*, is written above the girder in order to get it out of the way. Otherwise, it makes no difference whether the moments are written above or below the girder.

The signs of the fixed-end moments are determined by observing the direction of flexure at the ends of the members due to the loads. In order to apply to the columns the ordinary conventions for signs of bending moments it is necessary to turn the drawing of the structure.

The reader should realize that the solution is built up step by step. It is always the last figures showing that are to be operated on—distributed or carried over—so that in ordinary framework there is little chance for confusion as to what step should be taken next.

Distribute at each joint the unbalanced moment, as follows:

1.—At *A* there is no moment.

2.—At *B* there is an unbalanced moment of — 100 on one side of the joint. This moment is distributed to *B A* and to *B C* in the ratio, 2 : 4, so that the distributed moment to $B\ A$ is $\frac{2}{2+4}\ 100 = 33.33$ and to $B\ C$, $\frac{4}{2+4}\ 100 = 66.67$. **The signs are written in the only way possible to balance the joint by giving the same total moment (— 33.33) both to left and right of the joint.**

3.—At *C*, the unbalanced moments are, in *C B*, — 100, and in *C G*, — 50, giving a total of — 150 on the left of the joint; in *C F*, + 80, and in *C D*, — 200, giving a total of — 120 on the right of the joint. The total unbalanced moment at the joint, which is the difference between the total moment on the left and on the right of the joint, is 30. This is now distributed in the respective proportions, as follows:

To $C\ B$,

$$\frac{4}{4+2+5+1}\ 30 = 10$$

to *C F*,

$$\frac{2}{4+2+5+1}\ 30 = 5$$

to *C D*,

$$\frac{5}{4+2+5+1}\ 30 = 12.5$$

and, to *C G*,

$$\frac{1}{4+2+5+1}\ 30 = 2.5$$

There is only one way to place the signs of the distributed moments so that the total is the same on both sides of the joint. This is done by reducing the excess negative moment on the left and increasing the negative moment on the right.

4.—At *F*, the unbalanced moment is + 60. The hinge has no stiffness. The moment, then, is distributed between the member, *F C*, and the hinge in the ratio, 2 : 0; all of it goes to the member. The total balanced moment is + 60 — 60 = 0, as it must be at a free end.

5.—At *G*, the abutment is infinitely stiff and the unbalanced moment, — 50, is distributed between the member, *G C*, and the abutment in the ratio, $1 : \infty$. The member gets none of it; the end stays fixed.

6.—At *D*, the unbalanced moment, — 100, is distributed to *D C* and to *D E* in the ratio of 5 : 3.

7.—At *E*, the unbalanced moment is — 10 in the cantilever. Since the cantilever has no stiffness, this unbalanced moment is distributed between the

beam, *E D*, and the cantilever in the ratio, 3 : 0. This means that all of it goes to *E D*.

All joints have now been balanced. Next, carry over from each end of each member one-half the distributed moment just written, reverse the sign, and write it at the other end of the member. Thus, carry over, successively, in *A B*, 0 from *A* to *B* and + 16.67 from *B* to *A*; in *B C*, — 33.34 from *B* to *C* and — 5.0 from *C* to *B*; in *C F*, + 2.5 from *C* to *F* and + 30 from *F* to *C*; in *C G*, 0 from *G* to *C* and — 1.25 from *C* to *G*; in *C D*, + 6.25 from *C* to *D* and — 31.25 from *D* to *C*; and in *D E*, + 18.75 from *D* to *E* and + 5.00 from *E* to *D*.

Distribute the moments just carried over exactly as the original fixed-end moments were distributed. Thus, at *A*, + 16.67 is distributed 0 to *A B* (fixed-ended); at *B*, — 5.0 is distributed as — 1.67 and + 3.32; at *C*, the unbalanced moment is (— 33.34 + 0) — (+ 30.00 — 31.25) = — 32.09 which is distributed as + 2.67, + 10.68, — 5.34, and — 13.35; at *F*, + 2.50 is distributed as — 2.5 to the member; *G* is fixed-ended; at *D*, + 1.25 is distributed as — 0.78 and + 0.47; at *E*, the unbalanced + 18.75 is distributed to the member as — 18.75.

The moments distributed are now carried over as before and then re-distributed; and the process is repeated as often as desired. The procedure should be stopped after each distribution, however, and a check made to see that statics ($\Sigma M = 0$) is satisfied.

When it is felt that the process has gone far enough, all moments at each end of each member are added to give the total moment at the joint. After the moments at the joints have been determined, all other quantities, such as moments and shears, may be obtained by applying the laws of statics.

CONVERGENCE OF RESULTS

The distribution herein has been carried out with more precision than is ordinarily necessary, in order to show the convergence of the results. To show the rate of convergence, the successive values of the moments at the joints after successive distributions are given in Table 1.

TABLE 1.—CONVERGENCE OF RESULTS.

Successive values of bending moment at joint		After one distribution (two rows of figures)	After two distributions (four rows of figures)	After three distributions (six rows of figures)	After four distributions (eight rows of figures)	After five distributions (ten rows of figures)	After six distributions (twelve rows of figures)
A	....	0	+ 16.67	+ 17.50	+ 18.39	+ 18.48	+ 18.55
B	....	— 33.34	— 35.01	— 36.79	— 36.97	— 37.10	— 37.13
C	In *C B*	— 90.00	—112.66	—113.22	—114.24	—114.23	—114.26
	" *C F*	+ 75.00	+ 99.66	+100.36	+101.32	+101.36	+101.41
	" *C D*	—212.50	—257.10	—258.09	—259.89	—259.88	—259.93
	" *C G*	— 47.50	— 44.83	— 44.55	— 44.36	— 44.31	— 44.28
D	....	— 37.50	— 32.03	— 23.66	— 23.48	— 23.15	— 23.15
E	....	— 10.00	— 10.00	— 10.00	— 10.00	— 10.00	— 10.00
F	....	0	0	0	0	0	0
G	....	— 50.00	— 51.25	— 52.59	— 52.73	— 52.83	— 52.86

For most purposes the computations might as well have been stopped after the second distribution. Had this been done, the solution would have appeared as shown in Fig. 2.

For any practical purpose the computation might in this case have been stopped after the third distribution. In general, two or three distributions are sufficient. This is not true in all instances, but in any case the exactness of the solution at any stage will be indicated by the magnitude of the moments carried over in the members.

VARIATIONS OF THE METHOD

The writer has developed and used at different times several variations of the method shown, but the original method is itself so simple and so easy to remember that he finds himself inclined to discard the variants.

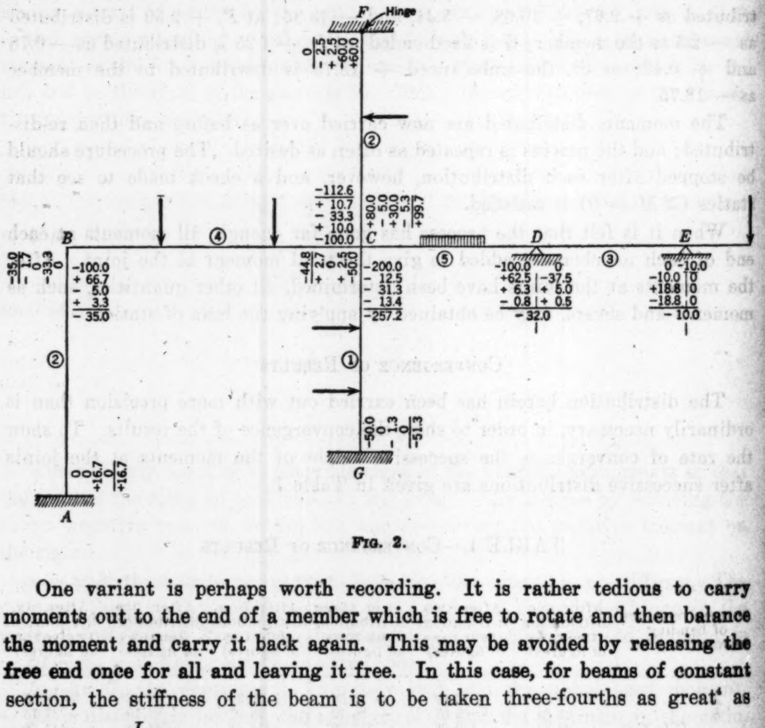

F
Hinge
+60.0
−60.0
+ 2.5
− 2.5
0
②
−112.6
+ 10.7
− 33.3
+ 10.0
−100.0
+80.0
− 5.0
+30.0
− 5.3
+99.7
B
④
C
⑤
D
③
E
−35.0
− 1.7
0
−33.3
0
−100.0
+ 66.7
− 5.0
+ 3.3
− 35.0
−44.8
+ 2.7
0
+ 2.5
−50.0
−200.0
− 12.5
− 31.3
− 13.4
−257.2
−100.0
+62.5
+ 6.3
− 0.8
0
−37.5
+ 5.0
+ 0.5
− 32.0
0
−10.0
+18.8
−18.8
−10.0
0
0
0
− 10.0
②
①
−50.0
0
− 1.3
0
−51.3
0
0
+16.7
0
+16.7
G
A

FIG. 2.

One variant is perhaps worth recording. It is rather tedious to carry moments out to the end of a member which is free to rotate and then balance the moment and carry it back again. This may be avoided by releasing the free end once for all and leaving it free. In this case, for beams of constant section, the stiffness of the beam is to be taken three-fourths as great[4] as the relative $\frac{I}{L}$-value would indicate. After the end of the beam is once released, no moments are carried over to it.

---

[4] The moment needed to produce a given rotation at one end of a beam when the other end is free is three-fourths as great as if the other end is fixed.

CORRECTING FOR SIDE-SWAY

Single square or trapezoidal frames, portals, L-frames, box culverts, and similar structures act as simple continuous beams if there is no transverse deflection. If they are symmetrical as to form and loading, they will not deflect sidewise and if they are restrained against sidewise movement, they cannot so deflect.

Side-sway of frames due to dissymmetry of the frame is rarely an important factor in design. Correction for side-sway may be made by a method which may be applied also in cases of transverse loading on bents. The method is to consider that the bent does not sway sidewise and analyze it as a series of continuous beams. The total shear in the legs will not now, except by accident, equal the shear which is known to exist. The difference must be a force which prevents side-sway.

Now, assume all joints held against rotation, but the top of the bent moved sidewise. Assume any series of fixed-end moments in the legs such that all legs have the same deflection. In this case for members of uniform section fixed-end moments in columns vary as $\frac{I}{L^2}$. Distribute these fixed-end moments and find the total shear in the legs. The changes in moments due to side-sway will then be to the moments just computed in the same algebraic ratio as the total unbalanced horizontal shear in the legs due to side-sway when the frame is analyzed as a continuous girder, is to the shear just computed.

MULTI-STORIED BENTS

Bents of more than one story, subject to side-sway, either as a result of unbalanced loading or due to horizontal forces, may be solved by this method. It is understood that exact solution of such problems is not commonly of great interest. It is the approximate effect that is desired rather than exact analysis.

To analyze by this method a two-story bent it will be necessary to make two configurations—one for each story. From the assumed shear in each story (producing, of course, shears in the other stories), a set of moment values may be obtained. These may be combined to obtain the true shears, and from the true shears the true moments follow.

GENERAL APPLICATION OF THE METHOD

The method herein indicated of distributing unbalanced moments may be extended to include unbalanced joint forces. As thus extended it has very wide application. Horizontal or vertical reactions may be distributed and carried over and thus a quick estimate made of the effect of many complicating elements in design. The writer has used it in studying such problems as continuous arch series, the effect of the deflection of supporting girders, and other phenomena.

An obvious application of moment distribution occurs in the computation of secondary stresses in trusses. Many other applications will doubtless suggest themselves, but it has been thought best to restrict this paper chiefly to continuous frames in which the joints do not move.

CONCLUSION

The paper has been confined to a method of analysis, because it has seemed wiser to so restrict it. It is not then an oversight that it does not deal with: (1) Methods of constructing curves of maximum moments; (2) methods of constructing curves of maximum shears; (3) the importance of analyses for continuity in the design of concrete girders; (4) flexural stresses in concrete columns; (5) methods of constructing influence lines; (6) the degree to which continuity exists in ordinary steel frames; (7) continuity in welded steel frames; (8) plastic deformation beyond the yield point as an element in interpreting secondary stress computations; (9) the effect of time yield on moments and shears in continuous concrete frames; (10) plastic flow of concrete as a factor in the design of continuous concrete frames; (11) whether in concrete frames it is better to guess at the moments, to take results from studies made by Winkler fifty years ago, or to compute them; (12) the effect of torsion of connecting members; (13) the relative economy of continuous structures; (14) the relative flexibility of continuous structures; (15) the application of methods of continuous frame analysis to the design of flat slabs; (16) probability of loading and reversal of stress as factors in the design of continuous frames; (17) the relation of precision in the determination of shears and moments to precision in the determination of fiber stresses: and a dozen other considerations bearing on the design of continuous frames.

A method of analysis has value if it is ultimately useful to the designer; not otherwise. There are apparently three schools of thought as to the value of analyses of continuous frames. Some say, "Since these problems cannot be solved with exactness because of physical uncertainties, why try to solve them at all?" Others say, "The values of the moments and shears cannot be found exactly; do not try to find them exactly; use a method of analysis which will combine reasonable precision with speed." Still others say, "It is best to be absolutely exact in the analysis and to introduce all elements of judgment after making the analysis."

The writer belongs to the second school; he respects but finds difficulty in understanding the viewpoint of the other two. Those who agree with his viewpoint will find the method herein explained a useful guide to judgment in design.

Members of the last named school of thought should note that the method here presented is absolutely exact if absolute exactness is desired. It is a method of successive approximations; not an approximate method.

DISCUSSION

C. P. VETTER,[5] ASSOC. M. AM. SOC. C. E. (by letter).—The author presents an ingenuous method for determining certain statically indeterminate quantities in continuous frames, one of the main advantages being the absence of any mathematics except the simplest arithmetic.

He illustrates the application of his theory by analyzing a certain hypothetical frame. In a way it is unfortunate that this particular type of structure has been chosen as an example, as the weaknesses of the principle are here so apparent. Had the author chosen, for example, an ordinary continuous girder, they would not have been evident, and it would have been more clearly demonstrated that he has added a new method to those already known for solving one of the most common problems in modern concrete engineering.

Considering Fig. 1, it is at once apparent that the vertical component of the reactions at F and G cannot be determined by the method outlined in the paper. In consequence, the direct stresses in the members, $F\,C$ and $C\,G$, are unknown.

Those familiar with the theory of least work will recognize that this is because of the author's neglect of deformations due to direct stresses and shear. While the latter may often be safely ignored, neglect of the former is dangerous in many cases, the given example being one of them.

In his "Conclusions", the author lists up to seventeen different points which have not been dealt with. The writer could add a great many more which have just as much, and more, to do with the subject under discussion, such as the influence of variation in temperature, the yielding of foundations, and the shrinkage or swelling of members if built of concrete.

The writer does not agree with the author's closing remarks concerning the value of a correct analysis of statically indeterminate structures. Stresses in most, although admittedly not as yet in all, such structures can be determined completely and correctly using those assumptions—and only those assumptions—which are also used when analyzing statically determinate, or simple, structures. Short-cuts and approximations are permissible if one is in a hurry and if the error is negligible and approximately known, but the value of an approximation is determined only by the number of assumptions involved; every time a new assumption is made the value of the findings is reduced. If enough new assumptions are added, the answer may be valueless.

The theory of least work and the slope-deflection method have not added any assumptions to those generally used in all engineering, and, therefore, should be preferred wherever possible. Also, an engineer should "hitch his wagon to a star."

L. E. GRINTER,[6] ASSOC. M. AM. SOC. C. E. (by letter).—In presenting this paper, Professor Cross has made generally available a highly valuable method of structural analysis, which has previously been known to relatively few engineers. This method of balancing fixed-end moments was suggested to the writer a number of years ago by Professor Cross. Subsequently, he has found need to develop it to care for the balancing of shears or deflections, of thrusts or joint forces, and of angle changes or joint rotations.

---

[5] Asst. Civ. Engr., Pacific Gas & Elec. Co., San Francisco, Calif.

[6] Prof. of Structural Eng., Agri. and Mech. Coll. of Texas, College Station, Tex.

Considered in its entirety, this is clearly a general method of successive corrections. In the form suggested by Professor Cross, one starts with fixed-end moments and the corrections are computed from the effect of allowing the joints to rotate in succession. The method is, therefore, either approximate or exact to any desired degree, depending only on the number of corrections one is willing to compute. It has the advantage over other methods of approximation in that the operator instinctively senses the amount of error involved in discontinuing the computation of the corrections at any given stage of the process, and naturally he continues until the desired degree of precision is reached. It has the further advantages: (1) Of speed, because one wastes no time in obtaining unnecessary precision; (2) of simplicity, because all computations involve only the use of arithmetic; and (3) of ease of remembering, since each step of the process has a definite physical significance.

These statements are borne out by the fact that the method in its simplest form—balancing moments on a continuous beam—is easily digested by junior engineering students and retained to the extent that, guided by a few suggestions, they readily make use of it in the senior year. Any one who has attempted to have senior students make use of the method of three moments, without considerable coaching, will appreciate the distinction.

In this connection, it should be noted that textbooks on strength of materials frequently do not give the three-moment theorem, except for equal spans and simplified loads. By balancing fixed-end moments, the student finds little added difficulty in handling unequal spans, complex loadings, or even a variation of moment of inertia in the several spans. Of course, variation of moment of inertia within a single span can be handled only by the advanced student.

The writer feels that much of the simplicity of this method is dependent upon a complete visualization of the corresponding physical action of the structure. In most methods of analyzing continuous frames, it is customary to allow rotation and translation to take place freely by inserting pins and sliding plates into the structure. Later in the process, continuity is reproduced by the application of forces at the cut sections. By the same reasoning it becomes clear that one may apply any number of restraining forces or moments to a structure as long as these restraints are removed later.

In the simplest case, that of balancing fixed-end moments where joint translation is not involved, the physical action is best described as the application and successive release of restraining moments at the joints. As these moments are released, each joint rotates until it takes its proper slope. When one joint is allowed to rotate, all others are held fixed. Every member meeting at the rotation joint, therefore, acts physically as a beam which is simply supported at the near end, fixed at the far end, and which, at the same time, is undergoing rotation at the near end. The resistance of such a beam to rotation is dependent upon its physical properties, or $\frac{I}{L}$ - value. As Professor Cross points out, this value must be replaced by the moment necessary to produce a unit rotation at the near end, if the beam is not of uniform section. Again, the physical action of such a rotating joint readily clears up the fact that the unbalanced moment will distribute to all members meeting at the joint in direct proportion to their stiffness or resistance to rotation.

The carry-over moment may best be looked upon as another characteristic of the physical action of the structure. When a simple beam of uniform section is rotated through a given angle at one end, the far end rotates through an angle which is one-half as great. To prevent this rotation, or to rotate the far end backward to its original position, clearly requires the application at the far end of one-half the applied moment at the near end. Using the ordinary sign convention of beams, the carry-over factor is accordingly—50% of the balancing moment. For a beam of non-uniform section, or for one not fully restrained at the far end, the imaginary angle change to be corrected at the far end would not be exactly 50% of that at the near end, and the carry-over factor would be different from 50 per cent.

The writer would like to emphasize the fact that it is not necessary to consider the far end of the member as fixed. It might be allowed to rotate freely and the angular change later be taken into consideration, or its restraint may be determined and a moment carried over in proportion to this restraint. The writer makes occasional use of the conception of free rotation in a method of balancing angle changes, and frequent use of the idea of estimating the end restraint in an approximate method of balancing moments. The viewpoint that he wishes to make clear, however, is that one may consider any joint as fixed, free, or partly restrained, at will, at any point in the process. It is only necessary that the recorded data be consistent with the physical action of the structure, that all artificial restraints be finally released, and that all discontinuities be removed.

The writer makes use of the term, "balancing," to indicate the process that Professor Cross prefers to call "distributing." This matter of terminology is of small moment, except that for clearness it has seemed desirable to reserve the term, "distributing," to explain the process of dividing a known moment in one member among the other members meeting at a joint. "Balancing" is used where an externally applied moment is divided among all members meeting at a joint. When so defined, "distributing" is a process to be applied when "running out" moments through a continuous frame of which only a single span is loaded. This method was mentioned in the preceding paragraph as an approximate method of balancing moments, where the carry-over moment is determined from the restraint at the far end of the member.

It is to be expected that among several investigators starting with the same conception as to balancing of fixed-end moments, each should finally develop a different system for accomplishing this purpose. The writer's convention of signs and method of recording the results are sufficiently different from those used by Professor Cross to justify their presentation.

The sign convention used by Professor Cross is that ordinarily given for horizontal beams in the study of strength of materials. This is ideally adapted to the discussion of moments in horizontal beams, but it has always seemed to the writer to be misplaced when applied to column moments. When thus used, some secondary sign convention must be adopted, such as the agreement that all columns will be considered as rotated through an angle of 90° in a clockwise direction. (Professor Cross suggests rotating the sheet instead.) When members meet at various angles, as in secondary stress computations, the convention of signs for strength of materials becomes still more confusing. In order to obviate this difficulty, the writer prefers to adopt the simple sign convention that positive moment at the end of any member tends to rotate the adjacent joint clockwise. This convention automatically cares for the signs of horizontal, vertical, and diagonal members.

In applying this system, it has been found to be a simple matter roughly to sketch the shape of the deflected structure with fixed ends. This process has the double advantage of fixing the signs beyond question and of enhancing the physical significance of the method. Sketches of typical deflected members are shown in Fig. 3. It is worth noting that a member which does not have a point of contraflexure has moments of opposite sign at the two ends. One point of contraflexure is indicated by moments of like sign and two points of contraflexure again by moments of opposite sign. The distinction between none and two points of contraflexure can not be drawn from the signs of the end moments, but is evident from the condition of loading.

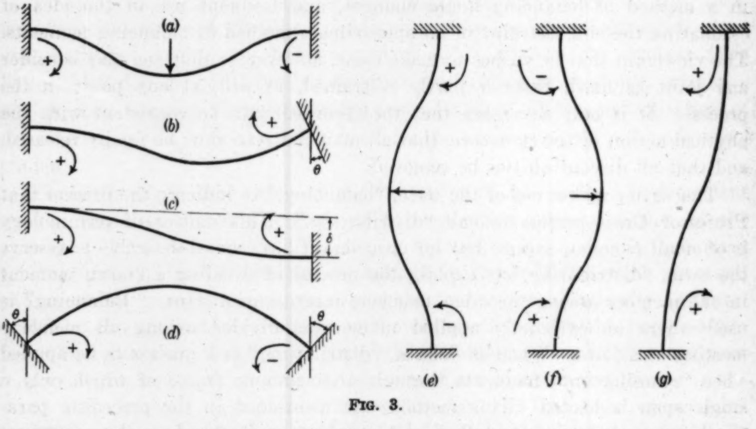


FIG. 3.

When this convention is used, the signs of the balancing moments around any joint are unusually simple. If the resultant unbalanced moment is positive, all balancing moments are negative, and their sum is equal to the total unbalanced moment. Thus, the total moments after balancing around the joint add up algebraically to zero. In comparison, one notices that the algebraic sum of all moments around a joint is not zero when written out according to the sign convention used by Professor Cross.

The carry-over moment, which is 50% of the balancing moment for a member of constant section, is of the same sign as the balancing moment. This fact is evident as the physical action of rotating one end of a member when the far end is restrained or fixed produces one point of contraflexure. (See Fig. 3(*b*).)

This convention of signs accordingly has the following advantages:

1.—Girders, columns, and diagonal members are handled in identically the same manner.

2.—The algebraic sum of all moments around a balanced joint is zero.

3.—All balancing moments at a joint caused by a single balance have like signs. Their sum is equal and opposite to the unbalanced joint moment.

4.—Carry-over moments are of the same sign as the balancing moment.

These advantages may seem somewhat unimportant, but they produce a simplification that reduces the possibility of error to an absolute minimum.

In finally comparing the sign convention used herein with that recommended by Professor Cross, one is naturally influenced in his decision of preference by his previous experience. One's personal system, even though more tedious, is always more satisfactory to him than another with which he is less familiar. It is of interest, however, that both sign conventions are actually used in either system. The signs set up by Professor Cross must be interpreted in terms of clockwise or counter-clockwise rotation in order to check $\Sigma M = 0$ at a joint. Similarly, the sign convention used by the writer must be transferred back into terms of fiber stresses if a moment diagram is to be drawn.

Another simplification which the writer has found advantageous is to complete the carrying over of moments from one joint before balancing the next joint. In this way one eliminates the possibility of neglecting to carry over certain moments. Furthermore, as the carry-over moment is balanced at the same time as the original fixed-end moment, a given number of balances will produce more nearly exact results.

It will further be found advantageous to balance in succession those joints which have the largest unbalanced moment. It is useless to balance a joint early in the process which has merely a slight unbalanced moment, because the carry-over moments from adjacent joints will later entirely disrupt the carefully calculated balance. If one adopts the system of drawing a horizontal line under all columns of moments at a joint each time the joint is balanced, a single glance will serve to point out the joint at any stage of the process which has the largest unbalanced moment.

The most rapid convergence is obtained, therefore, by carefully selecting the proper joint to balance at each step of the process, and then by carrying over the moments before passing on to another joint.

For handling fixed ends and pin ends, the writer prefers to follow the procedure indicated by a study of the physical action of the structure. A fixed end cannot rotate and, therefore, cannot be balanced. The original fixed-end moment plus the carry-over moments, accordingly, add up algebraically to give the final moment. A pin is entirely free to rotate and has zero moment. The fixed-end moments and all carry-over moments, therefore, must be released by allowing free rotation at the pin. In recording the data, this free rotation may be looked upon as produced by the application of equal and opposite moments at the pin.

In order to illustrate the system described herein, the writer has prepared Fig. 4, which is the identical problem worked out by Professor Cross. The balancing is started at Joint *B* where an unbalanced positive moment of 100 exists. This joint was selected because it offered as large an unbalanced moment as any other. Joint *D* might equally well have been chosen as a starting point. Since *BC* is twice as stiff as *BA*, the first correction moments are — 67 in *BC* at *B*, and — 33 in *BA* at *B*. After the balancing moments are recorded, horizontal lines are drawn under these figures to indicate that all moments above these lines are balanced; that is, they add up to zero around the joint. Now, one-half the — 67 at *B* in *BC* is carried over to *C* with the same sign, or — 33. Similarly, one-half the — 33 at *B* in *BA* is carried down to *A* with the same sign, or — 16. The next step is to balance Joint *D* where a large unbalanced moment of — 100 exists. Balancing moments of + 63 in *DC* and + 37 in *DE* are recorded, corresponding to five-eighths and three-eighths, respectively, of the unbalanced moment. Horizontal balance lines are again drawn, and these moments are carried over with one-half their value and like sign to the far ends, *D* and *E*.

The process is thus continued around the frame, the joints being balanced in the following rotation: *B, D, F, E, C, B, F*, and *C*. They might have been balanced with equal satisfaction in some other sequence, but it is not likely that a more rapid convergence will be found. It is well to note here that in returning to balance a joint for a second or third time, one balances only those figures below the last set of horizontal lines.

In Fig. 4 the first recorded figure in each group is the fixed-end moment. The last recorded figure in each group is the final moment in the adjacent member obtained by adding the column of figures. In order to clarify the diagram, column moments are written on the columns, and girder moments are written above the girders.

In justification of this method, it is necessary to show that the results obtained in this case by the balancing of only eight joints are within 4% of those obtained by the system suggested by Professor Cross in thirty balances. Also, after balancing each joint twice by Professor Cross' system, or a total of ten balances, one joint, *D*, still is seen to be 38% in error. A fairer comparison might be obtained by noting that Professor Cross' system gives satisfactory accuracy after balancing each joint three times, or a total of fifteen joints to be balanced. Thus, in this problem, nearly one-half the mechanical labor is saved by the system here suggested. At least a 25% reduction in labor will usually be found in the solution of all continuous frame problems.

The writer wishes to stress particularly the advantage that one sets his standard of accuracy before starting to balance joints. For instance, if

whole numbers are used as in Fig. 4, an error arises from the fact that 33 is recorded instead of 33.33, when 100 is to be divided into three equal parts. There is also an error where an uneven number is multiplied by 0.5 to obtain a carry-over moment. When dealing with whole numbers, a maximum error of 2 is a possibility. If the final moments are at least equal to 50, the maximum error is thus limited to about 4 per cent. Where smaller fixed-end moments are found, it may be desirable to use three significant figures. After one becomes familiar with the method, he naturally watches the figures closely and makes the errors balance. An excellent example of this may be seen in Fig. 4. Note that at Joint *A* the writer first carried down less than one-half the balancing moment and the second time he carried down more than one-half. No attempt was made to balance errors at other joints.

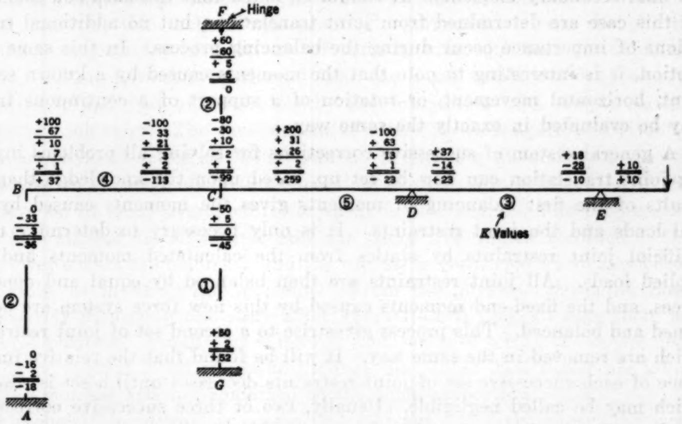


FIG. 4.

If these suggestions are followed, one merely continues to balance joints until the carry-over moments are smaller than unity in the last significant place used. When so reduced, the columns may be added, knowing that the results are within the limits of accuracy represented by a variation of 2 in the last significant figure. Slightly more care will reduce the variation to unity in the last significant figure.

Before closing this part of the discussion on balancing moments, it should be of interest to show that the correction of an error is usually possible without destroying the entire work. For instance, if the operator finds that he has failed to carry over a certain moment, he can carry it over later and then treat it as a single unbalanced moment to be balanced. The results of this balance are to be added algebraically to the first results. Other errors can be handled in a similar manner. It is clear that an error in balancing can be picked up by checking $\Sigma M = 0$ at each joint. An error in the original fixed-end moments or in the carry-over moments can be found by checking a part of the structure for equilibrium with the loads.

The system of balancing moments described herein has been stated to be a special case of the general method of successive corrections, and is directly applicable to all cases where the joints do not translate during the process of balancing. When translation from the balancing of the joints would normally occur, this process gives the moments caused by the original loads and the joint forces necessary to prevent additional translation. If these joint forces are negligible, the moments are nearly exact.

An interesting illustration of such a simplified case is that of secondary stress analysis. Knowing that the deflections of the pin-ended truss and of the riveted truss are essentially alike, one merely calculates the deflections of all panel points caused by the direct stresses; from these deflections one computes the fixed-end moments, and by balancing these moments arrives at the final secondary moments. It should be noted that the fixed-end moments for this case are determined from joint translations, but no additional translations of importance occur during the balancing process. In this same connection, it is interesting to note that the moments caused by a known settlement, horizontal movement, or rotation of a support of a continuous frame may be evaluated in exactly the same way.

A general system of successive corrections for solving all problems involving joint translation can now be set up, based upon the knowledge that the results of the first balancing of moments gives the moments caused by the real loads and the joint restraints. It is only necessary to determine these artificial joint restraints by statics from the calculated moments and the applied loads. All joint restraints are then balanced by equal and opposite forces, and the fixed-end moments caused by this new force system are determined and balanced. This process gives rise to a second set of joint restraints which are removed in the same way. It will be found that the relative importance of each successive set of joint restraints decreases until a set is reached which may be called negligible. Usually, two or three successive corrections will be sufficient for engineering accuracy. The final moments are obtained by adding algebraically the moment sets caused by the several force systems.

It is highly important to study the physical action of the structure, which corresponds to this method of analysis. The first deflected position of the structure does not correspond to the proper deflected position. As each successive force system is allowed to deflect the structure with fixed joints, that structure approaches nearer and nearer its true deflected position. After several successive corrections, it almost exactly coincides with its true deflected position, and the joint restraints are negligible.

The writer has used this method with entire success in the analysis of all continuous frames, including the determination of wind stresses in building frames, open web trusses, continuous arches on elastic piers, and more unusual academic problems. The method has a valuable application to continuous arch systems, in which the idea of balancing joint forces applies directly to the elimination of the troublesome unbalanced thrust at the pier tops.

The method simplifies itself when applied to a single-story bent or culvert in that the restraining force system is merely a single horizontal force applied to the top of the bent. Successive restraining forces are found to form a convergent series, geometric in form, which has a definite mathematical sum. From this knowledge, or from the physical action of the structure, one can readily see that there is a direct proportion between the true moments caused by a given lateral force and the first balanced set of moments caused by this force. This ratio is the same as that between the applied lateral force and the difference between this lateral force and the first joint restraint. Professor Cross arrives at this conclusion in a somewhat different manner.

When discussing multi-storied bents, Professor Cross suggests producing a unit shear successively in each story and determining the effect on adjacent stories in terms of both moment and shear. Then, by combining these effects, the proper shears may be obtained in all stories, and by combining the moment sets in the same proportion, the true final moments are obtained. When the writer first put this method into practical application, it was thought that it would be necessary to make use of simultaneous equations for combining shears. Later, however, he showed that the same result could be obtained more easily and just as accurately by the summation of a simple series of terms for each story. The development of such a series is again best visualized from the physical action of the structure. A shearing distortion in one story gives rise to shearing distortions in other stories. From the ratio of these shears a set of carry-over factors for shear is determined. The proper shears are then applied in all stories and the carry-over shears recorded. These carry-over shears are balanced, and the process is continued until they become negligible, as in balancing moments. From the physical viewpoint, one is successively distorting one story after another until they all take their final deflected position. Finally, by summing each series of terms, the shears necessary to be applied to all stories to produce the proper final shears are known, and the moment sets are combined in the same proportion.

Although this method is convenient and entirely satisfactory, the writer is now finding the general method of successive corrections, just described, to be more useful in most cases. Its main advantage is that it lends itself directly to an approximate solution as one may neglect joint restraints to give any desired degree of approximation, thereby allowing a saving of labor. It also offers an opportunity to the experienced analyst to take advantage of his knowledge of deflections to reduce materially the amount of work required. For instance, the writer seldom finds it necessary to balance more than one set of joint restraints. The method of balancing shears suggested by Professor Cross is more valuable for cases in which the effects of several force systems are to be evaluated. When the carry-over factors for balancing shears are once determined, the moments caused by several force systems may be obtained with relatively little additional labor.

This paper constitutes an outstanding contribution to the field of structural analysis. Without question the method of balancing fixed-end moments and the other closely related methods of analysis are destined largely to supplant the old classic methods dependent upon the solution of simultaneous equations. The writer is not alone in his belief that Professor Cross may be regarded as the leader in a new school of thought in the field of structural analysis.

S. S. GORMAN,[7] ASSOC. M. AM. SOC. C. E. (by letter).—This paper is a clear exposition of a simple and practical solution for many complicated problems. The method presented is very simply and easily applied and is just what its author claims it to be—a method of successive approximations, giving reasonable precision speedily. The writer has applied it to a number of analyses and has found it to be very useful, but he realizes that it must accompany sound judgment in its application.

This discussion will be confined to the method of distribution at a joint similar to *B*, in Fig. 5.* While the author's description has been very general, possibly a further elaboration on such a joint will save others from the confusion experienced by the writer.

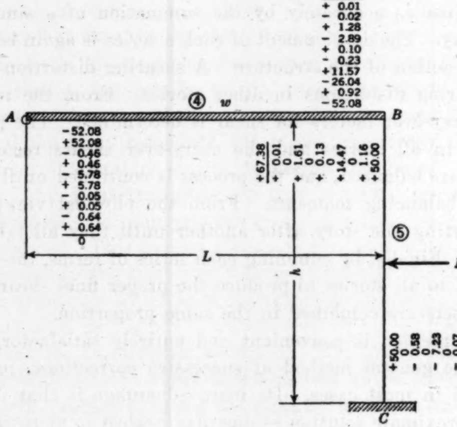


FIG. 5.

Assume an L-frame as shown (Fig. 5), in which, $p = 20\,000$ lb.; $w = 1\,000$ lb. per ft.; $L = 25$ ft.; and, $h = 20$ ft. The relative values of $\frac{I}{L}$ for the members are indicated in circles. End *A* is hinged and End *C* is fixed.

After first writing the fixed-end moments for each member, consider Joint *B*. At first, one might believe that the unbalanced moment is −102.08, and proceed to distribute this moment, ultimately obtaining an answer which would be incorrect. The unbalanced moment is actually −2.08, and is distributed between the members, *A B* and *B C*, as on the left side of the joint, and imaginary members of zero stiffness on the right side of the joint, giving the moments as shown in Fig. 5. This procedure is continued, as outlined by the author, always balancing the left-side moments against zero for the right-side moments, until the final moments are obtained.

---

[7] Cons. Structural Engr., San Francisco, Calif.

A similar condition is present in the author's illustration at the joint, C, Fig. 1, if the members, $C D$ and $C F$, are without load and are considered of zero stiffness. In this case the left-side moments (— 150) would be distributed so as to balance with zero on the right side of C ($B C$ would get + 120 and $C G$ would get + 30).

As the author has pointed out the selection of signs is most important, and as the moments are balanced at a joint algebraically, it is absolutely essential to a proper solution of any problem that the signs are properly selected; this is fundamental. Simplification of analysis is highly commendable, but it must not be spoiled by careless application.

The writer wishes to express his gratitude to the author for his splendid presentation, which eliminates formerly cumbersome methods of analysis of statically indeterminate frames.

A. A. EREMIN,[8] ASSOC. M. AM. SOC. C. E. (by letter).—A number of the analytical and semi-graphical methods of moment distribution in continuous frames with rigid connections have been developed in the last few decades. These methods are generally based on what is commonly known as Mohr's principle,[9] namely "the angular deformations of the elastic line at supports are equal to the reactions of the beam loaded by its moment area divided by $E I$".

This principle applies to beams resting on unyielding supports. However, when a load is applied to continuous frames the joints move from their original positions, due partly to direct stresses in the members, or lateral forces at the frame, and partly to changes in the lengths of the members as the result of variations in temperature, or a combination of all these factors.

The author restricts his analysis to frames in which the joints do not move. If it is required to determine the moments at the joints of a continuous frame under an unsymmetrical static loading the author's method will be the simplest and probably the shortest. However, for distributing moments due to moving loads or for constructing the influence lines for moments at the joints, Strassner's semi-graphical method is more convenient.

In Strassner's method the characteristic points, J and K, are determined for the unloaded condition and the procedure for determining the moments for each position of the external load is accomplished by simple graphics.

To facilitate the ordinary manipulations in the Strassner method, tables of the necessary quantities for different methods of loading and shapes of beams generally used in practice are prepared.[10]

In the limitations of his method the author omitted to state that the modulus of elasticity of materials was considered to be constant for all members of the frame and also that the lateral rigidity of the members of a continuous frame is sufficient to prevent buckling. The second statement is important to remember in the analysis of steel frames with rigid connections.

---

[8] Asst. Designing Engr., Bridge Dept., State Highway Comm., Sacramento, Calif.

[9] "Neuere Methoden zur Statik der Rahmentragwerke," by A. Strassner, Vol. 1, 1921, p. 11.

[10] *Transactions*, Am. Soc. C. E., Vol. 90 (1927), p. 167.

The author defined three schools of designers. There is no justification for the first of these because many experiments on existing reinforced concrete and steel frames with rigid connections have proved that there is a distribution of stresses from one member to adjacent members.

On the other hand, imperfections in workmanship and incomplete knowledge as to the actual elastic properties of structural materials prevent the development of an exact method of determining a real distribution of the stresses in frames.

In order to maintain a reasonably uniform margin of safety for all members in the frame it should be designed for stresses as determined by one of the approximate methods based on the elastic properties of materials. In the formulas for the final stresses in the members of the frame, constants may be introduced to eliminate the errors due to neglecting some minor factors in the development of the method. These constants may be determined by experimental study of the various frames with rigid connections.

ELMER F. BRUHN,[11] ASSOC. M. AM. SOC. C. E. (by letter).—The writer has long been familiar with this method of distributing fixed-end moments and has used it extensively. He strongly recommends it because it has developed for him a much better understanding of the action and relative rigidity of indeterminate structures than is given by the common methods of analysis. These latter, in general, involve the writing of sufficient elastic and static equations, after which the work is entirely an algebraic mechanical process. The structure is entirely forgotten and although the various terms in the equations may represent certain rigidity characteristics or distortional influences, it is no doubt true that this significance often escapes the designer's attention.

Using the author's method one assumes each member of the structure in a definite restrained state and then, realizing that continuity must be preserved and statics satisfied at each joint, the structure is gradually but definitely relieved, joint by joint, from its restrained and unbalanced state and is guided into a state of equilibrium. The structure is at all times the center of thought, a fact that merits emphasis.

In the aeronautic structural field the method of least work seems to be deeply rooted. For analyzing complex continuous structures it gives long and unwieldy equations and has proved to be very unsatisfactory. Of many uses, the writer has found the author's method of particular value in the following three problems:

(1) For making rapid comparative studies of continuous closed frames, a type of structure which is being used more and more in airplane bodies.

(2) For assisting in digesting test results. Government regulations generally require tests to destruction of all major airplane units and new types

---

[11] Engr. of Tests, B/J Aircraft Corporation, Baltimore, Md.

of structures. It is difficult to compute the ultimate strength of a continuous structure accurately. This is due, in part, to the fact that the ordinary elastic formulas apply only for stresses in the materials below the elastic limit. Tests of continuous structures generally show a higher ultimate strength than that computed. This result can be explained as follows: When a continuous structure is loaded some portions of it reach the elastic limit stress first and lose a considerable fraction of their original stiffness or rigidity. The result is that these overstressed portions are relieved of part of their load by the other portions of the structure which still retain their original stiffness. Since it is impossible or impractical to proportion structures for equal stress intensity, this drifting tendency of the internal loads to the portions of higher stiffness has considerable influence upon the ultimate strength, amounting in many cases from 25 to 40% more than the strength computed by the ordinary methods. The author's method has proved valuable in estimating this internal re-adjustment.

(3) For solving extremely complex problems as, for example, the determination of secondary moments due to deflection in a continuous beam with variable moment of inertia and carrying both transverse and axial loads. In airplane structures these moments prove of real importance. To attack such a problem with mathematics is practically impossible.

The question of correcting for joint displacements is less important in airplane work, as, in general, the loads and structures are symmetrical. On the other hand, space frameworks are quite common. This further complicates matters because of torsional forces, but there seems to be no reason why such forces could not be distributed in a similar manner if the torsional characteristics of the members can be determined.

It seems probable that in the aeronautic structural field will be found many problems that will lend themselves to the convenience and advantages of Professor Cross' method.

A. H. Finlay,[12] Assoc. M. Am. Soc. C. E. (by letter).—As the author states, a method of analysis is of value only if it is ultimately useful to the designer. The method of analysis so briefly and yet so fully described in Professor Cross' paper amply fulfills this condition, and structural engineers are fortunate in having it made available to them. Since 1922 Professor Cross has used in his classes the method of distributing unbalanced moments and, since 1924, when he developed the method in its present form, he has taught it to his graduate students with conspicuous success.

The authors of the many papers which have recently been published in connection with this general subject have interested themselves particularly in the analysis of continuous girders and have all approached the problem from more or less conventional directions—directions long since indicated by Clapeyron, Fidler, and others. It is probably sanguine to expect anything inherently new to result from such an approach. To Professor Cross must go the credit for abandoning the paths of his predecessors and for opening up a refreshingly new and eminently useful avenue of approach to the general problem of continuity.

---

[12] Asst. Prof. of Civ. Eng., Univ. of British Columbia, Vancouver, B. C., Canada.

Professor Cross himself states that the mathematically minded may object to solving a set of simultaneous equations by trial, which, of course, is what the successive approximations amount to. Such an objection from the mathematical standpoint is perfectly valid; indeed, it is the sort of objection which a mathematician would be certain to raise. From the standpoint of the designer, however, the trial solution constitutes one of the most important points in favor of the method since, at any time, even after one distribution, the designer is able to form a good idea of the final values of the various moments. The values thus obtained may indicate a complete revision of the elastic properties of the structure. If this is the case he is then at liberty to make the necessary changes, and the only time lost is the negligible time required to make the first distribution. On the other hand, had the equations been set up no idea could have been formed of the final values until the solution had been completed. Only the experienced designer of indeterminate structures will appreciate the importance of this point. Between the analysis of a given structure, which is essentially mathematics, and the design of a required structure, which is essentially art, lie many difficulties.

As the author states the adopted sign convention permits the retention of the usual convention regarding positive and negative moments in the girders. By an ingenious twisting of the sheet this convention is stretched to fit the columns. So far, so good. The writer's criticism is that this sign convention makes it necessary, first, to distinguish between members on the right of a joint and members on the left of the same joint (this distinction when columns are involved is purely arbitrary and may well lead to confusion in complicated structures); second, it requires during the distribution of the unbalanced moment at a joint, that those portions of the unbalanced moment which are distributed to members on one side of a joint be given a different sign from those portions distributed to members on the other side of the same joint. This requires unnecessary thought during the distribution and may well be a contributory source of error. These two characteristics are inevitably associated with any convention based upon retaining that used by the author with regard to positive and negative girder moments. The question arises, is this retention worth while? Professor Cross states that, in his opinion, it is. With due deference the writer feels that it is not and suggests the following: Any moment which tends to rotate any joint clockwise is to be considered positive; the reverse, negative.

As an illustration of the application of this latter method consider Joint *C* in Professor Cross' example (Fig. 1). In the first instance the unbalanced moment will be —30, being the algebraic sum of the fixed-end moments at the joint in accordance with the writer's convention. In other words, the locked joint is tending to turn counter-clockwise and will so turn when released. This unbalanced moment is distributed exactly as in Professor Cross' example with the important exception that all distributed portions have the same sign—

in this case, plus. Consideration will show that, in all cases, the sign given to the distributed portions of the unbalanced moment is opposite to the sign of the unbalanced moment. It is thus apparent that there is no need to consider on which side of a joint a member occurs; and thus an arbitrary distinction is avoided.

If this sign convention is adopted a little consideration will show that the carry-over factor becomes + ½ instead of — ½, as used by Professor Cross. In other words, one-half the distributed moments are carried over to adjacent joints as before, but the signs remain unchanged.

In order to clarify the foregoing, Fig. 6 has been drawn to show the distribution of moments in the frame used by Professor Cross (Fig. 1), in accordance with the writer's sign convention. Selecting the member, *C D*, Fig. 6 shows that the final value of the bending moment at the end, *C*, is + 257.2;

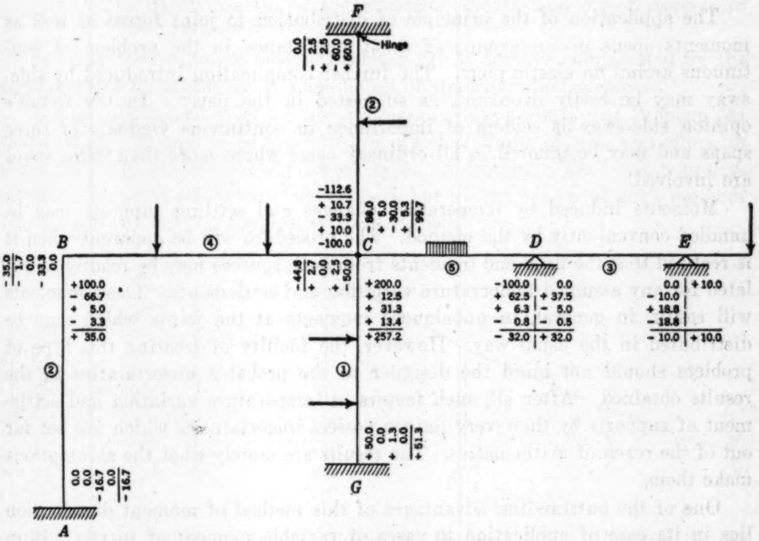

F
0.0 | - 2.5 + 2.5 - 60.0 + 60.0
Hinge
②
-112.6
+ 10.7
- 33.3
+ 10.0
-100.0
- 80.0 + 5.0 - 30.0 + 5.3 | - 99.7
B
④
C
⑤
D
③
E
- 35.0 | - 1.7 0.0 - 33.3 0.0
+100.0
- 66.7
+ 5.0
- 3.3
+ 35.0
- 44.8 | + 2.7 0.0 + 2.5 - 50.0
+200.0
+ 12.5
+ 31.3
+ 13.4
+257.2
-100.0 | 0.0
+ 62.5 | + 37.5
+ 6.3 | - 5.0
- 0.8 | - 0.5
- 32.0 | + 32.0
0.0 | + 10.0
- 10.0
+ 18.8
- 18.8
- 10.0 | + 10.0
②
①
+ 50.0 0.0 + 1.3 0.0 | + 51.3
0.0 0.0 - 16.7 0.0 | - 16.7
G
A

FIG. 6.

that is, the member is tending to turn Joint *C* clockwise. It is evident that tension must exist on the top of the member, *C D*, at Joint *C*. The signs of all other moments in the frame may be similarly translated.

Professor Cross has wisely restricted the paper to the bare description of the method. Those who use his method no doubt have their own particular procedure or technique developed in practice or teaching; indeed, if such is the case it is a healthy sign. The writer, who has used the method both in teaching and in practical design since 1926, has found himself less likely to make errors if, as soon as the moments are distributed at any joint, the correct proportions are at once carried over to adjacent joints before any further

distributions are made. Often this procedure is of advantage in so far as the carried-over moments may reduce any unbalance that previously existed at such joints, thereby hastening the convergence.

In practice it will be found quicker, and generally more advantageous, to treat those members having a hinge at one end as the paper suggests, by recognizing the existence of the hinge from the outset. As Professor Cross states, the stiffness of such a member must be modified in keeping with existing restraints, and it must be remembered that no moment can be carried over to a hinged end. It is perhaps hardly necessary to point out that the fixed-end moment for such a member is that obtaining when one end is hinged.

The speed of the method should be at once apparent. Assuming rollers at the support, *D* (Fig. 1), and a pin support at *E*, the frame is eight times statically indeterminate with respect to the external reactions, and yet the solution requires practically no effort.

The application of the principle of distribution to joint forces as well as moments opens up an avenue of great importance in the problem of continuous arches on elastic piers. The further complication introduced by side-sway may be easily overcome, as suggested in the paper. In the writer's opinion side-sway is seldom of importance in continuous viaducts of three spans and may be ignored in all ordinary cases where more than three spans are involved.

Moments induced by temperature changes and settling supports may be handled conveniently by the method. The procedure will be apparent when it is realized that the fixed-end moments from such sources may be readily calculated for any assumed temperature variation and settlements. These moments will result, in general, in unbalanced moments at the joints which may be distributed in the usual way. However, the facility of treating this type of problem should not blind the designer to the probable uncertainties in the results obtained. After all, such factors as temperature variation and settlement of supports by their very nature possess uncertainties which are set far out of the reach of mathematics. The results are merely what the assumptions make them.

One of the outstanding advantages of this method of moment distribution lies in its ease of application to cases of variable moment of inertia within each span—a condition imposed by economic as well as esthetic considerations. Such variation introduces tremendous complications into most so-called standard methods of analysis. In applying the method of moment distribution to such a problem, three factors are changed: (1) The fixed-end moments; (2) the stiffness of the individual spans; and (3) the carry-over factors. Once these quantities have been evaluated, the problem is solved exactly as before. All three of these factors may be tabulated by means of influence lines and graphs and the results made available to designers at a glance. In most cases extraordinarily close results may be obtained by approximating the changes in the various factors outlined.

The writer feels that structural engineers at large will give Professor Cross' paper the attention it deserves. Among those who realize that engineers are now only on the fringe of the general problem of continuity and who welcome each new advance, his work will find enthusiastic support such as may even overcome the inertia of things as they are.

R. F. LYMAN,[13] ASSOC. M. AM. SOC. C. E. (by letter).—In the practical application of the principles of mechanics to the analysis of a rigid frame, the designer is confronted with several problems, some of which are quite often difficult to solve. Among these may be mentioned the following:

(1) The relative stiffness of the various members of the frame, which, in a design problem, must necessarily be estimated in advance.
(2) The value of the connections, whether fixed, hinged, or partly restrained, as, for example, the end condition at the juncture of column and footing.
(3) The extent to which unbalanced loading should be considered as a factor in the design.
(4) The determination of the moments, shears, and direct stresses in the members of the frame.

In reinforced concrete design Problem (1) is further complicated by the varying moment of inertia of some or all of the individual members.

The method described by Professor Cross provides a solution of Problem (4) that is simple enough to be used, within practical limits of time and effort, in the design of the more complicated types of frames. The writer prefers graphical methods where they can be used, but he has not found such methods feasible for any but the simplest types of frames.

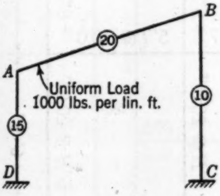


FIG. 7.

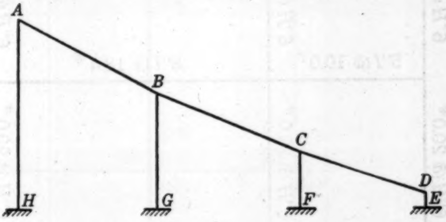


FIG. 8.

The writer believes that the subject of side-sway due to dissymmetry or transverse loading is of considerable importance. For example, in the frame shown in Fig. 7 the correction of the moment at *A* was found to amount to about 23 per cent. In the analysis of the frame shown in Fig. 8, which is typical of grand stand and stadium construction, it was found that the moment at *D* was very much affected by the correction for side-sway, due to dissymmetry, when the deck was uniformly loaded. Furthermore, the moments at other points are considerably affected by this correction when the span, *C D*, alone is loaded.

The author's method of correcting for side-sway is a little difficult to understand. It could, with profit, be explained more fully, and a numerical example given.

---

[13] Asst. Engr., H. J. Brunnier, San Francisco, Calif.

R. A. Caughey,[14] M. Am. Soc. C. E. (by letter).—This paper, in the writer's opinion, covers very important ground. As time goes on, and as people become more accustomed to using it, the value of the method described will be much more appreciated for use in problems, simple and complex.

One of the advantages of the method is that the application does not require complex mathematics; it requires nothing more than simple arithmetic. Another advantage is found in the time saved as compared with other methods; and still another is the automatic check of results.

The writer has presented to a class of undergraduate students, a problem involving a continuous beam of three spans with as many different moments of inertia and span lengths. After a brief presentation of the Cross method of analysis, supplemented by mimeographed notes, the problem was solved with practically no difficulty. The time involved was much less than the same class would require on the same problem by any other method known to the writer.

Mr. H. B. Gotaas, a graduate student working under the writer's supervision, computed moments in the members of a building frame loaded with vertical loads placed in various positions. Fig. 9 shows the dimensions and sizes of members in the bents of this building frame, which was actually built and used at Iowa State College to compare stress measurements obtained by strain gauges to stresses computed by theoretic methods. On this project, results obtained by the slope deflection method (ordinarily considered the most accurate of theoretic methods), were checked against those obtained by the method of Professor Cross, and it was found that the maximum variation in unit stresses, considering complete sets of stresses for ten loading conditions, was only about 400 lb. per sq. in. It is also interesting to note that the time necessary to compute moments in a bent by slope deflection was about 25 hours for each loading, while that required by the Cross method was 4 hours. It will

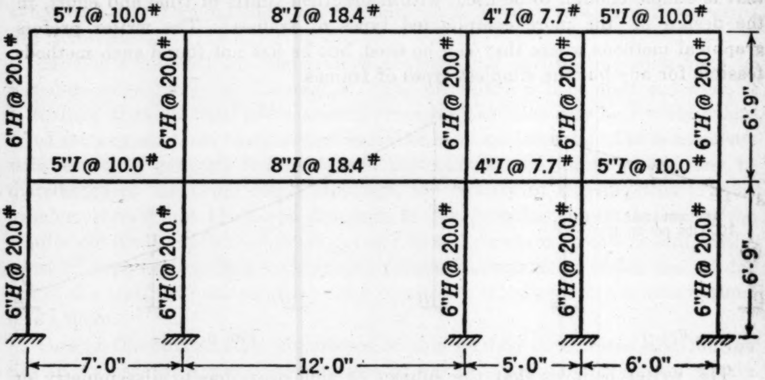


Fig. 9.—Diagram of Building Frame Constructed for Test Purposes at Iowa State College.

---

[14] Prof. of Structural Eng., Iowa State Coll., Ames, Iowa.

be agreed by all that the check on stress values is almost inconceivably accurate and that the time involved is reduced to allowable limits.

It might be well to state that in this comparison no correction was made for horizontal movements of joints. Had the frame been of great height as compared to width, this might have been necessary; but evidently it is not required in ordinary cases.

With concrete structures which are almost always constructed as rigid frames, with the welding of steel structures developing rapidly, and with an increasing desire on the part of designers to obtain stresses more accurately and to use material more economically, the writer prophesies a wide use for this method.

Orrin H. Pilkey,[15] Jun. Am. Soc. C. E. (by letter).—The Cross method of distributing moments is especially valuable in that it keeps intact the picture of the action of the structure. The user does not become lost in a maze of mathematics, rules, or formulas. However, the paper is entirely too brief. Some phases upon which entire chapters could be written with profit, are passed over with a concise statement.

For instance, the discussion of multi-storied bents subject to side-sway is limited to two short paragraphs. The writer feels that this is not sufficient, especially to one unfamiliar with moment distribution. He agrees with the statement that, except for the effect of transverse loads, the joints may be considered fixed in position in practically all rectangular frames. It should be recognized, however, that cases will occur in practice in which it will be important to make certain whether or not the effect of movement in the joints is appreciable. The following example will illustrate in detail a method of making the necessary correction.

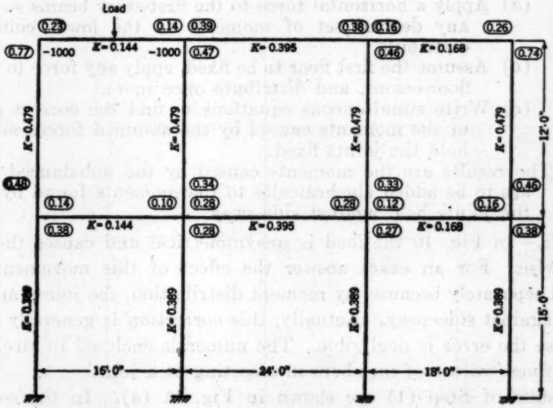


Fig. 10.—Diagram of a Two-Story Bent with a Vertical Unsymmetrical Load.

---

[15] New York, N. Y.

The bent in Fig. 10 presents a rather extreme case of loading, but even in this case the movement of joints is so small that the refinement obtained by correction for it is mostly of academic interest. The percentage of error at some of the joints is certainly great, but the actual magnitude of the error is negligible. It should be noted that it is the magnitude of the error rather than the percentage that is significant. For unsymmetrical loadings encountered in most buildings the movement of joints is so small as to affect the results very little.

Another point, illustrated by Fig. 10, is that the distribution of fixed-end moments is not practicable for analyzing multi-storied buildings subjected to appreciable side-sway, such as would be caused by wind pressure or other transverse loads. It would be more advantageous to distribute joint rotations.

Outline of Steps in Determining Moments Caused by Side-Sway.—Specifically, the method of correcting for side-sway may be conveniently elaborated and described in the following successive steps:

(1) Assume that the joints do not move sidewise and distribute moments;

(2) Find the magnitude of the transverse forces necessary to keep the joints from moving by determining the difference in the shears in the columns;

(3) Find moments in the frame as a result of these forces, by first subjecting each floor, successively, to an assumed load as follows:

  (*a*) Apply a horizontal force to the first-story beams so as to give any desired set of moments in the lower columns, and distribute.

  (*b*) Assume the first floor to be fixed, apply any force to the second floor-beams, and distribute once more.

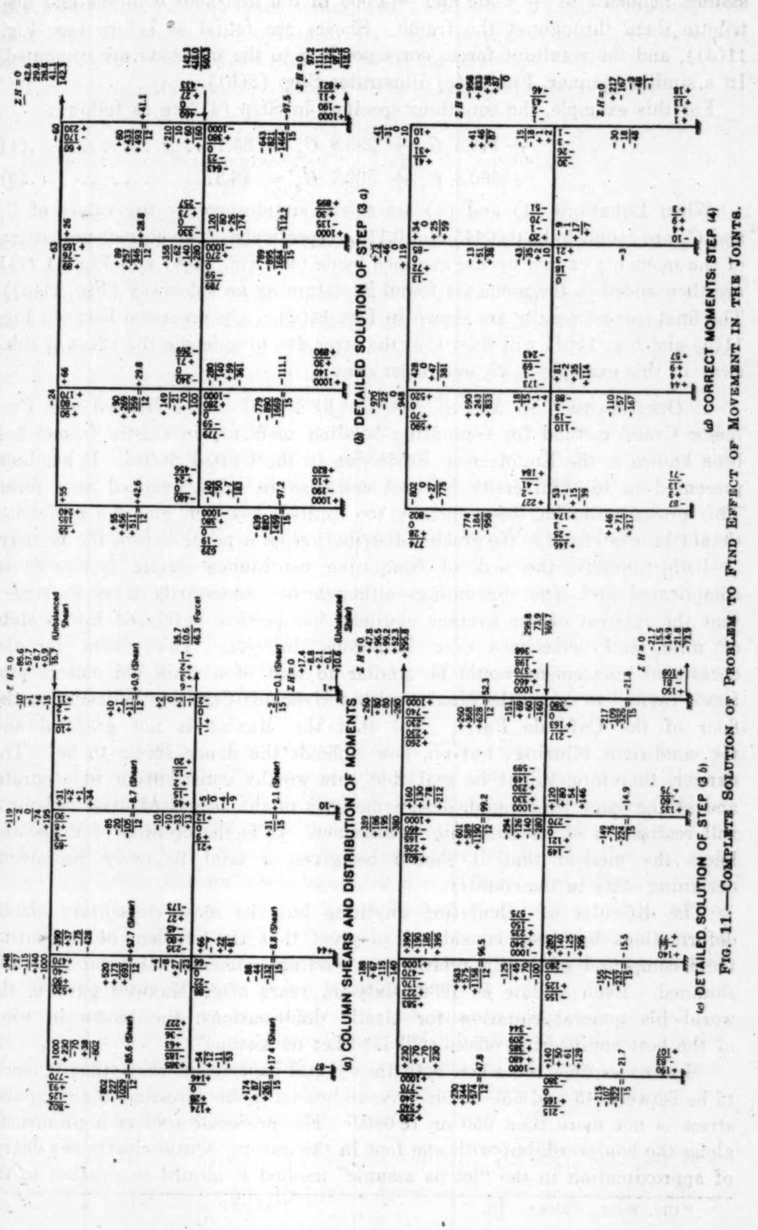
  (*c*) Write simultaneous equations to find the correct proportion of the moments caused by the assumed forces necessary to hold the joints fixed.

(4) The results are the moments caused by the unbalanced shear and are to be added algebraically to the moments found by assuming the joints held against side-sway.

Example.—In Fig. 10 the load is unsymmetrical and causes the joints to move sidewise. For an exact answer the effect of this movement must be determined separately because, by moment distribution, the joints are assumed to be held against side-sway. Actually, this correction is generally not necessary because the error is negligible. The numerals enclosed in circles are the relative stiffness values of members intersecting at a joint.

The results of Step (1) are shown in Fig. 11 (*a*). In the second-story columns the shears are found to be -85.6, -8.7, $+57.7$, and $+0.9$, respectively, which means that, if the condition, $\Sigma H = 0$, is to be satisfied, the unbalanced shear, -35.7, must be applied as a horizontal force restraining the bent at that level. Likewise, the unbalanced column shear in the first-story columns is $+17.4 - 8.8 + 2.1 - 0.1 = +10.6$; and for $\Sigma H = 0$, at the first story, the restraining force will be $35.7 + 10.6 = 46.3$. This constitutes Step (2).

Fig. 11(*b*) illustrates in detail the solution of Step (3(*a*)). If the load is assumed, the moment produced may also be assumed. The first step is to


(a) COLUMN SHEARS AND DISTRIBUTION OF MOMENTS

(b) DETAILED SOLUTION OF STEP (3 a)

(c) DETAILED SOLUTION OF STEP (3 b)

(d) CORRECT MOMENTS; STEP (4)

FIG. 11.—COMPLETE SOLUTION OF PROBLEM TO FIND EFFECT OF MOVEMENT IN THE JOINTS.

assume moments of + 1 000 and — 1 000 in the first-floor columns and distribute them throughout the frame. Shears are found as before (see Fig. 11(*a*)), and the resultant forces corresponding to the moments are computed. In a similar manner, Fig. 11(*c*) illustrates Step (3(*b*)).

For this example, the equations specified in Step (4) are as follows:

$$+142.3\ C_1 + 295.8\ C_2 = 35.7 \qquad (1)$$

$$+560.3\ C_1 + 369.7\ C_2 = 46.3 \qquad (2)$$

When Equations (1) and (2) are solved simultaneously the values of C_1 and C_2 are found to be 0.00445 and 0.119, respectively. The correct proportion of the moments caused by the assumed loads (see Fig. 11(*b*) and Fig. 11 (*c*)) are then added to the moments found by assuming no side-sway (Fig. 11(*a*)). The final correct results are shown in Fig. 11(*d*). A comparison between Fig. 11(*a*) and Fig. 11(*d*) will show that the error due to ignoring the effect of side-sway in this example is 27, or 3.4 per cent.

I. OESTERBLOM,[18] M. AM. SOC. C. E. (by letter).—In a limited way Professor Cross' method for computing bending moments in elastic frames has been known to the Engineering Profession in the United States. It has been presented in his University lectures and also in mimeographed note form. This presentation has been entirely too limited, however, and the profession should be grateful for its greater distribution as a paper before the Society.

Unfortunately, the task of computing continuous elastic frames is so complicated and time-consuming—although not necessarily time wasting—that the interest of the average engineer has become paralyzed into a state of mild, and sometimes even humorous, interest. The reason for the occasional amusement would be similar to that of a staid old college professor invited to dance the Maxixe with a vivacious Castilian ballerina on the floor of the Café de Paris. Not that the Maxixe is not graceful and the conditions alluring; but oh, how difficult the dance seems to be! The danger, therefore, might be real that this worthy contribution to accurate, and at the same time rapid, design methods might be passed over without a full realization of its economic significance. It is the opinion of those who know the method that it should be given a trial in every prominent designing office in the country.

The difficulty of calculating anything but the most elementary elastic deformations has been considered so great that the problem of computing their compound effects in relation to bending moments has been generally shunned. Even as late as 1930, sixty-six years after Maxwell gave to the world his general equation for elastic deformations, the motto in some of the best engineering offices still is: "Let us assume".

"Let us assume 100 when both theory and experiment show that it ought to be between 45 and 55—or *vice versa*—but let us be sure that the computed stress is not more than 650 or 16 000." The professor prefers a promenade along the boulevard, but with one foot in the gutter. Quite clearly the degree of approximation in the "let us assume" method is absurd in relation to the refinement of permissible stress assumption. To make Professor Cross' method potentially attractive it would seem desirable, therefore, to outline by comparison the real difficulties which one is trying to escape.

---

[18] Civ. Engr., Chicago, Ill.

The fundamental equation derived from Maxwell[17] reads, as follows:

$$\delta_a = \delta_{ao} - X_a\,\delta_{aa} - X_b\,\delta_{ab} - X_c\,\delta_{ac} \ldots\ldots + \delta_{at} \ldots\ldots\ldots\ldots (3)$$

Deflections are here represented by δ and the redundants by X. With a δ adjacent to an X, the deflection is for a unity, X. The indices have reference to the frame points, a, b, c, etc., and the temperature change, t. For a double index after δ, the first represents the location of the deflection; and the second, the location of its static or redundant cause. The symbol, δ_{ao}, signifies the deflection at a, with all redundants equal to zero. The equation represents the elementary fact that the deflection at any point is the sum of partial deflections, each attributable to a different cause. This is by the law of superposition, which is substantially true below the elastic limit of stress.

It should be noted further that for each redundant there is a sequence of partial deflections and that, therefore, one may write as many similar equations as there are redundants. Assuming every δ to be known, it would, therefore, be a simple problem of algebra to solve for the redundants.

There are two difficulties, however, to prevent a more common use of the universal method: First, one has to cope with the frequency of numerous redundants, which makes the use of determinants most necessary for the solution of the equations; and, second, each partial deflection, although purely a problem of statics, is difficult to ascertain.

To the average engineer the necessity for using determinants occurs so infrequently that he is very likely to get confused in his plus and minus signs. He is afraid, therefore, of the accuracy of the results and also afraid of the method.

The second objection is even more serious. There are many ways to compute the deflections: (1) By the equation of the elastic line; (2) by Mohr's method of moment areas; (3) by Castigliano's method of least work; (4) by equating the external work with the resilience; and (5) by the use of Maxwell's elastic weights. These methods are more or less similar or comparable and each has its advantages for given frames or elements. The unfortunate fact is that each partial deflection is quite a problem of its own; that there are as many[18] of them as the second power of the number of redundants; and that each is generally complicated through the compounding of several elements in more or less complex relations.

The elementary labor of integrating and summarizing each partial deflection is, therefore, very considerable, and multiplication of work to allow for all the redundants, together with the accumulation of these results into appropriate determinants, makes the task commercially impossible, except for very heavy, important, dangerous structures.

---

[17] This equation in some form occurs in all the textbooks and handbooks of Continental Europe; see, also, "Statically Indeterminate Stresses," by Parcel and Maney, New York, 1926, p. 112.

[18] By the use of Maxwell's law of reciprocal deflections the number of problems due to unity loads may be reduced by one-half.

The formulas for elementary frames are generally well known; some complicated frames have also been analyzed and reduced to a series of formulas,[19] but many of the more complicated frames, or simple frames with a few unusual features, must always be solved as individual problems.

The question then arises: Should not the continuous frame be abandoned for the sake of greater safety and simplicity? Obviously not; there are numerous most necessary frames which are inherently continuous and can be nothing else. They can not be abandoned therefore; neither should they be analyzed, as loosely as they often are, "by the method of assumption".

Here is where Professor Cross' method is certain to render conspicuous service to the profession. Using this method of successive approximations, the writer has checked a heavily loaded three-post reinforced concrete bent in 3 hours and with an error of less than 3 per cent. It was necessary to distribute and "carry over" only three times, whereas a very competent engineer had required several days to analyze it.

Knowing the universal method and its complications one is led to consider such a saving of time almost incredible and must necessarily ask in what manner such a miracle is possible. Professor Cross' method then appears to resolve itself into an interpretation and simplification of the universal method—and none can gainsay that it is both original and graceful. The reasons for simplicity and saving of time are outlined as follows:

> (1) The very annoying product, *EI*—which is scattered all over the calculation sheets of the universal method—has been combined with spans and heights into a "stiffness factor," on the basis of which—for any moment effect—from whatever cause, at any point, a distribution is made in proportion to resultant resisting moments of adjacent elements. This eliminates in one stroke numerous petty notations and calculations of multiple occurrence.
>
> (2) The travel of any moment induced at any point from any external source is then followed through along the lines of the frame by means of the "carry over factor" until such a point is reached that the effect is dissipated to an insignificant magnitude. This means the elimination from the universal method of all those elements of expression, which merely denote an unnecessary refinement of calculation, ordinarily concealed under an innocent looking form, but none the less present as a burden on the calculations.
>
> (3) Finally, there is the author's conventional grouping and summation, which takes the place—after all the "trimmings" have been eliminated—of the basic equations and their determinants.

In the analytical field Professor Cross' method takes a place with the method introduced in the graphical field by George E. Beggs, M. Am. Soc. C. E., of measuring the deflections of elastic models.[20] The two together form a very significant and admirable balance as against the profound theory of Continental Europe, which, excellent and extensively published, has none the less made little impression on the profession in the United States, except in designs of certain types of bridges and dams.

---

[19] See, for instance, Kleinlogel "Rahmenformeln" and "Mehrstielige Rahmen," Berlin, 1925 and 1924.

[20] *Transactions*, Am. Soc. C. E., Vol. 88 (1925), p. 1208.

Professor Cross himself has confessed to a limitation of his method: It is not apparent how it could be applied, if the supports were truly movable. Perhaps this gap may be eliminated as experience is gained and more thought is given the subject. However, at present, this is of less importance, as most commercial problems are dealing with unyielding supports, and, for those which are not, it would be economically feasible to use the universal method at full length. These are problems that generally involve large quantities of costly construction materials, and sometimes also foundation difficulties, so that the cost of careful analysis is, therefore, not only small by proportion, but also most necessary to safeguard the heavy investment.

Robert A. Black,[n] Assoc. M. Am. Soc. C. E. (by letter).—The writer has had occasion to apply the Cross method to the design of simple two-legged bents symmetrically loaded and to continuous beams simply supported, and he knows from experience with what ease, rapidity, and accuracy, indeterminate structures of the type mentioned may be solved. The paper, however, reveals the application of the method to bents of one or more stories subjected to side-sway due either to unsymmetrical or to horizontal loads.

It so happened that the writer was confronted with a problem concerning two-story bents subjected to traction forces and to unsymmetrical loading. The comparative ease and simplicity of the solution afforded by the method described, emphasizes the fact that the subject-matter of the paper offers an invaluable tool for the designer. A brief description of the problem and one phase of its solution follows.

In connection with the extension of a new Outer Drive over the Chicago River and of the development of the air-right territory over the tracks of the Illinois Central Railroad and the Michigan Central Railroad Companies, north of Randolph Street and east of Michigan Avenue, Chicago, the South Park Commissioners are required to build certain viaducts. These are known as the Randolph Street, the Field Boulevard, and the Wacker Drive, Viaducts. They all will be two-level structures having, in general, transverse girders at the column bents and longitudinal floor-beams between bents, and they will be built entirely over railroad-yard tracks. Because of certain sub-surface rights granted the Railroad Companies, it is required that the columns of these viaducts be carried down to Elevation — 19.0 (Chicago City Datum), at which point they will rest on caissons. As the top of the curb of the top deck is at Elevation + 47, the columns will be nearly 65 ft. long. They are limited in size in a direction normal to the center line of track to 2.0 ft., including the concrete encasing. Due to the multiplicity of tracks and their varied alignment, to loading platforms and team tracks, and to limited head-room allowances, the bracing which may be used will be confined entirely to small knee-braces.

The stiffness of these structures will depend almost entirely upon the slender columns and the rigidity of the connections. Therefore, it is important to know the effect of traction and of vertical loading upon bending in the columns and upon the strength of connections. A series of influence lines was considered to be the best means of studying the columns and their connections in these viaducts.

---

[n] Structural Engr., South Park Commrs., Chicago, Ill.

A typical cross-section of the Field Boulevard Viaduct is shown in Fig. 12. The letters refer to floor-beam points on the top deck and the numbers on the structure refer to points at which moments were computed, that is M_1, M_2, etc. The solution for the ordinate to an influence line with the load on the top deck only, may be outlined in six steps as follows, the unit load in this example being placed at Point C:

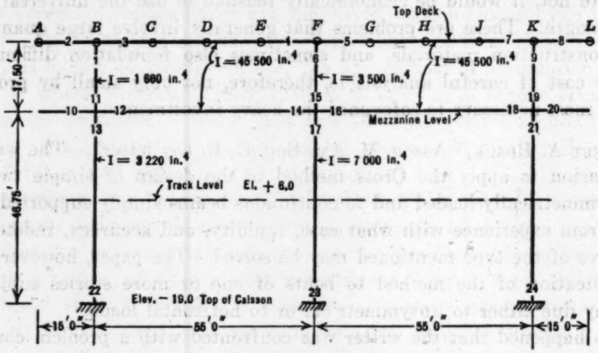

Top Deck
A
B
C
D
E
F
G
H
J
K
L
2
3
4
5
7
8
1
6
9
I = 45 500 in.⁴
I = 45 500 in.⁴
I = 1 680 in.⁴
I = 3 500 in.⁴
17.50'
11
15
19
10
12
14
16
18
20
Mezzanine Level
13
17
21
I = 3 220 in.⁴
I = 7 000 in.⁴
Track Level
El. + 6.0
46.75'
22
23
24
Elev. − 19.0 Top of Caisson
15' 0"
55' 0"
55' 0"
15' 0"

FIG. 12.—TYPICAL CROSS-SECTION, FIELD BOULEVARD VIADUCT, CHICAGO, ILL.

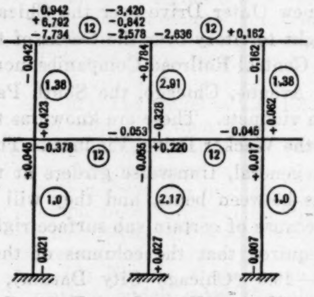

− 0.942
+ 6.792
− 7.734
− 3.420
− 0.842
− 2.578
− 2.636
+ 0.162
12
12
− 0.942
+ 0.784
− 0.162
1.38
2.91
1.38
+ 0.423
− 0.328
+ 0.062
− 0.053
− 0.045
+ 0.045
− 0.378
− 0.055
+ 0.220
+ 0.017
12
12
1.0
2.17
1.0
− 0.021
+ 0.027
− 0.007

FIG. 13.—VALUES OF THE MOMENTS, M'_C, WITH A LOAD AT POINT C. IN FIG. 15, AND WITH NO SIDEWISE DEFLECTION.

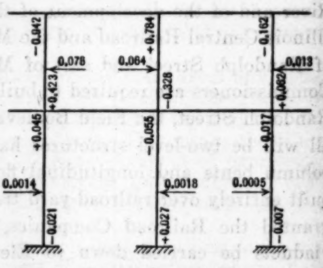

− 0.942
+ 0.784
− 0.162
0.078
0.064
0.013
+ 0.423
− 0.328
+ 0.062
+ 0.045
− 0.055
+ 0.017
0.0014
0.0018
0.0005
− 0.021
+ 0.027
− 0.007

FIG. 14.—COLUMN SHEARS. H'_{TC} AND H'_{BC}, WITH A LOAD AT POINT C, IN FIG. 15, AND WITH NO SIDEWISE DEFLECTION.

1.—With the unit load at Point C on the top deck, the fixed-end moments at B and F are — 7.734 and — 2.578, respectively (see Fig. 13). Distribute these moments as demonstrated by the author in Fig. 1, using a "carry-over factor" of $-\frac{1}{2}$, and considering no sidewise deflection of the bent. Only the final moments, M'_C, are given in Fig. 13.

2.—After the distribution operations, compute the column shears (see Fig. 14), H'_{TC} and H'_{BC}. (The subscripts refer to the top or bottom story with a unit load at Point C.)

3.—Take the same bent, but without the unit vertical load, and apply any convenient set of fixed-end moments to the top-story columns. The only restriction on these moments is that they shall be proportional to the $\frac{I}{L^2}$ of the columns to insure that the deflections at the tops of the columns will be the same. For the present purpose values of 100, 211, and 100, respectively, were selected and applied as shown in Fig. 15. Distribute these moments as before; sum them up and call the final moments, M''. Repeat Step 3, beginning with the lower-story columns. Call these final moments, M''' as in Fig. 16.

4.—From the moments, M'' and M''', compute the column shears in both stories. These shears are designated H''_T, H''_B and H'''_T, H'''_B, respectively. (See Figs. 17 and 18.)

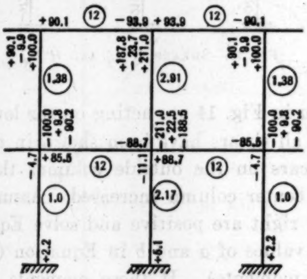

+90.1
(12)
−93.9
+93.9
(12)
−90.1
+90.1
−9.9
+100.0
+187.8
−23.7
+211.0
+90.1
−9.9
+100.0
(1.38)
(2.91)
(1.38)
−100.0
+9.8
−90.2
−88.7
−211.0
+22.5
−188.5
−85.5
−100.0
+9.8
−90.2
−4.7
+85.5
(12)
−11.1
+88.7
(12)
−4.7
(1.0)
(2.17)
(1.0)
+2.2
+5.1
+2.2

FIG. 15.—MOMENTS, M'', DUE TO DISTORTION IN THE TOP STORY.

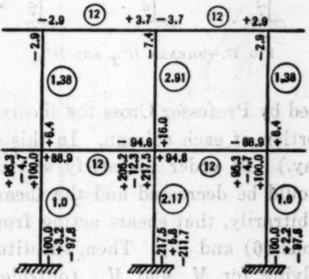

−2.9
(12)
+3.7
−3.7
(12)
+2.9
−2.9
−7.4
−2.9
(1.38)
(2.91)
(1.38)
+6.4
−94.6
+16.0
−88.9
+6.4
+95.3
−4.7
+100.0
+88.9
(12)
+205.2
−12.3
+217.5
+94.8
(12)
+95.3
−4.7
+100.0
(1.0)
(2.17)
(1.0)
−100.0
+3.2
−97.8
−217.5
+5.8
−211.7
−100.0
+2.2
−97.8

FIG. 16.—MOMENTS, M''', DUE TO DISTORTION IN THE LOWER STORY.

5.—The total shear in each story is made up of the shear due to the vertical load and that due to the assumed moments causing the side-sway, and this total shear must equal zero to satisfy statics; that is, the total shear is equal to H' plus some fraction of H'' and H'''. Therefore,

$$H'_{TC} + a\,H''_T + b\,H'''_T = 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (4)$$

$$H'_{BC} + a\,H''_B + b\,H'''_B = 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (5)$$

or, using the values from Figs. 14, 17, and 18, Equations (4) and (5) become,

$$-\,0.027 + 42.10\,a - 2.40\,b = 0 \quad \ldots\ldots\ldots\ldots\ldots (6)$$

$$+\,0.0001 - 0.643\,a + 17.18\,b = 0 \quad \ldots\ldots\ldots\ldots\ldots (7)$$

Solving Equations (6) and (7) simultaneously, $a = +\,0.000642$ and $b = +\,0.000018$.

The total moments, therefore, at Points 1, 2, 3, etc. (Fig. 12), due to the unit vertical load at C and corrected for side-sway are,

$$M_C = M'_C + a\,M'' + b\,M''' \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (8)$$

that is, using values from Figs. 13, 15, and 16, and the previously determined values of *a* and *b*,

$$M_1 = -0.942 + 0.00064 \times 90.1 - 0.000018 \times 2.9 = -0.884$$

and so on for all other moments, M_2, M_3, etc.

***Sign of a.*—By inspection of Fig. 14, it is seen that the resultant shear in the top story (0.027 lb.) acts from right to left. (According to the convention used by Professor Cross the shears as shown in Fig. 14 are acting on the lower portion of each column. In this example, all shears have been shown in this way.) In order to satisfy statics the shears on the outside columns then, should be decreased and the shear on the center column increased. Assume, arbitrarily, that shears acting from left to right are positive and solve Equations (6) and (7). Then, substitute these values of *a* and *b* in Equation (8), solving for M_1 and M_{11} (outside column moments). If these moments are decreased numerically from M'_1 and M'_{11}, respectively, then, the original assumption for the sign of the shears was correct and will hold for all other positions of the unit load on the top deck. A check on the work is obtained by noting that the sum of the column shears computed from the final moments, M_C, is equal to zero in each story.**

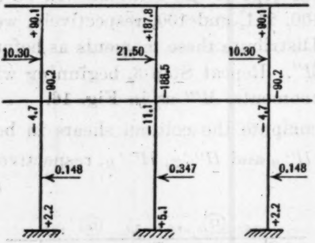


FIG. 17.—SHEARS, H''_T AND H''_B.

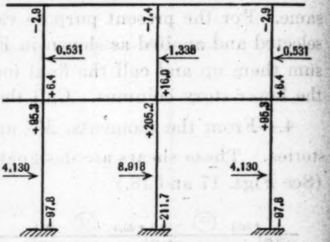
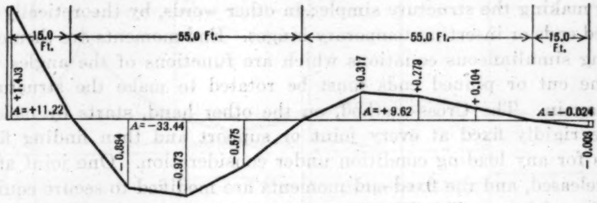


FIG. 18.—SHEARS, H'''_T AND H'''_B.

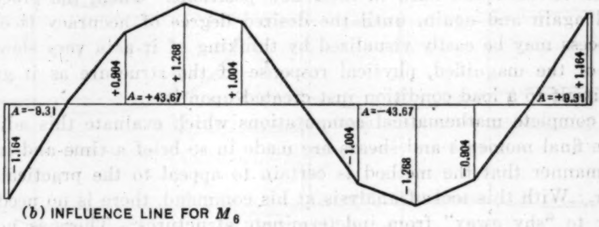
6.—By using Equation (8) to correct all M'_C moments at Points 1, 2, 3, etc., of Fig. 12, the material is derived with which to plot two ordinates (at *C* and at *J*) on the influence line for the moment at any one or all of the points shown in Fig. 12. It should be noted that the effect of the load at *J* on M_1, for example, will be the same as that of the load at *C* on M_9, the bent being symmetrical, except that the sign of the moment changes when dealing with column moments. (The sign remains unchanged for girder moments.) Examples of influence lines, selected at random, are shown in Fig. 19.

It should be noted, also, that in carrying the problem on for positions of the unit load at other points on either deck, the values of M'' and M''', and H''_T and H''_B, and H'''_T and H'''_B, herein used for Point *C*, may also be used for the load at all other points.

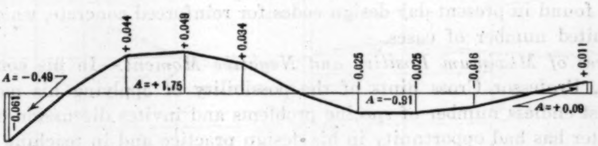
After the load has been run across the top deck, it can be placed on the lower deck and an independent set of influence lines obtained for moments at the same points, 1 to 24, inclusive, shown on Fig. 12. The conclusions and results which may be obtained from these influence lines are obvious and need not be mentioned here.



(a) INFLUENCE LINE FOR M_1



(b) INFLUENCE LINE FOR M_6



(c) INFLUENCE LINE FOR M_{13}

FIG. 19.—INFLUENCE LINES, IN FOOT-POUNDS UNITS, FOR M_1, M_6, AND M_{13}, WHEN THE TOP DECK ONLY IS LOADED.

In this example, tractive forces are acting normal to the bent and hence do not affect the moments herein computed. However, in any case where they are an element to be considered, their effect may be determined by Steps 3 to 6, inclusive, because in this case there are no moments, *M′*, and the shears do not equal zero, but are equal to the known tractive force. This alters Equations (4), (5), and (8). Otherwise, the procedure is the same.

In the example given, the moments of inertia of the various members were obtained from a preliminary design, and while their actual values may change somewhat their relative values should remain about constant. Almost without exception, all computations were made on a 10-in. slide-rule. The method has proved accurate, a great time saver, and a valuable aid to intelligent design.

H. E. Wessman,[22] Assoc. M. Am. Soc. C. E. (by letter).—Professor Cross' method of analyzing continuous frames is the most outstanding contribution made in many years to the field of structural engineering. Although it has for its fundamental basis the geometry of slopes and deflections of elastic members—which is common to all logical methods of analyzing indeterminate structures—it is developed in a much different way. Other methods usually begin by making the structure simple; in other words, by theoretically cutting restrained ends or inserting temporary hinges. End moments are then obtained by solving simultaneous equations which are functions of the angles through which the cut or pinned ends must be rotated to make the structure continuous again. The Cross method, on the other hand, starts by making the structure rigidly fixed at every joint or support and then finding fixed-end moments for any loading condition under consideration. One joint at a time is then released, and the fixed-end moments are modified to secure equilibrium at the released joint. This joint is temporarily fixed again until all other joints have been released and fixed in their new positions. Then, the procedure is repeated again and again, until the desired degree of accuracy is obtained. The process may be easily visualized by thinking of it as a very slow-motion picture of the magnified, physical response of the structure as it gradually adjusts itself to a load condition just created upon it.

The complete mathematical computations which evaluate this adjustment and give final moments and shears are made in so brief a time and in such a simple manner that the method is certain to appeal to the practical-minded engineer. With this tool of analysis at his command, there is no need for the designer to "shy away" from indeterminate structures. There is no excuse for "guessing" moments and shears by means of arbitrary coefficients such as those found in present-day design codes for reinforced concrete, which apply to a limited number of cases.

Curves of Maximum Positive and Negative Moment.—In his concluding remarks, Professor Cross hints of the possibility of applying his method to an almost endless number of specific problems and invites discussion on them. The writer has had opportunity in his design practice and in teaching to compare the application of several methods to some of these problems. He much prefers that of Professor Cross and wishes to illustrate its workability in constructing curves of maximum moments and in analyzing frames subject to lateral displacement. Curves of maximum positive and negative moment have been computed for a four equal-span continuous beam for various ratios of uniform live load to dead load (hereafter called the $\frac{L}{D}$ ratio). In the first case, there is no restraint at the supports from columns or other structural elements. In the second case, corresponding curves are computed for a four-span system having the same span lengths as the first, but in which rigidly connected columns at the supports affect the moments. This case is one in which analysis by previously accepted methods becomes very complex. It is just as easily treated by the author's method, however, as the first case in

---

[22] Structural Engr., Waddell & Hardesty, New York, N. Y.

which there are no restraining columns. A study of the two sets of curves will not only show the effect of column restraint in changing curves of maximum moments for the beams, but will raise some questions about present design codes for reinforced concrete.

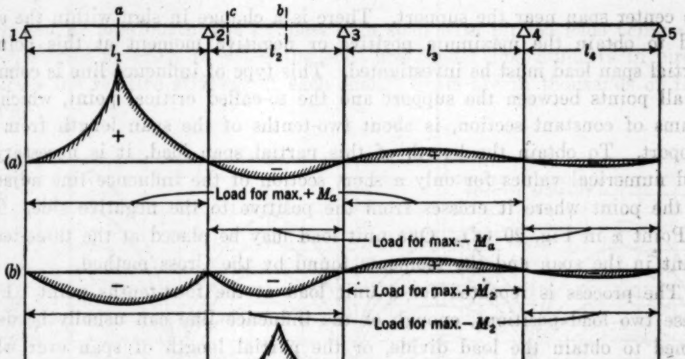

a
c
b
1
2
3
4
5
l_1
l_2
l_3
l_4
(a)
+
−
+
Load for max. + M_a
Load for max. − M_a
(b)
−
−
+
−
Load for max. + M_2
Load for max. − M_2
(c)
+
−
−
+
Load for max. + M_b
Load for max. − M_b
(d)
+
×
−
−
+
C
Load for max. + M_c
Load for max. − M_c
(e)
+
−
+
Load for max. + V_a
Load for max. − V_a
(f)
−
−
+
Load for max. + V_{2L}
Load for max. − V_{2L}

FIG. 20.—TYPICAL INFLUENCE LINES FOR MOMENTS AND SHEARS.

It is necessary to know where to place live loads in order to obtain the greatest effect at any point (or section) of the continuous system. Here is where the influence line performs its chief function. It is primarily a guide for the intelligent placing of live load. Fig. 20 shows typical influence lines for moments and shears at various governing points in a four-span continuous system. They are not quantitative; only qualitative. To most engineers familiar with indeterminate structures, sketching the influence line is only a moment's task. In most cases a sketch is sufficient to tell one where to place the loads for maximum effect. For the governing points of the design, that is, points along the middle one-half of the span, Fig. 20 (*a*) and (*c*), and

points at the supports, Fig. 20 (*b*), the influence lines for moment show that only full span loads are needed. They change sign (from positive to negative) only at the supports. In reinforced concrete design, these points are the only ones that need to be investigated in most cases.

Note the influence line, Fig. 20 (*d*), however. This is for *M* at a point in the center span near the support. There is a change in sign within the span and to obtain the maximum positive or negative moment at this point a partial span load must be investigated. This type of influence line is common to all points between the support and the so-called critical point, which, in beams of constant section, is about two-tenths of the span length from the support. To obtain the length of this partial span load, it is necessary to find numerical values for only a short section of the influence line adjacent to the point where it crosses from the positive to the negative side. This is Point *x* in Fig. 20 (*d*). One unit load may be placed at the three-tenths point in the span and the moments found by the Cross method.

The process is repeated for a unit load at the four-tenths point. From these two load positions, enough of the influence line can usually be determined to obtain the load divide, or the partial length of span over which live load must be placed. It is doubtful, however, whether this is of any practical significance. The part-span load combined with the other full-span loads to give maximum effect calls for a split loading that rarely occurs under actual conditions. In reinforced concrete design, there is certainly no practical necessity to consider these part-span loads for moments. In steel girder design, they may be needed for the longer spans in order to determine cover-plate cut-offs more accurately. It is largely a question of judgment as to the possibility of obtaining the split loadings in service conditions. For continuous steel trusses, however, it is advisable to determine these partial span loadings and their effects in order to obtain maximum chord stresses and provide for possible reversal of stress in the panels near the center piers. Consequently, for the maximum moment curves, illustrated in Fig. 21, only full span loads have been considered. Note how one span at a time is loaded and the results combined to give the final maximum moments for dead load plus live load.

Curves of maximum shear are not given in this discussion, although typical influence lines for shear at various points are shown in Fig. 20 (*e*) and (*f*). To obtain the maximum shear at any point within the span, a part-span load must be used. To get its length it is not necessary to compute any values for any part of the influence line, however, as in the case of moments. This is because the load divide point coincides with the point at which the shear is wanted or is in the same panel if a through girder is being analyzed. For reinforced concrete beams of constant section, there is no practical need to find the maximum shears at any interior point, except the center of the span. From this value and the maximum at the support, stirrup spacing may be determined closely enough. If the beams are haunched, the maximum shear at the quarter-point, or at the point of the haunch, should be found in order to keep the unit shear over the reduced section within specification limits.

Case I.—**Four 20-ft. spans are used. (See Table 2.) Each span has the** same cross-section and, consequently, the $\frac{I}{L}$ ratio is the same for all. As the ends are freely supported, however, the $\frac{I}{L}$ ratio for the end spans has been decreased by one-fourth, the stiffness of a span with pinned ends being only three-fourths as great as when the ends are fixed. The numbers in circles indicate the relative stiffness of each span. Units are in thousands of foot-pounds.

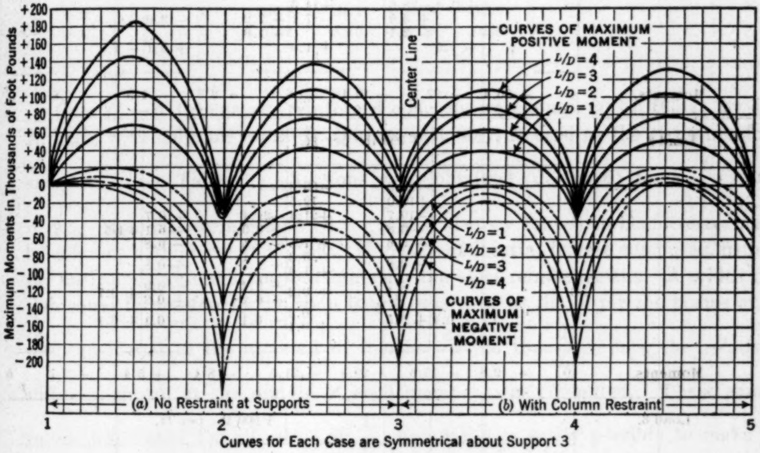
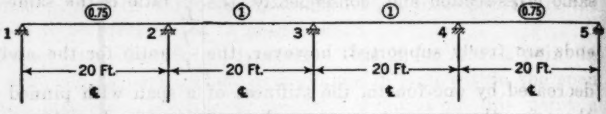


FIG. 21.—CURVES OF MAXIMUM MOMENT.

Moments are found at all supports and in the center of each span, due to a uniform load of 1 000 lb. per ft. over one span at a time. These moments are then combined to give maximum positive and maximum negative moments at each point, due to live load. These two values for each point are then added algebraically to give the moments due to a dead load of 1 000 lb. per ft. over the entire four spans of the continuous system. Live load moments are then combined with dead load moments to obtain the ordinates for the curves of total maximum positive and total maximum negative moments for a ratio, $\frac{L}{D}$, equal to 1. To get curves of moments for any other ratio of $\frac{L}{D}$, the live load moments are first multiplied by the necessary ratio before combining them with those of the dead load. These curves (Fig. 21 (*a*)), are quite easily plotted with no other points than those given, inasmuch as the middle three-fifths (approximately) for each span is a parabola. Curves are shown for $\frac{L}{D}$ ratios of 1, 2, 3, and 4.

TABLE 2.—COMPUTATION OF MAXIMUM MOMENTS IN A FOUR-SPAN BEAM.



Load 1.		1 000 lb. per ft.							
	−33.3 +33.3		−33.3 −16.7 +21.4 +1.5	−28.6 +3.6 −2.1	+14.3 −7.1 +1.0	+7.2 −1.0	−3.6 +2.1	−1.5	
Moments.	0	+36.4	−27.1	−10.4	+6.2	+2.3	−1.5	−0.8	0
Load 2.				1 000 lb. per ft.					
			−0.4	−33.3 +19.0 −10.7 −0.8 +6.6 −0.8 −0.1 +0.5	−33.3 −19.5 +21.4 +1.5 −3.3 +1.6 +0.2 −0.2 +0.1	−21.4 +3.0 −1.5 −1.7 +0.4 −0.2 −0.1	+10.7 −6.1 −0.8 +0.8 −0.9 +0.2 −0.1	+4.6 +0.7 +0.1	0
Moments.	0	−9.8	−19.6	+29.4	−21.5	−8.1	+5.4	+2.7	0
Load 3.						1 000 lb. per ft.			
Moments.	0	+2.7	+5.4	−8.1	−21.5	+29.4	−19.6	−9.8	0
Load 4.								1 000 lb. per ft.	
Moments.	0	−0.8	−1.5	+2.3	+6.2	−10.4	−27.1	+36.4	0
$+M_{L1}$	0	+39.1	−5.4	+31.7	+12.4				
$-M_{L1}$	0	−10.6	−48.2	−18.5	−43.0				
M_D	0	+28.5	−42.8	+13.2	−30.6				
$+M_{D+L1}$	0	+67.6	−37.4	+44.9	−18.2				
$-M_{D+L1}$	0	+17.9	−91.0	−5.3	−73.6				
$+M_{L2}$	0	+78.2	+10.8	+63.4	+24.8				
$-M_{L2}$	0	−21.2	−96.4	−37.0	−84.0				
						Symmetrical about center line.			
$+M_{D+L2}$	0	+106.7	−32.0	+76.6	−5.8				
$-M_{D+L2}$	0	−7.3	−139.2	−23.8	−114.0				

TABLE 2.—(*Continued.*)

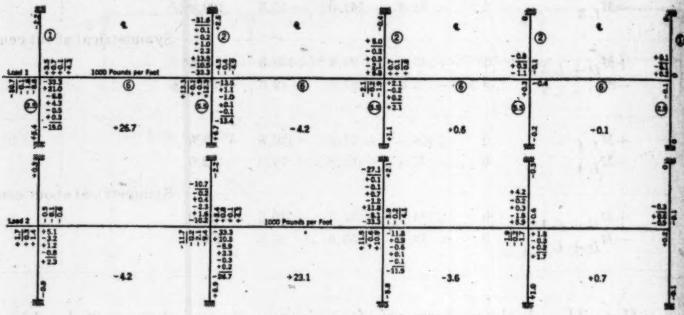
$+M_{L3}$	0	+117.8	+ 16.2	+ 95.1	+ 37.2	
$-M_{L3}$	0	− 31.8	−144.6	− 55.5	−129.0	
						Symmetrical about center line.
$+M_{D+L3}$	0	+145.8	− 24.6	+108.3	+ 6.6	
$-M_{D+L3}$	0	− 3.3	−187.4	− 42.3	−159.6	
$+M_{L4}$	0	+156.4	+ 21.6	+126.8	+ 49.6	
$-M_{L4}$	0	− 42.4	−192.8	− 74.0	−172.0	
						Symmetrical about center line.
$+M_{D+L4}$	0	+184.9	− 21.2	+140.0	− 19.0	
$-M_{D+L4}$	0	− 13.9	−253.6	− 60.8	−202.0	

Case II.—A three-story reinforced concrete warehouse, 80 by 160 ft. by 42 ft. high, with columns on 20-ft. centers each way, is used as the structure for analysis. (See Table 3.) Sections were obtained (designed) for a $\frac{L}{D}$ ratio of 2.5. Moments were computed for a second-story four-span beam restrained by columns both above and below. Moments of inertia, I, were computed, using the entire concrete section, but ignoring the steel reinforcement. A width of slab equal to one-fourth the span length was used as the T for the beams and was considered in the moment of inertia. From the $\frac{I}{L}$ values, the relative stiffness, shown at the middle of each member, was computed. These same values were used for all ratios of $\frac{L}{D}$, because it is easily possible to make a design for each loading without altering the relative stiffness of each member.

To obtain the curves of maximum positive and negative moment (Fig. 21 (*b*)), the same procedure used for Case I is followed. Maximum column moments are not shown in the final summation, but can easily be obtained by combining the values shown.

Case II is a good example of the form of computations recommended by the author. The method of "Moment Distribution" is admirably adapted to a simple and easily checked arrangement of figures. There is some difference of opinion as to the most desirable sign convention to be used. The writer made a somewhat detailed study of this question, comparing different conventions, and concluded that the one used by the author was to be preferred. It not only harmonizes with the usual practice of defining positive and negative moment in a beam, but, in complicated frames, there seems to be no chance of an error in sign remaining undetected when moments are distributed and the equilibrium of each joint is checked. The important factor, however, is that the method is just as workable, regardless of what sign convention is dictated by personal preference.

TABLE 3.—COMPUTATION OF MAXIMUM MOMENTS IN A THREE-STORY REINFORCED CONCRETE WAREHOUSE.

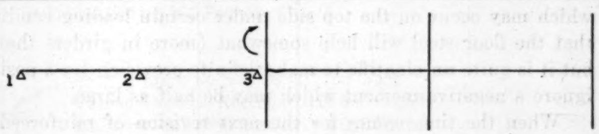


Load 3.							1 000 lb. per ft.					
Moments.	− 0.3	+ 0.7	+ 1.7	+ 4.2	− 3.6	− 11.5	−27.1	+23.1	−26.7	−10.7	− 4.2	+ 2.3
Load 4.										1 000 lb. per ft.		
Moments.	+ 0.1	− 0.1	− 0.4	− 0.8	+ 0.6	+ 2.1	+ 5.0	− 4.2	−13.4	−31.6	+26.7	−15.0
$+M_{L\,1}$	+ 2.4	+ 27.4	+ 1.7	+ 4.2	+ 23.7	+ 7.1						
$-M_{L\,1}$	−15.3	− 4.3	− 42.7	− 40.9	− 7.8	− 38.6						
M_D	−12.9	+ 23.1	− 41.0	− 36.7	+ 15.9	− 31.5						
$+M_{D+L\,1}$	−10.5	+ 50.5	− 39.3	− 32.5	+ 39.6	− 24.4	Symmetrical about center line.					
$-M_{D+L\,1}$	−28.2	+ 18.8	− 83.7	− 77.6	+ 8.1	− 70.1						
$+M_{L\,2}$	+ 4.8	+ 54.8	+ 3.4	+ 8.4	+ 47.4	+ 14.2						
$-M_{L\,2}$	−30.6	− 8.6	− 85.4	− 81.8	− 15.6	− 77.2						
$+M_{D+L\,2}$	− 8.1	+ 77.9	− 37.6	− 28.3	+ 63.3	− 17.3	Symmetrical about center line.					
$-M_{D+L\,2}$	−43.5	+ 14.5	−126.4	−118.5	+ 0.3	−108.7						
$+M_{L\,3}$	+ 7.2	+ 82.2	+ 5.1	+ 12.6	+ 71.1	+ 21.3	Symmetrical about center line.					
$-M_{L\,3}$	−45.9	− 12.9	−128.1	−122.7	− 23.4	−115.8						
$+M_{D+L\,3}$	− 5.7	+105.3	− 35.9	− 24.1	+ 87.0	− 10.2						
$-M_{D+L\,3}$	−58.8	+ 10.2	−169.1	−159.4	− 7.5	−147.3						
$+M_{L\,4}$	+ 9.6	+109.6	+ 6.8	+ 16.8	+ 94.8	+ 28.4	Symmetrical about center line.					
$-M_{L\,4}$	−61.2	− 17.2	−170.8	−163.6	− 31.2	−154.4						
$+M_{D+L\,4}$	− 8.8	+132.7	− 34.2	− 19.9	+110.7	− 8.1						
$-M_{D+L\,4}$	−74.1	+ 5.9	−211.8	−200.3	− 15.3	−185.9						

Results of Computations.—This discussion is submitted to show primarily the application of the author's method of analysis to a specific problem. It would not be complete, however, unless some pertinent conclusions about the particular problem investigated were included.

The correct values for the maximum moments at critical points are compared with the values obtained, using the conventional moment coefficients found in the design codes for reinforced concrete. (See Table 4.) The coefficients in Article 107 of the report of the Joint Committee on Standard Specifications for Concrete and Reinforced Concrete (1924)[23] are used for Table 4 (*a*) and the coefficients in Article 110 (*a*) and (*b*) are used for Case II, Table 4 (*b*). The length of span used in Table 4 (*a*) for code values is the full length, that is, 20 ft. In Table 4 (*b*), in which columns restrain the beams, code values are computed for a clear span of 18.5 ft. Therefore, in order to make a fair comparison, the correct moments at the face of the columns have been computed and tabulated. That explains the difference in values at the supports shown in Table 4 (*b*) and in Case II. The computations, Case II, are for the center line of the support.

TABLE 4.—COMPARISON OF CORRECT MAXIMUM MOMENTS WITH THOSE DETERMINED BY THE USE OF CONVENTIONAL MOMENT COEFFICIENTS.



		(*a*) No restraint.					(*b*) Column restraint.					
$\frac{L}{D} = 1$	Correct *M*.	0	+ 67.6	− 91.0	+ 44.9	− 73.6	− 15.4	+ 50.5	− 67.1	− 62.2	+ 39.6	− 55.3
	Code *M*.	0	+ 80.0	− 80.0	+ 66.6	− 66.6	− 57.0	+ 57.0	− 57.0		+ 42.8	− 57.0
	Error, percentage.	0	+ 18.3	− 12.1	+ 32.5	− 9.5	+270.0	+ 12.9	− 15.0	− 8.4	+ 8.1	+ 3.1
$\frac{L}{D} = 2$	Correct *M*.	0	+106.7	−139.2	+ 76.6	−114.0	− 24.0	+ 77.9	−101.5	− 95.1	+ 63.3	− 86.1
	Code *M*.	0	+120.0	−120.0	+100.0	−100.0	− 85.5	+ 85.5	− 85.5		+ 64.2	− 85.5
	Error, percentage.	0	+ 12.5	− 13.8	+ 30.6	− 12.3	+257.0	+ 9.8	− 15.8	− 10.1	+ 1.4	− 0.7
$\frac{L}{D} = 3$	Correct *M*.	0	+145.8	−187.4	+108.3	−159.6	− 32.6	+105.3	−135.9	−128.0	+ 87.0	−116.9
	Code *M*.	0	+160.0	−160.0	+133.3	−133.3	−114.0	+114.0	−114.0		+ 85.6	−114.0
	Error, percentage.	0	+ 9.7	− 14.6	+ 23.1	− 16.5	+249.0	+ 8.3	− 16.1	− 11.0	− 1.6	− 2.5
$\frac{L}{D} = 4$	Correct *M*.	0	+184.9	−235.6	+140.0	−203.6	− 41.2	+ 32.7	−170.3	−160.9	+110.7	−147.7
	Code *M*.	0	+200.0	−200.0	+166.6	−166.6	−142.5	+142.5	−142.5		+107.0	−142.5
	Error, percentage.	0	+ 8.2	− 15.1	+ 19.0	− 17.8	+246.0	+ 7.4	− 16.5	− 11.4	− 3.3	− 3.5

Figures showing the percentage of error in code moments should interest many designers. For both unrestrained and restrained beams, code coefficients give values too high for positive moment in the centers of the spans and values too low for negative moment at the supports. The difference is enough to warrant a revision in all the coefficients applied to continuous systems with little or no restraint, such as that of Case I. In systems like that of Case II, coefficients for the interior spans are sufficiently accurate for practical purposes. The coefficients for the end spans and particularly the end supports are decidedly in error, however, and should be revised.

---

[23] *Proceedings*, Am. Soc. C. E., October, 1924, Papers and Discussions. p. 1153 *et seq.*

The design codes make no reference whatever to the existence of negative moment at the center of spans. (See Fig. 21.) Even for a $\frac{L}{D}$ ratio as small as unity, negative moment may occur over the full length of interior spans subject to little end restraint. Consequently, reinforcing steel is needed at the top of the beam over its full length. Note the values for a $\frac{L}{D}$ ratio of 4. The maximum negative moment at the center of the second span is 60.8. The maximum positive moment at the same point is 140.0. Most designers will place steel in the bottom but not in the top, disregarding the fact that the top needs 43% of the total used in the bottom. Even for spans subject to column restraint, the negative moment is large enough over the greater part of the interior spans to justify a decided change in current design practice. Some will argue that there is no need for worrying about this because the floor steel in the slab will assist the beam in providing resistance to the tension which may occur on the top side under certain loading conditions. It is true that the floor steel will help somewhat (more in girders than in the beams), but it is quite unscientific to make definite provision for a positive moment and ignore a negative moment which may be half as large.

When the time comes for the next revision of reinforced concrete design codes, the writer hopes that the responsible authorities will omit all values for moment coefficients, thus placing the function of analysis entirely upon the shoulders of the designer. This will not only encourage the engineer to develop more self-reliance and initiative, but it will pay deserved premiums to the man with the technical ability to design structures intelligently. It must be remembered that too many designers will accept a "half-complete" code as the authority for a complete design. "Their's not to question why * * *". However, if the responsible authorities still insist that there must be more rules for the game, then it is advisable to define explicitly those limited cases to which the rules apply. It will impose some check on the designer who is inclined to take too much latitude.

With the author's method, maximum moments and shears for any design subject to any condition of uniform or concentrated loads are so readily computed that there is little need for using arbitrary coefficients in any case. Although the preceding discussion revolves chiefly around equal span systems subject to uniform loads, systems with equal or unequal spans subject to concentrated loads are just as easily treated.

Frames Subject to Lateral Displacement.—The writer also wishes to illustrate the facility of applying the author's method to a problem usually considered quite complex, that is, that of a mill-building bent, with a stepped column subject to a lateral thrust from a traveling crane. The problem is complicated by the variation in moment of inertia of the column due to the different sections used above and below the crane girder seat. (See Fig. 22.) The moments in the columns are wanted, due to the effect of a lateral concentrated load of 6 000 lb. on each column; this is the horizontal thrust of the crane, acting 27 ft. above the base. The moments are first found for the horizontal load acting on one column. Since there is a similar load acting on the other column, the final moments are obtained by combining those due to a single load on opposite hand bents. Vertical loads and wind loads are ignored in this illustration. Moments due to such loads would be computed in a similar manner. For a complete design, they would naturally have to be considered.

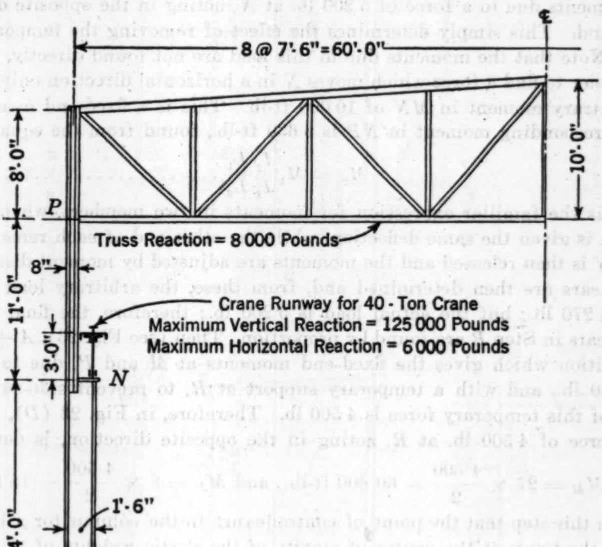

℄
8 @ 7'-6"=60'-0"
10'-0"
8'-0"
P
Truss Reaction = 8 000 Pounds
8"
11'-0"
Crane Runway for 40-Ton Crane
Maximum Vertical Reaction = 125 000 Pounds
Maximum Horizontal Reaction = 6 000 Pounds
3'-0"
N
1'-6"
24'-0"
M

FIG. 22.—HALF SKETCH OF A MILL-BUILDING BENT WITH A STEPPED COLUMN.

The base of the column is considered fixed. The ratio of the stiffness of the truss to the stiffness of the column is more than 10. This makes the upper end of the column practically fixed, so that in the following analysis the $\frac{I}{L}$ value for the truss is taken as infinity. The final moments are consequently slightly in error. If the exact moments are wanted, they may be found by using the correct stiffness factor for the truss and releasing Joints P and R (Fig. 23) after the fixed-end moments are found. The correction, however, is too small to have practical significance.

In the first operation, Fig. 23 (*A*), Column *MNP* is temporarily supported at *N*, where the change in column section occurs, and the moments on the resulting two-span continuous beam are found due to the load of 6 000 lb. Reactions at *M, N,* and *P* are found; that at *N* is 5 200 lb. In Fig. 23 (*B*), the moments due to a force of 5 200 lb. at *N*, acting in the opposite direction, are found. This simply determines the effect of removing the temporary support. Note that the moments due to this load are not found directly, however. It is easier to find a force which moves *N* in a horizontal direction only, causing an arbitrary moment in *MN* of 10 000 ft-lb. This is a fixed-end moment and the corresponding moment in *NP* is 3 630 ft-lb., found from the equation,

$$M_2 = M_1 \frac{I_2 L_1^2}{I_1 L_2^2} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(9)}$$

which is the familiar expression for moments in two members, when one end of each is given the same deflection, while the other end of each remains fixed. Joint *N* is then released and the moments are adjusted by moment distribution. The shears are then determined and, from these, the arbitrary load is found to be 1 270 lb.; but the actual load is 5 200 lb.; therefore, the final moments and shears in Step *B* are found by proportion. Then (see Fig. 23), $A + B = C$, a condition which gives the fixed-end moments at *M* and *P*, due to the load of 6 000 lb., and with a temporary support at *R*, to prevent side-sway. The value of this temporary force is 4 500 lb. Therefore, in Fig. 23 (*D*), the effect of a force of 4 500 lb. at *R*, acting in the opposite direction, is determined.

Thus, $M_M = 27 \times \frac{4\,500}{2} = 60\,800$ ft-lb., and $M_P = 8 \times \frac{4\,500}{2} = 18\,000$ ft-lb. Note in this step that the point of contraflexure in the column for a horizontal load at the top is at the center of gravity of the elastic weights of the column.

From the point, *P*, the distance down to the point of contraflexure is $\left(\frac{1}{6} \times 23 + 5.5\right) \div \frac{7}{6} = 8.0$ ft. The total elastic weight is $\frac{\Sigma L}{E I}$. This location is correct only when there is pure translation of the top and no rotation. The top member, *P R*, must be infinitely stiff to make this possible. A slight correction may be made inasmuch as *P R* is not infinitely stiff, but the correction is so small that it can be ignored.

In the next step, $C + D = E_R$, which gives the moments due to one of the 6 000-lb. loads. To account for the other load of 6 000 lb. on the other column, the moments shown in E_R are added to opposite-hand moments to obtain the final values in Fig. 23(*F*).

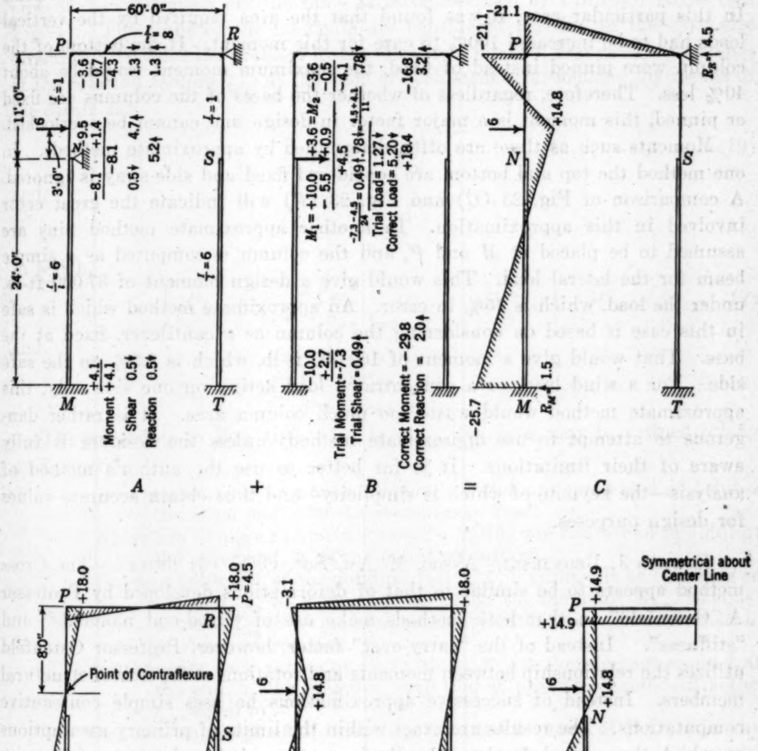

60'-0"
11'-0"
3'-0"
24'-0"
P
R
S
M
T
N
I/L = 1
I/L = 8
I/L = 6
-3.6
-0.7
-4.3
1.3
1.3
-9.5
+1.4
-8.1
-8.1
0.5
0.5
4.7
5.2
+4.1
+4.1
0.5
0.5
Moment
Shear
Reaction
A
+
B
=
C
M₁ = +10.0
-5.5
+4.5
+3.6 = M₂
+0.9
-3.6
-0.5
-4.1
.78
-16.8
3.2
Trial Load = 1.27
Correct Load = 5.20
+18.4
-10.0
+2.7
Trial Moment = -7.3
Trial Shear = 0.49
Correct Moment = -29.8
Correct Reaction = 2.0
-21.1
21.1
+14.8
-25.7
R_M 1.5
R_R 4.5
6
+18.0
8.0"
Point of Contraflexure
+18.0
P = 4.5
-3.1
+18.0
+14.8
-60.8
-60.8
-86.5
-60.8
-147.3
Symmetrical about Center Line
+14.9
+14.9
+14.8
D
E_R + C = D
E_R + E_L = F

FIG. 23.—MOMENTS IN A MILL-BUILDING BENT DUE TO LATERAL CRANE THRUST, IN THOUSANDS OF FOOT-POUND UNITS.

This problem not only illustrates the application of the Cross method to frames subject to side-sway, but, at the same time, points out some interesting conclusions about mill-building bents of this type.

Moments due to crane thrust demand large increases in column sections. In this particular case, it was found that the area required by the vertical loads had to be increased 100% to care for this moment. If the bottom of the column were pinned instead of fixed, the maximum moment would be about 40% less. Therefore, regardless of whether the bases of the columns are fixed or pinned, this moment is a major factor in design and cannot be overlooked.

Moments such as these are often determined by approximate methods. In one method the top and bottom are considered fixed and side-sway is ignored. A comparison of Fig. 23 (*C*) and Fig. 23 (*F*) will indicate the great error involved in this approximation. In another approximate method pins are assumed to be placed at *M* and *P*, and the column is computed as a simple beam for the lateral load. This would give a design moment of 37 000 ft-lb. under the load, which is 75% in error. An approximate method which is safe in this case is based on considering the column as a cantilever, fixed at the base. That would give a moment of 162 000 ft-lb. which is 9.5% on the safe side. For a wind load, or a concentrated load acting on one side only, this approximate method would waste too much column area. It is rather dangerous to attempt to use approximate methods unless the designer is fully aware of their limitations. It is far better to use the author's method of analysis—the keynote of which is simplicity—and thus obtain accurate values for design purposes.

EDWARD J. BEDNARSKI,[24] ASSOC. M. AM. SOC. C. E. (by letter).—The Cross method appears to be similar to that of deformations developed by Professor A. Ostenfeld[25] in that both methods make use of "fixed-end moments" and "stiffness". Instead of the "carry-over" factor, however, Professor Ostenfeld utilizes the relationship between moments and rotations at the ends of structural members. Instead of successive approximations he uses simple consecutive computations. The results are exact within the limits of primary assumptions on which the method developed by Professor Cross is based. It is of interest, therefore, to compare the actual work required to solve the same problem by both methods.

The writer has computed the problem shown in Fig. 1 by the Ostenfeld method, and finds that the computation and checking require less time than is needed by the Cross method. Besides, there is no confusion whatever regarding the signs of the moments because they are entirely independent of the position of the structural member under consideration.

The Ostenfeld method is not yet widely known in this country. It provides an adequate solution of the stresses in a statically indeterminate structure under unsymmetrical load, and subject to "side-sway". It can also be applied to continuous beams on tall columns as well as to multi-storied bents and multiple arches on high piers. The computation required by this method is outlined herein, as applied to the problem under discussion.

---

[24] Civ. Engr., Sacramento, Calif.

[25] "Deformations Methode," by A. Ostenfeld, Copenhagen, 1926.

The stiffness factors, μ, and the fixed-ended moments, M^0, to be used in the present demonstration are the same as those assumed by Professor Cross (see Fig. 1). The signs of the moments are considered positive in the clockwise direction. To solve the problem by the Ostenfeld method, first determine values of the rotation factors, ζ_{oa}, at the ends of each structural member, by means of the formulas:

$$\zeta_{oa} = \frac{Z_{oa}}{Z_{aa}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (10)$$

and,

$$\tau_a = \frac{1}{Z_{aa}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (11)$$

in which,

$$Z_{oa} = -\left[M^0{}_{am} + \frac{2\,\mu_{am}}{1 + 4\,\mu_{am}\,\tau_m}(\zeta_{om} - \tau_m\,M^0{}_{ma})\right] - M^0{}_{ai} - M^0{}_{ak} \,.. \quad (12)$$

and,

$$Z_{aa} = 4\,\mu_{am}\,\frac{1 + 3\,\mu_{am}\,\tau'_m}{1 + 4\,\mu_{am}\,\tau_m} + 4\,\mu_{ai} + 3\,\mu_{ak} \quad \ldots\ldots\ldots\ldots \quad (13)$$

The subscripts in Equations (10) to (13), inclusive, may be defined briefly as follows:[26]

$a\,i$ refers to one or more members, $a\,i$, that are connected at Joint a the other end, i, of the members is fixed.

$a\,k$ refers to one or more members, $a\,k$, that are connected to Joint a; the other end, k, of the members is fixed.

$o\,a$, as a subscript of the rotation factor, ζ, identifies it as referring to the end, a, of a member at which the moment, M_{am}, is equal to zero; Z_{oa} is the moment necessary at the end, a, of a member, to produce the rotation, ζ_{oa}.

$a\,a$ appears as a subscript to Z; thus, Z_{aa} is the moment applied at the end, a, of a member, such that it produces a rotation factor, ζ_{aa}, equal to -1.

$a\,m$, as a subscript of M, identifies it as the moment at Point a in any beam.

τ_m is the rotation factor at the end, m, of a member, $a\,m$, of such magnitude as to create a moment, M_{am}, equal to 1.

In the example under discussion, the necessary factors for the various beams and columns are found to be:

For Column $B\,A$: $B = a$ and $A = m$ in Equation (12) and Equation (13).

$Z_{OB} = 0$; $Z_{BB} = 8$; $\zeta = 0$; and $\tau_B = \dfrac{1}{Z_{BB}} = \dfrac{1}{8}$.

For Joint C: $C = a$, $B = m$, $G = k$, and $F = i$, in Equation (12) and Equation (13); hence,

$$Z_{OC} = -100 + \frac{2 \times 4}{1 + 4 \times 4 \times \frac{1}{8}} \times \frac{1}{8} \times 100 - 50 - 110 = -\frac{880}{3}$$

---

[26] See also "Deformations Methode," by A. Ostenfeld, Copenhagen, 1926.

$$Z_{CC} = 4 \times 4 \times \frac{1 + 3 \times 4 \times \frac{1}{2}}{1 + 4 \times 4 \times \frac{1}{8}} + 4 \times 1 + 3 \times 2 = \frac{70}{3}$$

$$\zeta_{CO} = \frac{88}{7}$$

and,

$$\tau_C = \frac{1}{Z_{CC}} = \frac{3}{70}$$

For Joint D: $D = a$ and $C = m$ in Equation (12) and Equation (13); $Z_{OD} = -\frac{1020}{13}$; $Z_{DD} = \frac{230}{13}$; $\zeta_{OD} = -\frac{102}{23}$; and $\tau_D = \frac{13}{230}$.

For Joint E: $E = a$ and $D = m$ in Equation (12) and Equation (13); $Z_{OE} = \frac{3060}{193}$; $Z_{EE} = \frac{2082}{193}$; $\zeta_{OE} = \frac{510}{347}$; and $\tau_E = \frac{193}{2082}$.

The actual rotation factors, ζ_m, are finally determined for each joint in succession by the formula:

$$\zeta_m = \frac{1}{1 + 4\,\mu_{am}\,\tau_m}\left(\zeta_{om} - 2\,\mu_{am}\,\tau_m\,\zeta_a - \tau_m\,M^0{}_{ma}\right) \ldots\ldots\ldots (14)$$

The actual values for the problem in Fig. 1 are thus found to be: $\zeta_E = \frac{2495}{1041}$; $\zeta_D = -\frac{1085}{347}$; $\zeta_C = -\frac{497}{347}$; $\zeta_B = \frac{3228}{694}$; $\zeta_P = \frac{1}{2}\,\zeta_C = \frac{497}{694}$; and $\zeta_A = \zeta_B = 0$.

The final moments are determined by means of the equations:

$$M_{am} = 2\,\mu_{am}\,(2\,\zeta_a + \zeta_m) + M^0{}_{am} \ldots\ldots\ldots\ldots\ldots\ldots (15)$$

and,

$$M_{ma} = 2\,\mu_{am}\,(2\,\zeta_m + \zeta_a) + M^0{}_{ma} \ldots\ldots\ldots\ldots\ldots\ldots (16)$$

in which, M^0, denotes the original fixed-end moment at the end of the beam indicated. The values of M, thus determined are, respectively,

$$M_{AB} = 18\,\frac{200}{347} = 18.576$$

$$M_{BA} = 2\,M_{AB} = 37.152$$

$$M_{BC} \text{ (computed independently)} = -M_{BA} = -37.152$$

$$M_{CB} = 114\,\frac{82}{347} = 114.236$$

$$M_{CF} = 101\,\frac{141}{347} = 101.406$$

$$M_{CD} = -259\,\frac{317}{347} = -259.913$$

$$M_{CG} = 44\,\frac{94}{347} = 44.271$$

$$M_{DC} = 23\,\frac{49}{347} = 23.141$$

$$M_{DE} \text{ (computed independently as a check)} = -M_{DC} = -23.141$$

$$M_{ED} = \frac{3470}{347} = 10$$

and,

$$M_F = -M_{ED} = -10$$

It should be noted as a check that the sum of all moments around Point *C* equals zero.

S. N. Mitra,[27] Jun. Am. Soc. C. E. (by letter).—Despite some limitations the method described in this paper is quite unique and satisfactory. In the writer's opinion, however, it has the decided disadvantage that it is not conveniently reducible to a general equation that can be used when the principles themselves have been mastered.

Furthermore, the writer believes that, for less complicated problems than the one the author has illustrated, such as those which occur in practice, other methods of analysis, as, for example, slope deflection and the graphical method of conjugate points, will be found more accurate, and quicker and easier to use.

Jens Egede Nielsen,[28] Assoc. M. Am. Soc. C. E. (by letter).—The large number of papers dealing with moment distribution in continuous beams and rigid frames is proof enough that this question needs a practical solution.

The method, as outlined by Professor Cross, is so easily memorized and so quickly applied that all approximate methods and guesswork profitably can be eliminated without the sacrifice of speed, which often is as essential as economy.

When balancing the moments at a point such as *C*, Fig. 1, in a system in which the signs are quite arbitrarily considered positive when causing sags downward in horizontal members and toward the right side of vertical members, it is not entirely evident which is the right and which the left side of the joint or, rather, which moments should be added and which should be subtracted in order to determine the unbalanced moment. If, however, it is borne in mind that moments in members which sag clockwise should balance moments in members which sag counter-clockwise, it is easy to visualize which moments should be added and which subtracted. Thus, in Fig. 24, — 200, — 50, and — 100, which members sag counter-clockwise, should be subtracted from — 400, which sags clockwise, giving an unbalanced moment of $-400 - (-200 - 50 - 100) = -50$ to distribute among the four members.

The determination of the fixed-end moments for different loadings can be greatly facilitated by the use of influence lines, such as those given in textbooks, one for concentrated loads and an integrated curve for uniformly distributed loads.

The limitations of the method should be realized at once. Systems in which shears are dominant (wind-bracing bents in tall buildings) and systems in which the longitudinal stresses are dominant (arches, etc.) and, particularly, systems in which moving loads are encountered, the time saving seems problematic, and other methods should be considered.

---

[27] Calcutta, India.

[28] Chicago, Ill.

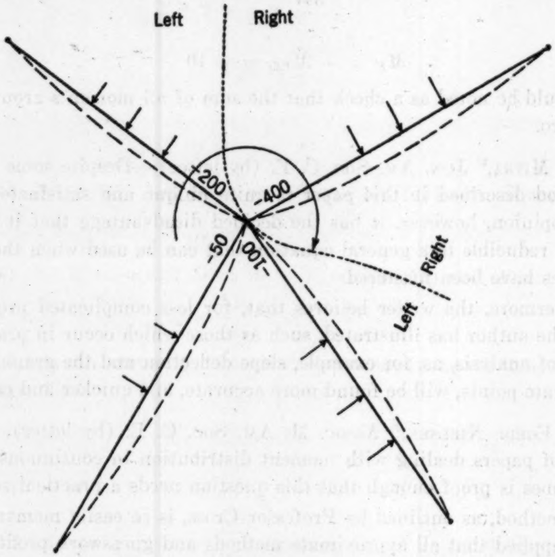


FIG. 24.

The paper is most valuable and should prove helpful to practicing structural engineers.

F. E. RICHART,[29] M. AM. SOC. C. E. (by letter).—The moment-distribution method of analyzing continuous frames originated by Professor Cross is one of the most important developments in this field in many years. The main advantage of the method is its simplicity, through which the designer can attain familiarity with a class of structures generally associated with long algebraic analyses. With this simplicity also comes rapidity of computation, which makes it possible for a structure to be analyzed under many different load conditions in the time previously taken for a single solution. In spite of its advantages, however, it will probably take designers in general a number of years to appreciate the method and to give it the widespread recognition that it deserves.

Because of a disinclination to confuse the reader's conception of the moment-distribution method, the writer hesitates to describe a variation which is nearly as simple in many cases and which leads directly to an exact solution. The scheme is based on an analysis by the slope-deflection method of various typical arrangements of flexural members, for which equations of a standard form may be written.

---

[29] Research Associate Prof., Theoretical and Applied Mechanics, Univ. of Illinois, Urbana, Ill.

In Fig. 25, the member, AB, meets the members, BC and BD, in a rigid joint at B. When a couple, M_{AB}, is applied at A (assuming there is no translation of the points, A, B, C, and D), the end, B, is partly restrained. The amount of restraint depends upon the stiffness, as measured by $K = \frac{I}{l}$ for each of the three members, and by the restraints at the points, C and D. In general, the moment, M_{AB}, is given by the equation,

$$M_{AB} = N_1 \, E \, K_1 \, \theta_A \ldots\ldots\ldots\ldots\ldots\ldots\ldots (17)$$

in which, N_1 is a constant, having a value between 3 and 4; E is the modulus of elasticity of the material; K_1 is $\frac{I}{l}$ for the member, AB; and θ_A is the slope of the elastic curve at A, due to the action of the couple.

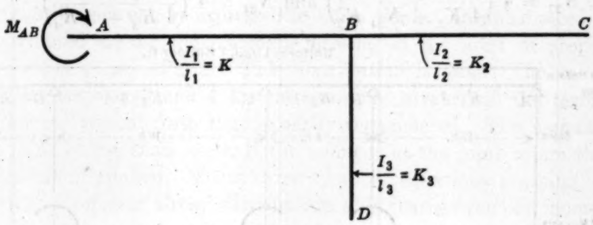


FIG. 25.

The value, N_1, may be written,

$$N_1 = 4 \left(\frac{3\,K_1 + N_2\,K_2 + N_3\,K_3}{4\,K_1 + N_2\,K_2 + N_3\,K_3} \right) \ldots\ldots\ldots\ldots\ldots (18)$$

N_2 and N_3 are constants similar to N_1; thus, if there is a hinge at C, $N_2 = 3$; if a fixed end, $N_2 = 4$; if the slopes at B and C are equal and opposite, $N_2 = 2$; and if the slopes are equal and alike, $N_2 = 6$. It will be noted that Equation (18) has terms in numerator and denominator which are alike except for the coefficients of K_1, which are always 3 and 4, respectively.

Another relation between the moments at A and B is as follows:

$$M_{BA} = \frac{M_{AB}}{2} \left(\frac{N_2\,K_2 + N_3\,K_3}{N_2\,K_2 + N_3\,K_3 + 3\,K_1} \right) \ldots\ldots\ldots\ldots (19)$$

The moment, M_{BA}, is equal and opposite to the sum of M_{BC} and M_{BD}, and the latter moments are proportional to the respective values of $N_2 K_2$ and $N_3 K_3$.

The application of Equations (17), (18), and (19), to a continuous beam or frame may be started just as in Professor Cross' procedure. For a given loading, the "fixed-beam" moments are found and the unbalanced moment is determined at each joint. In Fig. 26, the values of K for the four spans are denoted by K_1, K_2, K_3, and K_4, and the unbalanced moments at B, C, and D, by U_B, U_C, and U_D. An exact method of distributing these unbalanced moments throughout the beam has been published.[30]

---

[30] "Analysis of Statically Indeterminate Structures by the Slope-Deflection Method," by Wilson, Richart, and Weiss, *Bulletin 108*, Eng. Experiment Station, Univ. of Illinois, 1918, pp. 133 and 137.

Consider the unbalanced moment, U_B, at B. This is distributed between the members, BA and BC, as follows:

$$M_{BA} = U_B \left(\frac{3\,K_1}{3\,K_1 + N_{23}\,K_2}\right) \dots\dots\dots\dots\dots\dots\dots (20)$$

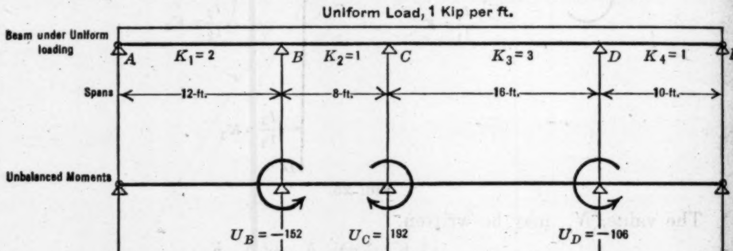
$$M_{BC} = U_B \left(\frac{N_{23}\,K_2}{3\,K_1 + N_{23}\,K_2}\right) \dots\dots\dots\dots\dots\dots\dots (21)$$

$$M_{CB} = \frac{M_{BC}}{2}\left(\frac{N_{34}\,K_3}{N_{34}\,K_3 + 3\,K_2}\right) = -\,M_{CD} \dots\dots (22)$$

and,

$$M_{DC} = \frac{M_{CD}}{2}\left(\frac{3\,K_4}{3\,K_4 + 3\,K_3}\right) \dots\dots\dots\dots\dots\dots\dots (23)$$

in which, $N_{23} = 4\left(\frac{3\,K_2 + N_{34}\,K_3}{4\,K_2 + N_{34}\,K_3}\right)$ and $N_{34} = 4\left(\frac{3\,K_3 + 3\,K_4}{4\,K_3 + 3\,K_4}\right)$.



Fixed Beam Moments with hinges at A and E	0	+ 216	−64	+ 64	−256	+ 256	−150	0
Couple, $U_B = -152$, at B	0	−94.2	−57.8	−22.0	+22.0	+2.2	−2.2	0
Couple, $U_C = 182$ at C	0	−17.5	+17.5	+ 52.4	+139.6	+17.5	−17.5	0
Couple, $U_D = -106$, at D	0	−3.9	+3.9	+11.6	−11.6	−81.0	−25.0	0
Resultant Moment	0	+ 100.4	−100.4	+106.0	−106.0	+194.7	−194.7	0

Note:- Plus sign indicates a clockwise couple acting upon member at a joint; negative sign a counter clockwise couple.

FIG. 26.

Similarly, at C, the unbalanced moment, U_C is distributed, as follows:

$$M_{CB} = U_C \left(\frac{N_{21}\,K_2}{N_{21}\,K_2 + N_{34}\,K_3}\right) \dots\dots\dots\dots\dots\dots (24)$$

$$M_{CD} = U_C \left(\frac{N_{34}\,K_3}{N_{21}\,K_2 + N_{34}\,K_3}\right) \dots\dots\dots\dots\dots\dots (25)$$

$$M_{BC} = \frac{M_{CB}}{2}\left(\frac{3\,K_1}{3\,K_1 + 3\,K_2}\right) = -\,M_{BA} \dots\dots\dots (26)$$

and,

$$M_{DC} = \frac{M_{CD}}{2}\left(\frac{3\,K_4}{3\,K_4 + 3\,K_3}\right) = -\,M_{DE} \dots\dots\dots (27)$$

in which, $N_{21} = 4\left(\frac{3\,K_2 + 3\,K_1}{4\,K_2 + 3\,K_1}\right)$.

At D, the unbalanced moment, U_D, is distributed, as follows:

$$M_{DC} = U_D \left(\frac{N_{32}\,K_3}{N_{32}\,K_3 + 3\,K_4} \right) \dots\dots\dots\dots\dots\dots (28)$$

and,

$$M_{DE} = U_D \left(\frac{3\,K_4}{N_{32}\,K_3 + 3\,K_4} \right) \dots\dots\dots\dots\dots\dots (29)$$

$$M_{CD} = \frac{M^{DC}}{2} \left(\frac{N_{21}\,K_2}{N_{21}\,K_2 + 3\,K_3} \right) = -M_{CB} \dots\dots\dots\dots (30)$$

in which, $N_{32} = 4 \left(\frac{3\,K_3 + N_{21}\,K_2}{4\,K_3 + N_{21}\,K_2} \right)$.

The first of the two subscripts used with N indicates the span nearest the origin of the moment being distributed.

Equations (17) to (30), inclusive, are given in full for further reference, but their significance can be expressed in a few words. An unbalanced moment, U, is distributed among the members meeting at the joint in proportion to their respective values of NK. This distribution is final. The value of the term, N, lies between 3 and 4 for the examples given, and the terms of the fraction have a typical form that is easily remembered. The moment at the adjacent joint is less than one-half the moment at the joint where the unbalanced moment is applied. These three types of equations are similar to the operations in Professor Cross' distribution and "carry-over" of moments, but the equations are applied only once to determine an exact final distribution of moments. However, it is necessary to distribute each unbalanced moment in turn, so that the final moments are the algebraic sum of the original "fixed-end" moments and the moments due to the three unbalanced couples.

As a numerical example, consider the four-span beam of Fig. 26 under a uniform load. With the ends, A and E, hinged, it is convenient to apply moments at B and D on the first and fourth spans, which apply to the case having one end hinged and the other fixed. Such "fixed-end" moments are three-halves as great as for the case in which both ends are fixed. The unbalanced moments at B, C, and D, are found to be —152, +192, and —106 units, and values of K are assumed at 2, 1, 3, and 1, respectively, for the four spans. The unbalanced moments are applied as couples producing a rotation in the direction indicated by the unbalance.

The values of the constants, N, are found to be $N_{21} = 3.6$; $N_{23} = 3.7$; $N_{32} = 3.2$; and $N_{34} = 3.2$. Using the convention as to signs followed in *Bulletin 108* previously referred to (a clockwise moment or attacking couple at the end of a member is considered positive), the moments at the ends of each span are given in Fig. 26 for the original fixed-end condition at B, C, and D, and for the successive application of the unbalanced moments at these three points. The algebraic sum gives the end moments in each span resulting from the original loading. It is a simple matter to sketch in the positive moments and to calculate shears and reactions by the methods of statics.

While familiarity with a method is likely to prejudice one in favor of it, the writer is inclined to grant that the method of Professor Cross is the simpler of the two to apply. However, both are much less cumbersome than the majority of methods of frame analysis usually encountered in the literature of the subject. It is to be kept in mind that the simplicity of both methods is due in part to the fact that they apply to the type of structure in which there is no displacement of joints or supports.

No originality is claimed by the writer for the idea of distributing the unbalanced moments, but the method of distributing such a moment throughout a frame was given a dozen years ago (1918) in *Bulletin 108*. The writer's method is quite similar to that[31] of H. M. Hadley, Assoc. M. Am. Soc. C. E., except that the procedure given herein for distributing the moment of the unbalanced couples is simpler and more easily followed. The equations given herein will apply to any type of beam or open frame, with various degrees of restraint at the ends of members, as well as for closed frames under conditions of symmetry.

WILLIAM A. OLIVER,[32] ASSOC. M. AM. SOC. C. E. (by letter).—The writer has read this paper and its discussion with much interest. There is a general human tendency to think that best which is one's own. This is as true of knowledge as of some more concrete possession. The structural analyst becomes familiar through use with one of the several methods of determining the moments in a statically indeterminate structure—slope and deflection, area moments, or what not. Being familiar with that method he concludes from first study that the new one is complicated and unwieldy. In this he is mistaken.

The writer does not intend to intimate that he wishes to detract from the value of the older methods. He received his first understanding of statically indeterminate structures from a thorough course in slope and deflection. However, being somewhat familiar with both methods, he realizes the immense saving of time that results with the use of Professor Cross' method. The short time required for the solution of a continuous beam by moment distribution and successive approximations, as compared with the time needed to solve the same beam by the equation of three moments, is surprising, and the first method may be just as accurate as the second.

Furthermore, the number of seemingly complex problems which may be successfully attacked by the method seem limitless—haunched beams (that is, beams continuous over supports with the moment of inertia varying within the span), continuous arches on elastic piers, and many others.

The writer believes that Professor Cross has discovered something which will add immeasurably to the field of structural analysis and design.

R. R. MARTEL,[33] M. AM. SOC. C. E. (by letter).—In his concise presentation, Professor Cross has restricted himself to a single, comprehensive, illustrative example. From this, the careful reader can grasp the two principles involved and apply them to other problems of the same general character without resorting to formulas of any kind. This method is a marked advantage over those others which require substitutions in formulas and subsequent algebraic manipulations.

---

[31] *Transactions*, Am. Soc. C. E., Vol. 93 (1929), p. 1396.

[32] Urbana, Ill.

[33] Prof. of Structural Eng., California Inst. of Technology, Pasadena, Calif.

The author's terse statement of the modifications by which the effect of side-sway, resulting from either unbalanced loading, or due to horizontal forces, may be evaluated, has led some of the discussers to illustrate the application of this by examples.

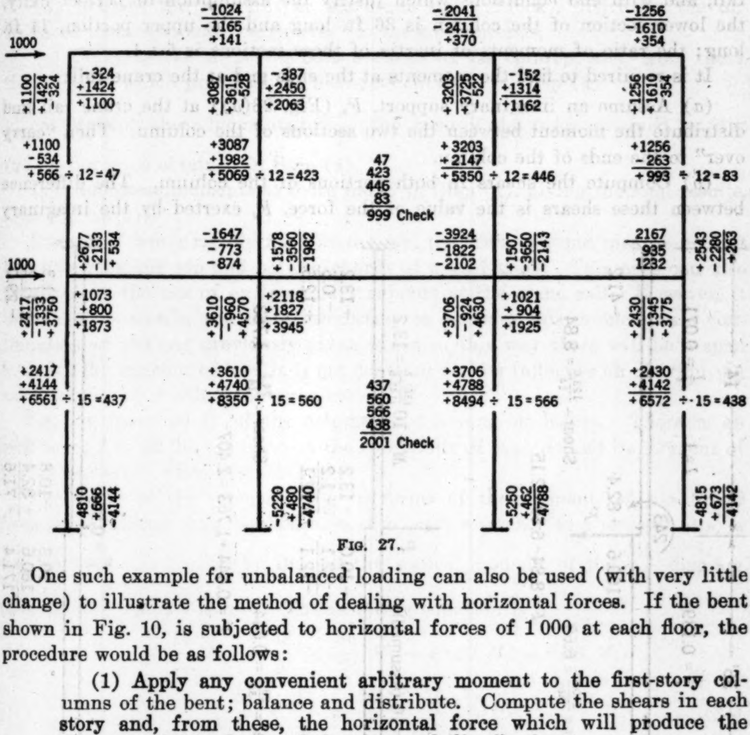

1000
-1024
-1165
+141
-2041
-2411
+370
-1256
-1610
+354
+1100
+1424
-324
-324
+1424
+1100
+3087
+3615
-528
-387
+2450
+2063
+3203
+3725
-522
-152
+1314
+1162
+1256
+1610
-354
+1100
-534
+566 ÷ 12 = 47
+3087
+1982
+5069 ÷ 12 = 423
47
423
446
83
999 Check
+3203
+2147
+5350 ÷ 12 = 446
+1256
-263
+993 ÷ 12 = 83
+2677
-2133
+534
-1647
-773
-874
+1578
-3560
-1982
-3924
-1822
-2102
+1507
-3650
-2143
2167
935
1232
+2543
-2280
+263
1000
+2417
-1333
+3750
+1073
+800
+1873
+3610
-960
+4570
+2118
+1827
+3945
+3706
-924
+4630
+1021
+904
+1925
+2430
-1345
+3775
+2417
+4144
+6561 ÷ 15 = 437
+3610
+4740
+8350 ÷ 15 = 560
437
560
566
438
2001 Check
+3706
+4788
+8494 ÷ 15 = 566
+2430
+4142
+6572 ÷ 15 = 438
-4810
+666
-4144
-5220
+480
-4740
-5250
+462
-4788
-4815
+673
-4142

FIG. 27.

One such example for unbalanced loading can also be used (with very little change) to illustrate the method of dealing with horizontal forces. If the bent shown in Fig. 10, is subjected to horizontal forces of 1 000 at each floor, the procedure would be as follows:

(1) Apply any convenient arbitrary moment to the first-story columns of the bent; balance and distribute. Compute the shears in each story and, from these, the horizontal force which will produce the moments obtained after balancing and distributing.

(2) Apply any convenient arbitrary moments to the second-story columns; balance and distribute. Compute the shears in each story and, from these, the horizontal force which will produce the moments obtained after balancing and distributing.

(3) Write simultaneous equations to find the ratios of the forces obtained in Steps (1) and (2) to the given forces.

(4) Multiply the distributed moments obtained in Steps (1) and (2) by the respective ratios found in Step (3). The algebraic sum of these two sets of moments gives the moments desired. Steps (1) and (2) are demonstrated in Figs 11(*b*) and 11(*c*).

The simultaneous equations of Step (3) would be:

$$+142.3\,C_1 + 295.8\,C_2 = +1\,000 \ldots\ldots\ldots\ldots\ldots (31)$$

$$+560.3\,C_1 + 369.7\,C_2 = -1\,000 \ldots\ldots\ldots\ldots\ldots (32)$$

from which the values of C_1 and C_2 are found to be -5.87 and $+6.20$, respectively. The operations called for in Step (4) are shown in Fig. (27).

A crane column subjected to moment due to the eccentricity of the crane girder, furnishes another simple illustration of the application of the general method with side-sway.

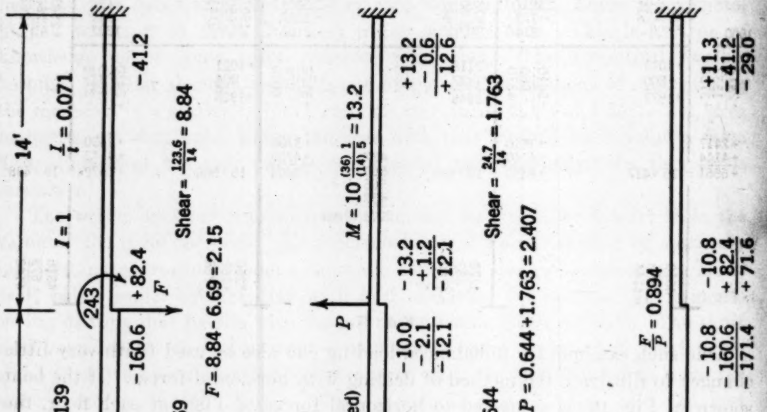
Given such a column subjected to a moment of 243 ft-kips at the crane rail, and with end conditions which justify the assumption of perfect fixity, the lower section of the column is 36 ft. long and the upper portion, 14 ft. long; the ratio of moments of inertia of these sections is 5 : 1.

It is required to find the moments at the ends and at the crane rail:

(*a*) Assume an imaginary support, *F*, (Fig. 28(*a*)) at the crane rail and distribute the moment between the two sections of the column. Then "carry over" to the ends of the column.

(*b*) Compute the shears in both portions of the column. The difference between these shears is the value of the force, *F*, exerted by the imaginary support. Removing this support will produce moments in the columns the same as by applying a transverse load at that point to the crane column considered as a fixed-end beam. These moments are most easily determined by a slight variant of the general method of distributing unbalanced moments.

(*c*) Assume arbitrarily a moment, say, 10, in one part of the column, which is temporarily considered direction-fixed (locked) at the crane rail, (Fig.



36'
14'
$I=5$
$\frac{I}{l}=0.139$
$I=1$
$\frac{I}{l}=0.071$
243
+80.3
−160.6
+82.4
−41.2
F
Shear $=\frac{240.9}{36}=6.69$
Shear $=\frac{123.6}{14}=8.84$
$F=8.84-6.69=2.15$
+10.0 +1.1 +11.1
−10.0 −2.1 −12.1
−13.2 +1.2 −12.1
+13.2 −0.6 +12.6
P
$M=10$ (Assumed)
$M=10\frac{(36)}{(14)}\frac{1}{5}=13.2$
Shear $=\frac{23.2}{36}=0.644$
Shear $=\frac{24.7}{14}=1.763$
$P=0.644+1.763=2.407$
$\frac{F}{P}=0.894$
+9.9 +80.3 +90.2
−10.8 −160.6 −171.4
−10.8 +82.4 +71.6
+11.3 −41.2 −29.0
(*a*)
(*b*)
(*c*)

FIG. 28.

28(*b*)). The moment in the other section (13.2), is then obtained from the relation,

$$M_1 \frac{l_1^2}{I} = M_2 \frac{l_2^2}{I_2}$$

(*d*) Remove the restraint at the crane rail (unlock), distribute the unbalanced moments, and "carry over" to the column ends.

(*e*) Compute the shears in both sections of the column, and, from these, the force, *P*, which produces the moments obtained in Step (4).

(*f*) Multiply these moments by $\frac{F}{P}$, the ratio of the force obtained in Step (2), to the force obtained in Step (4).

(*g*) Add these moments to those found in Step (1), giving the final results. (See Fig. 28(*c*).)

Instead of being rigidly fixed at the top, the crane column may be held at two points by the top and bottom chords of a roof truss. This case can also be solved by the use of an imaginary support at the crane rail. However, it is perhaps as simple and more interesting to consider this problem as a continuation of the one previously given, since in this way there will be a span in which the moment of inertia is not constant and its influence on distributing and carrying over will be brought out.

Let the lower 50 ft. of the column be the same as before. There is an additional 7 ft. at the top between the two points of support and its moment of inertia is unity. (See Fig. 29(*a*).)

The value of the moment, M_1, in terms of the moment, M_2, is found from the condition that the deflection of *o*, with reference to a tangent at *s*, is zero, as shown in Fig. 29(*b*); that is, the statical moment of the $\frac{M}{I}$-diagram (Fig. 29(*d*)) about *b* is zero. The resulting equation is:

$$\frac{14}{2} \times \frac{14}{3} M_2 + \frac{38}{5} \times \frac{36}{2} M_1 + (0.72\, M_2 + 0.28\, M_1) \left(\frac{28}{3} \times \frac{14}{2} + \frac{26}{5} \times \frac{36}{2}\right) = 0 \quad \text{.................(33)}$$

from which,

$$M_1 = -0.812\, M_2 \quad \text{.......................(34)}$$

Hence, the carry-over factor for this case is — 0.812 instead of — $\frac{1}{2}$.

The angle change at *o* produced by a moment, M_2, can now be determined from the area of the $\frac{M}{I}$-diagram. Thus,

$$\theta E = \frac{14}{2} M_2 + \frac{36}{(2 \times 5)} M_1 + (0.72\, M_2 + 0.28\, M_1) \left(\frac{14}{2} + \frac{36}{2 \times 5}\right) \quad \text{..(35)}$$

and, $\theta E = 9.28\, M_2$, or,

$$M_2 = 0.108\, \theta E \quad \text{.........................(36)}$$

Considering the upper section, Fig. 29(*c*), subjected to a moment, M_3, the angle change at *q*, is the deflection at *r*, with reference to a tangent at *q*, divided by the length, *c d*; that is, $\theta E = \frac{1}{3} \times \frac{7}{1} M_3$, or,

$$M_3 = 0.428 \, \theta E \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (37)$$

Hence, for a given angle change, θ, the ratio of M_2 to M_3 is 0.108 to 0.428. This is the ratio used in distributing the unbalanced moment between the two sections of the column. The remaining operations required to obtain the final moments are shown on Fig. 29(*e*).

Because so little additional work is involved, it is of interest to consider the effect of roof truss deflection on the moments in the crane column.

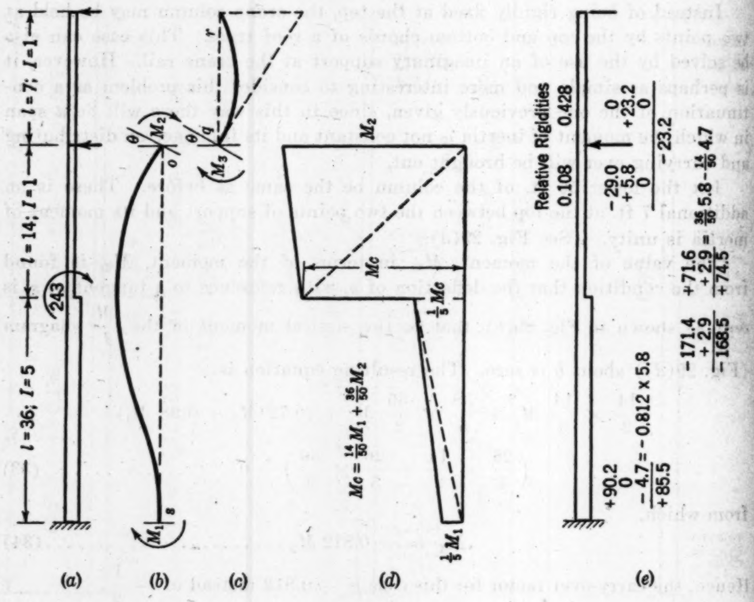
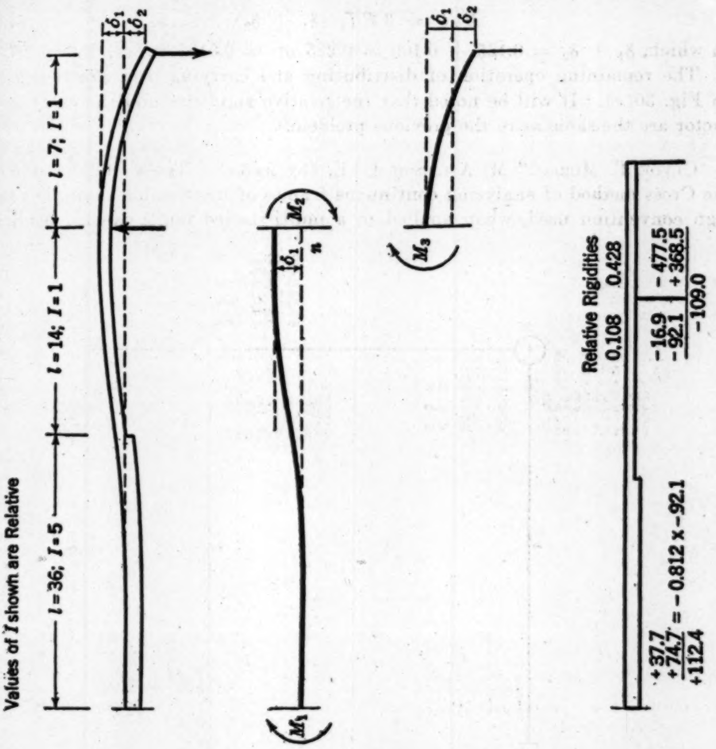

l = 36; I = 5
l = 14; I = 1
l = 7; I = 1
243
θ
M₂
o
s
M₁
θ
q
r
M₃
M₂
Mc
1/5 Mc
Mc = 14/50 M₁ + 36/50 M₂
1/5 M₁
Relative Rigidities
0.108
0.428
0
+23.2
0
−23.2
−29.0
+5.8
0
36/50 5.8 − 14/50 4.7
+71.6
+2.9 =
+74.5
−171.4
+2.9
−168.5
+90.2
0
−4.7 = −0.812 x 5.8
+85.5
(a)
(b)
(c)
(d)
(e)

FIG. 29.

Let the column be the same as before and assume that it is subjected to an outward deflection, δ_1 (by the bottom chord of the truss), of 0.125 in. and an inward deflection, δ_2 (by the top chord), of 0.100 in. (see Fig. 30(*a*)).

The moments of inertia of the column will be taken at 2 000 and 10 000 for the portions above and below the crane rail, thus maintaining the 1 : 5 ratio previously used.

It is convenient to consider that the column remains direction-fixed at the lower chord, Fig. 30(*b*), and on this basis to compute the moments resulting from the deflections in the upper and lower portions of the column. The moments thus found on each side of the bottom chord will not be equal. Removing the imaginary restraint, by means of which direction-fixity has been maintained, will produce changes in the moments that may be evaluated by distributing the unbalanced moment and carrying over in the usual way.


δ_1
δ_2
l=7; I=1
l=14; I=1
l=36; I=5
Values of I shown are Relative
M_2
δ_1
n
M_1
δ_1
δ_2
M_3
Relative Rigidities
0.108
0.428
-16.9
-92.1
-109.0
-477.5
+368.5
+37.7
+74.7 = -0.812 x -92.1
+112.4

(*a*) (*b*) (*c*)

FIG. 30.

For the lower portion of the column the statical moment of the $\frac{M}{I}$-diagram about *n* equals $E\,I\,\delta_1$. The left-hand side of the resulting equation is the same as in Equation (33) and $E\,I_1 = 30\,000\,000 \times 2\,000$ lb-in.^{2} = 417 000 kip-ft.^{2}; $\delta_1 = 0.125$ in. = 0.0104 ft.; M_1 and M_2 are in feet-kips; or,

$$147.2\,M_2 + 181.5\,M_1 = 4\,340 \quad \text{.................} (38)$$

Since this portion of the column is direction-fixed at both ends, θE, in Equation (35), is zero, or,

$$14.64\, M_2 + 6.57\, M_1 = 0 \ldots\ldots\ldots\ldots\ldots\ldots\ldots (39)$$

Solving Equations (38) and (39) simultaneously, $M_2 = -16.9$, and $M_1 = +37.7$.

For the upper portion of the column, the moment, M_3, is obtained from the relation,

$$M_3\, l^2 = 3\, E\, I_1\, (\delta_1 + \delta_2) \ldots\ldots\ldots\ldots\ldots\ldots\ldots (40)$$

in which, $\delta_1 + \delta_2 = 0.125 + 0.100 = 0.225$ in. $= 0.0187$ ft., or, $M_3 = 477.3$.

The remaining operation of distributing and carrying over are indicated in Fig. 30(*c*). It will be noted that the relative rigidities and the carry-over factor are the same as in the previous problem.

Clyde T. Morris,[34] M. Am. Soc. C. E. (by letter).—The writer has found the Cross method of analyzing continuous frames of inestimable value; but the sign convention used, when applied to a multi-storied multi-paneled bent, is more cumbersome than the following: "Any moment which tends to rotate a joint clockwise is to be considered positive, and the reverse negative".[35] After the solution is completed, it is a simple matter to translate these signs into the usual convention.

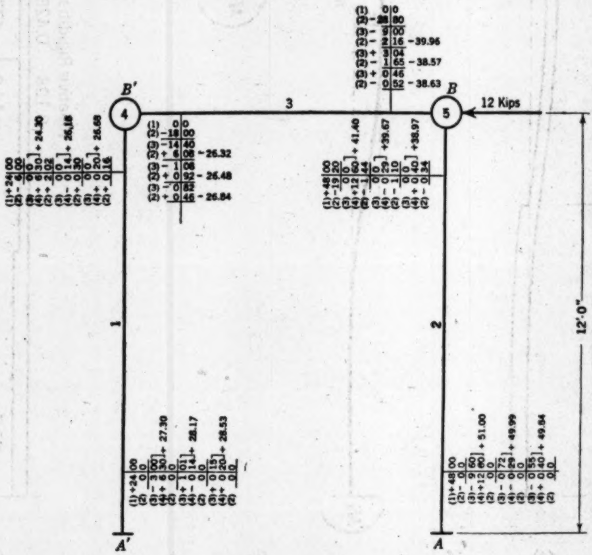


FIG. 31.—MOMENTS IN A TWO-LEGGED BENT.

---

[34] Professor of Structural Eng., Ohio State Univ., Columbus, Ohio.

[35] This is the convention suggested by L. E. Grinter and A. H. Finlay, Assoc. Members, Am. Soc. C. E., in their discussions of this paper.

The application of the method to the solution of stresses in building frames due to wind load is not explained. Professor Cross indicates that it is possible, and several of the discussers have mentioned the effects of side-sway due to eccentric loads. The method may be extended to lateral forces on a tall building frame with as much ease and accuracy as for vertical loads, in the following steps:

1.—Calculate the moments in the columns due to the lateral forces, considering the joints fixed against rotation, but free to deflect laterally. The sum of the moments at the top and bottom of all the columns of a story is equal to the shear in the story, multiplied by the story height, and, as the deflections of the columns in a story due to the lateral forces are equal, the column moments and shears are proportional to the $\frac{I}{L^2}$ values of the columns. (When all the columns of a story are of equal height the values of I, or $\frac{I}{L}$ may be used in proportioning the moments.)

2.—Distribute the moments at the joints, considering them free to rotate, but not changing their location. (Same process as for vertical loads.)

3.—Carry over the distributed moments using a carry-over factor of $+\frac{1}{2}$ as is done for vertical loads.

4.—Balance the column moments in each story by making their sum equal the shear in the story times the story height. This assumes the joints to be fixed against rotation, but free to deflect laterally as in the first step. This completes a cycle and the designer is now ready to repeat Steps 2, 3, and 4 as many times as the desired accuracy may require.

To illustrate the procedure, the calculations are given in Fig. 31 for a simple two-legged bent with the bottom of the columns fixed. The loads and dimensions are shown on the diagram. The relative magnitudes of the $\frac{I}{L}$ values for the members are indicated by the figures near the middle of the members. The factors resulting from the application of Steps 1, 2, 3, and 4 are indicated by the figures in parenthesis before the factors. Note that Step 4 does not apply to the girders.

TABLE 5.—COMPARISON OF MOMENTS IN A SIMPLE, TWO-LEGGED BENT

Methods	M'_A	M'_B	M_A	M_B
Elastic theory..........................	28.71	27.00	49.72	38.57
Cross method..........................	28.53	26.68	49.84	38.97

The correct figures for the moments, according to the elastic theory, compared to the corresponding moments by the Cross method, are given in Table 5. If greater accuracy is desired another cycle of corrections would require but a few minutes. After the moments in the members are determined, the shears and direct stresses are readily calculated.

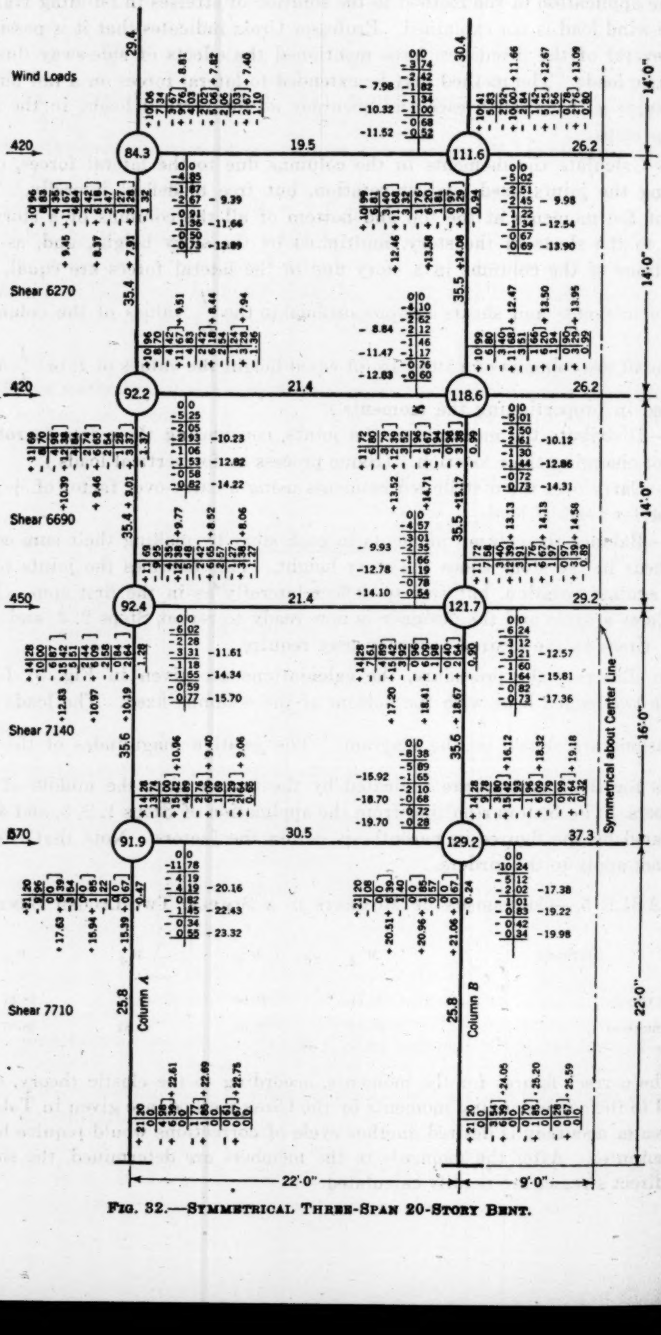

Wind Loads
420
420
450
570
Shear 6270
Shear 6690
Shear 7140
Shear 7710
29.4
30.4
84.3
19.5
111.6
26.2
35.4
35.5
92.2
21.4
118.6
26.2
35.4
35.5
92.4
21.4
121.7
29.2
35.6
35.6
91.9
30.5
129.2
37.3
25.8
25.8
Column A
Column B
14'0"
14'0"
14'0"
16'0"
22'0"
22'0"
9'0"
Symmetrical about Center Line

FIG. 32.—SYMMETRICAL THREE-SPAN 20-STORY BENT.

This method has also been applied to the lower four stories of the 20-story, symmetrical, three-span bent used by Messrs. Wilson and Maney in their paper on the slope-deflection method.[36] The calculations are shown in Fig. 32 and a comparison of the results, with those given by Messrs. Wilson and Maney by the slope-deflection method, is listed in Table 6.

TABLE 6.—COMPARISON OF MOMENTS IN A 20-STORY, SYMMETRICAL, THREE-SPAN BENT

Story	COLUMN *A*				COLUMN *B*				GIRDER *AB*				GIRDER *BB*	
	Top		Bottom		Top		Bottom		M_{AB}		M_{BA}			
	Slope-deflection method	Cross method	Slope-deflection method	Cross method	Slope-deflection method	Cross method	Slope-deflection method	Cross method	Slope-deflection method	Cross method	Slope-deflection method	Cross method	Slope-deflection method	Cross method
1	15.02	15.39	22.84	22.75	21.08	21.06	25.88	25.59	23.98	23.32	20.43	19.70	20.56	19.98
2	9.41	10.19	8.96	9.06	18.87	18.67	19.91	19.18	16.99	15.70	15.30	14.10	18.80	17.36
3	8.35	9.01	7.58	8.06	15.67	15.17	15.24	14.58	15.68	14.22	14.32	12.83	15.83	14.31
4	7.33	7.91	7.33	7.94	14.68	14.08	14.48	13.95	14.20	12.89	12.80	11.52	15.37	13.90

In computing the stresses in Fig. 32, it has been assumed that the carry-over terms for the columns of the fifth story were the same for the top and bottom. This introduces a small error at this story, which diminishes as the solution is carried down to the lower stories. This is shown in Table 6 by the diminishing errors in the lower stories.

The labor of calculating these stresses by the Cross method is little more than that required by the usual approximate methods in use, and infinitely less than for any of the other more exact methods.

FRANCIS P. WITMER,[37] M. AM. SOC. C. E. (by letter).—The author has described his method of analysis so clearly that its virtues are obvious. The writer believes that its simple underlying principles and surprisingly easy application make it the most convenient and expeditious method yet brought forward for the practical solution of numerical problems in internal bending stresses. For algebraic solutions it is, of course, not practically applicable as, without numerical values, one could not know when the process had been carried far enough for proper approximation.

The writer would like to comment on the subject of algebraic signs for moments. In 1917 he had occasion to make an investigation of indeterminate bending stresses resulting from eccentric connections in the trusses of an old elevated railroad structure. This led him to develop a method of analysis which has since become familiar as the "slope-deflection method". The necessity was quickly apparent for assuming that all moments in the same rotational direction were positive, regardless of where they were applied or whether they were external moments or internal resisting moments. Any other assumption was practically certain to result in errors and inconsistencies.

---

[36] "Wind Stresses in the Steel Frames of Office Buildings," *Bulletin No. 80*, Univ. of Illinois, Eng. Experiment Station.

[37] Director of Civ. Eng., Univ. of Pennsylvania, Philadelphia, Pa.

For illustration, consider the frame in Fig. 33. Assume that the various members have center bending from force action, as shown by the curved, dotted lines. The usual convention, as adopted by Professor Cross, would naturally lead to the algebraic designations shown for moments in all members except *B C*; but how shall moments in these members be designated? As they are nearly vertical, it may be logical to consider them similarly to the columns, *C D;* but this would be inconsistent with the adjacent members, *A B,* which are certainly analogous, structurally, to the members, *B C*. The adopted convention must not have limitations depending upon the angle of slope of members. Dilemmas such as these will continually occur in computations of this character, unless the convention adopted permits the assumption that all moments in one rotational direction are positive.

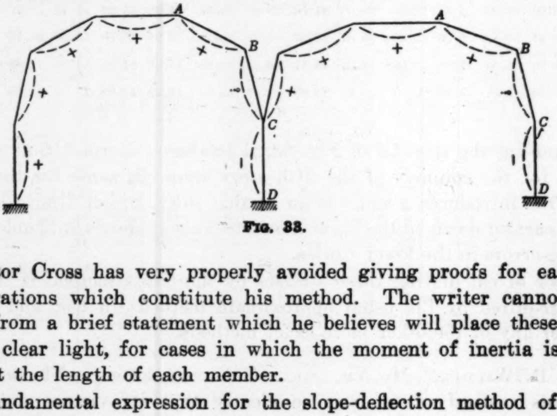


FIG. 33.

Professor Cross has very properly avoided giving proofs for each of the three operations which constitute his method. The writer cannot refrain, however, from a brief statement which he believes will place these relations in a very clear light, for cases in which the moment of inertia is constant throughout the length of each member.

The fundamental expression for the slope-deflection method of analysis, in cases where the straight-line axis joining the ends of a member does not change position, is:

$$M_1 = \frac{2\,E\,I}{l}\,(2\,\theta_1 + \theta_2) + C \ldots\ldots\ldots\ldots\ldots\ldots (41)$$

in which, *C* is a constant depending upon the particular loading. All the relations required in Professor Cross' method may be readily derived from Equation (41), as follows:

First.—For a beam fixed at both ends, the end, bending-deflection angles, θ_1 and θ_2, will both be zero, and $M_1 = C$. In other words, the fixed-end moment at either end is simply equal to the constant in the slope-deflection equation which corresponds to the existing loading. Tables of these constants are available in recent books describing this method of analysis.

Second.—Assume a moment applied at one end of a member, the other end being fixed. From the action of this moment alone (remembering that θ_2 at the fixed end is zero), Equation (41) becomes $M_1 = \frac{4\,E\,I\,\theta_1}{l}$. If two or more members meet and are rigidly connected at a point, and a total moment, *M*, is applied at this point, *M* will be so divided among the members that the ends of all will deflect through the same angle, θ_1, and these members, therefore, will carry moments proportioned thus:

$$M_1 : M_2 = 4\,E\,\theta_1\,\frac{I_1}{l_1} : 4\,E\,\theta_1\,\frac{I_2}{l_2}$$

or,

$$M_1 : M_2 = \frac{I_1}{l_1} : \frac{I_2}{l_2};\ M_1 : M_3 = \frac{I_1}{l_1} : \frac{I_3}{l_3};\ \text{etc.}$$

This proves the relation used in the second operation.

Third.—Assume, again, a member with a moment, M_1, at one end and a fixed moment, M_2, at the other end. From Equation (41),

$$M_1 = \frac{2\,E\,I}{l}\,(2\,\theta_1 + \theta_2)$$

and,

$$M_2 = \frac{2\,E\,I}{l}\,(\theta_1 + 2\,\theta_2)$$

Since θ_2 at the fixed end is zero, $M_1 = \frac{4\,E\,I\,\theta_1}{l}$, and $M_2 = \frac{2\,E\,I\,\theta_1}{l}$. Hence, $M_2 = \frac{1}{2}\,M_1$, and the "carry-over" factor is $+\,\frac{1}{2}$, as assumed in the third operation (with the proposed modification in the convention as to signs).

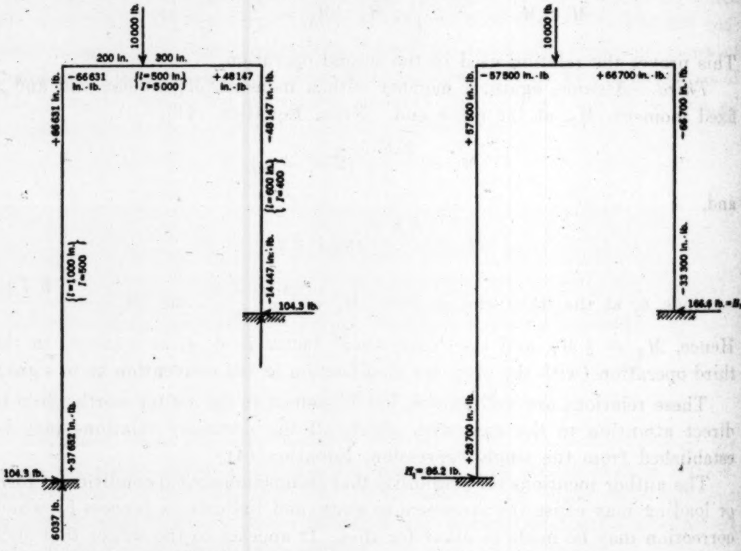
These relations are well known, but it seemed to the writer worth while to direct attention to the ease with which all the necessary relations may be established from the single expression, Equation (41).

The author mentions the possibility that an unsymmetrical condition of form or loading may cause the structure to sway, and indicates a process by which correction may be made to allow for this. It appears to the writer that such a condition is really the normal one, and that the case without sway is exceptional. It seems to him important that the process of correction be illustrated by simple numerical examples, in order to increase the availability of Professor Cross' method of analysis.

Consider, first, a frame without any applied horizontal force, but one which is unsymmetrical both in form and loading, as in Fig. 34(*a*). The correct moments and reactions, as computed by the slope deflection method, are shown. Fig. 34(*b*) shows the same frame with moments computed by Professor Cross' method, assuming that no sway occurs. From the moments in the posts, the shear reactions, H_1 and H_2, are computed to be of the values shown. Since these are unequal numerically, equilibrium does not obtain, showing that the structure has sway. The force, H_1, must be increased and the force, H_2, decreased so that they may equal each other, for equilibrium. Furthermore, the numerical sum of these two correcting quantities must be equal to the

numerical difference between H_1 and H_2 as first obtained and the two corrections must be proportioned to the values of $\frac{I}{l^3}$ for the respective columns, in order to equalize their sway. By observing these conditions it is found that H_1 must be increased by 17.1 lb. and H_2 must be decreased by 63.3 lb., thus bringing the resulting reactions to agreement at the value, 103.3 lb. This is practically the same as in Fig. 34(a).

The moments in the posts must now be corrected to correspond with these corrections in shear. The total correction for the left-hand post will be, 17.1 × 1 000 = 17 100 in-lb. and that for the right-hand post, 63.3 × 600 = 37 980 in-lb. These total correcting moments must be properly divided between the top and bottom of the posts. If the horizontal girder is infinitely rigid the tops will be perfectly fixed by these correcting moments, and the points of contraflexure produced thereby will be at mid-height, thus dividing the correcting moments equally between top and bottom. With this assumption for the present case, the resulting total moments will be:


10000 lb.
200 in.
300 in.
−66631 in.-lb.
{l=500 in. I=5000}
+48147 in.-lb.
+66631 in.-lb.
−48147 in.-lb.
{l=600 in. I=400}
{l=1000 in. I=500}
−14447 in.-lb.
104.3 lb.
+37652 in.-lb.
104.3 lb.
6037 lb.
10000 lb.
−57500 in.-lb.
+66700 in.-lb.
+57500 in.-lb.
−66700 in.-lb.
−33300 in.-lb.
166.6 lb.=H₂
+28700 in.-lb.
H₁=86.2 lb.

(a) SLOPE DEFLECTION METHOD (b) CROSS METHOD, NEGLECTING SIDE-SWAY

FIG. 34.

Left post:

At the top, 66 100 in-lb.; at the bottom, 37 300 in-lb.

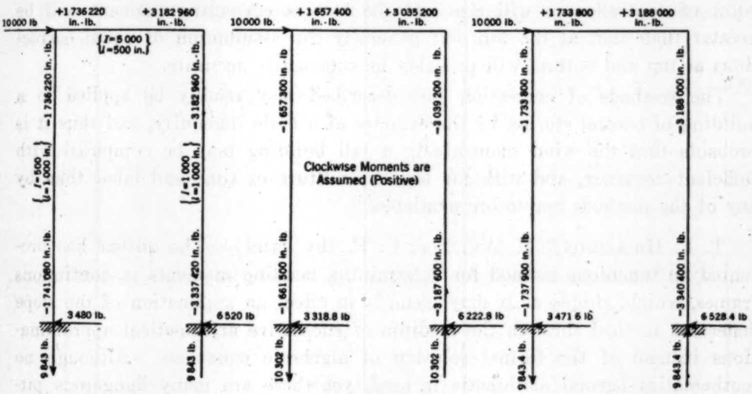
Right post:

At the top, 47 700 in-lb.; at the bottom, 14 300 in-lb.

These are very close to the values in Fig. 34(a), as they should be, since the girder is very rigid, as compared with the columns.

If the girder had no rigidity, the points of contraflexure, due to the correcting shears, would be at the top and the moment corrections would be wholly at the bottom. All cases will fall between these two extremes, and the correct proportion must be assumed by consideration of the relative rigidities of the girder and column. Probably, in most cases, the true corrections will not be far from equal at top and bottom, and, undoubtedly, the corrected moments may always be found in this manner, with a reasonably close approximation to the true values.

Consider, next, a frame with an applied horizontal force which will necessitate side-sway, whether or not the frame is symmetrical. Let Fig. 35(*a*) represent such a case, in which there also exists a lack of symmetry in size of columns. The moments and reactions shown on Fig. 35(*a*) are correct as computed by the slope-deflection method. First, divide the horizontal force between the two columns in proportion to their values of $\frac{I}{l^3}$, in order to equalize their sway, obtaining 3 333 lb. at the left and 6 667 lb. at the right column. Next, compute end moments for the columns, assuming them to be fixed top and bottom, following this by distributing unbalanced moments, according to the Cross method. Fig. 35(*b*) shows the resulting moments and forces. It is seen that the horizontal forces are not in equilibrium, showing that there has been sway in addition to that naturally resulting from the applied horizontal force; that is, due to lack of symmetry in the frame. If vertical loads had been applied in an unsymmetrical manner, these also would have contributed to this sway.


10000 lb
+1 736 220 in.-lb.
+3 182 960 in.-lb.
{I=5 000 l=500 in.}
−1 736 220 in.-lb.
{I=500 l=1 000 in.}
−1 741 080 in.-lb.
3 480 lb.
9 843 lb.
−3 182 960 in.-lb.
{I=1 000 l=1 000 in.}
−3 337 420 in.-lb.
6 520 lb
9 843 lb.
10000 lb.
+1 657 400 in.-lb.
+3 035 200 in.-lb.
−1 657 300 in.-lb
Clockwise Moments are Assumed (Positive)
−1 661 500 in.-lb
3 318.8 lb
10 302 lb.
−3 039 200 in.-lb
−3 187 600 in.-lb.
6 222.8 lb
10 302 lb.
10000 lb.
+1 733 800 in.-lb.
+3 188 000 in.-lb.
−1 733 800 in.-lb.
−1 737 900 in.-lb.
3 471.6 lb
9 843.4 lb.
−3 188 000 in.-lb.
−3 340 400 in.-lb.
6 528.4 lb
9 843.4 lb.

(*a*) SLOPE DEFLECTION METHOD (*b*) CROSS METHOD WITHOUT SIDE-SWAY (*c*) CROSS METHOD. CORRECTED FOR SIDE-SWAY

FIG. 35.

The amount of unbalanced horizontal force is found to be 458.4 lb. acting toward the left. Each horizontal reaction, therefore, must be increased in proportion to the value of $\frac{I}{l^3}$ of its column, resulting in corrections as follows: 152.8 lb. at the left column; and 305.6 lb. at the right column. Adding these to the horizontal reactions formerly obtained (see Fig. 35(*b*)), will balance the horizontal forces. The total moments for the columns must now be corrected by the following:

Left column	152.8 × 1 000	= 152 800 in-lb.
Right column	305.6 × 1 000	= 305 600 in-lb.

If the tops are assumed to be fixed by the girder, these moments will be equally divided between top and bottom, and the resulting moments and forces will be as shown in Fig. 35(*c*). These are seen to check very closely with the correct values in Fig. 35(*a*). If the girder is relatively less rigid, the point of contraflexure will rise and the bottom correcting moment will be greater than that at the top, but generally the assumption of equal corrections at top and bottom will probably be sufficiently accurate.

The methods of correction just described may readily be applied to a building of several stories, by the exercise of a little ingenuity, and thus it is probable that the wind moments in a tall building may be computed with sufficient accuracy, and with far less expenditure of time and labor than by any of the methods heretofore available.

T. F. HICKERSON,[38] M. AM. SOC. C. E. (by letter).—The author has presented an ingenious method for determining bending moments in continuous frames, which, simple as it may seem, is in effect an application of the slope deflection method through the medium of successive arithmetical approximations instead of the formal solution of algebraic equations. Although no mathematics beyond arithmetic is used, yet there are many dangerous pitfalls incident to an attempted application of this method by any one who is unversed in the basic principles of continuity of elastic structures.

The method is of most value in analyzing unsymmetrical frameworks (sidesway neglected), such as that given by the author in his Fig. 1, but for less complicated cases—the kind that ordinarily occur in practice—it appears to be too much of a juggling process for general acceptance in a design office.

Believing that the use of coefficients offers the simplest and quickest way of obtaining moments and shears at various sections in continuous beams and frames, the writer has prepared a comprehensive set of tables for this purpose. A simple outline, with a sample table of coefficients, was presented in connection with another paper.[39]

Four cases will now be cited illustrative of typical frameworks and their analyses by two methods: (1) The method of "Tabular Coefficients", proposed by the writer; and (2), the method of "Carry-Over Moments", proposed by the author.

---

[38] Prof. of Civ. Eng., Univ. of North Carolina, Chapel Hill, N. C.

[39] "Continuous Beams Over Three Spans," by I. Oesterblom, M. Am. Soc. C. E., *Transactions*, Am. Soc. C. E., Vol. 93 (1929), p. 1405.

***Case 1.—Symmetrical, Two-Legged Bents.*—The following notation**[40] **applies to this discussion:**

l = length of beam, AB.

I_1 = moment of inertia of beam, AB.

S_1 = stiffness of beam, $AB = \frac{I_1}{l}$.

h = length of column, AC (or BD).

S = stiffness of column, AC (or BD).

r = relative column stiffness $= \frac{S}{S_1} = \frac{I}{I_1}\,\frac{l}{h}$.

$M_A = M_{AB}$ = resisting moment at End A of Member AB (or AC).

The moment, C_{AB}, equals that part of M_{AB} which depends only on the loads; if the ends, A and B, are fixed on the same level, $M_{AB} = -C_{AB}$.

Referring to the frame shown in Fig. 36, where the columns are fixed at the bases,

$$M_A = -\frac{1}{2}\left[(C_{AB} + C_{BA})\left(\frac{2r}{1+2r}\right) + (C_{AB} - C_{BA})\left(\frac{r}{6+r}\right)\right] \cdot (42)$$

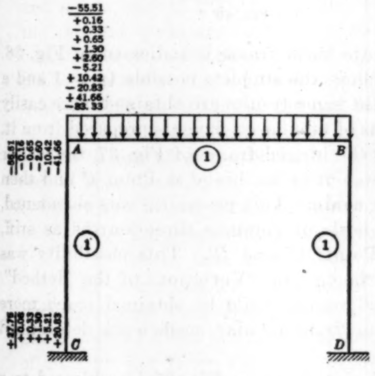

−55.51
+0.16
−0.33
+0.65
−1.30
+2.60
−5.21
+10.42
−20.83
+41.66
−83.33
−55.51
−0.16
−0.65
−2.60
−10.42
−41.66
A
B
①
①
①
+27.75
+0.08
+0.33
+1.30
+5.21
+20.83
C
D

Fig. 36

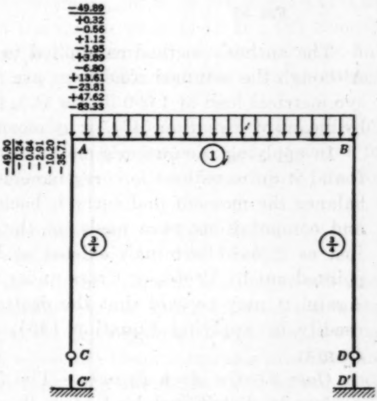

−49.89
+0.32
−0.56
+1.12
−1.95
+3.89
−6.80
+13.61
−23.81
+47.62
−83.33
−49.90
−0.24
−0.84
−2.91
−10.20
−35.71
A
B
①
$\frac{3}{4}$
$\frac{3}{4}$
C
D
C′
D′

Fig. 37

For symmetrical loads on AB, say, a uniformly distributed load of w lb. per ft., $C_{AB} = C_{BA} = \frac{1}{12}\,w\,l^2$; and, hence, Equation (42) takes the simple form:

$$M_A = -\frac{1}{6}\,w\,l^2\left(\frac{r}{1+2r}\right) \ldots\ldots\ldots\ldots\ldots\ldots (43)$$

Referring to Fig. 37 where the columns are hinged at the bases,

$$M_A = -\frac{3}{2}\,(C_{AB} + C_{BA})\left(\frac{r}{2+3r}\right) \ldots\ldots\ldots\ldots (44)$$

---

[40] This notation is the same as that used in *Bulletin 108*, Univ. of Illinois Eng. Experiment Station.

For a symmetrical loading on $A\,B$, say, a uniform load of w lb. per ft.,

$$M_A = -\frac{1}{4}\,w\,l^2\left(\frac{r}{2 + 3\,r}\right)\ldots\ldots\ldots\ldots\ldots\ldots(45)$$

Tables may be constructed based on Equations (42) to (45), inclusive, such that the corner moments for any vertical loading and for any condition of relative stiffness may readily be determined. Furthermore, such tables enable one to compare by inspection the difference between the effects of fixed and hinged terminals.

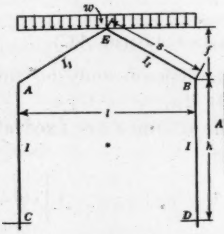


FIG. 38

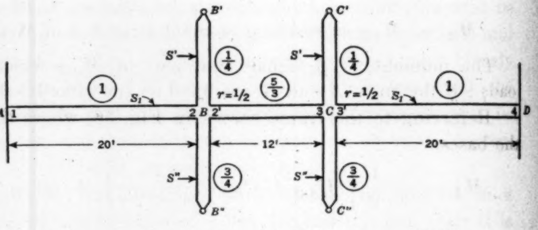


FIG. 39

The author's method as applied to the fixed frame is indicated in Fig. 36. Although the assumed conditions are about the simplest possible ($r = 1$ and a symmetrical load of 1 000 lb. per ft.), the same results are obtained more easily by means of Equation (43), or by means of tabular coefficients prepared from it.

In applying the author's method to the hinged frame of Fig. 37, the writer found it quite tedious to carry moments out to the hinge at Point C and then balance the moment and carry it back again. This procedure was abandoned, and computations were made on the basis of columns three-fourths as stiff, just as if fixed terminals existed at Points C' and D'. This possibility was pointed out by Professor Cross under the caption "Variations of the Method". Again, it may be said that the desired results could be obtained much more readily by applying Equation (45), or from tabular coefficients determined from it.

Case 2.—On Arch Frames.—The frame shown in Fig. 38 is subjected to a uniformly distributed load of w lb. per ft. A table may be used which is based on the formulas derived from the least work theorem; it would give coefficients for determining the corner moments,

$$M_A = \phi\,w\,l^2\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(46)$$

Thus, if $l = 30$ ft., $h = 20$ ft., $f = 6$ ft., $r = \frac{1}{2}$, $\frac{f}{h} = 0.3$, and $w = 1000$ lb. per ft., the values of ϕ given by the table (not given herewith) are —0.050 and —0.052 for the hinged and fixed frames, respectively. For the hinged frame, $M_A = -0.050 \times 1\,000 \times (30)^2 = -45\,000$ ft-lb.; and for the fixed frame, $M_A = -0.052 \times 1\,000 \times (30)^2 = -46\,800$ ft-lb. In Fig. 38, $S = \frac{I}{h}$; $S_1 = \frac{I_1}{S}$; and $r = \frac{S}{S_1} = \frac{I}{I_1}\,\frac{s}{h}$.

If the entire load is concentrated at the central ridge (at Point E, Fig. 38), the foregoing values would be multiplied by $\frac{3}{2}$, or increased by 50% ; and for other symmetrical loadings, the corresponding moments are obtained by merely multiplying the coefficients in the prepared table for this case, by known ratios.

The author's method seems inapplicable to the arched frame; in fact, the slope deflection method itself is hardly adaptable to this case.

Case 3.—Three Continuous Beams.—Fig. 39 shows a symmetrical arrangement of three continuous beams rigidly connected to columns at B and C. Tables of coefficients for determining bending moments at the central section of the middle span can likewise be compiled by adapting Equation (46) to this case, as previously explained elsewhere.[41] All column terminals have been assumed as hinged, but the tables are also adaptable to the case of fixed ends if the computed column stiffness is increased by one-third, that is, if it is multiplied by $\frac{4}{3}$.

Let it be required to find the maximum positive and negative moment at the central section of the middle span, due to a dead load of 1 200 lb. per ft., and a live load of 1 000 lb. per ft. Furthermore, let it be assumed: (1) That the two end spans are each 20 ft., and the middle span is 12 ft.; (2) that the beam terminals are one-half fixed (that is, the terminal moments are one-half the values for absolutely fixed conditions); and (3) that the ratio of average column stiffness to that of the end beam is $\frac{1}{2}$.

Referring to the proper table of coefficients (not given herewith), the following values of ϕ are obtained; -0.017 for first span loaded; $+0.026$ for second span loaded; and -0.017 for third span loaded; from which the following moments are easily computed;

Total maximum negative moment = − 17 440 ft-lb.
Total maximum positive moment = + 6 560 ft-lb.

To solve this problem by means of the author's method is a longer and more tedious process than that of tabular coefficients; so much so, that the writer got no farther than to find the moments (at the supports) due to the live load of 1 000 lb. per ft. on the middle span when the beam terminals are fixed and when they are hinged. The problem is then only about one-third solved, since the moment at the middle of the span must be determined from the equations of statics after the moments at the supports and the reactions are known.

Case 4.—Multiple Spans.—Consider a multiple-span arrangement, as, for example, the five continuous spans shown in Fig. 40(a). The writer has computed shear and moment coefficients upon the following bases: (1) The moment at any interior support and the shear at either side of it are greatest when the two spans adjacent to that support are covered with live and dead load, while the next span to the right and to the left is covered only with dead load; (2) the effect of loads on remote spans; that is, beyond the two spans from the support, in either direction, is neglected; and (3) the span arrangement beyond any support may be removed if a certain fixation factor is substituted therefor.

---

[41] *Transactions*, Am. Soc. C. E., Vol. 93 (1929), Table 12, p. 1407.

In order to find the maximum value of the bending moment, M_2, at the right end of the member, $B\,C$ (Fig. 40(*a*)), two sets of loads will be dealt with: (1) Live and dead load on the two spans, $C\,B$ and $C\,D$, adjacent to the support, C (Fig. 40(*b*));.and (2), dead load on the spans, $B\,A$ and $D\,E$ (Fig. 40(*c*)).

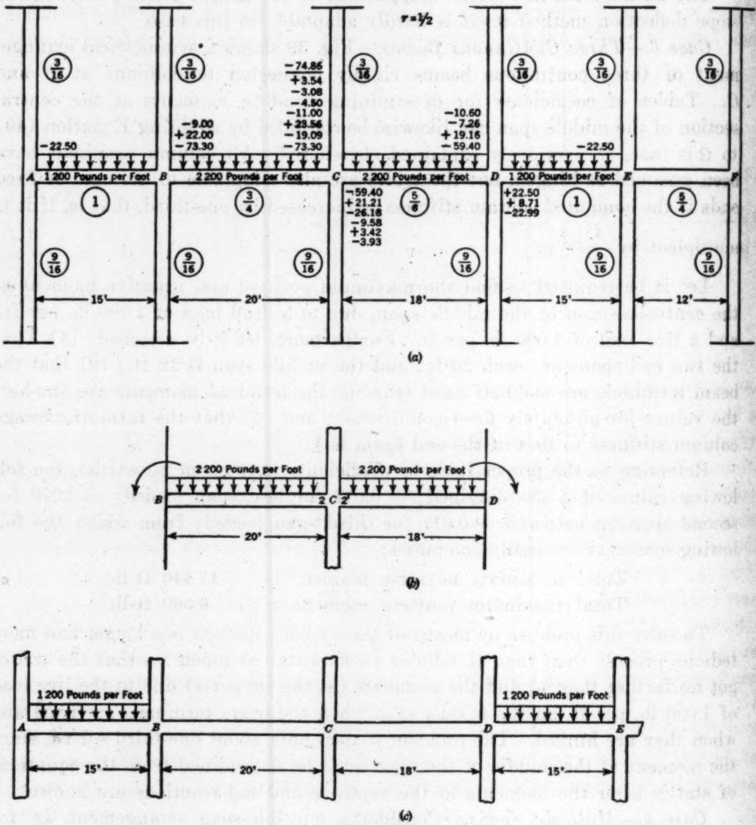

r = ½
3/16
3/16
−74.86
+3.54
−3.08
−4.50
−11.00
+23.56
−19.09
−73.30
−9.00
+22.00
−73.30
−22.50
3/16
−10.60
−7.26
+19.16
−59.40
3/16
3/16
3/16
−22.50
A
1 200 Pounds per Foot
B
2 200 Pounds per Foot
C
2 200 Pounds per Foot
D
1 200 Pounds per Foot
E
F
1
3/4
−59.40
+21.21
−26.18
−9.58
+3.42
−3.93
5/6
+22.50
+8.71
−22.99
1
5/4
9/16
9/16
9/16
9/16
9/16
9/16
15'
20'
18'
15'
12'
(a)
2 200 Pounds per Foot
2 200 Pounds per Foot
B
2
C
2'
D
20'
18'
(b)
1 200 Pounds per Foot
1 200 Pounds per Foot
A
B
C
D
E
15'
20'
18'
15'
(c)

FIG. 40

Referring to a table (not given herewith), with $x = \dfrac{18}{20} = 0.9$:

$$M_2 = [-0.117 + 0.29\,v + 0.010\,t]\,w\,l^2 \ldots\ldots\ldots\ldots (47)$$

in which, v and t are the fixation factors at the ends, B and D, in Fig. 40(*b*); that is, $v = 1$ and $t = 1$, if both ends are fixed. For this particular span arrangement, $v = 0.65$ and $t = 0.70$, hence, $M_2 = -0.091 \times 2\,200 \times (20)^2 = -80\,080$ ft-lb.

Referring to a table (not given herewith) for the proper coefficient to use in Fig. 40(*c*), the value of 0.019 is obtained; hence, $M_2 = +\,0.019 \times 1\,200 \times (15)^2 = +\,5\,130$ ft-lb. Combining the two values of M_2 thus obtained, the total maximum moment equals $M_2 = -\,80\,080 + 5\,130 = -\,74\,950$ ft-lb.

In applying the author's method to this problem, only those carry-over moments actually needed in evaluating M_2 are indicated in Fig. 40(*a*); and these were determined without much difficulty.

In conclusion, the writer wishes to commend the author for presenting this method, which is valuable to know even if it is tedious to apply.

F. H. CONSTANT,[42] M. AM. SOC. C. E. (by letter).—The treatment of side-sway in multiple-story frames, with intermediate vertical loads and horizontal joint loads (considered either separately or conjointly), may be generalized. For simplicity, a two-story frame bent, with any number of vertical columns, is assumed. The fixed-end moments due to the vertical loads, and the column moments due to the horizontal joint loads (resulting from side-sway and no joint rotation) are first found, and then distributed, carried over, and finally balanced.

As already explained in Professor Cross' paper, the moments thus found will be designated M_0. The M_0-column moments in any story will give rise to shears in those columns the sums of which will not, in general, be equal to the shear known to exist in that story (this latter is the sum of all the horizontal joint loads above the story in question). Let the known true shear in each story be V_1, V_2, etc., and the shear from the M_0-column moments in each story, V_{01}, V_{02}, etc. Finally, let any arbitrary set of column moments (hereafter called shear moments) be introduced in the first story and distributed among the columns of that story in the ratio of their $\dfrac{I}{h^2}$ values. Distribute, carry over, and finally balance these moments throughout all the members of the frame. Call the moments thus obtained, M_{11}, M_{12}, etc., the first subscript referring to the story in which the arbitrary shear moments are introduced, and the second to the moments in the other stories produced by them, after distribution and balancing.

The column moments, M_{11}, M_{12}, etc., in each story will produce shears in the corresponding stories. These shears will be V_{11}, V_{12}, etc., the subscripts having the same meaning as before. Then, introduce an arbitrary set of shear moments in the second-story columns, distribute, and balance throughout the frame as before. Call these moments M_{21}, M_{22}, etc., and the corresponding shears V_{21}, V_{22}, etc.

Stating this nomenclature more concisely:

M = required correct moment in any member.
V_1 = shear known to exist in the first-story columns.
V_2 = shear known to exist in the second-story columns.
M_0 = distributed and balanced moments, due to the fixed-end moments in the beams, and the shear moments in the columns, produced by the external loading.

---

[42] Prof. of Civ. Eng., Princeton Univ., Princeton, N. J.

V_{01} = shear in the first story, due to the M_0-moments in the first-story columns.
V_{02} = shear in the second story, due to the M_0-moments in the second-story columns.
M_1 = distributed and balanced moments due to an arbitary set of shear moments introduced in the first-story columns.
M_2 = distributed and balanced moments due to an arbitrary set of shear moments introduced in the second-story columns.
V_{11} = shear in first story due to the M_1-moments in the first-story columns.
V_{12} = shear in the second story due to the M_1-moments in the second-story columns.
V_{21} = shear in the first story due to the M_2-moments in the first-story columns.
V_{22} = shear in the second story due to the M_2-moments in the second-story columns.
C_1 and C_2 = constants.

Then,

$$V_1 = V_{01} + C_1 V_{11} + C_2 V_{21} \qquad (48)$$

and,

$$V_2 = V_{02} + C_1 V_{12} + C_2 V_{22} \qquad (49)$$

Solving for C_1 and C_2,

$$M = M_0 + C_1 M_1 + C_2 M_2 \qquad (50)$$

For a one-story bent subjected only to a horizontal force at the top joint, Equation (50) reduces to,

$$M = M_0 \frac{V_1}{V_0} \qquad (51)$$

In a multiple-story building there will be as many independent equations and constants as there are stories. To obtain the M_0-moments due to the external loading it will be necessary to distribute and balance throughout the frame. The moments, M_1, M_2, etc., are only great in and near the story in which the arbitrary moments are introduced. They decrease rapidly and become insignificant one or two stories away, in either direction. Shear moments are distributed between the columns of a story in proportion to their values of $\frac{I}{h^2}$.

The writer prefers the sign convention which designates a moment as positive when the member tends to rotate the joint in a clockwise direction. The shear at the foot of a column, A B, is, then, $V = \frac{M_A + M_B}{h}$, the sign and direction of V being automatically controlled by the signs of the numerical values of M_A and M_B. The work is also greatly expedited by balancing first the joints having the largest unbalanced moments, and carrying over the distributed moments at once, before proceeding to the next joint.

W. N. Downey,[43] Assoc. M. Am. Soc. C. E. (by letter).—The design of a rigid frame by the conventional methods generally in use is unquestionably a tedious and laborious process, because a clear conception of the frame action is obscured by a maze of algebraic manipulations. A method by which the designer can visualize step by step exactly what is happening is of great value. For this reason Professor Cross has made a valuable contribution to the literature on the subject of the statically indeterminate frame.

---

[43] Care, State Highway Dept., Frankfort, Ky.

The problems introduced in the paper may also be analyzed by successive corrections to the angular turns at the ends of the members. This method, as developed by the writer, has been found to be very rapid. The method of procedure may be stated briefly. All the angular turns, due to the load, are calculated approximately, then corrections are made to these approximate values to obtain more nearly the correct angles. As many corrections may be calculated as is considered necessary for the problem under consideration. From the magnitude of the corrections it can be judged when the computations have been carried far enough. The exactness of the final angles depends only on how many corrections the designer cares to make. The second correction generally gives results exact enough for ordinary design purposes.

Besides being rapid the method of successive corrections has other distinct advantages. It is particularly well suited to the calculation of influence lines for moving loads, which are almost indispensable in bridge design; the effect of horizontal deflection (and, consequently, temperature changes) and of horizontal loads at the joints is taken into consideration in a simple and direct manner. In addition, the method lends itself well to tabular arrangement of the computations, thus minimizing the chance of error in the numerical work. It applies equally as well to beams of variable moments of inertia as to beams of constant section.

Calculations Neglecting the Effect of Horizontal Deflection.—The slope-deflection equations[44] give a convenient way of visualizing the constants required in the approximations for the angles. These equations may be written:

For a beam restrained at both ends:

$$M_{AB} = E\,K_{AB}\left[\beta_{AB}\,\theta_A + \beta'_{AB}\,\theta_B - \beta''_{AB}\,\frac{d}{h_{AB}}\right] \mp C_{AB} \ldots\ldots (52)$$

$$M_{BA} = E\,K_{AB}\left[\beta_{BA}\,\theta_B + \beta'_{BA}\,\theta_A - \beta''_{BA}\,\frac{d}{h_{AB}}\right] \pm C_{BA} \ldots\ldots (53)$$

For a beam restrained at A and fixed at C:

$$M_{AC} = E\,K_{AC}\left[\beta_{AC}\,\theta_A - \beta''_{AC}\,\frac{d}{h_{AC}}\right] \mp C_{AC} \ldots\ldots\ldots\ldots (54)$$

$$M_{CA} = E\,K_{AC}\left[\beta'_{CA}\,\theta_A - \beta''_{CA}\,\frac{d}{h_{AC}}\right] \pm C_{CA} \ldots\ldots\ldots\ldots (55)$$

For a beam restrained at A and hinged at D:

$$M_{AD} = E\,K_{AD}\,\beta_{AD}\left[\theta_A - \frac{d}{h_{AD}}\right] \mp H_{AD} \ldots\ldots\ldots\ldots (56)$$

In Equations (52) to (56), $\beta + \beta' = \beta''$; $\beta'_{AB} = \beta'_{BA}$; and $K = \dfrac{I}{h}$.

For the case of a beam of constant section: $\beta_{AB} = \beta_{BA} = \beta_{AC} = 4$; $\beta'_{AB} = \beta'_{BA} = \beta'_{CA} = 2$; $\beta''_{AB} = \beta''_{BA} = \beta''_{AC} = \beta''_{CA} = 6$; and, $\beta_{AD} = 3$.

---

[44] *Bulletin 108*, Univ. of Illinois, Eng. Experiment Station.

The values of β and the fixed end moments for beams of varying moment of inertia may be found by any of the standard methods of calculation, or may be readily calculated from the tables[45] given by Strassner, or from the tables[46] given by Walter Ruppel, Assoc. M. Am. Soc. C. E. Both writers cover a wide range of conditions.

Sign Conventions and Notation.—A positive angular change is measured from the original position of the axis in a clockwise direction about the end of the member considered. A positive deflection is measured in the same direction as a positive angular change. An external moment acting in a clockwise direction about the end of a member is positive. An external load acting in a clockwise direction about a joint is negative. The following notation is used:

θ = change in slope of the tangent to the elastic curve at the end of a member, the subscript denotes the joint; θ_A = the change in slope at A.

d = relative displacement of the ends of a member with respect to the original direction of the member.

K = moment of inertia of the member divided by its length.

θ' = change in slope of the tangent to the elastic curve at the end of a member when no horizontal movement is considered, the subscript denotes the joint. θ'_A = the change in slope at A.

β, β', and β'' = constants determined by the elastic properties of the member.

α = angular turn at the joints produced by the deflection, $d = 1$. The subscript denotes the joint. α_A = the rotation at A.

H = horizontal reaction on the frame produced by the deflection, $d = 1$.

ρ = angular turn at the joints produced by $H = -1$. The subscript denotes the joint. ρ_A = the rotation at A.

d_A = deflection of the top of the columns due to a moment, $-X_a$. (The numerical value of $d_A = \rho_A$). Likewise, for d_B, d_C, etc.

X_a = external moment applied at the joint, A. Similarly, for X_b, X_c, etc.

Z_a = angular turn at A due to $X_a = -1$. All other joints held fixed. Similarly, for Z_b Z_c, etc.

X_{ba} = external moment at B, due to a unit angular turn at A, when Joint B is fixed against rotation. Similarly, for X_{ab}, X_{cb}, etc.

$X_{a(d)}$ = external moment about the end, A, of a member, due to the deflection, $d = 1$, when the end, A, is fixed against rotation, and the other end is either fixed or hinged.

Procedure.—The fundamental conception of calculating the angular turns by successive corrections is similar to that of distributing end moments. Consider one joint at a time as elastic and calculate the angular change at this elastic joint, due to the applied external load. Then proceed to the next joint, which is now considered as the only elastic joint, and calculate the angular turn at this joint, due to the angular change at the joint just calculated and the applied external load. In this manner, calculate angular turns at each joint in the structure and obtain the first approximation, $\Delta\, \theta'$.

---

[45] "Neuere Methoden," Vol. I.

[46] *Transactions*, Am. Soc. C. E., Vol. 90 (1927), p. 152.

For instance, consider the joints, A, B, C, and D, in sequence and calculate the first approximations, $\Delta\theta'_A$, $\Delta\theta'_B$, $\Delta\theta'_C$, and $\Delta\theta'_D$. To avoid confusion as to whether all effects have been considered, begin at the left joint and calculate to the right, or *vice versa*.

To obtain the corrections, θ'_1, begin with the last calculated $\Delta\theta'$ (for instance, $\Delta\theta'_D$), and compute the angular turn at C (θ'_{1C}), due to $\Delta\theta'_D$. The sum, $(\Delta\theta'_C + \theta'_{1C})$, would be a closer approach to the true value of θ'_C. Proceed to Joint B and calculate the angular turn, θ'_{1B}, due to angular change, $(\Delta\theta'_C + \theta'_{1C})$. Continue in this manner through the frame. These values constitute the first corrections, θ'_1.

Determine θ'_2 in the same manner as θ'_1, using only those increments of θ, which have not been previously used in going through the frame in this direction. That is, θ'_{2B} is obtained by calculating the angular turn at B due to θ'_{1A}; θ'_{2C} is obtained by computing the angular turn at C due to $(\theta'_{1B} + \theta'_{2B})$, etc.

All other corrections are made in the same way as θ'_2. When, from the magnitude of the corrections, it is seen that the angles are exact enough for the problem considered, sum up the first approximation and the corrections for each angle; that is:

$$\theta' = \Delta\theta' + \theta'_1 + \theta'_2 + \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (57)$$

Angle θ' is the rotation of the joint due to the effect of the external loads, the effect of horizontal deflection being neglected.

Computations of Constants for Determining Angular Changes.—Referring to Fig. 41, let Z_a equal the angular change at A due to the moment, $X_a = -1$, when all joints except A are considered fixed. Angular changes, Z_b, Z_e, etc., are similarly defined. For any value of X_a other than -1, the angular change at A is equal to $Z_a X_a$. For Joint A, Fig. 41:

$$M_{AB} + M_{AD} + M_{AC} = \frac{X'_a}{E} = X_a \ldots\ldots\ldots\ldots\ldots (58)$$

Therefore,

$$K_{AB}\,\beta_{AB}\,\theta_A + K_{AD}\,\beta_{AD}\,\theta_A + K_{AC}\,\beta_{AC}\,\theta_A = X_a \ldots\ldots\ldots (59)$$

When $X_a = -1$,

$$Z_a = \frac{-1}{K_{AB}\,\beta_{AB} + K_{AD}\,\beta_{AD} + K_{AC}\,\beta_{AC}} \ldots\ldots\ldots\ldots\ldots (60)$$

The denominator in Equation (60) is the moment at A due to $\theta_A = 1$.

In general:

$$Z_a = \frac{-1}{\Sigma\, K_{A(\ )}\,\beta_{A(\ )}} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (61)$$

in which, the summation is over all the members meeting at Joint A. In a like manner, Z_b, Z_e, etc., are computed.

The Determination of X_{ba}, X_{ab}, X_{be}, X_{eb}, Etc.—The moment at B, due to $\theta_A = 1$, is equal to X_{ba}. All other angles are considered zero. For any other value of θ_A than 1, the moment at $B = \theta_A X_{ba}$. The moments, X_{ab}, X_{be}, etc., are similarly defined; thus,

$$X_{ba} = X_{ab} = K_{AB}\,\beta'_{BA} = K_{AB}\,\beta'_{AB} \ldots\ldots\ldots\ldots\ldots (62)$$

From the quantities, Z_a, Z_b, etc., and X_{ab}, X_{be}, etc., the angles, θ', may be determined for any external load when the effect of the deflection, d, is neglected.

In using this method for fixed loads, one calculates the fixed-end moments at each joint due to the loads and applies to each joint the algebraic sum of the fixed-end moments at the joint as an external load with proper sign.

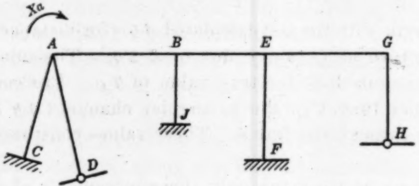


FIG. 41

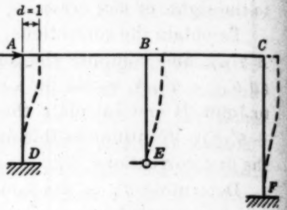


FIG. 42

If there are more than two load conditions to be considered, it is quicker to compute influence values for the moments. In the calculations for influence values, apply $X_a = -1$ (Fig. 41) as the only load on the structure, and compute the corresponding angular changes. Compute angular changes for $X_b = -1$, $X_e = -1$, and $X_g = -1$, respectively. Calculate the moments at the ends of the members for these conditions of loading. The actual moments are in direct proportion to the values of X_a, X_b, X_e, and X_g, due to the actual load. In computing influence lines, the following procedure gives the full solution very quickly. First, compute the angular changes for $X = -1$ for the joint farthest to the left (or right). For instance, in Fig. 41, compute the angular changes for $X_a = -1$. Then, apply $X_b = -1$ as the only load and compute the first approximation, $\Delta\, \theta'_b$. It would be necessary to perform the same operations on this value of $\Delta\, \theta'_b$ that were performed on $\Delta\, \theta'_b$ in the previous computation for $X_a = -1$. Therefore, the angular turns for $X_b = -1$ are equal to the angular turns for $X_a = -1$ multiplied by $\Delta\, \theta'_b$ due to $X_b = -1$, divided by $\Delta\, \theta'_b$ due to $X_a = -1$. In computing θ'_a, only the sum of the corrections from the first computations should be used.

Then, proceed to Joint E and apply Set $X_e = -1$. From Maxwell's theorem, the angular change at E due to a unit moment at A is equal to the angular change at A due to a unit moment at E; likewise, for B and E. Then, the values of θ'_A and θ'_B may be written at once for $X_e = -1$. Since this θ'_B is the final correct value of θ'_B, it is necessary only to make corrections for the joints to the right of B. Compute the angular turn at E due to $X_e = -1$. Compute the angular turn at E due to θ'_B and add the results, which equal $\Delta\, \theta'_e$; calculate $\Delta\, \theta'_g$; and make corrections only to those two angles. Finally, proceed to Joint G.

It is not necessary that the calculations for the load conditions $X_b = -1$; $X_e = -1$; and $X_g = -1$ be carried through as outlined herein, but it is believed that this method of computation requires the least amount of work.

The Effect of Horizontal Movement.—In the preceding discussion, the angles, θ', were obtained for the condition that the frame had no horizontal deflection. It is proposed now to calculate this deflection and corrections to the angles, θ', due to the deflection. The moments acting at the top and bottom of the columns in a frame produce a horizontal thrust, or shear in the columns which causes a deflection, *d*, and a change in the slope of the tangent to the elastic curve of the members. From Maxwell's law, a moment, —1, at B (see Fig. 42) produces a deflection, *d*, which is equal to the angular change at B due to a horizontal force, $H = -1$. Instead of applying a unit moment at a joint and computing the deflection, apply a unit deflection, *d*, and calculate the angular changes, α, at the joints. Knowing these angular changes, α, and knowing $d = 1$ (by assumption), the horizontal thrust, H due to $d = 1$, may be computed. The angular changes, ρ, due to $H = -1$, are then calculated by a simple proportion.

To obtain the angular changes, α, at the joints, due to $d = 1$, first assume the beam, ABC, as infinitely stiff; then, $\theta_A = \theta_B = \theta_C = 0$. For this condition, the moments at the top of the column will be, for $d = 1$ (see Fig. 42):

$$M_{AD} = -\frac{K_{AD}\,\beta''_{AD}}{h_{AD}};\ M_{BE} = -\frac{K_{BE}\,\beta_{BE}}{h_{BE}};\ \text{and}\ M_{CF} = -\frac{K_{CF}\,\beta''_{CF}}{h_{CF}}.$$

From which,

$$X_{a(d)} = -\frac{K_{AD}\,\beta''_{AD}}{h_{AD}} \qquad (63)$$

$$X_{b(d)} = -\frac{K_{BE}\,\beta_{BE}}{h_{BE}} \qquad (64)$$

$$X_{c(d)} = -\frac{K_{CF}\,\beta''_{CF}}{h_{CF}} \qquad (65)$$

The moments, $X_{a(d)}$, $X_{b(d)}$, and $X_{c(d)}$, are considered as external loads applied at their respective joints. The calculations for the angular changes, α, may be made exactly as in the case of neglecting horizontal deflection, previously described. If the computations for the θ' values have been carried through for influence lines, the more direct way is to proceed, as follows: Since the angular turns have already been found, due to a moment, —1, at each joint, the values of α may be determined by considering that $X_{a(d)}$, $X_{b(d)}$, and $X_{c(d)}$ act one at a time, and adding the effects. Multiply the moment considered acting, by the values of θ' previously obtained when the corresponding joint was loaded with $X = -1$, assuming no horizontal deflection. Repeat this process for all joints and add algebraically. It is obvious that these sums are the angular changes, α, due to $d = 1$. Note that the angles, θ', were calculated for $X = -1$, and the moments about the joints due to $d = 1$ are also negative.

The horizontal shear, $-H$, in the columns is the sum of the moments at the top and bottom of the columns divided by their respective heights. Since $-H$ is the horizontal shear due to $d = 1$, the angular changes, ρ, due to $H = -1$, equals $\frac{\alpha}{-H}$.

Computation of H.—It is best to separate the computations for H into two parts: First, the effect, H', produced by the angular changes, α; and, second, the effect, H'', produced by the deflection, $d = 1$.

For a member restrained at both ends:

$$H' = \frac{K_{AB}\,[\beta_{AB}\;\;\alpha_A + \beta'_{AB}\;\;\alpha_B]}{h_{AB}} + \frac{K_{AB}\,[\beta_{BA}\;\;\alpha_B + \beta'_{BA}\;\;\alpha_A]}{h_{AB}}$$

$$= \frac{K_{AB}\,\beta''_{AB}\;\;\alpha_A}{h_{AB}} + \frac{K_{AB}\,\beta''_{BA}\;\;\alpha_B}{h_{AB}} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (66)$$

$$H'' = -\;\frac{K_{AB}\,\beta''_{AB}}{h^2_{AB}} - \frac{K_{AB}\,\beta''_{BA}}{h^2_{AB}} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (67)$$

and,

$$H = H' + H'' \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (68)$$

For a single-story frame, such as that shown in Fig. 42:

$$H' = -\,X_{a(d)}\;\alpha_A - X_{\;(d)}\;\alpha_B - X_{c(d)}\;\alpha_C \ldots\ldots\ldots\ldots\ldots\ldots (69)$$

in which,

$$-X_{a(d)} = \frac{K_{AD}\,\beta_{AD} + K_{AD}\,\beta'_{DA}}{h_{AD}} = \frac{K_{AD}\,\beta''_{AD}}{h_{AD}} \ldots\ldots\ldots (70)$$

$$-X_{b(d)} = \frac{K_{BE}\,\beta_{BE}}{h_{BE}} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (71)$$

$$-X_{c(d)} = \frac{K_{CF}\,\beta_{CF} + K_{CF}\,\beta'_{FC}}{h_{CF}} = \frac{K_{CF}\,\beta''_{CF}}{h_{CF}} \ldots\ldots\ldots (72)$$

and,

$$H'' = +\,\frac{X_{a(d)}}{h_{AD}} + \frac{X_{b(d)}}{h_{BE}} + \frac{X_{c(d)}}{h_{CF}} + \frac{X_{d(d)}}{h_{AD}} + \frac{X_{f(d)}}{h_{CF}} \ldots\ldots\ldots (73)$$

The values of ρ may next be calculated by solving the formula:

$$\rho = \frac{\alpha}{-H} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (74)$$

that is, $\rho_A = \dfrac{\alpha_A}{-H}$, $\rho_B = \dfrac{\alpha_B}{-H}$, $\rho_C = \dfrac{\alpha_C}{-H}$, etc.

According to the sign system adopted, it will be seen that a negative H or a negative moment around a joint produces a positive deflection; therefore, for $X = -1$, $d = \rho$. For a horizontal load of —1 at the top of columns, $d = \dfrac{1}{-H}$.

The final values of the angular turns are obtained by adding the effects produced by the assumption that d equals zero, and the angular turns produced by considering the effect of d acting alone. These final angles may be written:

$$\theta = \theta' + \alpha\, d \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (75)$$

For instance, for the case of $X_\alpha = -1$: $\theta_A = \theta'_A + \alpha_A\; d_A$; $\theta_B = \theta'_B + \alpha_B\; d_A$; etc. The factor, d_A, signifies the deflection due to $X_a = -1$. The angles are now completely solved and the moments may be calculated by substitution into general slope deflection formulas. The method may be applied to multi-storied frames, secondary stresses, and other problems involving elastic deformations.

Example No. 1.—The writer has chosen the example solved by Professor Cross as an illustration of the method of making the calculations for a fixed condition of loading when the effect of horizontal movement is neglected.

(1).—The fixed-end moments in all the members are first calculated. Since Joint F is considered hinged, the moment at C in CF is computed from the relation: $H_{CF} = C_{CF} + \frac{C_{FC}}{2} = +\,80 + \frac{60}{2} = +\,110$, and the moment at $F = 0$.

The values of the external moments, X_b, X_c, etc., acting on the joints are next obtained by adding algebraically the fixed-end moments at the joints. In this connection it is to be noted that the external moment considered as acting around a joint is of the same sign as the fixed-end moment considered as acting on the loaded member. For example, $X_c = C_{CB} + H_{CF} + C_{CD} + C_{CG} = +\,100 + 110 - 200 + 50 = +\,60$. Similarly, $X_b = -\,100$; $X_d = +\,100$; and, $X_e = C_{ED} +$ cantilever moment $= 0 - 10 = -\,10$.

(2).—The angular changes at each joint due to a moment, $-\,1$, acting at the joint, while all other angles are considered zero, are next computed; thus $Z_c = \frac{-\,1}{4\,(4 + 5 + 1) + 3 \times 2} = -\,0.02174$. Similarly, $Z_b = -\,0.04167$; $Z_d = -\,0.03125$; and $Z_e = -\,0.08333$.

(3).—The moment at the fixed end of a member due to a unit rotation at the other end is given by the quantities, X_{cb}, X_{dc}, etc. For example, $X_{cb} = X_{bc} = 2 \times 4 = 8$; $X_{dc} = X_{cd} = 2 \times 5 = 10$; and $X_{ed} = X_{de} = 2 \times 3 = 6$.

(4).—The product, $X_{cb}\,Z_c$, therefore, gives the angular turn at C due to a unit rotation of Joint B. These quantities are computed for carrying over effects of angular turns from right to left, and from left to right. Table 7 gives these values for the author's Fig. 1. All the constants required for the determination of the angular turns have been found, and the computations for the actual rotations may be made.

TABLE 7.—COMPUTATION OF ANGLE CHANGES AT JOINTS INDICATED, CAUSED BY A UNIT ROTATION AT ADJACENT JOINTS

Joint	FOR CARRYING ANGLES TO THE RIGHT		FOR CARRYING ANGLES TO THE LEFT	
	Product $X\,Z$	Value	Product $X\,Z$	Value
B............	....	....	$X_{bc}\,Z_b$	$-\,0.3333$
C............	$X_{cb}\,Z_c$	$-\,0.1739$	$X_{cd}\,Z_c$	$-\,0.2174$
D............	$X_{dc}\,Z_d$	$-\,0.3125$	$X_{de}\,Z_d$	$-\,0.1875$
E............	$X_{ed}\,Z_e$	$-\,0.5$	....	....

For the first approximations, $\Delta\,\theta'$, the computations are as follows: At Joint B there is an external moment of $-\,100$; hence, $\Delta\,\theta'_B = -\,100 \times Z_b = -\,100 \times -\,0.04167 = +\,4.167$, which is the value of θ'_B for the moment, $-\,100$, at Joint B, when all other joints are held fixed.

At Joint C there is an external moment of $+\,60$, hence, $\Delta\,\theta'_C = +\,60 \times Z_c + (\Delta\,\theta'_B)\,(X_{cb}\,Z_c) = +\,60 \times -\,0.02174 + 4.167 \times -\,0.1739 = -\,2.029$, which is the value of θ'_C for the moment, + 60, at Joint C, when all other joints are held fixed, plus the effect of the angular turn ($\Delta\theta'_B = 4.167$) at Joint B.

The first approximations of θ'_D and θ'_E, computed exactly in the same manner as those for θ'_B, are found to be: $\Delta\,\theta'_D = -2.491$, and $\Delta\,\theta'_E = +2.078$, respectively (see Column (2), Table 8).

TABLE 8.—TOTAL ANGLE CHANGES, EXAMPLE NO. 1

Joint	FIRST APPROXIMATION	SUCCESSIVE CORRECTIONS					TOTALS	
	$\Delta\,\theta'$	θ'_1	θ'_2	θ'_3	θ'_4	θ'_5	θ' after two corrections	θ' after five corrections
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
B....	+ 4.1667	+ 0.4675	....	+ 0.01096	....	− 0.00087	+ 4.635	+ 4.644
C....	− 2.029	+ 0.626	− 0.0813	+ 0.0484	− 0.0019	+ 0.00452	− 1.484	− 1.433
D....	− 2.491	− 0.3898	− 0.1703	− 0.0525	− 0.0145	− 0.00628	− 3.051	− 3.124
E....	+ 2.0788	....	+ 0.2800	....	+ 0.0335	....	+ 2.358	+ 2.392

Corrections are made to these approximate quantities to obtain more exact values. The first corrections are made from the joint, E, to the left.

The first correction for the angle, D, is made by computing the angular change at D due to the angle, $\Delta\,\theta'_E = +\ 2.078$; thus, $\theta'_{1D} = \Delta\,\theta'_E\, X_{de}\, Z_d = +\ 2.078 \times -\ 0.1875 = -\ 0.3898$ (Table 8, Column (3)).

The first correction for the angle, C, is made by computing the angular change at C, due to $\Delta\,\theta'_D + \theta'_{1D}$; thus, $\theta'_{1C} = (\Delta\,\theta'_D + \theta'_{1D})\ X_{cd}\ Z = -2.879 \times -\ 0.2174 = +\ 0.626$.

All other corrections are made in the same manner. It is to be noted that they are made only on increments of the angles which have not been previously carried over to the adjacent joint. The total angular change is obtained for any joint by adding the first approximation and the corrections to the angle.

The simplicity of the arrangement of computations shown in Table 8 makes chance of error or failure to consider any term almost negligible. For comparison of results, the moments have been computed for the angles obtained after the second correction and after the fifth correction. The results which are given in Table 9 indicate that for all practical purposes the computations might properly have been stopped with the second correction.

TABLE 9.—COMPARISON OF MOMENTS AFTER TWO AND FIVE CORRECTIONS, RESPECTIVELY

Number of corrections	COMPARISON OF BENDING MOMENTS										
	M_{AB}	M_{BA}	M_{BC}	M_{CB}	M_{CF}	M_{CD}	M_{CG}	M_{GC}	M_{DC}	M_{DE}	M_{ED}
5.......	18.576	+37.152	−37.160	+114.224	+101.402	−259.90	+44.264	+22.132	+23.19	−23.136	+ 9.96
2.......	+18.54	+37.08	−37.71	+114.34	+101.10	−260.92	+44.06	+22.03	+24.14	−22.26	+10.40

Example No. 2.—The real simplicity and value of this method of computation is brought out in the calculations for influence lines for moving loads. The calculations are first carried through for the angles considering that the frame has no horizontal deflection. Then, these angles are corrected for the effect of the deflection. The computations are made for the frame shown in Fig. 43. An external moment, $X = -1$, is considered as the only load on the frame and the angular changes are computed for this condition of loading. The applied moment, $X = -1$, is assumed to act first at Joint A and then at each of the other joints in succession.

The quantities, Z_a, Z_b, etc., and X_{ba}, X_{cb}, etc., and the products, $X_{ba}\ Z_b$, $X_{cb}\ Z_c$, etc., are computed exactly as explained in Example No. 1, and are arranged conveniently in Table 10.

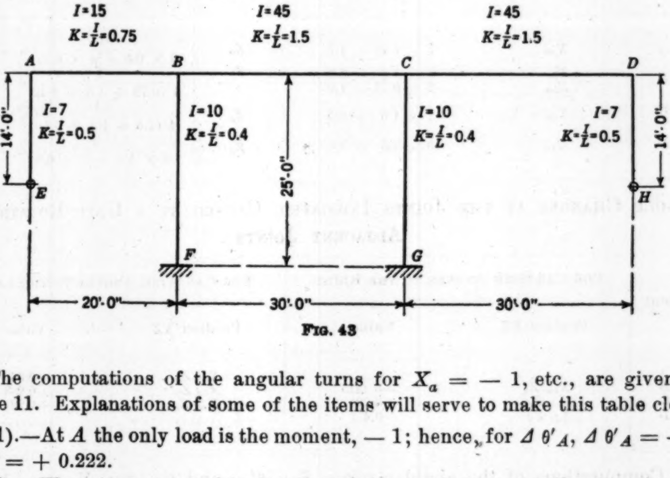


FIG. 43

The computations of the angular turns for $X_a = -1$, etc., are given in Table 11. Explanations of some of the items will serve to make this table clear:

(1).—At A the only load is the moment, — 1; hence, for $\Delta\ \theta'_A$, $\Delta\ \theta'_A = -1 \times Z = +0.222$.

(2).—There is no external load at B, and the rotation is caused by the effect of $\Delta\ \theta'_A$; therefore, for $\Delta\ \theta'_B$, $\Delta\ \theta'_B = \Delta\ \theta'_B\ X_{ba}\ Z_b = +0.222 \times -0.1415 = -0.0314$.

(3).—Likewise, the rotation at C is caused by the effect of $\Delta\ \theta'_B$, or $\Delta\ \theta'_C = \Delta\ \theta'_B\ X_{cb}\ Z_c = -0.0314 \times -0.221 = +0.00694$.

(4).—For $\Delta\ \theta'_D$, $\Delta\ \theta'_D = \Delta\ \theta'_C\ X_{dc}\ Z_d = +0.00694 \times -0.4 = -0.00278$. Corrections to these approximate values are made as in Example No. 1.

Computations of the angular turns for $X_b = -1$ are shown in Table 11(b). The value of θ'_A was obtained directly by Maxwell's law. The final values of θ'_B, θ'_C, and θ'_D, may be computed from the relation:

$$\theta' = \frac{\Delta\ \theta'_B\ (\text{for } X_b = -1)}{\Delta\ \theta'_B\ (\text{for } X_a = -1)} \times \theta'\ (\text{for } X_a = -1)$$

that is, the angular turns, θ'_B, θ'_C, θ'_D, for $X_b = -1$, are $\left(\frac{+0.0944}{-0.0314} =\right)$ 3.0 times as large as for $X_a = -1$.

Computations of the angular turns, θ'_A and θ'_B, when $X_c = -1$ (see Table 11(c)), may be written directly by Maxwell's law. Since they are the final values they require no correction, and computations must be made for θ'_C and θ'_D only, thus, $\varDelta\,\theta'_C = -1 \times Z_c + \theta'_B\, X_{cb}\, Z_c = -1 \times -0.0735 - 0.0256 \times -0.221 = +0.07916$, and, $\varDelta\,\theta'_D = \varDelta\,\theta'_C\, X_{dc}\, Z_d = +0.07916 \times -0.4 = -0.03170$. Corrections are made only to these two angles, and are carried through in the usual manner.

TABLE 10.—COMPUTATION OF ANGLE CHANGES, EXAMPLE NO. 2, CAUSED BY A UNIT ROTATION AT ADJACENT JOINTS

Joint	EXTERNAL MOMENTS, X		ANGLE CHANGE, Z, for $M = -1$	
	Symbol	Computation	Symbol	Computation
A	X_{ab}	$2 \times 0.75 = 1.5$	Z_a	$\dfrac{-1}{3 \times 0.5 + 4 \times 0.75} = -0.222$
B	X_{bc} X_{ba}	$2 \times 1.5 = 3.0$ $2 \times 0.75 = 1.5$	Z_b	$\dfrac{-1}{4\ (0.75 + 1.5 + 0.4)} = -0.0944$
C	$X_{cd} = X_{cb}$	$2 \times 1.5 = 3.0$	Z_c	$\dfrac{-1}{4\ (1.5 + 1.5 + 0.4)} = -0.735$
D	X_{dc}	$2 \times 1.5 = 3.0$	Z_d	$\dfrac{-1}{4 \times 1.5 + 3 \times 0.5} = -0.1333$

ANGLE CHANGES AT THE JOINTS INDICATED, CAUSED BY A UNIT ROTATION AT ADJACENT JOINTS

Joint	FOR CARRYING ANGLES TO THE RIGHT		FOR CARRYING ANGLES TO THE LEFT	
	Product XZ	Value	Product XZ	Value
A	....	....	$X_{ab}\,Z_a$	−0.333
B	$X_{ba}\,Z_b$	−0.1415	$X_{bc}\,Z_b$	−0.283
C	$X_{cb}\,Z_c$	−0.221	$X_{cd}\,Z_c$	−0.221
D	$X_{dc}\,Z_d$	−0.4	....	....

Computations of the angular turns, θ'_A, θ'_B, and θ'_C, for $X_d = -1$, may be written directly (see Table 11(d)), as follows: $\theta'_D = -1 \times Z_d + \theta'_C\, X_{dc}\, Z_d = -1 \times -0.1333 - 0.03475 \times -0.4 = +0.1472$. Table 11 gives the values of the rotations when the horizontal deflection is neglected.

To correct for the effect of horizontal deflection a unit deflection is assumed at the top of the columns, and the moments at the top and bottom of the columns are computed on the assumption that the beam, $A\,B\,C\,D$, is infinitely stiff; thus, $X_{a(d)} = X_{d(d)} = \dfrac{-3 \times 0.5}{14} = -0.1071$, and $X_{b(d)} = X_{c(d)} = X_{f(d)} = X_{g(d)} = \dfrac{-6 \times 0.4}{25} = -0.0961$.

The moments at the top of the column are considered as external moments on the joints. They are assumed to act one at a time, and the results are added for the total effect. The effect of $X_{a(d)}$ is obtained by multiplying the angles found for $X_a = -1$ by the value of $X_{a(d)}$; and likewise for all other moments, as shown in Table 12.

TABLE 11.—COMPUTATION OF ANGLE CHANGES, EXAMPLE NO. 2

θ'	(a) FOR THE LOAD, $X_a = -1$					(b) FOR THE LOAD, $X_b = -1$	
	$\Delta \theta'$	θ'_1	θ'_2	θ'_3	Total θ'	$\Delta \theta'$	Total θ'
θ'_A	$-1 \times Z_a = +0.222$	$+0.01118$	....	$+0.000615$	$+0.23379$	....	-0.03539
θ'_B	-0.0814	-0.00214	-0.00158	-0.000268	-0.03539	$-1 \times Z_b = +0.0944$	$-3 \times -0.03539 = +0.10650$
θ'_C	$+0.00694$	$+0.000614$	$+0.000821$	$+0.000127$	$+0.00850$	....	$-3 \times 0.0085 = -0.0256$
θ'_D	-0.00278	....	-0.000574	-0.000074	-0.003428	....	$-3 \times -0.003428 = +0.01032$
						$\frac{+0.0944}{-0.0814} = -3.0$	
	(c) FOR THE LOAD, $X_c = -1$					(d) FOR THE LOAD $X_d = -1$	
θ'_A	....	....	....	....	$+0.00850$	....	-0.003428
θ'_B	....	....	....	....	-0.0256	....	$+0.010320$
θ'_C	$-1\ Z_c = +0.07350$ $+0.00566$ $+0.07916$	$+0.0070$	....	$+0.000618$	$+0.08678$	....	-0.03475
θ'_D	-0.03170	....	-0.0028	-0.00025	-0.03475	$-1\ Z_d + 0.1333$ $= +0.0139$ $+0.1472$	$+0.14720$
	$-0.0256 \times -0.221 = +0.00566$					$-0.03475 \times -0.4 = +0.0139$	

TABLE 12.—Computation of Angular Turn, α

	A	B	C	D
$X_{a(d)}\ \theta'$ (for $X_a = -1$)	$+0.0250$	-0.0038	$+0.00091$	-0.000368
$X_{b(d)}\ \theta'$ (for $X_b = -1$)	-0.0034	$+0.0102$	-0.00246	$+0.000994$
$X_{c(d)}\ \theta'$ (for $X_c = -1$)	$+0.000816$	-0.00246	$+0.00833$	-0.00334
$X_{d(d)}\ \theta'$ (for $X_d = -1$)	-0.000368	$+0.001108$	-0.00372	$+0.0158$
α	$+0.022048$	$+0.005048$	$+0.003060$	$+0.013086$
$\rho = \frac{\alpha}{-H}$	$+0.843$	$+0.1929$	$+0.1170$	$+0.500$

TABLE 13.—Computation of Horizontal Reaction, H

	H'		H''
$-X_{a(d)}\,\alpha_A$	$+0.00236$	$X_{a(d)} \div h_{AE}$	-0.00766
$-X_{b(d)}\,\alpha_B$	$+0.000485$	$2\,X_{b(d)} \div h_{BF}$	-0.00769
$-X_{c(d)}\,\alpha_C$	$+0.000294$	$2\,X_{c(d)} \div h_{CG}$	-0.00769
$-X_{d(d)}\,\alpha_D$	$+0.00140$	$X_{d(d)} \div h_{DH}$	-0.00766
Total..........	$+0.004539$	Total........	-0.03070

$$H = H' + H'' = -0.026161$$

TABLE 14.—Final Angular Turns and Deflections for Various Load Conditions

	Load Conditions				
	$X_a = -1$	$X_b = -1$	$X_c = -1$	$X_d = -1$	$H = -1$
	$+0.2338$ $+0.0186$	-0.0354 $+0.0042$	$+0.0085$ $+0.0026$	-0.0034 $+0.0110$	
θ_A	$+0.2524$	-0.0312	$+0.0111$	$+0.0076$	$+0.8430$
	-0.0354 $+0.0042$	$+0.1065$ $+0.0010$	-0.0256 $+0.0006$	$+0.0103$ $+0.0025$	
θ_B	-0.0312	$+0.1075$	-0.0250	$+0.0128$	$+0.1929$
	$+0.0085$ $+0.0026$	-0.0256 $+0.0006$	$+0.0868$ $+0.0004$	-0.0347 $+0.0015$	
θ_C	$+0.0111$	-0.0250	$+0.0872$	-0.0332	$+0.1170$
	-0.0034 $+0.0110$	$+0.0103$ $+0.0025$	-0.0347 $+0.0015$	$+0.1472$ $+0.0065$	
θ_D	$+0.0076$	$+0.0128$	-0.0332	$+0.1537$	$+0.5000$
d	$+0.8430$	$+0.1929$	$+0.1170$	$+0.5000$	$+38.225$

Computations for H are as follows:

$$H = H' + H''$$

$$H' = -X_{a(d)}\,\alpha_a - X_{b(d)}\,\alpha_b - X_{c(d)}\,\alpha_c - X_{d(d)}\,\alpha_d$$

and,

$$H'' = \frac{X_{a(d)}}{h_{AE}} + \frac{X_{b(d)} + X_{f(d)}}{h_{BF}} + \frac{X_{c(d)} + X_{g(d)}}{h_{CG}} + \frac{X_{d(d)}}{h_{DH}}$$

The operations indicated are listed in Table 13. The angles, ρ, are computed by the formula, $\rho = \frac{\alpha}{-H}$, as follows:

$$\rho_A = \frac{\alpha_A}{-H} = \frac{+0.02205}{-(-0.02616)} = +0.843$$

$$\rho_B = \frac{\alpha_B}{-H} = \frac{+0.005048}{-(-0.02616)} = +0.1929$$

$$\rho_C = \frac{\alpha_C}{-H} = \frac{+0.00306}{-(-0.02616)} = +0.1170$$

and,

$$\rho_D = \frac{\alpha_D}{-H} = \frac{+0.013086}{-(-0.02616)} = +0.500$$

The values, ρ, in addition to being the angular turns at the joints due to $H = -1$, are also the deflections at the top of the columns due to $X = -1$ at the various joints; for instance: For $X_a = -1$, $d_A = \rho_A$; for $X_b = -1$, $d_B = \rho_B$; etc.

The angular turns at the joints, including the effect of the horizontal deflection, are computed from the equation: $\theta' = \theta' + \alpha\, d$. For instance, for $X_a = -1$: $\theta_A = \theta'_A + \alpha_A\, d_A$; $\theta_B = \theta'_B + \alpha_B\, d_A$; etc.

The final angular turns and deflections for the various load conditions are shown in Table 14.

The moments are computed by substitution into the general slope-deflection equations. From these values the effect of any load on the frame may be evaluated.

E. C. HARTMANN,[47] JUN. AM. SOC. C. E. (by letter).—The author's method of analyzing continuous frames by distributing fixed-end moments is so simple and direct that there seems to be small reason for suggesting a variation of the system outlined in his paper. However, the writer would like to point out a slightly different system for distributing the fixed-end moments which sometimes may prove useful where the separate effects of a number of loading conditions are to be studied. If it serves no other purpose, this discussion will at least emphasize the flexibility of the Cross method.

The system that the writer has in mind can probably be best illustrated by reference to the case of an ordinary continuous girder. Assume a four-span structure with the points of support lettered, successively, from left to right, A, B, C, D, and E; and assume that it is desired to study the moments caused by a number of different loading conditions. The solution suggested is as follows:

---

[47] **Research Engr., Aluminum Co. of America, New Kensington, Pa.**

1.—Assume any hypothetical unbalanced negative moment on the left side of Support *B*. Distribute this single unbalanced moment in the manner described in the paper, obtaining the resulting moments at each point of support.

2.—Apply the same hypothetical unbalanced moment at the right side of Support *B*. The resulting moments at each point of support can be determined in this case by inspection from the values obtained in Step 1.

3.—Repeat Steps 1 and 2 for Supports *C* and *D*.

4.—Determine the fixed-end moments for one of the loading conditions to be studied, say, Condition I.

5.—Take one of these fixed-end moments and determine the resulting moment at each point of support by proportion from the values obtained in Step 1, Step 2, or Step 3.

6.—Repeat Step 5 for each of the fixed-end moments for loading Condition I. The final moments at the supports for that loading condition may now be obtained by adding algebraically the results obtained for the several fixed-end moments.

7.—The moments at the supports resulting from other loading conditions may be obtained by repeating Steps 4, 5, and 6.

It is important to note in this example, that, regardless of how many loading conditions are investigated, the number of distributions of unbalanced moments never exceeds three. Having the results of these three distributions on hand, one can tabulate the remainder of the work for easy slide-rule calculation.

THOMAS C. SHEDD,[48] M. AM. SOC. C. E. (by letter).—The writer has had the good fortune to be closely associated with Professor Cross since the fall of 1922 when he first conceived the fundamental idea and began the development of his method of "moment distribution". Some inkling of the extent to which the author has developed this method has been given by several discussions contributed by his former students. More significant, it seems to the writer, is the number of discussers who, presumably unfamiliar with the method prior to the publication of the author's paper, have so quickly grasped the fundamental procedure as to be able to see at once its far-reaching utility, and, in many cases, to present discussions clearly illustrating applications of which the author little more than hinted. In view of the extremely condensed presentation this seems a remarkable tribute to the simplicity and directness of this new tool of analysis.

The development of the method and the manner of its presentation show that the author has strived to eliminate the academic "fiddle-faddle" and unnecessary complexity with which many of the classical methods of statically indeterminate analysis have always been embellished, and to change these from mathematical playthings for academicians to working tools for engineers.

Moment distribution by successive approximations is equally useful whether the engineer needs only a rough approximation or a precise analysis. One who uses the method frequently will be surprised at the number of cases in which an understanding of its fundamental principles enables him to make a mental estimate of indeterminate effects, which is sufficient for his purpose, and at the number of other cases in which a single distribution of the fixed-end moments yields adequate information; and, yet, the same procedure which so quickly gives an approximate answer, with the expenditure of very little additional labor (far less than is required for any kind of answer in other methods), will result in a solution of any desired precision.

---

[48] Associate Prof., Structural Eng., Univ. of Illinois, Urbana, Ill.

Appreciation of the power and utility of Professor Cross' method so increases with longer acquaintance that one is tempted to offer a multiplicity of examples of its application. The number of excellent discussions already presented makes this unnecessary, and the writer will content himself with expressing the opinion that "moment distribution" is the most powerful and versatile tool presented for the use of structural engineers in many years. In combination with the author's paper entitled "Column Analogy"[49] this paper will exert an increasingly widespread influence on thinking and design in the structural field. Finally, for the benefit of those who lament that "moment distribution" is not a collection of generalized formulas, the writer would like to quote from Percival Lowell:[50]

> "But worse still for their employed symbols tend to fictitious understanding. Formulæ are the anæsthetics of thought, not its stimulants; * * *."

DAVID M. WILSON,[51] ASSOC. M. AM. SOC. C. E. (by letter).—The author has described a method of analysis which is of special interest to one familiar with the slope-deflection method of analyzing statically indeterminate frames. In fact, the steps used in distributing fixed-end moments are based directly on the slope-deflection equation, and as Professor Cross states, the method is, therefore, not approximate, but a method of successive approximations. As the number of approximations is increased, the results approach nearer and nearer those obtained by rigorous application of the slope-deflection method. It should be noted, however, that in the slope-deflection method no account is taken of the change in length of any member due to axial stress. To this extent, both the slope-deflection method and the method of distributing fixed-end moments are approximate.

In the example presented by Professor Cross, joint displacement, due either to horizontal forces or to unsymmetrical vertical loads, has been disregarded. The writer wishes to submit a direct procedure which he believes is a general solution of the problem.

The writer prefers to use the same convention for signs of moments as ordinarily used in the slope-deflection method; that is, clockwise moments at the ends of members are positive and counter-clockwise moments are negative. The general slope deflection equation,

$$M_{AB} = 2EK\,(2\theta_A + \theta_B - 3R) \pm C \quad \ldots\ldots\ldots\ldots\ldots (76)$$

---

[49] "The Column Analogy," by Hardy Cross, *Bulletin 215*, Eng. Experiment Station, Univ. of Illinois, Urbana, Ill.

[50] "Mars and Its Canals," Preface, p. IX.

[51] Assoc. Prof. of Civil Eng., Univ. of Southern California, Los Angeles, Calif.

expressed in the form,

$$M_{AB} = 2E\left(2\frac{I}{L}\theta_A + \frac{I}{L}\theta_B - 3\frac{I}{L^2}d\right) \pm C \ldots\ldots\ldots\ldots\ldots (77)$$

suggests four steps for a general solution, as follows:

I (*a*).—Compute the fixed-end moments at the ends of all members.

I (*b*).—Compute and properly distribute the moments at the ends of the columns, due to their deflection, before rotation of the joints is assumed to have taken place.

I (*c*).—Add the moments obtained in Steps I (*a*) and (*b*) and place these values at the ends of members in a line diagram of the frame.

II.—Distribute the unbalanced moment at each joint of the frame so that the sum of the moments at each joint is equal to zero.

III.—"Carry over." In Steps II and III, rotation of the joints is assumed to have taken place.

IV.—Correct the column moments so that the sum of all moments acting on the columns is equal to zero. The distribution of unbalanced column moments is made in the same way as in Step I (*b*), and for the same reason.

The sum of the values found by Steps I, II, and III, for the girders and Steps I, II, III, and IV for the columns is the first approximate value of the moments sought. Better values may be obtained by repeating Steps II and III for the girders and Steps II, III, and IV for the columns, two or more times. Usually, three cycles of operations will give results which are sufficiently accurate for all practical purposes.

The foregoing steps can best be explained by examples selected for the purpose. Steps I (*a*), II, and III have been described by Professor Cross and further explanation of them is not necessary. Three examples are worked out completely, illustrating particularly Steps I (*b*) and IV, and the results, after three cycles of operations, are compared with those obtained by the slope-deflection method (see Table 15).

TABLE 15.—COMPARISON OF MOMENTS DETERMINED BY DISTRIBUTING FIXED-END MOMENTS WITH THOSE BY SLOPE-DEFLECTION METHOD.

Item	Description	MOMENTS AT JOINTS INDICATED, IN THOUSANDS OF INCH-POUNDS					
		M_{DA}	M_{AD}	M_{AB}	M_{BA}	M_{BC}	M_{CB}
				EXAMPLE 1			
1	Slope-deflection method..	−551.51	−127.27	+127.27	+200.00	−200.00	−321.21
2	By Fig. 44...............	−550.50	−126.80	+125.00	+197.00	−201.50	−321.20
				EXAMPLE 2			
3	Slope-deflection method..	+7.55	+18.14	−18.14	+16.14	−16.14	−9.55
4	By Fig. 45...............	+7.34	+17.68	−18.49	+16.49	−15.68	−9.34
				EXAMPLE 3			
5	Slope-deflection method..	−50.00	−33.33	+33.33	+16.67	−16.67	−16.67
6	By Fig. 46...............	−49.85	−34.23	+31.83	+16.79	−15.78	−16.17

Example 1.—The frame for this example and its solution, are shown in Fig. 44. The columns are assumed of equal length and the relative stiffnesses, $\frac{I}{L}$, of the various members are as shown in circles.

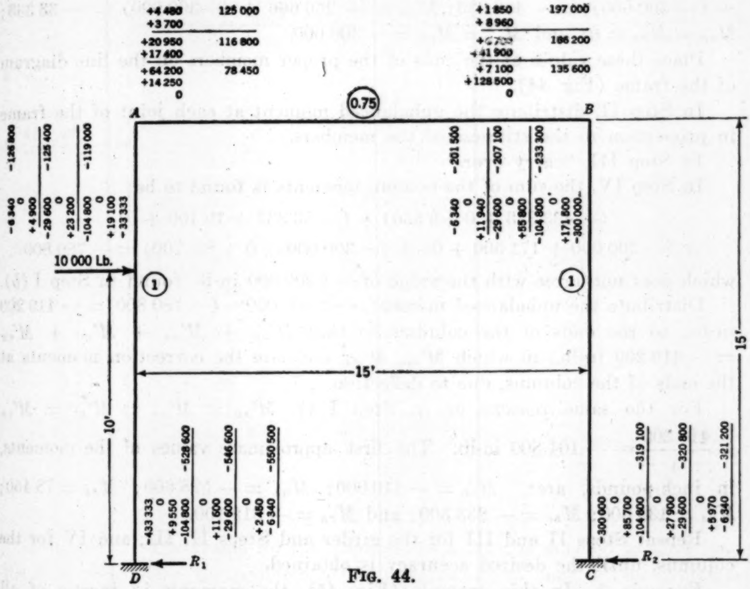

+4 480
+3 700
+20 950
+17 400
+64 200
+14 250
0
125 000
116 800
78 450
+1 850
+8 960
+6 700
+41 900
+7 100
+128 500
0
197 000
186 200
135 600
0.75
A
B
−128 800
−6 340
0
+4 900
−125 400
−29 600
0
+23 200
−119 000
−104 800
0
+19 100
−33 333
10 000 Lb.
1
−201 500
−6 340
0
+11 940
−207 100
−29 600
0
+55 800
−233 300
−104 800
0
+171 800
−300 000
1
15'
15'
10'
−433 333
0
+9 550
−104 800
−528 600
0
+11 600
−29 600
−546 600
0
+2 450
−6 340
−550 500
−300 000
0
+85 700
−104 800
−319 100
0
+27 900
−29 600
−320 800
0
+5 970
−6 340
−321 200
R_1
R_2
D
C

FIG. 44.

In Step I (*a*), fixed-end moments, in inch-pounds, are $M_{DA} = -133\,333$; $M_{AD} = +266\,666$; and $M_{AB} = M_{BA} = M_{BC} = M_{CB} = 0$. In Step I (*b*), the algebraic sum of all moments acting on the columns must be equal to zero. Therefore,

$$M_{AD} + M_{DA} + M_{BC} + M_{CB} = -1\,200\,000 \ldots\ldots\ldots\ldots (78)$$

Before rotation of the joints is assumed to have taken place, the following moments exist at the ends of the columns, due to their deflection. The deflections, *d* (slope-deflection notation), for Columns *AD* and *BC* are equal:

$$M'_{AD} = M'_{DA} = \frac{-3\,I_{AD}\,d\,(2E)}{L^2_{AD}} \ldots\ldots\ldots\ldots\ldots\ldots (79)$$

and,

$$M'_{BC} = M'_{CB} = \frac{-3\,I_{BC}\,d\,(2E)}{L^2_{BC}} \ldots\ldots\ldots\ldots\ldots\ldots (80)$$

Therefore,

$$\frac{M'_{AD}}{M'_{BC}} = \frac{I_{AD}\,L^2_{BC}}{I_{BC}\,L^2_{AD}} \ldots\ldots\ldots\ldots\ldots\ldots (81)$$

In this example, $I_{AD} = I_{BC}$ and $L^2_{BC} = L^2_{AD}$. Hence, $M'_{AD} = M'_{DA} = M'_{BC} = M'_{CB}$. Distribute the moment, − 1 200 000 in-lb., to the ends of the columns so that, $M'_{AD} + M'_{DA} + M'_{BC} + M'_{CB} = -1\,200\,000$ in-lb. Then, $M'_{AD} = M'_{DA} = M'_{BC} = M'_{CB} = -300\,000$ in-lb.

In Step I (*c*) by combining the moments obtained in Steps I (*a*) and (*b*) the initial values, in inch-pounds, are as follows: $M'_{DA} = -133\,333 + (-300\,000) = -433\,333$; $M_{AD} = +266\,666 + (-300\,000) = -33\,333$; $M_{AB} = M_{BA} = 0$; and $M_{BC} = M_{CB} = -300\,000$.

Place these values at the ends of the proper members on the line diagram of the frame (Fig. 44).

In Step II, distribute the unbalanced moment at each joint of the frame in proportion to the stiffness of the members.

In Step III, "carry over."

In Step IV, the sum of the column moments is found to be:

$$(-433\,333 + 0 + 9\,550) + (-33\,333 + 19\,100 + 0)$$
$$+ (-300\,000 + 171\,500 + 0) + (-300\,000 + 0 + 85\,700) = -780\,800$$

which does not agree with the value of $-1\,200\,000$ in-lb. found in Step I (*b*).

Distribute the unbalanced moment, $-1\,200\,000 - (-780\,800) = -419\,200$ in-lb., to the ends of the columns so that $M'_{AD} + M'_{DA} + M'_{BC} + M'_{CB} = -419\,200$ in-lb., in which M'_{AD}, M'_{DA}, etc., are the correction moments at the ends of the columns, due to deflection.

For the same reasons as in Step I(*b*), $M'_{AD} = M'_{DA} = M'_{BC} = M'_{CB} = \frac{419\,200}{4} = -104\,800$ in-lb. The first approximate values of the moments, in inch-pounds, are: $M_{AD} = -119\,000$; $M_{DA} = -528\,600$; $M_{AB} = 78\,450$; $M_{BA} = 135\,600$; $M_{BC} = -233\,300$; and $M_{CB} = -319\,100$.

Repeat Steps II and III for the girder and Steps II, III, and IV for the columns, until the desired accuracy is obtained.

Example 2.—In this example (Fig. 45), the moments of inertia of all members are assumed equal and the relative stiffnesses are shown in circles, as in Example 1.

In Step I (*a*), the fixed-end moments for the girder are -40 at *A* and $+20$ at *B*; and zero at both ends for each column.

In Step I (*b*), there are no horizontal forces acting on the columns so that the sum of the column moments is equal to zero, before rotation of the joints is assumed to have taken place. Therefore, no distribution of moments is required at this point.

Steps II and III are carried out as usual.

In Step IV, the sum of the column moments is found not equal to zero, because of the deflection of the frame caused by the unsymmetrical loading; that is, $(0 + 16 + 0) + (0 + 0 + 8) + (0 - 8 + 0) + (0 + 0 - 4) = 12$. Obviously, the unbalanced moment is -12, and this value is distributed to the ends of the columns so that, $M'_{AD} + M'_{DA} + M'_{BC} + M'_{CB} = -12$, as in Example 1, Steps I (*b*) and IV. The correction moments due to deflection are, for the same reasons, as in Example 1, $M'_{AD} = M'_{DA} = M'_{BC} = M'_{CB} = -3$. The first approximate values of the moments are determined next, and the prescribed cycle of operations is repeated.

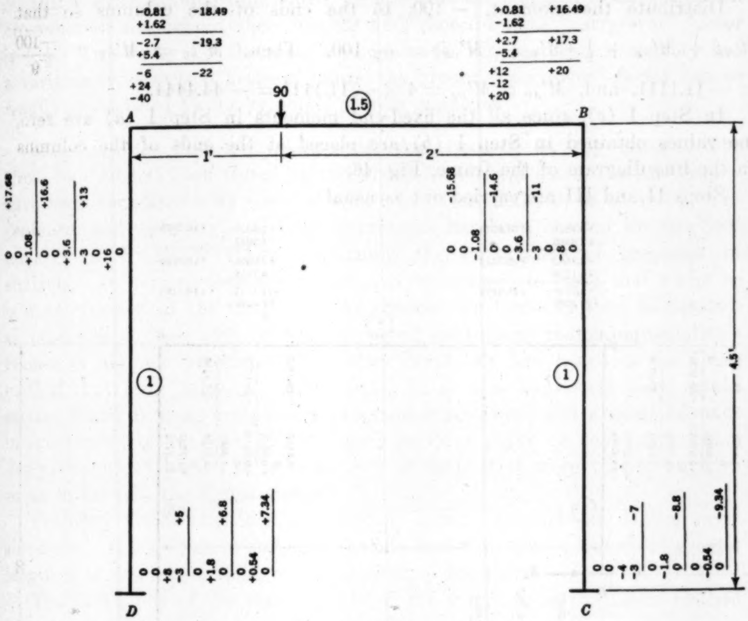

-0.81
+1.62
-18.49
-2.7
+5.4
-19.3
-6
+24
-40
-22
+0.81
-1.62
+16.49
+2.7
-5.4
+17.3
+12
-12
+20
+20
90
1.5
A
B
1'
2'
+17.68
0
0
+1.08
+16.6
0
0
+3.6
+13
-3
0
+16
0
-15.68
0
0
-1.08
-14.6
0
0
-3.6
-11
-3
0
-8
0
1
1
4.5'
+5
0
0
+6
-3
+6.8
0
+1.8
0
+7.34
0
+0.54
0
-7
0
0
-4
-3
-8.8
0
-1.8
0
-9.34
0
-0.54
0
D
C

FIG. 45.

Example 3.—The frame for this example (Fig. 46), is selected to show how the four steps solve the problem of an unsymmetrical bent. The moments of inertia of all members are assumed equal.

In Step I (*a*), all fixed-end moments are zero.

In Step I (*b*), the algebraic sum of all moments acting on the columns must be equal to zero. Therefore,

$$M_{AD} + M_{DA} + M_{BC} + M_{CB} + R_1 \times 10 + R_2 \times 20 = 0 \ldots\ldots\ldots (82)$$

Furthermore,

$$R_1 + R_2 = 10 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (83)$$

and,

$$M_{BC} + M_{CB} + R_2 \times 20 = 0 \ldots\ldots\ldots\ldots\ldots\ldots (84)$$

Eliminating R_1 and R_2 from Equation (82),

$$M_{AD} + M_{DA} + \tfrac{1}{2}(M_{BC} + M_{CB}) = -100 \ldots\ldots\ldots\ldots (85)$$

Before rotation of the joints is assumed to have taken place, the moments in Equations (79), (80), and (81) exist at the ends of the columns, due to their deflection. The deflections, *d*, for Columns *AD* and *BC* are equal.

In this example, $I_{AD} = I_{BC}$ and $L^2_{BC} = 4\,L^2_{AD}$. Hence, by Equation (80), $M'_{AD} = 4\,M'_{BC}$.

Distribute the moment, − 100, to the ends of the columns so that $M'_{AD} + M'_{DA} + \frac{1}{2}(M'_{BC} + M'_{CB}) = -100$. Then, $M'_{BC} = M'_{CB} = \frac{-100}{9} = -11.1111$, and, $M'_{AD} = M'_{DA} = 4 \times -11.1111 = -44.4444$.

In Step I (*c*), since all the fixed-end moments in Step I (*a*) are zero, the values obtained in Step I (*b*) are placed at the ends of the columns on the line diagram of the frame, Fig. 46.

Steps II and III are carried out as usual.

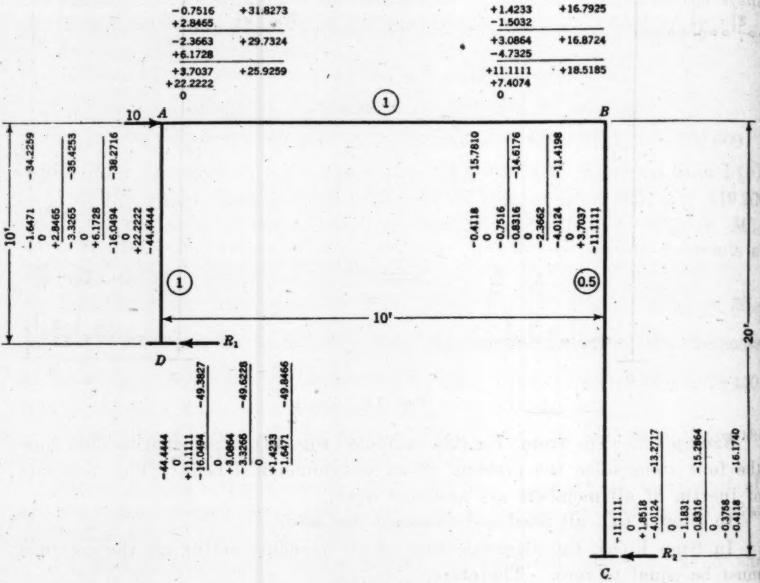

−0.7516 +31.8273
+2.8465
−2.3663 +29.7324
+6.1728
+3.7037 +25.9259
+22.2222
0
+1.4233 +16.7925
−1.5032
+3.0864 +16.8724
−4.7325
+11.1111 +18.5185
+7.4074
0
①
10
A
B
10'
−1.6471 −34.2259
0
+2.8465
−3.3265 −35.4253
0
+6.1728
−16.0494 −38.2716
0
+22.2222
−44.4444
①
−0.4118 −15.7810
0
−0.7516
−0.8316 −14.6176
0
−2.3662
−4.0124 −11.4198
0
+3.7037
−11.1111
(0.5)
10'
20'
R₁
D
−44.4444
0
+11.1111
−16.0494 −49.3827
0
+3.0864
−3.3266 −49.6228
0
+1.4233
−1.6471 −49.8466
−11.1111
0
+1.8518
−4.0124 −13.2717
0
−1.1831
−0.8316 −15.2864
0
−0.3758
−0.4118 −16.1740
R₂
C

FIG. 46.

In Step IV, the moment relation, as found in Step I (*b*) (see Equation (85)), is not satisfied, since $-22.2222 - 33.3333 + \frac{1}{2}(-7.4074 - 9.2593) = -63.8888$. Distribute the unbalanced moment, $-100 - (-63.8888) = -36.1112$, to the ends of the columns so that, $M'_{AD} + M'_{DA} + \frac{1}{2}(M'_{BC} + M_{CB}) = -36.1112$, in which, M'_{AD}, M'_{DA}, etc., are the correction moments at the ends of the columns, due to deflection.

For the same reasons as in Step I (*b*), $M'_{BC} = M'_{CB} = \frac{-36.1112}{9} = -4.0124$; and, $M'_{AD} = M'_{DA} = 4\,M'_{BC} = -16.0494$.

The first approximate values of the moments are as follows: $M_{AD} = -38.2716$; $M_{DA} = -49.3827$; $M_{AB} = 25.9259$; $M_{BA} = 18.5185$; $M_{BC} = -11.4198$; and $M_{CB} = -13.2717$. Repeat Steps II and III for the girder and Steps II, III, and IV for the columns, until the desired accuracy is obtained.

In the three examples presented, all members are assumed to have uniform cross-section throughout their lengths and, therefore, the "carry-over" factor is $+\frac{1}{2}$. This is by far the most common case. If, however, a member has a variable cross-section between joints, the proper "carry-over" factor can be readily determined by the principle of area moments.

MARSHALL G. FINDLEY,[52] ASSOC. M. AM. SOC. C. E. (by letter).—The Cross method has been tested by the writer for several cases, and the results have been compared with those obtained by other methods, as to speed, convenience, and accuracy. A false impression has been created by the final remarks of Professor Cross concerning the importance of precision in analysis. To correct this, the writer may be allowed to state that while he is a precisionist of the third type, he expects the Cross method to displace all others in at least 90% of all analyses of continuous beams or monolithic frames in his own practice. The writer previously had heard of the Cross method, but only vaguely. After trial, he is now convinced that within certain limits it is an instrument of extreme precision, and a great advance in indeterminate analysis. The writer's previous paper on continuous beam design practice[53] would be considerably revised, if it were to be rewritten so as to include the Cross method.

Professor Cross mentions, in passing, certain methods of dealing with side-sway. It would seem desirable that he deal with these methods at greater length in a separate article, or in his closing discussion.

The ambiguity of the sign-convention for moments in the Cross method is probably best corrected by the clockwise-positive convention, as developed in previous discussions.

The bent to which the method was applied in the paper (see Fig. 1) was evidently chosen to make clear several more or less difficult variations. An ordinary continuous beam of several spans or a monolithic frame, without these variations of arrangement, may be tentatively designed, sections may be selected and revised, and final analyses of great precision may be made, all in the time formerly taken for the tentative first analysis, and with a greater feeling of security concerning freedom from analytic errors. The usual procedure should be to make a number of analyses for the final design, one for dead load and one for each group or span of live loads, and then to use the method of summation to determine all maxima. This procedure has often been impossible on account of time limits when using other methods, but when the Cross method is used, it will be applied to many more situations where it is needed.

GEORGE E. LARGE,[54] ASSOC. M. AM. SOC. C. E. (by letter).—The writer believes the profession stands greatly indebted to Professor Cross for his development of the moment distribution method for analyzing continuous frames. A theoretically exact solution of many statically indeterminate structures can now be made with almost no mathematical drudgery. Moreover, the

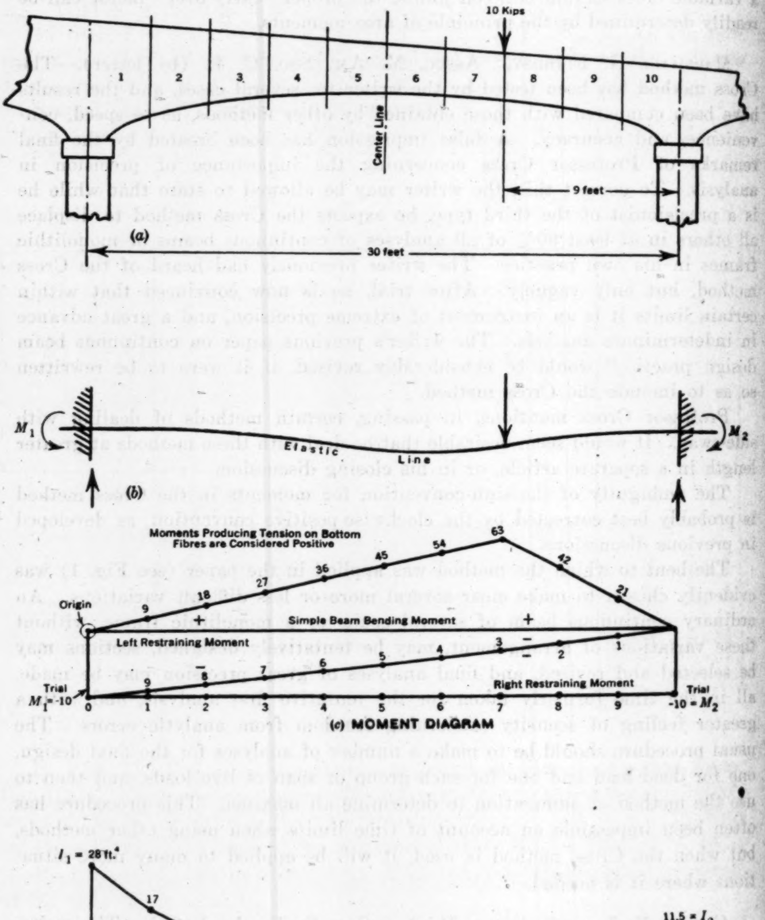
---

[52] Structural Designer, Harbor Commission, Milwaukee, Wis.

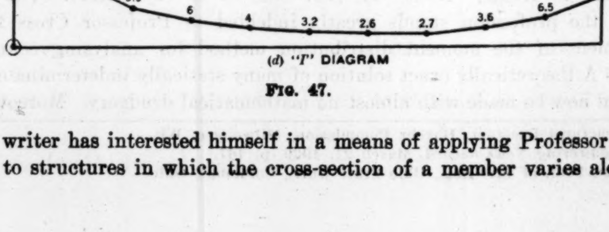
[53] *Engineering News-Record*, March 21, 1929, p. 467.

[54] Asst. Prof. of Civ. Eng., Ohio State Univ., Columbus, Ohio.

engineer can visualize the action of the structure while the solution is unfolding, and can thus develop a valuable sense of continuity seldom acquired in the use of calculus methods.


10 Kips
1
2
3
4
5
6
7
8
9
10
Center Line
9 feet
(a)
30 feet
M_1
M_2
Elastic Line
(b)
Moments Producing Tension on Bottom Fibres are Considered Positive
63
54
45
36
27
18
9
42
21
Origin
+
Simple Beam Bending Moment
Left Restraining Moment
+
3
2
1
−
4
5
6
7
8
9
Right Restraining Moment
Trial M_1 = -10
Trial -10 = M_2
0
1
2
3
4
5
6
7
8
9

(c) MOMENT DIAGRAM



(d) "I" DIAGRAM

FIG. 47.

The writer has interested himself in a means of applying Professor Cross' method to structures in which the cross-section of a member varies along its

length. Previous experience in analyzing and testing some such structures showed that an average value of the moments of inertia of a non-uniform member should not be substituted in the $\frac{I}{L}$ expression for stiffness of uniform members; neither are fixed-end moments and carry-over factors the same. Obviously, there is an infinite number of possible variations of the cross-section of members along their lengths. In comparatively few cases is it possible and feasible to write the equation of moment of inertia variation and perform the necessary integrations to find the required constants. The tables and diagrams prepared by some investigators cannot well cover all the possibilities.

A workable method is presented herein which permits of any conceivable variation of the moment of inertia of a member. It merely requires that the moment of inertia variation be known. Having found the fixed-end moments, and the stiffness and carry-over factors by this method, the solution is completed by moment distribution.

When members are unsymmetrical about the center line of a span there will be different stiffness and carry-over factors for the two ends of a member, and the two fixed-end bending moments due to symmetrically placed loads will have different values. In such cases, finding these six constants presents the chief problem. This general case is illustrated by the following example. When the member and the loading are each symmetrical, the work is shortened greatly.

The General Case: Moment of Inertia Variable; Unsymmetrical Member and Loading.—Fig. 47(*a*) represents a beam or slab construction of variable section involving complete dissymmetry. The fixed-end moments will be found first, then the carry-over and stiffness factors. To determine the fixed-end moments, let M_1 and M_2 be the unknown fixed-end moments at the large and small ends, respectively. Although ordinarily they will not be equal, it is convenient to assume arbitrary trial values, such as 10 kip-ft. for each, which are utilized to construct the negative parts of the composite moment diagram shown at Fig. 47(*c*). When the simple-beam bending moment due to the superimposed load is included on the positive side of such a diagram, the algebraic sum of the ordinates at any point gives the net bending moment due to the loading and assumed restraint.

Knowing the moment of inertia variation, as at *d*, the $\frac{M}{I}$ diagram, Fig. 48, must be constructed to scale, keeping the three parts separate as was done in the moment diagram at *c*, Fig. 47. The other diagrams need have no exact scale, but should be drawn in correct relation to each other.

The area under each curve of the $\frac{M}{I}$ diagram is found by a summation of the individual trapezoids. In this case, since all are of the same width, the sum of the average ordinates is conveniently taken as the area. (See Table 16.) The moments of the areas about the large end of the member are also found and the three centroids determined.

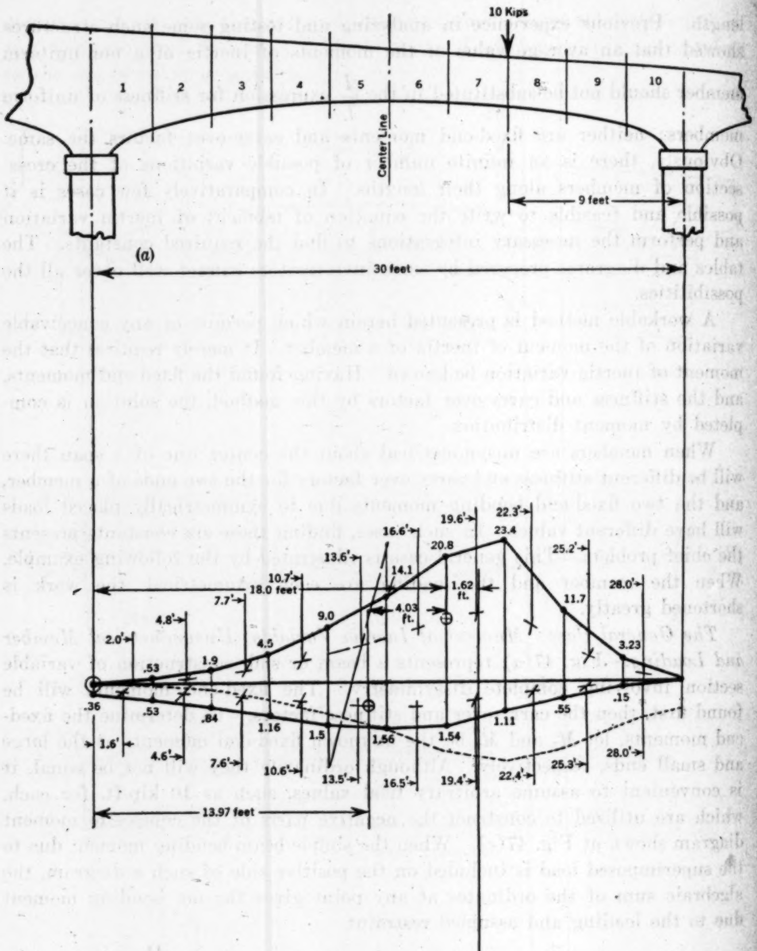

10 Kips
1
2
3
4
5
6
7
8
9
10
Center Line
9 feet
(a)
30 feet
22.3'
19.6'
23.4
16.6'
20.8
25.2'
13.6'
14.1
10.7'
18.0 feet
1.62 ft.
28.0'
7.7'
11.7
4.03 ft.
4.8'
9.0
2.0'
4.5
3.23
1.9
.53
.36
.53
.84
.15
.55
1.16
1.5
1.56
1.54
1.11
1.6'
4.6'
7.6'
10.6'
13.5'
16.5'
19.4'
22.4'
25.3'
28.0'
13.97 feet
0
.06
.21
0.5
1.0
1.56
2.30
2.60
2.22
1.39
0.87
2.0'
4.8'
7.8'
10.6'
13.6'
16.6'
19.5'
22.5'
25.4'
28.4'
19.62 feet

FIG. 48.—$\frac{M}{I}$ DIAGRAM.

By moment-area principles the algebraic sum of the true $\frac{M}{I}$ areas must equal zero:

$$\frac{9.12}{10} M_1 + \frac{12.27}{10} M_2 = 89.15 \text{............} (86)$$

The algebraic sum of the moments of the true $\frac{M}{I}$ areas about the large end of the member must also equal zero:

$$\frac{127.33}{10} M_1 + \frac{240.79}{10} M_2 = 1\,607.8 \text{........} (87)$$

Solving Equations (86) and (87) simultaneously, the true fixed-end moments (in kip-foot units) are, $M_1 = 27.95$, and $M_2 = 51.95$.

To determine the carry-over factors, consider a member fixed at one end and hinged at the other, as in Fig. 49(*b*). Professor Cross defined a carry-over factor as the ratio of the moment at the fixed end to the moment producing rotation at the rotated end. For members with constant moments of inertia, it is 0.5, but for non-uniform members its values must be calculated. In every case, the rotated end corresponds to the joint just released for moment distribution, while the fixed end is the remote end of the member. Unsymmetrical members have two carry-over factors, depending upon which end is rotated.

TABLE 16.—COMPUTATIONS TO DETERMINE FIXED-END MOMENTS AND CARRY-OVER FACTORS

Piece No. (Fig. 47)	Positive $\frac{M}{I}$			Trial $\frac{M_1}{I}$			Trial $\frac{M_2}{I}$		
	Average ordinate	Moment arm, in feet, from the large end	Moment about the large end	Average ordinate	Moment arm, in feet, from the large end	Moment about the large end	Average ordinate	Moment arm, in feet, from the large end	Moment about the large end
1..............	0.265	2.0	0.53	0.45	1.6	0.72	0.03	2.0	0.06
2..............	1.21	4.8	5.80	0.68	4.6	3.13	0.13	4.8	0.62
3..............	3.20	7.7	24.60	1.00	7.6	7.60	0.36	7.8	2.81
4..............	6.75	10.7	72.30	1.33	10.6	14.10	0.75	10.6	7.95
5..............	11.55	13.6	157.00	1.53	13.5	20.70	1.28	13.6	17.40
6..............	17.45	16.6	290.00	1.55	16.5	25.60	1.93	16.6	32.00
7..............	22.10	19.6	433.00	1.32	19.4	25.60	2.45	19.5	47.85
8..............	17.55	22.3	391.50	0.83	22.4	18.93	2.41	22.5	54.30
9..............	7.46	25.2	188.00	0.35	25.3	8.85	1.80	25.4	45.70
10..............	1.61	28.0	45.10	0.075	28.0	2.10	1.13	28.4	32.10
Area* ∝	89.15	......	......	9.12	......	......	12.27	......	....
M ∝	......	......	1 607.83	......	......	127.33	......	......	240.79
Distance of the centroid from the large end, in feet........	$\frac{1607.8}{89.15} = 18.00$			$\frac{127.33}{9.12} = 13.97$			$\frac{240.8}{12.27} = 19.62$		

* In computations for fixed-end moments and carry-over factors, the sum of the ordinates is conveniently called the "area" of the diagrams. The actual area is three times the value given.

Continuing the example, and referring to Fig. 49(*b*), the chief problem is to find what fixed-end moment, M_B, is necessary at the small end when the large end is rotated by a moment of 10 kip-ft. If a trial value of $M_B = -10$

is taken, the $\frac{M}{I}$ diagram (Fig. 48) may be sketched immediately from the areas, and the centroids may be found in the fixed-end moment calculations of Table 16. Then, by moment-area principles the algebraic sum of the moments of the true $\frac{M}{I}$ areas about the large end of the member equals zero; thus, $9.12 \times 13.97 - \frac{12.27 \times 19.62}{10} M_B = 0$, and the true $M_B = 5.29$ kip-ft. The carry-over factor from the large end to the small end equals 0.529.

By rotating the small end of the member, as in Fig. 50, and utilizing the same areas, but taking summation of moments about the small end, $12.27 \times 10.38 - \frac{9.12 \times 16.03}{10} M_D = 0$, and $M_D = 8.69$ kip-ft. The carry-over factor from the small end to the large end equals 0.869.

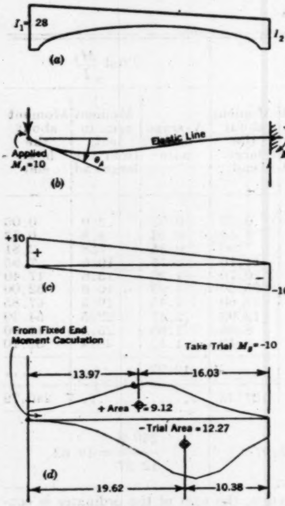

$I_1 = 28$
$I_2 = 11.5$
(a)
Applied $M_A = 10$
θ_A
Elastic Line
$M_B = ?$
(b)
+10
+
-
-10
(c)
From Fixed End Moment Caculation
Take Trial $M_B = -10$
13.97
16.03
+ Area = 9.12
- Trial Area = 12.27
(d)
19.62
10.38

FIG. 49.

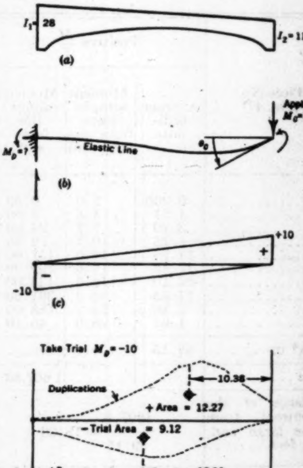

$I_1 = 28$
$I_2 = 11.5$
(a)
Applied $M_D = 10$
$M_D = ?$
Elastic Line
θ_D
(b)
+10
+
-
-10
(c)
Take Trial $M_D = -10$
Duplications
10.38
+ Area = 12.27
- Trial Area = 9.12
(d)
16.03

FIG. 50.

Stiffness Factor.—The stiffness of a member is measured by the moment necessary to turn the hinged end of a member through a unit angle when the other end is fixed. When a joint is released in the moment-distribution method, all the ends of the members connecting directly thereto are turned through the same angle. Of the entire moment to be distributed, the part that each member assumes is proportional to its stiffness.

It is proved in mechanics that, for a uniform member, $E\theta_A = \frac{M_A L}{4I}$, or $M_A = \frac{4EI\theta_A}{L}$. (See Fig. 49.)

Since, at any given joint, E and θ_A are constant for all members connecting thereto, the moment upon each will be proportional to its ratio, $\frac{I}{L}$. Accordingly, $\frac{I}{L}$ is used to state the relative stiffnesses of the uniform members of a structure, and is called the stiffness factor.

Since a structure may consist of both uniform and non-uniform members, it is necessary to state the stiffness factors for the non-uniform members in terms of $\frac{I}{L}$ also; thus,

$$S = \frac{k\,I_0}{L} \qquad \text{(88)}$$

in which, S is the stiffness factor for a non-uniform member; k is a constant to be determined for each member; and I_0 is the moment of inertia at a particular section of reference.

Furthermore, for non-uniform members,

$$E\,\theta_A = \frac{M_A\,L}{4\,k\,I_0} \qquad \text{(89)}$$

Referring again to Fig. 49, one may state concerning the algebraic sum of the positive and true negative areas, by moment-area principles: $E\theta_A = 3\,(9.12 - 0.529 \times 12.27) = \frac{10 \times 30}{k_1 \times 4 \times 28}$, using 3 to state the real area (see Table 16) and the largest moment of inertia, $I_1 = 28$, for reference Finally, $k_1 = 0.34$ and the stiffness factor for the large end $= 0.34\,\frac{I_1}{L}$.

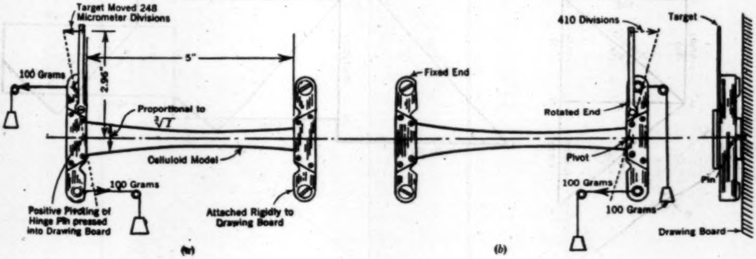

Target Moved 248 Micrometer Divisions
5"
2.96"
100 Grams
Proportional to $\sqrt[3]{I}$
Celluloid Model
100 Grams
Positive Pinching of Hinge Pin pressed into Drawing Board
Attached Rigidly to Drawing Board
(a)
Fixed End
410 Divisions
Target
Rotated End
Pivot
100 Grams
100 Grams
Pin
Drawing Board
(b)

FIG. 51.

Similarly, for the small end (see Fig. 50): $E\theta_0 = 3\,(12.27 - 0.869 \times 9.12) = \frac{10 \times 30}{k_2 \times 4 \times 28}$. Again, using the largest moment of inertia as a reference, $k_2 = 0.205$ and the stiffness factor for the small end $= 0.205\,\frac{I_1}{L}$.

Model Check of Stiffnesses Found.—In order to check the stiffnesses of the two ends of the member as computed herein, a model was made from $\frac{1}{16}$-in. sheet celluloid embodying the exact moment of inertia variation used in the calculations. A bar of a Beggs deformeter gauge was attached to each end of the model, as shown in Fig. 51. One bar was fixed to a drawing-board with screws, while the other was free to rotate about its hinge-pin which had been pressed into the drawing-board. A known torque was applied to the pivoted bar and the displacement of a target was observed with a micrometer microscope. The model was then reversed, and the other end rotated under the same torque.

The average displacements were 248 and 410 micrometer divisions for the rotation of the large and small ends, respectively. This indicates that the actual stiffness of the large end is $\frac{410}{248} = 1.652$ times that of the small end. The corresponding ratio of the k's found in calculating the stiffnesses, is $\frac{0.340}{0.205} = 1.658$. This remarkable correlation also verifies the values of the carry-over factors since the stiffnesses were calculated therefrom.

Sophus Thompson[55] and **R. W. Cutler**,[56] **Juniors, Am. Soc. C. E.** (by letter).—In this paper, mention was made of the application of the moment distribution method to the solution of secondary stresses in a truss. The writers have made a study of this phase. For the sake of simplicity and comparison of results, the truss analyzed herein has been chosen from among those treated in current textbooks by the "exact" method.

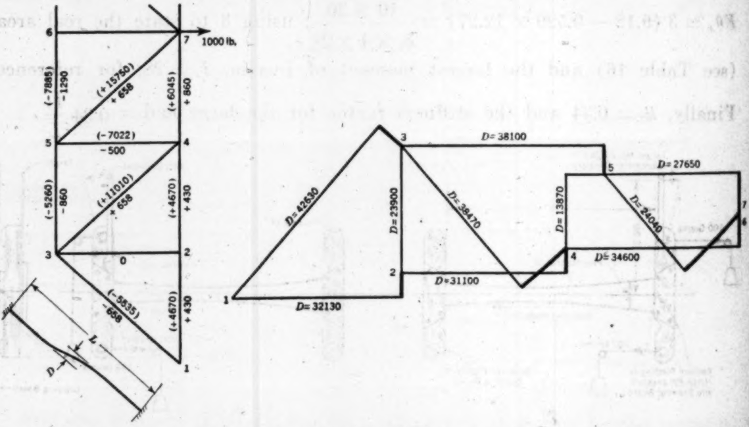


FIG. 52.—WILLIOT DIAGRAM FOR ANALYZING SECONDARY STRESSES BY CROSS METHOD.

The method used by the writers requires the construction of the Williot diagram (Fig. 52), from which the value of D for each member is obtained, D being the displacement, at right angles to the axis of the member, of one end with respect to the other. By cutting each member at the mid-point and

---

[55] Asst. Prof. of Civ. Eng., Southern Methodist Univ., Dallas, Tex.

[56] Dallas, Tex.

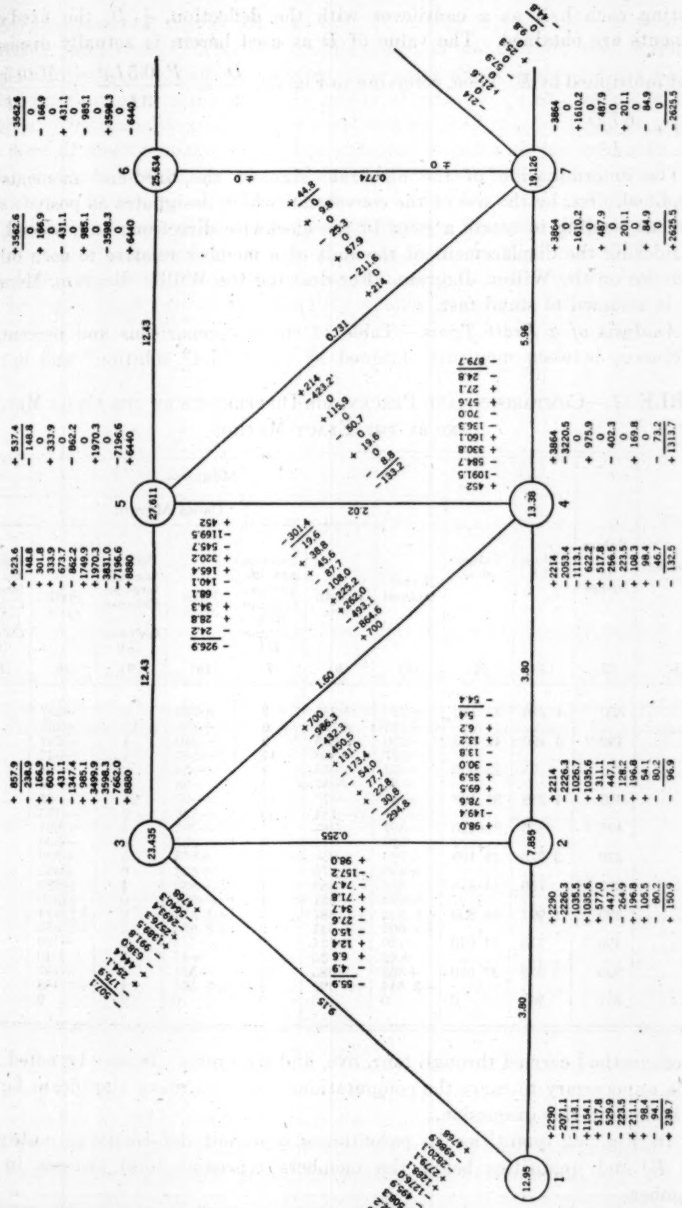

1
2
3
4
5
6
7
12.95
7.855
23.435
13.38
27.611
25.634
19.126
3.80
3.80
9.15
0.255
1.60
12.43
12.43
2.02
0.731
5.96
0.774
± 0
± 0
+ 857.9 − 238.9 + 166.9 − 603.7 − 431.1 − 1347.4 + 985.1 + 3499.9 − 3598.3 − 7662.0 + 8880
+ 523.6 − 148.8 + 301.8 + 333.9 − 673.7 − 862.2 + 1749.9 + 1970.3 − 3831.0 − 7196.6 + 8880
+ 537.4 − 148.8 0 + 333.9 0 − 862.2 0 + 1970.3 0 − 7196.6 + 6440
+ 3562.6 0 + 166.9 − 431.1 0 + 985.1 0 − 3598.3 0 + 6440
− 3562.6 0 − 166.9 + 431.1 0 − 985.1 0 + 3598.3 0 − 6440
+ 2290 − 2071.1 − 1113.1 + 1154.1 + 517.8 − 529.9 − 223.5 + 211.1 + 98.4 − 94.1 + 239.7
+ 2290 − 2226.3 − 1035.5 + 1035.6 + 577.0 − 447.1 − 264.9 + 196.8 + 105.5 − 80.2 + 150.9
+ 2214 − 2226.3 − 1026.7 + 1035.6 + 311.1 − 447.1 − 128.2 + 196.8 + 54.1 − 80.2 − 96.9
+ 2214 − 2053.4 − 1113.1 + 622.2 + 517.8 − 256.5 − 223.5 + 108.3 + 98.4 − 46.7 − 132.5
+ 3864 − 3220.5 0 + 975.9 0 − 402.3 0 + 169.8 0 − 73.2 + 1313.7
+ 3864 0 − 1610.2 0 − 487.9 0 − 201.1 0 − 84.9 0 − 2625.5
− 3864 0 + 1610.2 0 + 487.9 0 + 201.1 0 + 84.9 0 + 2625.5
+ 96.0 − 157.2 − 74.7 + 71.8 + 34.7 − 27.6 − 15.0 + 12.4 + 6.6 − 4.9 − 55.9
+ 98.0 − 149.4 − 78.6 + 69.5 + 35.9 − 30.0 − 13.8 + 13.2 + 6.2 − 5.4 − 54.4
+ 452 − 1169.5 − 545.7 + 320.2 + 165.4 − 140.1 − 68.1 + 34.3 + 28.8 − 24.2 − 926.9
+ 452 − 1091.5 − 584.7 + 330.8 + 160.1 − 136.3 − 70.0 + 57.6 + 27.1 − 24.8 − 879.7
+ 700 − 986.3 + 432.3 + 450.5 − 131.0 − 173.4 + 84.0 + 77.7 − 22.8 − 30.8 − 294.8
− 301.4 − 19.6 + 38.8 + 45.6 − 87.7 − 108.0 + 225.2 + 262.0 − 493.1 − 864.6 + 700
+ 214 − 423.2 0 + 116.9 0 − 50.7 0 + 19.6 0 − 8.8 − 133.2
+ 44.8 0 − 9.8 0 − 25.3 0 + 67.9 0 − 211.6 0 + 214
− 214 + 211.6 0 − 57.9 0 + 25.3 0 + 9.8 0 − 44.8
− 301.1 − 175.9 + 254.1 + 444.0 − 638.0 − 991.8 + 1389.5 + 2576.3 − 2493.4 − 5640.3 + 4766
− 239.7 − 226.5 + 222.3 + 495.9 − 508.8 − 1276.0 + 1288.1 + 2779.1 − 2820.1 − 4986.9 + 4766

FIG. 53.

treating each half as a cantilever with the deflection, ½ *D*, the fixed-end moments are obtained. The value of *D* as used herein is actually displacement multiplied by *E*. Then, referring to Fig. 52, $\dfrac{D}{2} = \dfrac{P\,(0.5\,L)^3}{3I} = \dfrac{M\,(0.5\,L)^2}{3I}$ or $M = \dfrac{6\,I\,D}{L^2}$.

The determination of the algebraic sign of the fixed-end moments is simplified, first, by the use of the convention which designates as positive any moment tending to rotate a joint in the clockwise direction; and, second, by considering the displacement of the ends of a member relative to each other, as shown on the Williot diagram. For drawing the Williot diagram, Member 6–7 is assumed to stand fast.

Analysis of a Pratt Truss.—Table 17 shows comparisons and percentage differences between moments obtained by the "exact" solution[57] and by the Cross method carried through four, five, and six cycles. It may be noted that it is unnecessary to carry the computations out to as many significant figures as is done in this discussion.

TABLE 17.—COMPARISONS AND PERCENTAGE DIFFERENCES BY THE CROSS METHOD AND BY THE EXACT METHOD

				MOMENTS						
					CROSS METHOD					
Member (1)	Values of *L*, in inches (2)	Values of *I* (3)	Values of *D* (4)	Exact method (5)	Fourth cycle (6)	Percentage difference, Column (6) to Column (5) (7)	Fifth cycle (8)	Percentage difference, Column (8) to Column (5) (9)	Sixth cycle (10)	Percentage difference, Column (10) to Column (5) (11)
1–2.......	320	1 218	32 130	+230	+235	2	+240	4	+237	3
2–1.......	......	......	....	+135	+126	6	+151	12	+139	3
1–3.......	490.7	4 490	42 630	—230	—235	2	—240	4	—237	3
3–1.......	......	......	....	—527	—586	11	—507	4	—542	3
2–3.......	372	95	23 900	—53	—55	4	—54	2	—54	2
3–2.......	......	......	....	—55	—57	4	—56	2	—56	2
2–4.......	320	1 218	31 100	—80	—70	12	—97	21	—85	6
4–2.......	......	......	....	—141	—184	30	—133	6	—153	8
3–4.......	490.7	805	38 470	—307	—287	7	—295	4	—291	5
4–3.......	......	......	....	—320	—321	3	—301	6	—309	3
3–5.......	320	3 978	38 100	+900	+930	3	+858	5	+889	1
5–3.......	......	......	....	+495	+370	25	+523	6	+463	6
4–5.......	372	750	13 870	—875	—882	1	—880	½	—882	1
5–4.......	......	......	....	—924	—931	1	—927	0	—930	1
4–7.......	320	1 907	34 600	+1 342	+1 387	3	+1 314	2	+1 344	0
7–4.......	......	......	....	+2 602	+2 541	2	+2 625	1	+2 589	½
5–7.......	490.7	358	24 040	—130	—124	5	—133	2	—130	0
7–5.......	......	......	....	+42	+35	17	+45	7	+40	5
5–6.......	320	3 978	27 650	+562	+685	22	+537	4	+597	6
6–5.......	......	......	....	+3 544	+3 396	4	+3 563	½	+3 488	2
6–7.......	372	288	0	0	0	0	0	0	0	0

In Fig. 52, quantities in parentheses represent deformations multiplied by *E*, and quantities below the members represent total stresses in the members.

---

[57] "Modern Framed Structures." Vol. II, by Johnson, Bryan, and Turneaure, p. 440.

ALFRED GORDON,[58] ASSOC. M. AM. SOC. C. E. (by letter).—The method of analyzing rigid frames, described in this paper, is one of those maddeningly simple solutions, of a baffling problem, that show sheer genius. By a flight of imagination amounting to pure poetry, Professor Cross has made the analysis of indeterminate structures even simpler than that of a simple truss. The flood of discussion his paper has unloosed is proof of its extraordinary verbal economy, nothing having arisen that was not indicated therein.

Although it may be only "dotting the i's and crossing the t's," the writer would point out more exactly the relation between Professor Cross's method and the tables[59] compiled by Walter Ruppel, Assoc. M. Am. Soc. C. E. The slope-deflection equations for any beam subjected to moments at each end may be represented (using Mr. Ruppel's nomenclature), as follows:

$$M_{AB} = \frac{1 - v}{pl\,[1 - (u + v)]}\,\theta_A + \frac{v}{pl\,[1 - (u + v)]}\,\theta_B \text{........} (90)$$

and,

$$M_{BA} = \frac{1 - u}{ql\,[1 - (u + v)]}\,\theta_B + \frac{u}{ql\,[1 - (u + v)]}\,\theta_A \text{........} (91)$$

or,

$$M_{AB} = \alpha\,\theta_A + \beta\,\theta_B \text{......................} (92)$$

and,

$$M_{BA} = \gamma\,\theta_B + \delta\,\theta_A \text{......................} (93)$$

in which, p is the area of the $\frac{M}{EI}$ diagram for $M = 1$ at the left support of the beam with unit span, having E and I (minimum) $= 1$; q is the corresponding area for $M = 1$ at the right support, with the same understandings; u is the distance from the left support to the left characteristic point when $l = 1$; and v is the distance from the right support to the right characteristic point when $l = 1$, and α and γ represent the numerators of the moment-distribution factors for the beams, AB and BA, the sum of such θ-coefficients for the members meeting at one point constituting the denominator.

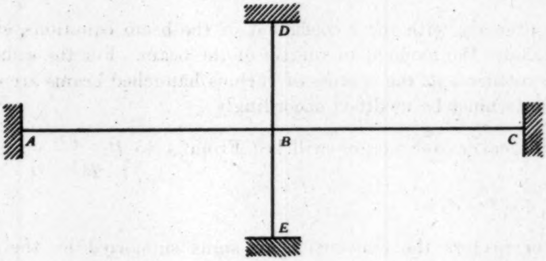


FIG. 54.

---

[58] Engr., Bridge Dept., C. P. Ry., Montreal, Que., Canada.

[59] *Transactions*, Am. Soc. C. E., Vol. 90 (1927), p. 167.

Thus, in Fig. 54, let the equations for the members be,

$$M_{BA} = \gamma\,\theta_B + \delta\,\theta_A \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(94a)}$$

$$M_{BC} = \eta\,\theta_B + \kappa\,\theta_C \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(94b)}$$

$$M_{BD} = \nu\,\theta_B + \epsilon\,\theta_D \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(94c)}$$

$$M_{BE} = \tau\,\theta_B + \psi\,\theta_E \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(94d)}$$

Then, locking the ends remote from their point of juncture, θ_A, θ_C, θ_D, and θ_E are all equal to zero and,

$$M_{BA} = \gamma\,\theta_B \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(95a)}$$

$$M_{BC} = \eta\,\theta_B \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(95b)}$$

$$M_{BD} = \nu\,\theta_B \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(95c)}$$

$$M_{BE} = \tau\,\theta_B \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(95d)}$$

and the moment-distribution factors are:

Beam	Moment distribution factor	Beam	Moment distribution factor
$BA =$	$\dfrac{\gamma}{(\gamma + \eta + \nu + \tau)}$;	$BC =$	$\dfrac{\eta}{(\gamma + \eta + \nu + \tau)}$
$BD =$	$\dfrac{\nu}{(\gamma + \eta + \nu + \tau)}$;	$BE =$	$\dfrac{\tau}{(\gamma + \eta + \nu + \tau)}$

Furthermore, when such general equations are combined with the particular equation for a column, PQ, fixed at the base, Q (that is, $M_{pa} = 4\,\dfrac{EI}{h}\,\theta_p$), care must be taken that the moment of inertia used, is expressed in terms of that used as unity for the entire frame; otherwise, the term, $\dfrac{4\,EI}{h}$, cannot be compared properly with the θ-coefficient of the beam equations, since that coefficient includes the moment of inertia of the beam. For the same reason, if the inertia moments at the centers of various haunched beams are different, the θ-coefficients must be modified accordingly.

Finally, the carry-over factors will be: From A to B, $\dfrac{p\,u}{q\,(1-v)}$; and, from B to A, $\dfrac{q\,v}{p\,(1-u)}$.

The writer prefers the convention in signs supported by the majority of the discussers—all clockwise moments positive.

A. W. EARL,[60] M. AM. SOC. C. E. (by letter).—Structures composed of members the cross-section of which varies from point to point along their length are not, fortunately, of very common occurrence, since an accurate analysis entails considerable work. When such structures do occur, the writer believes that the method so clearly and concisely stated by Professor

---

[60] Berkeley, Calif.

Cross, can be used to advantage and he, therefore, submits a development of the expressions for fixed-end moments, stiffness factors, and carry-over factors for members of variable cross-section, hoping they may prove useful.

The following notation will be used:

M_{AB} = moment at left end, A, of Member AB.

M_{BA} = moment at right end, B, of Member AB.

L = length of Member AB.

θ_A = change in slope of the tangent to the elastic curve of the member at A.

θ_B = change in slope of the tangent to the elastic curve of the member at B.

d = deflection of one end of a member with respect to the other end, measured perpendicularly to the original position of the member.

F_A = area of $\frac{M}{I}$ - diagram, due to the end moment, M_{AB}

F_B = area of $\frac{M}{I}$ - diagram, due to the end moment, M_{BA}

$m = \frac{F_A}{M_{AB}L}$ = area of $\frac{M}{I}$ - diagram for $M = 1$ and $L = 1$.

$n = \frac{F_B}{M_{BA}L}$ = area of $\frac{M}{I}$ - diagram for $M_{BA} = 1$ and $L = 1$.

s = stiffness factor.

u = distance of centroid of Area m, from the left end.

v = distance of centroid of Area n, from the right end.

F = area of $\frac{M}{I}$ - diagram for the applied loading.

$\bar{x}$ = distance of centroid of Area F from the right end.

The two basic propositions of the slope-deflection method may be expressed as follows:[61]

Area of $\frac{M}{EI}$ - diagram between B and A,

$$\theta_B - \theta_A = \int_B^A \frac{M\,dx}{EI} \qquad \text{.....................(96)}$$

and moment of $\frac{M}{EI}$ - diagram between B and A about B,

$$d - L\,\theta_A = \int_B^A \frac{Mx\,dx}{EI} \qquad \text{.....................(97)}$$

These expressions may be written,

$$\theta_B - \theta_A = \frac{1}{E}\left[- M_{AB}\,m\,L + M_{BA}\,n\,L - F\right] \text{.........(98)}$$

$$d - L\,\theta_A = \frac{1}{E}\left[- M_{AB}\,m\,(1 - u)\,L^2 + M_{BA}\,n\,v\,L^2 - F\bar{x}\right] \text{....(99)}$$

---

[61] For proof of the underlying propositions and development of the slope-deflection equations see "Analysis of Statically Indeterminate Structures by the Slope Deflection Method," by W. M. Wilson, F. E. Rickart, and Camillo Weiss, *Bulletin 108*, Eng. Experiment Station, Univ. of Illinois. The argument and sign convention of this *Bulletin* have been followed.

Solving simultaneously for M_{AB} and M_{BA} and replacing mu with nv, or *vice versa*, as required:

$$M_{AB} = \frac{E}{mL\,(1-u-v)}\left[\theta_A\,(1-v) + \theta_B\,v - \frac{d}{L}\right] - F\,\frac{\bar{x} - Lv}{mL^2\,(1-u-v)} \quad (100)$$

and,

$$M_{BA} = \frac{E}{nL\,(1-u-v)}\left[\theta_A u + \theta_B\,(1-u) - \frac{d}{L}\right] + F\,\frac{L\,(1-u) - \bar{x}}{nL^2\,(1-u-v)} \quad (101)$$

Equations (100) and (101) are the equations of slope deflection for a member of varying cross-section.

That $mu = nv$ may be proved, as follows:

$$mu = \frac{F_A\,ul}{M_{AB}\,L} = \frac{\int_0^L \frac{M}{I}\,x\,dx}{M_{AB}\,L} \qquad nu = \frac{F_B\,vL}{M_{BA}\,L} = \frac{\int_0^L \frac{M}{I}\,(L-x)\,dx}{M_{BA}\,L}$$

$$M = \frac{M_{AB}}{L}\,(L-x) \qquad M = \frac{M_{BA}\,x}{L}$$

then, then,

$$mu = \frac{M_{AB}}{L}\,\frac{\int_0^L \frac{(L-x)\,x\,dx}{I}}{M_{AB}\,L} \qquad nv = \frac{M_{BA}}{L}\,\frac{\int_0^L \frac{(L-x)\,x\,dx}{I}}{M_{BA}\,L}$$

$$= \frac{1}{L^2}\int_0^L \frac{(L-x)\,x\,dx}{I} \qquad = \frac{1}{L^2}\int_0^L \frac{(L-x)\,x\,dx}{I}$$

therefore, $mu = nv$

Since, in the case of a member fixed at both ends, resting on unyielding supports, θ_A, θ_B, and $\frac{d}{L}$, each equal zero, the fixing moments at A and B, respectively, may be written equal to the last terms of Equations (100) and (101) as shown by Equations (102) (Table 18).

TABLE 18.—SUMMARY OF FACTORS FOR MOMENT DISTRIBUTION

Item.	End of beam	Fixing moment, M_F. (102)	Stiffness factor, S (103)	Carry-over factor, γ (104)
A..........	Left	$F\,\frac{\bar{x} - Lv}{mL^2\,(1-u-v)}$	$\frac{E\,(1-v)}{mL\,(1-u-v)}$	$\frac{v}{1-v}$
B.........	Right	$F\,\frac{L(1-u) - \bar{x}}{nL^2\,(1-u-v)}$	$\frac{E\,(1-u)}{nL\,(1-u-v)}$	$\frac{u}{1-u}$

The signs as given are correct for downward loading. In general, the sign of the fixing moment at the end of a member is opposite to that of the moment of the applied load about that end, clockwise moments being considered positive.

The stiffness factor is defined as the moment at one end of a member (resting on unyielding supports at both ends), necessary to produce unit rotation at that end when the other end is fixed. Then, the stiffness factor for the end, *A*, may be found by placing $\theta_A = 1$, $\theta_B = 0$, $F = 0$, and $\frac{d}{L} = 0$ in Equation (100) and the stiffness factor at the end, *B*, may be found by placing $\theta_B = 1$, $\theta_A = 0$, $F = 0$, and $\frac{d}{L} = 0$, in Equation (101). The results are as shown by Equations (103) (Table 18).

If E is constant, it may be made equal to unity in these equations, since comparative stiffness is all that is required.

The carry-over factor is defined as the ratio of the moment at the fixed end to the moment producing rotation at the rotating end of a member (which is on unyielding supports at both ends), if one end of the member is rotated while the other end remains fixed. Then, the carry-over factor at End *A* equals the ratio of M_{AB} to M_{BA} when θ_A, $\frac{d}{L}$, and F, are made equal to zero in Equations (100) and (101). Similarly, the carry-over factor at End *B* is the ratio of M_{BA} to M_{AB} when θ_B, $\frac{d}{L}$, and F, are made equal to zero in the same equations. Thus, they take the form of Equations (104) (Table 18.)

The quantities, m, n, v, u, F, and $\bar{x}$ having been determined, the fixing moments, stiffness, and carry-over factors are easily calculated and the analysis may proceed as for a structure with members of uniform cross-section. The "bulk" of the labor is in determining the six aforementioned quantities for each member of the structure. A well-ordered tabulation, Simpson's rule for approximate integration, and a calculating machine will be found very helpful.

The valuable tables[62] given by Walter Ruppel, Assoc. M. Am. Soc. C. E., may be used and the work thus greatly shortened. These tables give coefficients, s, t, u, v, p, and q, for use in the solution of continuous beams of variable cross-section by Fidler's method. Simple relationships exist which make them easily applicable to the slope-deflection method, or to the method of distributed moments. The tables cover a large variety of shapes and loadings.

As defined by Mr. Ruppel (using the writer's symbols) the heights, respectively, of the left and right characteristic points when L and W are equal to 1, are:

$$s = \frac{F\,\bar{x}}{L^3\,W m} \qquad \text{(105)}$$

and,

$$t = \frac{F\,(L - \bar{x})}{L^3\,W n} \qquad \text{(106)}$$

---

[62] "Moments in Restrained and Continuous Beams," *Transactions*, Am. Soc. C. E., Vol. 90 (1927), 155 *et seq.*

Solving simultaneously:

$$F = W\,L^2\,(s\,m + t\,n) \quad\text{.....................}(107)$$

and,

$$\bar{x} = \frac{s\,m\,L}{s\,m + t\,n} \quad\text{........................}(108)$$

Thus, in terms of the coefficients given in Mr. Ruppel's table, m and n are obtained by dividing p and q, respectively, by the minimum moment of inertia of the member; u and v are obtained directly from the tables and F and $\bar{x}$ by substituting values for s and t in Equations (107) and (108). The tables are strictly accurate for beams of rectangular cross-section, but they may be used with satisfactory accuracy for **T**-beams.

A. Floris,[63] Esq. (by letter).—The method of analyzing statically indeterminate structures by the aid of successive approximations, as proposed by the author, is not quite new.

The most systematic study, relative to the analysis of structures by means of statically indeterminate deformations instead of statically indeterminate forces, was made by the late Professor A. Ostenfeld, including a method of computation by steps for fixed as well as for movable joints.[64] This last method has been mentioned by one of the discussers. The writer would like to add two more methods due to well-known writers.

The method of Professor Akira Miura may be summarized as follows:[65] First, the joints are assumed to be restrained from rotating and the bending moments of the members are determined for this condition. In reality, the joints are not fixed, and it is necessary, therefore, to calculate, for each joint, the moments that resist rotation. These moments are termed "the moments of restraint." Now, if it is assumed that the moments of restraint are acting at the joints in the opposite direction, the corresponding rotations, and, consequently, the bending moments, of the members can be determined. By adding the values found in the first step to those obtained in the second step the final values for the actual conditions in the structure can be calculated. This analysis permits various simplifications in its detailed development and is, according to its author, more simple and practical than the method devised by Professor Ostenfeld.

The reviewer of an American textbook on structural analysis in which Professor Cross' method is given, states that: "The procedure is somewhat similar to that due to Dr. F. Hartmann [Vienna, Austria], in which the joints are locked by eliminating the end slopes of the members."[66] Since reference is not made to the original source the writer could not verify this statement.

The writer wishes merely to put these comments on record without going into detail as to the relative merits of the methods under consideration.

---

[63] Civ. Engr., Los Angeles, Calif.

[64] "Die Deformationsmethode," von A. Ostenfeld, Berlin, 1926, pp. 17, 27, and 37.

[65] "Theory of Rigid Frames," by Akira Miura, Tokyo, 1929 (in Japanese). Also, "Die neuzeitliche Entwicklung der Rahmentheorie in Japan", von Akira Miura, *Proceedings*, World Eng. Congress, Tokyo, 1929, Vol. VII, Pt. I, p. 422.

[66] *Concrete and Constructional Engineering* (London), August, 1931, Vol. XXVI, No. 8, p. 468.

I. M. NELIDOV,[67] Assoc. M. Am. Soc. C. E. (by letter).—Rigid frames can be classified as structures consisting of a number of members, which is usually less than that required by the theorem of Cauchy,[68] that is, the number of members equals three times the number of joints minus six. As a consequence, equilibrium under the action of a given set of forces is accomplished only after considerable deformation takes place. In this respect, the point of view sometimes expressed that side-sway of the frame is of no importance apparently is in error.

Rigid frames generally fall into two distinct classes: First, those in which a given deflection of the joint at one end of the frame produces a known deflection at the remainder of the joints; and, second, those in which a given deflection of the joint at one end produces unknown deflections at the remainder of the joints.

Frames of the first class are: Two members connected at a joint and simply supported at opposite ends; three members meeting at a joint, or portals with parallel and non-parallel legs, saw-roof frames, and closed-ring frames; four members meeting at a joint, or closed rings, such as culverts, bins, etc.; and, finally, more than four members, such as structural building frames.

Portals consisting of four or more members with parallel or non-parallel legs, closed rings, and polygons, belong in the second class. Arches are included in the polygon classification. It should be noted that any frame may have rigid connections, but strictly rigid frames proper must have them in order not to become moving mechanisms.

All the frames of the first class can be solved with the aid of the method proposed by Professor Cross, by deflecting the structure a unit of distance, finding the resulting moments and shears, multiplying them by a certain coefficient, and equating them to acting forces.

With frames of the second class this procedure cannot be followed exactly because the deformations of the ring are known only when the total number of members is not greater than three. When this number is exceeded, the intermediate members "give" if the end members are undergoing a certain deformation.

To demonstrate the application of the Cross method to the second class of frames, with the number of members greater than three, the writer selected as the simplest case a symmetrical portal frame, consisting of four members and subjected to the action of a horizontal force, $P = 1$ kip, as shown on Fig. 55. Clockwise moments acting on the ends of a member are considered positive; moments in beams to the left of a section, or those which are plotted in a moment diagram, are assumed positive when the curvature of the elastic line is convex downward.

The deflection of the frame under the load can be resolved into two component deflections: First, two equal deflections to the right, measured at the top of the columns; and, second, two equal but opposite deflections, one at the left column toward the right and one at the right column toward the left.

---

[67] Senior Engr., Hydr. Structural Design, State Dept. of Public Works, Sacramento, Calif.

[68] "Redundant Forces in Structural Mechanics," by the late V. L. Kirpitcheff, Prof. Emeritus at Polytechnic Institute at Kiev, 1903 Edition, p. 10.

The top members of the portal above the columns can be replaced with one straight imaginary member connecting the top of the columns. This member should have properties equal to the replaced top of the portal, and its use will simplify the procedure. The properties to be found are: The new carry-over ratio; the new rigidity; and the forces and moments resulting from the deformation of the imaginary member.

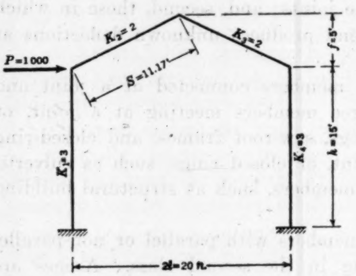


FIG. 55

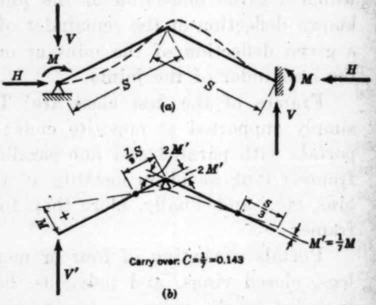


FIG. 56

To determine the carry-over ratio imagine the top portion of the frame hinged at the left end and fixed at the right, as shown on Fig. 56(*a*). From the condition which produces no displacement at the hinge, M' is determined with the aid of the conjugate-beam method: $\Delta\, L_v = 0$; therefore,

$$\frac{1}{EI}\left[\frac{1}{2}\,(M + 2M')\,s\,\frac{l}{3} - 2 \times 2M'\,sl + \frac{1}{2}\,(2M' + M')\,s\,\frac{5}{3}\,l\right] = 0 \quad (109)$$

Wherefrom: $M' = \frac{1}{7}\,M$, which gives the carry-over ratio, $c_i = -0.1429$, instead of $c = +0.5$ for a straight beam with constant moment of inertia. The moment diagram is shown on Fig. 56(*b*).

The deflection angle at the hinge is:

$$\Delta\,\phi = \frac{1}{EI}\left[\frac{1}{2}\,(M + 2M')\,s - 2 \times 2M's + \frac{1}{2}\,(2M' + M')\right]$$

$$= \frac{2}{7}\,\frac{Ms}{EI} = \frac{2}{7}\,\frac{M}{Ek} \quad (110)$$

in which, $k = \frac{I_2}{s_2} = \frac{I_3}{s_3} = 2$. This deflection angle must be equal to that of a straight imaginary beam hinged at the left end and fixed at the right, which is:

$$\Delta\,\phi_i = \frac{1}{4}\,\frac{M}{Ek_i} \quad (111)$$

Since $\Delta \phi = \Delta \phi_i$:

$$\frac{2}{7}\,\frac{M}{Ek} = \frac{1}{4}\,\frac{M}{Ek_i} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (112)$$

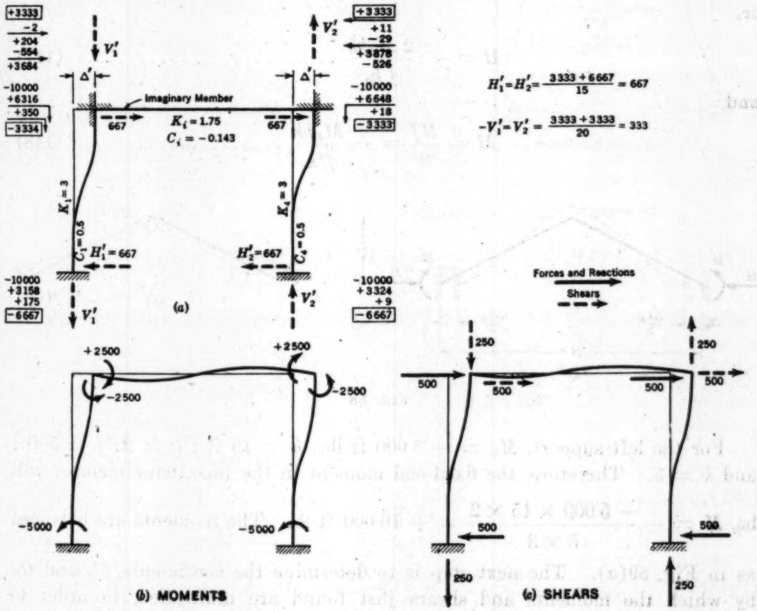
and $k_i = \frac{7}{8}\,k = \frac{7}{8}\,2 = 1.75$.

After the carry-over ratio and rigidity of the imaginary member are determined, the procedure is as follows.

Step 1.—Deflect the frame toward the right an equal amount, Δ', at both columns, compute the fixed-end moments, release the joints, and balance the moments. Fixed-end moments for columns in this case are:

$$M = -\,6\;Ek_c\,\frac{\Delta'}{h} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (113)$$

in which, $k_c = \frac{I_1}{l_1} = \frac{I_4}{l_4} = \frac{I}{h} = 3$. Assume an arbitrary value, $M = -\,10\,000$ ft-lb. Fixed-end moments for the imaginary member in this case are equal to zero. All computations for Step 1 are shown on Fig. 57(*a*).


+3333
-2
+204
-554
+3684
V_1'
Δ'
-10000
+6316
+350
-3334
Imaginary Member
667
$K_i = 1.75$
$C_i = -0.143$
667
V_2'
+3333
+11
-29
+3878
-526
Δ'
-10000
+6648
+18
-3333
$H_1' = H_2' = \frac{3333 + 6667}{15} = 667$
$-V_1' = V_2' = \frac{3333 + 3333}{20} = 333$
$K_1 = 3$
$C_1 = 0.5$
$H_1' = 667$
$K_4 = 3$
$C_4 = 0.5$
$H_2' = 667$
-10000
+3158
+175
-6667
V_1'
(a)
V_2'
-10000
+3324
+9
-6667
Forces and Reactions
Shears
+2500
-2500
+2500
-2500
-5000
-5000
250
500
500
500
500
250
500
500
250
250

(b) MOMENTS

(c) SHEARS

FIG. 57

Step 2.—Deflect the posts of the frame inward an equal amount, Δ''; compute the fixed-end moments; release the joints; and balance the moments. The fixed-end moments for columns in this case are expressed by:

$$M = \pm\,6\;E k_c\,\frac{\Delta''}{h} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (114)$$

In this case a value is assumed arbitrarily as $M = \pm 5\,000$ ft-lb. Fixed-end moments in the imaginary member are computed in accordance with Fig. 58(*a*) and Fig. 58(*b*), as follows:

From the condition that $\Delta\phi = 0$, it follows that $M' = -M$; furthermore, $V = 0$ by condition of loading; and, since $M' = M - Hf$, it follows that $M = +\frac{Hf}{2}$. The horizontal shortening of the span, $2l$, is expressed by:

$$\Delta L_h = \frac{1}{EI}\left[2\,Ms\,\frac{1}{2}f - 2 \times \frac{1}{2}Hfs\,\frac{2}{3}f\right] = -\frac{Hf^2 s}{6\,EI} = -\frac{Hf^2}{6\,Ek} \quad \ldots (115)$$

in which, $k = \frac{I}{s} = 2$.

On the other hand: $\frac{\Delta L_h}{2} = \Delta'' = -\frac{M_c h}{6\,Ek_c}$. As $\Delta'' = -\Delta L_h$:

$$\frac{Hf^2}{6\,Ek} = -\frac{2\,M_c h}{6\,Ek_c} \quad \ldots\ldots\ldots\ldots\ldots\ldots (116)$$

or,

$$H = -\frac{2\,M_c hk}{f\,k_c} \quad \ldots\ldots\ldots\ldots\ldots\ldots (117)$$

and,

$$M = \frac{Hf}{2} = -\frac{M_c hk}{f^2 k_c} \quad \ldots\ldots\ldots\ldots\ldots\ldots (118)$$

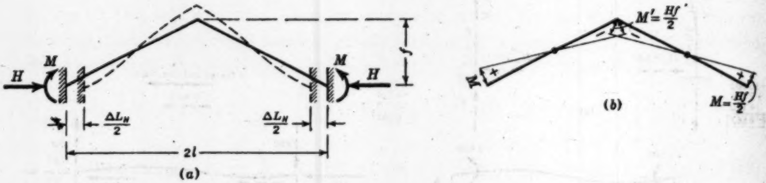


FIG. 58

For the left support, $M_c = -5\,000$ ft-lb.; $h = 15$ ft.; $k = 2$; $f = 5$ ft.; and $k = 3$. Therefore, the fixed-end moment in the imaginary member will be, $M = -\frac{-5\,000 \times 15 \times 2}{5 \times 3} = +10\,000$ ft-lb. The moments are balanced as in Fig. 59(*a*). The next step is to determine the coefficients, C_1 and C_2, by which the moments and shears just found are multiplied, in order to obtain the true moments and shears. When the top of the left column is in equilibrium:

$$C_1 H_1' + (H_1'' + H)\,C_2 = P \quad \ldots\ldots\ldots\ldots\ldots (119)$$

and, when the top of the right column is in equilibrium:

$$C_1 H_2' - (H_2'' + H)\,C_2 = 0 \quad \ldots\ldots\ldots\ldots\ldots (120)$$

In the present case, $H_1' = H_2'$ and $H_1'' = -H_2''$; therefore, $C_1 = \frac{P}{2\,H_1'}$ and $C_2 = \frac{P}{2\,(H_1'' + H)}$.

Substituting the numerical values: $C_1 = \frac{1\,000}{2 \times 667} = 0.75$; and $C_2 = \frac{1\,000}{2\,(967 + H)} = \frac{500}{967 + H}$, Factor H is the comprehensive force in the imaginary member corresponding to the balanced moments at its ends, as found in Step 2.

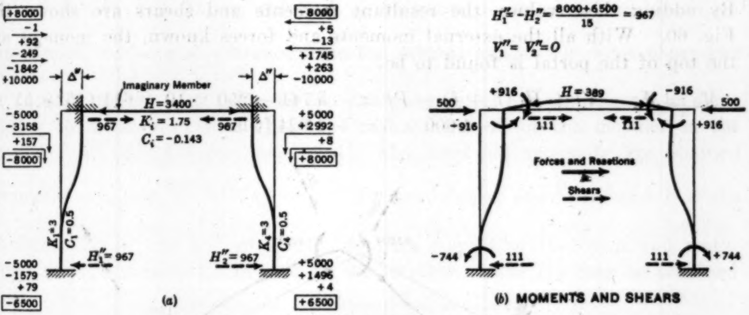

+8000
-1
+92
-249
-1842
+10000
Δ''
Imaginary Member
H=3400'
967
K_i = 1.75
967
C_i = -0.143
-8000
+5
-13
+1745
+263
-10000
Δ''
-5000
-3158
+157
-8000
+5000
+2992
+8
+8000
K_1=3
C_1=0.5
K_4=3
C_4=0.5
H_1''= 967
H_2''= 967
-5000
-1579
+79
-6500
+5000
+1496
+4
+6500
(a)
H_1'' = -H_2'' = (8000+6500)/15 = 967
V_1'' = V_2'' = 0
+916
H = 389
-916
500
500
-916
111
111
+916
Forces and Reactions
&
Shears
-744
111
111
+744
(b) MOMENTS AND SHEARS

FIG. 59

From Fig. 58(a), the angle of rotation at the left support is:

$$\Delta\,\phi = \frac{1}{EI}\,(Ms - \frac{1}{2}\,Hfs) = \frac{1}{Ek}\,(M - \frac{1}{2}\,Hf)\;\ldots\ldots\ldots(121)$$

For the adjacent column, $M'' = 2\,E\,k_c\left(2\;\Delta\;\phi_c - 3\;\frac{\Delta''}{h}\right)$, and, therefore,

$$\Delta\;\phi_c = \left(M'' + 6\,E\,k_c\,\frac{\Delta''}{h}\right)\frac{1}{4\,Ek}\;\ldots\ldots\ldots\ldots\ldots(122)$$

Substituting $\frac{\Delta''}{h} = -\frac{M_c}{6\,Ek_c}$,

$$\Delta\;\phi_c = \frac{M'' - M_c}{4\,Ek_c}\;\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(123)$$

From the condition that $\Delta\;\phi = \Delta\;\phi_c$,

$$\frac{1}{Ek}\,(M - \frac{1}{2}\,Hf) = \frac{M'' - M_c}{4\,Ek_c}\;\ldots\ldots\ldots\ldots\ldots(124)$$

and as $M'' = -M$,

$$H = \frac{2\,Mk}{f}\left[\frac{1}{k} - \frac{-1-u}{4\,k_c}\right]\;\ldots\ldots\ldots\ldots\ldots\ldots\ldots(125)$$

in which, u represents the ratio, $\frac{M_c}{M}$, for convenience.

The vertical reaction, V, equals zero, as before. Let $M = +\ 8\,000$ ft-lb.; $M_c = -\ 5\,000$ ft-lb.; $k = 2$; $k_c = 3$; and $f = 5$ ft.; then,

$$H = 2 \times 8\,000 \times \tfrac{2}{5}\left[\tfrac{1}{2} - \frac{-1 - \dfrac{-\ 5\,000}{8\,000}}{4 \times 3}\right] = +\ 3\,400 \text{ lb.}$$

and,

$$C_2 = \frac{500}{967 + 3\,400} = 0.1145$$

The actual moments and shears are shown on Fig. 57(a) and Fig. 59(b). By adding these values, the resultant moments and shears are shown in Fig. 60. With all the external moments and forces known, the moment at the top of the portal is found to be:

$$M_C = M_A - Vl + H_1\,(h + f) - Pf = -\ 5\,744 - 250 \times 10 + 611\ (15 + 5) \\ -\ 1\,000 \times 5 = -\ 1\,024 \text{ ft-lb.}$$

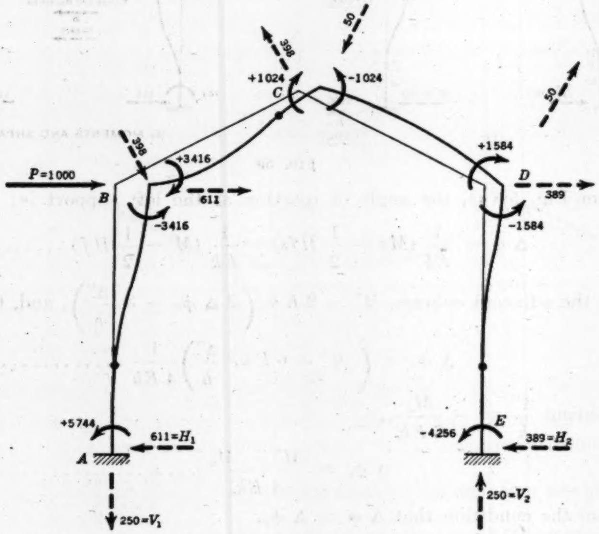

50
398
+1024
-1024
C
50
398
+3416
+1584
P=1000
B
611
-3416
D
389
-1584
+5744
611=H_1
A
-4256
E
389=H_2
250=V_1
250=V_2

FIG. 60

An analytical check[69] gives the following results: $M_A = 5\,744$ ft-lb.; $M_B = 3\,431$ ft-lb.; $M_C = -\ 1\,036$ ft-lb.; $M_D = -\ 1\,590$ ft-lb; and $M_E = -\ 4\,256$ ft-lb.

The foregoing computations show that Professor Cross' method can be applied to both classes of rigid frames.

---

[69] "Rahmenformeln," von A. Kleinlogel, Fifth Edition, p. 295.

E. A. MacLean,[70] Esq. (by letter).—The following example of the Cross method of distributing moments, as applied to a truss, is submitted for determining secondary stresses. In spite of the simplicity of the Cross method, it becomes tedious when applied to a large truss, but the chances of error can be greatly reduced by systematizing the work. The method of laying out the computations, as shown in the example herein, will be found to be very helpful, not only in the computation of the secondary stresses in a truss, but in computing the moments in a multi-story bent or any other complicated structure.

The problem is taken from a standard textbook[71] in which a solution of the secondary stresses is made by one of the older methods. The necessary data are shown in Fig. 61, in which, I = moments of inertia; l = length, in inches; $K = \frac{I}{l}$; c = distance from the neutral axis to the outer fiber; and s = unit stress. In Table 19 the "Percentage of the unbalanced moment going to member" (Item 1), is the ratio of the K of the member to the total of all the K's for that joint. The fixed-end moments are obained from the equation, $\frac{6\,D\,E\,K}{l}$, in which, E = modulus of elasticity; and D = the deformation of one end of a member with respect to the other end measured at right angles to the axis of the member. The D's may be obtained from a Williot diagram or by computation. Clockwise rotation of the joint causes positive moments in the members entering the joint. In Table 19, the computations for Joints D and E are omitted because the moments are the same as for Joints B and A, respectively, with opposite signs.

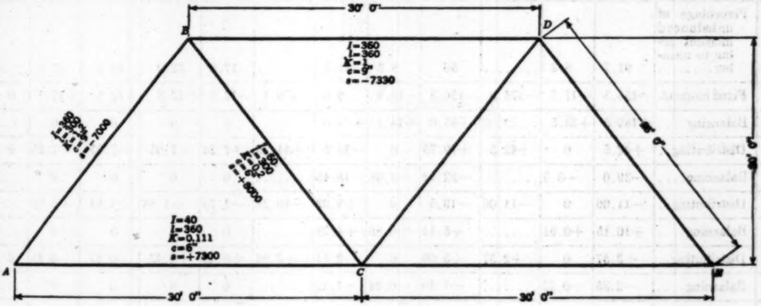


FIG. 61

The fixed-end moments are entered in the table (Item 2), under the proper member and are added across algebraically for each joint, in order to obtain the unbalanced moment. This unbalanced moment is balanced at each joint (Item 3). Each member will get the percentage shown at the top of the table (Item 1). These "balancing" moments are then distributed on the following line, one-half the "balancing" moment at one end of a member being the "distributing" moment at the other end. In making up Table 19 it should be noted that a member (for example, the end-post, *AB*) is identified as *AB* under Joint *A* and as *BA* under Joint *B*. The "distributing" moments (Item 4) are added across to obtain a new unbalanced moment, and one cycle is finished. As the work proceeds, every box in the table is filled, except those with the dotted lines, or better still, diagonal lines. This system eliminates as far as possible the chances of making an error by omitting a part of a step, and it also enables the work to be checked easily. The final moment in the members is obtained by adding each column of moments algebraically (Item 13).

---

[70] Designer, State Highway Comm., Augusta, Me.

[71] "Steel Mill Buildings," by Milo S. Ketchum, M. Am. Soc. C. E., p. 309.

The following secondary stresses were obtained which are, of course, very close to the results shown in the original problem: $BA = -630$; $CA = +1\,780$; $CB = +826$; and $BD = -700$.

In teaching work the writer found that undergraduate as well as graduate students readily grasped the Cross method and obtained a much clearer conception of the action of undeterminate structures as they followed through this method, than is possible by the more or less blind application of formulas. Students are appreciative, of course, of the ease of its application, particularly after using some of the more formal methods. Once the Cross method is grasped, it can be called to mind after only a few minutes' thought without any reference to notes or complicated formulas.

TABLE 19.—BALANCING MOMENTS BY CROSS METHOD

Item	Member	Joint *A*			Joint *B*				Joint *C*				
		AB	*AC*	Unbalanced	*BA*	*BC*	*BD*	Unbalanced	*CA*	*CB*	*CD*	*CE*	Unbalanced
1	Percentage of unbalanced moment going to member........	91.7	8.3	....	50	8.3	41.7	....	17.8	32.2	32.2	17.8	.....
2	Fixed moment.	−156.3	−17.7	−174.0	−156.3	−13.8	0.0	−170.1	−17.7	−13.8	+13.8	+17.7	0
3	Balancing....	+159.5	+14.5	....	+85.0	+14.1	+71.0	....	0	0	0	0	.....
4	Distributing..	+42.5	0	+42.5	+79.75	0	−35.5	+44.25	+7.25	+7.05	−7.05	−7.25	0
5	Balancing....	−39.0	−3.5	....	−22.12	−3.68	−18.45	....	0	0	0	0	.....
6	Distributing..	−11.06	0	−11.06	−19.5	0	+9.22	−10.28	−1.75	−1.84	+1.84	+1.75	.....
7	Balancing....	+10.15	+0.91	....	+5.14	+0.86	+4.28	....	0	0	0	0	.....
8	Distributing..	+2.57	0	+2.57	+5.06	0	−2.14	+2.92	+0.45	+0.43	−0.43	−0.45	0
9	Balancing....	−2.35	−0.22	....	−1.46	−0.24	−1.22	....	0	0	0	0	.....
10	Distributing..	−0.73	0	−0.73	−1.17	0	+0.61	−0.56	−0.11	−0.12	+0.12	+0.11	0
11	Balancing....	+0.67	+0.06	....	+0.28	+0.05	+0.23	....	0	0	0	0	.....
12	Distributing..	+0.14	0	+0.14	+0.34	0	−0.12	+0.22	+0.03	+0.025	−0.025	−0.03	.....
13	Totals, in kips.	+6.09	−5.95	....	−24.98	−2.71	+27.91	....	−11.83	−8.26	+8.26	+11.83	.....

George M. Dillingham,[73] Assoc. M. Am. Soc. C. E. (by letter).—For the analysis of continuous frames subjected to changes in loading, the method developed by Professor Cross is made readily applicable by the following steps:

(1) Determine the structural properties for a given frame; that is, dimensions; conditions of end restraint; the values of *I*, *K*, and the "carry-over" factors of the members, etc.

(2) Apply a "trial fixed-end moment" at the end of a member, either positive or negative, and of any convenient magnitude and unit.

(3) Distribute this "trial fixed-end moment" throughout the frame by the author's method, and tabulate the "trial final moment" thus induced for each end of all members.

(4) In like manner apply, in turn, "trial fixed-end moments" at the other end of the member mentioned in Step (2), and at each end of all other members; and tabulate for each application the corresponding "trial final moments."

(5) Determine all the "actual fixed-end moments" induced by following Step (3) for a specific loading.

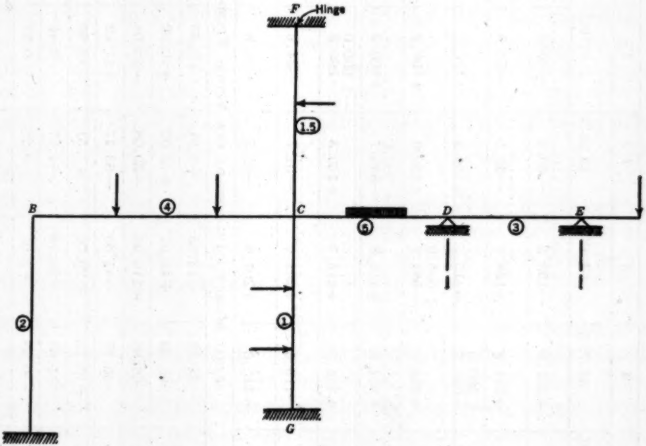


Fig. 62

(6) Determine the "actual final moments" for the given loading by the use of the following proportion:

$$\frac{\text{Actual final moment}}{\text{Actual fixed-end moment}} = \frac{\text{Trial final moment}}{\text{Trial fixed-end moment}}$$

Tabulate the final moments caused by each of the several actual fixed-end moments; add these algebraically for the end of each member, and record the total final moment.

(7) Change the loading, and repeat Steps (5) and (6).

---

[73] Evanston, Ill.

TABLE 20.—Arrangement of Computations for Example

Item	A	B		C				D		E	F	G	Joint
1	*AB*	*BA*	*BC*	*CB*	*CD*	*CF*	*CG*	*DC*	*DE*	*ED*	*FC*	*GC*	Member
					Final Moments Induced by the Trial Fixed-End Moment of 1 000 Foot-Pounds								
2	2	2	4	4	5	1.5	1	5	3	3	1.5	1.0	*K*.
3	100.0%	33.33%	66.67%	34.77%	43.48%	13.05%	8.70%	62.5%	37.5%	0.0%	0.0%	100.0%	Distribution factor.
4	....	1 000.0	....	....	....	....	....	....	....	....	....	....	Trial moment at *BA*.
5	+177.6	−644.6	+644.6	+221.9	−138.5	−50.1	−33.3	−26.1	+26.1	0.0	0.0	−16.6	
6	....	....	−1 000.0	....	....	....	....	....	....	....	....	....	Trial moment at *BC*
7	+177.6	+355.4	−355.4	+221.9	−138.5	−50.1	−33.3	−26.1	+26.1	0.0	0.0	−16.6	
8	....	....	....	−1 000.0	....	....	....	....	....	....	....	....	Trial moment at *CB*.
9	−33.2	−66.8	+66.8	−665.7	+415.3	+150.3	+100.2	+77.8	−77.8	0.0	0.0	+50.2	
10	....	....	....	....	−1 000.0	....	....	....	....	....	....	....	Trial moment at *CD*.
11	−33.2	−66.8	+66.8	+334.3	−584.7	+150.3	+100.2	+77.8	−77.8	0.0	0.0	+50.2	
12	....	....	....	....	....	−1 000.0	....	....	....	....	....	....	Trial moment at *CF*.
13	−33.2	−66.8	+66.8	+334.3	+415.3	−849.7	+100.2	+77.8	−77.8	0.0	0.0	+50.2	
14	....	....	....	....	....	....	−1 000.0	....	....	....	....	....	Trial moment at *CG*.
15	−33.2	−66.8	+66.8	+334.3	+415.3	+150.3	−899.8	+77.8	−77.8	0.0	0.0	+50.2	
16	....	....	....	....	....	....	....	−1 000.0	....	....	....	....	Trial moment at *DC*.
17	+11.5	+22.9	−22.9	−115.3	+201.6	−51.7	−34.6	−337.3	+337.3	0.0	0.0	−17.2	
18	....	....	....	....	....	....	....	....	−1 000.0	....	....	....	Trial moment at *DE*.
19	+11.5	+22.9	−22.9	−115.3	+201.6	−51.7	−34.6	+662.7	−662.7	0.0	0.0	−17.2	
					Final Moments Induced by the Actual Fixed-End Moments								
20	−17.76	−35.54	+35.54	−22.19	+13.85	+5.01	+3.33	+2.61	−2.61	0.0	0.0	+1.66	*BC* + 100.
21	−3.32	−6.68	+6.68	−66.58	+41.53	+15.03	+10.02	+7.78	−7.78	0.0	0.0	+5.02	*CB* — 100.
22	+6.64	+13.36	−13.36	−66.85	+116.95	−30.06	−20.04	−15.56	+15.56	0.0	0.0	−10.04	*CD* + 200.
23	−3.65	−7.35	+7.35	+36.77	+45.68	−93.47	+11.02	+8.56	−8.56	0.0	0.0	+5.52	*CF* — 110.
24	−1.66	−3.34	+3.34	+16.72	+20.76	+7.51	−44.99	+3.89	−3.89	0.0	0.0	+2.51	*CG* — 50.
25	+1.15	+2.29	−2.29	−11.53	+20.16	−5.17	−3.46	−33.73	+33.73	0.0	0.0	−1.72	*DC* — 100.
26	+0.06	+0.11	−0.11	−0.58	+1.01	−0.26	−0.17	+3.31	−3.31	0.0	0.0	−0.09	*DE* — 5.
27	....	....	....	....	....	....	....	....	....	−10.0	.....	....	*ED* —10.
28	....	....	....	....	....	....	....	....	....	....	.....	+50.00	*GC* + 50.
29	−18.54	−37.15	+37.15	−114.24	+259.94	−101.41	−44.29	−23.14	+23.14	−10.0	0.0	+52.86	Final moments.

Example.—In demonstration of Steps (1) to (7) the structure shown in Fig. 62 is analyzed herewith. The signs of moments are determined by the convention which designates, as positive, clockwise rotation at a joint due to externally applied forces.

Step (1): For greater ease in distributing the fixed-end moments, the *K* of the member, *CF*, is reduced from 2.0 to 1.5; the fixed-end moment at *F* is eliminated; and the fixed-end moment at *C* in *CF* is changed from −80 to −110 (see Fig. 62). These changes are made possible by the absence of restraint at the hinge, *F*. In all other respects, the data given by the author are used unchanged.

Steps (2), (3), and (4): In Table 20, Items 1 to 19 show the final moment induced at each end of each member by the trial fixed-end moment of −1 000.0, which is applied successively at Joints *B*, *C*, and *D*, as indicated. In preparing Table 20, the trial moment was applied to *AB* at *B*, to *CB* at *C*, and to *CD* at *D*; and the distribution process was carried out in the usual manner for each of these applications. From the results of these three applications, the values of the final moments for the other locations of the trial fixed-end moment were written by inspection.

Step (5): See Step (1).

Step (6): In Table 20, Items 20 to 29 show the final moments induced at each end of each member by the actual fixed-end moments given in Fig. 62.

The similarity of the general procedure described in this example to influence line diagram construction is easily recognized. Having once performed Steps (2), (3), and (4), loading changes may be made as desired, and final moments for each loading determined quickly and with a minimum of effort.

The distribution of Step (3) may be performed either by the method given in the paper under discussion, or by the method (termed the "Exact Method") described by Professor Cross in his paper[73] entitled "Continuity as a Factor in Reinforced Concrete Design."

Donald E. Larson,[74] **Jun. Am. Soc. C. E.** (by letter).—In presenting the method of moment distribution, Professor Cross has barely hinted at the possibilities of the more general method of moment and force distribution which includes the distribution of unbalanced joint forces as well as moments. This general method must be used in analyzing structures in which the joints are subject to linear displacement as well as rotation. A continuous series of arches supported on long slender piers is a structure of this type. Loads applied to the arches produce both rotation and horizontal translation of the joints. For this reason, the processes of moment distribution and force distribution cannot be carried out independently of each other. When the moments at a joint are balanced, the horizontal forces at the ends of the members intersecting at that joint are changed and must be adjusted accordingly. In the same way, the moments at the ends of these members must be adjusted when the horizontal forces at the joint are balanced.

---

[73] *Proceedings*, Am. Concrete Inst., 1929, p. 689.

[74] With Chicago Bridge & Iron Works, Chicago, Ill.

After considerable experimenting, the writer has worked out a fairly convenient procedure for analyzing multiple-span arches by distributing fixed-end horizontal forces and fixed-end moments. The method has several desirable features even if it is complicated to a certain extent by the adjustments previously mentioned. An outline of the procedure follows.

Step 1.—

(*a*) Compute the fixed-end horizontal forces at the ends of each arch, due to loads on the arch.

(*b*) Compute the fixed-end moments at the ends of each arch, resulting from loads on the arch.

Step 2.—

(*a*) Distribute the unbalanced horizontal force at each joint among the connecting members in proportion to their resistance to horizontal deflection. The percentage of the unbalanced force distributed to each member is equal to $\frac{J}{\Sigma J}$, in which, J is the horizontal force required to deflect the near end of the member one unit when applied along the centroidal axis of the member, the far end of the member being fixed.

(*b*) Distribute the unbalanced moment at each joint among the connecting members in proportion to their resistance to rotation. The percentage distributed to each member is equal to $\frac{N}{\Sigma N}$, in which, N is the moment required to rotate the near end of the member one unit when the far end is fixed.

Step 3.—

(*a*) The horizontal force distributed to each member in Step 2 (*a*) is carried over to the opposite end of the member with opposite sign. This is to keep the member in static equilibrium.

(*b*) A certain proportion, r, of the moment distributed to each member in Step 2 (*b*) is carried over to the far end of the member. This "carry-over factor," r, is the same as the one defined by the author. It may be either positive or negative, depending upon the geometric properties of the member.

Step 4.—

(*a*) Compute the horizontal thrust at the end of each member produced by the moments distributed to that member in Step 2 (*b*). These thrusts are equal to the product of this moment by h, in which, h is the horizontal force produced by a unit moment applied to one end of the member.

(*b*) Compute the moments at the ends of each member produced by the horizontal force distributed to that member in Step 2 (*a*). These moments are equal to the product of the force by d, in which, d is the vertical distance from the joint to the elastic center of the member.

Step 5.—

(*a*) The horizontal forces obtained in Step 3 (*a*) and Step 4 (*a*) are added algebraically.

(*b*) The moments obtained in Step 3 (*b*) and Step 4 (*b*) are added in the same manner.

These steps form a complete cycle in the process of moment and force distribution. Step 5 (*a*) gives a new set of horizontal forces at each joint. The resulting unbalanced force is again distributed, as in Step 2 (*a*). Step 5 (*b*) gives a new set of moments at each joint and the unbalanced moment is again distributed as in Step 2 (*b*). The foregoing procedure is then repeated as many times as required to produce results of the desired accuracy.

Formulas for Constants.—The constants defined in Steps 1 to 5, inclusive, may be computed by the relations given in Equations (126) to (134). The derivation of these relations is based upon the assumptions: First, that plane sections remain plane; and, second, that the material obeys Hooke's law. The subscript, *a,* is used to indicate that the constant applies to an arch, and the subscript, *p,* to indicate that the constant applies to a pier.

Constants for arches are expressed by the following equations:

$$d_a = \frac{\sum y \frac{ds}{I}}{\sum \frac{ds}{I}} \quad \dots\dots (126)$$

$$J_a = \frac{1}{\sum y^2 \frac{ds}{I} - d^2_a \sum \frac{ds}{I}} \quad \dots\dots (127)$$

$$N_a = \frac{1}{\sum \frac{ds}{I}} + \frac{L^2}{4 \sum x^2 \frac{ds}{I}} + d^2_a J_a \quad \dots\dots (128)$$

$$r_a = 1 - \frac{L^2}{2 N_a \sum x^2 \frac{ds}{I}} \quad \dots\dots (129)$$

and,

$$h_a = \frac{d_a J_a}{N_a} \quad \dots\dots (130)$$

Constants for piers are:

$$d_p = \frac{\sum y \frac{ds}{I}}{\sum \frac{ds}{I}} \quad \dots\dots (131)$$

$$J_p = \frac{1}{\sum y^2 \frac{ds}{I} - d^2_p \sum \frac{ds}{I}} \quad \dots\dots (132)$$

$$N_p = \frac{1}{\sum \frac{ds}{I}} + d^2_p J_p \quad \dots\dots (133)$$

and,

$$h_p = \frac{d_p J_p}{N_p} \quad \dots\dots (134)$$

The factor, r_p, corresponding to r_a in Equation (129) is not required when piers are fixed at the base. The geometric properties which are needed for computing the arch constants are shown in Fig. 63. The properties used in computing constants for the piers are shown in Fig. 64.

Signs.—The convention of signs adopted by the writer is similar to that introduced by the author; that is, positive moment in the arches corresponds to compression at the extrados; positive moment in the piers corresponds to compression on the left side of the pier; horizontal forces acting inward on the end of the arch are positive; and a force acting to the right on the top of the pier is positive. In balancing moments and horizontal forces at the joint, the pier is considered to be on the same side of the joint as the arch at the left. A positive moment applied to an arch or a pier produces a positive horizontal force; and a positive horizontal force applied to an arch or pier produces a positive moment. The directions of positive moments and forces are indicated by arrows in Fig. 63 and Fig. 64. The constants, except *r*, are always positive when used with the foregoing convention of signs.

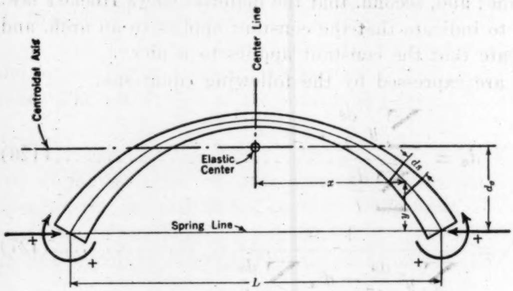


FIG. 63

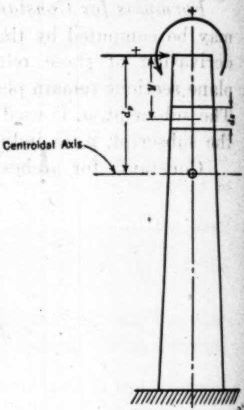
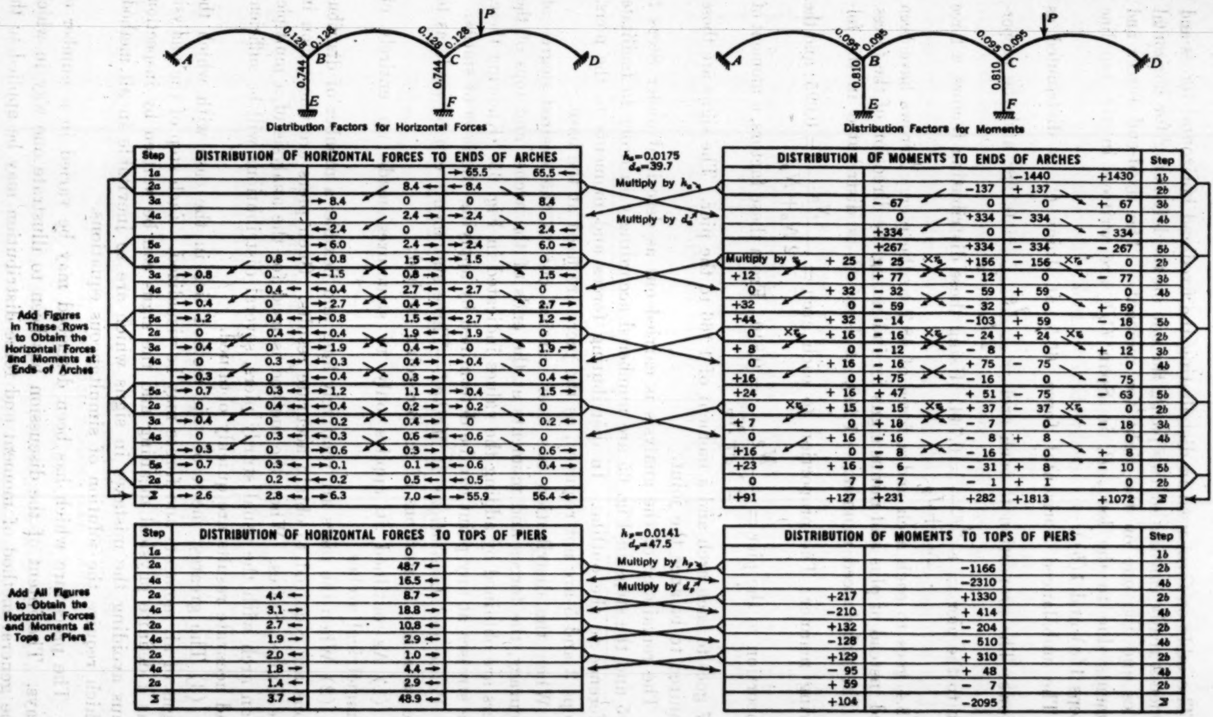


FIG. 64

Illustrative Example.—The analysis of a three-span arch by the method outlined, is illustrated in Fig. 65. For the sake of simplicity, a structure with arches all alike and piers of the same height has been chosen. The ends of the outer arches and the bases of the piers are fixed. The problem is to determine the moments and horizontal forces at the ends of the arches and piers.

Since the only purpose of the problem is to illustrate the method used in distributing the fixed-end moments and forces, the actual dimensions of the arches and piers are not given. The constants required for the analysis are: For arches, $d_a = 39.7$; $J_a = 0.00111$; $N_a = 2.53$; $r_a = +\,0.491$; and $h_a = 0.0175$; and for piers, $d_p = 47.5$; $J_p = 0.00643$; $N_p = 21.60$; and $h_p = 0.0141$. These constants represent a continuous arch series supported by piers which are relatively long and slender.


A
0.128
0.128
B
0.744
E
0.128
0.128
C
0.744
F
P
D
Distribution Factors for Horizontal Forces
A
0.095
0.095
B
0.810
E
0.095
0.095
C
0.810
F
P
D
Distribution Factors for Moments
$h_a=0.0175$
$d_a=39.7$
Multiply by h_a
Multiply by d_a
$h_p=0.0141$
$d_p=47.5$
Multiply by h_p
Multiply by d_p
Multiply by r_a
$\times r_a$
Add Figures in These Rows to Obtain the Horizontal Forces and Moments at Ends of Arches
Add All Figures to Obtain the Horizontal Forces and Moments at Tops of Piers

Step	DISTRIBUTION OF HORIZONTAL FORCES TO ENDS OF ARCHES					
1a					→ 65.5	65.5 ←
2a				8.4 ←	← 8.4	
3a			→ 8.4	0	0	8.4
4a			0	2.4 →	→ 2.4	0
			← 2.4	0	0	2.4 ←
5a			→ 6.0	2.4 →	→ 2.4	6.0 →
2a		0.8 ←	← 0.8	1.5 →	→ 1.5	0
3a	→ 0.8	0	← 1.5	0.8 →	0	1.5 ←
4a	0	0.4 ←	← 0.4	2.7 ←	← 2.7	0
	→ 0.4	0	→ 2.7	0.4 →	0	2.7 →
5a	→ 1.2	0.4 ←	→ 0.8	1.5 ←	← 2.7	1.2 →
2a	0	0.4 ←	← 0.4	1.9 ←	← 1.9	0
3a	→ 0.4	0	→ 1.9	0.4 →	0	1.9 →
4a	0	0.3 ←	← 0.3	0.4 →	→ 0.4	0
	→ 0.3	0	← 0.4	0.3 →	0	0.4 ←
5a	→ 0.7	0.3 ←	→ 1.2	1.1 →	→ 0.4	1.5 →
2a	0	0.4 ←	← 0.4	0.2 →	→ 0.2	0
3a	→ 0.4	0	← 0.2	0.4 →	0	0.2 ←
4a	0	0.3 ←	← 0.3	0.6 ←	← 0.6	0
	→ 0.3	0	→ 0.6	0.3 →	0	0.6 →
5a	→ 0.7	0.3 ←	→ 0.1	0.1 →	← 0.6	0.4 →
2a	0	0.2 ←	← 0.2	0.5 ←	← 0.5	0
Σ	→ 2.6	2.8 ←	→ 6.3	7.0 ←	→ 55.9	56.4 ←

DISTRIBUTION OF MOMENTS TO ENDS OF ARCHES						Step
				−1440	+1430	1b
			−137	+ 137		2b
		− 67	0	0	+ 67	3b
		0	+334	− 334	0	4b
		+334	0	0	− 334	
		+267	+334	− 334	− 267	5b
	+ 25	− 25	+156	− 156	0	2b
+12	0	+ 77	− 12	0	− 77	3b
0	+ 32	− 32	− 59	+ 59	0	4b
+32	0	− 59	− 32	0	+ 59	
+44	+ 32	− 14	−103	+ 59	− 18	5b
0	+ 16	− 16	− 24	+ 24	0	2b
+ 8	0	− 12	− 8	0	+ 12	3b
0	+ 16	− 16	+ 75	− 75	0	4b
+16	0	+ 75	− 16	0	− 75	
+24	+ 16	+ 47	+ 51	− 75	− 63	5b
0	+ 15	− 15	+ 37	− 37	0	2b
+ 7	0	+ 18	− 7	0	− 18	3b
0	+ 16	− 16	− 8	+ 8	0	4b
+16	0	− 8	− 16	0	+ 8	
+23	+ 16	− 6	− 31	+ 8	− 10	5b
0	7	− 7	− 1	+ 1	0	2b
+91	+127	+231	+282	+1813	+1072	Σ

Step	DISTRIBUTION OF HORIZONTAL FORCES TO TOPS OF PIERS		
1a		0	
2a		48.7 ←	
4a		16.5 ←	
2a	4.4 ←	8.7 →	
4a	3.1 →	18.8 →	
2a	2.7 ←	10.8 ←	
4a	1.9 →	2.9 ←	
2a	2.0 ←	1.0 →	
4a	1.8 →	4.4 →	
2a	1.4 ←	2.9 ←	
Σ	3.7 ←	48.9 ←	

DISTRIBUTION OF MOMENTS TO TOPS OF PIERS			Step
			1b
	−1166		2b
	−2310		4b
+217	+1330		2b
−210	+ 414		4b
+132	− 204		2b
−128	− 510		4b
+129	+ 310		2b
− 95	+ 48		4b
+ 59	− 7		2b
+104	−2095		Σ

FIG. 65.—COMPUTATIONS FOR A CONTINUOUS SERIES OF ARCHES SUPPORTED ON LONG SLENDER PIERS.

The table or form used in distributing the fixed-end horizontal forces and moments really consists of two parts, the one on the left being for horizontal forces and the one on the right for moments. The fixed-end forces and moments due to the load, *P*, in Span *CD*, are written in the top line (Steps 1(*a*) and 1(*b*)).

The unbalanced horizontal force, 65.5, at Joint *C*, is distributed as follows: The proportion to each arch $= \frac{J_a}{2J_a + J_p} = 0.128$; and the proportion to the pier $= \frac{J_p}{2J_a + J_p} = 0.744$. Using these distributing factors, a force of 8.4 goes to each arch and a force of 48.7 to the pier. Arrows have been used instead of plus and minus signs for indicating the direction of the forces.

The unbalanced moment, $-1\,440$, at Joint *C*, is distributed in the following manner: The proportion to each arch $= \frac{N_a}{2N_a + N_p} = 0.095$; and the proportion to the pier $= \frac{N_p}{2N_a + N_p} = 0.810$. From these factors, a moment of 137 goes to each arch and a moment of 1 166 to the pier. The signs are those required to balance the joint.

The remainder of the analysis is carried out as described under Steps 1 to 5 and the steps in Fig. 65 are numbered accordingly in order to facilitate reference to the outline. In distributing forces and moments to the piers, Steps 3 and 5 are not required, as the piers are fixed at the base.

When the distribution has been carried out to the desired degree of accuracy, the forces and moments at the ends of the arches and tops of the piers are obtained by adding the values indicated in Fig. 65. Knowing these, the stresses at any point can be determined by applying the laws of statics.

Value and Limitations of Method.—Three statements can be made as to the value and limitations of the foregoing method:

(1) As outlined it applies only to structures made up entirely of symmetrical arches.

(2) When the piers are relatively long and flexible, a number of distributions are required to obtain accurate results. Such piers are uncommon in the United States. When the method is used for the analysis of a multiple-span arch with the usual sturdy piers, several distributions will be sufficient and accurate results are quickly obtained.

(3) The greatest merit of this method lies in the ease with which the signs of all moments and forces may be visualized. Each step of the analysis has a definite physical meaning and the signs can be written by inspection, thus avoiding the mistakes in signs which are so inevitable in all methods which require the solution of simultaneous equations.

The procedure which has been described may be varied in a number of ways. The intent of the discussion has been to illustrate one way in which the general method of moment and force distribution may be applied to the analysis of multiple-span arches, or similar structures. The steps outlined need not be considered as a "hard- and-fast" set of rules which must always be applied in the exact order described. The idea of using arrows instead of plus and minus signs to indicate the direction of the horizontal forces was suggested to the writer by Professor Cross, and experience has proved this to be a valuable expedient as it decreases the probability of errors in the signs.

J. A. VAN DEN BROEK,[75] ASSOC. M. AM. SOC. C. E. (by letter).—Possibly the greatest single contribution to the theory of stress analysis of redundant structures in a generation is that presented by Professor Cross in this paper. Engineers have long been aware of the fact that the influence of a load on a support, several spans removed from the load, is negligible; yet, they have not found a legitimate way by which to avoid the cumbersome process of writing as many simultaneous equations as there are unknowns before attempting a solution. In a large number of problems commonly encountered, Professor Cross' method overcomes this objection. It appears to have been known and used for a number of years. When the subject was first called to the writer's attention, his immediate reaction was highly favorable. How simple! A veritable Columbus' egg! Upon closer study this first enthusiastic appreciation has grown rather than diminished.

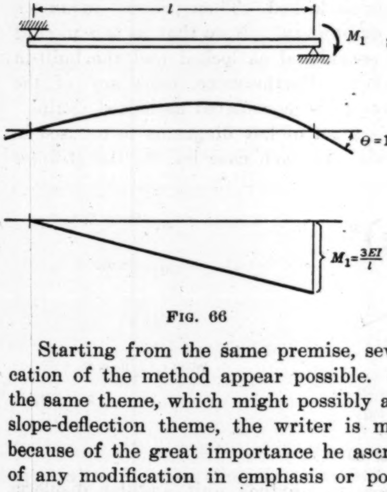


FIG. 66

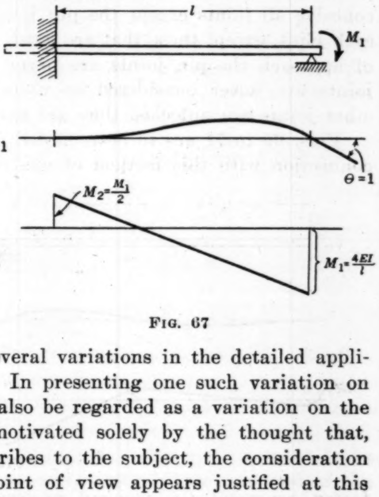


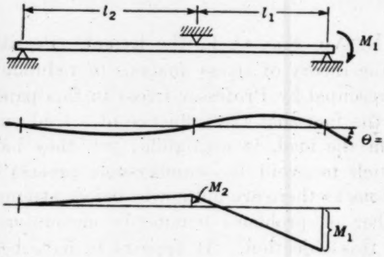
FIG. 67

Starting from the same premise, several variations in the detailed application of the method appear possible. In presenting one such variation on the same theme, which might possibly also be regarded as a variation on the slope-deflection theme, the writer is motivated solely by the thought that, because of the great importance he ascribes to the subject, the consideration of any modification in emphasis or point of view appears justified at this stage.

The basic principle in Professor Cross' method appears to be that first all joints are considered locked, and then released one joint at a time. As each joint is released all other joints are assumed as being locked. Each time a joint is unlocked the structure will deform toward the shape it will ultimately assume in its natural state. If all joints are successively unlocked and locked in turn, an infinite number of times, the structure will approach its final natural shape. Professor Cross appears to assume that this final shape will be approximated very closely after only two or three operations of locking and unlocking the joints. This premise appears reasonable. It is not certain, however, that in exceptional cases this convergence will be achieved. A rigorous proof of the aforementioned premise, would be of much value at this stage.

---

[75] Prof. of Eng. Mechanics, Univ. of Michigan, Ann Arbor, Mich.



$$\frac{E_1 I_1}{l_1} = k_1$$

$$\frac{E_2 I_2}{l_2} = k_2$$

$$M_1 = \frac{12(k_1 k_2 + k_1^2)}{4k_1 + 3k_2}$$

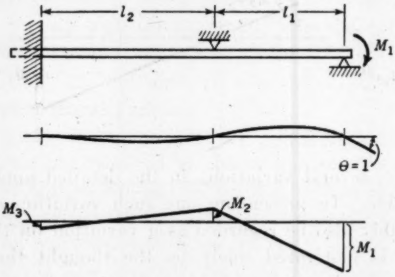
$$M_2 = \frac{k_2 M_1}{2(k_1 + k_2)}$$

When $k_1 = k_2$ then $M_1 = \frac{24EI}{7l}$; $M_2 = \frac{M_1}{4}$

FIG. 68

Instead of conceiving joints successively locked and unlocked, one might conceive all joints except the pin joints as locked. Then proceed to unlock each joint, except those that are fixed, or built in. Note that in this method of approach the pin joints are never considered as locked and the built-in joints are never considered as unlocked. Furthermore, once any of the other joints are unlocked they are never to be considered as locked again.

Figs. 66 to 71 are to be regarded as a set of key diagrams to be used in connection with this method of analysis. In each case let M_1, the stiffness factor, represent the moment necessary to produce unit angular displacement. The elastic coefficient for any member is expressed by $k = \frac{EI}{l}$.



$$M_1 = \frac{4k_1 k_2 + 3k_1^2}{k_1 + k_2}$$

$$M_2 = \frac{2k_2 M_1}{3k_1 + 4k_2}$$

$$M_3 = \frac{M_2}{2}$$

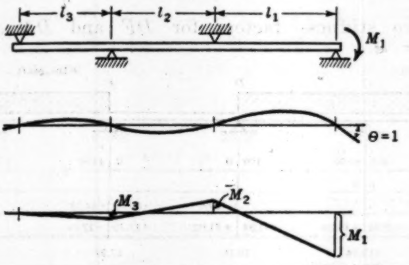
When $k_1 = k_2$ then $M_1 = \frac{7EI}{2l}$;

$$M_2 = \frac{2}{7} M_1$$

FIG. 69

Let Fig. 72 represent a continuous beam of five equal spans, with E and I constant. The left end, Joint A, is fixed; the right end, Joint F, is freely supported. The first three spans and the last span are loaded with a uniformly distributed load, w lb. per ft. The fourth span is unloaded.

The first line represents the moments at the respective joints, with all joints except Joint F locked. The fixed-end moment, $\frac{wl^2}{12}$, at Joints $A, B, C,$ and D, is represented by 100, while $\frac{wl^2}{8}$ at Joint E is represented by 150. The second line represents the effect of unlocking Joints B and C. Since there are no unbalanced moments at these points, no elastic deformations will take place when these joints are unlocked. Next, consider unlocking Joint E, keeping in mind that Joint D is still considered as locked. Fig. 66 provides the key for the stiffness factor of Span EF, while Fig. 67 provides the key for the stiffness factor of Span ED. Let M_1 in each case represent the moment necessary to produce unit angular displacement. Since, in this example, E, I, and l, are assumed equal for all spans, the stiffness factors for Spans EF and ED are in the ratio of 3 to 4. The total moment to be supplied at E, when the joint is unlocked, is ($-$ 150). The right end of the beam, DE, supplies $\frac{4}{7} \times (-150) = -85.72$, while the left end of the beam, EF, supplies $\frac{3}{7} \times (-150) = -64.28$. This increment of moment ($-$ 64.28) at the left end of EF produces no moment effect at the point, F, which was from the start a pin-connected joint. The moment, $-$ 85.72,



$$k_1=\frac{E_1 I_1}{l_1};\quad k_2=\frac{E_2 I_2}{l_2};\quad k_3=\frac{E_3 I_3}{l_3}$$

$$M_1=\frac{3k_1(4k_1k_2+4k_2k_3+4k_2^2+3k_1k_3)}{3k_2^2+4k_1k_2+3k_2k_3+3k_1k_3}$$

$$M_2=\frac{2k_2(k_2+k_3)M_1}{4k_1k_2+4k_2k_3+4k_2^2+3k_1k_3}$$

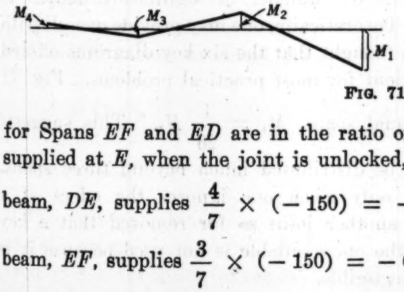
$$M_3=\frac{k_3M_2}{2(k_2+k_3)}$$

When $k_1=k_2=k_3$ then $M_1=\frac{45EI}{13l}$;

$$M_2=\frac{4}{15}M_1;$$

$$M_3=\frac{M_1}{15}$$

FIG. 70



$$M_1=\frac{4k_1(3k_2^2+4k_2k_3+3k_1k_2+3k_1k_3)}{3k_2^2+4k_2k_3+4k_1k_2+4k_1k_3}$$

$$M_2=\frac{k_2(3k_2+4k_3)M_1}{2(3k_2^2+4k_2k_3+3k_1k_2+3k_1k_3)}$$

$$M_3=\frac{2k_3}{3k_2+4k_3}M_2$$

$$M_4=\frac{M_3}{2}$$

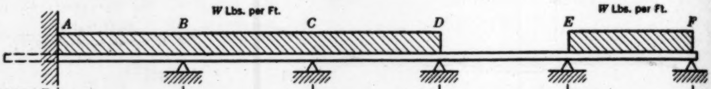
When $k_1=k_2=k_3$ then $M_1=\frac{52EI}{15l}$; $M_2=\frac{7}{26}M_1$;

$$M_3=\frac{2}{26}M_1;\quad M_4=\frac{1}{26}M_1$$

FIG. 71

at the right end of *DE*, induces a moment of $\frac{1}{2} \times (-85.72) = -42.86$ at *D*. These values, induced by the unlocking of Joint *E*, are recorded on Line 3, Fig. 72.

The only joint that remains to be unlocked is the joint, *D*. The stiffness factors for the right and left parts of the structure are taken from Fig. 68 and Fig. 71. In this case, in which the values of $k = \frac{EI}{l}$ for all spans are equal, the respective stiffness factors for *DF* and *DA* are $\frac{24}{7}$ and $\frac{52}{15}$, or 3.428 and 3.467. As Joint *D* is unlocked, a total balancing moment of $+142.86$ is to be supplied. The left end of *DF* supplies $\frac{3.428}{3.428 + 3.467} \times (+142.86) = +71.025$, while the right end of *AD* supplies 71.84. These values are recorded under *D* on the fourth line of Fig. 72.



	A	B		C		D		E		F
All Joints except *F*, are Locked	+100	-100	+100	-100	+100	-100	0	0	+150	0
Unlock *B* and *C*		0	0	0	0					
Unlock *E*							-42.86	-85.72	-64.28	0
Unlock *D*	+2.76	+5.52	-5.52	-19.34	+19.34	+71.84	+71.02	+17.75	-17.75	0
Resultant Moments at All Joints	102.76	94.48		119.34		18.16		67.97		0

FIG. 72

The effect of change in the moment at Joint *D* on the other joints can be obtained directly from Figs. 68 and 71. Thus, $\frac{7}{26} \times 71.84 = 19.34$ is the effect on *C* of the 71.84-moment change at *D*. The effect on Joint *E* of the moment change of 71.02 at Joint *D* is obtained from Fig. 68. This amounts to $\frac{1}{4} \times 71.02 = 17.75$. The final moments are obtained by adding the values under each joint. The answers in this case involve no approximation other than that involved in the number of significant figures to which the computations are carried. Theoretically, one may provide oneself with any number of key diagrams. It is thought that the six key diagrams offered herewith (Figs. 66 to 71) are sufficient for most practical problems. Fig. 71 shows that for a beam of three equal spans, $M_4 = \frac{1}{26} M_1$. This suggests that the moment change need not be distributed much beyond three spans, if at all. Approximation occurs only when one ignores the effect of a moment change at some joint on another joint so far removed that a key diagram is not available, or when the one available is not used because it is known that the effect involved is negligible.

Consider Fig. 73: Assume Joint *E* as locked, and let M_{BC} be represented by $+100$, and M_{CB} by -100; let $M_{BD} = +60$; $M_{DB} = -60$; and

$M_{EA} = M_{AE} = M_{EB} = M_{BE} = 0$. If $k_1 = k_2 = k_3 = k_4$, then, as the unbalanced moment at E is $+ 160$, each member supplies an equal balancing moment, which is $\frac{-160}{4} = -40$. The key diagram (Fig. 67) applies to all four legs. Thus, one-half the balancing moments is carried along each member to its extremity. The answers, then, are: $M_A = -20$; $M_B = -20$; $M_C = -100 - 20 = -120$; $M_{DE} = -60 - 20 = -80$; $M_{EC} = +100 - 40 = +60$; $M_{ED} = +60 - 40 = +20$; $M_{EA} = -40$; and $M_{EB} = -40$.

Suppose k_1, k_2, k_3, and k_4 are in the ratio, 1:2:3:4; they, as Joint E, Fig. 73, is unlocked, $M_{EA} = \frac{1}{10} \times (-160) = -16$; $M_{AE} = \frac{1}{2} \times (-16) = -8$; $M_{EB} = \frac{2}{10} \times (-160) = -32$; and $M_{EC} = +100 + \frac{3}{10} \times (-160) = +52$.

Suppose Joint C in Fig. 73, is pin-connected, and $k_1 = k_2 = k_3 = k_4$. With the same loading as shown, when Joint E is assumed to be locked, while Joint C is pin-connected, $M_{EC} = +150$. The unbalanced moment at Joint E then is $+150 + 60 = +210$. The stiffness factor for EC now is $\frac{3EI}{l}$, while, for the other three legs, it is $\frac{4EI}{l}$. The ratio, therefore, is 3 to 4. As Joint E is now unlocked, $M_{EA} = \frac{4}{15} \times -210 = -56$; $M_{AE} = -28$; $M_{EC} = +150 + \frac{3}{15}(-210) = +108$; $M_{CE} = 0$; $M_{ED} = +60 + \frac{4}{15}(-210) = +4$; and $M_{DE} = -60 + \frac{1}{2} \times \frac{4}{15} \times (-210) = -88$.

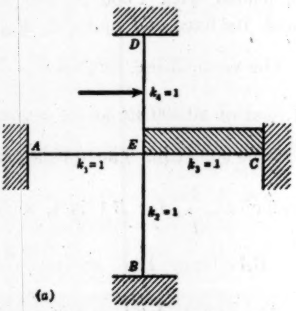


FIG. 73

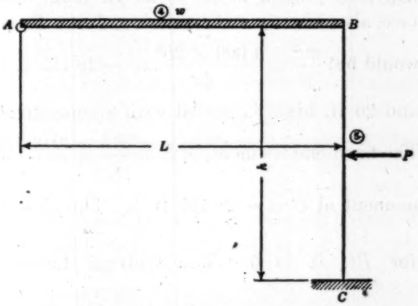


FIG. 74.

108
4
112

-56
-28
-88
172

The writer's engineering vocabulary does not include the much-loved word "exact"; otherwise, he would feel like applying this adjective to the answers given in the foregoing example. He has little objection to approximations, and certainly no objection thereto worth mentioning in design problems in which the number, 2, is as low as designers dare go in selecting a factor of safety. Professor Cross suggests that two approximations are generally sufficient. However, in his Table 1, it appears that the increment between the third and fourth approximation is greater than that between the second and third. If there is reason to carry on to three approximations, the same reason would then compel one to continue to at least four approximations.

The writer does not want to take space to work Professor Cross' example by the method he has herewith suggested. Joints *B, D, E,* and *F,* of Professor Cross' Fig. 1, might be unlocked successively. The stiffness factor of *CBA* may be taken from Fig. 69. Fig. 66 gives the stiffness factor for *CF;* Fig. 67 gives the stiffness factor for *CG;* while Fig. 68 gives the stiffness factor for *CDE.* After Point *C* is balanced, the carried-over moments for *B* and *A* may be taken from Fig. 69, while that for *D* is taken from Fig. 68. The factor carried over to *F* is zero, while the factor carried over to *G* is taken from Fig. 67. This procedure completes the entire solution. The results thus obtained check the values given by Professor Cross.

One more example might be given. In the L-frame, Fig. 74, the horizontal leg, *AB,* pinned at *A,* is 25 ft. long, and is loaded with 1 000 lb. per ft. (see, also, Fig. 5). Considering Joint *B* as fixed, its fixed-end moment, M_{BA}, would be: $\frac{wl^2}{8} = \frac{1\,000 \times 25^2}{8} = -\,78\,125$ ft-lb.. The vertical leg, *BC,* fixed at *C,* and 20 ft. high, is loaded with a concentrated load of 20 000 lb. at its center. The fixed-end moment, $M_{BC} = \frac{Ph}{8} = \frac{20\,000 \times 20}{8} = 50\,000$ ft-lb. The unbalanced moment at *C* is $-\,28\,125$ ft-lb. The elastic coefficient, $\frac{I}{l}$, for *BA* is 4, while for *BC* it is 5. The stiffness factor for *BA* from Fig. 66 is $\frac{3\,EI}{l} = 3 \times 4\,E = 12\,E$, while that for *BC* from Fig. 67 is $\frac{4\,EI}{l} = 4 \times 5 \times E = 20\,E$. As the joint is unlocked, the unbalanced moment distributes itself between *BA* and *BC* in the ratio of 12 to 20.

The final moment, $M_{BA} = -\,78\,125 + \frac{12}{32} \times 28\,125 = -\,67\,578$ ft-lb. The final moment, $M_{BC} = +\,50\,000 + \frac{20}{32} \times 28\,125 = +\,67\,578$ ft-lb. which compares with the value of 67 360 ft-lb. as determined by the more formal procedure. Once familiar with this method, one obtains in round figures the answer to a problem as simple as the foregoing without putting a figure on paper.

HARDY CROSS,[76] M. AM. SOC. C. E. (by letter).—The writer is indebted to those who have been kind enough to discuss the paper and have added so much to its value by illustrating the application of the principle involved.

Mr. Pilkey thinks the paper too brief. Pascal[77] is quoted as apologizing to a correspondent for writing a long letter on the ground that he did not have time to write a short one. The writer took time to make the paper short; as he looks at the mass of technical literature on his desk, he sincerely hopes the idea becomes popular. He thought best to confine the paper strictly to the fundamental principle and method involved, and not digress into the unlimited variations of this method or the unlimited applications of it. Secondary stresses, wind stresses, Vierendeel girders, continuous arches on elastic piers are special problems; while the elementary procedure is applicable to them, certain modifications are advantageous. The writer has indicated subsequently some of these modifications, but he has not felt justified in elaborating them in this closure.

---

[76] Prof., Structural Eng., Univ. of Illinois, Urbana, Ill.

[77] "Provincial Letters," December 14, 1656.

Questions raised in some of the discussions have been clearly answered in others. The questions of Mr. Lyman have been answered in detail by Professors Martel, Morris, and Witmer, and Messrs. Pilkey, Black, Wessman, and Wilson. The limitations of the method suggested by Mr. Nielsen have also been discussed by others.

Mr. Oesterblom's discussion is interesting and valuable, especially his explanation of the fundamental simplicity of Maxwell's method and of the difficulties which arise in applying it.

The discussions of Professor Shedd and Mr. Oliver call for little special comment. The writer appreciates them and heartily concurs in Professor Shedd's remarks with regard to formulas.

With the discussion of Mr. Vetter the writer differs in practically every statement. Mr. Vetter seems to have some serious objection to the method, but he does not make it clear. His interest in the vertical reactions at F and G is entirely academic. Of course, the total vertical reaction at C may be found by statics. This reaction is distributed between CF and CG in proportion to the "vertical stiffness" of these paths if "vertical stiffness" is defined as the vertical force at C along either path necessary to produce unit vertical deflection at C. Presumably it all goes to CG, unless CF is a "skyhook." If the members are of constant section and immovable at G and F, the vertical stiffness is $\frac{A\,E}{|L}$, and it should involve no difficulties if the section varies.

The effect of shearing distortions is not involved in the problem of the distribution of the total vertical reaction at C between the points, F and G. In the paper the writer makes no assumption at all as to the effect of shearing distortions; if they are taken into account, the fixed-end moments, the stiffness values, and the carry-over factors are different from those used if these distortions are neglected. Of course they should be neglected except perhaps in the most unusual cases. In the rare cases in which their effect in beams should be included, one should be guarded in accepting the conventional mechanics of internal stress in beams.

Whether longitudinal distortions shall be neglected depends on circumstances. It is a simple matter to correct for the effect of shearing and of longitudinal distortions by the method indicated in the paper. After the end moments are found in the beams, one computes the shears and the longitudinal forces and from these the distortions produced. He then finds that the frame is distorted by these displacements so that discontinuities would exist. The fixed-end moments needed to eliminate such discontinuities are then applied and these moments distributed throughout the frame. Theoretically, the procedure should be repeated, but except in the most unusual case, one trial will show the complete futility of the computation.

Mr. Vetter thinks that the engineer should "hitch his wagon to a star." He would do better to keep his feet on the ground. The conception of "mathematical accuracy" of the man on the street is decidedly amateurish; what the engineer needs is to get an answer with the requisite degree of accuracy without a prohibitive amount of labor.

The idea held by Mr. Vetter, that in some way (which he does not make clear), the method of moment distribution is approximate and involves assumptions which the theory of least work and the method of slope deflection escape, shows a fundamental misconception of these methods. Any one who computes the moments in a frame of reinforced concrete will make many assumptions as to the properties of materials, conservation of plane sections, and other matters; E has no definite value and I has no definite meaning in the analysis of frames of reinforced concrete. This is true by whatever method the frame is analyzed. Whether the computer knows that such assumptions are made is quite another matter.

Mr. Vetter seems also to object to the writer's list of incidental problems involved and states that he could add a great many more, "which have just as much, and more, to do with the subject under discussion [moment distribution?]" That is excellent, or it would have been excellent had he done so; one of the things needed in connection with continuous frames is a clear recognition of the complications involved. Another need is for a simple method of studying the effect of such complications; moment distribution is such a method. The three samples of complications furnished by Mr. Vetter are not of the type listed by the writer. Change in length of members due to shrinkage and to change of temperature and the settlement of supports are clearly recognized phenomena, the effect of which, for given data, are easily evaluated by moment distribution exactly as indicated in the text, thus:

> "Imagine any joint in a structure, the members of which are being deformed by loads, or in some other way, to be first held against rotation and then released."

The method is not restricted to analysis for loads on the structure.

Mr. Eremin thinks that,

> "In the limitations of his method, the author omitted to state that the modulus of elasticity of materials was considered to be constant for all members of the frame and also that the lateral rigidity of the members of a continuous frame is sufficient to prevent buckling."

The writer thinks that the first statement is in error. One does not necessarily assume either that E is constant for all members or from section to section in any one member or over any one cross-section of the member. It is merely necessary to have an accurate stress-strain diagram for the material.

If it is first assumed that E is constant and all stresses are computed on this basis, the fixed-end moments, stiffness values, and carry-over factors may then be revised for the values of E indicated by these stresses. The operation may be repeated to any degree of precision. The problem is evidently one for research, not for the office. The writer has studied it in relation to the effect of chance variation of the modulus of elasticity,[78] the effect of plastic flow and time yield in concrete, and the effect of secondary stresses beyond the elastic limit of steel. The common assumption of constant E is a matter of convenience and is not a limitation of method.

The statement that the method is restricted by the assumption that the members do not buckle seems to the writer to mean that the method will give the moment in a member if the member is still there—if it has not failed. This is correct.

Mr. Mitra deplores the fact that the method does not lead to any general equation. Since the avoidance of such general equations was the object of the paper, the writer is pleased by the comment. That the method of slope deflection and the graphical method of conjugate points are more accurate is not true; that these methods are quicker and easier to use is opposed to the experience of hundreds of engineers in the United States.

Professor Hickerson evidently does not like the paper at all, even though he "commends the author." Consequently, he introduces the subject of "Tabular Coefficients." Of course, it is better to use the answer previously determined for a problem than it is to re-work the problem, provided one is sure that the answer is correct and provided the answer is easily found. The trouble with tabular coefficients is that one cannot be sure, unless he checks them, that they are correct, however distinguished their author, and that the one using them may not agree with the basis on which they are computed. Unfortunately, few published tables explain clearly the basis on which they are prepared. Even so careful a scholar as Professor Hickerson fails to state whether the negative moments which he has computed are at the face of the support or at the center, and what allowance, if any, he has made for the increased flexural resistance of the members between the faces of any support.

Professor Hickerson gives a coefficient for an arch frame uniformly loaded between columns. He then states that "if the entire load is concentrated at the central ridge, * * * the foregoing values would be multiplied by $\frac{3}{2}$." This is not quite correct either here or in general. In this case it is nearly true but any one using the tables must determine for himself when it ceases to be approximately true. Thus, any one using the tabular coefficients will still need a method of analysis.

Mr. Floris thinks that the idea of using fixed-end moments in the analysis of a continuous frame is not new. Of course it is not, but Mr. Floris has gone far afield for his references. Fixed-end moments have been used in slope deflection in this country for years and apparently by Manderla before that and probably by others before Manderla. Mr. Floris, moreover, has overlooked the paper presented by Professor W. M. Wilson at the World Engineering Congress at Tokyo. The book in Japanese is not available to the writer and would do him little good if it were. The method of the late Professor Ostenfeld referred to by Mr. Bednarski and by Mr. Floris is apparently an extension of slope deflection along the lines indicated by the writer in his monograph on the column analogy.[79] The presentation by Mr. Bednarski is, unfortunately, not very attractive. The writer fails to find anything like the method of the paper in Hartmann's "Statisch Unbestimmte Systeme."

---

[78] "Dependability of the Theory of Concrete Arches," *Bulletin 203*, Eng. Experiment Station, Univ. of Illinois.

Two conventions of sign are possible for moments at the ends of the members of a frame. The convention of rotation for statical moments is indicated where rotations are involved in the solution and, therefore, is used in the method of slope deflection. The alternative convention is for bending moments. Where a structure is to be designed from the moments found by the analysis the writer still feels that it is best to adopt throughout the analysis the convention commonly used in design. Several who have preferred the convention of rotation have used it in connection with problems of wind stress or of secondary stress. In such problems the writer also prefers the convention of clockwise rotation, for with wind stresses and secondary stresses it usually makes no difference which side of a member is in tension. The object of analyses for secondary stresses is to determine whether they are too high, not to use them in design; the wind may blow in either direction. In any case, one should consistently follow one method, as Mr. Gorman notes.

Some misunderstanding has appeared in the discussions and some misstatement has appeared elsewhere as to the restriction given in the paper for the method. The writer does not confine the method to frames in which the joints do not move, but merely states that these movements must take place in finite jumps so that "the joints do not move during the process of moment distribution." The method is applicable, without special complication, to all frames for any condition of loading, transverse as well as vertical, and for any condition of deformation, whether due to shrinkage, change of temperature, or settlement of support. Several discussions illustrate such applications.

The first part of the paper is restricted to structures in which the joints do not move. The writer explained that if such movement actually takes place, allowance may be made for it as indicated later in the paper. This procedure has been explained in considerable detail by several contributors. With regard to this movement of the joint (referred to in the paper as side-sway), four questions are important: (1) Can side-sway occur in the structure? (2) Does side-sway occur for those conditions of loading which control the design of the structure? (3) Is this side-sway a matter of importance in design and is an approximate solution sufficiently accurate? (4) How shall one determine the exact moments produced by this side-sway?

In most structures side-sway produced by vertical loads only is either impossible or improbable. Thus, the probability of the lateral movement of a floor of a building due to vertical loads is negligible. Even where the forces act transversely, side-sway of individual bents is commonly either prevented or checked. Analysis for the moments produced by crane loads in a mill-building bent are often incorrect. If the load produces side-sway of each individual bent, there would occur, as the load moves, so much lateral vibration as to impair, seriously, the usefulness and the durability of the structure; such movement must be prevented or checked by some system of bracing and the bent then is not free to sway sidewise. Similarly, the bent of an elevated railway is not free to sway because of vertical live loads, but is restrained by horizontal bracing of the girders which frame into or rest upon the bent. The writer has also seen solutions presented for stresses produced in lateral frames of bridges due to unsymmetrical loading of the floor-beams which neglected the restraint against relative movement of the planes of the top and bottom chords of the bridge offered by lateral bracing.

---

[79] "The Column Analogy," *Bulletin 215*, Eng. Experiment Station, Univ. of Illinois.

In those cases in which side-sway due to the vertical loads is possible, it is frequently most improbable. An unsymmetrical vertical load may indicate lateral movement of any one loaded bay in a building of reinforced concrete, but for such lateral movement to occur freely it would be necessary for the entire floor to be loaded in this same way; the probability is that lateral sway will not occur. Attention should be called to the fact that side-sway is often a mitigating influence tending to decrease the maximum stress in certain parts of the structure, especially in the more highly stressed column; the assumption of side-sway where it does not exist may be on the side of danger.

In those cases in which side-sway may occur, it is neither necessary nor desirable to take account of it for each condition of loading. It is more convenient to correct for it only for those combinations of loading which produce maximum controlling moments.

Side-sway is much like the effect of wind. If a frame is analyzed on the assumption that the joints do not move and movement of the joints will actually occur, it will be found that the sum of the shears in the columns is not equal to the known shear. Movement of the joints must then have been prevented by forces acting at these joints. These forces reversed in direction may then be treated as wind loads. Moments may then be found by some of the approximate methods of wind-stress analysis to determine how large are the stresses involved.

In many cases side-sway does not exist; in other cases, it is of no importance; in most of the remaining cases. it is sufficiently unimportant to justify the use of approximate methods of wind-stress analysis without hesitation. In the remaining cases in which it requires precise analysis an indirect method may be used.

"Assume any series of fixed-end moments in the legs such that all legs have the same deflection. * * * Distribute these fixed-end moments and find the total shear in the legs." Add these moments to those assuming no side-sway in such ratio that statics is satisfied. The subject has interested several contributors. The procedure may be by direct proportion as indicated by the writer in another paper,[80] and as shown by several discussers; it may be by finding first the effect of unit shear in one story at a time and combining the solutions by a method of successive convergence, as suggested by Professor Grinter; one may write a series of formal equations, as Professor Constant suggests; or one may use successive approximations to the true values, as indicated in detail by Professor Morris.

It does not seem to the writer that any one of these variations of technique has any general advantage; sometimes one and sometimes another is the more useful. Thus, in a single-story frame, it is undoubtedly more expeditious to find the effect by direct proportion, but in a high building the method used by Professor Morris offers some advantages.

Another variation of method saves time in analyses for wind stresses. The joints are not assumed to be fixed-ended, but it is assumed that all joints not fixed have the same rotation, the moments in the girders being proportional to their K values and the moments in the columns, due partly to rotation of the joints and partly to displacement of the floors, being those determined by the shears. If the ratio of girder moments to K values is chosen with discretion, all the joints may often be nearly balanced before any moments are distributed. In the columns in the first story an adjustment must be made for the fixation at the base.

Fig. 75 shows the revised computations for the building discussed by Professor Morris. Only one distribution and one shear adjustment have been made. The moments given by Wilson and Maney are also given on the diagram (in brackets, thus: [24.0]); it will be seen that the agreement for girder moments is much more satisfactory than in Professor Morris' solution. Too many conclusions should not be drawn from this case, however; the building is too regular to be taken as typical of those variations of framing arrangement which present, in modern design, the real problems of wind stress.

In Fig. 75, two devices have been used to simplify the procedure. In the first place certain moments proportional to K have been thrown arbitrarily into the girders so that, to begin with, all joints are nearly balanced. These represent the effect of equal rotations at all joints except the bases of the columns; this exception is provided for in the columns of the bottom story by use of the factor, $\frac{1}{C}$, discussed later.

In the second place, the analysis is abbreviated by writing only the moments carried over and making one distribution at the end of the procedure. The moments produced by each distribution which would need adjustment to balance the shears are evidently equal to three times the moment carried over in the columns in any story.

The solution is presented here as an interesting variation of the technique of moment distribution and not necessarily as a procedure recommended for the analysis of wind stresses in all cases.

---

[80] "Continuity as a Factor in Reinforced Concrete Design," by Hardy Cross, *Proceedings*, Am. Concrete Inst., 1929.

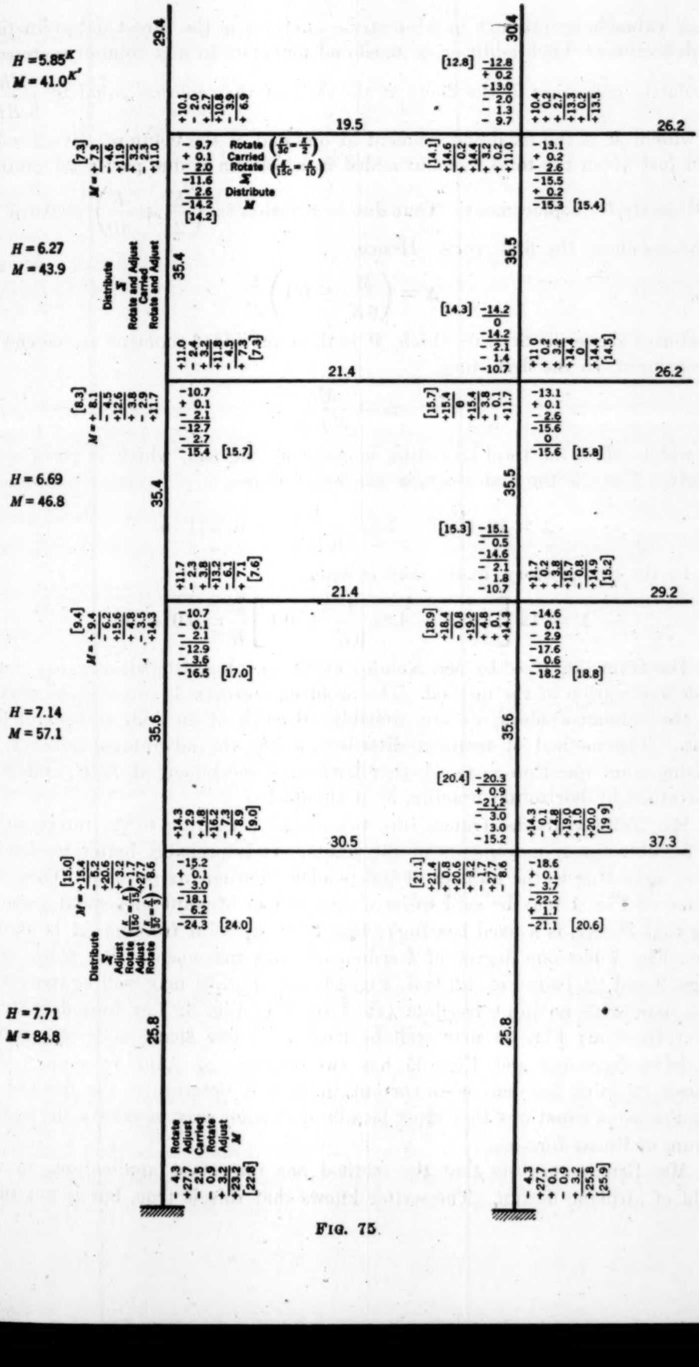

H = 5.85^K
M = 41.0^K'
H = 6.27
M = 43.9
H = 6.69
M = 46.8
H = 7.14
M = 57.1
H = 7.71
M = 84.8
29.4
30.4
19.5
26.2
35.4
35.5
21.4
26.2
35.4
35.5
21.4
29.2
35.6
35.6
30.5
37.3
25.8
25.8
[12.8]
[14.2]
[15.4]
[14.3]
[15.7]
[15.8]
[15.3]
[17.0]
[18.8]
[20.4]
[24.0]
[20.6]
Rotate
Carried
Rotate
Σ
Distribute
M
Distribute
Σ
Rotate and Adjust
Carried
Rotate and Adjust
Adjust
Rotate
Adjust
Rotate
Rotate
Adjust
Carried
Rotate
Adjust
M

FIG. 75

A valuable by-product in wind-stress analyses is the direct determination of deflections. Each addition of fixed-end moments in any column represents a relative deflection of the floors at the ends of that column equal to $\frac{Mh}{6\,EK}$, in which, M is the fixed-end moment at one end of the column. In the solution just given the total moment added in a column is due partly to rotation and partly to displacement. That due to rotation is $\left(\frac{K}{2} + \frac{K}{10}\right) = 0.6\,K$ in columns above the first story. Hence,

$$\Delta = \left(\frac{M}{6K} + 0.1\right)\frac{h}{E}$$

in stories above the first, in which, M is the total added moment at one end of the column. In the first story,

$$\Delta = \frac{Mh}{6\,EK}$$

in which, M is the total adjusting moment at one end, which is given separately. Thus, is the first story, in the wall column,

$$\Delta = 12\ (27.7 + 3.2)\ \frac{h}{6\,EK} = 0.0211 \text{ in.}$$

In the second story, in the wall column,

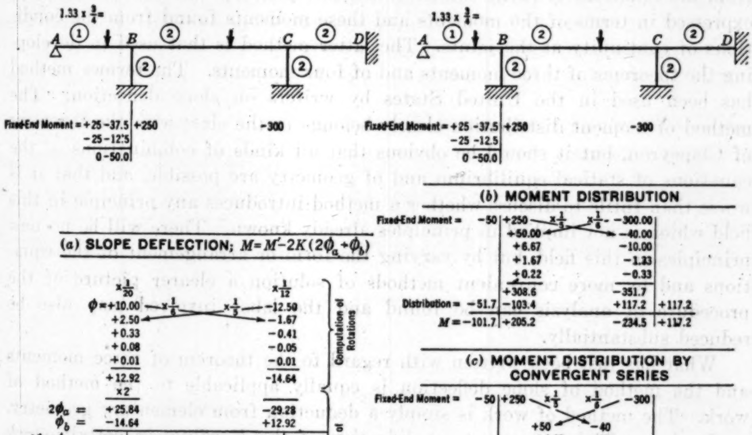
$$\Delta = 12\left[(14.3 + 4.8)\,\frac{1}{6K} + 0.1\right]\frac{h}{E} = 0.0146 \text{ in.}$$

The frame analyzed by Mr. Nelidov offers an interesting illustration of the wide application of the method. The problem, however, is solved more readily by the column analogy, as are probably all cases of an arch or bent of one span. The method of moment distribution has the advantage, however, of raising some question as to whether horizontal movement at B, C, and D is restrained by horizontal bracing, as it should be.

Mr. Nelidov divides frames into two classes according to the interrelation of possible linear movements of the joints. It is probably better to classify them according to the number of independent movements possible. Thus, the frame of Fig. 1 may be said to have no freedom of joint movement (assuming that D, say, is a fixed bearing); Fig. 5 has no joint freedom (A is fixed); also, Fig. 7 has one degree of freedom of joint movement; Fig. 8 has one; Figs. 9 and 10, two; Fig. 12, two; Fig. 22, one; Fig. 31 may well be treated as one span with no joint freedom (at A or A'); Fig. 32 has four degrees of joint freedom; Fig. 33 may well be treated as two spans with one degree of joint freedom; and Fig. 65 has two degrees of joint freedom. The degree of joint freedom is important in that it determines the number of simultaneous equations that must be solved in some way to satisfy the equilibrium of linear forces.

Mr. Bruhn suggests that the method has important applications in the field of airplane design. The writer knows that this is true, but is not sufficiently familiar with the problems involved in this field to illustrate its usefulness; he hopes that Mr. Bruhn will find time to do so. Certainly the method should facilitate the analysis of continuous struts subject to flexure. As Mr. Bruhn suggests, there is no difficulty in extending the method to include torsional effects. The writer has used the method in special problems of space frames in which the effect of torsional resistance is included in the analysis.

Mr. Downey has presented an interesting discussion of the application of successive convergence to the method of slope deflection. Slope deflection has more than justified its place in America and Mr. Downey has made a valuable addition to this literature. If, however, Mr. Downey will arrange his computation in the way shown by the writer and, further, if in using moment distribution one writes only the moments carried over in each member, it will be found that his computations parallel closely those of moment distribution; but in slope deflection the moments are first translated into rotations which are finally translated back into moments and this represents additional work not used in the method of moment distribution.


1.33 x $\frac{3}{4}$=
A
①
B
②
C
②
D
②
②
Fixed-End Moment = +25 −37.5 | +250
−25 −12.5
0 −50.0
−300
$x\frac{1}{20}$
$x\frac{1}{12}$
φ = +10.00
$x\frac{1}{6}$
$x\frac{1}{5}$
−12.50
+2.50
−1.67
+0.33
−0.41
+0.08
−0.05
+0.01
−0.01
+12.92
−14.64
x2
2φa = +25.84
−29.28
φb = −14.64
+12.92
2φa+φb = +1292 | +11.20
−16.36 | −14.64
2K = x4 | x4
x8
+51.70 | +44.80
−65.44 | −117.2
Fixed-End Moment = −50 | +250
−300
M = −101.7 | +205.2
−234.5 | +117.2
Computation of Rotations
Computation of Moments
Fixed-End Moment = −50 | +250
$x\frac{1}{5}$
$x\frac{1}{6}$
−300.00
+50.00
−40.00
+6.67
−10.00
+1.67
−1.33
+0.22
−0.33
+308.6
−351.7
Distribution = −51.7 | −103.4
+117.2 | +117.2
M = −101.7 | +205.2
−234.5 | +117.2
Fixed-End Moment = −50 | +250
$x\frac{1}{5}$
$x\frac{1}{6}$
−300
+50
−40
+6.67
−10
+56.67
−50.0
+58.6
$\left(x\frac{1}{1-\frac{1}{30}}\right)$
−51.6
+308.6
−351.6
Distribution = −51.7 | −103.4
+117.2 | +117.2
M = −101.7 | +205.2
−234.4 | +117.2

(a) SLOPE DEFLECTION; $M = M' - 2K(2\phi_a + \phi_b)$

(b) MOMENT DISTRIBUTION

(c) MOMENT DISTRIBUTION BY CONVERGENT SERIES

FIG. 76.

These relations are brought out by the problem shown in Fig. 76. This problem has been solved by slope deflection (Fig. 76(*a*)) and by moment distribution (Fig. 76(*b*)). As an additional check it has been solved also by the abbreviated method of moment distribution for a three-span beam in which the converging series are summed at once (Fig. 76(*c*)). By the method of slope deflection, using the method of successive convergence, the values of ϕ

are first found. These values of ϕ are next converted into moments and added to the original end moments. In moment distribution the unbalanced moments are carried over just as the ϕ values were carried over, the amount of computation being the same; but in moment distribution the quantities dealt with are always moments, which are the thing sought; by the method of slope deflection these moments are converted into rotations, from which the moments must be recomputed in the end.

Any frame must satisfy two general groups of conditions. The forces acting on it must satisfy the laws of statics so that the frame shall be in equilibrium. The forces acting on it must also satisfy the conditions of geometry, so that the rotations produced in the frame by these forces shall maintain continuity in the frame. The moments at the joints may be expressed either in terms of the joint rotations in the first place and these rotations found from the equations of static equilibrium at the joints, or the rotations may be expressed in terms of the moments and these moments found from the conditions of continuity at the joints. The latter method is that used in developing the theorems of three moments and of four moments. The former method has been used in the United States by writers on slope deflection. The method of moment distribution clearly belongs in the class with the theorems of Clapeyron, but it should be obvious that all kinds of combinations of the equations of statical equilibrium and of geometry are possible, and that it is worse than futile to discuss whether a method introduces any principle in this field which is not implied in principles already known. There will be no new principles in this field, but by varying the form or arrangement of the equations and by more convenient methods of solution a clearer picture of the procedure of analysis can be found and the labor involved may also be reduced substantially.

What has just been written with regard to the theorem of three moments and the method of slope deflection is equally applicable to the method of work. The method of work is simply a deduction from elementary geometry.

Professor Richart suggests modification of the *K* values in certain stock cases. General relations to be used in direct distribution may be stated as follows. The moment needed to produce unit rotation at one end of a member may be written in terms of the physical properties of the member and the ratio of the changes in end moments in the member due to rotation.

Let M_a = change in moment from the fixed-end condition at the end, *A*, under consideration;
M_b = change in moment from the fixed-end condition at the other end of the member, *B*;
S_a = stiffness of the member at *A*, defined as the moment necessary to produce unit rotation at *A* when *B* is fixed;
r_a = carry-over factor at *A* (from *A* to *B*); and,
r_b = carry-over factor at *B* (from *B* to *A*).

Then, for unit rotation at *A*,

$$M_a = S \frac{1 - r_a r_b}{1 - r_b \dfrac{M_b}{M_a}} = S_a \frac{1}{C}$$

If, then, the ratio, $\frac{M_b}{M_a}$, is known for all members at a joint, the unbalanced moment at that joint may be distributed directly.

In certain cases the ratio, $\frac{M_b}{M_a}$, is known from inspection. When the member is fixed at B, $\frac{M_b}{M_a} = r_a$; when the member is free at B, $M_b = 0$; in cases symmetrical as to form and loading, $M_b = M_a$; and in anti-symmetrical cases, $M_b = -M_a$.

Values of the constant, $\frac{1}{C}$, are shown in Table 21. The general expression may be deduced, of course, in several ways; many will deduce it from area moments. This general relation leads also to a check on the analysis. It is easy to see whether the conditions of statics are satisfied by observing the balance of the joints. The conditions of geometry are satisfied if $M_a - r_b M_b$ is proportional to $\frac{S_a}{1 - r_a r_b}$, or, in the case of beams of constant section, if $M_a + \frac{1}{2} M_b$ is proportional to K at any joint.

As one uses the method of moment distribution it soon becomes evident that it is not necessary to write the distributed moment each time. One may write only the moment carried over and make all distributions in one operation at the end of the process. Time is saved, since almost one-half the figures are omitted, but the simple physical picture of the operation is considerably obscured.

TABLE 21.—VALUE OF CONSTANT, $\frac{1}{C} = \frac{1 - r_a r_b}{1 - r_b \frac{M_b}{M_a}}$

Description of beam	Unsymmetrical haunching, $r_a >$ or $< r_b$	Symmetrical haunching, $r = r_a = r_b$	Uniform section, $r_a = r_b = -\frac{1}{2}$
Beam fixed at far end	1	1	1
Beam simply supported at far end	$1 - r_a r_b$	$1 - r^2$	$\frac{3}{4}$
Symmetrical	$\frac{1 - r_a r_b}{1 - r_b}$	$1 + r$	$\frac{1}{2}$
Anti-symmetrical	$\frac{1 - r_a r_b}{1 + r_b}$	$1 - r$	$3/2$
Beam continuous at far end	$(1 - r_a r_b)$ to 1	$(1 - r^3)$ to 1	$\frac{3}{4}$ to 1

This procedure may be carried further in any case where the value, $\frac{1}{C}$, is known for all spans but one in a frame. Carry over the proper proportion of the unbalanced moment in this span and repeat. Let a and b be the proportions carried over at the two ends. If the procedure of carrying over moments is continued it will be seen that they run to an infinite converging series the sum of which equals $\frac{1}{1-ab}$ times the sum of the first two terms. Thus, the total moments carried over may be written after two terms and the total unbalanced moment may be distributed.

Fig. 76(*c*) shows the procedure. The unbalanced moments at *B* and *C* are carried over twice. The sum of the moments thus carried over is then multiplied by $\frac{1}{1-ab} = \frac{1}{1-\frac{1}{5}\times\frac{1}{6}}$. The sum thus found for the moments carried over is then added to the original fixed-end moments and the total is distributed at the joint.

This method of writing at once the sum of the converging series may be extended to frames in which the value, $\frac{1}{C}$, is unknown for more than one span in the frame. The general procedure of exact moment distribution by summing the convergent series is of great interest and, in some cases and for some purposes, of great value; but such methods are not properly a part of a paper the object of which is to present a single general method involving no special paraphernalia.

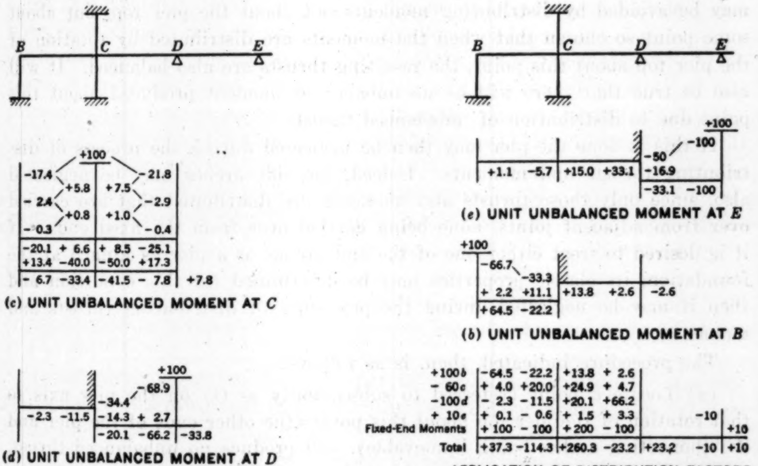
It is interesting to note that, to determine moments in any span, any system whatever may be reduced with close approximation to an equivalent structure of three spans, such as *A B C D* in Fig. 76, by estimating or computing the fixation of the outer ends of all members except *BC*. Professor Van den Broek indicates the use of such substitute frames; so also does Professor Richart. The writer also used the concept of equivalent stiffness in his paper, "Continuity as a Factor in Reinforced Concrete Design," previously mentioned.

As another illustration, in the truss analyzed by Mr. MacLean, the members meeting at *C* are known to remain fixed at the end at this joint; also, the member, *BD*, is known to be symmetrically distorted and $\frac{1}{C} = \frac{1}{2}$. The value, $\frac{1}{C}$, is then known for all members except the end posts. By twice carrying over distributed moments in this member and then summing the series by multiplying by $\frac{1}{1-ab}$, the total change in end moment due to carry over is at once determined and can be directly distributed.

In secondary stresses this method of convergence is usually only of academic interest. In the problem solved by Messrs. Thompson and Cutler it is not applicable, but the procedure used by them is much shortened if all distribution is made once for all at the end of the analysis.

Extension of the method of moment distribution to the study of the effect of gusset-plates on the secondary stresses is of some interest and presents no great analytical difficulty, the members being treated as of varying section. Any one who has analyzed on this basis will recognize more clearly the futility of great precision in computations on the usual basis.

Mr. Hartmann suggests that time may be saved by distributing unit unbalanced moments at the individual joints and then combining for any combination of loading. Mr. Dillingham uses the method in connection with moving load systems. This procedure has much merit where many combinations of loading are to be considered. It is not necessary, however, to distribute a unit moment in each member, but only at each joint. The distribution at each joint being known, any unbalanced moment from any of the members meeting at that joint may be distributed at once. The writer always uses this method where moving concentrated loads are under consideration. The fixed-end moments are found from influence lines. These are available for beams of varying section in a recent paper by Mr. L. T. Evans.[81] The unbalanced end moments are then distributed by the factors already found. On a string polygon for the load system, one then lays off the end moments at the supports, draws the closing lines in each span, and thus constructs the entire curve of moments for any position of the loads.


B
C
D
E
B
C
D
E
+100
-17.4
-21.8
+5.8
+7.5
- 2.4
-2.9
+0.8
+1.0
-0.3
-0.4
-20.1 + 6.6
+ 8.5 -25.1
+13.4 -40.0
-50.0 +17.3
- 6.7 -33.4
-41.5 - 7.8
+7.8
+100
-100
-50
+1.1 -5.7
+15.0 +33.1
-16.9
-33.1 -100
+100
-66.7
-33.3
+ 2.2 +11.1
+13.8 +2.6
-2.6
+64.5 -22.2
+100
-68.9
-34.4
-2.3 -11.5
-14.3 + 2.7
-20.1 -66.2
-33.8
+100*b* -64.5 -22.2 +13.8 + 2.6
- 60*c* + 4.0 +20.0 +24.9 + 4.7
-100*d* - 2.3 -11.5 +20.1 +66.2
+ 10*e* + 0.1 - 0.6 + 1.5 + 3.3 -10
Fixed-End Moments + 100 - 100 + 200 - 100 +10
Total +37.3 -114.3 +260.3 -23.2 +23.2 -10 +10

(*c*) UNIT UNBALANCED MOMENT AT *C*

(*e*) UNIT UNBALANCED MOMENT AT *E*

(*b*) UNIT UNBALANCED MOMENT AT *B*

(*d*) UNIT UNBALANCED MOMENT AT *D*

APPLICATION OF DISTRIBUTION FACTORS, IN (*b*) (*c*) (*d*) AND (*e*), TO FIG. 1.

FIG. 77.

This procedure may be abbreviated by distributing moments at a support near the center of the structure. This joint is then held rigid while distribution is made at adjacent joints on either side, the resulting unbalanced moment at the first joint being then distributed from the values already found. Thus, the distribution of individual moments becomes more rapid as the ends of the structure are approached.

Fig. 77 shows the application of this procedure to the problem solved in the original paper. An unbalanced moment is first distributed at *C*. Then, when unbalanced moments are distributed at *B* and *D*, *C* is first treated as fixed and the values previously found are used to distribute the unbalanced moment at C. Similarly, when an unbalanced moment is distributed at E, Joint D is first treated as fixed. The total end moments got by distributing the unbalanced fixed-end moments are shown at the end of the problem. The advantage of the method, however, seems to be chiefly in those problems where many different combinations of moments are to be considered. The abbreviated method of distributing moments only once is used; the convention of signs herein is that for statical moments.

---

[81] "The Modified Slope-Deflection Method," by L. T. Evans, *Journal*, Am. Concrete Inst., October, 1931.

Mr. Larson has made a valuable addition to the paper. The method of moment distribution is applicable to beams which are curved as well as to those which are straight, and is applicable to the distribution of forces as well as of moments. In order to use it conveniently, however, it is important that the convergence be rapid.

In the solution indicated by Mr. Larson it will be seen that the convergence is not very rapid. This is due to the large unbalanced thrusts and moments thrown back into the system each time by the pier. This disturbing factor may be avoided by distributing moments not about the pier top, but about some point so chosen that when the moments are distributed by rotation of the pier top about this point, the resulting thrusts are also balanced. It will also be true that there will be no unbalanced moment produced about this point due to distribution of unbalanced thrusts.

If this is done the pier may then be neglected during the process of distributing thrusts and moments. Indeed, the side arches may be neglected also, since only those thrusts and moments are distributed that are carried over from adjacent joints, none being carried over from the fixed ends. If it is desired to treat either one of the end arches or a pier as hinged at the foundation, its elastic properties may be determined for this condition and then it may be neglected during the procedure of distributing thrusts and moments.

The procedure indicated, then, is as follows:

(*a*) Locate a point (referred to subsequently as O) on the pier axis so that rotation of the pier top about this point (the other ends of the pier and the connecting arches being immovable), will produce no unbalanced thrust.

(*b*) Determine the thrust needed in each member alone to produce unit horizontal displacement at the pier top without rotation. Determine from this the distribution factors to the connecting arches.

(*c*) Determine the moments needed in each member alone to produce unit rotation of the end of the member about the point, O. Find from these the distributing factors for moments about O for the connecting arches. Locate also the thrust lines for such rotations. From the percentage of unbalanced moment distributed to each arch find the percentage carried over to the other end.

This gives all the essential data to be used in the distribution of the thrusts and moments. Now, proceed as follows:

(1) Distribute and carry over the unbalanced thrusts. Do not write the distributed values, but only the values carried over. Continue to convergence and find the total thrusts carried over.

(2) These thrusts produce unbalanced moments equal to the thrust multiplied by the vertical distance from the point, *O*, to the thrust line.

(3) Distribute and carry over the unbalanced moments just found, together with the original unbalanced moments about the points, *O*. Write only the moments carried over. Continue to convergence and find the total moments carried over.

(4) These moments produce unbalanced thrusts equal to the moments divided by the vertical distance from *O* to the thrust line.

Repeat the cycle to any desired accuracy. Next, by addition, find the total thrusts produced by displacements and the total thrusts produced by rotations. Since their lines of action are known, any other quantities may be determined by statics.

Since it is now necessary to deal only with the arches during the process of distribution, a somewhat more convenient convention of signs for thrusts than that used by Mr. Larson may be adopted. If the usual convention of signs is followed for moments used in the design of arches and girders that positive moment produces tension on the lower side, then a thrust acting above the axis—on the positive side—produces positive moment. Thrust in the arch is positive and tension negative. This makes it possible to add thrusts carried over to opposite ends of the arch without confusion.

Fig. 78 shows the solution of the problem given by Mr. Larson, using this technique. The percentages to be carried over and the lever arms of the thrusts have been determined from the data given by Mr. Larson. The final solution is obtained by adding the different thrusts and is indicated at the bottom of the diagram. The solution is shown here only to illustrate how, in this special case, a modification of the technique facilitates the convergence. The writer hopes to elaborate this procedure, elsewhere. It has a wide field of application in studies of continuous arch series in bridges or in buildings and of combinations of girders and arches.

It is not the function of this paper to discuss methods of deriving the elastic properties to be used in distributing the fixed-end moments and thrusts to the piers and arches. The writer prefers to use the theorems of the column analogy, according to which, the stiffness values and lines of thrust are defined as follows: (1) The stiffness for thrust is the stress at unit vertical distance from the neutral axis of the analogous column section for unit moment about the horizontal axis of this section; (2) the stiffness for moment is the stress on the analogous column section at the center of rotation for unit load at this center; and (3) the lines of thrust are the neutral axes of the analogous column section for these loadings. These theorems are equally applicable whether the arches are symmetrical or unsymmetrical.

The point about which rotation should be applied in order that the resulting thrusts shall balance may be found as "the centroid of the values for horizontal stiffness," or so that the sum of the products of vertical distances to the thrust lines for displacement multiplied by the horizontal stiffness shall equal zero.

Professor Large, and Messrs. Earl and Gordon have discussed cases in which the moment of inertia of the members is variable. As stated in the paper, there is no difficulty in applying the method to such structures; the fixed-end moments, the values for stiffness, and the carry-over factors are different if the members are of varying *I*. It may be well to observe that, at least in reinforced concrete, the case of variable section is the rule rather than the exception, although this fact is not yet generally recognized. Even where the members seem to be of constant section, the moment of inertia is very much increased beyond the faces of the supports. Note carefully that the sum of fixed-end moments at the center of intersections is to be distributed, not the sum of moments at the faces of supports. Neglect of this requirement of statics invalidates the entire analysis. Tables or curves that purport to give the moment at the face of the supports of continuous frames without reference to the ratio of the width of support to span length are necessarily incorrect and may be very seriously in error. Note, also, that traditional practice in the use of moment coefficients is based on the clear span, as is pointed out by Mr. Wessman in his interesting discussion.

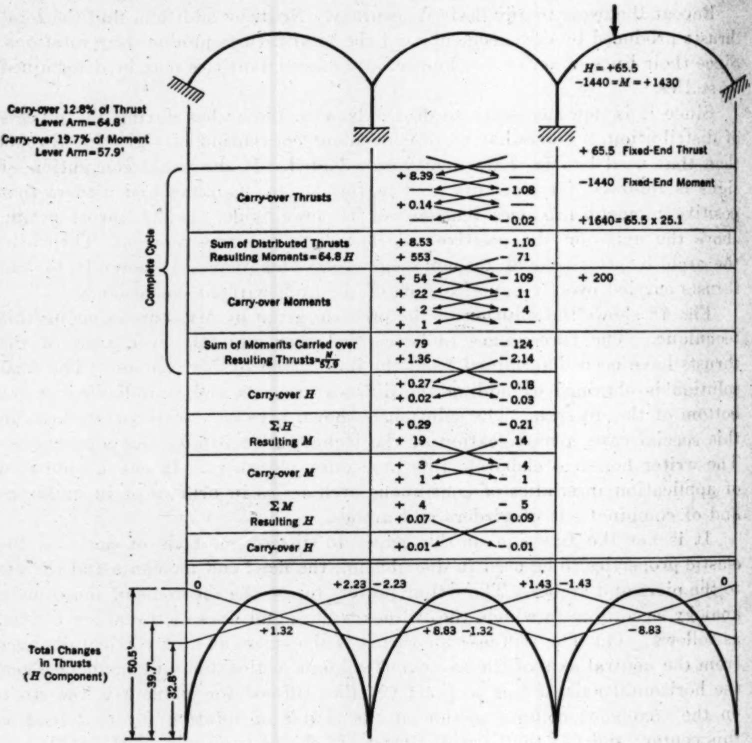

H = +65.5
−1440 = M = +1430
Carry-over 12.8% of Thrust
Lever Arm = 64.8'
Carry-over 19.7% of Moment
Lever Arm = 57.9'
+ 65.5 Fixed-End Thrust
−1440 Fixed-End Moment
+1640 = 65.5 x 25.1
Complete Cycle
Carry-over Thrusts
+ 8.39
—
+ 0.14
—
—
− 1.08
—
− 0.02
Sum of Distributed Thrusts
Resulting Moments = 64.8 H
+ 8.53
+ 553
− 1.10
− 71
Carry-over Moments
+ 54
+ 22
+ 2
+ 1
− 109
− 11
− 4
—
+ 200
Sum of Moments Carried over
Resulting Thrusts = M/57.9
+ 79
+ 1.36
− 124
− 2.14
Carry-over H
+ 0.27
+ 0.02
− 0.18
− 0.03
ΣH
Resulting M
+ 0.29
+ 19
− 0.21
− 14
Carry-over M
+ 3
+ 1
− 4
− 1
ΣM
Resulting H
+ 4
+ 0.07
− 5
− 0.09
Carry-over H
+ 0.01
− 0.01
0
+2.23
−2.23
+1.43
−1.43
0
+1.32
+8.83
−1.32
−8.83
Total Changes in Thrusts (H Component)
50.5'
39.7'
32.8'

FIG. 78.

The writer has discussed elsewhere the computation of the constants for beams of varying section, and thought it best to exclude such material from the paper.[79] [80] There is no great difficulty in determining these constants. In any given case five quantities are involved, the area, centroid and moment of inertia of the $\frac{1}{EI}$ diagram, and the area and centroid of the $\frac{M}{EI}$ diagram, in which, M is from any trial moment curve. These quantities are then to be combined. There are evidently endless variations of detail in doing this, the principal rule being to follow a definite procedure and avoid repetition of computation. The writer prefers to use the column analogy which converts the entire procedure into the familiar process of computing fiber stresses in beams.

In his paper previously mentioned, Mr. Evans has given constants for members of varying sections. It will be of interest to those who use moment distribution to note that the values for moment coefficients given by Mr. Evans are for fixed-end moments, the beam coefficients, C_1 and C_3, are for stiffness, and the carry-over factor at one end is $\frac{C_2}{C_1}$, and at the other end, $\frac{C_2}{C_3}$.

Curves and constants for the properties of haunched beams of reinforced concrete based on the assumption that I varies as the cube of the beam depth, however, should not be taken too literally. The effect of cracking, and the action of the flange in tension and in compression, introduce many uncertainties.

All kinds of ingenious tricks may be used to shorten the procedure. This is all right for the specialist who is engaged daily in this field. The general method presented in the paper originated from studies made by the writer in the analysis of three-span frames and is his final choice for ordinary daily use from many variations of method that he has developed and used. Any one can develop an unlimited number of such variations. Their novelty is no virtue; what is wanted for daily use is maximum simplicity consistent with reasonable facility. Such variations present a fascinating occupation to dilettantes who amuse themselves in the field of indeterminate structures; for the ordinary designer who analyzes such frames not as a vocation or for amusement, but as a passing incident to the design of the structures, it is essential that the procedure be simple and without special exceptions. The structural engineer has so many things to think about that it seems very unfortunate to load up his "tool chest" with a great many pretty but rather complicated "gadgets." Although developed in a class-room, the method of this paper was intended for use in offices.

The writer is especially pleased that so many have found the method useful in the design of actual structures. Professor Caughey and others have given estimates showing savings of from 80 to 90% of the time required by other methods; the writer knows of cases in which complete studies have required only one-twentieth of the time formerly used.

The writer is indebted to Mr. Findley for emphasizing the accuracy of the method. The writer called it a method of successive approximation; a better term which the writer has often used in this connection is "successive convergence." As Professor Van den Broek states, the word "exact" scarcely has a place in the engineer's vocabulary; but the term "approximate" is one which many people misunderstand. The quest of the absolute is a beautiful thing; but he who seeks in engineering analysis a precision that cannot be ultimately translated into such units as pounds of steel and yards of concrete is misled. Structures are analyzed so that they may be designed; not for the pleasure or practice of analyzing them. As Professor Finlay well states, "between the analysis of a given structure, which is essentially mathematics, and the design of a required structure, which is essentially art, lie many difficulties."

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
TRANSACTIONS

Paper No. 1794

SOME ASPECTS OF WATER CONSERVATION[1]

BY R. A. SUTHERLAND,[2] ASSOC. M. AM. SOC. C. E.

WITH DISCUSSION BY MESSRS. E. C. EATON, A. W. SIMONDS, W. H. R. NIMMO, C. R. OLBERG, JOHN W. PRITCHETT, A. J. GILARDI, AND R. A. SUTHERLAND

SYNOPSIS

Development of water conservation in the past has frequently been made in a piecemeal and haphazard manner, resulting in a less effective and economical exploitation of rivers than could be obtained by a systematic and coordinated study of all the factors concerned. In this paper formulas are developed for studying, in a preliminary manner, the merits of various storages and dam sites. The formulas have been tested in practice and were found to give reasonably correct results in most cases. Methods are also given for studying the relative economy of different types of dam, where the site admits of alternative types. The economic limits of specific capacity, or water stored per cubic yard of masonry in the dam, are discussed by means of examples from practice.

The paper represents an attempt to reduce to general and simple terms those factors which admit of generalization, but it is recognized that each case has peculiarities which must be studied in the light of local conditions.

The original manuscript of this paper, containing many tables, curves, and illustrations, which are not published herein, may be found on file at the Engineering Societies Library.[3] An abstract that forms the nucleus of it was published in Australia in 1928.[4]

IMPORTANCE OF WATER CONSERVATION

Water conservation, especially in countries where rivers show marked irregularity of flow, has become a question of such importance that Government departments have been set up to handle the whole question, particularly with regard to irrigation and water power. In addition, many private agencies or local authorities are actively undertaking conservation works in connec-

---

[1] Published in September, 1930, *Proceedings*.

[2] San Francisco, Calif.

[3] Approved by the Senate of the Univ. of London for the degree of Doctor of Philosophy in the Faculty of Engineering.

[4] *The Commonwealth Engineer*, Vol. 15, No. 11, June 1, 1928, p. 419.

tion with irrigation, water power, or water supply projects. Immense sums of money have already been expended on such works, and with increasing population the necessity for the most economical and effective organization of all works for water conservation will become ever more insistent.

In most countries the systematic collection and recording of data with regard to water conservation has been entirely neglected, due probably to the fact that so many different authorities are represented in the field. Notable exceptions are the publications of the United States Bureau of Reclamation and other Federal Departments, and those of a branch of the Italian Ministry of Public Works ("Consiglio Superiore delle Acque").

The total storage for irrigation already developed or proposed by the U. S. Bureau of Reclamation is equal to 13 792 500 acre-ft. in 49 reservoirs. In California, power companies and municipalities have created for power purposes alone no less than 76 reservoirs with an aggregate storage of 1 724 402 acre-ft.[5]

Canada is richly endowed with water resources; the total available storage of 22 of the most important lakes and artificial reservoirs used for power purposes is 30 030 000 acre-ft.

South Africa, which geologically is a country ill-adapted to water conservation, has storages exceeding 750 000 acre-ft.

The States of New South Wales and Victoria in Australia are, in proportion to population, among the foremost in the conservation of water, which is mainly used for irrigation. The total storage for this purpose in New South Wales is about 1 900 000 acre-ft., or 0.84 acre-ft. per capita, while a further storage of 2 500 000 acre-ft. is proposed. Victoria has a total storage of 900 000 acre-ft. for irrigation, or 0.54 acre-ft. per capita, and an added quantity of 950 000 acre-ft. is proposed.

The State of Queensland, which is the most recently developed of the Eastern States of Australia, has, at present (1930), storage equal to only about 30 000 acre-ft., while projects involving a total storage of about 3 000 000 acre-ft. are either proposed or being developed.

In Europe, the climate does not in general require, nor does the geological nature of the continent usually permit, the formation of large storages. Italy, however, has an immense number of storages of moderate size, totaling more than 675 000 acre-ft. in 86 reservoirs used for power, while works under construction will add a further 626 000 acre-ft. in 27 reservoirs. The largest of these reservoirs is that of the Tirso, with a capacity of 285 000 acre-ft.

Switzerland has many storages of small size, the total capacity of the 9 reservoirs which exceed 10 000 acre-ft. being 271 900 acre-ft.

An immense number of small storages exist in various countries for purposes of irrigation and water supply, but no complete list of these storages has ever been compiled. Some appreciation of the magnitude of the conservation of water for irrigation may be obtained from the fact that the estimated world area irrigated is more than 137 000 000 acres.[6]

---

[5] The data here given were up to date in 1928. For notes on large dams, see "*New Reclamation Era,*" January, 1930, "The Highest and Largest Dams in the United States and Abroad," by P. I. Taylor.

[6] *New Reclamation Era,* September, 1927.

The storage required per acre irrigated varies from zero to as much as 20 acre-ft. An average figure for projects under the U. S. Reclamation Bureau appears to be about 4 acre-ft. of storage per irrigated acre, while, in order to cope with the prolonged droughts occurring in Australia, a proposed plan there involves a storage of about 30 acre-ft. per irrigated acre. Taking a conservative all-round figure of only 0.5 acre-ft. of storage per irrigated acre, the world storage for irrigation would amount to about 69 000 000 acre-ft.

FUTURE DEVELOPMENT

Great as has been the progress already made in water conservation, it is only a fraction of that required to achieve the complete hydraulic development of the various countries of the world. Some countries are already so highly developed in this regard that the completion of their program of hydraulic development, if not already in view, will be so within the next ten or twenty years. Of such countries, Switzerland and Italy may be cited as examples. Italy has now (1930) reached such a stage that all the most promising storage sites have already been exploited, and a Government department is actively collecting data relating to the stream flow and potential storage sites of the remaining streams. Practically all the main rivers and streams have been accurately leveled, storage sites surveyed, and catchment basins defined. Other countries such as Brazil, Central Africa, Queensland, New Zealand, etc., have as yet developed only a small percentage of their hydraulic resources.

When it is realized that the present total world area irrigated is just a fraction of that for which water would be available with adequate conservation, and that in the field of hydro-electric power the present world development is only 25 000 000 h.p. out of an estimated total of about 440 000 000 h.p. available, it is considered safe to say that present water conservation is probably not more than 10% of that which will ultimately be required. Accurate data, however, are lacking on which to base any forecast of the rate of development.

In addition to storages needed for irrigation and water power, an increasing number will be required also for water supply and flood control; but development on these lines depends so entirely on sectional requirements that it is useless to attempt any forecast of what may be required. It is interesting to note, however, that, in recent times,[7] considerable sums of money have been spent on flood control by the construction of detention basins. The works of the Miami Conservancy District in Ohio, of the City of San Antonio, Tex., and of Los Angeles County, California, are examples.

The investigation and development of water conservation in the past have been frequently haphazard, unscientific, and uneconomical. Common errors have been:

1.—The use of insufficient stream-flow data;

2.—The use of rainfall records and assumptions as to run-off without adequate data showing the relationship between them.

3.—An under-estimation of flood flows, and insufficient spillway provision;

---

[7] The Furen's Dam (1866) was the first large modern dam built for flood control. Until recently, large expenditure for this purpose alone was exceptional.

4.—Insufficient geological examination of reservoir basin and dam site;
5.—Incorrect design or faulty construction of dam; and
6.—Choice of site, and design of dam without regard to economic considerations or future development of other sites on the same stream.

NOTATION

The following notation is used in the paper:

a = area of a cross-section at a dam site, to any height, h.
A = total area of a cross-section at a dam site, to height, H.
w = capacity of a reservoir to any height, h_1, measured at the dam.
w_s = specific capacity. Other subscripts refer to the type of dam.
W = total capacity of a reservoir to height, H_1.
s = submerged area of a reservoir, corresponding to a height, h_1, measured at the dam.
l = width of a dam site at any height, h.
t = thickness of dam at any height, h.
t_1 = crest thickness of a dam with a trapezium section.
K = sum of face batters (ratio of horizontal to vertical) of a dam. Proper subscripts refer to type of dam.
K_R = sum of face batters of a dam with a trapezium section.
h = height of dam site to any elevation.
h_1 = depth of water at the dam.
H = total height of a dam of any type.
V = volume. Proper subscripts refer to type of dam.
V_R = volume of a dam with a trapezium section.
R_u = up-stream radius of an arch dam.
R_1 = up-stream radius of a constant-angle arch dam, at the crest.
R_2 = up-stream radius of a constant-angle arch dam, at the base.
R_h = up-stream radius of a constant-angle arch dam, at any elevation, h.
f = cylinder stress in an arch dam, in pounds per square inch.
ϕ = a subtended angle in an arch dam.

C_A = a coefficient (Equation (1)) $= \frac{a}{h^n}$.

C_w = a coefficient (Equation (2)) $= \frac{w}{h_1{}^m}$.

D = a coefficient (Equation (6)) $= \frac{l}{h^v}$.

n = an exponent (Equation (1)) $\doteq \frac{\log a - \log C_A}{\log h}$.

m = an exponent (Equation (2)) $= \frac{\log w - \log C_w}{\log h_1}$.

v = an exponent (Equation (6)) $= \frac{\log l - \log D}{\log h}$.

J = a coefficient (Equation (20))

$$\doteq \frac{0.217\ \phi}{n\ C_A\ H^{n-1} \sin \frac{\phi}{2}} \left[\frac{n\ R_1 + 2\ R_2}{n\ (n+1)\ (n+2)} \right].$$

J_A = a coefficient (Equation (14)) $= 0.434\ \frac{R_u}{L} \left(\frac{n}{n+1} \right) \left(\frac{\phi}{2 \sin \frac{\phi}{2}} \right)$.

J_{CA} = a constant $= J\ n^2$.

Q = a constant (Equation (25)) $= L \dfrac{V_A}{V_g}$.
g = a subscript denoting gravity dams.
A = a subscript denoting arch dams.
CA = a subscript denoting constant-angle arch dams.

NOTES ON RIVER SECTIONS AND PROFILES

Koechlin[8] has shown that for a river in a state of equilibrium, the shape of the bed is sinusoidal and depends only on the slope and the nature of the ground traversed. This shape he calls the "normal cross-section." In order that this section may be attained, the following conditions must be fulfilled:

(1) The river must have arrived at a state of equilibrium; that is, the longitudinal profile must be constant.
(2) The flow must be as constant as possible.
(3) The ground must be homogeneous.
(4) The banks must be bare.
(5) The river must be straight.
(6) The river, even at high flood, must be contained entirely within the bed.

Actually few of these conditions are ever satisfied simultaneously, so that the theoretical profile does not hold good in practice. The effect of departures from the conditions mentioned is examined by Koechlin in some detail.

Over certain limited tracts of river, all the conditions cited except Condition (2) may be fairly well fulfilled. This is generally the case in gorges or valleys suitable for the construction of dams, and, here, it might be expected that the sinusoidal shape of section might be most frequently found.

The area enclosed within a sine curve, can be shown to vary nearly as the $\frac{3}{2}$ power of the height measured from the crest of the curve. By computing areas carefully and plotting them against heights, on logarithmic paper, the writer obtained an index of 1.51. The cross-sectional areas of dam sites can generally be expressed closely in terms of height by means of an equation:

$$a = C_A\, h^n \qquad (1)$$

The mean value of n for thirty-seven sites examined was found to be 1.54. This close agreement between theory and average practice appears to confirm the correctness of Koechlin's work.

It is a matter of common experience that the longitudinal profile of rivers is approximately parabolic in shape. Fig. 1(*a*), Fig. 1(*b*), Fig. 1(*c*), and Fig. 1(*e*) show the profiles of various rivers and illustrate the parabolic shape.[9] Land surfaces as a whole vary in slope in a similar manner.[10]

In the vast majority of cases, a river flows through country of varying geological character, and the profile is far from regular. At certain points the river breaks through more or less well-defined ridges, the areas above these points constituting self-contained catchment areas. When the ridges are well defined, the river is usually in a gorge, and such a formation is frequently suitable both for a dam site and for an efficient storage basin. Each catchment above its respective gorge has its own characteristics, which depend on the geology of the region and a variety of other factors. The general land slope above each gorge will be parabolic. Hence, the profile of many rivers consists of a series of parabolas, the whole again forming roughly one large parabola.

---

[8] "Mécanisme de l'Eau," by René Koechlin, Vol. 1, 1924, p. 101.

[9] See, also, "Mécanisme de l'Eau," by René Koechlin, Vol. 1, 1924, Fig. 48, p. 136.

[10] "Géographie Physique," by Martonne, 1913, p. 49.

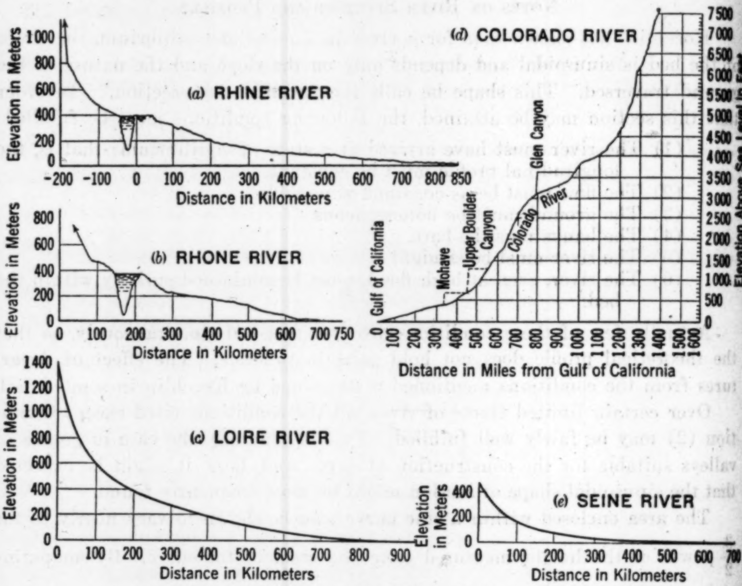


FIG. 1.—LONGITUDINAL PROFILES OF VARIOUS RIVERS.

Fig. 1(*d*) shows the profile of the Colorado River,[11] in which two major and several minor parabolas are readily distinguishable. The ideal case is shown in Fig. 2 in which the river is assumed to break through two successive ridges. The longitudinal profile then consists of three curved sections, each of approximately parabolic shape.

In the upper basin, the river has a relatively small flow but carries considerable gravel and many boulders, and, in general, the stream bed will be deeply eroded and the landscape scarred by numerous tributaries. The general nature of the cross-section, therefore, will be as shown by Section *A*.

In the second basin the stream bed will be wider and less deeply eroded, somewhat as shown at Section *B*, while even farther down the land may be almost level, and the river bed very wide, the normal dry-weather flow occupying only a small winding channel in a wide bed (Section *C*).

THE RESERVOIR

Suitable dam sites generally occur at points such as those marked *A* and *B* (Fig. 2); that is, some distance down the transition from one parabola to the next. An examination of the slopes above and below almost any storage dam will verify this fact.

---

[11] *Transactions*, Am. Soc. C. E., Vol. 88 (1925), Fig. 13, p. 341.

From a consideration of the profile and cross-sections of Fig. 2 it will be seen that the storage obtained for a given height will generally be less for the upper reservoir shown than for the lower one. On the other hand, the rate of increase of storage with increasing depth of water will be greater. Hence, the reservoirs will have certain well-defined characteristics and may be classified accordingly. The classification adopted will be: (1) Gorge type; (2) hill type; (3) foothill and flood-plain type; and (4) lake type.

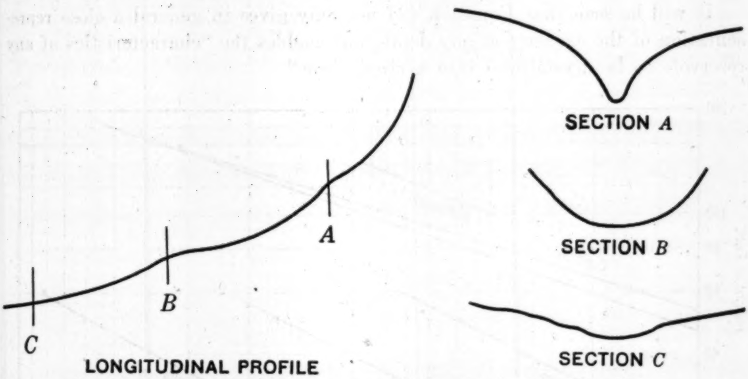


FIG. 2.—PROFILES AND SECTIONS OF AN IDEAL RIVER (DIAGRAMMATIC).

A classification such as this was proposed by W. H. R. Nimmo, M. Am. Soc. C. E., when this paper was first discussed before the Institution of Engineers (Australia), but the criteria now adopted differ from those proposed by Mr. Nimmo.

Reservoir *A* would be of the first class, Reservoir *B* of the second, and a reservoir at a site, such as *C*, of the third class. The lake type of storage is a special case and may occur at any elevation. As the name implies, it is a reservoir formed either by artificially raising an existing lake, or by impounding water in what was once a lake or in a basin of similar general form.

CHARACTERISTICS OF RESERVOIRS

In 1911,[12] Capt. A. ff. Garrett, R. E., defined the capacity of a reservoir as a function of the depth, by expressing the area and capacity as series in ascending powers of the depth, and showed that the first three or four terms of such series sufficed to give quite accurate values of the quantities considered. Hence, he deduced rules for determining the capacity curve with the minimum amount of survey work. For this purpose Garrett's method is valuable. Later, Hillman[13] showed that the capacity of a reservoir can be expressed closely as an exponential function of the depth of water, but he made practically no use of this fact.

---

[12] "The General Theory of the Storage Capacity and Flood Regulation of Reservoirs," Tech. Publication, Govt. of India, 1911.

[13] "The Capacities and Capacity Equations of Some Western Australian Dam Sites," *Transactions*, Inst. of Engrs. (Australia), Vol. 11, 1921.

Fig. 3 represents a few of the large number of capacity curves which have been plotted by the writer on logarithmic paper, and shows that the relationship first suggested by Hillman is generally very close. When the capacity curve has been drawn as a straight line on logarithmic paper (with horizontal and vertical scales equal) the slope of this line will give the index of the height in the general capacity equation:

$$w = C_w h_1^m \quad \ldots\ldots (2)$$

It will be seen that Equation (2) not only gives in general a close representation of the capacity at any depth, but enables the "characteristics of any reservoir to be crystallized into a single line."

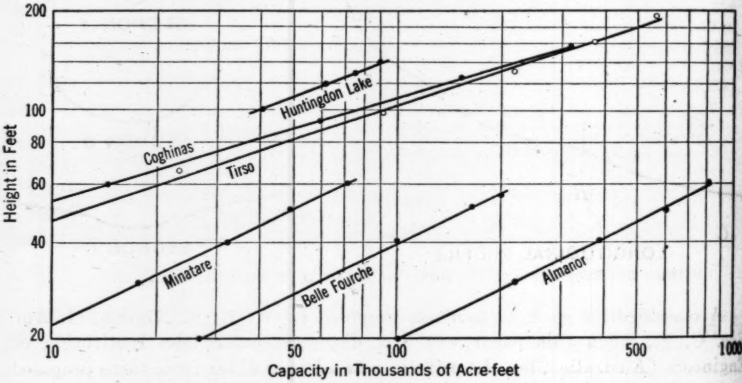


FIG. 3.—LOGARITHMIC CAPACITY CURVES.

In Table 1 a number of reservoirs have been classified into types, and the characteristics of each reservoir (as represented by C_w and m) have been tabulated. The basis of classification arbitrarily adopted is as follows:

	Values of m.
Lake type	1.0 to 1.5
Flood-plain and foothill type	1.5 to 2.5
Hill type	2.5 to 3.5
Gorge type	3.5 to 4.5

This classification is not intended to be hard and fast, since the different types merge into each other, but is a sufficiently definite basis for describing the general nature of any reservoir.

The constant, C_w, may be readily determined by taking the capacity from the straight line for a depth of 100 ft., then:

$$\log C_w = \log w_{100} - 2\,m \quad \ldots\ldots (3)$$

The value of m is the ratio of maximum depth to average depth, the latter being defined as maximum capacity divided by maximum water-spread area.

TABLE 1.—Characteristics of Storage Reservoirs

Item No.	Name of reservoir.	Purpose.	m.	C_w.	Maximum depth, in feet.	Capacity, in acre-feet.
	Lake Type: m = 1.0 to 1.5.					
1	Great Lake	Power	1.04	29 500	35	1 157 000
2	Jackson Lake	Irrigation	1.13	13 330	39	789 000
3	Matese	Power	1.37	2 350	13.1	11 600
4	Strawberry	Irrigation	1.26	2 390	44	278 000
5	Vannino	Power	1.22	33.12	78.5	6 900
	Weighted mean (for capacity)		1.104			
	Arithmetical mean		1.204			
	Flood-Plain and Foothill Type: m = 1.5 to 2.5.					
6	Lake Almanor (1930)	Power	1.9	347	35	300 000
6(a)	Lake Almanor (future)	Power	1.9	347	59	700 000
7	Assuan	Irrigation	2.03	171.5	98	1 970 000
8	Belle Fourche	Irrigation	1.98	71.2	75	203 000
9	Boulder Canyon	Power and irrigation	2.35	9.07	.....	....
10	Ceresole Reale	Power	2.32	0.325	134.3	26 900
11	Codelago	Power	1.66	9.43	79	13 100
12	Elephant Butte	Irrigation	2.39	10.75	193	2 637 000
13	Gibson	Irrigation	2.24	1.02	161	90 000
14	Huntington Lake	Power	2.38	0.695	140	88 700
15	Little Nerang	Water supply	2.24	0.1365	197	18 500
16	Lake Manchester	Water supply	2.03	3.7	70	21 100
17	Minitare	Irrigation	1.99	20.54	55	60 766
18	Mojave	Power and irrigation	1.76	1 330	.....	....
19	Lake Spaulding	Power	2.43	0.0858	275	74 488
20	Sherburne Lake	Irrigation	1.65	63.4	73	75 000
	Weighted mean		2.2			
	Arithmetical mean		2.09			
	Hill Type: m = 2.5 to 3.5.					
21	Arrowrock	Irrigation	2.92	0.02848	249	280 000
22	Bon Accord		3.15	0.04134	75	32 000
23	Bull Run	Water supply	2.57	0.058	180	37 000
24	Lower Buller	Water supply	2.63	0.0324	57	....
25	Burrenjuck	Irrigation	3.04	0.0773	200	771 641
26	Calitzdorp		2.77	0.01731	100	600 000
27	Coghinas	Power	3.18	0.3272	157	320 000
28	Coomera	Water supply	2.70	0.1592	170	175 000
29	Lake Cushman	Power	2.62	0.288	250	465 000
30	Hume	Irrigation	3.00	1.58	90	1 098 500
31	Kamanassie		2.50	0.310	100	31 100
32	Nepean	Water supply	3.41	0.000518	207	38 800
33	Roosevelt	Irrigation	3.04	0.0847	220	1 367 300
34	Shoshone	Irrigation	3.42	0.00433	220	460 000
35	Lower Stanley	Water supply	3.37	0.0273	120	290 000
36	Stony Gorge	Irrigation	3.05	0.0286	111	50 000
37	Tirso	Irrigation and power	2.87	0.1638	196	574 000
	Weighted mean		2.99			
	Arithmetical mean		2.78			

TABLE 1.—(*Continued.*)

Item No.	Name of reservoir.	Purpose.	*m*.	C_w.	Maximum depth, in feet.	Capacity, in acre-feet.
	GORGE TYPE: $m = 3.5$ TO 4.5.					
38	Avon	Water supply	4.25	0.000023	210	174 000
39	Upper Buller	Water supply	3.94	0.000131	60	1 340
40	Cataract	Water supply	4.11	0.0000915	150	77 000
41	Cordeaux	Water supply	4.33	0.0000165	170	76 000
42	Glenn Canyon	Power	3.97	0.00092	.....	....
43	Grass Ridge	....	3.64	0.01118	100	210 000
44	Mt. Edwards	Irrigation	3.56	0.00497	75	30 000
45	Nathan	Irrigation	4.35	0.001566	130	2 485 000
46	Pathfinder	Irrigation	4.28	0.0001376	202	1 070 000
47	Silverwood	Water supply	3.62	0.00075	70	3 600
48	Upper Stanley	Water supply	3.77	0.00317	110	159 754
49	Tarka	....	3.61	0.00374	100	59 000
50	Van Ryneveld's Pass	....	3.93	0.0009	100	32 500
51	Warragamba	Water supply	4.22	0.0000682	260	578 000
	Weighted mean		4.23			
	Arithmetical mean		3.97			

The ratio of mean depth to maximum depth has been given by Charles Slagg[14] for seventy-five different reservoirs. The maximum value of this ratio is 0.612, corresponding to $m = 1.64$, and the minimum value is 0.281, corresponding to $m = 3.57$, the mean values being 0.456 and 2.2, respectively. In Table 1, fifty-one reservoirs are arranged according to the writer's classification. The arithmetical mean value of *m* is 2.75 and the weighted mean, 2.87, for the fifty-one cases. Both these values are greater than the figure deduced from Slagg's table.

Certain of the reservoirs cited occur on the same river, and a study shows that their values of *m* are functions of elevation. The following reservoirs are arranged in order of descent of the river, and the values of *m* show a corresponding decrease:

	Values of *m*.
Upper Buller River	3.94
Lower Buller River	2.63
Upper Stanley River	3.77
Lower Stanley River	3.37
Glenn Canyon, Colorado River	3.97
Boulder Canyon, Colorado River	2.35
Mojave Reservoir, Colorado River	1.76

Furthermore, the general deduction may be made that for reservoirs of similar type, C_w will vary usually with the catchment area.

REPRESENTATION OF RESERVOIR CAPACITY

Equation (2) may be represented by means of an alignment chart, such as Fig. 4, in which the outside scales are graduated for height and capacity, the

---

[14] "Water Engineering," by Charles Slagg, First Edition, 1888, p. 24, Crosby Lockwood and Son.

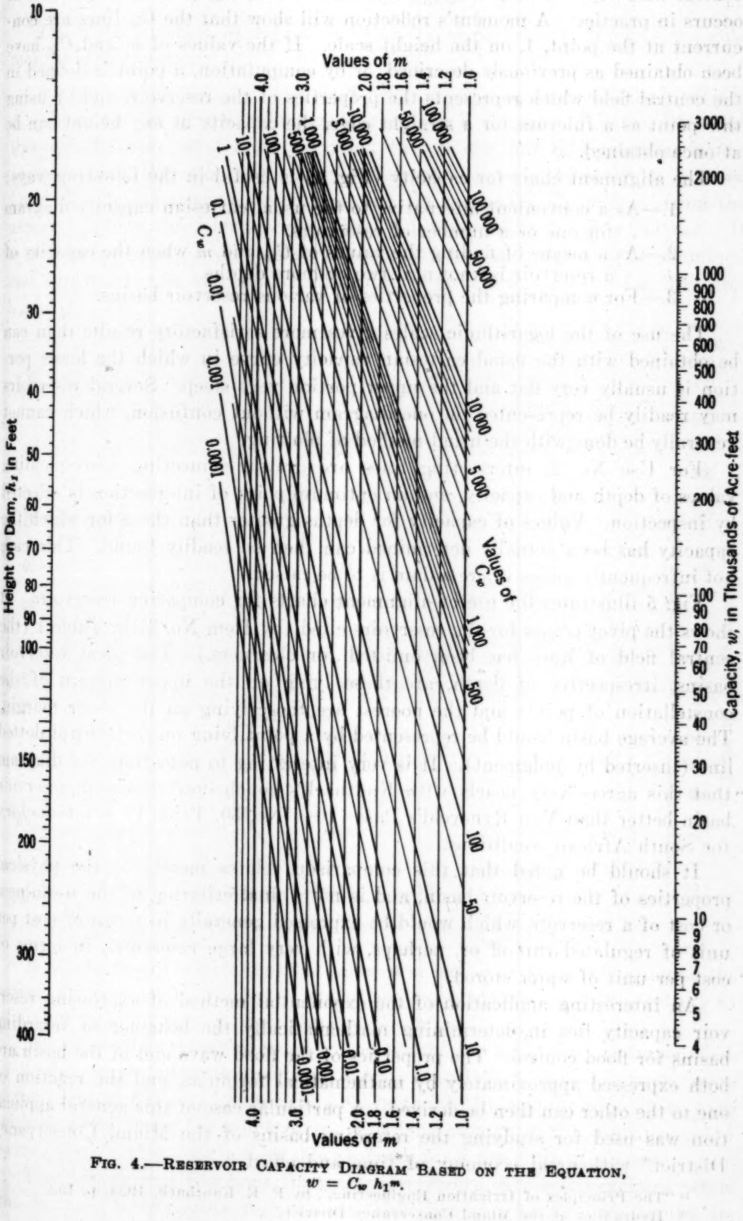


FIG. 4.—RESERVOIR CAPACITY DIAGRAM BASED ON THE EQUATION, $w = C_w h_1^m$.

central field representing various values of m and C_w, over a range such as occurs in practice. A moment's reflection will show that the C_w lines are concurrent at the point, 1, on the height scale. If the values of m and C_w have been obtained as previously described, or by computation, a point is defined in the central field which represents the properties of the reservoir, and by using this point as a fulcrum for a straight edge, the capacity at any height can be at once obtained.

The alignment chart for capacity (Fig. 4) is useful in the following ways:

1.—As a convenient alternative to the usual cartesian capacity diagram for one or a number of reservoirs.
2.—As a means of finding the values of C_w and m when the capacity of a reservoir is known at two or more depths.
3.—For comparing the properties of various reservoir basins.

The use of the logarithmic scales gives more satisfactory results than can be obtained with the usual cartesian capacity curve in which the lower portion is usually very flat and the upper portion very steep. Several reservoirs may readily be represented on one diagram without confusion, which cannot generally be done with the usual method of plotting.

For Use No. 2, intersecting lines are drawn connecting corresponding values of depth and capacity, and an optimum point of intersection is selected by inspection. Values of capacity for depths greater than those for which the capacity has been actually determined can then be readily found. This case not infrequently arises where a dam is to be raised.

Fig. 5 illustrates the use of alignment charts for comparing reservoirs. It shows the pivot points for the reservoirs cited (by Item No. 3) in Table 1 (the central field of lines has been omitted for clearness.) The ideal reservoir basins, irrespective of depth, are those lying on the upper margin of the constellation of points and the poorest are those lying on the lower margin. The average basin would be represented by a point lying on the central dotted line (inserted by judgment). It is very interesting to note from the diagram that this agrees very nearly with Kanthack's conclusion[15] that any reservoir basin better than Van Ryneveld's Pass (Item No. 50, Table 1) is satisfactory for South African conditions.

It should be noted that this comparison relates merely to the physical properties of the reservoir basin, and is not a final criterion of the usefulness or cost of a reservoir, which would be expressed generally in terms of cost per unit of regulated run-off or, perhaps, with very large reservoirs, in terms of cost per unit of water stored.

An interesting application of the exponential method of expressing reservoir capacity lies in determining mathematically the behavior of retarding basins for flood control. The properties of the flood wave and of the basin are both expressed approximately by mathematical formulas, and the reaction of one to the other can then be derived. A particular case of this general application was used for studying the retarding basins of the Miami Conservancy District[16] with great economy of time and effort.

---

[15] "The Principles of Irrigation Engineering," by F. E. Kanthack, 1924, p. 153.

[16] "Hydraulics of the Miami Conservancy District."

SUBMERGED AREA

The differentiation of Equation (3) with respect to depth gives the submerged area equation:

$$s = m\ C_w\ h_1^{m-1} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (4)$$

A comparison of actual submerged areas for a number of cases, with the theoretical areas found from Equation (4), shows that this equation is less accurate than Equation (3). It is sufficiently accurate, however, to be used for preliminary purposes for determining the constants, C_w and m, from the areas at two or three levels only. Fig. 6 is convenient for this purpose. It is well known that for one reservoir proposal adopted, many may be examined and discarded, and the methods mentioned are quite suitable for this preliminary study.

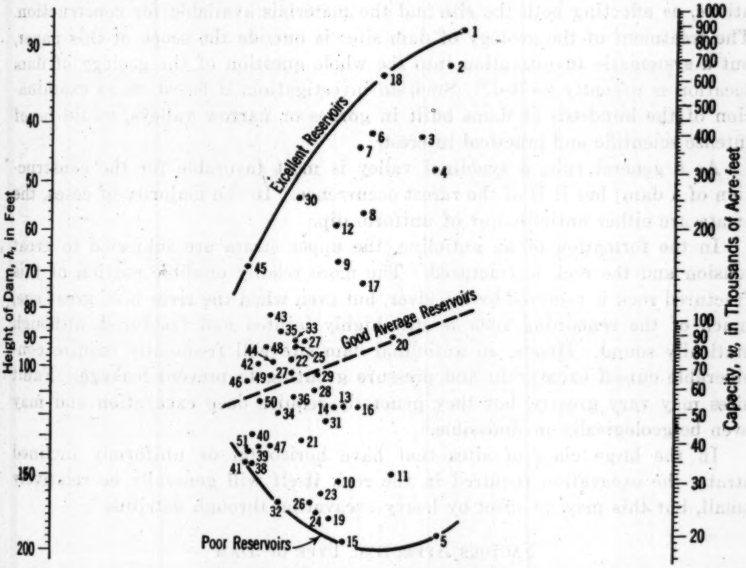


FIG. 5.—COMPARISON OF RESERVOIRS.

It is interesting to note that the value of m has an important bearing on the effect of evaporation, which has sometimes to be considered very carefully in studying large storages in dry climates.

The effect of evaporation being to remove the top slice of water impounded, it is clear that, other things being equal, evaporation causes a much more serious loss in the case of reservoirs characterized by a high value of m, than with those having a low value of m. For long storage in dry climates, a low value of m, is, therefore, a desirable property for a reservoir to possess.

The Assuan and Elephant Butte Reservoirs are excellent examples, while the Nathan Reservoir is unfortunately handicapped in this respect by its high value of m. To illustrate this point it will be sufficient to state that the evaporation losses for the Elephant Butte Reservoir over a period of twenty years would be equal to about 20% of the inflow, while for the Nathan Reservoir over a period of fifteen years these losses would be 40.5% of inflow. The depth lost annually is practically the same for each reservoir, so that the cases are comparable. The losses over a shorter drought period would be even more to the disadvantage of the Nathan Reservoir. It is unnecessary to add that sites suitable for the creation of such large storages are rare, so that there is not usually any opportunity of selecting the reservoir site with regard to its desirability or otherwise from the point of view of evaporation losses.

THE DAM

The location and design of a dam depend primarily on geological considerations, as affecting both the site and the materials available for construction. The treatment of the geology of dam sites is outside the scope of this paper, but a systematic investigation into the whole question of the geology of dam location is urgently needed.[17] Such an investigation, if based on an examination of the hundreds of dams built in gorges or narrow valleys, would be of intense scientific and practical interest.

As a general rule, a synclinal valley is most favorable for the construction of a dam; but it is of the rarest occurrence. In the majority of cases, the strata are either anticlinal or of uniform dip.

In the formation of an anticline, the upper strata are subjected to great tension and the rock is fractured. The more readily erodible portion of this fractured rock is removed by the river, but even when the river is of great age, much of the remaining rock is still highly jointed and fractured, although statically sound. Hence, an anticlinal dam site will frequently require considerable cut-off excavation and pressure grouting to prevent leakage. Fault sites may vary greatly, but they generally require deep excavation and may even be geologically inadmissible.

In the large class of sites that have horizontal or uniformly inclined strata, the excavation required in the rock itself will generally be relatively small, but this may be offset by heavy excavation through detritus.

FACTORS AFFECTING TYPE OF DAM

The selection of the most economical type of dam for any site, which geological examination has shown to be feasible, depends on a large number of factors, such as the purpose for which the storage is intended, economic considerations, flood conditions, the dimensions of the sites available, etc.

Purpose.—In certain cases the purpose of storage may influence the choice of a dam. Thus, the gravity dam is frequently preferred for water supply projects, because of its monumental proportions. In other cases, a hollow dam may be chosen, partly because it may afford housing for a power station or filter plant.

---

[17] A good sketch of the subject is that of Segré, "Cenni sugli studi geognostici applicati all'impianto di Laghi Artificiali", *Annali,* 1920; see, also, *Technical Publication No. 215,* Am. Inst. Min. and Metallurgical Engrs., "Geology and Engineering for Dams and Reservoirs."

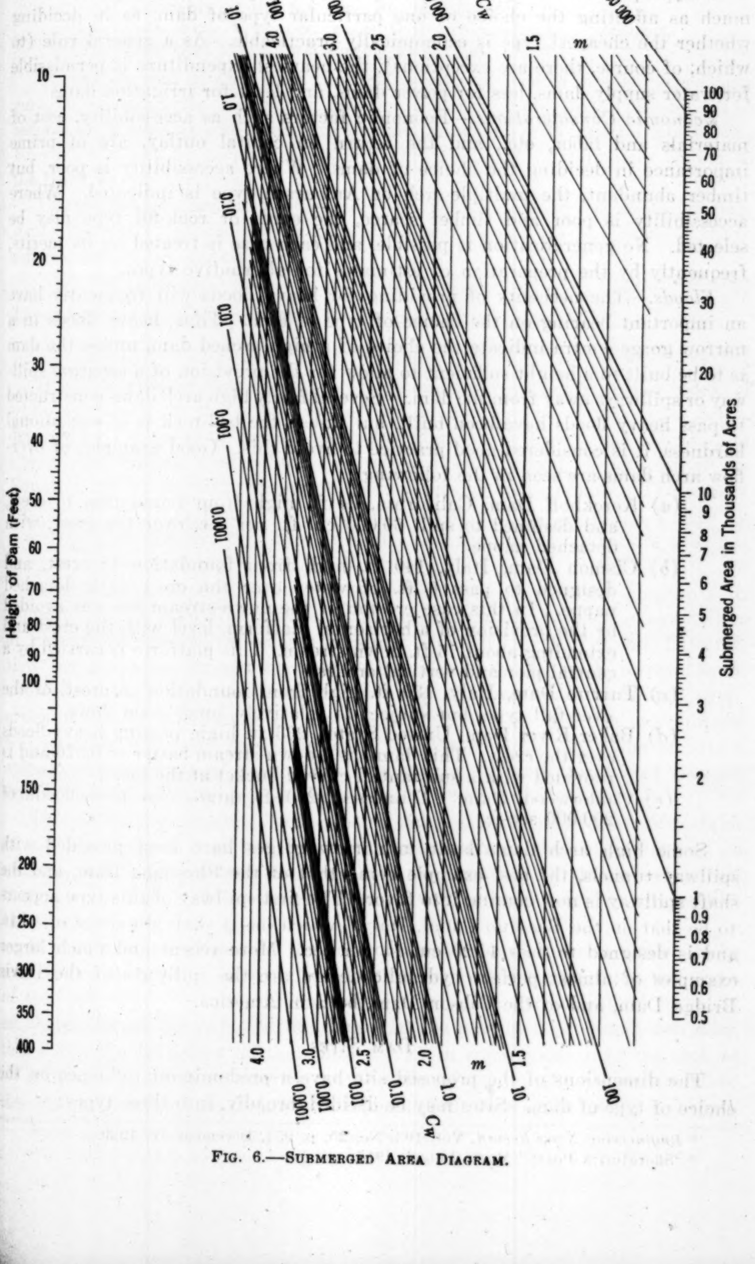


FIG. 6.—SUBMERGED AREA DIAGRAM.

The purpose of storage has an important bearing on the subject, not so much as affecting the choice of one particular type of dam, as in deciding whether the cheapest type is economically practicable. As a general rule (to which, of course, there are exceptions), the heaviest expenditure is permissible for water supply dams, less for power dams, and least for irrigation dams.

Economic Considerations.—Economic factors such as accessibility, cost of materials and labor, etc., and the return on capital outlay, are of prime importance in deciding the choice of dams. Where accessibility is poor, but timber abundant, the multiple-arch or Ambursen type is indicated. Where accessibility is poor and timber scarce, the earth or rock-fill type may be selected. No generalization is possible, and each case is treated on its merits, frequently by the preparation of estimates for alternative types.

Floods.—The necessity of providing for heavy floods will frequently have an important bearing on the choice of type of dam. Thus, heavy floods in a narrow gorge contra-indicate the choice of a high arched dam, unless the dam is to be built to a height sufficient to allow for the provision of a separate spillway or spillways away from the dam. Several fairly high arch dams constructed to pass heavy floods have been built, but unless the bed-rock is of exceptional hardness, it is considered good practice to avoid this. Good examples of overflow arch dams are seen in the following:

(*a*) Kerckhoff Dam, California, 94 ft. high from foundation to crest, and designed to spill 60 000 cu. ft. per sec. over the crest, with detached nappe.

(*b*) Cismon Dam, Italy, 140 ft. high from foundation to crest, and designed to pass 6 ft. of water over the crest with detached nappe. In this case erosion of the down-stream toe was avoided by the provision of a horizontal platform, level with the crest and extending about 75 ft. down stream. This platform is carried by a central pier and vertical arches.

(*c*) Turrite Dam, Italy, 130 ft. high from foundation to crest, of the over-fall type, but subject to relatively small flood flows.

(*d*) Baker River Dam, United States, 233 ft. high, passing heavy floods over the crest. This dam has a down-stream batter of 0.376 and is provided with a substantial curved bucket at the toe.

(*e*) Calderwood Dam,[18] Tennessee, 230 ft. high, can pass floods of 200 000 sec-ft.

Some high arch dams built in narrow gorges have been provided with spillway tunnels, the best example being that of the Shoshone Dam, and the shaft spillway is now coming into favor. The first spillway of this type appears to be that on the Bassano Canal, Italy,[19] which has a shaft diameter of 8.4 ft. and is designed to pass 1 120 cu. ft. per sec. More recent and much larger examples of almost perfect hydraulic design are the spillways of the Davis Bridge Dam, and of the Gibson Dam, both in America.

DAM SITES

The dimensions of the proposed site have a predominant influence on the choice of type of dam. Sites may be divided, broadly, into three types:

---

[18] *Engineering News-Record,* Vol. 103, No. 25, p. 954, December 19, 1929.

[19] "Sfioratori a Pozzi," Mario Luiggi, 1917.

(1) Those which are very narrow at the bottom but which broaden considerably at the top (see Fig. 2, Section A).
(2) Those with V-shaped walls that are practically straight.
(3) Those that are approximately U-shaped.

It may be said at once that the first or convex type of site is exceedingly rare, three examples being the Morena, Pacoima, and Jogne Dams. The second or V-type is also quite infrequent, an example being the site of the Stevenson Creek Experimental Dam.[20] The majority of dam sites belong to the third or concave type.

Characteristic canyon profiles may be represented by parabolic curves, using the word, "parabolic," in its mathematical sense[21] as the shape of curve represented by the equation,

$$y^p = Q\,x^q \dotfill (5)$$

These curves may be represented by the general equation,

$$l = D\,h^v \dotfill (6)$$

It will be obvious that the shape of the curve depends only on the index, v, while the scale, or the width at any height for curves of the same shape, depends on the value of the constant, D. Hence, the dimensions of any dam site, if the natural surface were perfectly smooth, could be represented by Equation (6) by selecting the appropriate values of the constants. The convex type would be characterized by values of v greater than unity, the V-type by $v = 1$, and the concave type by values of v less than unity. The site need not be symmetrical to comply with Equation (6).

Any attempt to determine from the section of a site the constants in Equation (6) is generally unsatisfactory, because of the marked irregularities of natural surface which generally occur. The writer has found, however, that the equation derived by integrating Equation (6) with respect to h, affords a satisfactory means of representing the average dimensional properties of almost any dam site. Equation (1) which is used for this purpose will be called the "site area equation." In this equation the value of n is a criterion of the type of site. Thus, the convex, V, and U-types of site are typified by values of n greater than, equal to, and less than 2.

The writer has computed and plotted site areas for more than fifty dams of all descriptions, and has found that although marked irregularities of natural surface are present the hypothesis of Equation (1) is well supported in practice. Fig. 7 shows the site-area curves for only a few of the profiles examined.

It should be noted that in the case of arched dams, the site section to be used is not that taken on a vertical section across the gorge, since the whole of the arch dam does not rest on this section. The section should be taken on chords across the curve of the dam. In the case of a proposed arch dam, this curve can be drawn on the contour plan in such a way that the arch at all points will abut satisfactorily on the sides of the gorge. Of course, when the contour lines are parallel, a vertical cross-section may be taken.

---

[20] *Proceedings*, Am. Soc. C. E., May, 1928, Pt. III.

[21] Frost in "Curves and Curve Tracing."

It may be thought that the use of the site-area curve is a somewhat roundabout way of defining mathematically the properties of a dam site. The method, however, has the great advantage that when the area is plotted against height on logarithmic paper, the straight line which best represents the points may readily be drawn by inspection, and this cannot easily be done if width is plotted against height. As a method of representation, the site-area equation is, of course, equally as good as the site-width equation, from which it is derived by integration. The process is seen to consist practically in replacing the actual site by another theoretical site having the same general properties.

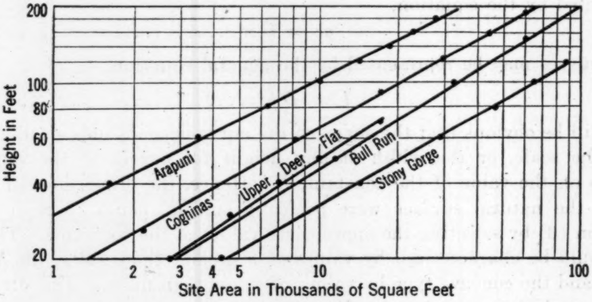


FIG. 7.—LOGARITHMIC SITE AREA CURVES.

Equation (1) may be represented graphically by means of an alignment chart (Fig. 8). This chart may be used to find C_A and n for any site by drawing straight lines between different points on the height scale, and corresponding points on the area scale, and selecting an average of the points of intersection. This point will define C_A and n.

VOLUME OF DAMS

Having seen that the dimensions of most dam sites may be approximately represented by a mathematical equation, it will be clear that the volume of dams built, or to be built, in such sites may be determined mathematically, since the cross-sectional dimensions of dams may also generally be represented by mathematical equations.

Gravity Dam.—The early profiles proposed by Rankine, Krantz, Harlacher, and others, which had curved faces, were dealt a severe blow by Atcherley,[22] and the modern gravity dam is practically triangular in cross-section. Wegmann's[23] practical profiles, and those recently proposed by Koechlin[24] are in effect triangular (except for the additional thickness of the crest), and where variations do occur, the profile may be represented for the present purpose by an equivalent triangular profile.

---

[22] "On Some Disregarded Points in the Stability of Masonry Dams," 1904.

[23] "The Design and Construction of Dams."

[24] "Mécanisme de l'Eau," Vol. 2, 1926.

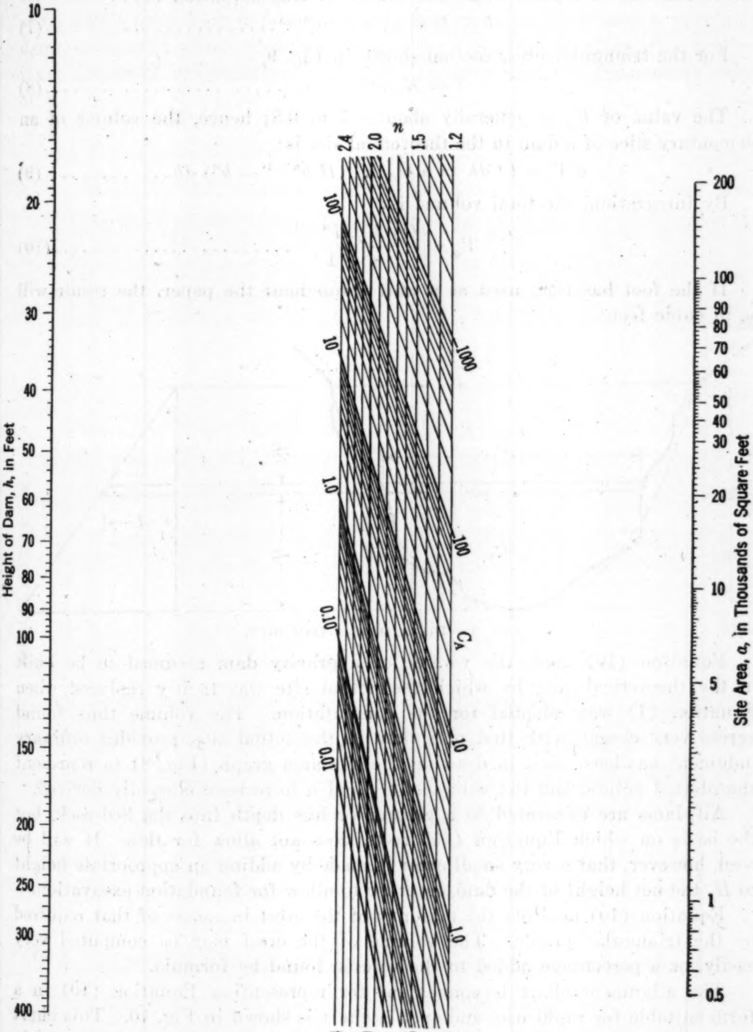


FIG. 8.—SITE-AREA DIAGRAM FROM EQUATION (1).

Referring to Fig. 9 in which the actual site is shown in full lines and the theoretical site in broken lines, and differentiating Equation (1):

$$l = D\,h^{v} = n\,C_A\,h^{n-1} \qquad (7)$$

For the triangular cross-section shown in Fig. 9,

$$t = K_g\,(H - h) \qquad (8)$$

The value of K_g is generally about 0.7 to 0.8; hence, the volume of an elementary slice of a dam in the theoretical site is:

$$d\,V = l\,t\,dh = n\,C_A\,K_g\,(H\,h^{n-1} - h^{n})\,dh \qquad (9)$$

By integration, the total volume is:

$$V_g = \frac{C_A\,K_g\,H^{n+1}}{n+1} \qquad (10)$$

If the foot has been used as a unit throughout the paper, the result will be in cubic feet.

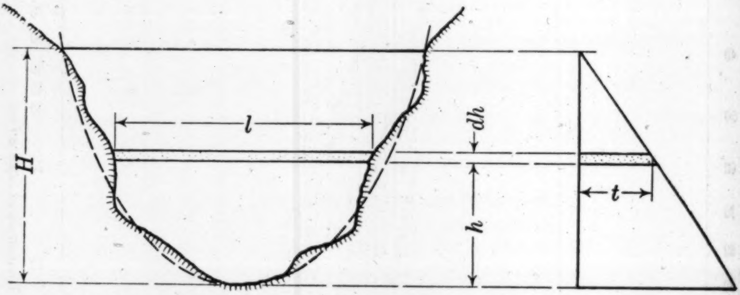


FIG. 9.—DIAGRAM OF A DAM SITE.

Equation (10) gives the volume of a gravity dam assumed to be built in the theoretical site, by which the actual site was tacitly replaced when Equation (1) was adopted for the computation. The volume thus found agrees very closely with that of a dam in the actual site, provided ordinary judgment has been used in drawing the site-area graph (Fig. 8) to represent the plotted points, and the values of C_A and n have been correctly derived.

All dams are excavated to a greater or less depth into the bed-rock, but the basis on which Equation (10) rests, does not allow for this. It will be seen, however, that a very small error is made by adding an appropriate height to H, the net height of the dam, in order to allow for foundation excavation.

Equation (10) neglects the masonry of the crest in excess of that required by the triangular profile. The volume of the crest may be computed very easily, or a percentage added to the volume found by formula.

The alignment chart is convenient for representing Equation (10) in a form suitable for rapid use, and such a chart is shown in Fig. 10. This chart will give the same results as the formula, with an error (in reading) within 2 per cent.

Fig. 10 is based on a value of K_g equal to 0.7 and includes 5% excess volume for crest thickness, this having been found by experience to be an average value. The chart may be used as it stands to determine approximately the volume of a gravity dam of moderate height in a site for which the values of C_A and n have been determined. In other cases, as, for example, very high dams, or those in which allowance is made for a high degree of uplift, a preliminary design of cross-section is required to determine the appropriate value of K_g.

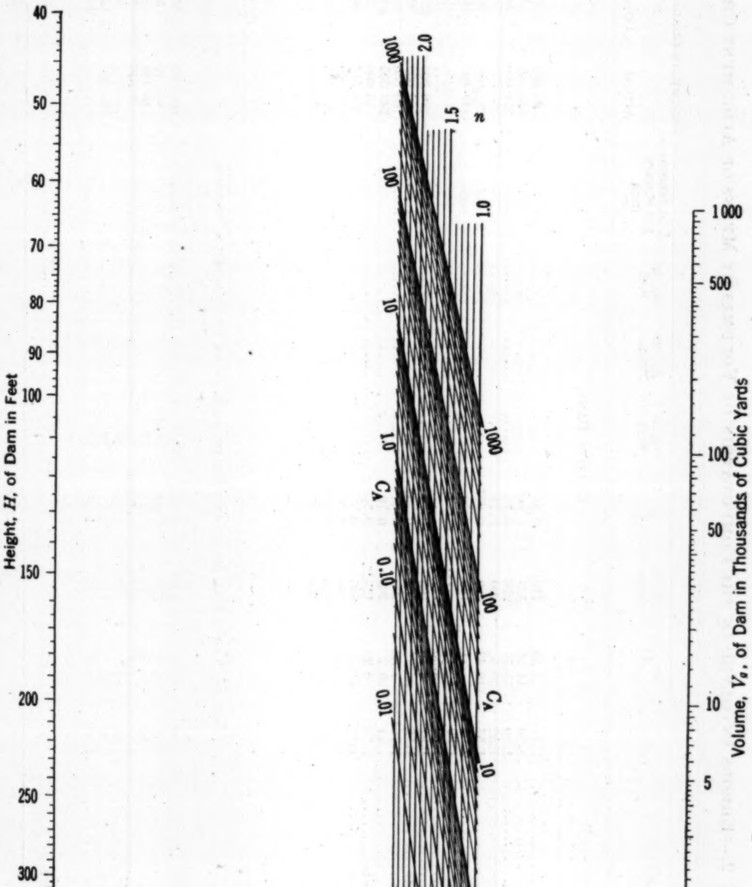


FIG. 10.—VOLUME OF GRAVITY DAM.

It may be stated at once that the writer does not suggest that the equation or chart should ever be used for an accurate computation of quantities, as for a contract, since the mensuration, although itself mathematically accurate,

TABLE 2.—ERRORS INVOLVED IN THE COMPUTATION OF VOLUMES BY MEANS OF ALIGNMENT CHARTS.

Item No.	Name of dam.	n.	C_A.	H, in feet.	K.	t_1, in feet.	R_1, in feet.	R_2, in feet.	f, in pounds per square inch.	VOLUME, IN CUBIC YARDS.		Percentage of error.
										Actual.	Computed.	
	GRAVITY DAMS.											
1	Arapuni	1.89	1.59	210	0.81	......	......	......	......	95 000	70 000	......
2	Arrowrock	1.66	14.36	323	0.680	......	......	......	......	585 165	591 000	1.01
3	Avon	1.60	15.17	226	0.70	......	......	......	......	214 000	200 000	6.5
4	Bull Run	1.52	30.5	200	0.74	......	......	......	......	230 000	222 000	3.48
5	Camarasa	1.57	10.43	299	0.827	......	......	......	......	283 000	286 000	1.05
6	Cataract	2.09	1.71	185	0.70	......	......	......	......	149 800	146 200	2.4
7	Ceresole Reale	1.30	140.4	......	......	......	......	......	......	....	....	......
8	Coghinas	1.69	8.55	......	......	......	......	......	......	....	....	......
9	Cordeaux	1.87	6.46	195	0.70	......	......	......	......	216 000	220 000	1.82
10	Elephant Butte	1.52	36.7	264	0.735	......	......	......	......	500 000	502 600	0.52
11	Little Nerang	1.764	7.83	200	0.65	......	......	......	......	165 000	156 700	......
12	Nepean	1.44	35.7	268	0.70	......	......	......	......	342 000	340 000	0.6
13	Lake Manchester	1.78	8.26	125	0.73	......	......	......	......	53 000	54 000	1.85
14	O'Shaughnessy	1.37	35.3	306	0.75	......	......	......	......	398 516	386 000	3.14
15	O'Shaughnessy	1.37	35.3	395	0.78	......	......	......	......	660 000	630 000	4.50
16	Roosevelt	1.53	25.2	275	0.600	......	......	......	......	342 325	346 302	1.15
17	Lower Stanley	1.56	30.3	150	0.67	......	......	......	......	155 000	110 000	......
	Average error	......	......	......	......	......	......	......	......	....	....	2.5
	ROCK OR EARTH-FILL DAMS AND PLAIN ARCH DAMS OF TRAPEZIUM SECTION.											
18	Goose Creek	1.52	48.3	145	5.0	20	......	......	......	1 000 000	1 059 700	5.97
19	Pathfinder	1.48	10.69	218	0.40	10.9	......	......	......	60 210	57 800	3.65
20	Silverwood	1.56	26.85	72.5	3.5	12	......	......	......	85 180	83 000	2.55
21	Shoshone	1.30	14.07	328	0.40	10	......	......	......	78 576	72 300	8.00
22	Upper Deer Flat	1.41	41.7	66	5.0	20	......	......	......	....	89 180	......
23	Upper Stanley	1.80	30.15	110	5	20	......	......	......	1 130 000	1 136 200	0.55
24	Walnut Grove	1.45	9.06	110	1.07	10	......	......	......	....	479 000	......
	Average error	......	......	......	......	......	......	......	......	....	....	4.14

TABLE 2.—(*Continued.*)

Item No.	Name of dam.	n.	C_A.	H, in feet.	K.	t_1, in feet.	R_1, in feet.	R_2, in feet.	f, in pounds per square inch.	VOLUME, IN CUBIC YARDS.		Percentage of error.
										Actual.	Computed.	
					PLAIN ARCH DAMS.							
25	Gibson	1·67	13·7	195	0.44	......	......	......	......	160 700	142 000	11
26	Mt. Edwards	1·5	27·0	90	0.33	......	......	......	......	22 400	20 300	9·35
27	Melones	1·595	12·27	210	0.38	......	......	......	......	91 000	78 000	......
28	Salmon River	1·33	44·8	220	0·55	......	......	......	......	....	121 000	......
29	Turrite	1·54	6·57	130	0.44	......	......	......	......	....	10 100	......
					CONSTANT-ANGLE DAMS.							
30	Bullards Bar	1·60	10·5	183	......	......	240	94	350	47 000	44 170	6·3
31	Coomera	1·468	23.39	170	......	......	290	140	300	....	36 500	......
32	Gerber	1·53	25.25	70	......	......	197	155	300	9 500	9 200	3
33	Jogne	2·28	0·175	170	......	......	129	90	......	....	....	......
34	King River	1·87	1·68	240	......	......	250	100	305	56 675	52 900	6·7
35	Nathan	1·68	10·43	205	......	......	380	200	335	110 000	110 000	......
36	Pacoima	2·12	0.2924	375	......	......	820	115	290	160 000	149 000	6·9
37	Salmon Creek	1·68	11.34	168	......	......	331	147·5	330	52 000	53 000	1.92
38	Kerckhoff	1·53	20·4	108	......	......	......	......	......	23 000	....	......
39	Lake Spaulding	1·99	1·466	275	......	......	440	250	420	192 000	165 000	1.50
40	Warragamba	1·51	21·8	310	......	......	......	......	......	......	261 000	......
	Average error	......	......	......	......	......	......	......	......	......	....	4·38

is based essentially on an approximation, and no engineer would stake his reputation on an approximate method. The method is valuable, however, for many preliminary studies, and the use of the chart enables a volume to be found in a few minutes, which by the usual methods of calculation would require many hours' work.

In Table 2 are given the values of C_A and n for a number of sites as determined from the site area graphs, such as Fig. 8. These data, together with the actual height above foundation, and the total batter, are used to obtain the theoretical volume, which is compared with the actual volume.

The values of n and C_A for Item No. 2 of Table 2 refer to the developed up-stream face. A correction was made for arch curvature in this case by multiplying the computed volume by a factor,

$$\frac{\text{Up-stream radius} - \frac{1}{4}\text{ base width}}{\text{Up-stream radius}} = 0.92$$

Thus, an allowance is made for the reduced radius, measured to the center of gravity.

The computed volumes for Items Nos. 4 and 16 were multiplied by 1.03 and 1.06, respectively, to allow for curvature. Due to an added width of base, 64 000 cu. yd. were added to the volume of Item No. 14 as computed by the charts.

The actual volumes given for Items Nos. 1, 11, and 17 include estimates of quantities in the spillways, access bridges, etc., the amount of which was not known.

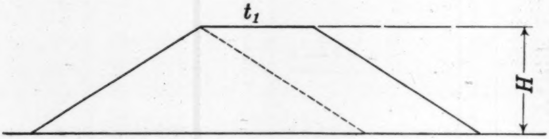


FIG. 11.—TYPICAL TRAPEZIUM CROSS-SECTION.

DAMS OF TRAPEZIUM CROSS-SECTION

The earth and rock-fill dam is almost always trapezoidal in cross-section. In some cases, one or more berms are provided and the batter may vary slightly from crest to base. In such instances, an equivalent trapezium section may be drawn. Gravity dams are also sometimes of trapezium cross-section. For example, the cross-section in Fig. 11 may be divided into two parts. One part is triangular, and the equivalent volume may be found from the same formula as for a gravity dam (Equation (10)), by inserting K_R for K_g. In this paper the batter is defined as the ratio of horizontal increment to vertical increment. The reciprocal of this ratio is sometimes referred to as the batter. The volume of the second part is simply the site-area to height, H, multiplied by t_1.

The total volume in cubic feet, therefore, is,

$$V_R = C_A \left(\frac{K_R H^{n+1}}{n+1} + t_1 H^n\right) \dots\dots\dots\dots\dots\dots (11)$$

The volume in cubic yards can also be readily found from the charts by taking $\frac{K_R}{0.735}$ times the volume of a gravity dam to height, H, from Fig. 10, plus $\frac{t_1}{27}$ times the site-area to height, H, from Fig. 8. In Table 2, Items Nos. 18 to 24, inclusive, have been prepared to furnish a comparison between the actual volume of a number of dams and the volume obtained by Equation (11). In the case of arch dams, a factor of 1.05 is introduced to allow for the additional length along the mean arc. For Items Nos. 19 and 21, the value of H is listed as 218 and 328, respectively. In computing the volumes by Equation (11), the values used were 223 and 338, respectively, in order to make allowance for excavation.

PLAIN ARCH DAMS

A plain arch dam with vertical up-stream face and uniform cylinder stress is theoretically of triangular section. The cylinder stress is the arch stress computed by the cylinder formula. As is well known, it may differ considerably from the stress that actually exists. The down-stream batter is uniform and is equal to,

$$K_A = \frac{62.4\ R_u}{144\ f} = 0.434\ \frac{R_u}{f} \ldots\ldots\ldots\ldots\ldots\ldots\ldots (12)$$

The volume may best be found by the same formula as for a gravity dam by inserting K_A for K_g in Equation (10) and multiplying by a factor to allow for curvature. The factor, 1.05, which corresponds to a mean subtended angle of about 60°, may be used for this purpose. For other values of the mean subtended angle the factor may be read from the curve in Fig. 12.

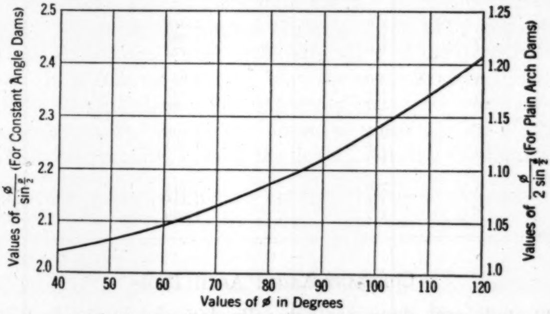


FIG. 12.—CURVATURE FACTORS FOR ARCH DAMS.

In Table 2, Items 25 to 29, inclusive, show volumes for five arch dams. In all cases, 5% has been added for additional material in the crest. Another 5% has been added in all cases to take account of curvature. The quantity given as actual volume in Item No. 27 includes the volume of the bucket on the down-stream side of the dam.

From the fact that the use of the formula for the volume of a gravity dam has been recommended for finding that of an arch dam, it must not be assumed that this volume varies directly as C_A, or as the $(n + 1)$ power of the height. This is not so. A formula can be derived as follows: Take a crest angle of 110° and write,

$$R_u = 0.613\, n\, C_A\, h^{n-1} \ldots\ldots\ldots\ldots\ldots\ldots (13)$$

Inserting this value of R_u in Equation (12) and the resulting value of K_A in Equation (10), and also introducing a curvature factor, $\dfrac{\phi}{2 \sin \dfrac{\phi}{2}}$, the result is,

$$V_A = \frac{0.266 \left(\dfrac{n}{n+1}\right) C_A^2\, H^{2n}}{f} \times \frac{\phi}{2 \sin \dfrac{\phi}{2}} = \frac{J_A\, C_A^2\, H^{2n}}{f} \ldots\ldots (14)$$

To determine the values of J_A in terms of n for different crest angles, several "type" designs were developed for a crest span of 600 ft. and a height of 100 ft. (the values used are immaterial). The values of the curvature factor were obtained by weighting the value of the ratio, $\dfrac{\text{arc}}{\text{chord}}$, at different points by the volume per foot of height at that point and thus obtaining the weighted mean. Values of this curvature factor and of J_A are given in Table 3.

TABLE 3.—VALUES OF CURVATURE FACTORS AND J_A.

Crest angle, in degrees.	n.	$\dfrac{\phi}{2 \sin \dfrac{\phi}{2}}$.	J_A.
120	2.0	1.048	0.1750
	1.6	1.064	0.1642
	1.2	1.122	0.1532
100	2.0	1.036	0.1953
	1.6	1.047	0.1826
	1.2	1.086	0.1676
80	2.0	1.025	0.2300
	1.6	1.082	0.2140
	1.2	1.054	0.1986
60	2.0	1.014	0.2934
	1.6	1.018	0.2721
	1.2	1.029	0.2440

CONSTANT-ANGLE ARCH DAMS

Constant-angle arch dams are generally designed in the first place by the use of the cylinder formula, modifications being then made on the basis of more accurate and frequently involved analyses. In any case the cylinder stress is readily determinable.

It will be assumed that the up-stream radius varies linearly from crest to base, an assumption which is generally valid.[25] It will also be assumed that the cylinder stress and the subtended angle are uniform from crest to base. Actually, this is not always true,[25] but average values may be chosen to represent, fairly well, the actual value. In the case of a preliminary design, the cylinder stress is taken to be uniform throughout.

---

[25] Due to Mr. Nimmo, in his discussion previously mentioned.

Now, the thickness at the height, h, is,

$$t = \frac{0.434}{f} R_h (H - h) \qquad \text{(15)}$$

in which,

$$R_h = R_2 + \frac{h}{H} (R_1 - R_2) \qquad \text{(16)}$$

Substituting this value of R_h in Equation (15):

$$t = \frac{0.434 (H - h)}{f} \left[R_2 + \frac{h}{H} (R_1 - R_2) \right] \qquad \text{(17)}$$

The length of arc is $\dfrac{n\, C_A\, h^{n-1}\, \phi}{2 \sin \dfrac{\phi}{2}}$. The volume of an arch element at a height, h, is, therefore,

$$d\,V = \frac{0.434\, n\, C_A\, \phi\, h^{n-1} (H - h) \left[R_2 + \dfrac{h}{H} (R_1 - R_2) \right]}{2 f \sin \dfrac{\phi}{2}}\, dh \qquad \text{(18)}$$

Integrating between $h = 0$ and $h = H$ and reducing:

$$V_{CA} = \frac{0.217\, n\, C_A\, \phi\, H^{n+1} \left[\dfrac{n\, R_1 + 2\, R_2}{n\,(n + 1)\,(n + 2)} \right]}{f \sin \dfrac{\phi}{2}} \qquad \text{(19)}$$

Theoretically, the constant-angle dam has the same subtended angle from crest to base. The most economical angle is about 133°, but comparatively wide variations do not cause very much alteration in the quantity of material in the arch. It is due to this fact that the method of analysis leading to Equation (19) is legitimate. The subtended angle for constant-angle arch dams is generally about 110° to 120° over the upper portion of the dam, the angle falling away rapidly toward the base. For use in Equation (19), a value of 85° to 95° gives satisfactory results. Values of the factor, $\dfrac{\phi}{\sin \dfrac{\phi}{2}}$, for various angles are given in Fig. 12.

Table 2, Items Nos. 30 to 40, inclusive, shows a comparison between actual quantities for a number of dams and those obtained by Equation (19). In Item No. 32, the writer added 15 ft. for foundations and reached an assumed value of H, equal to 70 ft. Likewise, for Items Nos. 35 and 36, 10 ft. were added to the net height to allow for foundations. For Items Nos. 37 and 39, 5 ft. were added for foundations. The heights of these four items were further increased 5% to make allowance for the crest. The actual volume listed for Item No. 39 includes approximately 24 500 cu. yd. of extra material in the gravity base. This dam (Lake Spaulding) was partly constructed as a gravity dam and later was altered to a constant-angle type. The concrete in the constant-angle dam is, therefore, about 167 500 cu. yd. The computed volume of Item No. 40 is as presented by Mr. Nimmo.[26]

A comprehensive formula for the volume of a constant-angle arch dam can also be found in much the same way as Equation (14) was derived for the volume of an arch dam. Three "type" designs were prepared on the following basis:

Height, in feet	100
Crest span, in feet	800
Crest angle, in degrees	116
Cylinder stress, in pounds per square inch	300

$n = 1.6$, 2.0, and 2.4, respectively.

The up-stream radii were made to reduce linearly from the crest to one-fifth of the height, at which point the radius was taken as R_2. Furthermore, the radii were naturally made as small as possible without undercutting on the up-stream face. From these trial designs, the factors involving radii and angle in Equation (19), can be expressed in terms of n and of crest span, on which alone the radii and angles depend, under the assumptions stated.

In Equation (19) let, $0.217 \dfrac{\phi}{\sin \dfrac{\phi}{2}} \left[\dfrac{n R_1 + 2 R_2}{n (n+1)(n+2)}\right]$ be made equal to $J \times$ the span, or $J \times n\, C_A\, H^{n-1}$, in which, J is a function of n only.

Inserting this value in Equation (19),

$$V_{CA} = \frac{J\, n^2\, C_A^2\, H^{2n}}{f} = \frac{J_{CA}\, C_A^2\, H^{2n}}{f} \dots\dots\dots\dots\dots (20)$$

which is similar to Equation (14). For practical purposes, J_{CA} may be taken as 0.147. Equation (20), therefore, may be reduced to,

$$V_{CA} = \frac{0.147\, C_A^2\, H^{2n}}{f} \dots\dots\dots\dots\dots (21)$$

which is a very simple expression for the volume of a structure of relatively complicated design.

Equation (21) may be solved by the use of the alignment chart, Fig. 13. For this purpose, f is assumed as 300 lb. per sq. in. For other values of f the volume can be adjusted proportionately. To allow for crest thickness increase the volume from 2 to 5 per cent. The error introduced by using this equation is not greater than that by Equation (19), and the alignment chart introduces only a small reading error.

USE OF VOLUME FORMULAS

Equations (10), (11), (14), and (20) have been evolved for gravity, rock-fill (or trapezoidal dams of any type), arch, and constant-angle dams, respectively. They have been found by test to give errors in most cases within 5%, as compared with the actual volume of constructed dams, or the carefully computed volume of proposed dams. For many preliminary purposes these equations, or the charts derived from them, therefore, may be used to estimate

---

[26] In his discussion previously mentioned.

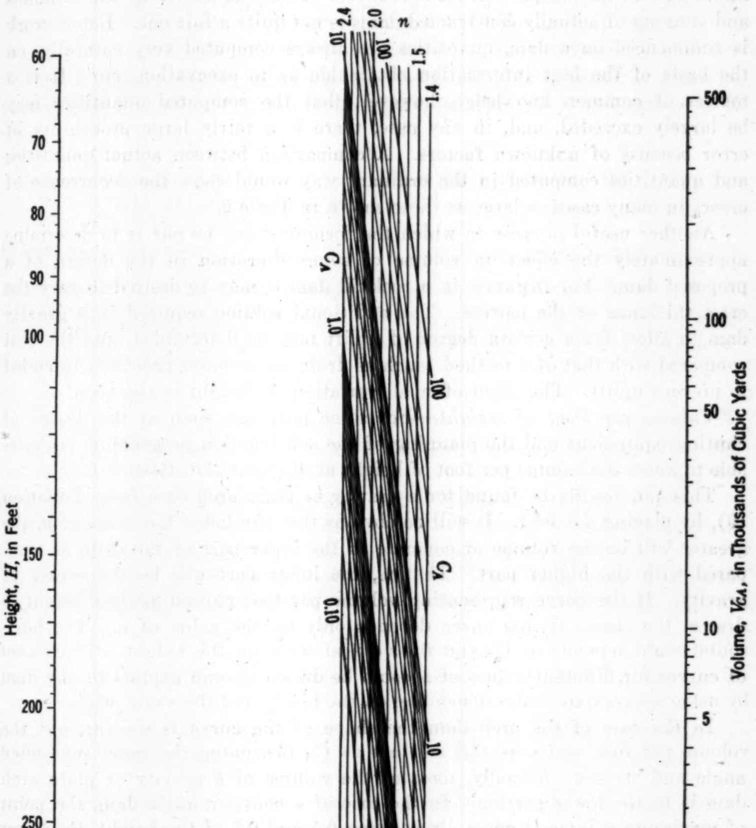


FIG. 13.—VOLUME OF CONSTANT ANGLE ARCH DAM.

roughly the volume and cost of a proposed dam. In this connection it should be stated that the comparison made between volumes computed by the formulas and volumes of actually constructed dams is not quite a fair one. Before work is commenced on a dam, quantities are always computed very carefully, on the basis of the best information obtainable as to excavation, etc. It is a matter of common knowledge, however, that the computed quantities may be largely exceeded, and, in any case, there is a fairly large probability of error because of unknown factors. A comparison between actual quantities and quantities computed in the ordinary way would show the occurrence of errors in many cases as large as those shown in Table 2.

Another useful purpose to which the formulas may be put is to determine approximately the effect on volume of some alteration in the design of a proposed dam. For instance, in a rock-fill dam it may be desired to vary the crest thickness or the batters. The additional volume required in a gravity dam to allow for a certain degree of uplift may be determined, and the cost compared with that of a method (such as drainage or water-proofing) intended to prevent uplift. The effect of small variations in height is also seen.

Volume per Foot of Height.—For some purposes, such as the design of chuting equipment and the planning of the construction program, it is desirable to know the volume per foot of height at different elevations.

This can readily be found for a gravity or plain arch dam from Equation (9), by placing $dh = 1$. It will be obvious that the lower the value of n, the greater will be the volume of concrete in the lower part of the dam, as compared with the higher part. That is, the lower part will be the center of gravity. If the curve representing volume per foot plotted against height is drawn, the shape of this curve depends only on the value of n. The horizontal scale depends on C_A and the vertical scale on the height. Thus, a set of curves for different values of n could be drawn up and applied to any dam by using appropriate scales depending on the height and the value of C_A.

In the case of the arch dam the shape of the curve is similar, but the volume per foot varies as C_A^2 and not as C_A (assuming the same subtended angle and stress). Actually, most of the volume of a gravity or plain arch dam is in the lower portion. In the case of a constant-angle dam, the point of maximum volume is generally between 0.5 and 0.7 of the height, the lower value occurring with relatively low values of n and the higher value with high values of n.

SUITABILITY OF SITES FOR DAMS OF THE ARCH TYPE

There is a large class of sites that apparently may be suitable for a dam of the arch type, although the site is not so narrow as to indicate at once the choice of an arch dam. It is convenient to have a criterion for the rapid examination of such sites.

If,

$$J_A = 0.434 \frac{R_u}{L} \left(\frac{n}{n+1}\right) \left(\frac{\phi}{2 \sin \dfrac{\phi}{2}}\right)$$

Equation (14) may be written,

$$V_A = 0.434 \frac{R_u}{L} \left(\frac{C_A^2 H^{2n}}{f}\right) \left(\frac{n}{n+1}\right) \left(\frac{\phi}{2 \sin \frac{\phi}{2}}\right) \ldots\ldots\ldots (22)$$

The volume of an arch dam may then be determined in terms of that of a gravity dam and relative to the theoretical crest span. For this purpose note that,

$$V_G = C_A K_G \left(\frac{1}{n+1}\right) H^{n+1} = \frac{n K_G \left(\frac{1}{n+1}\right) C_A^2 H^{2n}}{L} \ldots\ldots (23)$$

Hence,

$$\frac{V_A}{V_G} = \frac{0.434 \frac{R_u}{L} \frac{\phi}{2 \sin \frac{\phi}{2}}}{f K_G} L \ldots\ldots\ldots\ldots\ldots\ldots (24)$$

or,

$$V_A = V_G Q_A L \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (25)$$

Let $f = 300$ lb. per sq. in. and $K_g = 0.7$, and insert in Equation (24) the values already found for the other terms; then successive values of Q_A for various crest angles and spans are as listed in Table 4.

TABLE 4.—VALUES OF Q_A IN EQUATION (25).

Crest angle, in degrees.	n.	Q_A.	Maximum span.
120	2.0	0.125	800
	1.6	0.127	787
	1.2	0.134	746
100	2.0	0.140	715
	1.6	0.141	710
	1.2	0.147	680
80	2.0	0.1646	608
	1.6	0.1654	603
	1.2	0.169	591
60	2.0	0.209	477
	1.6	0.210	476
	1.2	0.212	472

The constant, Q_A, is seen to express the volume of an arch dam (cylinder stress, 300 lb. per sq. in.) in terms of that of a gravity dam (total batter, 0.7) as a percentage per foot of the theoretical crest span. The figures given for "maximum span" represent the spans which would make the volume of the arch dam equal to that of the gravity dam. The practical span, of course, will be less.

For a constant-angle dam, a similar procedure can be followed, but because the volume is, within practical limits, only very slightly dependent on the subtended angle, the expression derived is simpler. Thus,

$$\frac{V_{CA}}{V_G} = \frac{100 \; J_{CA} \; (n+1)}{210 \; n} L = Q_{CA} L \ldots\ldots\ldots\ldots (26)$$

Values of Q_{CA} for use in this equation are listed in Table 5. The practical use of these results (Tables 4 and 5) is to determine rapidly for any given site and height of dam, what degree of economy may be effected by the use of an arch as against a gravity dam. The arch dam will be required to show considerable economy in material for the following reasons: (1) The design and setting out of the dam is more complicated; (2) the cost of form work per cubic yard is higher, since the faces are curved and frequently warped and the sections are less massive; (3) the concrete is frequently made either richer in cement or with a less quantity of large rock added; (4) greater care is generally required in construction; and (5) the cost of spillways and accessory works may be higher since the arch dam is less conveniently adapted for passing floods over the crest.

TABLE 5.—Values of Q_{CA} in Equation (26)

n.	Q_{CA}.	Maximum span.
2.4	0.1055	958
2.0	0.1054	959
1.6	0.1138	878
1.2	0.1288	776

Table 6 has been prepared to show the relative suitability of various sites on which dams of the plain arch or constant-angle arch type have been constructed, or for which they have been proposed. The table is based on the following assumptions:

1.—The cylinder stress in all cases is 300 lb. per sq. in.
2.—The correct values are given for the crest angle where it is known. Other values are assumed.
3.—In Items Nos. 1 to 10, inclusive, the dams have a vertical up-stream face.
4.—In Items Nos. 11 to 27, inclusive, the up-stream radii at other points than the crest are made as small as possible without undercutting.
5.—The gravity dam used for comparison has a value, $K_g = 7$.

Table 6, therefore, compares, on a uniform basis, the merits of the sites only, and has no reference to existing or proposed dams on the sites. This is considered the most reasonable method of comparison, since the design of the actual dams may vary considerably from the bases assumed, because of a variety of factors, such as the strength of rock, the predilections of the engineers, or local peculiarities of the site, and it would be difficult to make just allowance for all these factors.

Items Nos. 8, 9, 10, and 22 to 27 inclusive, contain data relating to sites at which actually other types of dam are constructed or proposed, but where dams of the arch or constant-angle arch type, under certain circumstances, might be suitable.

It appears evident that, for economy, the volume of an arched dam should not exceed 80% of that of a gravity dam, on the assumptions mentioned. In other words, the theoretical crest span should not exceed 80% of the maximum permissible span. It must be remembered, however, that favorable circumstances such as very resistant rock may influence the design of an arched dam to a much greater extent than in the case of a gravity dam. The profile of the latter may be varied only between comparatively narrow limits, no matter how excellent the site, but an arch dam may be designed so as to use more effectively the strength of the foundations and of the material of construction. The Gibson Dam exemplifies this fact, and it represents no doubt the first step in the direction of broadening the scope of arch dams by a more rational system of design and technique of construction.

TABLE 6.—RELATIVE SUITABILITY OF DAM SITES AS REGARDS ECONOMY OF MATERIALS.

Item No.	Name of dam.	*n*.	C_A.	Height, in feet.	Theoretical crest span, in feet.	Crest angle, in degrees.	Maximum span, in feet.	Volume expressed as a percentage of the volume of a gravity dam.
PLAIN ARCH DAMS.								
1	Pathfinder...........	1.48	10.69	218	209	80	600	34.8
2	Shoshone...........	1.30	14.07	328	104	65	510	20.4
3	Gibson	1.67	13.7	195	782	120	792	98.6
4	Mt. Edwards.........	1.50	27.0	90	385	80	600	64.2
5	Melones..............	1.595	12.27	210	470	110	750	62.7
6	Salmon River........	1.33	44.8	220	353	100	690	51.2
7	Turrite..............	1.55	6.57	130	140	80	604	23.2
	Mean	1.49	.....	....	....	....	....	50.7
8	Camarasa	1.57	10.43	299	422	100	708	59.7
9	Coghinas	1.69	8.55	180	520	100	712	73.0
10	Nepean..............	1.44	35.7	210	542	100	700	77.5
CONSTANT-ANGLE ARCH DAMS.								
11	Bullards Bar	1.60	10.5	183	384	116	877	43.8
12	Coomera.............	1.468	23.39	170	379	116	845	44.8
13	Gerber..............	1.53	25.25	70	368	116	860	42.8
14	Jogne...............	2.28	0.175	170	287	116	972	29.6
15	King River..........	1.87	1.68	240	370	116	935	39.5
16	Nathan	1.68	10.43	205	653	116	895	73.0
17	Pacoima.............	2.12	0.2924	375	479	116	970	49.4
18	Salmon Creek.......	1.68	11.34	168	620	116	895	69.2
19	Kerckhoff............	1.53	20.4	108	374	116	860	43.5
20	Lake Spaulding	1.99	1.466	275	758	116	955	79.4
21	Warragamba........	1.55	21.8	310	618	116	855	72.1
	Mean	1.75	.....	....	....	....	....	53.4
22	Camarasa	1.57	10.43	299	422	116	870	48.5
23	Coghinas	1.69	8.55	180	520	116	900	64.6
24	Nepean..............	1.44	35.7	210	542	116	840	65.0
25	Gibson..............	1.67	13.7	195	782	116	895	57.8
26	Mt. Edwards	1.50	27.0	90	385	116	853	45.2
27	Melones..............	1.595	12.27	210	470	116	876	53.7

MOST ECONOMICAL SIZE OF INSTALLATION

For every installation involving storage there exists a size of storage which gives the most economical result in terms of cost per second-foot of regulated outflow.[27] By means of the procedure outlined in this paper, and following the general lines of the method previously used by the writer,[28] the curve of specific cost (total cost of installation per second-foot of regulated outflow), may be found very readily for any particular case, and the most economical size of installation determined.

---

[27] "The Most Economical Size of a Hydro-Electric Installation," by A. B. Mead, *New Zealand Journal of Science and Technology,* 1921.

Whether the installation adopted will be of the size indicated by considerations of lowest specific cost depends on a large number of local conditions, which must be considered for each particular case. In general, there will be a tendency to push the size somewhat beyond the point of lowest unit cost, although the installation may be developed in stages.

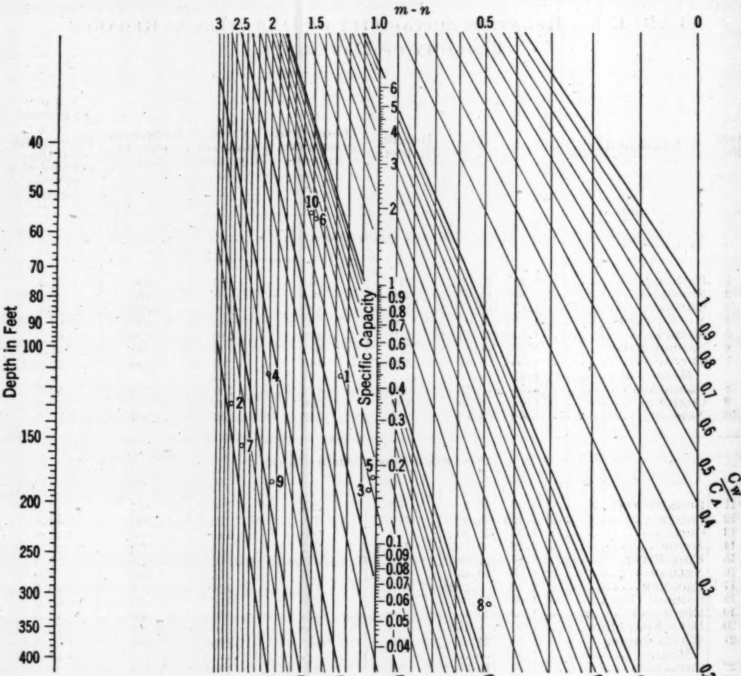


FIG. 14.—SPECIFIC CAPACITY OF GRAVITY DAMS.

The method is useful in the study of any proposed plan, but is not intended to constitute a hard and fast rule by which the size of installation is to be fixed. It may be stated, however, that many economically and technically unsound projects would never have been adopted had such a method of study been in general use in the past.

---

[28] "The Economic Height of a Dam," *New Zealand Journal of Science and Technology*, 1923, Vol. 6, No. 1, pp. 36-39.

TABLE 7.—COMPUTATIONS TO DETERMINE SPECIFIC CAPACITY.

Item No.	Name of dam.	Depth, in feet.	C_w.	C_A.	C_x.	m.	n.	$m - n$.	$m - 2n$.	Stress, in pounds per square inch.	W_S, in Acre-Feet per Cubic Yard.	
											From chart.	Actual.
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)
					Gravity Dams.							
1	Arrowrock	249	0.2848	14.36	0.00198	2.92	1.66	1.26	......	......	0.54	0.48
2	Avon	210	0.000023	15.17	0.00000152	4.25	1.60	2.65	......	......	0.80	0.81
3	Bull Run	180	0.058	30.5	0.0019	2.57	1.52	1.05	......	......	0.16	0.16
4	Cataract	150	0.0000915	1.71	0.0000535	4.11	2.09	2.02	......	......	0.60	0.51
5	Ceresole Reale	134	0.325	140.4	0.00232	2.32	1.30	1.02	......	......	0.18	0.154
6	Coghinas	157	0.03272	8.55	0.00383	3.18	1.69	1.49	......	......	2.8	3.37
7	Cordeaux	170	0.0000165	6.46	0.00000256	4.33	1.87	2.46	......	......	0.27	0.35
8	Little Nerang	197	0.1365	7.83	0.0174	2.24	1.76	0.48	......	......	0.076	0.11
9	Nepean	207	0.000518	35.7	0.0000145	3.41	1.44	1.97	......	......	0.195	0.185
10	Roosevelt	220	0.0847	25.2	0.00336	3.04	1.53	1.51	......	......	4.0	4.0
......	Mean	174	......	......	......	......	1.64	......	......	......	0.96	1.02
					Arch Dams.							
11	Coghinas	157	0.03272	8.55	0.000447	3.18	1.69	......	−0.20	300	5.15	......
12	Gibson	161	1.02	13.7	0.00542	2.24	1.67	......	−1.1	470	0.99	0.56
13	Nepean	207	0.000518	35.7	0.000000406	3.41	1.44	......	+0.53	300	0.21	......
14	Mt. Edwards	75	0.00497	27.0	0.0000068	3.56	1.5	......	+0.56	300	2.3	1.36
15	Pathfinder	202	0.0001376	10.69	0.0000012	4.26	1.48	......	+1.32	122	17.1	17.84
					Constant-Angle Arch Dams.							
16	Coghinas	157	0.03272	8.55	0.000447	3.18	1.69	......	−0.2	300	6.0	......
17	Coomera	170	0.1592	23.39	0.000291	2.70	1.47	......	−0.24	300	3.2	4.85
18	Gibson	161	1.02	13.7	0.00543	2.24	1.67	......	−1.1	600	0.78	......
19	Mt. Edwards	75	0.00497	27.0	0.0000068	3.56	1.5	......	+0.56	300	2.8	......
20	Nepean	207	0.000518	35.7	0.0000004	3.41	1.44	......	+0.53	300	0.25	......
21	Lake Spaulding	275	0.0858	1.47	0.0397	2.43	1.99	......	−1.55	420	......	0.45

SPECIFIC CAPACITY

Specific capacity will be defined as the acre-feet of reservoir capacity per cubic yard of material in the dam. By means of formulas developed herein, the specific capacity can be determined approximately for gravity, arch, and constant-angle arch dams. The results are useful in enabling an opinion to be formed as to the economic nature of any storage proposal, or for comparing quickly the merits of different reservoirs and sites.

Gravity Dams.—Let $h_1 = 174$, $H = 194$, and $n = 1.64$ (average values from Table 2). Then Equations (2) and (10) can be modified by the insertion of these quantities, and combined to form the following formula for specific capacity of a gravity dam:

$$w_{SG} = 72.9 \frac{C_W}{C_A} h_1^{m-n-1} \ldots\ldots\ldots\ldots\ldots\ldots\ldots (27)$$

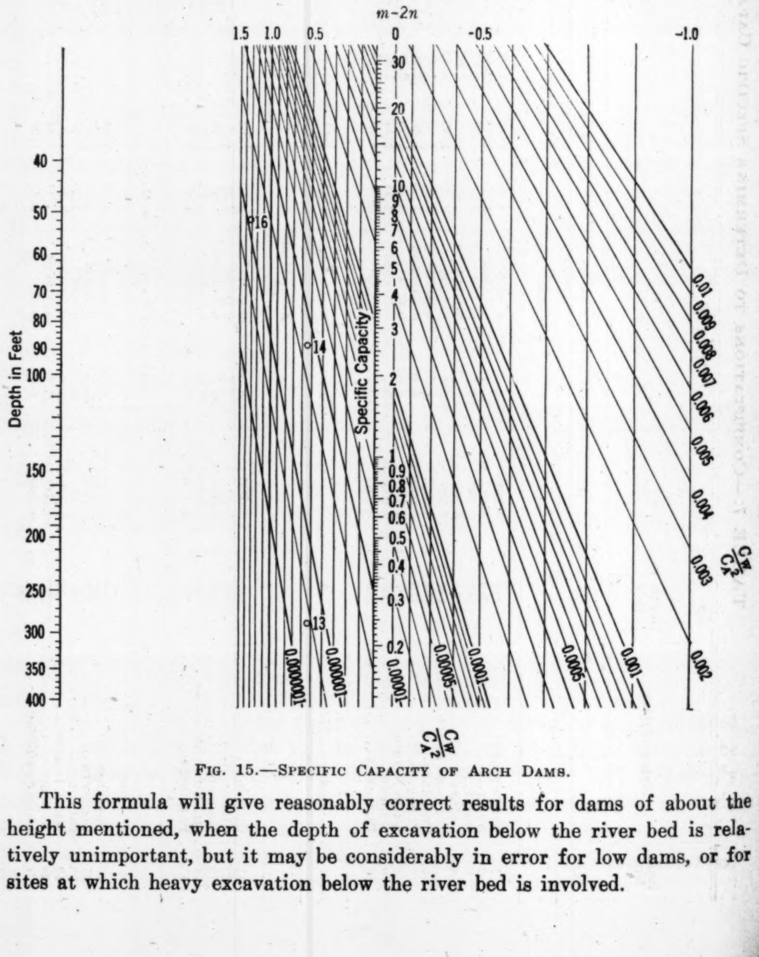


FIG. 15.—SPECIFIC CAPACITY OF ARCH DAMS.

This formula will give reasonably correct results for dams of about the height mentioned, when the depth of excavation below the river bed is relatively unimportant, but it may be considerably in error for low dams, or for sites at which heavy excavation below the river bed is involved.

Equation (27) has been represented as an alignment chart in Fig. 14, and a comparison of the results obtained from the chart with actual values is shown in Table 7, Items Nos. 1 to 10, inclusive. No allowance has been made for the fact that in some cases the batter differs somewhat from the value of 0.7, on which the formula is based. Discrepancies are due to this variation in K_g, and to variations of n, height, and difference between height and depth, from the values assumed. The "actual" specific capacity for Items Nos. 5, 6, and 9 (see Column (13), Table 7) are estimated. Values of the quantity, $\frac{1}{n+1}$, vary from 0.322 at Item No. 4 to 0.435 at Item No. 5. In solving Equation (27) for values in Column (13) a mean value of 0.380 was used. Errors due to this cause alone may therefore be equal to about 20 per cent.

Arch Dams.—The assumptions necessary to derive a simple formula for specific capacity for gravity dams have been seen in Table 7 to lead to only approximate results. In the case of arch dams the variables are even more numerous, and the degree of approximation, therefore, is even less close. Let $\phi = 90°$, $n = 1.50$, $h_1 = 200$, and $H = 220$. Then, as before, Equations (2) and (14) may be combined to form an equation for specific capacity, in acre-feet per cubic yard, as follows:

$$w_{SA} = 30\ 000 \frac{C_W}{{C_A}^2} {h_1}^{m-2n} \quad \text{(28)}$$

This formula is represented graphically in Fig. 15, and a comparison of results obtained by its use with the actual values of specific capacity is shown in Table 7, Items Nos. 11 to 15, inclusive. Unfortunately, the number of arch dams for which particulars are available regarding both the storage and the site is rather limited. Table 7, therefore, shows also the values of specific capacity for certain other sites in which dams of the arch type might be constructed (see Items Nos. 11 and 13). The values for Items Nos. 11 to 15, inclusive, in Column (12), were corrected for stress only.

Constant-Angle Arch Dams.—Let $n = 1.75$, $h_1 = 210$ ft., and $H = 230$ ft. Then, as before, the expression for specific capacity for a constant-angle dam may be written:

$$w_{SCA} = 38\ 000 \frac{C_W}{{C_A}^2} {h_1}^{m-2n} \quad \text{(29)}$$

This formula is represented in Fig. 16 and the results obtained are given in Table 7, Items Nos. 16 to 21, inclusive.

It is also interesting to compare values of the specific storage for actual dams which have been built for different purposes, irrespective of the type used. In Figs. 17 and 18 only masonry dams have been included, since for dams built of this material the relative costs of storage can be compared roughly on the same basis. These diagrams may be taken as showing broadly values of the specific capacity which have been found economic for the purposes named, although naturally a number of local factors will alter the economic limits in each particular case. The upper and lower limits for specific storage are such that, for storages of 10 000 to 100 000 acre-ft., a much lower specific storage (and, hence, higher unit cost) is permissible for water supply purposes than for irrigation or power.

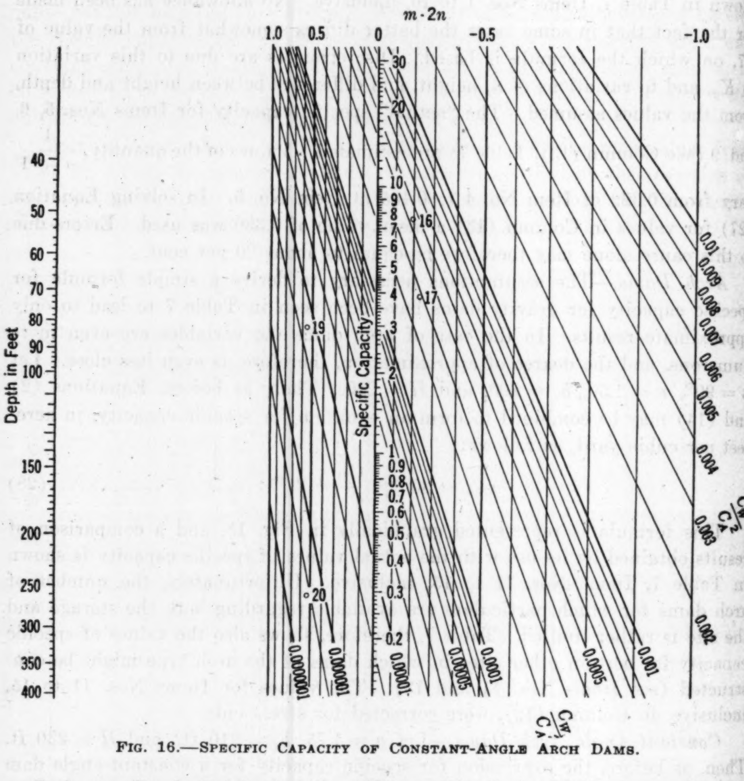


FIG. 16.—SPECIFIC CAPACITY OF CONSTANT-ANGLE ARCH DAMS.

It is usually desirable that "specific capacity," should increase with the height of dam; in other words, that, for gravity dams, m should exceed $n + 1$, and, for arch dams, m should exceed $2n$; but in practice this is quite frequently not so, as will be seen from the values given in Tables 1 and 2. When these conditions are not fulfilled, it would theoretically be cheaper to build two or more dams of less height instead of one; but suitable sites are generally lacking.

AVERAGE COST OF STORAGE

A number of writers have quoted figures for the cost per acre-foot of various storages. Thus, Wilson[29] quotes unit costs for a large number of storages, mainly for irrigation, in different countries. The reservoirs are arranged in order of increasing magnitude, and with increasing capacity the unit cost tends to fall. Exceptions are due to the reservoirs having been constructed in different countries, at different times, and under a variety of different conditions.

---

[29] "Manual of Irrigation Engineering," 1893, Table XI.

A better method of stating "unit costs" is by means of the volume of dam per acre-foot of storage. Actually, the cost of a dam is not strictly proportional to the volume, but it is approximately so. By this means, the physical merits of various storages may be compared, irrespective of the time at which they were completed or the economic conditions of the country.

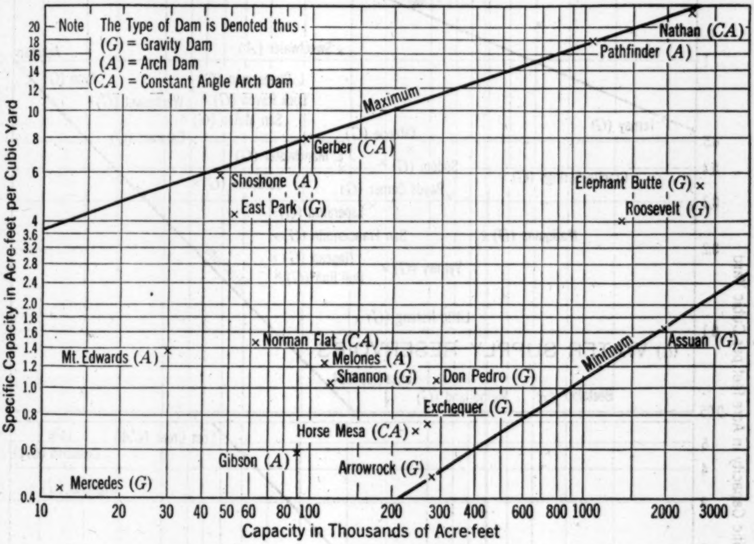


FIG. 17.—COMPARISON OF SPECIFIC CAPACITY FOR DAMS BUILT FOR IRRIGATION PURPOSES.

The various types of dam, such as masonry, earth-fill, and rock-fill, are then comparable in each group, but not between groups without introducing factors depending on the relative costs of the materials of construction.

CONCLUSION

It is the writer's hope that this outline, although "sketchy" in parts, will point the way to a more scientific treatment of the many aspects of water conservation. Some at least of the factors which have been treated in the past by rule-of-thumb or trial-and-error methods have here been shown to be susceptible of graphical or mathematical treatment.

Many agencies are engaged in taking stock of water resources in various countries, and the use of the methods outlined will enable development to be planned in the most economical manner.

ACKNOWLEDGMENTS

The writer wishes to acknowledge the use of information culled from many sources. Some of these are cited in footnotes, but much matter noted during

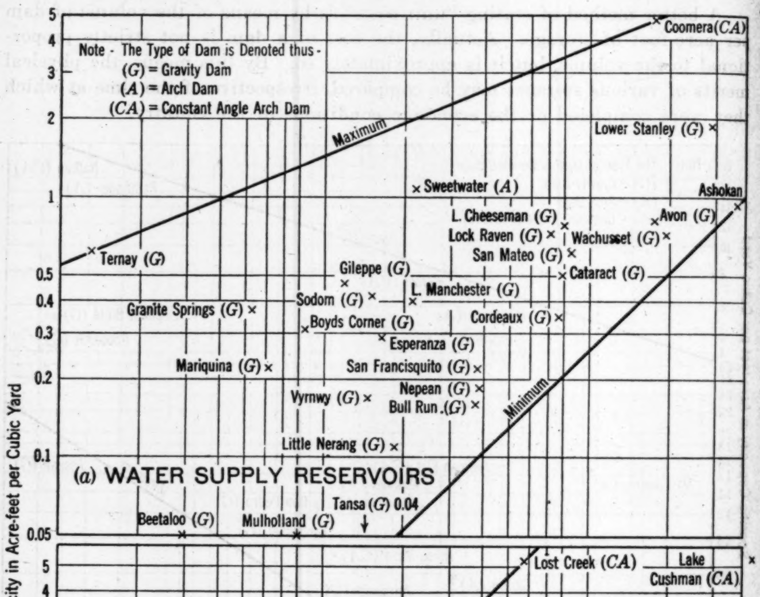


FIG. 18.—COMPARISON OF SPECIFIC CAPACITY FOR DAMS BUILT FOR WATER SUPPLY AND POWER RESERVOIRS.

the course of his professional work, from current and past technical literature, from information privately supplied by friends, notably W. H. R. Nimmo, M. Am. Soc. C. E., and from other sources, has been incorporated.

He also wishes to thank A. F. Partridge, Commissioner for Irrigation and Water Supply, Queensland, for permission to use information relating to proposals in Queensland, and R. J. Spowart, Jun. Am. Soc. C. E., for much valuable assistance in the preparation of diagrams and with the computations.

DISCUSSION

E. C. Eaton,[30] **M. Am. Soc. C. E. (by letter).**—The writer has read with interest and benefit the excellent paper by Mr. Sutherland. The subject is particularly applicable to many of the problems facing engineers in Los Angeles County, California, where lack of water is the only limitation to expansion and prosperity, and where regulation and conservation by dams of various types are absolute necessities.

California is known generally in the East as an arid or at least a semi-arid State, while the truth is that the State enjoys within itself many unparalleled possibilities for water conservation, the development of which has hardly been scratched on the surface.

Take Los Angeles County as an example; it consists of a broad sloping valley area with a background of high mountains roughly equal to the valley area, from which ages of erosion have built deltas to known depths of more than 1 000 ft. These tremendous cones of gravel, sand, cobbles, and large boulders, forming the underlying strata of practically the entire valley areas, have been built by erosion of the many violent stream flows from more than one hundred and fifty separate water-sheds in the County, each with its individual stream channel debouching on the plains. Many of these watersheds are small, more than 10% of them being individually less than 3 sq. miles in area, while the largest is only 222 sq. miles in total area. The annual fluctuation of stream flows is great, ranging from 8 to 358% of normal in the thirty-five years for which authentic records are available.

The low average annual rainfall (about 21.1 in.) has been insufficient to wash off completely the heavy cover of disintegrated loose material which, were the rainfall greater, would have already been washed down, lowering the mountain areas and correspondingly building up the valley areas. This loose material of disintegrated granite is held together solely by the root growth of the mountain vegetation, consisting of grasses, chaparral, and some timber. During eight months of the year rain is practically unknown and during this period the fire risk is imminent, requiring a most thorough system of policing and fire patrols by Federal and County agencies.

Fires leave the mountain areas without the binders of brush, and even moderate storms following such fires cause a heavy débris flow on to the valleys. A storm having a duration of 10 min. and an intensity of less than 2 in. in 1 hour has been known to bring down as much as 25 000 cu. yd. of débris from a single square mile of water-shed.

The tremendous area and great depth of the gravel cones are in reality of the greatest benefit to this southern County. These gravel areas, sloping from the base of the foothills to the ocean, are cut by dikes of clay and impervious materials which by damming the underflow to the ocean form a series of underground reservoirs, fed partly by the water issuing from the mountains and partly by the direct valley rainfall. These underground reservoirs are responsible for the entire early development of the County and supply two-thirds of the water used in the County to-day. By the construction of the impermeable covering of the valley, modern development of buildings and roads has not only deprived the underground reservoirs of the natural replenishment, but has increased the rate of run-off.

---

[30] Chf. Engr., Los Angeles County Flood Control Dist., Los Angeles, Calif.

The keynote to the flood protection and conservation of waters in this County lies in a series of regulating reservoirs in the mountains ironing out the peak flows to a rate that will not be of a destructive nature, but will be such that the gravels may absorb the stream flow and cause it to pass into the underground basins.

Twelve of these structures have been built to date (1930) and twice this number will ultimately be required ranging from almost every known type of arch dam, including the highest dam in the world (the Pacoima Dam), to earth, rock-fill, gravel-fill, and gravity concrete dams.

The remaining dam sites in Los Angeles County mountain territory are relatively costly and of small capacity when compared to storage sites in many other localities, the cheapest mountain dam built by the Flood Control District costing about $60 per acre-ft., a few being as much as $500 and more, with an average of more than $100. Many "off stream" storages are available, however, ranging from $20 to $50 per acre-ft. of storage space, some of which have been constructed and many more of which are still to be built.

While these costs may appear high the reservoirs are not, in actual practice, used for direct storage. They are merely used for regulation or "ironing out" of peak flows down to a rate such that they can be absorbed by spreading grounds below. These spreading grounds are built over pervious areas overlying the underground reservoirs of which the capacity for water is enormous compared with surface storages. Each of the flood-control regulating dams, therefore, may regulate several times its total capacity in a single year.

In its search for additional regulating reservoirs which are recognized as necessary for the fullest development of the underground reservoirs, the Los Angeles County Flood Control District has investigated scores of dam and reservoir sites, and has felt the need of some satisfactory short-cut method of comparing the capacities and approximate costs of a number of possible sites. The methods proposed by Mr. Sutherland, therefore, have been studied with great interest and tested to a limited extent as to their application to local conditions.

Mr. Sutherland sets forth a method by which it is possible to compare quickly the merits of various locations and types of dams as regards cost per unit of storage capacity alone. This applies equally well to all uses, whether conservation, regulation for flood control, or power, or any other use for which water is stored. The amount by which the results attained by this approximate method differs from those by the more exact and slower methods depends upon the variation from a straight line of the capacity curve platted on logarithmic paper and upon the judgment used in establishing a dam site cross-section which is equivalent to the true cross-section. The method appears to be quite satisfactory in the case of large reservoirs.

In the case of relatively small reservoirs, requiring high dams, such as those built by the Los Angeles County Flood Control District, the errors introduced by the approximations are likely to be relatively large. Mr. Sutherland cites three uses for the alignment chart, Fig. 4. Puddingstone Reservoir, which is the only County Flood Control Reservoir constructed to date that falls within the limits of the chart, was studied from the standpoint of Use No. 1. Adopting the value of m determined by a depth of water of 132 ft. at the deepest point, the capacities corresponding to various depths as shown by the alignment chart and by the ordinary capacity curve are as given in Table 8 for comparison.

TABLE 8.—COMPARISON OF RESERVOIR CAPACITIES

Maximum depth of water, in feet	From alignment chart, Fig. 4, in acre-feet	From capacity curve, in acre-feet	Variation, percentage
57	920	1 056	12.9
82	3 493	3 639	4.0
107	9 267	9 406	1.5
132	20 012	20 012	0

The variation results from the fact that for any of the reservoirs of the District, the value of m varies when different depths of water are used in its determination.

Many considerations determine the justified expenditure for a dam. The benefits to be derived from, and the cost of, the project as a whole are the determining factors. The cost of channels below flood-control dams depends on the degree of control effected by the reservoir. The justified total cost for the system depends on the amount of damage prevented through its operations. It is estimated that without protection, $50 000 000 of direct and indirect damage would result from a flood on the San Gabriel River equal to that which occurred in 1914. It is obvious that a reservoir on this river would be in reality of greater value than a reservoir on some other stream on which little or no damage would result, but which was classified as excellent. From the conservation standpoint the cost per unit of water delivered to the consumer is the determining factor. Systems designed for conservation may be beneficial in two ways:

1.—By causing the retention for use of water which would otherwise be wasted; and,

2.—By effecting a decrease in cost of delivery to consumers through holding the water at more favorable locations until such time as it is to be used.

The expense justified in carrying out the first of these items depends upon the use to which the water is to be put and the value of crops which it is possible to raise under the existing climatic and soil conditions. In general, it might be stated that the least cost at which water can be supplied in sufficient quantity to the entire area under consideration is the factor which fixes the cost justified in any case. Thus, in the case of domestic supply, a reservoir costing much more per acre-foot stored might be more necessary and, therefore, more valuable than a cheaper reservoir supplying some area where reservoir sites are plentiful and water is easy to obtain.

In the area covered by the Los Angeles County Flood Control District, the storage capacity available is limited, not only by the general steepness of the topography, but by the fact that foundation conditions preclude the use of many dam sites. This limited storage must be used for two purposes, flood control and conservation. In many cases the total storage space available is insufficient to control the stream completely. The storage required for economical control is of greater importance than the cost of this storage per acre-foot.

The foregoing shows that reservoir sites can not be classified as excellent, good, average, or bad, depending upon their cost per acre-foot of capacity. However, where other conditions are equal, two or more dam sites can be compared and one can be shown to be better in this respect than the others. Mr. Sutherland states that these other factors must be considered, but their importance far outweighs that of cost of storage per acre-foot, in the case of flood-control dams in an area of high productivity, such as this one.

Nevertheless, a comparison of capacities and costs of possible sites, between two or more dam sites has often been found necessary here, even though the unit capacity costs of such sites are so high as to eliminate their consideration in other parts of the United States where conditions are more favorable.

In general, to make such a comparison, it is desirable to answer either or both of two questions, other things being equal:

(*a*) Of two or more dam sites the reservoirs of which overlap and which serve about the same water-shed, which will be most economical?

(*b*) Where two or more small reservoirs compare with one larger reservoir for the same water-shed, what combination will be the most economical?

The following short-cut method has been developed by the Los Angeles County Flood Control District, largely through the efforts of C. B. Gibbons, Designing Engineer, and has proved very satisfactory for preliminary studies, serving to eliminate from further field investigation all the less advantageous sites. The method may be described in fifteen successive steps as follows:

1.—A topographic map is prepared. Aerial surveys give results close enough for such investigation, and have been used extensively.

2.—The topographic map is divided into zones along the stream bed by vertical planes (sometimes broken in plan), cutting along every important ridge and roughly segregating portions of the reservoirs between successive bottle-necks or narrows. (See Fig. 19.)

3.—For a distance of 300 to 400 ft. vertically above the stream bed, each contour, by zones, is measured by a planimeter.

4.—Beginning at the point at which each contour crosses the stream bed, the partial areas within this contour are cumulatively totaled for each zone, so that at each bottle-neck or narrows, the relation between depth of water and reservoir area is determined.

5.—Beginning at the stream bed at each bottle-neck or narrows, the cumulative areas are cumulatively totaled, and these totals, multiplied by the contour interval, give the usual data for a reservoir capacity curve at each such point.

6.—To serve as abscissas in Fig. 20 (which is described in Step 8), distances along the stream bed are roughly measured, and both bottle-necks and contours noted.

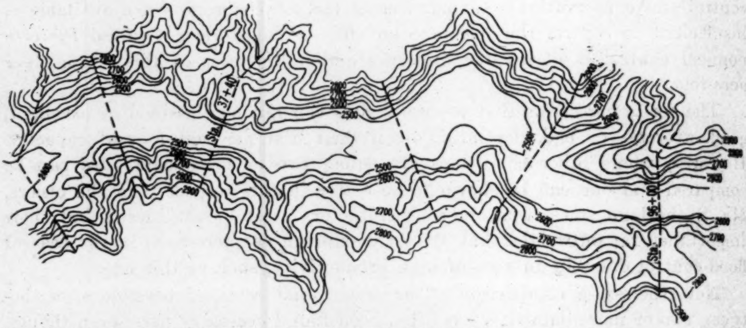


FIG. 19.—EXAMPLE OF TOPOGRAPHICAL MAP FOR USE IN COMPARING CAPACITIES AND COSTS OF DAM SITES.

7.—Using stream-bed distances as abscissas and acre-feet of reservoir capacity as ordinates, graphs are drawn (to 300 or 400 ft. above stream bed) for each contour measured. (See solid lines in Fig. 20.)

8.—From the reservoir capacity curves thus drawn and the points at which each contour crosses the stream bed, other graphs are drawn for each height of dam (in even multiples of the contour interval). (See broken-line curves in Fig. 20.)

9.—Graphs such as Fig. 20 portray continuously, over the range being investigated, the relation between reservoir capacity and height of dam. This graph has been called a capacity profile. By inspection, certain narrows or bottle-necks may be selected, which are favorable because the capacity is large as compared with the cross-section.

10.—At such selected points, a cross-section of the canyon is drawn, and for different heights of dam, a trapezoid is constructed, which roughly averages the cross-section. Increasing weight must be given to the lower portions where the dam is thicker. A reasonable amount of stripping or excavation must be assumed.

11.—Prior to field inspection and geological study, it may be assumed that these sites are equally feasible. Some type of structures, such as a rock-fill or concrete gravity dam, is then estimated at each site to various elevations, care being taken to allow for increased average width at the stream bed where higher dams straddle a rounded nose.

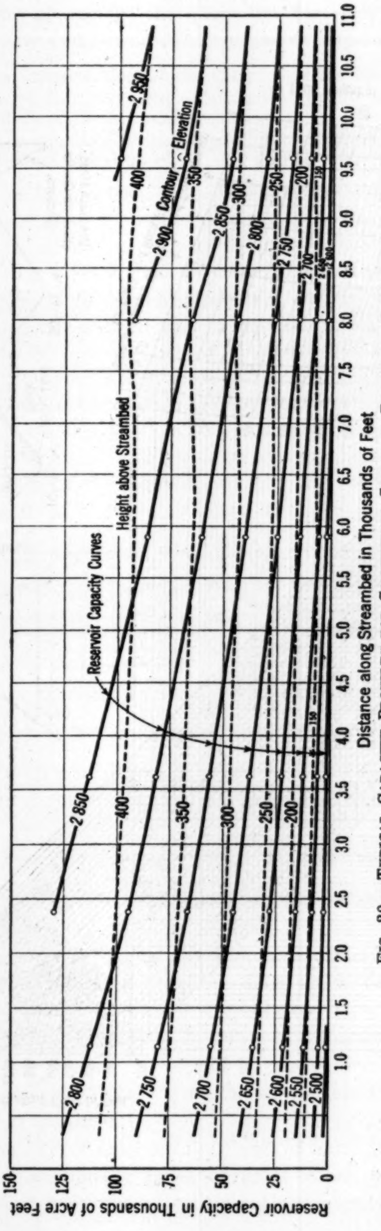


FIG. 20.—TYPICAL CAPACITY PROFILES FOR CONTINUOUS STUDY OF RESERVOIR CAPACITY.

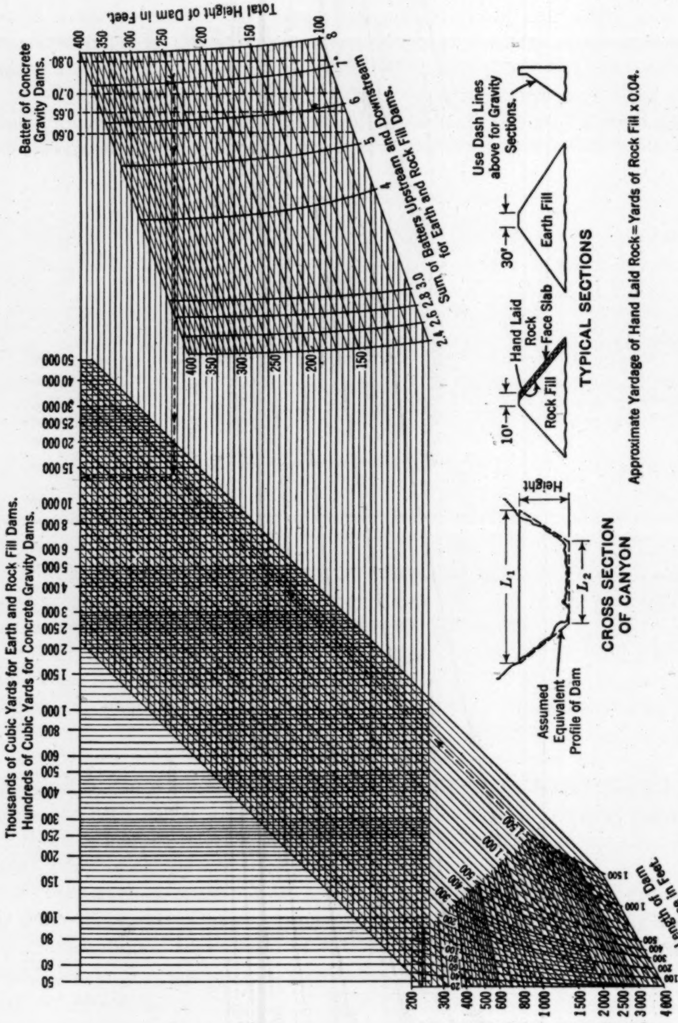


FIG. 21.—DIAGRAM FOR ESTIMATING DAMS.

12.—By the use of Fig. 21 the quantities for various heights of dam at each feasible site are computed. The successive steps are:

(*a*) Draw a cross-section of the canyon on the axis of the dam.
(*b*) Construct an equivalent straight-line profile of the dam, neglecting small deviations near the top.
(*c*) Find the yardage at the top of the diagram for the given height and assumed values of L_1, L_2, and batter.

For example, let $L_1 = 2\,000$ ft.; $L_2 = 1\,400$ ft.; height = 250 ft.; and the batters, 4 on 1 and 3 on 1 (total 7). Following the dotted lines, for this example in Fig. 21, a volume of 13 400 000 cu. yd. is indicated.

13.—By assigning a reasonable unit cost, the cost per acre-foot (at intervals of height equal to the contour interval) may be determined. For purposes of comparison the costs in Fig. 22 are estimated for gravity dams with a 10 on 8 batter at $8 gross per yd. of concrete. The depth of excavation is assumed as 50 ft. Quantities are estimated from Fig. 21.

14.—From the foregoing data assuming that the total reservoir capacity has been determined, a schedule can be prepared which makes possible a study of the various combinations of high dams and low dams at all feasible sites, with a view to obtaining the requisite capacity for the minimum total expenditure.

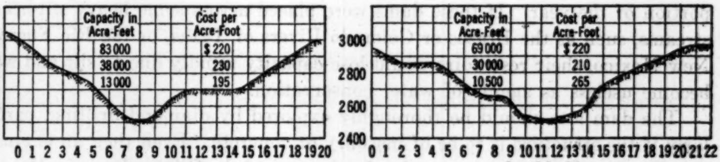


FIG. 22.—COMPARISON OF COSTS PER ACRE-FOOT AT VARIOUS HEIGHTS.

15.—Field inspection and geological study of the obviously desirable sites or combinations may reveal a wide discrepancy due to either:

(*a*) The availability of an arch dam;
(*b*) A considerable difference in the requisite excavation or stripping; or,
(*c*) The existence of faults which exclude all except a more expensive flexible dam.

In the latter case a revision of the comparative cost figures at the different sites may be made, retaining only the data which correlate capacity with height of dam.

The foregoing method has been described as an alternative means of arriving at the same objective as that sought by Mr. Sutherland's methods, and is felt to be more applicable perhaps to the special conditions existing in Los Angeles County as outlined.

Mr. Sutherland's paper has been the object of much interest in Los Angeles County and will undoubtedly prove of great value, particularly where the method is applied to reservoirs of considerable size and uniform cross-section.

A. W. SIMONDS,[n] Assoc. M. Soc. C. E. (by letter).—This paper briefly covers a great many points of interest in the problem of water conservation, indicating that the author has spent much time in the study of this subject. Water conservation is a geologic as well as an engineering problem. Too much emphasis can not be placed on this phase, particularly where a failure would involve the loss of considerable life and property. The engineer who disregards the geologic factors may invite disaster to his work. Some glaring examples of this are the failures of the Colorado River Dam at Austin, Tex., and the St. Francis Dam in California. The Colorado River Dam was built across a badly shattered fault zone which was inadequately excavated and not grouted. The result was that a section of the dam over the fault zone slid out. Some years later the dam was repaired, but as a storage reservoir the project was a failure, owing to the rapid loss of capacity due to silt deposition. If the services of a competent geologist or an engineer with a knowledge of geology had been engaged before the dam was built, these results might have been foreseen.

Successful projects in one locality might be failures elsewhere, due to different geologic conditions. Such dams as the Cheesman and Barker Dams in Colorado and the Gibson Dam in Montana are successful as a means of conserving water in their respective localities. They are situated high in the Rocky Mountains where their reservoirs are fed with clear water for the greater portion of the year. If these dams were placed across some heavy silt-bearing streams, such as the Brazos or Colorado Rivers of Texas, or the Rio Puerco of New Mexico, their reservoirs would lose capacity rapidly due to silt and would become useless as agents of water conservation.

The dam site should be thoroughly explored by diamond drilling, particularly if the structure is to be of masonry or concrete. Even then an engineer's eyes cannot see very far into rock. In the case of the Gibson Dam, in Montana, sub-surface exploration was made by drilling. In excavating for the structure, it was found unnecessary to cut as deep at the crown section as was originally intended, but at a point about half way between the crown section and the north abutment, it was necessary to cut deeper than the original line of assumed excavation.

The curves for submerged area (Fig. 6), site area (Figs. 7 and 8), and volume (Figs. 10 and 13), are interesting, but are of doubtful practical value. Dams and reservoirs are expensive structures, and their cost warrants a careful engineering investigation. A topographic survey of the reservoir and dam site is one of the first requirements. A carefully made topographical map will be found much more reliable for computing volumes in dams than the use of curves. This is particularly true in the case of the Deadwood Dam in Idaho. If the location of this dam were to be shifted a short distance either up stream or down stream its volume would be materially changed.

In connection with Table 2, it is of interest to note that the engineer's pre-construction estimate of the volume of Gibson Dam (Item 25) was 160 000 cu. yd. The contractor's final estimate for payment was 160 700 cu. yd.

---

[n] Assoc. Engr., U. S. Bureau of Reclamation, Denver, Colo.

W. H. R. NIMMO,[32] M. AM. SOC. C. E. (by letter).—The part of this paper dealing with reservoir capacities is based on the fact (which has been demonstrated by Hillman[33]) that the capacity of a reservoir at any level may be expressed very closely as an exponential function of the depth of water, and it is of interest to inquire why this should be so.

The size of the particles which a stream can transport depends to a considerable extent on their shape and the material of which they are composed, but if they were all spherical and of the same material, their diameter would vary as the square of the velocity of the stream and their volume as the sixth power of the velocity.[34] If it is assumed that all particles are spherical then the force exerted by the stream on a stationary particle is proportional to the change of momentum in that portion of the stream passing the particle and varies as the square of the mean diameter, D_p, of the particle and the square of the velocity, V, of the water, that is,

$$F = k\, D_p^2\, V^2 \qquad (30)$$

The frictional resistance due to the weight of the particle varies as the cube of the diameter so that, when the particle is about to move,

$$F = f\, D_p^3 \qquad (31)$$

Equating Formulas (30) and (31),

$$V^2 = \text{constant} \times D_p \qquad (32)$$

but from the Chezy formula,

$$V^2 = \text{constant} \times r\,s \qquad (33)$$

therefore, $r\,s$ varies as D_p and, if the hydraulic radius does not vary greatly, the slope, s, will vary as D_p.

If it is assumed that the size of particles increases uniformly in an up-stream direction, then the slope must also increase at a uniform rate which is a property of a parabola. The foregoing appears to provide a physical explanation for the observed fact that the longitudinal section of a stream tends to be parabolic throughout most of its length.

Considering the case of the denudation of a slightly sloping block of land, such as a plain ($k\,m$) uplifted above the sea, as represented in Fig. 23, it will be noted that, at first, there can be no permanent stream, and denudation will take place only by weathering and the intermittent removal of material when rain falls. Gradually, run-off from storms will collect into runnels and, finally, into consequent streams running in the direction of the slope. Once formed, these streams will increase in volume and velocity as they proceed. Therefore, their erosive power will increase also, and they will steepen their beds to a convex curve such as $a\,b$ (Fig. 23). The stream will continue to erode until a narrow valley has been cut back to the position, $c\,d$; and the process will tend to continue to the position, $e\,f$ (Fig. 23), etc., back to the water-shed.

---

[32] Designing Engr., Main Roads Comm., Brisbane, Queensland, Australia.

[33] "The Capacities and Capacity Equations of Some Western Australian Dam Sites," *Transactions*, Inst. of Engrs., Australia, Vol. II, 1921.

[34] "Physical Geography," by P. Lake, Cambridge Univ. Press, 1925.

Erosion, however, cannot take place below the sea level at *c* and, having reached this point, the stream will commence to deposit material. At any point, such as *e,* erosion cannot take place below a level determined by the slope necessary to maintain a velocity that will transport the load which the stream is carrying. Consequently, a concave curve, such as *c e g,* will now be superimposed on the previous convex curve and the line of the valley will be eroded in gradual stages back to the position, *c e h k,* the portion, *c e h,* being concave and parabolic and the portion from *h* to the water-shed at *k* being convex. The term, concave, is used relative to the surface of the earth, which is considered to be flat over a limited area. Usually, the bed of a river will not be concave to the sky. The concave curve cannot proceed beyond the actual source of the stream, which will usually be a spring at about the point, *h,* since above this point there will not be sufficient accumulation of ground-water to maintain a flow during the interval between storms. Therefore, denudation here will proceed by weathering and the water-shed will usually remained rounded.[35]

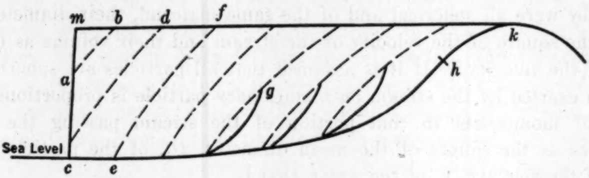


FIG. 23.

A young stream which is eroding the bed of a narrow gorge near the head of its drainage area will usually be fully loaded with detritus. It will not also be eroding the sides of the valley which will be of a convex shape. The shape of a storage basin in such a locality will be similar to that shown in Fig. 24. Adopting the author's notation as far as possible, the area of any typical cross-section, such as *b c d,* is given by,

$$a = \int l\,dh = \int D\,h^{v}\,dh = \frac{D}{v}\,h^{v+1} = C_A\,h^{n} \dots\dots\dots\dots (34)$$

and the capacity of the reservoir will be,

$$w = \int a\,d\,l' = \frac{D}{v}\int h^{v+1}\,d\,(P\,h^{z}) = \frac{D\,P}{v}\int h^{v+z}\,dh = \frac{D\,P}{v\,(v+z)}\,h^{v+z+1}$$

$$w = C\,w\,h^{m} \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots (35)$$

Equations (34) and (35) are identical with Equations (1) and (2).

As the author points out, the value of *n* for a convex valley, such as Fig. 24, will be greater than 2 and, therefore, the value of *v* will be greater than unity. Since dams are usually built at control points such as *A, B,* or *C,* of Fig. 2, the longitudinal section of the stream may be considered to be parabolic and the value of *z,* therefore, will be 2. Hence, the value of $m = v + z + 1$ will be greater than 4; values of *m* greater than 4.5 do not seem to occur. In a previous discussion of this subject before the Institution of Engineers (Australia), the writer suggested the classification of reservoirs into gorge, hill, foothill, flood-plain, and lake types according to their values of m. The author has developed the idea in rather more detail, combining the flood-plain and foothill classes into one type, and adopting a definite range of values of m for each type, and his classification appears to be satisfactory.

---

[35] See, also, "The Curve of Stream Erosion," by John Challinor, *The Geological Magazine,* Lond., Vol. LXVII, No. 2, February, 1930.

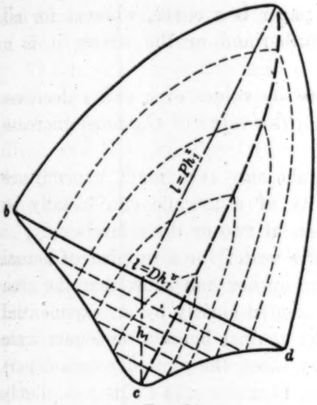


FIG. 24.

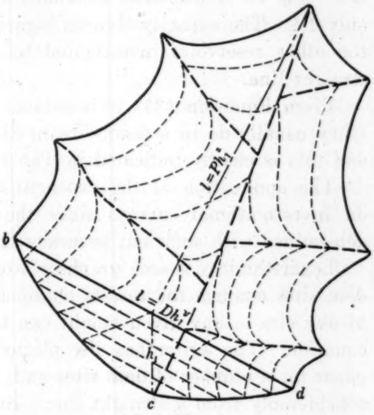


FIG. 25.

If a reservoir is located some distance from the water-shed, but in hilly country, there will be a larger flow of water to transport eroded material than in the foregoing example and the stream will have cut lateral valleys of parabolic section. Generally, the lower portion of the sides of the main valley will have become parabolic and concave although the actual water-shed will still be convex, so that the value of v will be 2. The longitudinal section of the main stream will now be on a flattened portion of the parabola so that z will be nearly unity. Such a reservoir is illustrated in Fig. 25 and its value of m would be about 4. Following down the main stream still farther, the lateral valleys will also become flattened, and the valley will be V-shaped with $v = 1$, $z = 1$, and $m = 3$, corresponding to the author's hill type. Still farther down stream the grade of the main stream may be so slight that z approaches zero while v may still be nearly unity, and $m = 2$, corresponding to the author's flood-plain and foothill type. For the lake type the floor is approximately flat in all directions and for large shallow lakes both v and z will be nearly zero, so that m will be rather greater than unity, but it can never be less than unity. The lowest value of m thus far encountered by the writer was 1.04 for the Great Lake, in Tasmania.

A true U- or box-shaped canyon having a nearly flat floor and nearly vertical sides may occur at any portion of a stream where it has been forced by uplift to erode its channel through hard horizontally bedded sedimentary rocks. In such a case, the value of v will be small and that of z may approach 2, so that m will probably be between 3 and 3.5.

The writer suggests the addition to the classification of a compound type to cover such reservoirs as Lake Cushman, Washington, which for the downstream half of its length consists of the narrow gorge of the Skokomish River and for the up-stream half of the expanse of Lake Cushman itself. For the lower portion of the capacity, water is stored in the gorge only and $m = 4.09$. As the water level rises, a greater percentage of the storage is in the lake and the value of m decreases gradually until for the upper 100 ft. its value is only 1.4. The capacity line on logarithmic paper is a curve, whereas for all the other reservoirs investigated by Mr. Sutherland or the writer it is a straight line.

From Equation (35) it is evident that as the values of v and z decrease (they usually do in a down-stream direction), the value of C_w must increase and this is clearly indicated in Table 1.

The application of the exponential formula may save much expenditure on investigational surveys since the capacity of a storage can usually be determined with sufficient accuracy from areas at two or three levels only.

Logarithmic site-area graphs plotted by the writer for a number of actual dam sites confirm the results obtained by the author and show that the area of any site to any given height can be represented closely by an exponential equation. The writer has also plotted widths against heights on logarithmic paper for a number of dam sites and, in many cases, the plotted points depart considerably from a straight line. Evidently, Equation (7) holds less closely than Equation (1) from which it is derived and, since the author's formulas for the volume of dams are based upon Equation (7), it might be expected that they would yield results which were considerably in error.

The writer computed the volume of a number of existing or proposed gravity dams by means of the formula given by Mr. Sutherland and compared the result with the volume calculated by the usual methods. It was found that, for sites which were fairly symmetrical, the formula gave quantities agreeing closely with the actual quantities. Even for decidedly unsymmetrical sites the formula gave results which were sufficiently accurate for preliminary estimates, since the error did not exceed that usually present owing to uncertainty as to the amount of foundation excavation which will be necessary. For earth or rock-fill dams, the formulas will also give satisfactory results.

The formulas for the volume of constant-angle arch dams (Equation (26)) involve the assumption that the contours of the foundation are both straight and parallel to the axis of the valley, or that the line, on which the cross-section is taken and from which the site-area equation is derived, is such that the effect of convergence or divergence of the contours is neutralized. Some of the most favorable sites for constant-angle arch dams exhibit considerable convergence of the contours and for such sites errors exceeding 30% in the computed volume of the dam may occur if the site-area equation is determined from a single cross-section taken along a chord. The composite section which is now proposed by the author will usually eliminate most of the convergence error.

The factor, $\dfrac{\phi}{2 \sin \dfrac{\phi}{2}}$, represents the ratio of the arc to the chord. The writer computed the ratio at the crest of fourteen dams and found that the value varied from 1.40 for the Pathfinder Dam to 1.015 for the Upper Otay Dam. He concluded that a constant factor of 1.05 would not be suitable for all dams. The factors given by the author in Table 3 vary with the value of n and the value of the crest angle, and their introduction will eliminate much of the error due to the use of a constant factor.

The author's equations also involve the assumptions that the radius varies uniformly from crest to base and that the subtended angle and the cylinder stress are constant from crest to base. For the Warragamba, Pacoima, King River, and Bullards Bar Dams, the writer has found by plotting curves that the value of the radius does vary almost uniformly from crest to base, and that the subtended angle varies greatly near the base. In high dams it is only approximately constant for the upper half of the dam. For the Warragamba and Pacoima Dams, the sites of which are fairly symmetrical, the cylinder stress is approximately constant at all levels; but for the Bullard's Bar, Salmon Creek, King River, and Lake Cushman Dams, it varies by as much as 50 per cent.

The site of the proposed Warragamba Dam is a straight gorge cut in sandstone, and the contours are practically straight lines parallel to the river. The computed volume of 261 000 cu. yd., given in Table 2, is for a preliminary design of a constant-angle arch dam made some years ago by the writer for this site, by means of the cylinder formula. The site is unusually symmetrical, and the probable depth of foundation excavation was allowed for in determining the site-area equation. The radii vary uniformly from crest to base, and the cylinder stress is unusually constant throughout. The actual volume calculated by ordinary methods is 284 000 cu. yd., so that the result given by the formula is 8% too small. The subtended angle is not constant, and the site-area equation was obtained from a section along a chord, instead of along the arc, and the error must arise from these two causes since all the other conditions required by the formula were satisfied.

Notwithstanding the various assumptions on which the author's formula for the volume of constant-angle arch dams is based, it appears from practical applications that in most cases the formula will give a quantity that is sufficiently near the true volume to be useful in making preliminary investigations, but the results may be expected to be usually less accurate than those obtained by the formula for the volume of gravity dams.

With the exception of the Gibson Dam, the actual volume of which is not shown, Table 2 does not include quantities for any dams designed by the trial-load or similar methods, or for any of the recently proposed high dams in which high compressive stresses are adopted. There is, therefore, no evidence to show what error would be involved in computing the volume of such dams by the author's formulas. The volume of the earlier arch dams, designed before the publication of the Cain formulas, are now of little practical interest.

A V-shaped valley, for which $n = 2$, is the most favorable for a constant-angle arch since the subtended angle can be kept approximately constant. For a valley with flat floor and vertical sides, $n = 1$, and a constant-angle arch becomes identical with a constant-radius arch.

For any valley with sloping sides (no matter how steeply), the constant-angle type of arch will contain less material than the constant-radius type. The comparison of specific capacities for different purposes in Figs. 17 and 18 is very instructive.

The writer has read this paper with much interest, and its value for practical use is much enhanced by the addition of the alignment charts.

C. R. OLBERG,[38] M. AM. SOC. C. E. (by letter).—The author has performed a distinct service to the profession in calling attention to the use of logarithms as providing a means of comparison between reservoir sites and dam sites, respectively. As Mr. Sutherland advises, the quantities derived from his methods cannot be used as a basis for final estimate, but they promise to be invaluable for the purpose of preliminary estimates and they also afford a method for direct comparison between various dam sites, and between different types of dams of various heights of the same site, with a minimum of work.

It is a definite achievement, since heretofore this was not possible, without days or even weeks of arduous labor, as those who have to pass on the relative excellence of proposed dam sites, or the respective merits of various types of dam at the same site, are well aware.

Most of the items considered are clearly and ably presented, but the author himself confesses that through the necessity of reducing the length of his original paper, the discussion of some of the subjects is somewhat "sketchy". It is to be regretted that Mr. Sutherland felt constrained to limit his discussion on "economic installation" to references to previous publications, which unfortunately are not readily available.

Reservoir Sites.—It has long been observed that capacity and area curves of reservoirs, follow some form of exponential curve, but it has remained for the author to demonstrate this fact in the application of logarithms to determine the equation.

The general form of an exponential curve in the nomenclature of the author (see Equation (2)) is $w = C_w h^m$.

Passing to logarithms,

$$\log w = \log C_w + m \log h \qquad (36)$$

Obviously, this is the equation of a straight line, in which, m is the tangent of the slope and $\log C_w$, the intercept on the y-axis.

Regarding a reservoir, if the logarithms of corresponding depths and capacities are plotted on co-ordinate paper with equal horizontal and vertical scales, it will be found that they will be nearly on a straight line. The straight line that best satisfies the points is the logarithmic equation, from which the algebraic formula, Equation (2), may readily be written.

---

[38] Tiflis, Union of Socialistic Soviet Republics.

Since two points are required to determine a straight line, the capacities corresponding to two depths are all that is necessary to obtain the approximate capacity of a reservoir at any other depth. However, three or more points will permit a better adjustment of the theoretical to the actual curve. As pointed out by the author, it is remarkable how close the theoretical agrees with the actual capacity.

The general equation can readily be used to extend the capacity curve of a reservoir beyond the limit of the original survey, a necessity that sometimes arises. The first derivative of the general equation with respect to h_1 is, $dw = C_w\ m\ h_1^{m-1}\ dh_1$, or, $s = C_w\ m\ h_1^{m-1}$, which is Equation (4) in Mr. Sutherland's paper. From Equation (4) the area of the reservoir, s, at any depth, h_1, may be computed.

The corresponding logarithmic equation is,

$$\log a = \log C_w + \log m - (m - 1) \log h_1 \ldots\ldots\ldots\ldots (37)$$

Equation (37) is also a straight line for which $m + 1$ is the tangent of the slope and $\log C_w + \log m$, the intercept on the y-axis. Equations (4) and (37) may be of assistance in problems of evaporation and seepage losses and, also, in extending the area curve.

The author's classification of reservoirs into types in accordance with the value of m is interesting, but in the writer's opinion this is important only as a means of definition and comparison.

It is not clear what particular means of mathematical legerdemain was used to demonstrate that m is the ratio of the maximum depth to the average depth of the reservoir. The writer must also confess ignorance of the real meaning of the constant, C_w, except that h_1^m may be regarded as a unit of measurement for each particular type of reservoir, and C_w, a scalar number denoting the number of times the quantity is taken as a unit.

The preparation and use of line charts is an ingenious method of determining quickly the values of C_w and m, but the writer believes that there is an advantage in plotting the logarithmic line, in order to permit an adjustment of the theoretical to the actual curve. Possibly the same results may be secured by adjusting the points within the polygon formed by three or more lines on the line chart somewhat analogous to a solution of simple three-point problems in plane-table work.

Dam Sites.—Mr. Sutherland points out that the theoretical area of a dam site may be found in a manner similar to that for determining the theoretical capacity and area curves of a reservoir. To the writer the simplest method of procedure would be to plot the logarithm of the width of the dam site at critical elevations against logarithms of the corresponding depths.

The tangent of the slope of the straight line adjusted through the points will be the exponent, p, of the independent variable, h, and the intercept on the y-axis will be the coefficient, B, in the general equation,

$$w = B\ h^p \ldots\ldots\ldots\ldots\ldots\ldots (38)$$

in which, w is the theoretical width of the dam site at any height, h. Equation (38) is the trace, on a vertical plane, of the theoretical dam site.

Integrating Equation (38) with respect to h,

$$A = p\,B\,h^{p-1} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (39)$$

which is the general area equation.

In Equation (38) letting w equal l, $B = nC_a$, and $p = n - 1$; then $l = n\,C_a\,h^{n-1}$, which appears to be derived by the author directly from the capacity equation for the reservoir (see Equation (7)). Until the relation between reservoir and dam site is more definitely established, the writer feels that this is putting assumption to rather a severe test.

***Dams.*—Equation (9) is proposed by the author for determining the volume of a triangular dam. Substituting for t and l in this equation, their equivalents, $Kg\ (H - h)$ and $n\ C_a\ h^{n-1}$, respectively, and integrating with respect to n, he derives Equation (10) as an expression for the volume of the dam. Substituting for n and C_a in this equation, their equivalents, $p + 1$ and $\frac{pB}{p+1}$, respectively, results in the formula,**

$$V_g = \frac{B\,K_g\,H^{p+2}}{(p+1)\,(p+2)} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (40)$$

in which, the exponent, p, and the coefficient, B, are derived from Equation (39), the integration of the theoretical curve of the dam site. The same result could have been obtained by integrating Equation (39).

If suitable allowances are made for the concrete in the foundations along the base and abutments, and an appropriate quantity is added for crest width, Equation (40) will give a close approximation of the volume in a gravity dam, at the site under consideration.

The author has done a large amount of work and displayed considerable ingenuity in developing approximate formulas for arch and constant angle dams. These two types, with the variable radius dam, are more dependent than others on the shape of the dam site for their stability and volumetric contents, and for this reason cannot be compared directly with a gravity dam.

The proposed formula and the presentation in alignment charts will be of great assistance since it renders this comparison possible, without the labor required to design a series of curved dams for each site. The error involved in the use of the cylinder formula, however, may be considerable, and the writer suggests the possibility of applying a factor to the volume as determined by the cylinder formula that will reduce, if not eliminate, the error.

The volumes of other types of dams, including the nail-head, multiple-arch, multiple-dome, and Ambursen, can be compared almost directly to the volume of a gravity dam of equal height, since they are only slightly affected by the shape of the dam site. It is probable that some of the factors will vary as a power of the height, but the relation should approximate a straight line.

***Specific Capacity.*—The writer believes that a more accurate determination of the "specific capacity", as defined by the author, could be secured by making an adjustment, providing for the clearance of the crest of the dam above the reservoir surface and the depths of the foundations below the bottom of the reservoir, so that the variable, h, of the reservoir would correspond to**

the variable, *h*, of the dam. It would complicate the equations somewhat, but it is not believed the difficulty would be insurmountable.

While the specific capacity will afford a ready means of comparing dams of variable heights of the same type, the reciprocal of the specific capacity (or the number of cubic units of the dam per unit of storage capacity) would afford a better means of comparison between dams of different types.

Costs.—A factor, as suggested by Mr. Sutherland, applied to the various materials will then give a direct comparison of cost between the various types of dams. This information could be plotted on a logarithmic chart, on which the variation in costs of the several types of dams of varying heights, for the same site, could be clearly and concisely shown.

The cost of the accessories of a dam, including spillways, outlet gates, etc., depends directly upon the type selected, and it is evident that the size, which influences the cost, is more a function of the reservoir than the dam. For dams of the same height of different types, the capacities of the spillways and outlet gates will be the same, and if the same type of accessories is used, the cost will be a constant. For different heights of dams of the same type, the capacity of the spillway will vary as some negative power of the depth of reservoir, and the capacity of the outlet gates will remain practically constant.

On account of these varying conditions, it appears to be hopeless to apply a factor to the cost of a dam to determine the cost of the accessories, except as the roughest kind of an approximation. These costs, like the cost of rights of way, must be computed separately and added to the cost of the dam, to give the total cost of the project.

Conclusion.—The form in which the author presents his equations dispels any lingering hope the writer may have had respecting a maximum specific capacity. It was a fond hope that had been long entertained. The economic height must be sought in other relations than those which exist between the height of a dam and storage capacity. The loss in this respect, however, is more than compensated for by the assistance which the data, so ably presented by the author, have been to the work at hand. It must be recognized that the results are approximate, but in an instance under observation they have been close approximations and serve as a firm basis, on which to form an opinion respecting the relative merits of various heights and types of dams for the same site, and to form a means of comparison between various dams and reservoir sites.

JOHN W. PRITCHETT,[37] ASSOC. M. AM. SOC. C. E. (by letter).—The author's estimate of total storage for irrigation already developed or proposed by the U. S. Bureau of Reclamation does not include proposed reservoirs in Texas which have not been investigated by the Federal Reclamation Bureau. The developed storage for irrigation in Texas is more than 1 000 000 acre-ft., not including the Lower Rio Grande Valley, where permits have been granted for the storage of about 275 000 acre-ft. Surveys and estimates have been made of sites that show favorable opportunities for the development of more than 3 500 000 acre-ft. of storage. The storage developed for water power in Texas amounts to about 25 000 acre-ft. Storage being developed for water power, or for which State permits have been granted for water power, are equal to more than 2 500 000 acre-ft. The storage developed or being developed (1930) for municipal water supply equals 900 000 acre-ft. That being developed for flood control in connection with other uses amounts to 500 000 acre-ft. In addition to the foregoing, a preliminary district has been created by the Legislature, looking toward the development of the entire Brazos River water-shed for irrigation, municipal supply, power, and flood control. The area of this water-shed is about 40 000 sq. miles, and the mean annual run-off is 5 500 000 acre-ft. The reservoir sites that have been surveyed in this water-shed give a total storage capacity of more than 4 500 000 acre-ft.

---

[37] Engr., State Board of Water Engrs., Austin, Tex.

Some of the common errors noted by the author, in the investigation and development of water conservation, are among those generally encountered in the investigations in this section of the United States, namely:

1.—The use of insufficient data on stream flow.
2.—The use of rainfall records and assumptions as to run-off without adequate data showing the relationship between them.
3.—An under-estimation of flood flows, and insufficient spillway provision.

Mr. Sutherland's method of studying the characteristics of storage reservoirs has been applied to thirty-four reservoir areas in Texas that have been surveyed by the Topographic Branch of the U. S. Geological Survey, co-operating with the Texas Board of Water Engineers. Some of the areas studied are mapped on a scale of 1 : 24 000, with a contour interval of 5 ft.; others are mapped on a scale of 1 : 48 000, with contour intervals of 10 and 20 ft. At some of these areas two or more locations for the dam were considered, making a total of fifty-five reservoir sites studied. The development of some of them would be impractical; only a few of them have been, or are being, developed. They merely represent some of the investigations that have been made by the Texas Board of Water Engineers, looking toward the development of the water resources of the State. In this investigation, some of the dams were carried to maximum height for the purpose of applying the author's method of study, and not as an indication of what might be practical in the economic development of these reservoirs.

The capacity curves of the reservoirs in Table 9 were drawn as straight lines on logarithmic paper, as shown by the author. For most of these reservoirs the plotted points showing capacity, fit closely the lines thus drawn. For some the plotted points give a uniform upward curve; while others give a uniform downward curve. Some of them give a decided curve at the lower heights, but soon develop into a tangent as the height increases. The extension of some of these lines to the 100-ft. height, in order to calculate the constants in the author's formula, give storage values that are probably not reliable and would affect the accuracy of the values of the constants, C_w, calculated therefrom.

In the arbitrary classification of the reservoirs into four types, the particular class into which a reservoir may fall depends upon the shape of the cross-section of its area, and somewhat upon the height of the dam considered, as the value of *m* may be varied considerably by changing the height of the dam. This classification may have little to do with the general topography of the surrounding country outside the stream channel. However, with the foregoing observations in mind, the classification as made and used by the author appears very satisfactory and should be useful.

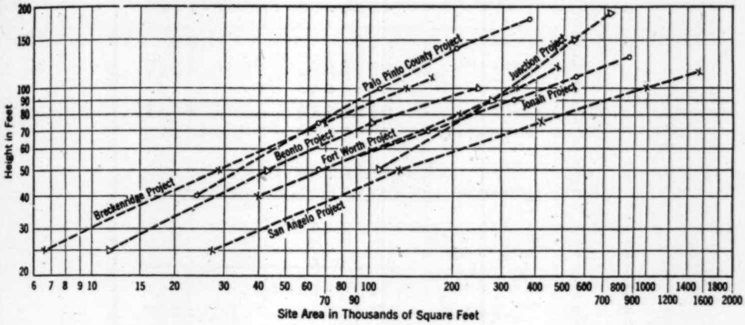


FIG. 26.—SITE AREA CURVES FOR PROJECTS IN TABLE 9.

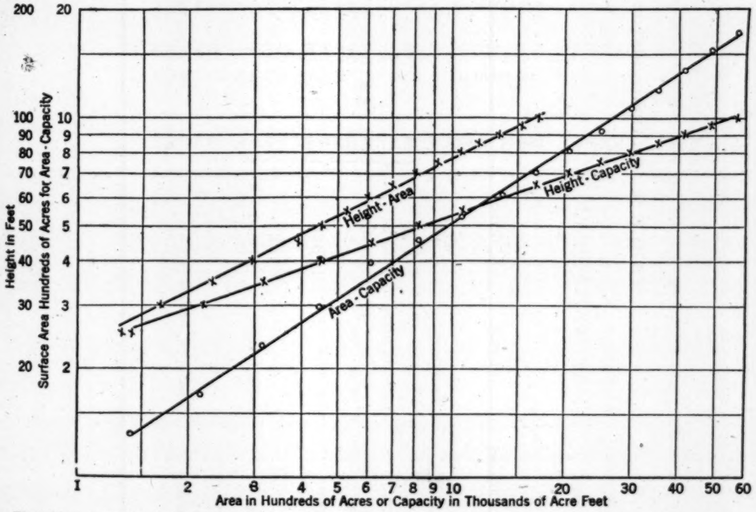


FIG. 27.—CURVES FOR THE CHARACTERISTICS OF A RESERVOIR ON THE SAN SABA PROJECT.

Table 9 shows the characteristics of seven of the fifty-five projects studied. The elevation of the bottom of the reservoir is given for each point so that

TABLE 9.—CHARACTERISTICS OF STORAGE RESERVOIRS

				Capacity, in Acre-Feet			Area, in Acres				Values of m		Values of C_w		
Item No.	Name of project	Stream	Type	Depth, in feet	Computed	By diagram*	Computed	By diagram*	Average depth, in feet	Ratio: Mean depth / Maximum depth	Computed	By diagram	Computed	By diagram	From depths
1	Bronte	Colorado River	Gorge	25	2 936	....	330	....	....	0.267	....	....	....	....	....
				50	27 202	22 500	1 985	1 800	13.70		3.65	3.75	0.02033	0.015	50–75
				75	131 887	105 000	7 168	5 400	18.40		4.08	3.90	0.00353	0.006	50–100
				100	405 652	300 000	15 175	11 500	26.73		3.74	4.15	0.01343	0.002	75–100
2	San Angelo	Concho River	Hill	25	4 660	....	481	....	....	0.334	3.25	3.39	0.2098	0.15	50–100
				50	60 778	80 000	3 953	4 500	15.38		3.15	3.85	0.3294	0.15	50–115
				75	258 176	275 000	10 857	10 300	23.78		3.02	3.30	0.5994	0.20	75–100
				100	657 193	610 000	19 830	18 200	33.14		3.00	3.30	0.6572	0.25	75–115
				115	1 020 357	1 000 000	26 585	24 000	38.38		....	....	....	....	....
3	Junction	South Llano	Hill	50	25 090	28 000	1 447	1 250	17.33	0.405	2.89	2.7	0.2539	0.7	50–90
			Hill	90	124 990	120 000	3 647	3 000	34.27		2.63	2.6	0.8408	1.0	50–150
			Flood-plain	150	430 850	420 000	6 579	6 200	65.49		2.29	2.5	4.024	2.0	50–190
			Flood-plain	190	754 130	775 000	9 829	8 800	76.88		2.47	2.47	1.757	2.5	90–150
			....	....	....	....	....	....	....		....	2.35	....	3.5	90–190
4	Breckenridge	Clear Fork, Brazos	Gorge	25	1 130	....	140	....	....	0.311	....	....	....	....	....
			Gorge	50	16 159	34 000	1 554	2 100	10.40		4.81	4.18	0.000069	0.0015	50–100
			Hill	75	103 567	130 000	5 331	5 100	19.43		3.86	4.05	0.005518	0.002	50–110
			Hill	100	289 594	320 000	9 749	10 000	29.70		3.37	3.50	0.0527	0.03	75–100
			....	110	395 524	430 000	11 565	12 500	34.20		3.22	3.50	0.1051	0.03	75–110

* By author's Figs. 4 and 6, using values of m and C_w computed by using maximum depth shown in this table for the particular site.

TABLE 9.—(*Continued*)

CAPACITY, IN ACRE-FEET							AREA, IN ACRES				VALUES OF m		VALUES OF C_w		
Item No.	Name of project	Stream	Type	Depth, in feet	Computed	By diagram*	Computed	By diagram*	Average depth, in feet	Ratio: Mean depth / Maximum depth	Computed	By diagram	Computed	By diagram	From depths
5	Palo Pinto County	Brazos	Hill	40	27 970	25 000	1 921	1 975	14.56	0.336	2.75	3.58	2.355	0.060	40–100
			Hill	75	220 260	220 000	8 088	9 700	27.23		2.75	3.39	2.355	0.105	40–140
			Hill	100	744 816	600 000	19 206	19 200	38.78		2.58	3.39	5.153	0.105	40–180
			Hill	140	2 031 900	2 000 000	47 500	45 000	42.78		3.27	4.41	0.2148	0.0015	75–100
			Gorge	180	4 753 070	5 000 000	89 910	86 000	50.86		3.54	3.58	0.06195	0.05	75–140
			....	....	....	....	....	....	....		....	2.82	....	1.5	100–140
			....	....	....	....	....	....	....		....	3.00	....	0.70	100–180
6	Jonah	San Gabriel	Hill	50	18 100	13 000	1 175	800	15.40	0.282	3.25	....	0.6167	....	....
			Hill	70	56 880	41 000	2 703	1 900	21.07		3.32	3.35	0.04467	0.045	50–90
			Hill	90	133 280	100 000	4 937	3 500	27.00		3.33	3.40	0.04169	0.04	50–100
			Hill	110	267 680	200 000	8 503	5 700	31.48		3.49	3.40	0.02042	0.035	50–130
			Gorge	130	485 100	360 000	13 247	8 800	36.62		3.54	3.55	0.01672	0.02	90–130
7	Forth Worth	Clear Fork, Trinity	Gorge	40	6 704	12 700	813	1 050	8.25	0.308	4.85	3.9	0.000242	0.0035	40–80
			Hill	80	96 786	125 000	4 049	5 200	23.90		3.35	3.75	0.02422	0.006	40–100
			Hill	100	206 934	260 000	7 195	8 600	28.76		3.48	3.70	0.02269	0.007	40–120
			Hill	120	382 380	460 000	10 335	13 000	37.00		3.24	3.20	0.08628	0.09	80–100
			....	....	....	....	....	....	....		....	3.35	....	0.045	80–120
			....	....	....	....	....	....	....		....	3.55	....	0.02	100–120
8	Bee Mountain	Brazos	Gorge	50	15 860	11 000	929	810	....	....	....	3.8	....	0.005	50–100
				100	228 170	215 000	10 498	7 700	21.73	0.217	4.60	4.0	0.0005731	0.002	150–120
				120	519 250	480 000	18 610	14 000	27.90	0.249	4.30	4.5	....	0.0003	100–120
9	Brazos—Madison County	Navasota	Hill	20	63 485	72 000	7 971	4 300	....	....	....	2.25	....	70.0	20–30
				30	165 380	182 000	12 408	6 900	....	....	....	2.30	....	60.0	20–40
				40	318 660	350 000	18 248	9 500	17.46	0.436	2.29	2.40	71.02	50.0	30–40

* By author's Figs. 4 and 6, using values of m and C_w computed by using maximum depth shown in this table for the particular site.

comparison may be made of the values of m in reference to elevation on any particular stream. The computed values of the constants, m and C_w, for different heights of dam, are given in order that comparison may be made. In order to compute the value of C_w in Item No. 9 (Table 9) by Equation (3), it was necessary to extend the height-capacity curve from Height 40 to Height 100. This may have given a capacity for 100-ft. depth that would be considerably in error, which, in turn, would cause errors in the computed value of C_w and also in the capacities and areas taken from the diagrams.

The author's reservoir capacity diagram, Fig. 4, as well as the other alignment charts are very ingenious, and should often be found useful in the preliminary examination of reservoirs. Comparing the computed values of m and C_w with values taken from Fig. 4, by using intersecting lines from two known depths and capacities, quite a difference in values is often noted. Where such differences are found, capacities taken from the diagram for other depths may be considerably in error, as may be seen from Table 9.

Computing the value of the ratio, the average or mean depth divided by the maximum depth, to any given elevation for the fifty-five sites considered, the mean value is 0.3148, corresponding to $m = 3.18$. Most of the reservoirs fell into the "hill type" classification, which probably accounts for the smaller variation in the values of m.

Areas were computed for the sites in Table 9 and for different depths. These are plotted in Fig. 26. The plotted points have been connected by dotted lines for easier identification, no attempt being made to fit a straight line to them. Table 9 also shows a comparison of the computed values with the values taken from the author's diagrams for capacity, surface area, m, and C_w. In computing the site area it is sometimes found that there is a low sag at one side or the other, that will require the construction of a levee, spillway, or other structure. Whether or not to include this value in computing the site area is a problem that is not always easy to determine, because it may be a continuation of the main site area, adjacent to it, or widely separated from it.

Fig. 27 represents the curves of height plotted against capacity, height plotted against area, and area plotted against capacity for the San Saba Project as determined in this study. These curves are fairly well represented by straight lines on logarithmic paper for this particular site. The scale on which these values are plotted has much to do with the distance of certain points from a straight line drawn on the logarithmic chart.

A. J. GILARDI,[38] ASSOC. M. AM. SOC. C. E. (by letter).—The method of comparing and classifying reservoirs, dam sites, and the various types of dams described in this paper, is very useful, because, to a certain extent, it makes possible the visualization of their respective merits in comparison with such standards or "yardsticks" as those with which the hydraulic engineer may be most familiar.

Probably, the constant use of the charts given in the paper would overcome some difficulties which the writer has experienced in attempting to apply a few of them in his studies of dams. It would seem possible, however, to introduce a few simplifications and thus increase the facility and accuracy of application of the charts on the part of engineers.

---

[38] Cons. and Civ. Engr., Seattle, Wash.

In the course of his work of making numerous preliminary investigations of dam sites and reservoirs, the writer has prepared two simple and helpful charts to determine the approximate volume of dams for a wide range of sizes and types.

Fig. 28 gives directly, as abscissas, the total volumes of concrete required for gravity dams of a given height, $H_{\text{max.}}$, and for a given distance between abutments, measured at the water surface. This diagram is based on the following assumptions:

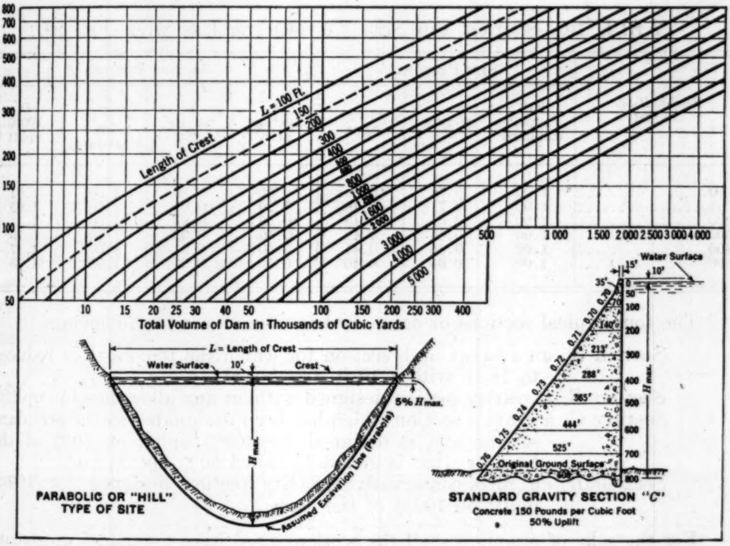


FIG. 28.—TOTAL VOLUME OF GRAVITY DAMS FOR STANDARD SECTION, *C*, AND PARABOLIC OR HILL TYPE OF SITE

(*a*) That the dam has a gravity section which will be designated as the standard section, *C* (see Fig. 28), and is designed for uplift of 100% on 50% of the area, or what is usually called 50% uplift. The weight of the concrete has been assumed at 150 lb. per cu. ft. The section has a top width of 15 ft. and a free-board of 10 ft., which would correspond to about the same volume as a top width of 20 ft. and a free-board of only 5 ft.

(*b*) That the actual dam site has such a shape as to be replaceable by a theoretical site of the parabolic or "hill" type. This assumption is similar to that suggested by Mr. Sutherland, with the exception that two modifications of a practical nature are introduced. The first is that the vertex of the parabola is at the deepest part of the excavation (estimated or actual, as the case may be), the vertical distance from this point to the water surface being designated as the height of the dam, $H_{max.}$. The second assumption is that the foundation of the dam or excavation line is assumed to follow a parabolic line and end at the abutments with a depth of excavation of 5% of $H_{max.}$.

Fig. 28 covers a wide range of sizes, from heights of 50 to 800 ft. and lengths of 100 to 5 000 ft.

Fig. 29 is intended for the purpose of modifying the volumes obtained from Fig. 28, whenever it is necessary to take into consideration sections and sites other than those used as a basis for that chart. This desirable result is arrived at by the introduction of factors, which will be designated as "section factors" and "site factors" (see Table 10).

TABLE 10.—SECTION AND SITE FACTORS FOR USE WITH FIG. 29.

Height, $H_{max.}$, in feet	Site Factors			Section Factors			
	Hill type, standard	V-type	Gorge type	*A*, average arch	*B*, no uplift	*C*, 50% uplift (standard)	*D*, 100% uplift
50	1.00	0.76	0.58	0.50	0.96	1.00	1.04
100	1.00	0.71	0.49	0.50	0.94	1.00	1.05
200	1.00	0.67	0.43	0.50	0.93	1.00	1.11
300	1.00	0.66	0.42	0.50	0.92	1.00	1.15
400	1.00	0.65	0.41	0.50	0.91	1.00	1.17
600	1.00	0.64	0.40	0.50	0.90	1.00	1.18

The four typical sections of dams used in Fig. 29 are the following:

Section *A*, an average arch section for which the top width is reduced to 7½ ft. with a 10-ft. free-board;
Section *B*, a gravity section designed without any allowance for uplift;
Section *C*, a gravity section which has been designated as the standard section and is designed for 100% uplift on 50% of the base, or what is generally called 50% uplift; and,
Section *D*, an ultra-conservative gravity section designed for 100% uplift on 100% of the base.

For the sake of simplicity, all the sections used have a vertical up-stream face throughout and have practically a triangular shape. While it is probable that for the higher dams the triangular section would be replaced by a more complicated section, this would mean essentially that the masses would be distributed somewhat differently, although approximately the same total volume would be required.

The "section factors" in Table 10 are intended to be applied to the volume obtained from Fig. 28 for the standard section, *C*, the given $H_{max.}$, and length of crest, for the purpose of allowing for a change in section, such as the engineer may consider desirable in his particular case. For instance, a dam, 250 ft. high ($H_{max.} = 250$ ft.) with a crest length of 1 500 ft., on a site of the parabolic or "hill" type, would have a total volume of 685 000 cu. yd., as determined from Fig. 28 for a standard section designed for 50% uplift. A dam for the same site, designed without any allowance for uplift, would have a section factor of 0.925. Therefore, the corresponding volume would be reduced to 685 000 cu. yd. by 0.925, or 633 000 cu. yd.

Fig. 29 shows the three typical sites investigated, which are the parabolic or "hill" type, the V-type, and the "gorge" type.

The "site factors" are intended to be applied to the volume obtained from Fig. 28 for a site of the parabolic or "hill" type, for the purpose of determining the volume of a dam of the same crest length and height, $H_{max.}$, but for a different type of site. For instance, a dam of standard section, *C*, 1 500 ft. long at the crest and with $H_{max.} = 250$ ft., located on a parabolic or "hill" type of site, would have a volume of 685 000 cu. yd. A dam with the same height, $H_{max.}$, and same crest length, *L*, and designed on the same basis, but located in a site of the "gorge" type, would have a site factor of 0.425 and, therefore, would require 0.425 by 685 000 cu. yd., or about 291 000 cu. yd.

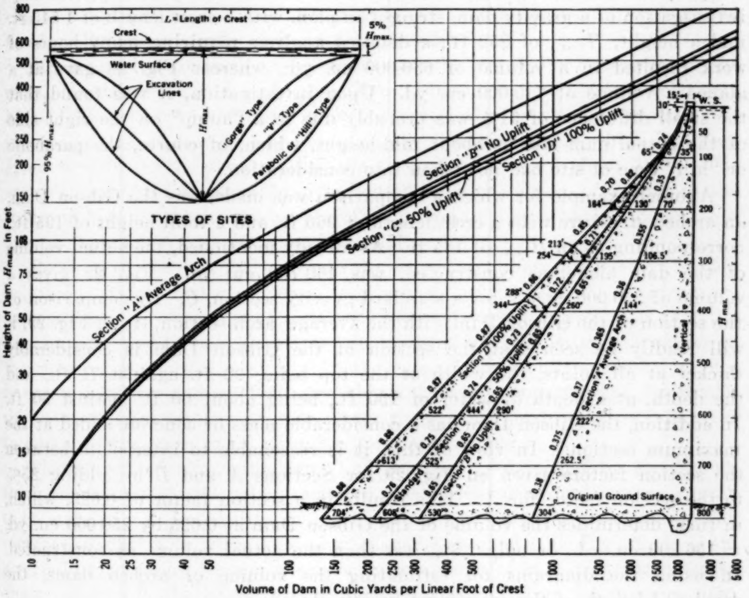


FIG. 29.

Both section and site factors may of course be applied simultaneously to determine the desired volume when both the basis of design and the type of site are changed from those used in Fig. 28. It is also readily possible to interpolate for such intermediate types of sites and sections which are not shown directly in Fig. 29.

For trapezoidal sites, the volume of the dam can be determined readily by segregating into a V-type and adding to this volume that obtained by multiplying the volume per linear foot given in Fig. 29 by the length of the part of the dam which has the constant height, $H_{max.}$. The same procedure of segregation and summation may be applied to such others for which the three typical sites may not be directly applicable. The diagrams presented herewith are intended, of course, to be used for preliminary purposes only, and as approximate checks in other cases.

The writer has not had the time required to prepare a detailed tabulation to compare the volumes obtained from the diagrams with those of dams actually constructed. However, a few comparisons have shown to his satisfaction that the diagrams give results which are sufficiently accurate for the purposes for which they are intended. For instance, in a preliminary investigation of a gravity dam, straight in plan, with a crest length of 1 545 ft. and a height, $H_{\text{max.}}$, of 252 ft., a detailed analysis requiring many hours of work resulted in a volume of 680 000 cu. yd., whereas Fig. 28 gave at a glance a volume of 720 000 cu. yd. Upon investigation, it was found that the small difference of 5½% was probably due to a "hump" on the right side of the actual dam site at about mid-height, which, of course, the parabolic or "hill" type of site had not taken into consideration.

Another example for which a comparison was made, was the Gibson Dam, an arched structure with a crest length of 960 ft. and a total height of 195 ft., corresponding to an $H_{\text{max.}}$ of 185 ft. As already mentioned, the actual volume of the dam alone, as constructed, was 160 700 cu. yd. Fig. 28 gives a volume of 250 000 cu. yd. for a standard gravity section, *C*. By comparison of the section of the Gibson Dam with the average arch section, *A*, on Fig. 29, it will readily be seen that the section of the Gibson Dam is considerably thicker at all points, the width at the top being 20 ft. against 7½ ft., and the depth, at a depth of water of 150 ft., being about 60 ft. against 50 ft. In addition, the Gibson Dam has a considerable mass of concrete added at the maximum section. In view of this, it is reasonable to interpolate between the section factors given on Fig. 29 for Sections *A* and *B* by adding 25% to the section factor for *A*. This results in a section factor of 0.625, which, in turn, determines the volume of the Gibson Dam at 0.625 by 250 000 cu. yd. = 156 200 cu. yd., or only 2.8% less than the actual volume as constructed. In using the diagrams for estimating the volume of arched dams, the developed length of the crest must be introduced.

The diagrams will sometimes be found useful in reversing the usual procedure of determining the volume from given dimensions. Assuming, for instance, that, for a given dam site, only a limited sum of money is available and that, therefore, the maximum volume of concrete is fixed, it is possible to determine in a few minutes what maximum height of dam could be constructed. The diagrams have been prepared for non-overflow sections of dams. However, they have been found quite useful also for overflow sections, with minor arithmetical adjustments. Diagrams similar to those presented herewith, could be prepared for earth dams, rock-fill dams, and other types.

In the course of the extensive computations and other work required for the preparation of the diagrams, the writer has had the valuable co-operation of Mr. L. S. Moore, of the U. S. Engineer Office, at Seattle, Wash., to whom thanks are extended herewith.

R. A. SUTHERLAND,[39] ASSOC. M. AM. SOC. C. E. (by letter).—The discussion by Mr. Eaton is particularly interesting in that it deals with problems similar in many ways to those met by the writer in Australia. The increase in flood severity and in the quantity of detritus carried, due to destruction of natural cover, as mentioned by Mr. Eaton, has become a problem of real gravity in many countries. The writer believes that in America this problem has been faced for many years by the reservation of immense areas of forest, with adequate provision for safeguard from fire. In most countries, however, the destruction of forest and other cover is allowed to go on far beyond a safe point before anything is done to prevent it.

Mr. Eaton has touched a vital point when he states that "the costs of the remaining dam sites in Los Angeles County mountain territory are relatively high." This is now becoming true for practically every country, and the increasing cost of storage has prompted the intensive study which has been given to economy in dams during the twenty years since about 1910. The writer's paper was developed and presented with the object of aiding in the study of how to obtain the greatest "value for money," having regard not only to the dam but to the reservoir itself.

Relative to the justified expenditure on a dam, there are, as Mr. Eaton points out, many factors to consider; looking at the dam as an insurance against either an excessive or a defective flow of water, it is obvious that the premium which can be afforded for such insurance will be in proportion to the risks to be guarded against. In the case of dams built solely or mainly to create head (and these form probably a growing proportion of the total), the value of such head is another factor to be considered, whether it is used to create power or to avoid pumping for water supply.

The method of comparing dams and storages, as described by Mr. Eaton, is of great interest and value, enabling a comprehensive preliminary survey to be made of a series of sites on any one stream or on adjacent streams. The increase of unit cost with increasing capacity as shown by Fig. 22, is characteristic of storages having a relatively low value of m. The cost of most dams varies with the height according to a power of from 2.5 to more than 4, depending on the site and the type of dam, while many storages have lower values of m.

The writer is gratified that his paper has been of interest in a district which he has always admired for its progressive water administration policy.

The writer agrees fully with Mr. Simonds as to the paramount importance of the geological aspect of water conservation. It is essential that a dam should form a stable and integral part of the landscape, or to use a mechanical analogy, that it should be a "weld" and not a "patch." Scarcely less importance should be attached to the geological properties of the reservoir basin. This phase of the subject is treated more thoroughly in the comprehensive manuscript from which the material for this paper was condensed.[40]

As Mr. Simonds points out, expenditure on diamond drilling is generally not only necessary, but is a true economy. The exploration of the dam site should not stop at the examination of the rock strata, but should be sufficiently extensive to enable the history and probable stability of the entire surrounding terrain to be visualized. The writer can cite a case in which the site of a large dam to be built for power generation was most excellently prospected by drilling and tunneling, but in which, nevertheless, the completed works had to be shut down soon after commencing operation, on account of unsuspected weaknesses in the adjacent terrain, under the altered conditions. Provision against this meant a large further expenditure in remedial works, in addition to heavy losses through idle plant.

---

[39] San Francisco, Calif.

[40] Bound copy on file at Engineering Societies Library, New York, N. Y.

The writer regrets that Mr. Simonds considers the curves to be of doubtful practical value. The topographic survey is essential when the project has reached a certain stage, and the writer has never suggested otherwise, but he has seen thousands of dollars wasted in careful surveys of projects which were never begun. In many cases this useless expenditure could be avoided by a common-sense application of the broad principles introduced in this paper.

Mr. Nimmo's able discussion of the reasons for the characteristic properties of different types of reservoir is very interesting, and appears to be founded on a scientifically correct basis.

Mr. Olberg considers that the classification of reservoirs into different types is of importance only as a means of definition and comparison. The original idea of this classification is due to Mr. Nimmo, and it has been fairly well established by theoretical considerations and from actual practice. It seems desirable to be able to recognize any given reservoir as belonging to a certain type, since each type has distinct characteristics of its own as regards seepage, evaporation, and flow-regulating properties. This aspect of the subject has been merely touched on in the paper, but might be considerably elaborated.

No mathematical legerdemain is required to prove that m is the ratio of maximum depth to average depth. The average depth equals $\frac{\text{capacity}}{\text{water-spread area}}$ $= \frac{C_w \; h_1{}^m}{m \, C_w \; h_1{}^{m-1}}$, and dividing this into the maximum depth, h_1, gives m. As regards the constant, C_w, this is the theoretical capacity at unit depth, but this fact is of no practical importance.

Mr. Olberg is correct in preferring the plotting of the logarithmic line to the use of the alignment charts (Figs. 4, 6, and 8) as a means of finding the correct constants for a reservoir or dam site. The alignment charts, however, are sufficiently accurate for practical purposes and give the desired result at a glance without calculation or the chance of arithmetical error.

Referring to the remarks of both Mr. Olberg and Mr. Nimmo regarding the use of the site-area and the site-width equations, the writer has evidently not made the fact sufficiently clear that the use of Equation (1) as the starting point in deriving the formulas for the volume of dams is purely empirical. It was found to give satisfactory results, and the use of the width of dam site was not found satisfactory. It should also be pointed out that

Equation (7), derived from Equation (1), really applies not to the site itself, but to the equivalent theoretical site as shown in dotted lines in Fig. 9. If the width is measured at a large number of short-height intervals (that is, making *dh* very small), and the logarithmic line is plotted, it should give exactly the same result as the plotting of site areas. In practice, it is easier to let the planimeter assist in striking the average, than to plot so many points.

Mr. Olberg's assumption that Equation (7) was derived from the capacity equation for the reservoir is in error. The writer has not endeavored to derive any relationship between the capacity and site-area equations. It is doubtful whether even a quantitative relationship holds between the constants of the two equations; and examples will readily come to mind of large storages being impounded at a narrow gorge, and, conversely, of long and costly dams being required for a relatively small storage. The fact is that the properties of dam sites are frequently determined by some rather local geological condition, while the properties of reservoir basins are determined by the geological and climatological conditions on a fairly extensive terrain.

Mr. Nimmo has pointed out the existence of a compound type of reservoir, as exemplified by Lake Cushman. The writer believes this to be very exceptional, and has never encountered a similar case. He is glad to note that Mr. Nimmo and Mr. Olberg have found his formulas to give results sufficiently accurate for preliminary investigations. For such purposes the data available are usually not sufficient to warrant the more laborious methods frequently used in computing quantities, and the accuracy of these methods is often more fictitious than real.

The volume of dams designed by the Cain formulas or by the trial-load method, might be found as a certain percentage of the volume of the corresponding plain arch dam having a given "cylinder" stress, such percentage being determined from experience. The writer has not sufficient data to enable him to indicate a coefficient for this use. Mr. Simonds has given the actual volume of the Gibson Dam as 160 000 cu. yd. The volume computed by the writer's method is 142 000, which indicates an error of 11 per cent.

Mr. Olberg suggests that an adjustment might be made in the formulas for "specific capacity," to allow for the crest and foundation margins. The difficulty is that by altering the arguments in the numerator and denominator of the expression, the possibility of deriving a simple expression is lost. The writer made an adjustment in the coefficient to allow for the average conditions and margins stated, and the formulas will be approximately correct under those conditions, but will be increasingly in error as the dam considered is very low or very high.

The method of determining the most economical size of installation, of which Mr. Olberg would like to have more information, depends on a use of the Rippl curve, or other means of determining the storage required to provide a given regulated flow. For various values of regulated flow, various sizes of dam are required, the quantity of material in which can be found approximately from the formulas given in the paper; and, by applying proper unit costs, the total cost of the different dams can be found. Dividing these costs by the values of the regulated flow, therefore, enables the point to be found, which will give the lowest cost per unit of regulated flow.

Data for developed and proposed storage in Texas for different purposes, as given by Mr. Pritchett, are of great interest. The large value of 2 500 000 acre-ft. being developed or approved for water power is remarkable. In view of the arid nature of Texas climate, it would have been of interest to have figures showing the acre-feet of storage considered necessary per acre of irrigated land, and the writer presumes that this figure would be relatively high. It is noted from Table 9 that the computed and diagram capacities for the larger storages generally agree fairly closely. The values of m also agree reasonably well, while those of C_w show wide discrepancies. Actually, it would be found that the computed values would give an exact agreement of capacity for one value of height, with greater or smaller errors at other values of height, and the object in selecting values of m and C_w should be to spread the errors so that they are nowhere excessive.

Mr. Gilardi's very useful and practical diagrams should be of great value in investigations of the cost of dams when the leading particulars, such as height, crest length, and type have been decided. They should give practically the same results in parallel cases, as the writer's Fig. 10, for any particular case. For example, consider a site for which $n = 1.5$, $C_A = 10$, and $H_{\text{max.}} = 100$. In that case, $L = 150$. This gives, from Mr. Gilardi's Fig. 28, a volume of 15 500 cu. yd. and from Fig. 10, 15 390 cu. yd. (total height $= 110$ and mean $K_g = 0.73$, as shown on Fig. 28).

The use of site and section factors, applied to the results obtained for a standard gravity section, is a distinctly useful artifice, although interpolation in two senses may not be always easy. For sites of the composite type, not covered by the writer's procedure, Mr. Gilardi's proposal to divide the site into its component parts and determine the separate volumes from his two diagrams, is a practical method of procedure.

It is necessary to correct the impression that the "gorge" and "hill" types of dam site mentioned by Mr. Gilardi, have any necessary connection with the classification of reservoirs into these types adopted by the writer. The type of reservoir depends on the average of the cross-sections of an extensive stretch of river, including any tributaries in the reservoir basin. Where these cross-sections are predominantly of the gorge type, the reservoir is a gorge type. In many cases the nature of the dam site may be determined by purely local geological conditions, and it may be of a different type as, for example, the hill type. In point of fact dam sites below reservoir basins of the gorge type are frequently of the hill type, and the gorge type of site may occur below a lake type of basin. For this reason the writer did not attempt to draw any connection between the type of basin and its dam site.

Regarding the writer's statement following Table 1 that in the cases examined by him, the value of m for reservoirs occurring on the same river was a function of the elevation, Mr. Nimmo has kindly pointed out a case in which a large number of reservoir sites on one river were investigated, and in which this relationship does not hold. This is the Umpqua and North Umpqua River, in Oregon. Fig. 30 and Table 11 were plotted by Mr. Nimmo from data supplied by the U. S. Geological Survey.[41] While it is true that the relationship does not hold in this case, Table 11 does show in a general manner that, for reservoirs of similar type, C_w varies with the catchment. For example, compare Sites 10 and 17; 16, 21, and 22; and 8, 23, and 24. Site 14 does not conform to this generalization.

TABLE 11.—CHARACTERISTICS OF PROPOSED RESERVOIR SITES ON THE UMPQUA AND NORTH UMPQUA RIVERS, OREGON

Dam Site No.	Mileage from Scottsburg, Ore.	Elevation, in feet	Value of C_w	Value of m
25	1	0	70.8	1.75
24	11.5	41.5	149.2	1.60
23	29.5	100	91.6	1.64
22	45	166	53.3	1.72
21	49.5	186	48.5	1.72
20	63	255	1.162	2.51
19	69	294.5	1.022	2.68
17	91	439	23.2	1.82
16	97.5	506	42.2	1.73
15	106	596	24.25	1.89
14	112	660	38.5	1.80
12	122.5	847	16.6	1.76
10	137.5	1 187	6.82	1.82
9	145.5	1 450	10.6	1.68
8	152	1 750	10.97	1.62

Referring to that part of the writer's paper, headed "Arch Dams," the following should be added: The Gibson Dam (Table 7, Item No. 12), was designed by the trial-load method and the use of Fig. 15 is not strictly applicable. The discrepancy between the actual and chart values of W_s shows that the actual volume of concrete is much more than when computed from the stress derived from the batter of the mid-section. This increase is due to the considerable thickening of the flanks of the dam and the excess section over the theoretical triangular section.

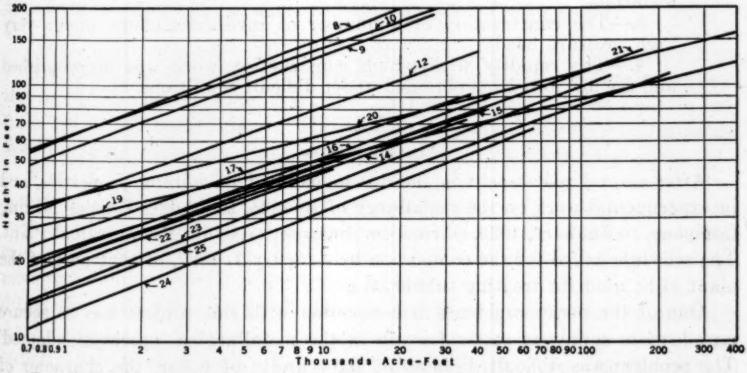


FIG. 30.—RESERVOIR CAPACITIES, UMPQUA AND NORTH UMPQUA RIVERS, OREGON.

---

[41] *Water Supply Paper, 636,* U. S. Geological Survey.

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
TRANSACTIONS

Paper No. 1795

CONSTRUCTION OF LA OLA PIPE LINE, IN CHILE[1]

BY W. B. SAUNDERS,[2] M. AM. SOC. C. E.

WITH DISCUSSION BY MESSRS. ERNEST B. NELSON, H. L. THACKWELL, WILLIAM E. RUDOLPH, AND W. B. SAUNDERS

SYNOPSIS

The purpose of building the pipe line described in this paper was to supply water to the Metallurgical Plant of the Andes Copper Mining Company, at Potrerillos, Chile. The line is 32.3 miles long and the pipe varies in diameter from 22 to 36 in.

The unique and outstanding features are as follows:

1.—The fabrication of riveted steel pipe in the field.

2.—The unusual method of field transportation for construction materials.

3.—The construction of a number of inverted siphons under very high pressure heads.

4.—The rapidity with which construction work was accomplished and the low unit costs obtained under difficult conditions.

INTRODUCTION

After several years spent in the development of its mining properties, and in experimental work on the metallurgy of its ores, the Andes Copper Mining Company, in January, 1925, started the construction of its Metallurgical Plant. The schedule called for the completion by January 1, 1927, of that part of the plant to be used for treating sulfide ores.

One of the major problems in connection with this project was to secure an adequate supply of water for use in the metallurgical processes adopted. The problem was difficult, because of the scarcity of water, the character of the country, and the high altitude of the plant.

The estimated requirements were 29 cu. ft. per sec., of which about 9 cu. ft. could be reclaimed.

Potrerillos is about 596 miles south of the northern boundary of Chile, and about 74.5 miles east of the Port of Chañaral. (See Fig. 1.)

---

[1] Published in September, 1930, *Proceedings*.

[2] Hydr. Engr., Andes Copper Min. Co., Barquito, Chañaral, Chile.

About 2 miles south of Chañaral, the Andes Copper Mining Company maintains its own port—Barquito—where its ocean-cargo handling facilities are located, a steam power plant of 30 000-kw. capacity, a warehouse, the Chilean Custom House, the railway terminal, and living quarters for employees engaged in port activities.

The Chilean Government maintains and operates a railway from Chañaral to Pueblo Hundido, a village 36 miles east of Chañaral. The Andes Copper Mining Company operates over this railway, under an agreement with the Government. It has constructed its own line between Pueblo Hundido and Potrerillos, a rail distance of 57.8 miles.

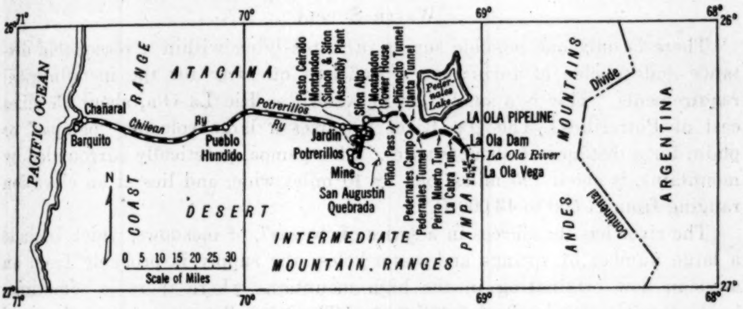

71°
70°
69°
68°
26°
27°
PACIFIC OCEAN
COAST RANGE
ATACAMA DESERT
Chañaral
Barquito
Chilean Ry
Pueblo Hundido
Potrerillos Ry
Pasto Ceirado
Montandon
Siphon & Sifon Assembly Plant
Sifon Alto
Montandon Power House
Piñoncito Tunnel
Uanta Tunnel
Jardin
Potrerillos
Mine
San Augustin Quebrada
Piñon Pass
Pedernales Camp
Pedernales Tunnel
Perro Muerto Tun
La Cabra Tun
Pedernales Lake
LA OLA PIPELINE
La Ola Dam
La Ola River
La Ola Vega
PAMPA
INTERMEDIATE MOUNTAIN. RANGES
ANDES MOUNTAINS
Continental Divide
ARGENTINA
N
0 5 10 15 20 25 30
Scale of Miles

FIG. 1.—REGION OF LA OLA PIPE LINE.

Potrerillos, the site of the Metallurgical Plant, is the terminus of La Ola Pipe Line, described in this paper. The mine is connected with the Metallurgical Plant by an electrified railway, 6.2 miles in length.

DESCRIPTION OF COUNTRY

The Continental Divide of South America is formed by the main range of the Andes Mountains which parallels the Pacific Coast; the summit is only from 100 to 150 miles inland. This range of mountains is the highest in the Western Hemisphere, the peaks reaching more than 22 000 ft. in altitude.

From a point about 300 miles south of Chañaral to beyond the northern boundary of Chile, the Humboldt Current (extremely cold), touches the western coast of South America and tempers the climate of that area. This cold-water area coming in contact with cold air currents from the high mountains and with cold ocean winds from the south, creates a physical condition that eliminates all possibilities of any great precipitation of moisture. As a consequence Potrerillos and the entire area surrounding it, is almost entirely devoid of rainfall. This area is known as the Atacama Desert, and receives a smaller amount of precipitation than the Sahara Desert.

This arid condition has produced a marked effect on the country. Incomplete erosion has left a topography of extreme roughness with considerable variation in elevation. The entire area is practically devoid of vegetable growth, and consequently geological formations are strikingly exposed.

The Metallurgical Plant is at an altitude of about 9 500 ft. and the mine at about 10 500 ft. The plant and the mine are both in what might be classed as the foothills of the main Andes Range, the summit of which is about 50 miles farther east.

The area between Potrerillos and the Continental Divide is rough in the extreme; it is cut by "quebradas", or canyons, thousands of feet deep, which have created large areas inaccessible to any form of transportation other than by saddle or pack animals. Water holes are few and far between, and the entire area is incapable of supplying food of any character for either man or beast.

The Ola Pipe Line was constructed within this area during 1925 and 1926, under the direction of the writer.

WATER SUPPLY

There is only one possible supply of water lying within a reasonable distance and capable of furnishing a sufficient quantity for the metallurgical requirements. This is a small stream, known as Rio La Ola, about 25 miles east of Potrerillos. The Ola River traverses a large volcanic "pampa," or plain, for a distance of about 9 miles. This pampa, practically surrounded by mountains, is about 125 miles long by 10 miles wide, and lies at an elevation ranging from 11 500 to 13 000 ft.

The river has its sources in a series of "vegas", or meadows, which include a large number of springs and water-holes; the supply is probably from an artesian flow originating in the high mountains where there is sometimes heavy precipitation in the form of snow. The "vegas" cover an area of several thousand acres and are almost entirely surrounded by a water-bearing formation. (See Fig. 2.)

The Ola River flows north above the surface for a distance of about 9 miles and then gradually disappears into the loose sands and gravels of which the pampa is formed. The indications are that from its point of disappearance it follows an underground seepage course and again rises about 12 miles farther north in Pedernales Lake, which is at the most northerly end of this pampa. Pedernales Lake is a salt bed with an area of about 115 sq. miles.

Necessarily any diversion of water from the Ola River must be accomplished at some point within the 9 miles where the water flows above the surface. Only one site was found that was suitable for the construction of a diversion dam. At this place, the formation of the pampa changes from loose sand and gravel to a conglomerate which has the characteristics of a very fair grade of concrete. Through this formation the river cut a narrow channel, leaving a good site for a low diversion dam. This formation appears for only a short distance up stream and down stream from the dam site, and seems to form a cut-off or barrier that throws all the Ola River water to the surface at this point.

The Spanish word, "ola," means ocean wave or billow; so this stream very probably received its name from the fact that the water is slightly salt or brackish to the taste. It does not contain sufficient salt to be unpalatable, however, and it was used throughout the construction period for camp drinking water and construction purposes. An analysis of the water is given in Table 1.

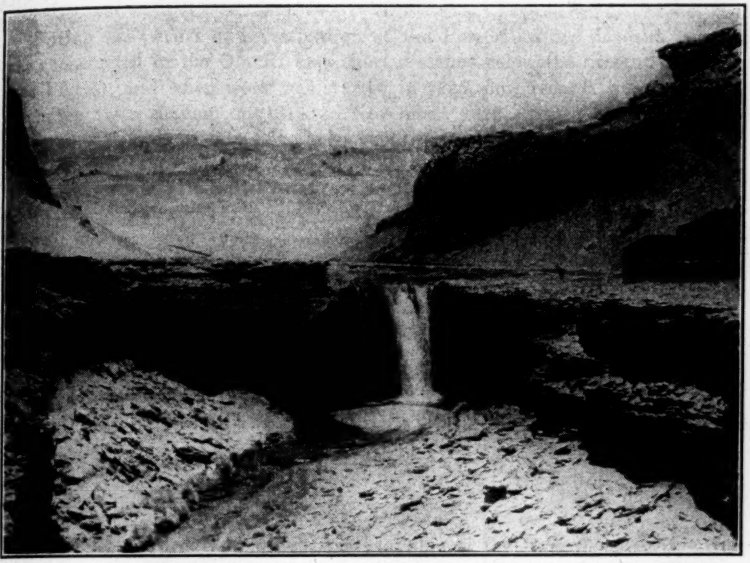


FIG. 2.—JUNCAL FALLS IN THE DRAINAGE AREA ABOVE LA OLA RIVER. THIS WATER DISAPPEARS BEFORE IT REACHES THE RIVER.

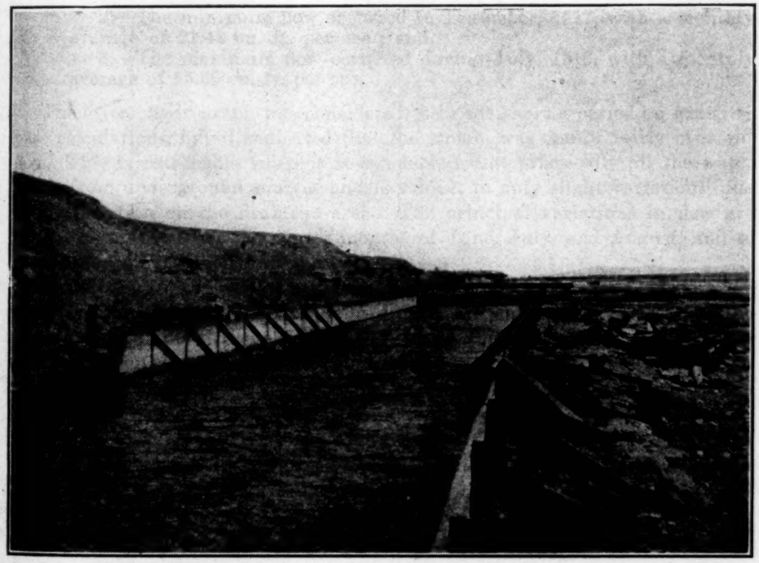


FIG. 3.—WEIR FOR MEASURING FLOW OF STREAM ABOVE LA OLA DAM.

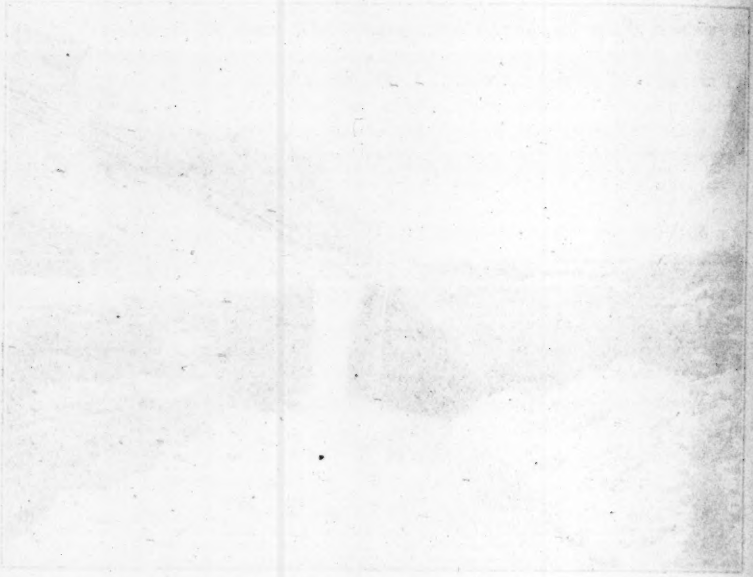


Fig. 2.—[illegible]

Fig. 3.—[illegible]

During the period of investigation of the Potrerillos ore deposits, a weir was constructed in the Ola River a short distance below the present dam site (see Fig. 3), and when work was started in 1925, flow records were available extending from September, 1915, to November, 1920. Since February, 1925, continuous records of flow have been kept. (See Fig. 4.)

TABLE 1.—ANALYSIS OF RIO LA OLA WATER.

Analysis.	Parts per million.	Probable combination.	Parts per million.
Ca	147.0	SiO_2	64.0
Mg	43.0	FeO	8.0
Na	661.0	$NaNO_3$	48.0
SiO_2	64.0	NaCl	1 646.0
FeO	8.0	$MgCl_2$	122.6
NO_3	35.0	$CaSO_4$	405.2
Cl	1 089.0	$Ca(HCO_3)_2$	112.5
SO_4	286.0	$Mg(HCO_3)_2$	68.6
HCO_3	141.9		
Total	2 474.9	Total	2 474.9
Residue	2 502.0		

The 5-year hydrograph which was available at the time the pipe line was designed, shows the following characteristics:

1.—The average flow for the 5-year period was 27.87 cu. ft. per sec.;
2.—The minimum flow occurred in December, 1917, with a monthly average of 21.44 cu. ft. per sec.; and,
3.—The maximum flow occurred during July, 1916, with a monthly average of 55.50 cu. ft. per sec.

The 5-year hydrograph was considered to be too short a period on which to base calculations, but it indicated that the stream was one of fairly constant flow. This is reasonable when it is considered that practically all the water comes from underground sources and is subject to only slight variations, due to precipitation on the drainage area. The principal variations in flow are due to freezing during the winter months of June, July, and August, and to evaporation during the summer months.

Variations from freezing are characterized by daily peaks caused by the vegas freezing at night and thawing during the day. This variation often reaches 40% of the average flow of the stream; but it is easily controlled by the pondage up stream from La Ola Dam.

Climatic conditions in this area are such that a maximum of evaporation may be anticipated. The atmosphere is absolutely dry, the sun shines practically every day of the year, the air is brilliantly clear, there is no foliage and no grass, except in the vegas, and the wind reaches a velocity of about 20 miles per hour on practically every day of the year. During the warmer months, from November to April, these conditions have a marked effect on the flow of this stream, and it is believed that almost the entire seasonal variation is due to evaporation.

The annual variation, no doubt, is due to the precipitation on the catchment area that supplies the underground watercourses. However, since the rate of underground flow is undetermined and the catchment area is unknown, no approximations or calculations can be made regarding this factor.

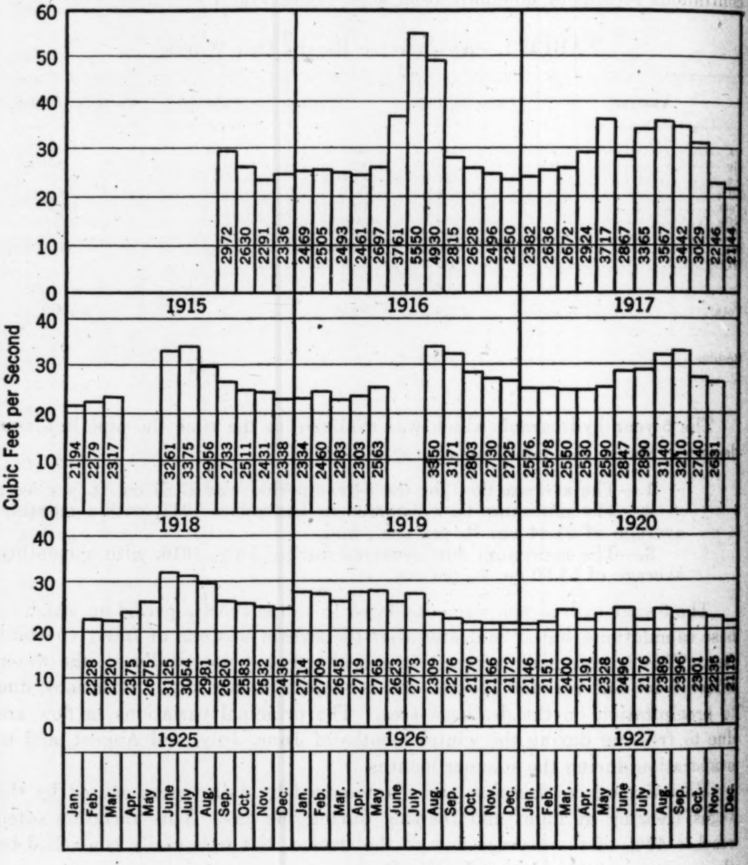


FIG. 4.—HYDROGRAPH OF LA OLA RIVER.

An excellent demonstration of this variation is shown by the hydrograph for 1927 (Fig. 4). During the early part of July of that year an unusually heavy snowfall occurred on the pampa and on the mountains surrounding it (see Fig. 1). The average depth of snow was more than 3.0 ft. on the pampa and undoubtedly reached a depth of many feet on the western slope of the main Andes Range; yet the month of June, just prior to this storm, shows a greater run-off than any following month of that year. A similar snowfall had not occurred since the winter of 1919. It is impossible, therefore, to forecast accurately the annual variations of flow. Hence, it was necessary to assume, from past performance, that the stream has a fairly consistent and constant flow and that it is not subject to sudden great increases or decreases.

From the 5-year period it may be seen that the stream flow is less than 25 cu. ft. per sec. for about 30% of the total time; that the duration of low flow extends each year over a period of from 3 to 6 months; and that it is more than 26.5 cu. ft. per sec. for 50% of the total time.

The storage capacity of the reservoir above La Ola Dam is only 1 025 000 cu. ft., which is not sufficient for any appreciable quantity of storage; but it will supply a regulation covering a reduction of flow of 40% for a period of 30 hours. This 30-hour regulation is more than sufficient to take care of all daily fluctuations.

With the plant requirements estimated at 29 cu. ft. per sec., of which 9 cu. ft. per sec. might be recovered by pumping, and with a flow of 26.5 cu. ft. per sec. recorded for 50% of the time, it was deemed to be economical to develop this project with the object of securing 26.5 cu. ft. per sec. for as much of the time as possible. Therefore, the pipe line was designed with that quantity in view.

DETERMINATION OF TYPES OF PIPE LINE

Prior to 1925, various studies had been made to determine possible means of conducting water from La Ola River to Potrerillos. Three plans were considered: One for a high-pressure steel pipe line; one for a reinforced concrete pipe; and the third for a continuous wood-stave pipe.

The result of the preliminary work was a recommendation to construct a diversion dam about 1.5 miles up stream from the present site of the La Ola Dam and to use continuous wood-stave pipe for the pipe line. Through the greater part of the distance this line was to follow a location for light pressures, and the plan called for the use of lap-welded steel pipe at points of heavy pressure and in inverted siphons. This arrangement involved a line 39 miles long and was considered of sufficient merit to warrant the establishment of a definite location, and the driving of tunnels. A total of seven tunnels had been planned; two were more than a mile long and the others varied from 250 to 1 270 ft.

On January 1, 1925, the work of driving these tunnels had been in progress for several years and they had all been completed except about 2 000 ft. in Pedernales Tunnel which has a total length of more than 1 mile.

During the prosecution of this work, a considerable length of 6-in. wood-stave pipe failed. It had been laid in soil having a chemical content similar to that to be traversed by the Ola line. This resulted in a decision to use steel in the construction of the new line. The difference in ocean freight and customs duties on fabricated pipe and steel plates made it necessary to manufacture as much of the pipe as possible on the site of the work. This is illustrated in Table 2.

Factors recommending the use of riveted pipe in preference to electrically welded pipe were: (1) The class of labor available; (2) the speed of fabrication that might be maintained; (3) the quality of workmanship to be obtained with unskilled labor; and (4) the joining of pipe in the field where electric power would not be available. In order that the life of the pipe would be equal to the requirements of the plant, and in order to construct pipe of sufficient rigidity, for handling and transportation, it was deemed advisable to specify $\frac{3}{16}$ in. as the minimum thickness of metal.

TABLE 2.—Comparison of Costs Between Lap-Welded and Riveted Pipe.

Pipe.	Per foot.	Per ton.
Riveted pipe assembled on job	$ 4.33	$ 84.20
Light-weight lap-welded joint pipe purchased in United States	8.20	167.50
Heavy-weight lap-welded joint pipe purchased in United States	19.15	172.90

At the beginning of construction work the problem was encountered of using the tunnels which already had been driven at a grade laid down for the location of a wood-stave pipe line, and to adapt them to a hydraulic grade that would be consistent with the use of steel pipe. To complicate this problem the investigations made in 1925 indicated that the upper dam site, previously approved for use with the wood-stave pipe design, was unsuitable. This made necessary the use of the lower dam site where a previous structure had failed. This site was very close to the grade of the tunnels.

A bold location, therefore, was adopted. As few horizontal angles as possible were used; heavy pressures were taken in preference to unnecessary length; and the line was made as short as the topography would permit. This threw the line into a series of inverted siphons, some of which are under far greater pressure heads than any previously attempted in a pipe line of this character.

The line, as designed and constructed, therefore, is unique in two features: (*a*) It involved the use of heavy siphons; and (*b*) the greater part of the pipe was fabricated on the ground.

Available Labor

The Chilean laborer, when properly managed, compares very favorably with the laborer in the United States; but to obtain efficiency, certain expedients must be used in making work an object. With work on an hourly or daily rate of pay the results obtained are not satisfactory; but when placed on a piece-work system, or "contrato", the Chilean laborer works steadily and efficiently. With these characteristics in view, methods of handling labor on the Ola Pipe Line were revised to suit the needs of the situation. Much of the work was done on a strictly piece-work basis. Often a "capataz", or foreman, would be given a contract and he, in turn, would divide his profits with the laborers. Special contracts were entered into with men or groups of men. In other cases bonuses were paid for certain work accomplished.

A separate and important class of labor on this work were the cartmen or "carreteros" who drove the mule carts which were used exclusively for transportation of pipe and supplies. These men are almost a distinct class, and their work is a profession among them, which is handed down through generations in the same family. They consider themselves superior to the ordinary laborer, and will not do his work. They are a tough, hardy set of men who provide their own meals on the road and sleep under their carts at night, regardless of the weather.

There are few rough-work, or form, carpenters in Chile, but high-grade carpenters are rather plentiful. They are especially adapted to cabinet-making or work of a similar nature. They are slow, but they turn out a splendid product.

Mechanics are not plentiful, and it became necessary to recruit men from the better grade of laborers and train them as riveters. These people are naturally quick and skilful with their hands, so that by confining them to one class of work and putting that work on a contract basis, they would quickly become efficient.

The higher class of engineers and foremen on this work were in some cases contract men brought from the United States, and the others were secured in Chile.

The money values quoted in this paper are reduced to United States currency at the various rates of exchange existing throughout the period that the work was in progress. For practical purposes 8.25 pesos equal $1.00. The average 8-hour daily wages paid throughout this entire work were as follows:

Blacksmiths and helpers	$1.41
Boiler-makers and helpers	1.61
Carpenters and helpers	1.47
Chauffeurs	1.38
Clerks	2.66
Cooks and helpers	1.53
Electricians and helpers	1.74
Engineers and draftsmen	8.36
Foremen	4.52
Harness makers and helpers	1.32
Iron workers and helpers	1.53
Laborers	1.22
Machinists and helpers	$1.69
Masons and helpers	1.44
Miners: Hand	1.28
Miners: Machine	1.54
Painters and helpers	1.40
Pipe-fitters and helpers	1.85
Rodmen and levelmen	1.71
Teamsters and stablemen	1.50
Tinsmiths and helpers	1.43
Watchmen	1.36
Welders and helpers	1.69
Wheelwrights	1.48

The average wages, including bonus on contracts, were $1.57 U. S., or 12.95 Chilean pesos.

When work began on the Ola Pipe Line there was a decided scarcity of labor, due to the fact that all the work, in and about Potrerillos, was being started at that time. The ultimate requirement for the Company's entire project was to be about 10 000 men. The isolated location, slow transportation facilities, and incomplete organization in the beginning made the mobilization of labor forces slow and difficult. The quality of labor secured in the beginning was also of a low order. There was no choice except to take all who came, many of whom were drifters, and men who had no intention of doing a day's work. This, combined with the high altitude which affected many, the severe climatic conditions, and lack of proper clothing, made the turnover of labor reach as high as 40% each month at the beginning of the work.

There was a continual demand for more labor on this work from its start until September, 1926, when the first necessary reduction of forces was begun. The maximum number of men on the job was reached in April, 1926, at which time the payroll showed 1 310 employed. Shortly after that date an epidemic of influenza made a considerable reduction in the forces, and that number was not reached again.

PIPE ASSEMBLY PLANT: STEPS IN THE PROCESS

All riveted pipe and special shapes were fabricated at the assembly plant, which was located in the bottom of Pasto Cerrado Canyon where the pipe line intersects the Potrerillos Railway (see Fig. 1). This is the lowest point on the entire line and is about 2 000 ft. lower than the outlet.

The canyon is about 650 ft. wide at the bottom and has a gradient of about 4 per cent. The walls are 1 800 ft. high on the north side and 2 500 ft. on the south. The Potrerillos Railway runs along the northern edge of the canyon bottom, and a spur was run at the end of the plant in such a way that, by utilizing the 4% grade, tracks were dropped from this spur so that all materials could be transported through the assembly plant by gravity.

The plant equipment consisted of layout benches, punches, shears, drill-press, forges, rolls, air compressors, oil rivet-heaters, assembly platforms, dipping-tanks, and the necessary trackage, trolleys, and derricks for handling the materials through the plant.

Twelve distinct operations carried the steel plates through the plant and finished them into pipe of approximately 20-ft. lengths. Each operation was performed by men specially trained for that particular work. Practically every step was done on a piece-work basis and the gangs of men were so proportioned that the work ran through the plant at about an equal speed for each operation. The itemized costs given in Table 3 are the averages for the eighteen different classes of pipe manufactured.

Receipt of Materials.—All steel plates and other materials were received on railway cars, dropped into the assembly plant yard, and "spotted" at the proper unloading point. Many cars of plates contained loads of mixed sizes and thicknesses. Laborers were kept busy sorting this material but the size of the gang varied with the requirements. These men would work at any time, night or day, when cars were received for unloading. The pay for unloading and sorting plates and other material was from 40.00 to 60.00 pesos ($4.85 to $7.25) per car, depending on the class of material to be unloaded. (See Table 3, Item 4.)

Layout Work.—The second process consisted of transporting plates on push-cars from the stock piles to the layout tables and there marking them for the punches.

For straight pipe, pilot holes were first made with screw punches from a template which squared the plate. Later, these were used to hold the plate in proper position in the gang punches. This was done by drift-pinning the plate through the pilot holes to a corresponding hole in a spacing table erected in front of each punch. These pilot holes were placed in the regular position of rivet holes and, later, fell into the line and spacing of the regular punching.

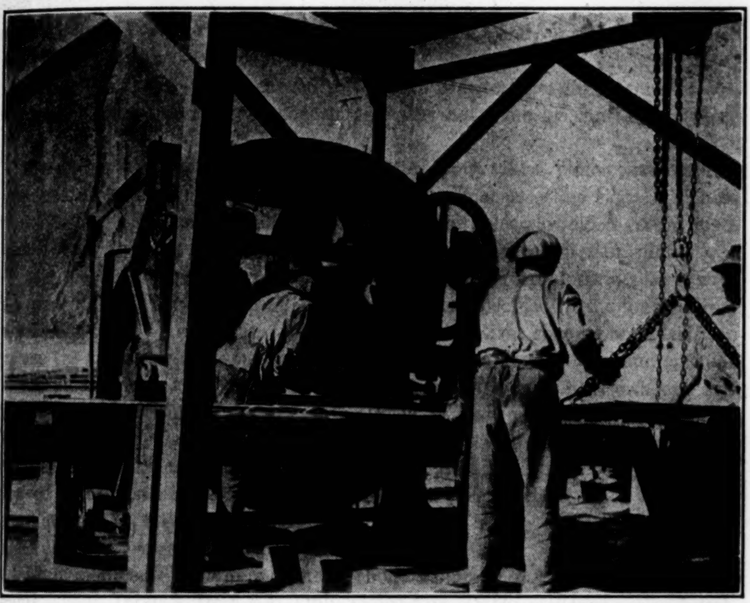


FIG. 5.—PUNCHING PIPE PLATES WITH GANG PUNCH.

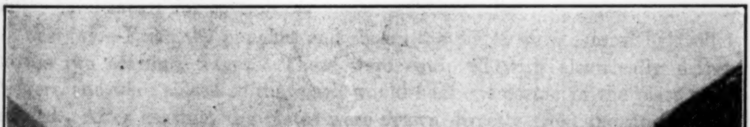


FIG. 6.—ROLLING PLATES IN THE BENDING ROLLS.

(See Table 3, Item 5.) The price paid for handling and pilot-punching plates for straight pipe was 0.60 peso per plate.

Special layout work for curved pipe, valve connections, flange connections, expansion joints, and special length pipe was not done by contract or piece work.

Punching and Bevel Shearing.—From the layout tables, plates were handled over chain-block trolleys to the two 62-in. gang punches. (See Fig. 5.) These punches had a capacity to make thirty-six $\frac{7}{16}$-in. holes through a $\frac{7}{16}$-in. plate when evenly spaced, or as many as nine $\frac{13}{16}$-in. holes through $\frac{3}{8}$-in. plate.

All the pipe designs were adapted to using male and female plates with no taper in the plate sections; double riveting was specified in the longitudinal seams and single riveting in the circular seams. Rivet spacing was designed to use the same size and spacing of rivets in both the longitudinal and circular joints so that one type of plates could be completely punched with one setting. One gang punch was always assigned to male plates and the other to female plates of the same type; and they were set so that the holes would correspond on the circular seams of the two plates. This method of punching proved to be fast and accurate, and scarcely any reaming of holes was necessary, except at the scarfed corners of the plates where the accuracy of the holes had been upset in the process of scarfing.

Four men could handle and punch about 8 plates per hour with this equipment. (See Table 3, Item 6.) Only plates $\frac{1}{2}$ in. thick, and more, were bevel-sheared for caulking. These plates went directly from the punches to the rotary bevel shears for cutting.

Scarfing.—From the punches and shears the plates were carried by trolley to the two scarfing forges. These were equipped with electrically driven blowers, and were placed so that each would heat one corner of the plate to be scarfed. After heating, the plates were drawn directly from the fires to two anvils where the scarfing was done by hand. Four men would scarf about 9 plates per hour at the rate of 0.50 peso per plate. (See Table 3, Item 7.)

Bending Ends of Plates.—Before being turned through the rolls it was necessary to bend the end of the plates to conform with the pipe radius; otherwise, the rolls would leave a flat section at each end of the plate. For doing this work, forms were constructed with the proper bend, upon which the plates were held by clamps. The ends were bent by hammering the plates cold over these forms with 12-lb. hammers. Four men could bend both ends of the plates at the rate of about 9 plates per hour. The piece-work price paid for this work was 0.50 peso per plate. (See Table 3, Item 8.)

Rolling.—The plant was equipped with a 5-ft., motor-driven, reversible bending roll for rolling plates to the pipe radius. (See Fig. 6). One operation through these rolls would bend the lighter plates to the proper radius, the heavier plates requiring two or three operations to bend them to shape. A force of four men could handle and roll an average of 9 plates per hour at the rate of 0.50 peso per plate. (See Table 3, Item 9.)

Assembling.—From the bending rolls the plates were rolled by hand to the assembly platforms. These platforms were constructed in such a manner that the pipe would rest on a system of disk rollers, on which the pipe could be turned without being moved from the platforms. Pipe of four plate lengths, or a laying length of 19 ft. 5 in., were assembled and bolted together on these platforms. Circular joints were made without belling and all joints were assembled, metal to metal, with no paint between. Such little reaming as was necessary at the scarf corners was done immediately after the assembly of the pipe, and this cost was included in the unit price. Four men would average the assembly of 1 pipe made up of 4 plates, per hour. The price paid for this work was 6.50 pesos per pipe of 4 plates. (See Table 3, Item 10.)

TABLE 3.—MANUFACTURE OF RIVETED STEEL PIPE IN THE FIELD.

Item.	Steps in manufacture.	COST, U. S. DOLLARS.		Man-hours per mile.	Remarks.
		Per ton.	Per mile.		
1	Flat plates (F. A. S. Chanaral)	$42.50	$11 560.00		
2	Customs duties	6.30	1 713.60		
3	Delivery to job	7.50	2 040.00		Lighterage, dockage, and railroad freight.
4	Unloading and sorting	0.18	48.96	177	
5	Layout work	0.33	89.76	326	
6	Punching and bevel shearing	0.44	119.68	544	
7	Scarfing	0.28	76.16	484	
8	Bending ends of plates	0.28	76.16	484	
9	Rolling	0.28	76.16	484	
10	Assembling	0.87	236.64	1 088	
11	Riveting	7.04	1 914.88	5 712	Cost includes rivets.
12	Caulking and welding	0.32	87.04	340	
13	Inspection	0.09	24.48	245	
14	Dipping	7.37	2 004.64	272	Cost includes asphalt and fuel.
15	Assembly plant equipment	6.16	1 675.52		Cost and erection.
16	Miscellaneous supplies and expenses	4.26	1 158.72		Power, tools, etc.
	Totals	$84.20	$22,902.40	10 156	

Riveting.—Rivets, $\frac{3}{8}$ and $\frac{7}{16}$ in. in size, were driven cold and by hand, and the heads were snapped and finished to a reasonably smooth button. Those more than $\frac{7}{16}$ in. in size were driven hot with air hammers.

All rivets were bucked up from the inside of the pipes by means of a bucking iron held in place by a wooden lever. The weight of the man who sat on the end of the lever would provide the proper pressure to the bucking iron.

Air for rivet hammers and forges was provided by two motor-driven compressors with a total capacity of 550 cu. ft. per min. A gasoline-driven compressor of 250 cu. ft. capacity was used as a stand-by for times when no electric power was available. Rivets were heated in a series of oil forges distributed through the riveting sections of the plant.

The riveting gangs consisted of three men, and the average number of rivets driven was 65 per hour per gang. The prices paid ranged from 7.00 to 9.00 pesos per hundred rivets, according to size. (See Table 3, Item 11.)

Caulking.—All joints between plates heavier than $\frac{3}{16}$ in. were caulked with air hammers. In the beginning of the work and before the first hydraulic test was made in the field, all pipes, including those made of $\frac{3}{16}$-in. plate, were caulked at the scarfs. The first hydraulic test showed scarf caulking to be ineffective and many pin-leaks developed. Thereafter, all scarfs were electrically welded, which completely eliminated this defect. Caulking and welding were done on a "per day" wage basis because inspection of this class of work is difficult. (See Table 3, Item 12.)

Inspection.—General inspection of the work was made by the foremen in the plant and special inspection of rivets, caulking, punching, and assembly was made by men who also kept accounts on piece-work contracts. Test pipes were also made, with bulkheads in the ends, and these were subjected to hydraulic pressure to twice their working capacity. (See Table 3, Item 13.)

Dipping.—After they were riveted and inspected, all pipes were numbered and marked to show the type and were then placed in a storage yard. From this yard they were rolled by hand to the dipping tanks as they were required for use.

The two dipping tanks, with a capacity of four pipes each, were constructed of steel plates, surrounded with adobe. These tanks rested upon furnaces built of fire-brick; they were equipped with odometers and were heated by oil burners.

The pipe dip used was a high-grade asphalt. When this was heated, a special flux was added to an amount of about 2% of the asphalt. The temperature of the dip was maintained at from 375° to 400° Fahr. It has a flash-point of about 425° and caught fire twice, due to carelessness of the furnace operators. The first fire resulted in the loss of practically all the mastic in the tank as well as the pipe that was in the bath. Prior to the second fire a steel cover for the tanks had been constructed, and with this the fire was quickly smothered.

The pipe was dipped cold and allowed to remain in the tank about 20 min. to bring the temperature of the metal up to the temperature of the bath. This resulted in a very smooth, shiny coating, about $\frac{1}{32}$ in. thick, which had an excellent bond to the metal, was not brittle in cold weather, not too soft in hot weather, and withstood well the rough handling received in transportation. After almost four years of use this coating has now (June, 1930) resisted the abrasive and other effects of the water and is in excellent condition.

The two dipping tanks were served by a derrick which was used for placing the pipes in the tanks, for removing them, and for standing them on the drying platform. The pipes were held over the tanks until the drip had stopped and were then stood on end to cool. As soon as they reached the temperature of the air they were ready for use, because this dip required no further drying. The labor of dipping was paid for at the rate of 2.00 pesos per pipe. (See Table 3, Item 14.)

Special Pipe.—All special pipes were laid out and fabricated to fit the particular point for which they were made. Curves were limited to a maximum of a 5° angle for a single joint between 5-ft. plates; hence a length of pipe, of four plates, was enough for any angle up to 15 degrees. Angles of less than 5° were made in one joint. Angles between 5° and 15° were made in three equal joints, and those of more than 15° were extended to as many joints as were necessary to cover the angle. Field surveys and excavations followed

these rules, so that when the Special Plant received an order for a curve of a certain degree angle, no further information was necessary to show how that pipe should be laid out to fit the ground.

Special lengths of pipe were required for making closures between sections of pipe-laying and for fitting curves of high degree to the ground. These had to be specified, together with the type of ends, whether male or female, and had to be turned out at the plant in a short time to prevent delays in the pipe-laying. Holes for air and drain valves were cut out with acetylene torches and all saddles were caulked after riveting. Expansion joints were manufactured in the United States and shipped to Chile with blind companion flanges. These flanges had to be punched to fit the type of pipe to which they joined, and were riveted at the plant, to one plate length of pipe.

The fabrication of "specials" was carried on in a manner similar to that of straight pipe except that the Special Department was equipped with a double-end punch and shear for cutting and punching curved joints, and with acetylene welders and torches for cutting and welding. All "specials" were tagged as they passed through the dipping tanks, and as soon as they cooled from the dip they were lettered with white paint with their line station numbers.

TABLE 4.—SUMMARY OF PIPE LINE COSTS.

	Item.	UNIT COSTS, IN DOLLARS (8.25 PESOS).			
		Materials and supplies.	Labor.	Expenses.	Total.
	Cost of Pipe, Including Specials:				
1	Assembly per foot	$ 2.88	$ 0.85	$ 0.27	$ 4.00
2	Dipping per foot	0.24	0.09	0.00	0.33
3	Total, Items 1 and 2	$ 3.12	$ 0.94	$ 0.27	$ 4.33
4	Total, Item 3, in dollars per ton	$60.60	$18.40	$ 5.20	$84.20
	Cost of Field Transportation:				
5	Per ton-mile	$ 0.37	$ 0.32	$ 0.08	$ 0.77
	Cost of Excavation and Back-Filling:				
6	Total per linear foot	$ 0.08	$ 0.61	$ 0.02	$ 0.71
7	Total per cubic yard	0.17	1.28	0.04	1.49
	Cost of Tunnel Driving (1 942 ft.):				
8	Total per linear foot	$13.72	$12.34	$ 0.54	$26.60
9	Total per cubic yard (3 100 cu. yd.):	8.59	7.73	0.34	16.66
	Cost of Tunnel Lining:				
10	Total per linear foot (7 196 ft.)	$ 2.87	$ 3.88	$ 0.38	$ 7.13
11	Total per cubic yard (5 300 cu. yd.)	3.83	5.27	0.52	9.68
	Cost of Pipe Laying:				
12	Total cost per foot	$ 0.12	$ 0.84	$ 0.09	$ 1.05

Approximately 8% of the total pipe fabricated was in "specials." In Table 4, showing a summary of pipe-line costs, "Expenses" include electric power, railway freights, railway passages, and various other small items.

Rate of Progress.—Activities began at the "Sifon Assembly Plant" in March, 1925, but such work as was possible at that time was confined to the construction of camp buildings, and the laying out and grading of the assembly plant and yard. At first, the shortage of labor and material retarded progress.

It was not until July, 1925, that the first of the equipment was received, and it was nearly the end of August before sufficient equipment had been received and erected to start assembly work on the pipe.

The total period of manufacture extended from August, 1925, to August, 1926. In that time 131 200 ft. of riveted pipe were fabricated, or an average of 360 ft. per day. This rate required the use of 75 plates and the driving of about 8 100 rivets per day.

The maximum rate of fabrication was reached in April, 1926, when the monthly average was 640 ft. per day, which required the daily use of 130 plates and the driving of 15 000 rivets.

TRANSPORTATION

The largest single item of labor cost for the construction of the Ola Pipe Line was that of transportation, and except for the cost of pipe material, it was the largest item of the entire work.

Those who are unfamiliar with South American and Chilean conditions can hardly realize the tremendous undertaking required to transport the 22 040 tons of materials and supplies that were used.

The Assembly Plant was the freight-receiving station for all materials and supplies for the entire job. At this point the general store house, and the yards with sufficient capacity for handling all materials, were located. It was 28 miles, by pipe line, from the intake at La Ola Dam, and 4.3 miles from the outlet of the pipe at Potrerillos. The distances by rail were 83.9 miles from the Port of Barquito and 12.4 miles from Potrerillos. As previously stated, the plant was built at the bottom of a deep canyon and at the point of lowest elevation on the entire pipe line, which meant an uphill haul from there to all parts of the line. The location chosen, however, was the best possible because railway facilities were necessary for incoming materials, and this location was at the only intersection of the pipe line and the railway.

Existing roads, which in reality were little more than trails, extended from the Assembly Plant down Pasto Cerrado Canyon to Jardin, a distance of 4.3 miles, and from there zigzag roads had been constructed up the canyon sides, one leading south to Potrerillos and the other going up the opposite canyon wall, and thence to Pedernales and La Ola. From the Assembly Plant to the rim of the canyon the vertical distance is 1 700 ft., and the haul by road is 12.4 miles, of which 3.7 miles is a switch-back road on a 10% grade. This haul is a full day's work for a mule cart. The construction of the inverted siphon across Pasto Cerrado Canyon required the construction of inclined tracks paralleling both legs. It was decided, therefore, to build the north leg track with the purpose of using it for hoisting all pipe and supplies to the top of the canyon wall.

This inclined railway was built with a vertical lift of 1 600 ft. in a horizontal distance of 2 950 ft. The average slope was 27° and the maximum slope, 46 degrees. The roadbed was cut through limestone and conglomerate throughout the entire distance. The lower end of the track began at a sloping trestle about 328 ft. long which made an overhead crossing over the railway and landed at the top of an almost vertical cliff of limestone. The track had a gauge of 39.37 in., which corresponded with the gauge of the Potrerillos Railway. The rails were laid on ties in the usual manner, but were anchored at each rail-joint to an eye-bolt grouted into the rock of the roadbed. On the trestle the ties were drift-bolted to the stringers.

This incline was equipped with a 10-ton electric hoist having a drum, 5 ft. in diameter, which carried 3 800 ft. of 1-in. wire rope. Flanged cast-iron rollers, 2 ft. long and mounted in bronze bearings, were placed at all points where the loaded cable would drag.

The car used on this incline was 20 ft. long and had a capacity of 12 tons. Its traction pull was from the center through a bridle that could be raised and lowered at the cable end to conform with the irregular grade of the track and to prevent lifting the front wheels of the car from the rails.

The hoist had a rope speed of 200 ft. per min. and the average trip, up or down, consumed about 20 min. With loading and unloading, a round trip of the car required about 1 hour. For much of the time this car ran night and day and handled as much as 2 314 tons in 1 month.

All the materials for the construction of the La Ola Dam, 27.9 miles of pipe line, the Montandon Power Station, tunnel lining, the stock feed, and camp supplies for all the work at the dam end of the line, passed over this incline.

Sifon Alto Camp was at the head of this incline railway on a sloping mesa or ridge about 0.62 mile wide which drops away to the north into another canyon of about the same depth as Pasto Cerrado. At this camp there was ample room for a large storage yard for pipe, corrals for mules, shops for blacksmiths, cart repairmen, and harness makers, and for the necessary camp buildings for housing the men.

Sifon Alto was headquarters for all the animals used on the job and for all cartmen. All cart and harness repairs were made here and all feed for the animals passed through this camp. The maximum number of animals in service was more than 600. All water for animals and men had to be hoisted to this camp from the Assembly Plant which was supplied with water from a pipe line constructed to serve the railway.

From Sifon Alto the cart road followed a ridge for about 4½ miles at about a 4% grade; then it meandered and zigzagged along side hills at about a 10% grade for a distance of 7½ miles to the top of Piñon Pass which has an elevation of about 13 000 ft. From Piñon Pass the road first drops and then rises a second time to Llanta Pass in a distance of about 2½ miles, and then drops abruptly to an elevation of about 11 250 ft. at Pedernales Camp, which is at the bottom of a deep valley between two mountain ridges.

Pedernales and Llanta Tunnels were driven through these two ridges. They were each more than 1 mile in length and were the destination of much trans-

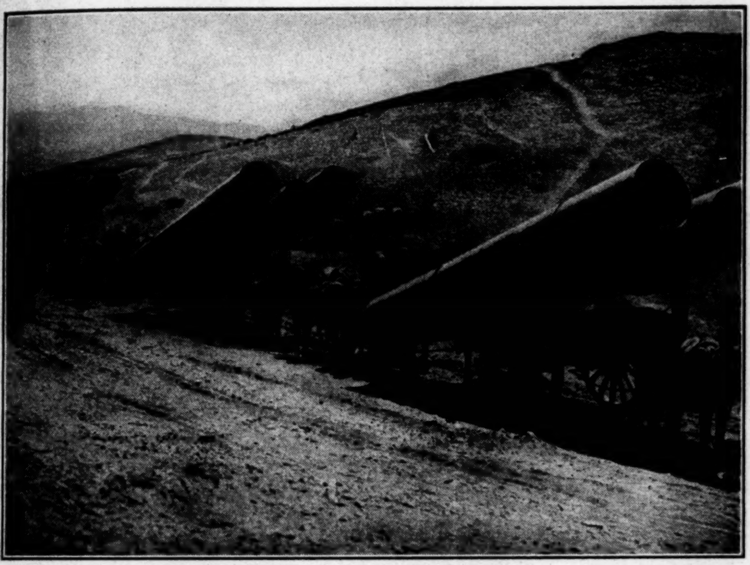


FIG. 7.—THE NOON STOP AT REFRESCO.

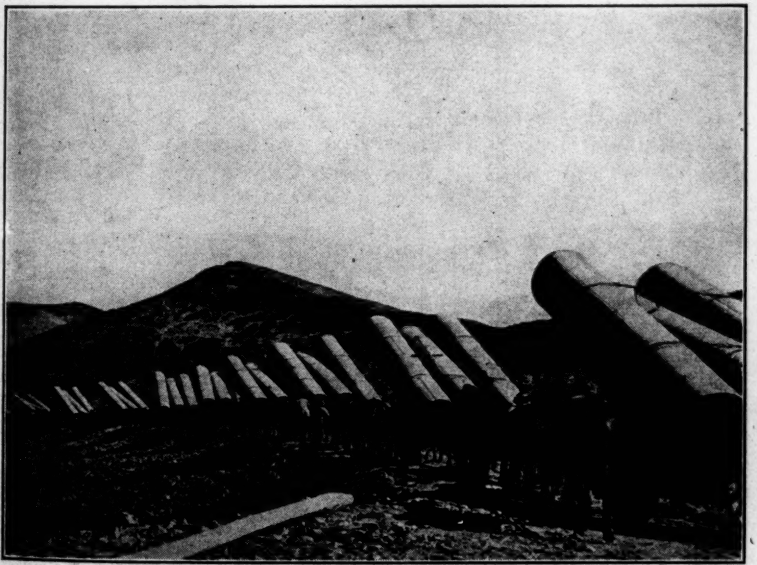


FIG. 8.—TRAIN OF PIPE CARTS RESTING ON PINON PASS. ELEVATION, 13 000 FEET (3 962 METERS), AND THE HIGHEST POINT ON THE ROAD.

[illegible]

[illegible]

ported material. The camp consisted of a staff house, bunk houses, store, warehouse, mess house, first aid station, corral, and blacksmith shop. Water in sufficient quantities to supply the corrals and camp, was available from shallow wells. A slaughter house was also located at this camp where all meat for the upper end of the line was butchered. Beef and mutton provided their own transportation to this point.

From Pedernales Camp the road followed down the course of Pedernales Canyon for about 3¾ miles, on about a 2% grade, and from there it led to the open La Ola Pampa, which is a flat valley 6 to 12 miles wide, with a sand and gravel floor. This pampa rises at about a 1% grade toward La Ola Dam and the intake of the pipe line. About 15.5 miles of this pampa had to be crossed to reach the dam and no attempt was made to maintain roads because the character of the country was such that carts could break new tracks as fast as the old ones became impassable. The pipe line was located along the western edge of this pampa and gradually reached the bounding ridge of mountains as the grade dropped away. The cart roads, therefore, followed the pampa as far as practicable, and then turned in to accessible points on the pipe line, with no attempt to follow any established trails.

The Chilean mule cart, which was used exclusively on the work of transportation of pipe and other materials, is at first glance a very inefficient piece of equipment. However, it has been developed through generations of use, and proved in this instance to be well adapted to the bad roads, steep grades, deep ruts, and sharp turns over which material had to be hauled. It is a heavy vehicle, with two wheels about 6 ft. in diameter, and a gauge of 5 ft. (See Fig. 7.) The tires are 5 in. wide and 1 in. thick. The hubs are of cast steel, and the axles are made of 3-in square metal. The car is so constructed that the load is balanced equally, fore and aft, on the axle. At the front and rear are two swinging props on which the cart is balanced while being loaded or when the team is resting on the road. A brake-beam with large wooden brake shoes operates against the rear of the wheels by means of a screw stem and large wing nut called a "mariposa," or butterfly. To operate this brake the driver has to run behind the car and turn up the screw. The brake is powerful and can be set at any tension desired.

When in motion the cart and load are balanced by one mule that is hitched between rigid shafts. This mule's harness consists of a heavy halter, a collar with chain traces hooked to the front of the shafts, and a padded saddle with a heavy cinch to which the shaft ends are bound in a manner to balance the cart by the weight of the mule. The mule also wears a heavy breeching which is attached forward to the shafts so that he is hitched firmly forward, back, up, and down. He acts as a flyball for the entire unit and is either carrying a heavy load on his back or is scratching for a toehold. The drivers are remarkably clever in balancing their loads, however, and the outfit performs better than might be expected. The mules are hitched three abreast to these carts, one on each side of the shaft mule.

The harness of each of the other mules in the team consists of a collar and chain traces which are held up by a padded saddle. The traces of the wheel mules are hitched to single-trees and no eveners are used. The traces of the lead mules are hitched to the traces of the mule behind at the saddle. No bits are used, and no lines, except a jerk-line to the lead mules' halters which is pulled to stop the team or when making a turn. The driver rides the left-hand wheel mule on a high saddle and drives his team with a whip. This hitch is ingenious and very flexible, providing for quick changes and additions to a team, and can be strung out to any number of animals. On this work as many as twenty-four mules were used on one hitch.

A driver will ordinarily handle six or seven animals and the best drivers, nine. In transporting pipe, with a load of about 5 500 lb., in addition to the weight of the cart, one driver and one helper were used for a 9-mule team. On this work it was necessary to have one man to handle the brake while the other drove the team.

Pipe carts were constructed with a rack in front, wide enough to carry three 36-in. pipes side by side. This rack raised the front end of the pipes and held them in a slanting position with the rear end high enough to clear the irregularities of the road. The pipes were bound to the carts with chains tightened with turnbuckles and were held from slipping back by chains hooked into the ends. The pipes, being about 20 ft. long, extended over the wheel mules and driver and produced such a teetering motion that an extra shaft was added to the right side of these carts so that two mules could be used in the shafts to balance the load.

Each cart carried its ration of hay and feed for the entire trip which, on the longest end of the haul, took four days to complete. The mules were allowed 15½ lb. of hay and 6½ lb. of grain per day; working seven days a week, they kept fat on this ration.

At some points of the line, particularly in the neighborhood of La Tabla and Huacho Tunnels, the road was as much as 1¼ miles from the line of the pipe. At these points the loads would be carted as far as possible along the ridges and from the point of unloading the pipes would be rolled to the line. At many points, it was impossible to deliver pipe by cart above the line, and at these places it had to be rolled uphill. Where any great number had to be handled in this manner the pipes were rolled with ropes and blocks.

Between the Assembly Plant and Potrerillos the pipe traverses the canyon side across the roughest and most inaccessible part of the line. To make deliveries throughout this section, pipe had to be loaded on railway cars and transported part of the distance by rail, and then hauled to the line on carts. In order to do this, 4⅓ miles of new, side-hill road had to be constructed through a steep and difficult country. This road had one main entry to the railway where the greater part of the pipe was taken from the cars, hoisted to the top of a ridge about 100 ft. high, and there loaded on the carts with a steam derrick. From this entry the road followed the top of a long ridge at a grade of about 10%, to a point above the pipe line where it branched in both directions, going at a level grade to Sifon Sur at one end and to San Augustine Canyon at the other.

San Augustine Canyon presented the most difficult problem for the delivery of pipe of any point on the line. A double siphon crossed this Canyon and the ridge between the two outside legs was inaccessible to any form of transportation except pack animals. It would have been necessary to construct about 3 miles of difficult road to reach the central ridge. The problem of delivering pipe to this section was solved by the use of a low two-wheeled cart. The pipe was balanced on the cart and was pulled to the top of the ridge with a hand winch. It required the work of 2 men for 16 hours to hoist 1 pipe to the top. The vertical lift was 325 ft. in a horizontal distance of about 500 ft. About 1 000 ft. of pipe were transported in this manner and it took about 30 days to make the delivery.

Trains of pipe and supply carts (see Fig. 8) were started each morning from Sifon Alto under the supervision of a leader who had complete charge of the train en route. These leaders were men who had been advanced from drivers, were thoroughly competent and reliable, proud of their responsibility, and who would take their trains through the trip and without delay under almost any circumstances. Each day's trip was laid out for the drivers, and if they completed that distance they were paid a bonus of 25% more than their regular pay. Regardless of the reason, the driver, unless he completed his trip, received no bonus. This made them care for their carts, harness, and mules, and assume entire responsibility for their outfits. The result of this bonus system was that very few drivers would fail to finish their trips.

The average day's haul, up grade and with a load, was about 15½ miles. Returning, down grade and with no load, the average was about 25 miles per day.

The haul to La Ola Dam and the upper end of the line required four days for the round trip. From La Cabra Tunnel the time was cut to three days and from Perro Muerto Tunnel, to two days. Train leaders would start from Sifon Alto at 7:00 A. M. with as many as twenty-four carts and would arrive at Pedernales at about 5:00 P. M.

Pipe carts were unloaded in the field by throwing off the binding chains and sliding the pipe to the rear until the ends rested on the ground. This would throw them out of balance so that the cart could be driven out from under them. Table 4, Item 5, includes the construction of 4.34 miles of road, the maintenance of 19.88 miles, the purchase of carts and harness, all maintenance of equipment, the loading, unloading, and operation of the incline car, and all regular transportation expenses.

TRENCHING AND BACK-FILLING

In the high altitudes where the Ola Pipe Line was constructed an extreme daily variation of temperature exists. To prevent freezing and to reduce expansion and contraction it was decided that the pipe would be placed in a trench and covered throughout its entire length, except across the Montandon Siphon, where it would be supported on piers and anchors.

On the Ola Pampa, where the line is exposed to extreme weather and violent winds, it was planned to cut a trench of sufficient depth to place the pipe entirely below the surface of the ground. In less exposed locations and at lower altitudes the depth of the trench was planned to be equal to about two-thirds the diameter of the pipe, and the excavated material was to be used for covering. Figs. 9 and 10 illustrate conditions along parts of the line.

All trench excavations were made by hand. The material was such that only very heavy excavating equipment would have sufficed, and the use of such equipment was prevented by the roughness of the country. The ground along about 50% of the line required blasting before it could be excavated, and much of the trench was through solid rock.

Practically all the excavation was done by contract, the men being paid according to the classification and quantity of material moved. The earth was classified as earth, hardpan, or solid rock, and measurements of the different classifications were made after the excavation had been completed. A gang or group of men would be given a contract for excavating the trench between certain stations on the line. They would be required to excavate and finish that section, and would receive no liquidation of their contract until the work was finally finished.

The proper width and shape of the trench was maintained by the use of light wooden templates which were marked along the sides in such a manner that the area of the cross-section could be read from them direct. By this method the quantities and classification of excavated materials could easily be recorded and checked.

Trench excavation was started in July, 1925, and was finished in August, 1926. During the greater part of this time two gangs of from 60 to 100 men were employed.

Through the greater part of the work the back-filling was done by hand. The dirt was tamped under the invert of the pipe with shovels to prevent settling. Where the pipe followed side-hills, the fill on the down-hill side was maintained with dry rubble walls. On the pampa where the pipe was laid below the level of the ground, the top-fill was placed with teams and scrapers. The costs are shown in Table 4, Items 6 and 7. These items include the cost of constructing rubble walls.

LA OLA DAM

A diversion dam was constructed in La Ola River during the summer season of 1925 and 1926, between the months of October and April. (See Fig. 1.)

The dam is approximately 160 ft. long between abutments and raises the water about 10 ft. above its natural level. It is a concrete structure of the gravity type, with a spillway 65 ft. long, an outlet chamber and head-gate, and one sluice-gate for drawing off sediment from the reservoir.

TUNNELING

When work on the Ola Pipe Line was begun in March, 1925, 1 942 ft. of Pedernales Tunnel remained to be driven. This section was 1 190 ft. from the west portal and 1 670 ft. from the east portal of the tunnel. The completed portion at the west end was through a hard blue limestone which stood up well without timbering and required no lining to make it water-tight. The section of completed tunnel at the east end was through bad ground for 1 640 ft., but

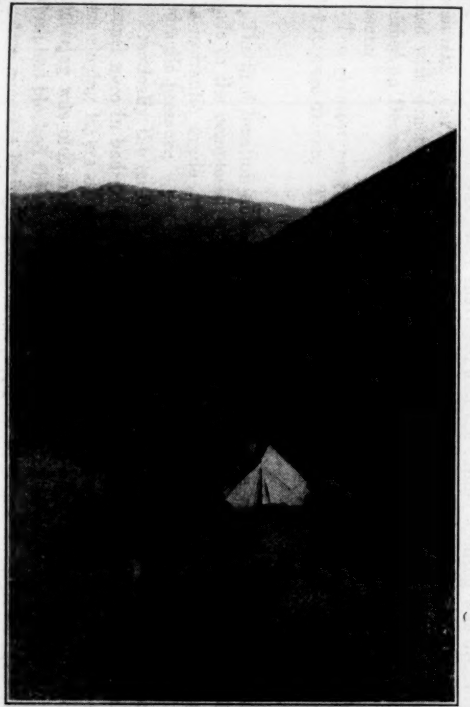


FIG. 9.—PIPE TRENCH EAST FROM PEDERNALES TUNNEL.

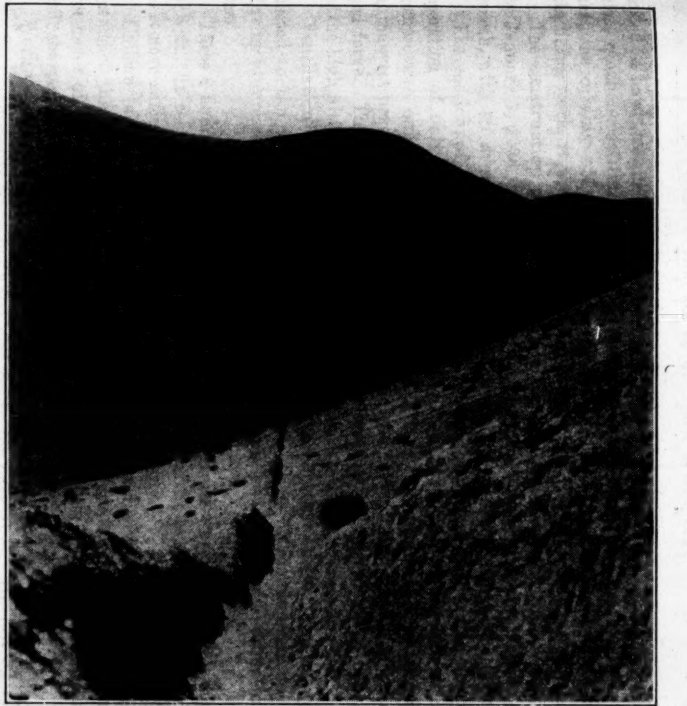


FIG. 10.—PIPE TRENCH BETWEEN PINONCITO AND LA TABLA TUNNELS.

ended in hard granite. The timbering in the first 1 640 ft. was in bad condition, which necessitated the lining of that section to make it safe. In order that the driving of the remainder of this tunnel should not conflict with the lining work, it was decided to complete all the driving from the west face.

The construction program called for the completion of the tunnel driving by January 1, 1926. In order to do this it was necessary to drive 220 ft. per month, with an average haul of more than 2 000 ft. for the material broken.

The section of tunnel was 6.56 ft. square, which allowed room for the operation of only one drill at the face. Two portable gasoline-driven air-compressors, one of 250 cu. ft. and one of 175 cu. ft. capacity, were installed at the west portal to provide air for drilling and for running a drill sharpener and forge. The efficiency of these compressors was only about 60%, because of the high altitude.

Ventilation was provided by a gasoline-driven blower, and fresh air was conducted to the face through canvas vent tubes; these were extended as the driving progressed. Broken rock was hauled by mules in 1-yd., side-dump mine cars.

Three 8-hour shifts were worked at the face. Holes were drilled to a depth of 5 ft. It required two shifts to drill a round of holes, to blast them, and to remove sufficient material to permit further drilling.

Chilean labor, on a contract basis, was used exclusively. The Company furnished the equipment, tools, air, timber, and powder, and paid 150.00 pesos per meter of tunnel completed ($5.53 per ft.), including all labor for drilling, drill sharpening, shoveling, and timbering.

Work was started March 1, 1925, by hand drilling. It was not until May 1 that the compressors were installed and ready for operation. During September, 1925, a fault plane was intersected which opened up quite a heavy flow of water. The ground was badly shattered and the tunnel opening was difficult to maintain. For about 820 ft. the tunnel had to be heavily timbered and, later, lined with concrete. This condition slowed up progress, but the tunnel was finished on December 29, 1925, two days ahead of schedule, and with a perfect closure.

The greatest progress made in any one month was during August when 376 ft. were driven. The total cost is shown in Table 4, Items 7 and 8.

TUNNEL LINING

The final location of the Ola Pipe Line with the necessary adjustments of grade to the completed tunnels, threw Pedernales and Llanta Tunnels below the hydraulic grade of the line, and, consequently, they are under a light hydraulic pressure.

Practically the entire length of Llanta Tunnel and 2 890 ft. of Pedernales Tunnel were in bad ground, composed of a shattered limestone through which were sloping layers of soapstone. Some water seeped into these tunnels; the timbering was old, and several falls of rock had occurred in Llanta Tunnel which had blocked the entries and flooded it for almost its entire length.

As a concrete lining seemed better adapted to inspection and maintenance than the process of running the pipe through these tunnels, it was decided to line the bad sections. On account of the small size of the tunnels it was necessary to use concrete forms that would not obstruct the passage in any manner to prevent the placing of concrete, or the removal of forms from the completed sections. To accomplish this, forms were developed that needed no center bracing, and that left the tunnel bore entirely open between the form ribs. This was done by making the ribs in three sections, consisting of an invert section and two side sections.

A concrete floor was first laid to grade, and upon this, while the concrete was still green, the invert ribs were placed, held in position by bracing against the roof, and then lagged. Near each end of these invert ribs were holes which held a ¾-in bolt. The bolts were fitted with nuts and O. G. washers which were embedded in the invert concrete when the forms were filled. Upon removal of the invert forms the bolts would be tapped lightly and then unscrewed, leaving holes with the nuts and washers embedded in the concrete. After the invert concrete was set and the invert forms were removed, the side forms were erected and held in place at the bottom by screwing back the bolts through corresponding holes in the upper ribs. The two side ribs were held in position at the top by a wedge, which when dropped would loosen the ribs for removal. The lagging for the upper ribs was built up as the sides were filled with concrete. The top lagging was put on in 5-ft. sections, and the concrete was shoved and spaded back from the end until the top was thoroughly filled.

The concrete in these tunnels was reinforced with a ⅜-in., round, reinforcement placed on 12-in. centers both ways and short ⅜-in., round, bars were used for bonding at the construction joint between the invert and the sides.

Sand and gravel were available at the portals of nearly all these tunnels, but had to be separated and screened by hand. Water for concrete was available from the inside except for about one-half the east end of Pedernales Tunnel. For this work the water had to be hauled about 2 miles up a 15% grade. The cement was carted from Sifon Alto to Llanta Tunnel and to Pedernales Camp, and packed from there on burros to both ends of Pedernales Tunnel.

All the concrete for the latter was mixed by hand inside the tunnel and shoveled from the mixing platforms directly into the forms. The concrete for Llanta Tunnel was mixed in a steam mixer at the east portal, and hauled by mules in end-dump cars to the forms.

All the work was done on a contract basis and was paid for at the rate of 80.00 pesos per lin. m. ($3.00 per ft.) This price included the removing of timber, cleaning up the tunnels, screening sand and gravel, placing and removing forms, cutting and placing reinforcement, and mixing and placing concrete.

The progress of the work was more rapid in Llanta Tunnel where the concrete was mixed outside. In Pedernales Tunnel a delay was caused by bad air from the setting concrete. This was before the tunnel had been holed through, and there was no ventilation until a canvas vent-tube and a blower were installed.

The 1 640 ft. of tunnel lining in Pedernales, East, was started in May, 1925, and completed in August. During July, 853 ft. were completed, which was the best monthly average on that work.

The Llanta Tunnel lining was started in August and completed in November. The length of tunnel lined was 4 308 ft., and the best run was made in November, during which 1 801 ft. were finished.

Pedernales, West, was lined in January and February, 1926, the best run being made in February when 1 014 ft. were completed by the 20th of that month.

The moist cool air of these tunnels retarded the setting of the concrete to some extent; but it contributed in producing a splendid lining.

Due to the cheap labor and to the fact that salvaged steel transmission-line cable was used for reinforcement, for which no charge is included, the costs of this tunnel lining are very low. These costs include all labor and material, but do not include transportation. (See Table 4, Items 10 and 11.)

PIPE LAYING

The laying and jointing of the pipe in the field were the final steps in the construction of La Ola Pipe Line, and required the co-ordination of all the work up to this stage of the construction. This co-ordination included preparation of the ground, the receipt of materials and supplies (including expansion joints, saddles, air and drain valves, and those sections of lap-welded pipe which were purchased in the United States), the proper order of assembly, and the transportation of material from the Assembly Plant.

The pipe-laying work extended over so much territory that, even when material was ready for shipment from the Assembly Plant, it often would take two days, or more, to make deliveries to the points where the materials were needed. This part of the work included the laying of eighteen different classes of riveted pipe, fabricated at the Assembly Plant, and seventeen different types of lap-welded pipe, purchased in the United States. Much of the latter was pipe that had to be joined in its proper order because each joint had been manufactured to fit its special location. The delivery of this pipe from the United States was erratic, and a delay in the receipt of a few sections held up the work of laying for about two months. This co-ordination proved to be the most difficult problem with which the field organization had to deal.

Pipe-laying started at the intake end of the line with 36-in. riveted pipe. The first part of the work was across the flat pampa where there were no horizontal and very few vertical curves. The system of laying adopted was to rivet together several 20-ft. sections of pipe outside the trench, and to join such a section to the last that had been laid. As shown in Fig. 11, the end of the finished section and the section being joined to it were held up in the trench with tripods and chain-blocks until the joint had been riveted, and then were lowered into place with the chain-blocks. It was found that a long string of this pipe had sufficient flexibility to permit the use of this method without putting undue strain on the rivets. After riveting and caulking, all field joints were painted inside and outside with hot mastic.

In the beginning of this work more than 12½ miles of pipe were laid before the expansion joints and valves were received. Consequently, the laying was done in sections leaving openings at points where the expansion joints were to be placed. Saddles for air and drain valves had been received in time to be connected to the riveted pipe at the time of fabrication, and the valves were bolted in place after the pipe was laid. Inability to install expansion joints and valves at the time the first pipe was laid, delayed the hydraulic tests on the line until the construction had reached Pedernales Tunnel, a distance of 14¾ miles from the intake.

The first test included the section between La Ola Dam and La Cabra Tunnel, a distance of about 5 miles. This disclosed no breaks in the line and no faulty pipe, but showed a large number of pin-leaks, particularly at the points where the plates had been scarfed and lapped. After this test all the pipe assembled at the plant was electrically welded at the scarfs and much greater care was used in caulking all field joints.

In the rougher country it became necessary to decrease the number of lengths of pipe connected together outside the trench, and it was found that, in very rough sections and on steep grades, better progress could be maintained by handling only one length of pipe at a time.

When expansion joints were finally installed it was found that the openings, which had been left between pipe sections, varied considerably in length, due to expansion and contraction. In practically all cases of installing expansion joints special lengths of pipe had to be provided to produce a proper fit. It was by this method, and at expansion joints, that all closures were made between different sections of pipe laying. Four gangs were necessary to keep this part of the work up to schedule.

Two types of lap-welded pipe were used: (*a*) Bell and spigot pipe with leaded joints was used in sections of lower pressure; and (*b*) Van Stone flange-joint pipe was used at points of high pressure on the line—in general, for pressures greater than 400 lb. per sq. in. Angles in this latter pipe were made either with curved sections for the higher degree or with wedge-shaped "Dutchmen" for the low-degree angles. As much of this pipe was laid in the Montandon Siphon, where the slopes exceeded 45°, the fitting of the joints required much care and accuracy.

Each second joint was supported on a concrete pier (see Fig. 12), and anchors were spaced about 164 ft. apart, with an expansion joint near each anchor. Reinforced concrete anchors were used where solid-rock footings were available. The reinforcement was extended into the rock foundations by being grouted into drill holes, and concrete anchors of the gravity type were used where solid-rock footings were not available.

A special cross-casting, equipped with high-pressure drain-valves, was installed at the invert of Montandon Siphon where the working pressures were 1 000 lb. per sq. in. This drain is equipped with two high-pressure 10-in. valves in tandem, which are interconnected by a by-pass pressure relief. Two men can operate them by hand under the full pressure head.

When hydraulic tests were put on the lap-welded pipe sections it was found that the leaded joints would not withstand the combined effect of expansion

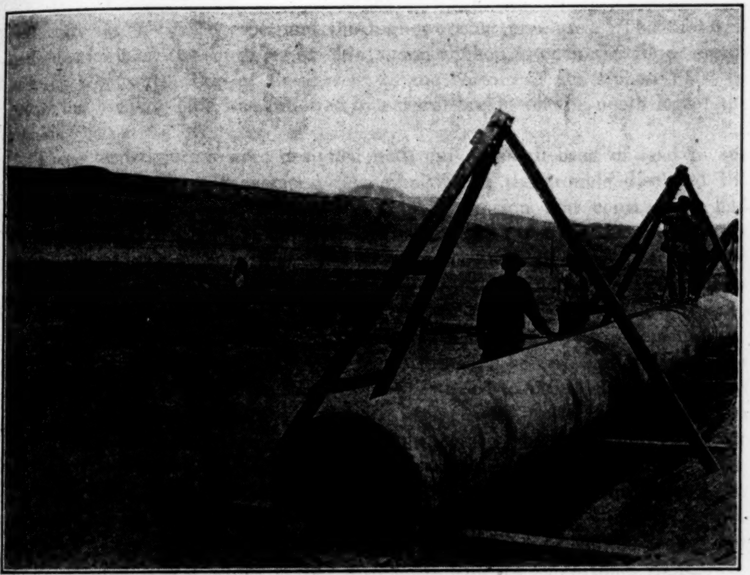


FIG. 11.—LAYING PIPE ON LA OLA PAMPA; SHOWING METHOD OF SUPPORTING PIPE WHILE MAKING FIELD JOINTS.

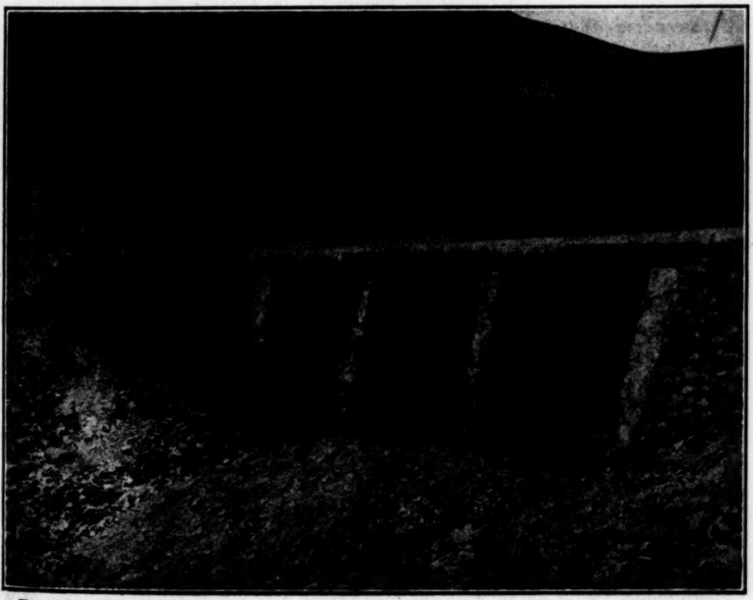


FIG. 12.—PIPE LINE CROSSING A CANYON; SHOWING METHOD OF SUPPORT FOR DRAINAGE.

and contraction of the pipe, and the high working pressures. The first test caused a joint on the south leg of Montandon Siphon to open and four lengths of pipe blew out. Two of these went to the bottom of the siphon 1 500 ft. below the break. This was followed by several breaks on the north leg of the siphon.

These leaded joints were designed with only a small bead of lead in the spigot end to prevent movement. An analysis of the trouble disclosed the fact that the movement of the pipe from expansion and contraction had sheared this bead of lead, and subsequent movements had worked it loose in the joint. The high pressures then blew it from the joint. This analysis showed that any process that might be used to hold the lead in place would remedy the difficulty. It was found that the quickest and simplest method was to construct reinforced concrete collars. Three rings of $\frac{3}{8}$-in. reinforcement were made, one to lie in front of the lead, one behind the bell of the joint, and the third around the bell. These three rings were laced together with heavy wires of sufficient cross-section to withstand a stress equal to that exerted on the lead joint. This reinforcement was held in place by pouring a concrete collar around it. About 1 100 of these collars were made to protect leaded joints. To date (June, 1930), none of these joints has broken and only one has shown necessity for repair.

Only four of the Van Stone joints in the entire line showed leaks, and these required only a slight taking up on the bolts to make them tight. No leaks have developed at these joints to date (June, 1930).

Pipe laying was started on November 22, 1925, and the last pipe was laid November 21, 1926. The average rate of laying was 3 280 ft. per week; the maximum laid in any one month was 6.52 miles, or 1 150 ft. per day; and the maximum laid in one day was 1 312 ft. The cost items are shown in Table 4, Item 12.

Montandon Power Station

The original construction plans and estimates made for the Ola Pipe Line did not contemplate the construction of the Montandon Power Station, although it had been considered. It was only after the arrival of the construction forces in Chile, and the financial possibilities of the plant being presented by them, that the construction of this plant was authorized.

The idea of locating a hydro-electric plant in the middle of a desert is at first thought rather fantastic, and would not be practical unless the water was brought to it free of cost. In this case 1 250 ft. of head were available in the line, in excess of the required pressure head necessary to deliver the water to Potrerillos. This excess head and hydraulic drop occurs between the points where the line is tunneled through the mountain range which intervenes between La Ola Pampa and Potrerillos, and the head of Montandon Siphon; that is, in a distance of 4.40 miles beginning at Piñoncito Tunnel and ending at a point 525 ft. in elevation above the Potrerillos outlet of the line.

Piñoncito Tunnel was used as a forebay for the penstock and was provided with a spillway to care for any overflow of the line which might occur during a shut-down of the plant. The penstock between Piñoncito Tunnel and the Montandon Power Plant consists of 1 mile of 36-in. riveted pipe, 2.4 miles of 26-in. riveted pipe, 0.6 mile of 26-in. Matheson joint lap-welded pipe, and 0.3 mile of 26-in. Van Stone joint lap-welded pipe. The latter is used on the lower end of the line. This penstock contains a total of twenty inverted siphons, the greatest of which is under a pressure head of 430 ft.

As the primary purpose of the pipe line was to furnish water to Potrerillos, the power station was a secondary consideration and therefore was designed to by-pass water when the plant was shut down. Provision for this feature was made by governing the plant with a deflector which cuts the jet at the nozzle, allowing the water to pass, but deflecting it from the wheel. The plant is also provided with a by-pass for diverting the total stream away from the wheel. The needle-valves of both wheel and by-pass nozzles are hand-operated and interconnected in a manner to close one valve while the other one opens.

The hydraulic equipment in the plant consists of a single-nozzle, overhung, impulse wheel, 53 in. in diameter, propelled by a 4¼-in. jet. This wheel is designed to operate under 1 132 ft. of head, at a speed of 514 rev. per min. It is mounted on the end of the generator shaft which runs on 9 by 15-in. bearings. The exciter unit is direct-connected to the other end of the generator shaft through a flexible coupling.

The tail-race and by-pass flumes discharge into a steel tank 25 ft. in diameter in which baffles have been constructed to check the velocity of the water. This tank is provided with a spillway and an outlet gate that discharges into the pipe line to Potrerillos. When closed, this gate diverts the flow over the spillway into a canyon which takes the water entirely away from the pipe line. The capacities of this plant for corresponding quantities of flow, are as follows:

Quantity of flow, in cubic feet per second.	Capacity, in kilowatts.	Quantity of flow, in cubic feet per second.	Capacity, in kilowatts.
21	1 575	26	1 950
22	1 650	27	2 025
23	1 725	28	2 100
24	1 800	29	2 175
25	1 875	30	2 250

Excavation for the foundations of this plant was started on March 15, 1926, and was completed by the end of that month. During April, all the substructure concrete was poured up to the level of the operating floor. During May, the work was delayed for lack of the superstructure steel and plant equipment, but during that month a permanent building for housing operators was constructed. By the end of June all the building steel had been received and erected, and the equipment had been transported to the site. On July 27, 1926, the plant was completed, and has been in continual operation since that date, except for minor interruptions.

The only difficult part of this work was the transportation of the plant equipment from the assembly plant yard to the power plant site. The heaviest piece to be transported was the generator shaft and rotor, which weighed more than 10 tons and was of a very difficult shape for handling because the rotor was mounted on the shaft. The weight of the piece taxed the capacity of the incline railway and was of such shape that the crate had to be bridled ahead to the haulage cable of the car, in order to prevent it from tipping over backward. From the top of the incline the transformers and other heavy pieces were carried in a four-wheeled wagon drawn by eighteen mules.

CONCLUSION

The difficulties attending this construction make a story in themselves. The organization of the necessary supervising staff, the selection and training of a large number of men in all departments of the work, the effect of climatic conditions at high altitudes, the difficulties in getting together and organizing the transportation facilities, and last, but not least, the delay in the receipt of necessary material were the disturbing factors encountered on this work. To appreciate the task, it is necessary to see the country traversed. The last pipe was in place and all work completed on November 21, 1926. To date (June, 1930), La Ola Pipe Line has demonstrated its fitness to serve the purpose for which it was built.

DISCUSSION

Ernest B. Nelson,[3] **Esq.** (by letter).—The author is to be commended for this very interesting description of La Ola Pipe Line and especially for the valuable unit construction costs presented. These low unit costs indicate a very efficient organization, as is likewise evident from the rapid progress of the work which was carried out in the face of many adverse and difficult conditions far from the source of supplies. As is the case on many construction jobs, transportation proved to be a major problem in maintaining progress to the required schedule.

The original design for this pipe line (1917-18) called for continuous wood-stave pipe for the low-pressure portions, about 80% of the total length, with lap-welded steel pipe at points of high pressure, including of course the Pasto Cerrado (now Montando) Siphon. Conditions at that time, with the World War in progress, made it necessary to reduce to a minimum the use of steel in all units of the projected plant development, since deliveries of steel plate, structural shapes, etc., were very slow and uncertain. During this period, due to the fact that steel construction had to be virtually abandoned, the last 20 km. of the railroad from Pueblo Hundido to the plant at Potrerillos—a difficult section in the mountains—was relocated in order to eliminate several proposed steel bridges. Four additional tunnels were thus introduced, making a total of eleven in this portion of the line, but the work was continued without serious interruption so that the railroad was completed and placed in operation early in 1919, a few months after the signing of the Armistice.

Work on the tunnels for the pipe line had been discontinued. Later, it was resumed, another tunnel having been introduced in the meantime by an alternate location through Pedernales Ridge. Late in 1920, with six tunnels completely driven and the Pedernales Tunnel 60% through, a general shutdown of construction put a stop to further activities.

When the work was opened up again early in 1925, the change to steel pipe throughout had been adopted, and the project was carried through to completion as described by the author. Steel pipe besides being more dependable and less likely to give trouble in operation than either wood-stave or concrete pipe, has the further advantage of a minimum loss or waste of water. Everywhere in a desert region, such as Northern Chile, water is scarce and inadequate, and in this case the entire flow of the Ola River is utilized during half the year, or more.

For a wood-stave pipe line of the size and length here considered, leakage, including seepage and evaporation losses due to saturation of the staves, may reasonably be taken at about 4% of the total flow, or well over 1 cu. ft. per sec., which would have meant just that much less water available for use at the plant. It is apparent that for conditions in which the supply of water is strictly limited and possibly insufficient for the demands, this item of leakage may be a determining factor in the selection of the type of pipe to be used.

---

[3] Supt. of Concrete Constr., Anglo-Chilean Consolidated Nitrate Corp., Tocopilla, Chile.

The author may have some data from observations of the carrying capacity of this line, which would be of general interest for comparison with some of the accepted formulas. If the writer remembers correctly, at several points in the line, provision was made for measuring the flow by means of a sharp-crested weir in a short section of open canal.

H. L. THACKWELL,[4] ASSOC. M. AM. SOC. C. E. (by letter).—The local conditions, trials, and labors of constructing a bold and daring water-supply project in the high pampas of Northern Chile, has been ably presented in this paper.

The casual reader can better appreciate this work by comparing it with a similar undertaking, such as crossing the Grand Canyon of Arizona with a pipe line, except that in Chile the "quebrados", or canyons, do not have running streams.

The transportation problem is particularly difficult to solve. All foodstuffs must be imported due to the arid climate, which is unsuitable for agriculture. This very lack of vegetables. has made the Northern Chilean a meat eater, and largely a protein feeder.

As stated under the heading, "Water Supply", the "atmosphere is absolutely dry"; in fact, it is so much so that the mucus membrane in one's nostrils dries up, and will crack and bleed unless greased with vaseline or similar ointment. The statement that "the air is brilliantly clear" can be better understood by the study of photographs taken of mountains 25 and 40 miles away, which clearly show geographical demarcations and variations in texture and conformity. The only fuel is the root of a small "wind-blown" bush somewhat similar to the sagebush found in the Western United States.

The wind not only "reaches a velocity of about 20 miles per hour * * * every day", but it carries sand and small pebbles which cut like emery. Facing a high desert wind on La Ola Pampa is veritable torture; even burros turn their backs and seek shelter. Engineers' stakes have been eroded by the pampa winds to a depth of 0.25 in. in a year's time, erasing all marks of identification.

Chilean laborers are strong and capable and, in many ways, remind one of the Japanese; they are short, stocky, and able to carry large loads easily without tiring. The "rotos" are a distinct class of labor, and when recruited from the desert districts they become efficient workmen. Chilean miners working by hand in the tunnel headings, make as good progress as the average Western miner of the United States, using the same tools and methods.

The water supply from the Rio Ola is fairly constant as can be seen from Fig. 4. It is quite possible to obtain 29 cu. ft. per sec., provided a storage of 4 000 acre-ft., is available. A study of Fig. 13, will show that a maximum possible draft is available with adequate storage, and that if the pipe line is designed for 26 sec-ft., there will be periods every year when the mill will have to reclaim several second-feet of water. A mean draft of 22.5 cu. ft. per sec. is practical with a storage of 100 acre-ft. at the mill site, near Potrerillos. In

---

[4] Cons. Engr., Jacksonville, Tex.

Fig. 13, the hatched area below 26 cu. ft. per sec. indicates used water that should be reclaimed at the mill. Water flow will fall below the requirements about one-half the time.

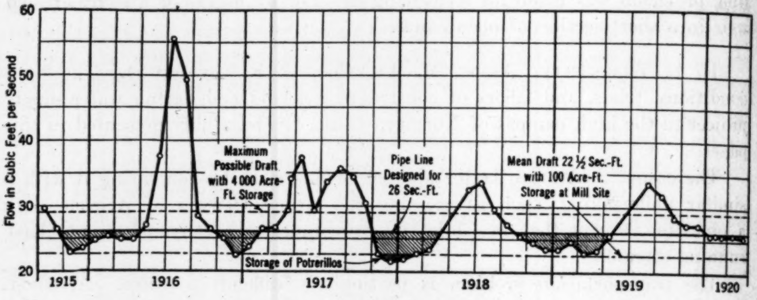


FIG. 13.—HYDROGRAPH OF RIO OLA.

The author is to be congratulated for the excellent manner in which the final construction work was completed, and, in particular, he should be commended for the difficult organization work and effective training of inexperienced labor and the economical results obtained by skillful supervision.

WILLIAM E. RUDOLPH,[5] M. AM. SOC. C. E. (by letter).—Some costs compiled by the writer might be of interest in comparison with those tabulated by Mr. Saunders. This particular undertaking was not a new pipe line, but concerned the construction of looping lines to increase the capacity of an existing water supply line.

The line in question was 55 miles long, 47 miles of which was 9-in. pipe and the remainder, 6-in., with all screw couplings. Its capacity had been impaired about 30% by nodular incrustations, and at one section the hydraulic grade had fallen below the level of the pipe. The measures taken to regain a portion of the lost capacity consisted of substituting a section of new 9-in. pipe for the 6-in. pipe at one location and a section of new 12-in. pipe for the 9-in. pipe at another. In each case the pipe that was removed was relaid beside the existing line in the adjoining section, this division of the flow between two parallel lines further reducing the flow friction losses.

Unit costs on this work are shown in Table 5. The following notes are in explanation of certain units in the tabulation:

Item 1.—The cost of housing and feeding a laborer in camp was about equal to the average wage paid. As this expense was considered part of the general overhead, it has not been charged against the various units. However, it can be distributed by doubling the units for labor under each of Items 3 to 10, inclusive.

Item 2.—The unit cost of pipe would appear high as compared to costs given by Mr. Saunders. This was partly due to the large number of specials required. On work of greater magnitude, and on which larger sizes of pipe were used, it would undoubtedly have been advantageous to assemble the pipe in the field as described by Mr. Saunders.

---

[5] Chf. Engr., Green-Muth Bldg. Corporation, Rockville Center, N. Y.

TABLE 5.—UNIT COSTS OF CONSTRUCTING "LOOPING" LINES FOR INCREASING CAPACITY OF TOCONCE PIPE LINE, SUPPLYING CHUQUICAMATA, CHILE, APRIL TO SEPTEMBER, 1927

No.	Item	UNIT COSTS, IN DOLLARS (8.18 PESOS)			
		Materials	Labor	Expenses	Total
1	Cost of maintaing men in camps per man-day:				
	(*a*) Camp supervision		$ 0.03		$ 0.03
	(*b*) Food and water	$ 0.62	0.11	$ 0.01	0.74
	(*c*) Fuel	0.03	0.01	0.01	0.05
	(*d*) Camp maintenance, including depreciation of equipment	0.14	0.06		0.20
	(*e*) Transportation of camp equipment		0.03		0.03
	Total	$ 0.79	$ 0.24	$ 0.02	$ 1.05
2	Cost of new pipe, unloaded from cars per foot	$ 3.65	$ 0.04	$ 0.03	$ 3.72
	Cost of new pipe unloaded from cars per ton	158.95	1.75	1.30	162.00
3	Cost of uncovering old pipe per foot		0.01		0.01
4	Cost of taking up old pipe per foot		0.03		0.03
5	Cost of motor-truck haulage per ton-mile	0.18	0.02	0.04	0.24
6	Cost of haulage by mule and by hand per ton-mile	3.21	8.00	0.93	12.14
7	Cost of excavation per foot	0.02	0.14		0.16
8	Cost of pipe-laying per foot	0.01	0.05		0.06
9	Cost of covering pipe per foot		0.06		0.06
10	Cost of building road per mile		381.25		381.25
11	Engineering and supervision, percentage				9

Item 5.—The trucks hauled average loads of 2 tons on outgoing trips and returned empty.

Item 6.—The average length of haul for each pipe, after being unloaded from a truck, was 0.65 mile, of which about one-third was accomplished by mules and two-thirds by hand. This hauling was in rugged country, on steep slopes of canyon enclosures for the greater part. The cost of hay and grain for the mules was $0.51 per animal day.

Item 10.—Motor-truck haulage on this work effected large savings, but it did involve considerable road construction. Such roads were build in large part by merely driving a truck over the barren pampa to consolidate the surface materials, and by removing projecting rocks. Grades as steep as 15% were used. At a dry canyon crossing the new road showed a tendency to become blocked by snow drifts, and it was found to be cheaper to relocate the road than to shovel snow from the earlier location.

High labor turnover was experienced. Little difficulty was encountered in recruiting men at the start, because although it was late April (the beginning of the winter season), Chilean workmen sought employment where lodging and board were free, even if it did involve living in tents at an elevation of 13 000 ft. In most cases, such enthusiasm lasted only until the first pay day. Later, the depleted ranks of workers were in part filled with native Bolivians, or more exactly, Indians, from the small farming centers of the mountain slopes. These "pueblos" exist upon the community system as in the days of the Inca, and they have little, if any, connection with the outside world. It was necessary to obtain permission from the head of each village to give employment to his men.

One would have expected that native Indians, accustomed to the altitude and the severe climate of the Andean highlands, would prove far superior to the average Chilean laborer. Such was not the case. Generations of inbreeding had worked its effects upon the Indians. The Chileans averaged 25% and more bonus on their contracts, the Indians scarcely earned their day's pay. As the job neared completion, it was the Chilean laborers who had remained through devastating snow and wind storms, constructing pipe lines on exposed mountain slopes. The Indians had returned to the less rigorous life of repairing their irrigation ditches, down in the valleys.

W. B. SAUNDERS,[6] M. AM. SOC. C. E. (by letter).—Since the completion of La Ola Pipe Line in November, 1926, operating history over a period of five years has answered in part the discussions by Messrs. Nelson, Thackwell, and Rudolph.

The general interest which would be accorded data concerning the carrying capacity of La Ola Pipe Line is mentioned by Mr. Nelson. For purposes of efficient operation and observation daily weir readings have been recorded at two different points on the line since December, 1926, from which some interesting data have been accumulated and, without going into minute details, the following observations have been made.

The section of pipe from the intake at La Ola Dam through to La Cabra Tunnel is that best adapted for experimentation and observation. In this section the hydraulic grade is 1.5 per 1 000 when the reservoir is full, and the pipe is of a uniform diameter throughout, with an average of 36.83 in. between male and female plates.

At the beginning of operation, when the pipe was new and the asphalt coating was very smooth, the records show a capacity of 28.5 cu. ft. per sec., but this capacity fell off rapidly at the beginning of operation to about 27.0 cu. ft. per sec. By July, 1929, the discharge had decreased to 25.5 cu. ft.

---

[6] Hydr. Engr., Andes Copper Min. Co., Barquito, Chañaral, Chile.

per sec., and an examination of the inside of the pipe showed a nodular incrustation forming, which materially increased the friction and reduced the inside diameter of the pipe.

After some experimenting a "go-devil" was devised which successfully cleans the pipe. The main frame of this apparatus is of wood, bound at the joints and contact points with steel plates, to which are attached steel brushes. The total weight and displacement is such that it will just float in water. It is moved through the pipe by the pressure head of water exerted on the circular disks. These disks are of such size, and their lengths are such, that the "go-devil" will easily pass through the flat curves in the pipe line. In theory, the disks form sufficient obstruction to allow the building up of about 10 lb. per sq. in. pressure head over their areas before the openings around them will pass the full flow of the pipe line. In practice, it is not fully known what pressure is exerted to move this apparatus, but when in use, the flow capacity of the line is reduced about 22%, and the "go-devil" moves at an average rate of about 2.5 ft. per sec., which approximates the velocity of the water at the reduced flow capacity.

In operation, this apparatus takes a rotary motion and successfully brushes out the incrustations from the line. By using it every six months the line capacity is maintained at 27.0 cu. ft. per sec., or more.

The coefficient of roughness, *C* (Hazen and Williams' formula), as applied to this pipe line and its operation, has been observed to be as indicated in Table 6.

TABLE 6.—QUANTITIES, VELOCITIES, COEFFICIENTS, AND CONDITIONS OBSERVED AND COMPUTED FROM LA OLA PIPE LINE OPERATION

Observed quantity of flow, in cubic feet per second	Computed velocity, in feet per second	Coefficient (Hazen and Williams), *C*	Condition
28.5	3.83	115	New pipe
27.5	3.73	112	Immediately after cleaning
27.0	3.66	110	60 days old and after cleaning
26.5	3.59	108	2 years old
25.5	3.46	104	2½ years old
24.5	3.32	100	Not yet reached

Mr. Thackwell presents a hydrograph (Fig. 13) which indicates the possibility of obtaining a continuous flow of 29 cu. ft. per sec. of water, provided 4 000 acre-ft. of storage might be made available at the head of the pipe line. The topography of the country and the available dam sites do not permit the use of such a hydrograph, within a reasonable cost. The present (1931) storage or regulation at La Ola Dam is about 25 acre-ft., and the site of the dam does not permit this quantity of storage to be increased at that point. The character of the country is such that an appreciable quantity of storage at any other available point would be prohibitive in cost. However, after two years of operation, additional water was added to La Ola supply by the construction of 4.2 miles of pipe line, ranging in diameter from 14 to

18 in., which diverts 12 cu. ft. per sec. of water from the Juncal River (see Fig. 2) and puts this water into a small stream leading into the head of La Ola Vega. After losses from seepage and evaporation, which are very large, this quantity of water is sufficient to regulate the flow of La Ola River at the intake of the pipe line so as to release a larger quantity than that which will pass through the pipe.

Mr. Rudolph's discussion and tabulated costs are interesting and show efficient management of that work. It is apparent that living and labor conditions must have been very similar on the two jobs and that the cost of labor was much the same.

A direct comparison of other costs between the two jobs is difficult, except between the cost of steel pipe purchased in the United States. The small difference in cost for this pipe is easily accounted for by a probable difference in ocean and rail freight charges and a different date of purchase. The comparison of these costs with those of riveted steel pipe fabricated on the Ola Line are all in favor of the local fabrication. On the large tonnage of pipe used in this line the saving effected by local fabrication was approximately $500 000.

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
TRANSACTIONS

Paper No. 1796

THEORY OF SIMILARITY AND MODELS[1]

BY BENJAMIN F. GROAT,[2] M. AM. SOC. C. E.

WITH DISCUSSION BY MESSRS. H. BATEMAN, W. P. ROOP, A. V. KARPOV, JASPER O. DRAFFIN, ED S. SMITH, JR., F. J. EISNER, AND BENJAMIN F. GROAT.

SYNOPSIS

The paper aims to present a complete, concise treatise on the theory of similarity and models. Newton's theorem, which is not generally understood, is shown to cover every possible type of dynamic model. A misconception which has been unduly propagated, is disclosed and traced to its source. It is shown that models can be built to specification, as to materials and their properties, so as to perform with theoretical exactness. It is suggested that materials, including fluids, should be sought, made, or arranged, so as to adapt them especially for service in models, thus opening a new field in engineering. The theory is based upon data given in a previous paper by the writer.[3]

INTRODUCTION

Every engineer should know and understand that a model, constructed in accordance with Newton's theory of similarity, is not only the means for reproducing the complexities of flow, or other mechanical action, on a small scale, but that such model may be made the best means of designing works of great magnitude. In the present state of knowledge, a properly constructed and properly tested model will answer the difficult questions of hydrodynamics much more quickly and accurately than the most profound mathematical analysis.

Engineers and other groups of society incline too little toward making the most of past experience. It is about 250 years since Newton enunciated the principles of similarity and the theory of models; yet it is only during the last very few years that engineers have made any use of models, as a reliable means for determining a design, especially in hydraulics.

---

[1] Published in October, 1930, *Proceedings*.

[2] Cons. Engr., Brookline, Mass.

[3] "Ice Diversion, Hydraulic Models, and Hydraulic Similarity," *Transactions*, Am. Soc. C. E., Vol. LXXXII (1918), p. 1138.

The cause of this inertia in the advancement and dissemination of science and truth, is the tendency to place a blind reliance upon the authority who makes a statement, instead of basing belief on analysis of what is said. Civilization should be standardized on the basis of reason; it should not be allowed to develop along a mere line of natural social evolution. Were it not that Lincoln said it, the slogan, "Government of the people, for the people, and by the people," would be a forgotten generalization to-day. It is "harped" upon because a great man said it, but, in reality, too little has been done toward a sufficient application of this, one of the greatest generalizations of all time. Is not the Engineering Profession best prepared to overcome inertia of every kind all along the front?

NOTATION

The following notation will be used consistently in this paper and its discussion:

$\propto$ = varies as.
g = acceleration due to gravity; also, acceleration in model generally.
V = velocity in prototype, or ratio of homologous velocities according to context.
D = diameter, or ratio of diameters, according to context.
l = linear magnitude in model.
p = pressure.
μ = absolute coefficient of viscosity, usually in centimeter-gramme-second units.
ρ = density in model, or ratio of homologues, according to context.
P = property of a material, or ratio of such. In Appendix I, it is ratio of homologous pressures.
T = time corresponding to a. (Stokes.)
T = time dimension, or ratio of homologous times. Sometimes as $[T]$.
W = specific weight in prototype, or ratio of homologues.
L = linear dimension, or ratio of such homologues. Sometimes as $[L]$.
F = force dimension, or ratio of such homologues. Sometimes as $[F]$.
$\nu = \dfrac{\mu}{\rho} =$ kinematic coefficient of viscosity of the fluid.
R = scale ratio.
G = acceleration in prototype, or acceleration ratio.
N = kinematic coefficient of viscosity of the fluid in the prototype.
E = ratio of homologous elastic moduli.
k = elastic spring constant.
Δ = density in prototype.
M = mass ratio.
M = absolute coefficient of viscosity in prototype, or ratio of homologues.
f = sign of functionality.
P = pressure of gas. (See, also, previous definition.)
c = gas constant in Appendix I.
w = specific weight of a gas.
θ = absolute temperature, in degrees centigrade.
K = a constant. Reynolds' number in Equation (9).
ϕ, ϕ' = signs of functionality in Equations (20) and (21).

HISTORIC BACKGROUND

History may now be recited to show how the theory of models and similarity made its appearance, and what the situation is at present (1930) relative to the use of models. It was more than 100 years after the greatest natural philosopher, Sir Isaac Newton, adduced the theory—in fact, the whole idea—expressed in true scientific simplicity, before any use was made of it. Most individuals fail to remember that the entire theory of terrestrial mechanics was given by Newton in three simple rules, or "laws," which can be written in about as many lines. The entire basis of celestial mechanics was delivered by that same philosopher in a statement of two or three more brief lines. Newton propounded the theory of models in a single paragraph, and nothing has since been added to any of these theories, other than their natural consequences.

In his "Principia Mathematica" (original in Latin), Newton published the theory of similarity[4] as follows:

> "Suppose two similar systems of bodies consisting of an equal number of particles, and let the correspondent particles be similar and proportional, each in one system to each in the other, and have a like situation among themselves, and the same given ratio of density to each other; and let them begin to move among themselves in proportional times, and with like motions (that is, those in one system among one another, and those in the other system among one another). And if the particles that are in the same system do not touch one another, except in the moments of reflexion; nor attract nor repel each other, except with accelerative forces, that are as the diameters of the correspondent particles inversely, and the squares of the velocities directly, I say that the particles of those systems will continue to move among themselves with like motions and in proportional times."

The term, "accelerative force," means the intensity of a force per unit mass acted upon by the force. The most common example is the acceleration of gravity, g, which is the acceleration of a freely falling body at the earth's surface. If this is multiplied by the measure of the mass acted upon, the result is the total force acting upon the mass.

Hence, Newton's Law of Similarity supplies the fundamental equation of similitude, namely,

$$g = \frac{V^2}{D} \qquad (1)$$

in which, the symbols relate to ratios of homologues.

Two systems of forces in space are dynamically similar if the lines of action of the forces (produced if necessary) form similar geometrical figures in space, while the measures of the two individuals of every pair of homologous forces in the two systems are in a fixed ratio (called the "force ratio of the homology"). Thus it is clear that proportionality of homologous forces, and geometrical similarity of the figures formed by the systems of lines representing their magnitudes and loci, are both involved in the idea of dynamical similarity.

Every engineer is familiar with the force diagram and the space diagram of graphical statics. Proportionality of homologous forces in dynamically similar systems corresponds to similar force polygons, while similarity of applications of forces to masses, similarly situated in two similar systems, corresponds to similar space polygons. Thus, two geometrically similar framed structures, bearing proportional loads similarly distributed, would sustain similar internal stresses, any two homologues of which would be in the force ratio. One such frame, so analyzed, would be the solution of all such similar frames bearing similar systems of loading.

---

[4] Book II, Proposition XXXII, Theorem XXVI.

Correspondingly, the term, "mass similarity," may be used to express the idea that two systems of masses occupy space in such configurations that the measures of homologous masses are in a fixed ratio called the "mass ratio of the homology", while the loci of the homologous masses are similarly situated in the two systems.

The complete notion of mechanical similarity embraces the idea of "similar motions," in addition to the ideas of dynamical similarity and mass similarity. Similar motions require that the paths described by homologous masses be geometrically similar, and that the velocities of homologous masses be in a fixed ratio, which may be called the "velocity ratio."[5]

Newton's law simply states the conditions which are necessary and sufficient to establish and maintain dynamic similarity, including geometrical similarity of both form and motion. It is to be emphasized that any one of these conditions must be satisfied in order to have similarity, but any one of them alone would not be sufficient without the others. For example, it is necessary to establish initial similarity, or the theorem would not apply at all. This means, furthermore (taking a practical example), that if a model of a section of a river is built, it is necessary that the water enter and leave the model with motions and levels exactly similar to the homologues in the actual river. This condition is as necessary as any of the others. Observe, in particular, that accelerative forces must be proportional to the squares of velocities directly and to the linear dimensions (diameters, for example) inversely. This part of the conditions relates especially to similar motions. However, the theorem does not require that accelerative forces be constant throughout any given system. Indeed, they may vary discontinuously from point to point. They may be concentrated at a point, for example. The theorem does require that the ratios of all pairs of homologous accelerations be the same.

This relation between accelerative forces and velocities has been erroneously called "Froude's law," because of his use of it in the celebrated tests of "skin friction" on boards (sides of ships), and especially in his tests of ship models to determine the wave-making resistances to their motion through the water. Froude's law has been written in the form of a dimensionless number, $\left(\frac{V^2}{l\,g}\right)$. The statement that it is constant is nothing more or less than Newton's law, $g \propto \frac{V^2}{D}$ (see Equation (1)). Note that l may be inserted in the formula in the place of D, since the ratio of similar dimensions is constant, regardless of what dimension is used.

Newton's law should be distinguished from the law of skin friction, which states that resistances per unit area vary as the squares of velocities. Equation (1) expresses the relation between model and prototype, while the frictional resistances may follow another law; for example, they may vary directly as the velocity. This last relationship is quite consistent with Equation (1). When the common law of friction (V^2 law) holds, the model may be of any size, but when friction varies with a power of the velocity different from 2, the scale of the model ceases to be arbitrary.[6]

---

[5] "Ice Diversion for St. Lawrence River Power Co.," by B. F. Groat, M. Am. Soc. C. E., *The Canadian Engineer*, November 25, 1920.

Newton's law of similarity was stated so as to cover models in general, independent of size, shape, and materials. To make this sweeping statement, its originator was careful not to permit the possibility of unknown reactions between the moving bodies. Consequently, he excluded every contactual influence except instantaneous contact of the moving particles, not because of any fundamental necessity for such restriction, but to emphasize that the forces of dynamic similarity must be proportioned so as to cause, or develop, the accelerations specified by him in the law, as he stated it.

It was not until 1850 that Stokes,[7] in the course of his investigations, noticed certain important mathematical relations involved in the theory of similarity and models. Through a slight oversight, however, he rendered an account of similarity involving viscosity which has led a number of investigators into the same error.

As its title shows, Stokes' paper was primarily concerned with viscosity. This is a special type of force which Newton wisely did not mention specifically, although his theorem, or law, does include any force which can be made to conform to it.

Stokes was an able pioneer investigator in hydrodynamics, a subject full of fundamental mathematical forms devised by him. He essayed to study the laws of similarity in viscous fluids, since it was necessary to ascertain the effect of changing the sizes, shapes, and conditions of bodies moving in a viscous fluid, such as air. He showed that the sizes of bodies did not change the mathematical treatment. In other words, the same differential equations would represent each of two, or more, similar linear systems.

His paper includes one of the most remarkable contributions touching upon similarity and models since Newton's great generalization. It covers the entire question of viscosity; but its author did not make his account of similarity in the presence of viscous fluids as complete as this part of the investigation can be made, as will appear after considering his very simple equations relative to similarity in the presence of viscous fluids.[7] This part of Stokes' paper requires some knowledge of coefficients of viscosity, which has been previously explained by the writer.[8]

ANALYSIS OF STOKES' REMARKS UPON SIMILARITY

Consider any number of similar systems, composed of similar solids oscillating in a similar manner in different fluids or in the same fluid. Let

---

[6] This follows from the writer's article in *Engineering Record,* September 25, 1915, p. 377.

[7] "On the Effect of Internal Friction of Fluids on the Motion of Pendulums," *Transactions,* Cambridge Philosophical Soc., 1850, p. 19, Section 5.

[8] In his discussion of the paper entitled, "Experiments on Discharge over Spillways and Models, Keokuk Dam," by Floyd A. Nagler and Albion Davis, Members, Am. Soc. C. E., *Transactions,* Am. Soc. C. E., Vol. 94 (1930), p. 833, *et seq.*

a, a', ..., be homologous lines in the different systems; T, T', ...; corresponding times, such, for example, as the times of oscillation from rest to rest. Let x, y, and z, be measured from similarly situated origins, and in corresponding directions, and t, from corresponding epochs, such, for example, as the commencements of oscillations when the systems are beginning to move from a given side of the mean position.

If any dynamical equations (including differential equations, of course) are satisfied for one system they will be satisfied for all the systems provided $u \propto v \propto w$; $x \propto y \propto z$; and $p \propto \frac{\mu u}{x} \propto \frac{\rho u x}{t}$, when passing from system to system. Stokes did not say that $p \propto \frac{\rho u x}{t}$ is the statement of Newton's law. The variations, $x \propto y \propto z$, merely signify that similarly situated points must be compared in inferring, from the circumstance that equations are satisfied for one system, that they will be satisfied for all the systems. If c, c', ... , are the maximum displacements of similarly situated points of the fluids, then $u \propto \frac{c}{T}$; $x \propto a$; and $t \propto T$.

Quoting Stokes,[7] "the sole condition to be satisfied, in addition to that of geometrical similarity, in order that the systems should be dynamically similar, becomes" $\frac{a^2}{T} \propto \frac{\mu}{\rho}$, or $\frac{a^2}{T} \propto \nu$. This condition being satisfied, similar motions will take place in the different systems; therefore, $p \propto \frac{\rho a c}{T^2}$.

It follows from equations that might be written down by symmetry, that the pressures, such as normal or tangential, vary in the same manner as p; whence it appears, from the relation, $p \propto \frac{\rho a c}{T^2}$, that the resultant or resultants of the pressures of the fluids on the solids, acting along similarly situated lines, which vary as $p a^2$, vary as ρa^3 and $\frac{c}{T^2}$ conjointly. In other words, these resultants in two similar systems are to one another in a ratio compounded of the ratio of the masses of fluid displaced, and of the ratio of the maximum accelerating effective forces belonging to similarly situated points in the solids.

Stokes' relation, $u \propto v \propto w$, means simply that (if capitals relate to the first system and minuscules to the second),

$$\frac{U}{u} = \frac{V}{v} = \frac{W}{w}$$

or, as Stokes writes it, $u \propto v \propto w$, in which the u's relate to x components of velocity, and the v's and w's, respectively, to the y and z components. The simplest way to see this is to note generally that the "dimensions" of velocity are, in all cases, $\frac{\text{distance}}{\text{time}} = \frac{L}{T}$. Thus, if the two systems are geometrically similar, and the times of motion over homologous distances are proportional, $u \propto v \propto w$ is satisfied for all the motions in two similar systems; that is, for all the homologous motions.

This is a general truth. If any two or more quantities have the same "dimensions" in F, L, and T, then, if every ratio of two homologues measured in a given simple dimension, L, F, or T, is in a given ratio which depends upon the type of the dimension, the quantities themselves are also in a determinate ratio, and that ratio is found by calculating the product of the known powers of the fixed ratios of their dimensions. Thus, the ratio, $\frac{V}{v}$, equals the product, $\left(\frac{L}{l}\right) \times \left(\frac{T}{t}\right)^{-1}$, and it will be constant from molecule to molecule if each of the factors is constant. Hence, one may substitute the sign of equality for the sign of variation when the symbols relate to ratios of homologues, as will frequently be the case hereafter.

In passing, it is well to call attention to the fact that confusion of ideas results from two distinct uses of the word, "dimension." One use relates to architectural dimensions, so familiar to all engineers; the other, to "dimensions of units," which is much less common. The first use relates to a measure in terms of a unit; the latter to the nature of the unit. Both the magnitude and its unit have the same dimensions (latter use). The measure of a magnitude is, therefore, a pure number (no dimensions). The measure of a fixed magnitude varies inversely as the magnitude of its unit, that is, of the unit employed in the measure. The complete specification of an architectural dimension consists of a measure followed by a specification of the unit, as 200 ft. Correspondingly, N ft.^{2}[9] names the unit as 1 sq. ft.; $N \frac{\text{ft.}}{\text{sec.}}$[9] names the unit as 1 ft. per sec.; etc. The dimensions (latter use) just specified are, respectively, feet, square feet, and feet per second. To avoid naming the unit along with a specification of its dimensions (latter use), the following forms are used, respectively, for the three cases mentioned, L, L^2 and $\frac{L}{T}$, or, $[L]$, $[L^2]$, and $\left[\frac{L}{T}\right]$. It is unfortunate that the word has been used in the latter sense, because architectural dimensions are measured in terms of units of certain dimensions. Thus, two different meanings of the word may occur in the same sentence.

If the three fundamental units, for measurements in the model, are so chosen as to bear the same respective magnitudinal ratios to their homologues as do corresponding homologous magnitudes, then the measurement of any quantity in the model will be precisely the same numerically, as in the prototype. One-twenty-fourth full size gives measures equal to those in full size when the scale is "½ in.=1 ft." Homologous ratios are merely cases of "con-

---

[9] This is the correct form in which the unit should be specified; it shows just how the unit is constituted. The notation is now universal when the nature of the unit is involved in the analysis.

version factors." In the example just mentioned the conversion factor is unity, by reason of the choice of the model unit.

To illustrate the nature of dimensions, take Stokes' requirement that $p \propto \frac{\mu u}{x} \propto \frac{\rho u x}{t}$. The respective dimensions of the individual quantities are:

$$p = \frac{F}{L^2} \qquad (2)$$

$$\mu = \frac{F\,T}{L^2} \qquad (3)$$

$$u = \frac{L}{T} \qquad (4)$$

$$x = L \qquad (5)$$

$$\rho = \frac{F\,T^2}{L^4} = \frac{M}{L^3} \qquad (6)$$

$$t = T \qquad (7)$$

$$\nu = \frac{\mu}{\rho} = \frac{L^2}{T} \qquad (8)$$

Supplying these dimensions to all the terms, it may be shown that each one has the dimensions of pressure; that is, $p \propto \frac{F}{L^2}$, if L, F, and T are con**sidered the fundamental dimensions of units, length, force, and time. Of course, it would have been found that all the terms were of the same dimensions, had mass been utilized as a fundamental unit, instead of force. Therefore, all that is required to insure these equal proportions is that forces be proportional to the areas upon which they act, the dimensions of area being L^2. This applies to shearing stress (viscosity) as well as to normal pressure.**

It is clear generally, without further discussion, that since all the terms of any equation must necessarily have the same dimensions (apples cannot be added to peaches, so to speak), any equations expressing relations between homologous quantities in two similar systems will be the same for either system, except for a common constant factor in all its terms, which factor, of course, may be removed. If homologous units are suitably chosen the factor will be unity, as previously explained.

The relation, $\frac{a^2}{T} \propto \nu$, is the principal one for present purposes, since **it imposes a new condition resulting from the introduction of viscosity into the calculation. It may be written in the form:**

$$\frac{V\,L}{\nu} = K \qquad (9)$$

in which, the constant, K, is a dimensionless number since $V = \frac{L}{T}$, and ν is defined as the ratio of μ to ρ, which is the kinematical coefficient of viscosity (see Equation (8)).

Of course, the constancy of Equation (9) is maintained only for homologous values in similar systems, as model and prototype. It expresses the well-known Reynolds' number, which, as it appears, was first recognized by Stokes; in fact, it was defined by Stokes, before Reynolds made any use of it. It would seem more appropriate to call it "Stokes' number." In this relation, the real debt of science to Reynolds is for his discovery of "turbulence," as compared to stream-line flow, and for his additional discovery of the "critical velocity" and its relation to Stokes' number as the criterion of the existence of turbulence.

The reason why Equation (9) (or its equivalent, Equation (10)) puts a new limitation on models is that the introduction of viscosity demands a new, and additional, relation (Equation (9) or Equation (10)) between two of the three fundamental ratios of the homology, *L, F,* and T, or *L,* M, and T, if preferred. These ratios are considered fundamental because they relate to the three units which are chosen as the "fundamental" units of the system of measures involved. Briefly, viscous fluids in similar systems require that,

$$\nu = \frac{\mu}{\rho} = \frac{L^2}{T} = \frac{a^2}{T} \; \ldots., \text{etc} \ldots\ldots\ldots\ldots\ldots\ldots (10)$$

The symbol, ν, is the kinematic coefficient of viscosity. It is clear why this coefficient is called "kinematical," since, in finding the ratio of μ to ρ (see Equation (10)) the dynamical dimension, "force" (or "mass," when mass is one of the three "fundamental" units, or dimensions), is cancelled, leaving the ratio dependent upon "length" and "time" only, and, therefore, "kinematical."

While the writer has the greatest admiration for Stokes' valuable analysis, he can find no justification for the statement[7] that Equation (10) as derived by Stokes, is "the sole condition to be satisfied, in addition to that of geometrical similarity, in order that the systems should be dynamically similar."

If Stokes had said: "It is the sole condition to be satisfied in addition to those of geometrical and dynamical similarity in the absence of viscous fluids, in order that the systems shall remain geometrically and dynamically similar after introducing viscous fluids," there could have been no reasonable objection. His analysis conforms exactly with this statement, and it is probably what he intended to say, "geometrical similarity", doubtless including geometrically similar motions, which latter condition demands dynamical similarity.

In other words, Stokes' systems were dynamically and geometrically similar, in accord with Newton's law $\left(g \propto \frac{V^2}{D}\right)$. This condition is also expressed in Equation (12), before introducing viscous fluids, or, what is the same thing, before causing the fluids to become viscous. His principal equations (which need not be discussed here) show this, as they correctly determine geometrically and dynamically similar systems without the terms involving viscosity, and without those involving "extraneous forces" (gravity). Introducing viscosity may, or may not, change ratios of homologues, but the similarity continues, provided the conditions of Newton's theorem (law) are imposed throughout the entire time.

Consequently, Equations (9) and (10), are not essentially criteria of dynamic similarity at all, but are merely necessary conditions that must be satisfied by any viscous fluids in the systems. They are conditions appurtenant to viscosity, not to similarity.

Were this a matter of opinion there would be little hope of interesting the engineer and analyst; but it is not a matter of opinion. It is a matter of mere mathematical relations, and it is the writer's purpose to show that, although these relations were plainly visible in Stokes' paper, they were overlooked by him in giving an account of his investigation. This was doubtless due to his greater concern with the more intricate mathematical analysis of the pendulum problems in the remainder of his paper, which need not be examined here.

FORMAL THEORY OF SIMILARITY

Let two systems be constructed with the requirement that they shall be merely geometrically similar. Clearly, the sole requirement for this is that homologous linear dimensions be proportional. Stokes writes this in the form, $x \propto y \propto z$, or more formally, it may be written:

$$L = R \text{.............................} (11)$$

in which, R simply indicates the constancy of the ratio, $\frac{L}{l}$ (which is represented by L) of any pair of homologous linear dimensions, as $\frac{A}{a}$ for example. The value of the scale ratio, R, is perfectly arbitrary until the two systems are chosen; but Equation (11) shows, further, that one does not destroy the validity of any correct equation in one of the geometrically similar systems by applying it to any of the others, independent of the value of R. That is what Stokes says so far, and it is evidently quite correct; and with general application to all manner of quantities which have a characteristic constant ratio between their homologous dimensions (whether linear, or otherwise) in the two systems.

Suppose it is now required that there shall be similar motions, in addition to the similarity of forms as required by Equation (11). (This equation is supposed to be applied to all homologous linear dimensions, thus representing an infinite number of equations, but it will be referred to as one equation hereafter.) This is specified by adding Newton's general theorem of similarity, $g \propto \frac{V^2}{D}$ (see Equation (1)). These necessary and sufficient conditions for simultaneous geometrical and dynamic similarity require that Equation (11) shall be satisfied and that,

$$\left(\frac{V^2}{L} = \frac{L}{T^2}\right) = \frac{G}{g} \text{.......................} (12)$$

in which, the notation used on the left relates, respectively, to the ratios of homologues, and of homologous "dimensions" of the homologues, in accordance with the notation on the left of Equation (11). For example, either of the terms, $\frac{V^2}{L}$, and, $\frac{L}{T^2}$, indicates that the ratios of homologous accelerations are equal to certain functions of the ratios of the simpler dimensions, V, L, and T.

Equations (11) and (12) state simply, that Newton's theorem of similitude puts no restriction upon the scale ratio of two similar systems, nor upon the respective values of the homologous accelerations in the two systems, whether such accelerations arise from "extraneous forces" (gravitation, say), or from forces within the system, or from both combined. Relative to these equations, furthermore, Newton's theorem states that, if the experimenter takes care to establish similar motions initially, the similarity will continue. This is entirely independent of the presence or absence of viscous fluids. If Newton's law is satisfied, the systems must be similar, whether viscous fluids are present, or otherwise.

Observe that the extraneous forces may be eliminated, or annihilated, but that there must be left a ratio of homologous accelerations, just as there must be a constant ratio between any two homologues of any other type in similar systems. Stokes makes this necessary assumption, by providing for it in the form, $\frac{c}{T^2}$, and also in his equation,

$$p = \left(\frac{\rho\, u\, x}{t}\right) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (13)$$

all of which arise from terms which do not contain the viscosity. In order to make the situation perfectly clear, Equation (13) may be expressed in the form:

$$\frac{p}{x} = \frac{\rho\, u}{c}\left(\frac{c}{t}\right) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (14)$$

or, substituting for $\frac{c}{t}$, its equivalent, u, and transposing:

$$\frac{p}{\rho\, x} = \frac{u^2}{c} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (15)$$

In other words the ratio, force to mass, varies as $\frac{u^2}{c}$.

Equation (15) is nothing more or less than Newton's law of similarity, $g \propto \frac{V^2}{D}$, or Equation (12), since Stokes intends it to be applied to homologues, and since the ratio of force to mass (left-hand member of Equation (15)) has the dimension, acceleration.

Equation (11) and Equation (12) involve the ratios, L and T, in terms of the two quantities, R and $\frac{G}{g}$, the respective scale ratio and accleration ratio of the two systems. Clearly, these ratios may be chosen arbitrarily without interfering with the consistency of the equations, since the first equation contains only L and does not contain T. In other words, geometric and dynamic similarity may exist regardless of the particular values of the scale ratio and acceleration ratio in the two systems.

Newton went farther than this in demonstrating his theorem. In the briefest possible language he says (see the theorem as quoted previously) that, if the two equations (Equations (11) and (12)) hold, and the mass ratio is properly adjusted, initial geometrical and dynamical similarity will continue as a necessary consequence.

It is important to emphasize that nothing less than this will do, for the similarity must exist initially (as expressly required by the theorem) so that Equation (11) and Equation (12) are not only sufficient, but necessary; that is, they must hold throughout the entire time and throughout the systems.

For example, if the systems are similar initially, as in the case of Stokes' two pendulums at the points of maximum displacement, the subsequent motions of surrounding frictionless fluids will be similar; or, the motions of surrounding viscous fluids will be similar, if the viscous forces can be adjusted so as to satisfy Newton's theorem, or law of similarity, or, more formally, Equation (11) and Equation (12). One cannot disregard Newton's absolute, necessary, and sufficient condition to sustain dynamical and geometrical similarity.

The only way to explain Stokes' silence on this point is to assume that he really did not mean just what he said, being more immediately concerned with his pendulum analysis than with similarity. However, there can be no doubt that he made the oversight and that it has caused many to repeat his remark. Others, less fortunately, based formulas and calculations of a practical nature upon a misconception. Such oversights are frequently made by writers of unquestionable authority. For example, Rankine has stated[10] that the coefficient of elasticity of a gas has the form:

$$A = \frac{d\,p}{d\,w} = \frac{\gamma\,p}{w}$$

The slightest consideration shows that this cannot be, since A and p have like dimensions (those of pressure), γ is an abstract number, and w, the specific weight of the fluid (Rankine calls it "density"). The nature of the correction is obvious.

This error has been running for the last seventy-five years without correction, apparently without notice, but, thanks to analysis of what is said, rather than reliance upon authoritative literature, the error is self-disclosing by inspection. No better textbook was ever written and none more carefully edited.

Proceeding further, take Stokes' limitation on ratios of homologues, Equation (10), as the additional (not the "sole") condition that systems containing viscous fluids shall be both dynamically and geometrically similar. Then, the three conditions may be represented by Equations (10), (11), and (12). This gives three equations between two unknown ratios, L and T; but only one of the unknowns appears in Equation (11).

---

[10] Rankine's "Applied Mechanics" (any edition including that current in 1930), p. 563. See, also, the footnote in *Transactions*, Am. Soc. C. E., Vol. 94 (1930), p. 836, for another such error by another author.

Equations (10) and (12), therefore, present two conditions in terms of two unknowns, L and T. These two ratios, consequently, become determined; so that the condition defined by Equation (11), is no longer arbitrary. Thus, the three ratios, R, $\frac{\mathrm{N}}{\nu}$ and $\frac{G}{g}$, are not entirely independent, but any one depends upon the other two.

Solving Equations (10), (11), and (12):

$$L^{\frac{3}{2}} = R^{\frac{3}{2}} = \frac{\frac{\mathrm{N}}{\nu}}{\left(\frac{G}{g}\right)^{\frac{1}{2}}} \dots\dots\dots\dots (16)^{11}$$

and,

$$T = \frac{\left(\frac{\mathrm{N}}{\nu}\right)^{\frac{1}{3}}}{\left(\frac{G}{g}\right)^{\frac{2}{3}}}. \dots\dots\dots\dots (17)$$

The conditions of Equations (10), (11), and (12) (all of which are present in Stokes' paper) determine the necessary scale-ratio and time-ratio (Equations (16) and (17)), between the two similar systems in terms of the physical properties of the fluids and the physical conditions existing at the places of the two systems. There can be no dynamically and geometrically similar systems (model and prototype) without all these restrictions, when viscous fluids are present.

This is probably what Stokes intended to read from his paper, but he accidentally overlooked the fact that Newton's theorem must be applied, whether or not the extraneous forces (gravitation, say) are eliminated.

The practical difficulties which arise in constructing models are apparent from Equations (16) and (17). If it is attempted to use the same fluids in model and prototype, the ratio of the coefficients of viscosity is then unity, and it is found that:

$$L = \left(\frac{G}{g}\right)^{-\frac{1}{3}} \dots\dots\dots\dots (18)$$

In this case it is theoretically wrong to test model and prototype at the same place on the earth's surface, for then $L =$ unity, and the model should be of the same size as the prototype, which would rarely be useful. If the same fluid is present in the two systems, but at different temperatures, such as to change the coefficient of viscosity, then the two conditions are equivalent to two different fluids, and should be so regarded.

On the other hand, if instead of $L =$ unity, $\left(\frac{G}{g}\right) =$ unity (testing at the

---

[11] These results were obtained by the writer in studies leading up to his paper entitled "Ice Diversion, Hydraulic Models, and Hydraulic Similarity," *Transactions*, Am. Soc. C. E., Vol. LXXXII (1918), p. 1138. The results of an application to large ice diversion works on the St. Lawrence River, and to the construction of other features, were also described by the writer under the title, "Ice Diversion for St. Lawrence River Power Co.," *Canadian Engineer*, November 25, 1920. See, also, *Engineering Record*, September 25, 1915, p. 377, and *Canadian Engineer*, February 14, 1918, p. 139.

same place on the earth's surface, say), then,

$$L^{\frac{3}{2}} = \left(\frac{N}{\nu}\right) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (19)$$

in which, N and ν are, respectively, the kinematic coefficients of viscosity of the two fluids in prototype and model. Therefore, the two fluids must be different or, if they are the same fluid, then it must be at different temperatures in the model and prototype. Of course, one may have a model of the same size as the prototype with the same fluid at the same temperature in the model and prototype; but that again might not be very useful.

Thus, in this case, the conclusion is reached that, to have the model as small as possible, the ratio, $\frac{N}{\nu}$, should be as large as possible, and the ratio, $\left(\frac{G}{g}\right)$, as small as possible.

Examining Equation (19) more closely, the rule to secure a small-sized model evidently is to select a fluid having as small a kinematic coefficient of viscosity as possible, and also to test the model where the value of g is as large as possible (see Equations (16) and (17)).

Air would not likely make a small-sized model because, owing to the very small value of its density, which occurs in the denominator of the kinematical coefficient, the value of the coefficient is relatively large. Water would serve better, and mercury still better, owing mainly to their increasing densities. Hence, one reaches the astonishing conclusion that an airplane propeller might be tested in a small-sized model in mercury.[12]

As another example, consider a ship and its model. At corresponding speeds—whether greater or less than critical velocities—the total resistances to a ship and its true model of proper scale in true "model" fluid (say, mercury) would vary exactly as the respective densities, wet areas, and squares of corresponding velocities. This relation between ship and model would hold for all velocities of the ship and model at corresponding speeds (constant-velocity ratio). It would no longer be necessary to treat the skin friction and wave-making resistances separately. This truth illustrates perfectly the real advantage of a true model, namely, that it reduces the entire problem to a comparison of simple ratios, whereas, in general, for arbitrary sizes and shapes, the ratios are not simple. If they were, there would be no need for models.

SUMMARY OF EQUATIONS OF SIMILITUDE

Properties of Materials.—Thus far, nothing has been said of the force ratio, or mass ratio. The truth is that both have been determined by the method of making the problem kinematical, which is another process of elimination that has obscured the true relations and added to the confusion previously discussed.

---

[12] As suggested in the paper on "Ice Diversion, Hydraulic Models, and Hydraulic Similarity," *Transactions*, Am. Soc. C. E., Vol. LXXXII (1918), footnote, p. 1187. It is shown that the model would be one-twenty-fifth full size and would run at one-fifth full linear speed.

As is well known, ordinary mechanics involves only three "fundamental" units, in terms of which all quantities of that science may be measured. At least three equations will be available connecting ratios of homologous quantities in similar systems with the ratios of homologous measures in each of the three fundamental units, L, F, and T (any three independent units may be taken as fundamental, but here, length, force, and time are selected). Procedure in sciences of more than three fundamental units is the same, there being simply an additional equation for each such unit beyond three.

Summary.—Hence, summarizing the equations of similitude, the four necessary conditions for dynamic similarity among ordinary mechanical systems are:

$$L = R \quad \text{(11)}$$

$$\left(\frac{V^2}{L} = \frac{L}{T^2}\right) = \frac{G}{g} \quad \text{(12)}$$

(first property equation),

$$f_1\,(L, F, T) = \phi'\,(P') \quad \text{(20)}$$

(second property equation),

$$f\,(L, F, T) = \phi\,(P) \quad \text{(21)}$$

To illustrate the use of the property equations it may be explained that, if P' in Equation (20) is to represent the density, this equation would reduce to the form:

$$\frac{F\,T^2}{L^4} = \rho$$

since the left-hand member expresses the dimensions of ρ. In Equation (21), P denotes the ratio of pertinent homologous properties, such as elastic moduli, electrical constants, etc. If more than one property, or material, must satisfy Equation (21) separately, suitable bodies must be found, made, or arranged, so to do, before similarity can be sustained. If a property (density, or load, say) satisfies the third equation (Equation (20)), then the fourth equation (Equation (21)) is free, as to that property, and may be used with some other property or properties. Thus, P may denote the ratio of homologous viscous, or elastic, pressures, or both, if they should have the same ratio, either by chance or by artifice.

Thus, the ratios L, F, and T, can each be found in terms of the conditions and properties of systems and materials by solving the equations. Conditions and properties include: Terrestrial gravity, internal accelerations, density, viscosity, elasticity, capillarity, mass attraction, electrical forces, etc. Therefore, solving Equations (11), (12), (20), and (21):

$$L = R = \phi_1\ (G, \rho, \text{and } P) \quad \text{(22)}$$

$$F = \phi_2\ (G, \rho, \text{and } P) \quad \text{(23)}$$

$$T = \phi_3\ (G, \rho, \text{and } P) \quad \text{(24)}$$

From these equations, the ratios of homologues of any kind can be found, since any such ratio can be "derived" from its dimensional equation. The specifications for the model may be written from this analysis. Then, if the model is caused to move with motions similar to those of the prototype, the similarity will continue, because all conditions of Newton's theorem are satisfied.

Example 1.—Write the conditions for dynamical similarity in a model elastic structure. Besides Equations (11), (12), and (20) (see illustration given after Equation (20)), Equation (21) takes the form,

$$\frac{F}{L^2} = E \qquad (25)$$

in which, E is the ratio of homologous elastic moduli. If shear as well as extension is taken into account, both E's must satisfy Equation (25). Engineering will be required. True models must be built to specification. They are not more likely to be picked up by chance than is a well-designed structure. If factors of safety enter, still more moduli must satisfy Equation (25). It is as great a mistake to use the same materials, or fluids, in a model and prototype, as it would be to use the same scale. Therefore, solving these simultaneous equations, and assembling relations,

$$L = R = \frac{E}{\rho\ G}$$

or,

$$\frac{R\ \rho\ G}{E} = 1 \qquad (26)$$

$$F = \frac{E^3}{\rho^2\ G^2} \qquad (27)$$

and,

$$T = \frac{1}{G}\sqrt{\frac{E}{\rho}} \qquad (28)$$

Then, also,

$$L = T\sqrt{\frac{E}{\rho}} \qquad (29)$$

$$F = E\ L^2 \qquad (30)$$

and,

$$V = \frac{L}{T} = \sqrt{\frac{E}{\rho}} \qquad (31)$$

These equations conform exactly to the dimensions of factors of safety given in the paper on "Ice Diversion, Hydraulic Models, and Hydraulic Similarity" (page 1188), previously mentioned. Equation (26) may be called the "characteristic."

Equations (26) to (31) include the velocities of sound in the solids as well as the velocities of oscillations. In Equation (31), V denotes the ratio of maximum velocities (components) due to vibrations alone; but it is the correct velocity ratio.

These results give great generality for elastic models of suitably selected, made, or arranged materials; but, if model and prototype are of the same materials, the model not only becomes non-representative (elastic properties change in small sizes of given material), but of limited utility, as may be seen by making $E = 1$ and $\rho = 1$, giving:

$$L = R = \frac{1}{G} \qquad (32)$$

$$F = \frac{1}{G^2} \qquad (33)$$

$$T = \frac{1}{G} \qquad (34)$$

$$L = T \qquad (35)$$

$$F = L^2 \qquad (36)$$

$$V = 1 \qquad (37)$$

or,

$$V = v \qquad (38)$$

This latter method ($E = 1$, $\rho = 1$) is to elastic models what "Reynolds' Model Law" is to fluid models; both methods lead to undesirable conditions for model tests. Notice in Equation (26) that $RG = 1$ is the characteristic, since $E = 1$ and $\rho = 1$ (see, also, Equation (32)).

Example 2.—Construct two similar systems, each combining mechanical, elastic, and viscous actions. Suspend a weight by a spiral spring, also making a similarly suspended weight to a smaller scale, such that all manner of oscillations and motions of solids and fluids will be similar. Equation (21) must satisfy two properties (forms two equations):

$$F = k\,L = \mu\,\frac{L^2}{T} \qquad (39)$$

Letters refer to ratios of homologues, unless considering one system separately, when they refer to indicated measures. The elastic spring constant and density are, respectively, k and ρ, the former being of the dimensions, pounds per foot. Then, since $R = L$, solve Equations (11), (12), (20), and (21), as follows:

$$\frac{k^3\,\rho}{G\,\mu^4} = 1 \qquad (40)$$

$$L = \frac{k^5\,\rho}{\mu^6\,G^2} = \left(\frac{k}{\rho\,G}\right)^{\frac{1}{2}} = \frac{\nu^{\frac{2}{3}}}{G^{\frac{1}{3}}}, \text{ etc} \qquad (41)$$

$$F = \frac{k^6\,\rho}{\mu^6\,G^2} = \frac{k^3}{\mu^2\,G} = \frac{\mu^2}{\rho}, \text{ etc} \qquad (42)$$

$$T = \frac{\mu^7\,G}{\rho^2\,k^5} = \frac{k}{\mu\,G} = \frac{\nu^{\frac{1}{3}}}{G^{\frac{2}{3}}}, \text{ etc} \qquad (43)$$

There are four equations involving the three quantities, L, F, and T. The solution demands that these equations be consistent, that is, that they apply simultaneously. The condition is found by eliminating L, F, and T, which gives Equation (40), the "characteristic." Had there been five equations instead of four, there would have been two characteristics instead of one. In fact, a superabundance of properties may introduce any number of characteristics according to the circumstances. It will be seen that some of the quantities involved are arbitrary, as the mass, or weight, of spring, or model;

and this is the case with the elastic constant, which may be assumed at 10 lb. per ft. for the model; or, after testing the full-sized spring, the model must be made to have a constant conforming to the ratio, k. The characteristic (Equation (40)) is equivalent to,

$$\frac{k_1^{\ 3}\ \rho_1}{G_1\ \mu_1^{\ 4}} = \frac{k_2^{\ 3}\ \rho_2}{G_2\ \mu_2^{\ 4}}$$

each member being dimensionless, and it contains all the properties involved—G, ρ, μ, and k. It corresponds to characteristics in other homologies, Reynolds' number, Froude's number, etc.

Adopting air as the fluid, the following data, in centimeter-gramme-second units, may be found in physical tables, such as International and Critical Tables, and those by Landolt and Börnstein, for the large scale and small scale, respectively. The values used in the problem are merely approximations:

Air temperatures, in degrees centigrade = 260 and 20
$10^7\ \mu$ = 2 800 and 1 830
$10^7\ \rho$ = 6 000 and 12 000
ν = 0.467 and 0.1526

Then, by slide-rule,

$$\frac{\mathrm{M}}{\mu} = 1.53 \pm;\ \frac{\varDelta}{\rho} = \frac{6}{12} = \frac{1}{2} \pm;\ \text{and}\ \frac{\mathrm{N}}{\nu} = 3.06 \pm$$

In this case, the density for air determines the density ratio for the homology, as all such ratios must be the same throughout for homologues. It will not be difficult to make springs and weights to satisfy this requirement, if the maker can grasp the idea that he is not making springs, but models. The centimeter-gramme-second ratios are the same as the English. The springs are to oscillate under the action of gravity, as well as by reason of their elasticity. Therefore, all homologous accelerations are equal, although they need not necessarily be the same, numerically, as gravity. Thus, the ratio, $G = 1$, and the characteristic gives:

$$k^3 = \frac{\mu^4}{\rho} = 10.96\Big]_{G=1};\ k = 2.22;\ R = L = \left(\frac{k}{\rho}\right)^{\frac{1}{2}} = \left(\frac{k}{\mu}\right)^2 = 2.104\Big]_{G=1}$$

and, since $G = 1$, the mass ratio is,

$$M = \frac{T^2\ F}{L} = \rho\ L^3 = 4.675\Big]_{\rho=\frac{1}{2}} = F$$

Now, make, calibrate, and adjust, the similar springs. The larger may be of spring steel and the other a model, such that its length, effective density, and elastic constant will be in the proper proportions, L, ρ, and k. The weight (arbitrary) on the smaller spring is 10 lb.; then, on the larger spring, it must be 46.75 lb., the respective masses being $\frac{10}{32.2}$ and $\frac{46.75}{32.2}$ slugs. Thus, to check the calculations, 1.5 ft. being an extension (to one side) of the larger spring,

$$G_1 = \frac{k\ L}{M_1} = 22.9 \text{ ft. per sec.}^2$$

and,

$$G_2 = \frac{10 \times 0.731}{M_2} = 22.9 \text{ ft. per sec.}^2$$

The final check is by Equation (16) thus:

$$\frac{L^3\ G}{\mathrm{N}^2} = \frac{9.35 \pm \times 1}{9.36 \pm} = 1$$

which checks.

Of course, Reynolds' equation will now be satisfied, thus:

$$\frac{V\,L}{\mathrm{N}} = \frac{v\ l}{\nu} = 18.9$$

This checks exactly, although it is not expressed entirely in English units. It is evident, however, that these results could not possibly have been attained, had Reynolds' equation been considered the sole requirement in addition to geometrical and elastic similarity, as, in that case, Newton's theorem would have been ignored, whereas, it must always be effective in every case of dynamic similarity.

If, now, the springs are extended in proper ratio, *R,* at any angle to the vertical, and if they are released simultaneously, in their properly tempered atmospheres, the motions will be similar, although compounded of various types of oscillation. By properly proportioned side impulses on the sides of the springs, flexural vibrations may be introduced, as well as longitudinal and pendular oscillations, without destroying the similarity, provided shearing has been introduced into the equations, as mentioned. The springs must be confined in similar spaces.

To attain this result means, of course, that the material, or form (without destroying theoretical similarity, both external and internal), of the model spring must be devised so as to develop an effective shearing modulus of elasticity, or, if necessary, an artificial "modulus", such as will satisfy Equation (21), simultaneously with k. An illustration is given by Equation (39), in which, k and μ are connected.

The resulting motions of the two springs are not synchronous (time scales being different), and would gradually subside unless maintained by some device (clockwork). It should be noticed, however, that if time is reckoned according to the model time-scale (as by a "model" watch), the motions would be synchronous in the sense of similarity. The elastic motions need not be, and probably are not, isochronous.

The problem has been solved by postulating that the air will act as an incompressible fluid. This is accurate for most motions of solids in air, such, for example, as the motions of ordinary machinery, automobiles, and airplanes. If fluid elasticity is to be involved (sound effects, cavitation, etc.), the ratios of corresponding elastic moduli must be made to satisfy Equation (21), just as E does in Equation (25) which is merely a form of Equation (21). In this way, sound, cavitation, etc., should be reproduced in the model.

It is worthy of notice that, whether the cubic strains are isothermal, adiabatic, or whether they vary according to an arbitrary law, the ratio of homologous moduli is the same as the ratio of the homologous atmospheric pressures. This assumes that the ratio of the ratios of the specific heats is equal to unity. In the case of gases, exhibiting a variance in this respect, the ratio of the ratios of the specific heats must be introduced into formulas.

By "specific heat" is here intended the general specific heat, isothermal and adiabatic conditions being merely special cases.

Whether it will be permissible to modify the natural atmospheric pressure, or introduce an artificial "atmosphere", in a model, depends upon conditions in the prototype. If the normal pressure in the prototype nowhere falls below p lb. per sq. ft., then the atmospheric pressure in it may be reduced by p lb. per sq. ft. without otherwise changing materially the dynamic action of the prototype. In such cases the model atmosphere may be adjusted to correspond to such hypothetically reduced atmosphere on the prototype, without making any actual change of the atmospheric conditions in the prototype.

Under these circumstances, there would be a point, or points, in the model at which the pressure would be nil. This last condition determines the least possible pressure of the model atmosphere, for there is no such thing as a negative pressure. It is only a difference in pressure, or its direction, which may be considered as being negative.[13] Such difficulties are not problems of theory. Among many problems, they are merely some which must be solved in the field by the Newtonian model engineer, a type of engineer yet to be developed, of whom much may be expected. A problem involving atmospheric pressures is treated in Appendix I.

Herein lie possibilities for creating a new department of engineering design, that of "model engineering", concerned with the technical methods of constructing models which will perform with theoretically exact similarity. In this way, the most difficult problems of mechanics may be solved by making and shaping materials (including fluids) so as to have desired properties and qualities. This care is especially necessary in designing important structures when the mathematical difficulties render solutions impracticable. This does not necessarily mean difficult constructive problems in making the models, since, in many cases, properly constituted materials and fluids are at hand, or may be made easily.

Many ordinary kinds of forces are properly constituted so as not to interfere with theoretically exact model operation.[14] Among these may be mentioned all those similar forces which are proportional to homologous areas and to the squares of homologous velocities, such as: (1) Pressures; (2) ordinary skin friction under the V^2 law; (3) reactions of moving masses; (4) "adjusted viscous forces"; (5) static and dynamic effects due to heads of water and of water falling over a spillway; (6) effects of wind on running ice; (7) transportation of detritus and carriage of floating ice; (8) time on a "model" watch; and (9) measurements in "model" units. Time given by the model watch will agree with the actual time. Weighings and measurements by the model weights and measures agree with actual weights and measures for the prototype. Even factors of safety for structures may be included, if models are properly constructed and properly tested. Moreover, if materials are found, made, or arranged to satisfy Newton's theorem of similarity, as many as may be so provided can be used in the model.

---

[13] See Lamb's "Hydrodynamics," Third Edition, p. 18, where such pressure changes are mentioned, but without notice of the limitation.

[14] *Transactions*, Am. Soc. C. E., Vol. LXXXII (1918), p. 1171.

The effect of varying values of g may be obtained by rotating the model about a vertical axis, if care is taken to place the model in a proper position relative to that axis. This has the effect of increasing g and may be desirable when the properties of the model leave the size to be determined by the acceleration ratio of the homology. Theoretically, when above the surface of the earth, the model should be nearer the earth's center than the prototype, supposing viscous forces to be present. When below the earth's surface, the reverse would be the case. Other conditions than viscosity also affect the size of the model, if similarity is to be obtained. (For effect of pressure, see Appendix I.)

It may be desirable to know the significance of "Reynolds' number." That dimensionless number occurs in many formulas for the flow of viscous fluids;[15] its form is given in Equation (9). It occurs also in Equation (10). It occurs implicitly in Equations (16) and (17). If L is calculated from Equation (16), then, upon forming $\frac{L\,V}{\frac{N}{\nu}}$, the latter will be found equal to unity. That will mean that Reynolds' number is constant as between similar systems, or, $\frac{V\,L}{N} = \frac{v\,l}{\nu} =$ constant. This may also be calculated from Equation (10) by simple division, having regard to the dimensions of velocity. Therefore, if similarity exists throughout the systems, Reynolds' number will be constant throughout.

It cannot be inferred from this, however, that the systems will be similar if Reynolds' number is constant. In order that this latter state of affairs may hold, the scale of the model relative to the scale of the prototype must be determined so that it will satisfy the three equations, (10), (11), and (12), which is the real reason why Reynolds' number is constant in Equations (16) and (17).[16]

Example 3.—Let it be supposed that a tank furnishes water to a 5-in. horizontal discharge under 25-ft. head, and that it is desired to make a model of the plant one-fifth of the full size to be operated with water. By Newton's theorem,

$$G = \frac{V^2}{D} = \frac{V^2\,D^2}{D^3} = \frac{1}{D^3} \ldots\ldots\ldots\ldots\ldots\ldots (44)$$

since Reynolds' number is to be taken constant, and because of the use of water in the model. The velocities are: $V = 8.02\sqrt{25} = 40$ ft. per sec.; and $v = 5 \times 40 = 200$ ft. per sec., since $VD = vd$. Thus, the effective test head is $\frac{200^2}{64.4} = 625$ ft. This head is altogether out of proportion. Reynolds' number does not here prove to be an infallible criterion. However, if the horizontal model discharge, alone, is supplied with water from a main under a head of 625 ft., it is supposed that the flows in the discharges will be similar, although the systems are not similar throughout. Thus, $\frac{G}{g} = \frac{200^2}{1} \div \frac{40^2}{5} = 125$, or the value of the acceleration in the model discharge is 125 times normal g, by Newton's theorem.

---

[15] *Transactions*, Am. Soc. C. E., Vol. 94 (1930), pp. 839 to 841, Equations (8) to (13), inclusive. The friction factor, F, in Equation (12), for example, has been shown by experiment to be a function of Reynolds' number; "Similarity of Motion in Relation to the Surface Friction of Fluids," by Stanton and Pannell, Collected Researches, British National Physical Laboratories.

[16] *Transactions*, Am. Soc. C. E., Vol. LXXXII (1918), p. 1187, first two paragraphs relative to Equation (20), where this important admonition is given by the writer after careful theoretical studies followed by several years of experimental investigation.

Now, try a model with mercury at 80° cent., the water in the tank being at 15° cent. The values of the kinematical coefficients are, respectively, $\nu_m = 0.00097$, $\nu_w = 0.0114$, (centimeter-gramme-second); giving $\frac{\mathrm{N}}{\nu} = 11.76$. Then, by Equations (16) and (17), $R = 11.76^{\frac{2}{3}} = 5.18$, so that a scale ratio of 1 : 5.18 is correct for the mercury model. Hence, $h = \frac{25}{5.18} = 4.82$ ft.; and $d = \frac{5}{5.18} = 0.965$ in. — the respective head and diameter of discharge for the model plant, which is a perfect model in all respects. The velocity ratio is $1 : \sqrt{5}$, or the velocity in the model discharge is less than one-half that in the larger discharge. No calculations have been made for friction, because the mercury model will reproduce it perfectly; nor does the friction head need to be proportional to the square of the velocity in creating the friction heads in the separate systems, because the similarity will exist between the systems at all corresponding velocities, including pure viscous flow. However, the friction heads, as between model and prototype, will vary as the squares of homologous velocities, also including pure viscous flow.

There are other ways by which to satisfy Newton's theorem, and to adapt the apparatus to viscous fluids. One may assume that the scale ratio and acceleration ratio are known; and then adapt the viscosity ratio to suit the requirements, if suitable fluids can be found, or made. If limited to certain fluids, as in the first part of Example 3, one may let the acceleration ratio be determined from Equations (16) and (17) by introducing the properties of the fluids and the chosen scale ratio, R. In every case, it is necessary to satisfy Equations (16) and (17), and to see that conditions conform to Newton's theorem, in order to have similarity. It is the adjustment of values to satisfy Newton's theorem that suffices to establish a model, and not solely the constancy of Reynolds' (Stokes') number.

In the last case mentioned (first part of Example 3), when the acceleration ratio is determined in terms of R and N, it may be necessary to use two given fluids, or one and the same fluid, in model and prototype. In either of these events, the force ratio is,

$$F = \rho\, \nu^2 = \frac{\mu^2}{\rho} \quad \text{.........................} (45)$$

which, of course, is intended to refer to homologues throughout. This force ratio may be found by the general process outlined in connection with and by Equations (20) and (21), as exemplified by Equations (25) to (31) inclusive;

or, that which is the same thing, by solving Equation (20), and the equation formed by the first and third members of Equation (39), for F. In the latter case the result is reached quickly by dividing μ^2 by ρ.

When the two fluids become one and the same fluid, the density ratio and viscosity ratio each become unity, thus also reducing the force ratio (Equation (45)), to unity. This means that the actual total operating force on the model must be the same as the total force operating upon the full-sized prototype. This is an awkward and unnecessary situation, for it frustrates the entire object of a model, in which it is desired to reduce the measures to moderate values. This difficulty has been commented upon by other writers,[17] and it is a little surprising that none of them has pronounced it an absurd extreme, for the purpose of designing by models. The parallel with elastic models was mentioned in connection with Equations (32) to (38), inclusive, in which the velocity in the elastic model was the same as in the full-sized structure.

Furthermore, the ideas have long been current in scientific and engineering literature that the method of allowing the acceleration ratio to adjust itself to the velocity ratio, is equivalent to "eliminating gravity"; and that the motion of the fluid is "non-accelerated," as in the case of steady flow along a pipe. It is true that the mean velocity of steady flow along a cylindrical pipe is uniform, but the fundamental idea of similarity has nothing directly to do with the nature of the flow in any separate system. The idea concerns itself only with a comparison of homologous conditions in two, or more, distinct systems. Thus, homologous velocities must be in fixed ratios, whether those velocities are uniform or variable, as to time or place, in any given system. The similarity applies to the motion of fluid particles, such that every eddy or irregularity of flow is to be repeated in the model. If this were not true, there would be no such thing as comparing two systems, for it is this elemental similarity which causes like actions. All these eddies and irregularities of flow are the vital agencies which produce the principal friction losses—the main resistances with which it is usually necessary to deal. Consequently, instead of being non-accelerated, the motion is highly accelerated, both longitudinally and transversely, to say nothing of the initial acceleration to introduce the flow, and to maintain it at joints, fittings, and bends. It seems to the writer that the theory of probability might be applied to the flow of water in a manner somewhat similar to that used in the kinetic theory of gases. The study of the nature of such flow (turbulence included) offers one of the great fields for hydraulic and hydrodynamic research.

The use of the same materials in model and prototype leads to certain limiting cases of models. Weber has tabulated[18] five such cases which compare the operations of models of gravity, mass attraction, viscosity, elasticity, and capillarity.

The writer prefers not to consider these cases as separate types of similarity, but merely as special forms of more general cases due to the operation of models of different types, when made of materials identical with those of the prototype. Mathematically, this amounts merely to assigning particular values to the parameters in the equations of each general type of model. This was illustrated when placing E and ρ, each equal to unity, in Equations (26) to (31), inclusive, to reduce them to Equations (32) to (38), inclusive. The latter condition corresponds exactly with Weber's elastic model, but, as shown, is merely a limiting case of one more general.

---

[17] "Model Experiments and the Forms of Empirical Equations," by Buckingham *Transactions*, Am. Soc. Mech. Engrs., Vol. XVII (1915), Paragraph 30; see, also, "Dimensional Analysis," by Bridgman, pp. 85 and 86.

[18] Jahrbuch der Schiffsbautechnischen Gesellschaft, 1919, p. 414.

Taking this view, it would seem better to compare general types of models, rather than special cases of those types. To make such comparison, it is not necessary to go beyond the use of three dimensions, since all homologous ratios may be derived from any three. It will be sufficient to indicate such a general synopsis by comparing the two types which have been previously treated. (See Table 1.)

TABLE 1.—COMPARISON OF ELASTIC AND VISCOUS MODELS.

Ratio.	Symbol.	ELASTICITY.		VISCOSITY.	
		Dimensions.	Equations.	Dimensions.	Equations.
Scale (linear)...............	L	$\frac{E}{\rho G}$	(26)	$\frac{\mu^{2/3}}{\rho^{2/3} G^{1/3}}$	(16), (41)
Force.......................	F	$\frac{E^3}{\rho^2 G^2}$	(27)	$\frac{\mu^2}{\rho}$	(39), (42)
Time	T	$\frac{E^{1/2}}{\rho^{1/2} G}$	(28)	$\frac{\mu^{1/3}}{\rho^{1/3} G^{2/3}}$	(17), (43)
Characteristics:					
General case.............	....	$\frac{R \rho G}{E} = 1$	(26)	$\frac{R^3 \rho^2 G}{\mu^2} = 1$	(16)
Special case :					
Same materials in model and prototype; values of constant ratios.....	....	$\left.\begin{matrix} R G = 1 \\ E = 1; \rho = 1 \end{matrix}\right\}$	(32)	$\left\{\begin{matrix} R^3 G = 1 \\ \mu = 1; \rho = 1 \end{matrix}\right.$	$\}$ (16), (18)

In some instances the special cases of characteristics can be associated with the names of celebrated investigators. For example, $R^3 G = 1$ indicates the constancy of Reynolds' (Stokes') number, as shown by linear scale ratio and acceleration ratio, thus,

$$R^3 G = L^3 \frac{L}{T^2} = L^2 \frac{L^2}{T^2} = (L V)^2 = 1 \ldots\ldots\ldots\ldots (46)$$

which is the same as $V L = 1$, or $V D = 1$; that is, respectively, $V L = v l$, and $V D = v d$, the usual form of statement that Reynolds' number (Stokes' number) is constant for a given fluid. For purposes of definition, it seems better to state such dimensionless numbers in terms of the constants of the particular problem involved; that is, in terms of given data. This has been done in Table 1.

It is instructive to notice that the dimensions of L, F, and T, are given in terms of the "dimensions"—acceleration, density, viscosity, elasticity, etc.—

in which, by making the problems kinematical, the number of arbitrary dimensional ratios may be reduced to two, the least possible number. As an example, the dimension, L, in viscosity may be expressed in the two dimensions, N and G, each being necessary dimensions in a viscous model. Theoretically, there are three dimensions, μ, ρ, and G. In elastic models the three dimensions are, E, ρ, G, etc. In this last case, to secure similarity, E must be the same for all types of moduli involved. Observe that in each case the ratio, E, is that of two similar moduli.

It will be noticed, of course, that the general case of the characteristic for viscous models, in Table 1, is precisely the form expressed by Equations (16) and (17).[19] Reynolds' number is a special case of this general characteristic of viscous models. It may be worth while to emphasize that this characteristic, and also that Equations (10), (11), and (12), are satisfied by straight-line viscous flow, as well as by turbulent flow, when similarity holds.[20] There is no essential difference in these two types of similarity; both of them conform exactly to Newtonian, which alone is dynamic similarity. If one will substitute the dimensions of N and ν in Equations (16) and (17), either of the two forms of Newton's theorem may be made to appear. When this criterion is satisfied throughout systems containing viscous fluids (model and prototype), and only then, can similarity exist throughout the systems.

It seems to be somewhat of an open question, whether similarity will establish itself, as in a pipe, but Stanton and Pannell's experiments, previously mentioned, go a long way toward showing that it will. These experiments, properly interpreted, are to hydraulics what Stokes' investigations were to hydrodynamics. It is hoped that experiments of the same nature will be made for differing degrees of roughness, and for differing shapes of channel boundary.[21] Every hydraulic engineer should study Stanton and Pannell's paper on "Similarity of Motion, in Relation to the Surface Friction of Fluids."[22] The word "similarity," in the title of this paper, is used in a somewhat different sense from that adopted in the writer's present paper unless the conditions of the test were such as to satisfy Newton's theorem.

Of course, Reynolds' number was satisfied in Stanton and Pannell's experiment, so that theoretically exact dynamic similarity may have been in operation; but it is certain that Newton's theorem must be shown to hold in any case, before an inference to that effect can be sustained.

Relative to the significance of Reynolds' number, the functional relation between the coefficient of hydraulic friction and the value of that number, as exhibited in the experiments of Messrs. Stanton and Pannell, shows that all hydraulic friction, of whatever nature, including turbulence, depends primarily and ultimately upon viscosity.[23] This is not a common view. It is, in fact, one that has been rejected by numerous writers of profund attainment,[24] but the writer clings to the affirmative for the reasons cited in the former reference.[23]

---

[19] See, also, *Transactions*, Am. Soc. C. E., Vol. LXXXII (1918), p. 1186, Equation (19).

[20] See *Transactions*, Am. Soc. C. E., Vol. 94 (1930), pp. 833–841, for an explanation of hydraulic resistance of all kinds.

[21] "Strömungswiderstand in rauhen Röhren," von K. Fromm, *Zeitschrift für angewandte Mathematik und Mechanik*, Band 3, Heft 5. Gives results of recent experiments on resistances in channels of different degrees of roughness and of different shapes.

[22] Collected Researches, National Physical Laboratory (British).

[23] *Transactions*, Am. Soc. C. E., Vol. 94 (1930), p. 841.

CONCLUSIONS

It is concluded that the present usage of models is not wholly consistent with sound theory. Newton's theorem for conditions of dynamical similarity has been gradually fading from view, while the fallacious idea that there are differing kinds of similarity has been unduly propagated. Nothing can be farther from the truth than the idea of differing kinds of similarity. All the variously designated "laws" of similarity reduce immediately to Newton's one unique definition and theorem. When that theorem is fully satisfied, Reynolds' number will be satisfied. Even in the absence of fluids of any kind, if Newton's theorem is satisfied under the conditions of acceleration in Equation (44), Reynolds' number will be satisfied ($V D = 1$), but that number may be satisfied without the existence of similarity. Moreover, except as just stated, and in the case mentioned, Newton's theorem may hold and similarity may exist, when Reynolds' number is not satisfied (for example, when, in the absence of viscous fluids, $v\,d$ is not the same in model and prototype).

Besides the establishment of initial similarity, the necessary and sufficient conditions to establish a model are, that the size of the model, the coefficient (kinematic) of viscosity, and the acceleration ratio, shall all be determined so that Equations (10), (11), (12), and (16)—in the general case Equations (11), (12), (20), and (21)—shall be satisfied. If this is not done, it is not known exactly what errors result, and one must rely upon the supposition that the effects of viscosity (and any other properties) in distorting the model are not great. Perhaps the important conclusion is that a thorough investigation should be made to ascertain what materials and fluids will serve best as materials for models, and to synthesize, if possible, materials which are capable of having their "physical properties" adjusted. By "physical properties" is meant, not only the usual properties of materials, but artificial properties produced in a practical way so as to serve in the manner desired. It is not improbable that, in due time, the stream lines and eddies of flowing water will be produced by mechanisms in the absence of fluids altogether.

There is a great field for the development of a corps of specialists to build models to the specifications indicated by the Newtonian theory, which is the only sound theory for models.

The final paragraph of Appendix I shows that an homology of elastic solids and incompressible fluids is mathematically identical with an homology of elastic fluids and incompressible solids. More generally, any homology of solids, liquids, and gases of varying (constant) characteristics (properties), will remain a homology, in the sense here used, providing the physical properties of the constituents and the other internal and extraneous conditions, instantaneously adjust themselves (or remain constant, as adjusted) to the requirements of Newton's theorem.

---

[24] *Transactions*, Am. Soc. M. E. (1915), p. 281, "Model Experiments and the Forms of Empirical Equations," by Buckingham; Encyclopædia Britannica, Eleventh Edition, Vol. 27, p. 736; "Dimensions of Units," by Larmor; "Dimensional Analysis," by Bridgman, p. 85; and "Hydrodynamics," by Lamb, Third Edition, 1906, p. 593.

APPENDIX I

As further illustration of the nature and use of the dimensional forms for *L, F,* and *T,* given in Table 1, and as illustrating the method of treating compressibility of fluids, such as mentioned after solving Example 2, involving Equation (39), it will be interesting to examine the possibility of testing an airplane propeller in air under pressure as has been suggested.[25] This, of **course, would increase the density and, consequently, diminish the kinematic viscosity, so as to admit of an accurate test of a model of reduced scale, assuming that the viscosity is not seriously affected adversely by an increase of pressure. It will then be of additional interest to see if the conditions of test can be improved by substituting a different gas in the model. The method of substituting a liquid (mercury, say) to represent the air, has been mentioned in the remarks following Equation (19). The question of "slip" is discussed in** treatises on the kinetic theory of gases, and is also referred to by Lamb.[26]

The specifications of a gas should always relate to standard conditions of pressure and temperature, usually 1 atmosphere, and 0° cent. Then the gas equation, $P = c\,w\,\theta$, determines the pressure for any other conditions, the standard conditions being indicated by $P_s = c\,w_s\,\theta_s$, assuming that c remains constant. Whence,

$$\frac{P'}{P_s'} = \frac{w'\,\theta'}{w_s'\,\theta_s'} \qquad (47)$$

and,

$$\frac{p_m}{p_s} = \frac{w_m\,\theta_m}{(w_s)\,\theta_s} \qquad (48)$$

in which, the primed symbols may relate to the prototype, and the other symbols to the model. The subscript, *s,* refers to standard conditions of pressure and temperature. The symbols, *w,* refer to the specific weight, or to the density, indiscriminately, as long as the acceleration of gravity has its standard value. In either case, *w* is the reciprocal of the specific volume of the gas, the volume in terms of which the gas equation is so frequently written. Observe that the model and prototype may contain either the same, or different, gases.

Then, the ratios of homologues, prototype to model, become:

$$P = \frac{P'}{p_m} = \frac{w\,\theta}{w_s}\left]\begin{array}{l} P_s' = p_s \\ \theta_s' = \theta_s \\ \theta = \dfrac{\theta'}{\theta_m} \\ w = \dfrac{w'}{w_m} \\ w_s = \dfrac{w_s'}{(w_s)_m} \end{array}\right. \qquad (49)$$

and,

$$\rho = w = P\,\frac{w_s}{\theta}\Bigg]_{G' = g_m = g} \qquad (50)$$

---

[25] See *Transactions*, Am. Soc. C. E., Vol. LXXXII (1918), footnote, p. 1187.

[26] "Hydrodynamics," Third Edition, p. 594.

In Equation (50), the equality of the specific weight ratio, w, to the density ratio, ρ, refers to the comparison of conditions in the prototype and model as affected by standard gravity (equality of g in model and prototype). If the acceleration is really different in the model, then the specific weight ratio, referred to the resulting different acceleration ratio, is not the same as the density ratio. Of course, the density ratio is unaltered. Thus,

$$\bar{w} = \rho\, G = w\, G \ldots\ldots (51)$$

in which, $\bar{w}$ indicates the new specific weight ratio. It may be represented by $w\, G$.

If it is assumed, as usual, that the viscosity of the gas is independent of the pressure,[27] then, for fixed temperatures in model and prototype, the absolute viscosity of each gas will be determined by its temperature. The resulting constant viscosity ratio will be indicated by μ, which, accordingly, becomes a constant of the problem.

Bearing in mind the foregoing relations, and recalling the solution of Equations (11), (12), (20), and (21), (Newton's conditions of similitude), as indicated in Table 1, the following simultaneous equations may be written:

$$R^3 = \frac{\mu^2}{\rho^2\, G} \ldots\ldots (52)$$

$$F = \frac{\mu^2}{\rho} \ldots\ldots (53)$$

$$T^3 = \frac{\mu}{\rho\, G^2} \ldots\ldots (54)$$

$$\rho = \frac{P\, w_s}{\theta} \ldots\ldots (55)$$

$$F = P\, R^2 \ldots\ldots (56)$$

Equation (56) represents a very obvious equality.

As mentioned in the text, any quantity of ordinary mechanics can be expressed in terms of any three "fundamental" dimensions. Let these three dimensions be the most important three constants of the problem, say, R, μ, and $\frac{\theta}{w_s}$; or, P, μ, and $\frac{\theta}{w_s}$, if the pressure is more important than the scale ratio. Hence, in the former case, eliminating ρ, G, and P, the dimensions L, F, and T, are expressed in terms of the former set of dimensions, as follows:

$$L = R \ldots\ldots (11)$$

$$F = \mu\, R \sqrt{\frac{\theta}{w_s}} \ldots\ldots (57)$$

$$T = R \sqrt{\frac{w_s}{\theta}} \ldots\ldots (58)$$

from which the ratio of any pair of homologues may be found. Thus, for example,

$$G = \frac{L}{T^2} = \frac{1}{R}\left(\frac{\theta}{w_s}\right) \ldots\ldots (59)$$

---

[27] "Hydrodynamics," by Lamb, p. 536.

$$\rho = \frac{T^2 F}{L^4} = \frac{\mu}{R}\sqrt{\frac{w_s}{\theta}} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(60)$$

$$P = \frac{F}{L^2} = \frac{\mu}{R}\sqrt{\frac{\theta}{w_s}} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(61)$$

$$V = \frac{L}{T} = \sqrt{\frac{\theta}{w_s}} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(62)$$

If it is desired to have a small model, R will be chosen, necessarily, as large as practicable, numerators of ratios relating to the prototype. This would tend to make the ratio, P, smaller, or the pressure in the model would be increased as the size of the model is decreased. This puts a practical limitation on the size of the model. However, by taking θ as large as possible, and w_s as small as possible, that is, keeping the temperature of the model atmosphere low, and selecting a fluid for the model with as great a density (under standard conditions) as practicable, the conditions of operation will be improved. If air is used in both model and prototype, the density of the air in the model may be increased by increasing its pressure. There does not appear to be any great changes in μ for ordinary changes of temperature, but one should use a model gas having as little viscosity as possible. Notice that diminishing θ (in case of most gases) has a like effect upon μ. In following the discussion, the text shows when a symbol relates to a ratio, and when it relates to a quantity in one of the systems only (model or prototype).

The investigator is now in a position to examine any homology which contains either the same, or different, fluids, especially for the purpose of determining the suitability of a particular fluid for use as a "model" of another fluid. Thus, let it be required to examine air for use as a model of itself, say, for the purpose of testing a model airplane propeller of small size in an atmosphere of modified air.

For this case, $w_s = 1$, since the specific weight of air is always the same for standard conditions; but θ may be modified, say, from 40° cent. (hot summer weather) to 0° cent., in the model; thus $\theta = \frac{313°}{273°} = 1.146$ (slide-rule), while μ, for these conditions, will be about $\frac{190}{170} = 1.118$; therefore, the dimension, $\sqrt{\frac{\theta}{w_s}}$, becomes 1.070. The size of the model may be tried at one-sixteenth the full size of the propeller, $R = 16$. Theoretically, the space in which the model is tested should be indefinitely large, all near-by objects being carefully made to scale, yet, for convenience, the model may be tested in a tunnel, or in a large pipe, provided the model is sufficiently small, relatively thereto. Equations (11), (57), and (58) then give, $L = 16$; $F = 19.14$; and $T = 14.94$, which, substituted in the first forms of Equations (59), (60), (61), and (62), or, if desired, the values of μ, R, and $\frac{\theta}{w_s}$, directly substituted in the second forms of Equations (59), (60), (61), and (62), give $G = \frac{1}{13.95}$; $\rho = \frac{1}{15.34}$; $P = \frac{1}{13.36}$; and $V = 1.07$.

Thus, the greatest difficulty with an air model is caused by the relatively high pressure in the model (about $13.36 \times 14.7 = 196.5$ lb. per sq. in.); but this would operate nicely if it were practicable to use such pressure.

In order to ascertain whether it is possible to improve the pressure condition in the model, another fluid, different from air, may be examined. An improvement in this respect will be made, if a gas can be found (it has been shown previously that a liquid such as mercury might serve) having a greater standard density than that of air. There are several such gases and vapors, which should be thoroughly investigated.

For present purposes, it will be assumed that a suitable gas can be found of viscosity (absolute) 1.111 times that of air under the actual conditions of the test. (It would be better to have its viscosity less, but it is assumed greater, for illustration). Thus, μ is about 0.9, which will be considered as the value applying for the temperature conditions of the test, since, as mentioned previously, the viscosity of the gas is independent of the pressure. The density ratio under standard conditions will be taken as $\rho = w_s = 0.133 = \frac{1}{7.525}$. The scale ratio, R, will be 16, as before.

With these three constants (μ, ρ, and G) fixed, the problem becomes determinate by Equations (11) and (57) to (62). Then, by introducing the numerical values of the constants, making the calculations as suggested, and as illustrated in the previous calculations, and by assembling all the formulas and results obtained in this Appendix, Table 2 may be compiled as shown.

A further improvement can be made if μ is reduced in the model. This suggests the idea of devising a method of synthesizing, or treating, a fluid in such manner as to secure a decreased viscosity without altering seriously the value of the density ratio. This makes the molecules of the gas (regarded as a homogeneous fluid) more "slippery," or, perhaps it would be more accurate to say, rendering the molecules less "coherent," so as to admit of rapid shearing distortion with less resistance.

Hence, it appears, that a different gas for the model may admit of reduced pressure in the testing pipe and, also, approximately, in the case examined herein, less than one-half the pressure required in the prototype, in this case, air. Moreover, the velocity in the model, for such change, is reduced to about one-third the velocity in the prototype; it is reduced to about one-third its value for the test of the model propeller in air.

The acceleration ratio, G, is not here due directly to gravity (terrestrial), but relates to homologues in the form of eddies and turbulence generally, as well as to centripetal accelerations at bends, etc. In fact, only the portion of space surrounding (and including) the model propeller, and along the testing pipe, or tunnel, forms the model. Whether the restricted dimensions of the pipe will have a material effect upon the action of the model, is a question which must be considered by the engineer.

TABLE 2.—CONDITIONS FOR TEST OF MODEL AIRPLANE PROPELLERS IN GASES UNDER PRESSURE (STUDY OF POSSIBILITIES).

Symbol.	DIMENSIONS.			ILLUSTRATIVE NUMERICAL VALUES. TEST OF MODEL IN:	
	In Terms of:			(*a*) Same Gas (Air).	(*b*) Different Gas.
	μ, ρ, and G.	μ, R, and $\frac{\theta}{w_s}$.	μ, P, and $\frac{\theta}{w_s}$.	$w_s = 1$; $\theta = 1.146$; and $\mu = 1.118$.	$w_s = 0.138$; $\theta = 1.146$; and $\mu = 1.111$.
L........	$\frac{\mu^{2/3}}{\rho^{2/3}\,G^{1/3}}$	R	$\frac{\mu}{P}\sqrt{\frac{\theta}{w_s}}$	16	16
F........	$\frac{\mu^2}{\rho}$	$\mu R\sqrt{\frac{\theta}{w_s}}$	$\frac{\mu^2}{P}\left(\frac{\theta}{w_s}\right)$	19.14	42.3
T........	$\frac{\mu^{1/3}}{\rho^{1/3}\,G^{2/3}}$	$R\sqrt{\frac{w_s}{\theta}}$	$\frac{\mu}{P}$	14.94	5.45
G........	$\frac{L}{T^2}$†	$\frac{1}{R}\left(\frac{\theta}{w_s}\right)$	$\frac{P}{\mu}\sqrt{\frac{\theta}{w_s}}$	$\frac{1}{13.95}$	$\frac{1}{1.856}$
ρ........	$\frac{T^2 F}{L^4}$†	$\frac{\mu}{R}\sqrt{\frac{w_s}{\theta}}$	$P\,\frac{w_s}{\theta}$	$\frac{1}{15.34}$	$\frac{1}{52.20}$
P*........	$\frac{F}{L^2}$†	$\frac{\mu}{R}\sqrt{\frac{\theta}{w_s}}$	P	$\frac{1}{13.36}$	$\frac{1}{6.056}$
V........	$\frac{L}{T}$†	$\sqrt{\frac{\theta}{w_s}}$	$\sqrt{\frac{\theta}{w_s}}$	1.07	2.94
Gauge pressure in model (14.7 lb. less than absolute pressure)				182.3.........	74.3

---

* Since *PR* is a constant in these similar systems, the bursting stress on the sides of pipes (or testing tunnels) of homologous thickness is constant; thus, the pipe may be made safer as the size of the model (including the pipe) is decreased. This favors small models.

† In terms of *L, F,* and *T;* may be used to reduce to results of the following columns.

It can easily be shown that Equations (11), (57), and (58) and their various forms as indicated in Table 2, are identical with Equations (40), (41), (42), and (43) in the problem of the elastic pendulums oscillating in gases, or fluids, of an incompressible nature. For example, observing that $k = PL$, in Equations (40), (41), (42), and (43), and that by Table 2, $\rho = \frac{P\,w_s}{\theta}$ and $G = \left(\frac{P}{\mu}\sqrt{\frac{\theta}{w_s}}\right)$, one may substitute in, say, Equation (41), for k, ρ, and G, so as to reduce that equation to an identity, that is, to $L = L$. A similar process reduces all the other equations, in this group, to identities, thereby showing the equivalence of the algebraic forms in Equations (41) to (43), and (11), (57) and (58); or in Equations (40) to (43) and Table 2.

In the case of Equations (11), (57), and (58), or Table 2, the fluid is assumed to be elastic and the solids, incompressible. In the case of Equations (40) to (43), the solids are elastic and the fluids incompressible. Of course, any, or all, the materials may have the property of elasticity or any other properties, if, as explained in Example 2, these properties are suitably "adjusted."

DISCUSSION

H. Bateman,[28] Esq. (by letter).—A numerical coefficient in a mathematical expression for a property of a complete physical system, S, can depend only on numbers which measure the system relative to itself. These numbers will be called the proportions of the system. In some cases they may be regarded as obtained by means of some measuring apparatus included in the system, but in most cases they are supposed to be known from some former measurements to which the system was subjected and from recognized laws relating to their constancy or mode of variation since these measurements were made.

In an ultimate physical analysis a proportion will be either the ratio of the measures of two entities of the same physical type (for example, two lengths or two velocities), or a definite function of such ratios (a product, for instance). A compound expression such as a Reynolds' number is in this sense a proportion of a system. By using the kinetic theory of gases, Dr. Th. von Kármán has shown that the Reynolds' number can be regarded as the ratio of two lengths multiplied by the ratio of two velocities. One of the lengths may be taken to be the mean free path of a molecule and one of the velocities the velocity of sound in the fluid.[29]

When a theoretical expression for a numerical coefficient is lacking one cannot strictly speak of a theory of similarity except in particularly simple cases. Generally, one must be content to discuss the validity of the working hypothesis that the coefficient depends essentially upon a certain finite set of proportions which may be called the dominant proportions of the system, S.

A theory of model testing tacitly assumes the existence of motions and configurations which are strictly similar in the geometrical and dynamical senses although sometimes the situation is simplified by the assumption of steady motion and then it may happen that a proportion involving gravity or an acceleration ceases to be dominant. Much depends, however, upon circumstances.

When a proportion is dominant it is customary to speak of a corresponding effect, such as scale effect, compressibility effect, etc. In this sense one can speak of a gravity or acceleration effect.

The writer agrees with Mr. Groat in his main contentions, but is inclined to say that, although Galileo[30] and Newton recognized the existence of the gravity effect, the credit belongs to W. Froude for the discovery that the associated proportion is often dominant when the physical system involves a solid body and two fluids which are separated by gravity. This discovery was essentially of an experimental nature because generally certain inequalities must be satisfied by the proportions, in order that one may be dominant, or that two may be dominant relative to the others. A valuable discussion of the theoretical basis of model strength tests for aeroplane structures has been given by Messrs. W. L. Cowley and H. Levy.[31] In laying down conditions for the similarity of structures these writers recognize that in many cases there is some latitude in the scope of selection of material for the model.

---

[28] Prof., Guggenheim Graduate School of Aeronautics, California Inst. of Technology, Pasadena, Calif.

[29] Abhandlungen aus dem Aerodynamischen Institut an der Technischen Hochschule, Aachen, Heft 4, p. 25 (1925).

[30] "Growth and Form," by D'Arcy Wentworth Thompson, Cambridge Univ. Press, 1917.

Physical considerations relating to the structure of matter and the texture of surfaces make it seem improbable that strict similarity can ever be attained in model testing.

When a model has a simple form the number of geometrical proportions needed for its description is small, but in order to specify accurately the shape of an airfoil a large number of proportions are needed. If these proportions are chosen at random without any guiding theory or experience, it is impossible to say which of them will be dominant. A model made with the aid of a template may seem to be of the right shape with a fair degree of accuracy, yet the errors in some proportions, needed for the accurate description of the shape, may be large while those in others may be small. If it happens that the great accuracy is attained in the case of unimportant proportions and the low accuracy in the case of proportions which are really dominant, tests with the model will give misleading results.

In hydraulics, model testing is often uncertain on account of the irregular form of the bed of a river and the substance of which it is composed.[32] Some interesting tests of the validity of the theory of similitude in hydraulics have been made by C. Camichel and his co-workers.[33]

W. P. ROOP,[34] M. AM. SOC. C. E. (by letter).—Mr. Groat's effort to make the experimental method based on the theory of similitude more accessible to members of the Society must command the liveliest sympathy from a member of the staff at the United States Experimental Model Basin where the theory of similitude constitutes the "breath of life."

There are, however, certain statements in his treatment of the subject which receive a degree of emphasis not altogether appropriate to a dispassionate consideration of the subject. For one, the writer must demur to the author's remark that nothing ever has been added to any of Newton's theories.

In Proposition III Newton[35] establishes the idea of terminal speed of a body falling through a resisting medium, but the writer does not find that he anywhere traces this effect to frictional, as opposed to dynamic, causes. It is not surprising, therefore, that he should believe that the "resistance of globes in infinite mediums is in * * * the duplicate ratio of the velocity * * * "; or that, "a cylinder, a sphere, and a spheroid of equal breadths equally hinder the passage of the water * * *", and that "the forementioned bodies are equally acted on by the water * * *". On the contrary he seems to suggest in the second paragraph of Proposition XL that it is the inertia of the fluid rather than its friction that is primarily concerned.

Detailed discussion of Newton's ideas is perhaps not in order here, but one or two further allusions are necessary. Proposition XL has as its purpose "to find by phenomena the resistance of a globe moving through a perfectly fluid compressed medium". Whether by this he meant an inviscid fluid is not clear. In the discussion of Experiment 12 he uses these words:

---

[31] *Engineering*, Vol. 105, p. 219 (1918).

[32] "Hydraulic Laboratory Practice," by John R. Freeman, Past-President and Hon. M. Am. Soc. C. E., Am. Soc. Mech. Engrs., 1929.

[33] *Engineering*, Vol. 123, p. 27 (1927); see, also, a paper by C. Camichel, L. Escande, and M. Ricaud, *Revue Générale d'Electricité*, t. 20, p. 54 (1926).

[34] Lieut. (C. C.), U. S. N., New York Shipbuilding Co., Camden, N. J.

[35] "Principia," Book II.

> "The more swiftly the globes move, the less are they pressed by the fluid at their hinder parts; and if the velocity be perpetually increased, they will at last leave an empty space behind them, unless the compression of the fluid be increased at the same time."

One might suppose from this that Newton was aware of all the intricate processes of eddy formation and cavitation that cause such inconvenient complexity in fluid dynamics.

However, this would be reading into his words, meaning which is certainly not contained in them in an unambiguous way. He did not know that a sphere moving at uniform speed would meet no resistance at all in an infinite frictionless fluid; that its actual resistance, including that due to eddies, owes its origin to the fact that fluids are not frictionless.

Apparently, Mr. Groat still is under the impression that frictional drag on a solid surface due to tangential motion of a fluid across it is ordinarily proportional to the square of the speed. At the Experimental Model Basin such a statement is considered completely erroneous, and if such a law of friction were used, it would lead to gross errors in power estimates.

Possibly some details as to this point may be of interest because they will illustrate the fact that the successful application of the model method depends on development of a suitable experimental technique as surely as upon sound theoretical analysis.

Even Froude,[36] sixty years ago, was aware that the friction on a plank towed edgewise was not proportional to the square of the speed of the plank through the water. He thought that it varied according to a speed exponent which departed from 2.00 by a fixed amount, and by the use of this assumption obtained for the first time, in a field where many others had failed, a practicable model method for estimating power requirements for ships.

More recently Gebers,[37] at Vienna, Austria, made careful friction tests and arrived at a result differing somewhat from that of Froude. A large number of friction tests have also been made at the U. S. Experimental Model Basin in the thirty years of its existence.

Data from these three sources have been summarized in a paper recently published,[38] which shows that their precision is not sufficient to permit investigators to distinguish between two types of friction formula, in one of which the exponent may be $\frac{5}{3}$, or even less.

Nevertheless, in spite of arbitrary model scales, which Mr. Groat states are permissible only if the exponent is 2.00, and in spite of variability of actual frictional resistances—in short, in spite of the impossibility of application of the model method for absolute power calculations—it is true that the model method excels all others so long as its function is restricted essentially to that of providing comparisons between existing and proposed full scale projects, not differing too much in type.

---

[36] Rept., British Assoc. for the Advancement of Science, 1872 and 1874.

[37] *Schiffbau,* Vol. XXII; translation by National Advisory Committee for Aeronautics, *Technical Memo. 308.*

[38] "Frictional Resistance of Ship Models," by W. P. Roop, M. Am. Soc. C. E., *Transactions,* Soc. of Nav. Archts. and Marine Engrs., Vol. 37 (1929), p. 45.

Mr. Groat's insistence that dynamic similitude is the one and only kind seems to be somewhat academic. In his "Conclusions" he states: "When that [Newton's] theorem is fully satisfied, Reynolds' number will be satisfied". As Newton says nothing of viscosity, the writer assumes that Mr. Groat must have it in mind that viscous action is itself ultimately a dynamic phenomenon, in which, however, the bodies are of molecular dimensions; but the scale on which molecules are built cannot be controlled. In order that two similar dynamic systems should have at the same time equal speed-length ratios and equal values of Reynolds' number it would be necessary to adjust the kinematic viscosity of the fluid separately for each speed. This is obviously impracticable, and as a matter of fact it is unnecessary. In practical work on ship resistance no attempt is made to satisfy rigorously the conditions of full similitude. It is true that a large part of the uncertainties would be eliminated if rigorous similitude could be achieved; but this cannot be done and engineers are dependent on a friction formula showing variation of friction when equality of Reynolds' numbers is not obtainable.

The really important point is that success in ship-model work is achieved, not by insistence on theoretical rigor, but by acceptance of a procedure that is only partly based on similitude, justified by the fact that it produces the desired results. In all discussions of similitude the fact should not be lost from sight that the power required to drive a ship actually can be predicted with rather high precision by means of model tests. The methods used in accomplishing this are the products of long evolution. It may not be generally known that model tests on ship resistance were made by Benjamin Franklin as early as 1768.

From the point of view of ship-power estimates the writer would say that the conditions for successful model work include long continuance of model tests under uniform circumstances as to technique, with frequent full-scale comparisons.

A person who expects to be able to draw valid conclusions from one or two or a dozen model tests in a new field in which the possibilities of unforeseen scale effects have not been explored fully had better prepare his mind for disappointment. It is unfortunate that such disappointments in the past have led some engineers to overlook the fact that, under the right conditions, the model method may be made to yield results otherwise unobtainable. Extension of its use on a long-term basis might well lead, as Mr. Groat suggests in his concluding remarks, to the creation of "a new department of engineering design". He is quite right also in referring to the necessity for development of a "corps of specialists". Some progress in this direction is being made at the U. S. Experimental Model Basin and as appreciation of the importance of the subject becomes more general, progress may be expected to become more rapid.

A. V. KARPOV,[89] M. AM. SOC. C. E. (by letter).—The use of models in connection with engineering studies is not a recent development; it has become extensive only in the past few years. The advent of aeronautics and the successful use of models in the investigations of the complicated problems of this new branch of engineering have brought out the importance of model studies, not only in aeronautics but in many other fields of engineering endeavor, and started the general trend of the extensive use of models in involved engineering studies and investigations which is evident to-day.

The excellent presentation of the underlying theoretical principles of model studies by the author will probably do much toward the removal of the misconceptions that are so common in the model work of the past. Mr. Groat discusses the models involving dynamic problems, which are naturally the more complicated.

In connection with the arch dam investigation that is being conducted by the Company with which the writer is connected, it was necessary to determine the conditions that will govern the building of a model of an arch dam, in order to study the structural and design problems arising in this investigation. This work stresses not only the theoretical aspects of the problem, but the practical ones as well, and is included in this discussion in its entirety.

Similarity Conditions Between an Arch Dam Model and Its Prototype.—Past experience in solving complicated engineering problems by model studies not only justifies the use of models, but shows that in many cases the only way in which reliable results can be obtained is by making proper model studies.

The first step in making such study is to determine what, in the engineering sense, is a model. A very common conception, even in engineering circles, is that any representation of the prototype in an arbitrary scale is already a model; but from the engineering point of view, a scalar representation of the prototype can be called a model only when, from a study of its behavior, conclusions can be derived that will disclose the behavior of the prototype. This means that a certain relationship is to exist between the behavior of the model and prototype, which necessarily must be of a rather simple nature and which is brought about by the fulfillment of the similarity conditions.

These similarity conditions are governed by the purpose for which the model study is to be made and may cover a wide range of different requirements. In a number of simpler problems the only similarity condition to be met is the proper scalar representation of the prototype; in more complicated problems additional requirements will be included and there are still other problems in which the similarity conditions will require that the model is not a true scalar representation of the prototype. In many cases the relationship between the properties of the material or materials that are used in the prototype and in the model is covered by similarity requirements that may be of no less importance than requirements covering the geometrical relationship between the shapes of the prototype and the model.

The scope of the similarity conditions depends not only on the problem itself, but often, to a larger extent, on the kind of information that is to be obtained from the model study. The similarity conditions may be simple if the study is restricted to the influence of some main factors. With the increase of the number of factors that are to be included in a study the similarity conditions increase very rapidly in number and in scope.

---

[89] Designing Engr., Hydr. Dept., Aluminum Co. of America, Pittsburgh, Pa.

In studies of the more complicated structural problems practically every model represents a certain compromise between the rigid theoretical requirements and some practical considerations.

The cost of the model study measured in terms of time and money is the most important of all the practical considerations; if it could be neglected, the best possible model on which to make the studies would be the prototype itself.

The following analysis of the similarity conditions of a model of a simply supported concrete beam will serve as an illustration:

A.—If the purpose of the study is to determine the deflections and stresses of a prototype beam under uniform or concentrated loads of such character and magnitude that the deflections due to shear and the dead weight of the beam can be neglected, then the similarity conditions can be formulated very simply:

(*a*) The model is to be a true scalar representation of the prototype.

(*b*) The material of the model is to be such that, in the range of stresses of the model, the variations of the modulus of elasticity are similar to the variations of the modulus of elasticity of the material of the prototype in its corresponding range of stresses.

(*c*) The loadings of the model are to be proportional to the loadings of the prototype, but no limitations apply to the value of the model loadings.

B.—If the same study is to be made under such conditions that the dead weight of the beam cannot be neglected, then the similarity requirements introduce the following additional conditions:

(*d*) The specific gravity of the material of the prototype and of the beam should be equal.

(*e*) The loadings of the model are to be not only proportional to the loadings of the prototype, but the ratio of the loadings of the prototype and model is to be equal to the square of the linear scale of the model.

C.—If the study is to be extended so that it includes the deflections due to shear, then the additional similarity condition will be:

(*f*) The ratio of the modulus of elasticity in shear and of the modulus of elasticity in flexure of the model material within the range of the stresses of the model is to be the same as that of the material of the prototype within its range of stresses. In other words, the Poisson's ratios of the model and prototype materials are to be equal at corresponding stresses.

D.—If the same study is to be made on a reinforced concrete beam, then the representation of the reinforcement in the model can be so inconvenient that the practical compromise would be to make a model that is a scalar representation of the prototype in its length only. Such a model would have cross-sections which would not be scalar representations of the prototype. It would represent the reinforcement by a proper increase of the size of the part of the beam section that includes the steel.

E.—The practical compromise that probably will be necessary to make Model Studies A to D possible would be to simplify the similarity conditions by assuming that the modulus of elasticity and Poisson's ratio of the material of the prototype are constant, and to choose a material for the model that is fairly uniform, the modulus of elasticity and Poisson's ratio of which can be assumed as constant.

F.—No restrictions apply to the linear scale of the model and to the value of the modulus of elasticity of model material, but for models that satisfy the similarity conditions in Model Study B, the following conclusions can be derived:

> (*g*) The deflections of the model are proportional to the square of the linear scale multiplied by the ratio of the moduli of elasticity of the prototype and model materials.
>
> (*h*) The stresses of the model are proportional to its linear scale.

G.—If it is desirable that the geometrical resemblance of the model and prototype is strictly retained after the application of the load, an additional condition will be imposed:

> (*i*) The ratio of the moduli of elasticity of the materials of the prototype and model is to be equal to the linear scale of the model.

H.—If the model is to be tested to destruction then, again, an additional condition will be:

> (*j*) The ratio of the moduli of rupture of the materials of the prototype and model is to be equal to the linear scale of the model.

An arch dam represents a more complicated problem than a simply supported beam and, keeping in mind the purpose for which the model study is to be made, the problem can be stated in a general form, as follows:

Determine the similarity conditions of an arch dam model in which the deflections and stresses are to have some simple relationship with the deflections and stresses of the prototype under conditions that are well below the destruction point of the prototype.

A certain amount of judgment must be exercised to classify and give the proper weight to the different similarity requirements and the most practical way would be to formulate the theoretical similarity conditions and, later, to bring them into agreement with the practical requirements. No single method is known that would determine all the similarity conditions and the entire problem is to be considered from different points of view. The first and most formal will be the conclusions that can be derived from the application of the dimensional theory, but this theory alone by no means covers all similarity conditions; some additional conditions that are not based on this theory, must be derived.

Similarity Conditions Derived by Application of Dimensional Theory.—The dimensional theory can be applied to the studies of an arch dam model, as follows: The deflection can be expressed as,

$$\delta_p = \psi' \left(\rho_p, \gamma_p, E_p, S_p, g, l_p\right) \dotfill (63)$$

and the stresses,

$$f_p = \psi \left(\rho_p, \gamma_p, E_p, S_p, g, l_p\right) \dotfill (64)$$

in which,

ψ' and ψ = some unknown functions;
ρ_p = mass per unit of volume of the material of the dam;
γ_p = mass per unit of volume of the loading liquid of the dam (water);
E_p = modulus of elasticity in flexure of the material of the dam;
g = acceleration due to gravity;
l_p = length scale; and
S_p = modulus of elasticity in shear of the material of the dam.

If μ_p is Poisson's ratio of the material of the dam, then:

$$S_p = \frac{E_p}{2\,(1 + \mu_p)} \tag{65}$$

and since μ_p is not a dimensional quantity, the dimensional Equations (63) and (64) can be written:

$$\delta_p = \psi'\,(\rho_p, \gamma_p, E_p, g, l_p) \tag{66}$$

and,

$$f_p = \psi\,(\rho_p, \gamma_p, E_p, g, l_p) \tag{67}$$

Expressing all the quantities of Equations (66) and (67) in terms of mass, *M*, length, *L*, and time, *T*, then:

$$\rho_p = \gamma_p = M\,L^{-3} \tag{68}$$

$$E_p = M\,L^{-1}\,T^{-2} \tag{69}$$

$$g = L\,T^{-2} \tag{70}$$

$$l_p = L \tag{71}$$

The deflections have the dimensions, *L*, and from Equation (66) the dimensional equation will be,

$$\delta_p = \rho_p^{\,x}\,\gamma_p^{\,y}\,E_p^{\,z}\,g^u\,l_p^{\,v} \tag{72}$$

making the necessary substitutions:

$$L = L^{(-3x - 3y - z + u + v)}\;M^{(x + y + z)}\;T^{(-2z - 2u)} \tag{73}$$

The following equations can be written by equalizing the powers of *L*, *M*, and *T*:

$$1 = -3x - 3y - z + u + v \tag{74}$$

$$0 = x + y + z \tag{75}$$

and,

$$0 = -2z - 2u \tag{76}$$

Solving these equations:

$$z = -u \tag{77}$$

$$v = 1 + u \tag{78}$$

and,

$$x + y = u \tag{79}$$

Substituting in Equation (72),

$$\delta_p = l_p \left(\frac{\gamma_p\, g\, l_p}{E_p}\right)^u \left(\frac{\rho_p}{\gamma_p}\right)^x \ldots\ldots\ldots\ldots\ldots\ldots(80)$$

Assuming that the material follows Hooke's law and that no buckling or other disturbances which would distort the deflections occur, then δ_p will be proportional to $\frac{1}{E}$ and, consequently, from Equation (80), $u = 1$, and,

$$\delta_p = \frac{\gamma_p\, g}{E_p}\, l_p^{\,2}\, \phi'_p \left(\frac{\rho_p}{\gamma_p}\right) \ldots\ldots\ldots\ldots\ldots\ldots(81)$$

in which, ϕ'_p is an unknown function.

Using the subscript, m, the deflections of the model can be written similarly to Equation (81),

$$\delta_m = \frac{\gamma_m\, g}{E_m}\, l^2_m\, \phi'_m \left(\frac{\rho_m}{\gamma_m}\right) \ldots\ldots\ldots\ldots\ldots\ldots(82)$$

From Equations (81) and (82),

$$\frac{\delta_m}{\delta_p} = \left(\frac{l_m}{l_p}\right)^2 \frac{\gamma_m}{\gamma_p}\, \frac{E_p}{E_m}\, \frac{\phi'_m \left(\frac{\rho_m}{\gamma_m}\right)}{\phi'_p \left(\frac{\rho_p}{\gamma_p}\right)} \ldots\ldots\ldots\ldots\ldots\ldots(83)$$

By making the ratio, $\frac{\rho_m}{\gamma_m}$, in the model the same as the corresponding ratio in the dam (that is, the ratio of the specific gravities of the material of the structure to the loading fluid), it will become a model constant, and,

$$\frac{\delta_m}{\delta_p} = \left(\frac{l_m}{l_p}\right)^2 \frac{\gamma_m}{\gamma_p}\, \frac{E_p}{E_m} \ldots\ldots\ldots\ldots\ldots\ldots(84)$$

because,

$$\frac{\rho_m}{\gamma_m} = \frac{\rho_p}{\gamma_p} \ldots\ldots\ldots\ldots\ldots\ldots(85)$$

In the same way from Equation (67),

$$f_p = \rho_p^{\,x}\, \gamma_p^{\,y}\, E_p^{\,z}\, g^u\, l_p^{\,v} \ldots\ldots\ldots\ldots\ldots\ldots(86)$$

and since the dimension of f_p is $M\,L^{-1}\,T^{-2}$,

$$f_p = E_p \left(\frac{\gamma_p\, g\, l_p}{E_p}\right)^u \phi_{fp} \left(\frac{\rho_p}{\gamma_p}\right) \ldots\ldots\ldots\ldots\ldots\ldots(87)$$

in which, ϕ_{fp} is an unknown function.

The stresses do not depend on E_p. Consequently, $u = 1$, and,

$$f_p = \gamma_p\, g\, l_p\, \phi_{fp} \left(\frac{\rho_p}{\gamma_p}\right) \ldots\ldots\ldots\ldots\ldots\ldots(88)$$

For the model, similarly,

$$f_m = \gamma_m\, g\, l_m\, \phi_{fm} \left(\frac{\rho_m}{\gamma_m}\right) \ldots\ldots\ldots\ldots\ldots\ldots(89)$$

and,

$$\frac{f_m}{f_p} = \frac{\gamma_m}{\gamma_p}\frac{l_m}{l_p}\frac{\phi_{fm}\left(\frac{\rho_m}{\gamma_m}\right)}{\phi_{fp}\left(\frac{\rho_p}{\gamma_p}\right)} \text{.......................} (90)$$

Considering, again, $\frac{\phi_{fm}\left(\frac{\rho_m}{\gamma_m}\right)}{\phi_{fp}\left(\frac{\rho_p}{\gamma_p}\right)}$ as a model constant,

$$\frac{f_m}{f_p} = \frac{l_m}{l_p}\frac{\gamma_m}{\gamma_p} \text{..........................} (91)$$

Equations (84) and (91) give the similarity conditions of the model that can be derived from the dimensional theory.

Introducing the linear scale,

$$R = \frac{l_m}{l_p} \text{..............................} (92)$$

These equations can be written:

$$\delta_m = R^2 \frac{\gamma_m}{\gamma_p}\frac{E_p}{E_m}\delta_p \text{.....................} (93)$$

and,

$$f_m = R\frac{\gamma_m}{\gamma_p}f_p \text{...........................} (94)$$

In order to arrive at these equations the two following assumptions were made, without which the similarity conditions would be so complicated that the model would be impractical:

(1) The materials of the prototype and model shall follow Hooke's law, which means that the modulus of elasticity, E, is constant in the prototype and in the model.

(2) The ratio of the specific gravities of the material of the prototype and its loading fluid, $\frac{\rho_p}{\gamma_p}$, is equal to the same ratio of the model, $\frac{\rho_m}{\gamma_m}$.

Similarity Conditions Covering Non-Dimensional Factors.—In applying the dimensional theory the factors that have no dimension cannot be covered, and the similarity conditions for such factors have to be investigated separately.

Poisson's ratio is a non-dimensional factor and is to be investigated by a different method. This ratio influences the conditions of the dam in two ways: First, directly, by the introduction of stresses at right angles to the stress developed by the applied force; and, second, due to the dependence of the modulus of elasticity in shear on Poisson's ratio and the consequent dependence of the shear deflections on this ratio. These factors will be considered separately.

The direct influence of Poisson's ratio can be handled in the following way. Due to Poisson's ratio, μ_p, of the dam material, a stress, f_p, will produce in a restrained unit cube of the dam a stress, f'_p, at right angles to it,

$$f'_p = \mu_p f_p \qquad (95)$$

and similarly for the model,

$$f'_m = \mu_m f_m \qquad (96)$$

To retain the similarity conditions between the dam and model it is necessary that,

$$\frac{f'_p}{f'_m} = \frac{f_p}{f_m} \qquad (97)$$

and, consequently,

$$\mu_p = \mu_m \qquad (98)$$

The indirect influence of Poisson's ratio can be treated in a similar manner:

The deflection of any part of the prototype due to shear will be,

$$\delta_{sp} = \frac{1}{S_p} \int s_p \, dl_p \qquad (99)$$

in which, s_p is unit shear; and dl_p, differential distance.

Similarly, for the model,

$$\delta_{sm} = \frac{1}{S_m} \int s_m \, dl_m \qquad (100)$$

From Equations (92) and (94),

$$dl_m = R \, dl_p$$

$$s_m = R \frac{\gamma_m}{\gamma_p} s_p$$

Substituting these values in Equation (100),

$$\delta_{sm} = \frac{1}{S_m} (R)^2 \frac{\gamma_m}{\gamma_p} \int s_p \, dl_p$$

and,

$$\frac{\delta_{sm}}{\delta_{sp}} = \frac{S_p}{S_m} (R)^2 \frac{\gamma_m}{\gamma_p}$$

To retain the similarity between the model and prototype, $\frac{\delta_{sm}}{\delta_{sp}} = \frac{\delta_m}{\delta_p}$, and, from Equations (93) and (65),

$$\frac{E_p}{E_m} = \frac{S_p}{S_m} = \frac{2\,(1 + \mu_m)}{2\,(1 + \mu_p)} \frac{E_p}{E_m}$$

and, $\mu_p = \mu_m$. This will add the condition:

(3) Poisson's ratios of the prototype and model materials are to be equal at corresponding stresses.

Similarity Conditions Derived from Geometrical Similarity.—In order to have the prototype and model similar in their geometrical shape before and after the loadings are applied, the linear deflections of the model are to be in the same linear scale as the model itself. Therefore, $\delta_m = R\,\delta_p$. Introducing this relation in Equation (93),

$$R\,\frac{\gamma_m}{\gamma_p}\,\frac{E_p}{E_m} = 1 \text{..........................} (101)$$

or,

$$\frac{E_m}{E_p} = R\,\frac{\gamma_m}{\gamma_p} \text{..........................} (102)$$

which introduces the additional condition:

> **(4) The ratio of the moduli of elasticity of the model and prototype is to be equal to the scale of the model multiplied by the ratio of the specific gravities of the loading liquids of the model and prototype.**

Similarity Conditions Covering Discontinuities.—An actual arch dam cannot be built as a continuous structure; depending on the methods by which it is built, it has a number of construction joints which may become planes of discontinuity.

The construction joints of the dam are to be represented in the model so as to maintain the similarity between the model and dam.

The dam has horizontal and vertical construction joints. The horizontal construction joints are kept uneven and rough across the block in order to insure a better bond between the successive layers of concrete and as long as the dam is not loaded no separation or opening of these joints can take place. The vertical construction joints divide the dam into separate blocks and although bonding boxes are provided, these joints are much more definite than the horizontal joints, and when the dam is not loaded they may open due to the contraction of the concrete. The horizontal construction joints can carry a limited tension that is much less than the tension that can be carried in a concrete block between the joints, but the vertical joints cannot carry any tension. In conformity with these conditions the horizontal and vertical joints of the model are to be treated differently.

The horizontal joints are construction joints only, and it can be assumed that after the dam is constructed no measurable horizontal joints are left. Even if the dam is loaded far below its destruction point, the tension stresses in some of the joints can be higher than the ultimate tension that can be carried over the joint. In this case, a rupture of such joints will take place starting at the surface and extending to some point at which the tension stress is low enough to be carried by the joint.

In indicating the ultimate unit tensile stresses that can be carried over the joints of the prototype and model by f_{jp} and f_{jm}, respectively, the requirement would be that,

$$\frac{f_{jm}}{f_{jp}} = \frac{f_m}{f_p} \text{..........................} (103)$$

and from Equation (94),

$$\frac{f_{jm}}{f_{jp}} = R\,\frac{\gamma_m}{\gamma_p} \text{..........................} (104)$$

which adds the additional condition:

> (5) The ratio between the ultimate unit tensile stresses that can be carried by the horizontal joints of the model and prototype is to be equal to the linear scale of the model multiplied by the ratio of the specific gravities of the loading liquids of the model and prototype.

The vertical joints of a dam can have a physical dimension and, in such case, have to be represented in the model as joints of a certain size.

Considering the vertical joints of an actual dam, the action of these joints can be represented by two steps: First, Poisson's ratio of the dam material causes a part, b'_p, of the width of joint, b_p, to be closed; and, second, the remaining part of the joint, b''_p, will increase the deflection of the arch.

Considering a block of the length, s_p,

$$b'_p = \mu_p \frac{1}{E_p} f_p \, s_p \qquad (105)$$

and, similarly, for the model,

$$b'_m = \mu_m \frac{1}{E_m} f_m \, s_m \qquad (106)$$

and,

$$b'_m = \frac{\mu_m}{\mu_p} \frac{E_p}{E_m} \frac{f_m}{f_p} R \, b'_p \qquad (107)$$

Substituting Equation (94),

$$b'_m = R^2 \frac{E_p}{E_m} \frac{\gamma_m}{\gamma_p} \frac{\mu_m}{\mu_p} b'_p \qquad (108)$$

If the action of the remaining part of the joint is to be represented in the model so as to result in deflections of the model that are of the same scale as the deflections due to elastic deformations, then, from Equation (93),

$$b''_m = R^2 \frac{E_p}{E_m} \frac{\gamma_m}{\gamma_p} b''_p \qquad (109)$$

and, since $b'_m + b''_m = b_m$,

$$b_m = R^2 \frac{E_p}{E_m} \frac{\gamma_m}{\gamma_p} \left[b''_p + \frac{\mu_m}{\mu_p} b'_p \right] \qquad (110)$$

Remembering Equation (98),

$$\frac{b_m}{b_p} = R^2 \frac{E_p}{E_m} \frac{\gamma_m}{\gamma_p} \qquad (111)$$

which adds the additional condition:

> (6) The ratio of the width of the vertical construction joints in the model and prototype is to be equal to the square of the scale of the model multiplied by the ratio of the specific gravities of the loading liquids and by the inverse ratio of the moduli of elasticity of the model and prototype.

Similarity Conditions Covering Foundations and Abutments.—The elastic deformations of an arch dam are by no means limited to the dam itself, but are carried through the foundations and abutments. The previously developed formulas and statements concerning the dimensional theory, non-dimensional factors, and geometrical shape can also be applied to the foundation and abutments. It is obvious, however, that the size of the elastic foundations that are practically infinitely large as compared with the size of the dam cannot be represented in a practical model, and some compromise must be made in the very beginning if the model is to be feasible.

The theoretical investigation of foundation deflections made by Fredrik Vogt, Assoc. M. Am. Soc. C. E., entitled "Ueber die Berechnung der Fundamentdeformation", disclosed the fact that the action of the infinitely large elastic rock foundation can be closely represented by an imaginary block of definite dimensions, made of elastic material and rested on a fixed non-elastic and non-yielding base.

The dimensions of such a foundation block and its elastic properties depend on the elastic properties of the foundation rock. The investigations available in this respect are very meager as far as the elastic properties of the ground rock and the relationship between these properties and the sizes and elastic properties of the imaginary foundation block are concerned. It can be stated in general terms that the following condition should be added:

> (7) The action of the infinitely large foundation rock may be represented by a block of elastic material of some definite dimensions, which has a fairly low modulus of elasticity so that by supporting it on a base of a material of a high modulus of elasticity, the conditions of an elastic foundation block supported on an unyielding non-elastic base can be represented.

Similarity Conditions Covering Influence of Temperature Changes.—In the study of the stresses and deflections of the model due to different temperatures the following simple relationships are obvious:

$$c_p\, T_p = c_m\, T_m \dots\dots\dots\dots\dots\dots\dots (112)$$

and,

$$T_m = \frac{c_p}{c_m}\, T_p \dots\dots\dots\dots\dots\dots\dots (113)$$

in which, c_p and c_m are the coefficients of thermal expansion of the prototype and model; T_p is the difference in temperature of the material of the dam between the time the dam was built and the time under consideration; and T_m, the corresponding temperature difference of the model.

The only theoretical complication that would enter is, that consideration should be given not only to the coefficients of thermal expansion of the materials of the prototype and models, but also to the materials of the actual foundation and the imaginary foundation block; but that is a practical impossibility, and being of no importance, is to be neglected. The additional condition to be included can be formulated, as follows:

> (8) The ratio of the temperature differences of the model material, between the time of testing and building of the model, and of the temperature difference of the material of the prototype, between the time for which the investigations are made and the time of the building of the prototype, should be equal to the ratio of the coefficients of thermal

expansion of the materials of the model and the prototype, provided that these coefficients are sufficiently uniform to be assumed as constant.

Type of Model.—These similarity conditions are general, and any model of an arch dam that is to be used to study the stresses and deflections should satisfy them.

An attempt to build a model covered by this set of similarity conditions will meet with many obstacles, and in order to build one with a reasonable expenditure of time and money a number of additional compromises are necessary. These must be determined, analyzing separately each of the similarity conditions as applied to a particular problem to be investigated.

The dam described in this discussion is the Calderwood Arch Dam on the Little Tennessee River, which is about 200 ft. high and about 800 ft. long at the crest. Two different kinds of models were considered: First, one in which, at a number of points, the deflections of the down-stream surface and the strains at the down-stream and up-stream surfaces of the model could be measured directly; and, second, a model in which some information could be obtained about the distribution of stresses through the thickness of the model.

The first type would represent a model along the more conventional lines; the second could be obtained by the use of a modification of the photo-elastic method of stress determination. The second model would look very attractive if practical results could be obtained, having the following advantages:

(*a*) Determinations of stress through the thickness of the model as against the determination at the up-stream and down-stream surfaces of the model only.

(*b*) The possibility of the use of a smaller model and, consequently, a possible saving in cost.

The disadvantages of such a model can be summarized as follows:

(*c*) This method has not been used on three-dimensional models. The possibilities are that it would require considerable time to make the improvements or modifications that would be necessary in order to adapt the method to the conditions of an arch dam model.

(*d*) The usual transparent materials used in this method have a high modulus of elasticity and the proper reproduction in small scale of the details of the prototype would be difficult.

(*e*) Considerable time would be necessary to determine by test and trial the material that would be suitable for such a model.

Considering the uncertain features of the photo-elastic method, it was decided to build a model of the Calderwood Dam according to the first method, and the following discussion is based on this decision.

Loading Liquid.—One of the most important points, which governs all other conditions, to a considerable degree, is the choice of the loading liquid.

Condition (2) requires that the ratio of the specific gravities of the loading fluids of the model and prototype shall be equal to that of the materials of the model and prototype. This requirement is fundamental, because if it is neglected, the relationship between the arch and gravity action of the model will be distorted and the study of the behavior of the dam will be made difficult and uncertain due to the necessity of correcting the results to take care of a number of secondary influences that will be introduced. From the practical point of view, the choice of the loading liquid of the model is rather limited. Either the same liquid must be used as in the prototype (which is water, the specific gravity of which is equal to 1), or the next reasonable possibility would be the use of mercury which has the specific gravity of 13.6.

Using water, the specific gravity of the material of the model should be about 2.4, the same as for concrete, the material of which the dam is constructed. Using mercury, the specific gravity of the material of the model will be $2.4 \times 13.6 = 32.84$. That would practically exclude the use of mercury, because no known material has such a high specific gravity, and the only way to approximate these conditions would be by the use of some artificial method of loading the model with additional loads or springs.

The choosing of water as the loading liquid will determine the specific gravity of the material of the model equal to about 2.4 and will simplify the similarity conditions considerably since $\frac{\gamma_m}{\gamma_p} = 1$, and Equations (93), (94), (102), (104), and (111) can be written:

$$\delta_m = R^2 \frac{E_p}{E_m} \delta_p \ldots\ldots\ldots\ldots (114)$$

$$f_m = R f_p \ldots\ldots\ldots\ldots (115)$$

$$E_m = R E_p \ldots\ldots\ldots\ldots (116)$$

$$f_{jm} = R f_{jp} \ldots\ldots\ldots\ldots (117)$$

and,

$$b_m = R^2 \frac{E_p}{E_m} b_p \ldots\ldots\ldots\ldots (118)$$

Scale of Model.—The choice of the scale of the model is entirely arbitrary from the theoretical point of view; but of course it is very much limited by practical considerations. The amount of material in the model, and probably the cost of it, is proportional to the cube of the scale and the area that is covered by the model. Probably also the time necessary to make the tests is proportional to the square of the scale. That would mean that the scale of the model must be chosen as small as possible in order to reduce the cost and time and increase the convenience of the use of the model.

On the other hand, the stresses are proportional to the scale and the deflections to the square of the scale so that the larger models will give better results as far as observations are concerned. Furthermore, if the model is to be a true representation of the prototype the horizontal, and particularly the vertical, construction joints must be represented and that can be done satisfactorily only if the scale of the model is large enough. For the model of the Calderwood Dam it was decided that the smallest scale that could be used was 1:50, and the following considerations are based on a model of such scale.

Modulus of Elasticity of Model Material.—Assuming the modulus of elasticity of the concrete used in the prototype as 3 800 000 lb. per sq. in., from Equation (116), $E_m = \frac{3\,800\,000}{50} = 76\,000$ lb. per sq. in.

Assuming that the maximum deflection of the prototype under consideration will be about 0.5 in., the maximum deflection of the model from Equation (114) will be $\delta_m = \frac{0.5}{50} = 0.01$ in.

The proper observation of the behavior of the model under load will include not only the measurements of the maximum deflections, but the series of measurements at a considerable number of points ranging from $\delta_m = 0$ to $\delta_m = 0.01$ in.

From a practical point of view the deflections that are to be measured at the many points must be kept reasonably large so that the measurements can be made with simple instruments and without the use of complicated and involved measuring devices.

Besides the possibility of measuring the deflections with a sufficient degree of accuracy, the following important point should be stressed: If the model is made of an elastic material and the deflections are relatively large, the secondary influences, such as uncontrollable changes of temperature, inaccuracies in the size of construction joints, displacements of the abutments, etc., have only a small influence on the total deflection, and can be partly accounted for and partly neglected. For stiffer models these secondary influences become more important, and the value of deflection measurements decreases rapidly due to secondary influences as well as to the decreasing degree of accuracy of the measurements.

The requirements of retaining the similarity of the geometrical shape of the dam and model before and after the load is applied, are theoretically correct, but, practically, a considerable increase in the deflections can take place before it could influence the results to a noticeable extent. From these considerations it was deemed advisable to use a material for the model with a much lower modulus of elasticity than that which is required theoretically.

Poisson's Ratio of Model Material.—Poisson's ratio for the concrete used in the prototype is about 0.20 and that for the material of the model should be the same.

If the material of the model has a different Poisson's ratio, then the results of the tests will be influenced in two ways: First, the direct influence that results in redistribution of stresses; and, second, the indirect influence that will manifest itself in a different value of the modulus of elasticity in shear and, consequently, disturb the behavior of the model as far as the shearing deformations and the part of the stresses due to such deformations are concerned.

The direct influence of Poisson's ratio can be taken care of in the model by a proper variation of the size of the vertical construction joints.

An arch dam as well as the model is restrained in the horizontal direction along the axes of the arches, is only partly restrained in the vertical direction due to the weight of the structure that rests on it, and has practically no restraint in the horizontal directions at the up-stream and down-stream faces of the dam. Consequently, Poisson's ratio can influence the stresses and deflections in the horizontal direction along the axes of the horizontal arches, but it is of no concern in the horizontal direction between the faces and in the vertical direction.

In the horizontal direction the influence of Poisson's ratio will be felt, due to the loads that are applied in the vertical direction and the direct water pressure at the up-stream face of the dam.

Selection of Model Material.—The known arch dam models of the past were built as follows: (*a*) Concrete loaded with mercury; (*b*) celluloid loaded with mercury; and (*c*) rubber loaded with water. None of these meets the similarity conditions with any degree of satisfaction.

The most important discrepancies are the improper ratio of the specific gravities of the loading liquid to the model material and the fact that all these models were built as continuous structures, which could carry considerable tension in all directions.

In selecting the material the first consideration is naturally given to the possibility of making the model out of concrete, the same as the prototype.

Concrete models loaded with water, however, will show very small and uncertain deflections. In order to increase the deflections in the models built in the past, mercury was substituted for water as the loading liquid. This increased the deflections manifold, but at the same time violated an important similarity condition concerning the ratio of the specific gravities of the loading liquid and the material of the model.

The celluloid models loaded with mercury and the rubber models loaded with water have the same drawback. The relationship between the arch and gravity action of the dam is greatly distorted and, consequently, from studies made on such models no proper conclusions can be derived as to the behavior and shape of the dam.

These considerations eliminated the use of concrete, celluloid, or plain rubber as a model material, and a search for a more suitable material was necessary.

For the chosen scale of the model (1:50), and by assuming the maximum range of possible stresses in the prototype to be from 300 lb. per sq. in. tension to 1 000 lb. per sq. in. compression, the corresponding range of stresses in the model material is from 6 lb. per sq. in. tension to 20 lb. per sq. in. compression. In general, this represents a very narrow range of stresses for most of the materials that could be considered.

Besides the requirements that are directly covered by the similarity conditions the material to be selected must satisfy a number of requirements that follow from the similarity conditions indirectly, and that have to be met within the range of stresses in the model.

These are the requirements that characterize the elastic and plastic properties of the model material. The condition of a constant modulus of elasticity must be extended to cover a uniform modulus of elasticity in all directions, which would preclude the use of materials with pronounced directional properties.

On the other hand, the constant modulus of elasticity will preclude materials with pronounced plastic properties that are characterized by a slow recovery or a high permanent set, and materials that have any considerable skin or internal stresses.

The necessary practical consideration will be to have the material such that it could be brought to the many required irregular shapes without prohibitive expenditure of time and money.

The specifications for the required material based on all previous considerations and in the stress range of the model are summarized in the order of their importance, as follows:

(1) Specific gravity of 2.4.
(2) Uniform elastic properties without any pronounced directional qualities.
(3) Absence of skin or internal stresses.
(4) Low modulus of elasticity.
(5) Reasonably uniform values of the modulus of elasticity at various unit stresses.
(6) Low permanent set.
(7) Low plastic properties.
(8) Properties that will permit the material to be easily furnished in the required irregular shapes.
(9) Poisson's ratio close to 0.20.
(10) Uniform coefficient of thermal expansion.

No material is commercially available or even known that would meet this list of specifications in its entirety, so that the problem resolved itself into the development of a material of the desired qualities. Although the work done to date (1930) in developing such material seems to be rather promising, it showed clearly that considerable additional time must be spent before the ideal material can be found, particularly if the low value of Poisson's ratio is to be maintained.

The practical compromise that was decided on was to drop this requirement and to develop a material that meets the more important demands reasonably close, leaving to the future the development of a material that meets all the specifications. This decision will probably make necessary some corrections in the results of the model test.

A preliminary search disclosed the fact that the combination of the two requirements—specific gravity of 2.4 and low modulus of elasticity—precludes a range of well-known and commercially used materials, most of which have low specific gravities at low moduli of elasticity.

The search made of different, less known materials having cellulose, cellulose acetate, etc., bases, showed that they all have a specific gravity much lower than 2.4 and that the new materials that could be obtained by loading the known materials to the desired specific gravities were unacceptable on account of the high modulus of elasticity and uncertain elastic properties.

Rubber compounds showed the best promise and an extensive co-operative research was made in this field. The main difficulty in this research was that all the customary rubber tests are made at high tensile stresses which approach the ultimate, and very little is known about the general behavior of rubber compositions at low stresses.

Practically nothing is known about the behavior of highly loaded rubber compounds of high specific gravity at low compressive stresses. This research can be summed up briefly as follows:

(a) A large number of rubber compounds of the desired specific gravity of 2.4 can be made by using the commercially available fillers.

(b) As a general rule, these heavily loaded rubber compounds are unstable and show uncertain elastic and highly developed directional properties at low compressive stresses.

(c) The values of Poisson's ratios of different compounds can be varied in rather wide limits; but the tests made, thus far, developed unsteady compounds with directional properties for low values of Poisson's ratio. The customary manufacturing processes seemingly do not give the refinement in control that is necessary in order to obtain material of the same quality by repeated fabrication of compounds of the same composition and manufacturing procedure.

(d) From the different compounds it was possible to pick some that, as long as the high values of Poisson's ratio were accepted, have steady and fairly uniform non-directional elastic properties and a low modulus of elasticity, which properties could be satisfactorily repeated.

(e) The attempt to find a material that would meet these requirements, having at the same time a low Poisson's ratio led to the conclusion that such material is possible and feasible, but that a considerable amount of research would be required, that would include not only the composition but the manufacturing procedure as well.

Adopting the research made as a basis, it was decided to use for the model a litharge rubber compound of the required specific gravity of 2.4, with a low modulus of elasticity of about 500 lb. per sq. in., and with a high Poisson's ratio approaching 0.5. This compound has uniform and stable elastic properties, and it is only slightly influenced by aging or repeated loading, showing very small differences at a time interval of about six months and at repeated loadings that ran as high as 5 000 lb. per sq. in. hydraulic pressure.

The fabrication of this material can be repeated infinitely, insuring the possibility of manufacturing any desired quantities similar to the original samples; it can be cemented together easily, or cut on a band-saw, insuring the possibility of making blocks of any desired shape and size.

Tension in Horizontal Joints.—In accordance with Equation (116) the ratio of the ultimate unit tension that can be carried over the horizontal joints of the model and prototype must be equal to the scale of the model. A number of difficulties will be encountered in an attempt to fulfill this condition: First, the value of the ultimate unit tension that can be carried over the horizontal joint of the dam is uncertain; and, second, it is rather difficult to provide a joint in the model that would be ruptured at a low tensile stress with any degree of accuracy. The compromise that was reached was to decrease the number of horizontal joints, placing them on the model corresponding to larger lifts than those that were actually used in the prototype.

Foundations of Model.—The proper representation of the foundation conditions of a model is uncertain, due to the fact that practically no information concerning this subject is available.

The practical compromise that was decided upon, was to make the foundations of the model of the same material as the model of the dam, in blocks of arbitrarily chosen sizes, but with provisions that will make it possible to change them, substituting blocks of different size. At the same time the foundation blocks were partly restrained in the horizontal direction by means of rubber struts, which could again be made of different size and located at different heights. The lower part of Fig. 1 shows this arrangement.

The expectations are that the proper conclusions can be obtained by repeating the tests with foundations of different sizes and with different amounts of restraint. Furthermore, a study of the influence of the elastic deformations of the rock on the behavior of the dam will be made possible.

General Arrangement and Plan of Model.—The model is to be an exact representation of the actual dam constructed to a scale of 1:50, of rubber compound, inside a concrete tank as shown on Fig. 1.

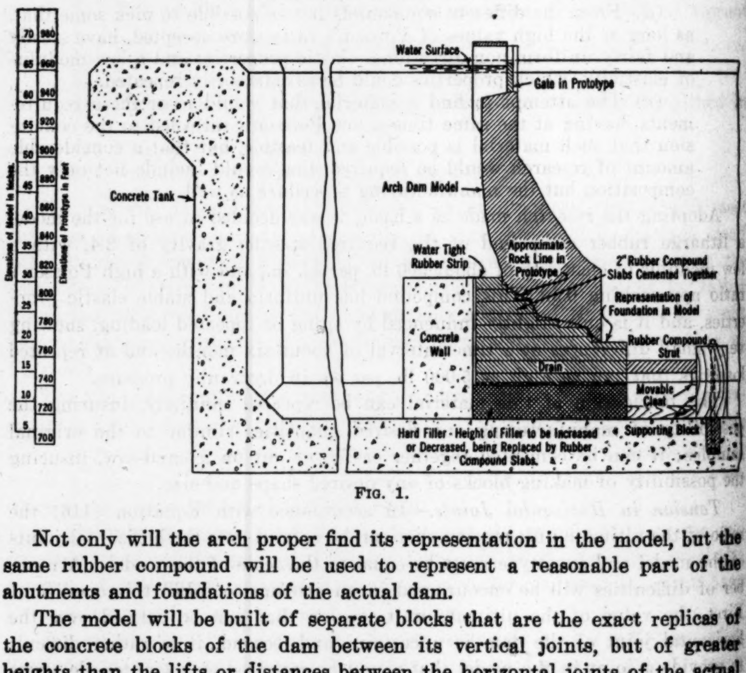

Elevations of Model in Inches
70
65
60
55
50
45
40
35
30
25
20
15
10
5
Elevations of Prototype in Feet
980
960
940
920
900
880
860
840
820
800
780
760
740
720
700
Water Surface
Gate in Prototype
Concrete Tank
Arch Dam Model
Water Tight Rubber Strip
Approximate Rock Line in Prototype
2" Rubber Compound Slabs Cemented Together
Representation of Foundation in Model
Concrete Wall
Rubber Compound Strut
Drain
Movable Cleat
Supporting Block
Hard Filler - Height of Filler to be Increased or Decreased, being Replaced by Rubber Compound Slabs.

FIG. 1.

Not only will the arch proper find its representation in the model, but the same rubber compound will be used to represent a reasonable part of the abutments and foundations of the actual dam.

The model will be built of separate blocks that are the exact replicas of the concrete blocks of the dam between its vertical joints, but of greater heights than the lifts or distances between the horizontal joints of the actual dam. The necessary modifications in the size of the vertical joints are to be made.

Water is to be used as the loading liquid and deflection and strain measurements are to be made at different points of the dam and are to be repeated for different conditions of foundation deflections.

As far as similarity is concerned, the study of this model can be made by considering the following conditions (see Conditions (1) to (8), previously defined):

(*a*) Condition (1) is assumed to be satisfied.
(*b*) Conditions (2), (7), and (8) are actually satisfied.
(*c*) Conditions (5) and (6) are partly satisfied; and
(*d*) Conditions (3) and (4) are not satisfied.

Conditions Assumed to Be Satisfied.—Condition (1), concerns the assumed constancy of the moduli of elasticity of the dam and model materials.

Conditions Actually Satisfied.—Condition (2) concerning the ratio of the specific gravities of the loading liquids and the material of the dam and model, is in this class.

Two other factors in this class are: Condition (7) concerning the representation of the foundations; and Condition (8) concerning the ratio of temperature differences, in model and prototype, between the time of building and the time of observation.

Conditions Partly Satisfied.—Condition (5) concerning the ratio of the ultimate unit tensions in the horizontal joints of the dam and model is partly taken care of by increasing the size of the horizontal lifts in the model, as compared with the prototype. The expectation is that no corrections will be necessary in the results of the tests on account of this discrepancy.

Condition (6) concerns the size of the vertical construction joints. Some corrections may be required in the results of the tests to take care of the discrepancy, due to the difference in Poisson's ratios.

Conditions Not Satisfied.—Condition (3) concerns the equal values of Poisson's ratio for the materials of the model and dam. Some corrections may be necessary in the results of the tests, and an attempt will be made to take care of part of the discrepancy by a proper dimensioning of the vertical construction joints.

Condition (4) concerning the ratio of the moduli of elasticity of the material of the model and prototype, is necessary to retain the geometrical similarity of the shape of the models and the prototype before and after the load is applied. The expectations are that the difference in observations due to this discrepancy will be so small that it can be neglected.

Acknowledgments.—This work is a part of the arch dam investigation that is being conducted by the Aluminum Company of America at the request of Edwin S. Fickes, M. Am. Soc. C. E., under the direction of James W. Rickey, M. Am. Soc. C. E., and Mr. J. P. Growdon. The research work in rubber compounds in the Research Laboratory of the Goodyear Tire and Rubber Company was made under the direction of Dr. L. B. Sebrell by Messrs. R. D. Evans and M. J. DeFrance. The Aluminum Research Laboratories of the Aluminum Company of America are assisting in the investigation and, under the direction of R. L. Templin, M. Am. Soc. C. E., R. G. Sturm, Assoc. M. Am. Soc. C. E., and Mr. R. L. Moore, tested the physical properties of the rubber compounds. Messrs. J. P. Den Hartog and J. G. Baker, of the Research Laboratory of the Westinghouse Electric and Manufacturing Company, assisted in formulating the similarity conditions based on the application of the dimensional theory.

JASPER O. DRAFFIN,[40] M. AM. SOC. C. E. (by letter).—Although the writer cannot add anything to the content of this paper, he wishes to express a feeling of satisfaction that this type of study is receiving attention. To examine the validity of assumptions concerning the principles underlying the use of models is very valuable, because so much hydraulic practice is based upon information obtained from models. More work of this character is needed and the author is to be congratulated for what he has done.

It is impossible to read the accounts[41] of the work done in European laboratories without being impressed by the large amount of space devoted to discussions of models. It is also of interest to note the increasing importance attached to the theory of models.

Undoubtedly, the study of river flow and control in the United States has lagged behind that of Europe, but this condition is changing. The 1927 floods in New England and in the Mississippi River Valley, added to the Ohio floods of two decades ago, have emphasized the growing importance of flood-control works and have resulted in a public demand for relief from such disasters. This demand has brought responsible opinion to the point of studying flood flow and control by means of models, as evidenced by the establishment of the National Hydraulic Laboratory for the study of these problems.

There are still, however, many people who doubt the possibility of predicting the action of a great river in flood flow by experiments on a small model of that river. If the confidence of the people in the ability of the engineer to solve such gigantic problems is to be retained, all the available theory must be brought to bear upon the design of the models which determine how such great works are to be constructed.

It may not be amiss to call attention to the fact that the early experiments with models were made in a somewhat crude fashion and that the methods have gradually improved. From the early work of Dubuat to that of Reynolds, of Vernon-Harcourt, and, more recently, the work of Mr. Groat, is a long step, and great progress has been made in the use of the model as well as a clearer understanding of the significance of the various elements involved in its design and use. Considering the great variety of engineering structures which are built after or tested by models, such as bridges, dams, machinery, ships, and airplanes, as well as structures involving the flow of water, it is desirable that there should be considerable diffusion of knowledge about the necessity of careful design of models and the methods by which it may be achieved. Some experimenters seem unaware of the limitations of their experiments. It is well that the elements of viscosity, temperature, and surface tension should be given prominence by the author since, as Mr. A. C. Chick points out,[42] these factors may be important under small heads or with small openings.

---

[40] Asst. Prof., Theoretical and Applied Mechanics, Univ. of Illinois, Urbana, Ill.

[41] "Hydraulic Laboratory Practice," by John R. Freeman, Past-President and Hon. M., Am. Soc. C. E., Am. Soc. Mech. Engrs., 1929.

[42] *Loc. cit.*, pp. 776-777.

ED S. SMITH, JR.,[43] ESQ. (by letter).—The author's paper attacks the present usage of models and unjustifiably urges that the broad generalization of Newton be used for all cases of fluid flow. This viewpoint seems to be a mistaken one since the correct and adequate basis for flow similarity in pipes is the Reynolds number and not that of Newton. The writer's experience with quantity-rate fluid meters shows Mr. Groat's theory to lead to an incorrect basis for flow similarity for pipes and confirms the present usage, based on the Reynolds number, which permits accuracies of within one-fourth of 1% to be realized with Venturi meters when metering one fluid-using datum obtained with another fluid, for example.

Excellent use has been made by Mr. Groat of Newton's basis for flow similarity in its proper field—open channel flow and the fluid transportation of sand and gravel (non-fixed shape and non-homogeneous fluid)—but the writer maintains that Newton's basis (with g as the acceleration of gravity, as used by Mr. Groat) simply does not apply to the ordinary case of the steady flow of homogeneous fluids in closed conduits of fixed shape, and that consequently Newton's basis is absolutely not generally applicable; it must be used only in its proper field. For the case mentioned it can only enter in so far as it may be considered in the internal acceleration of the fluid which is taken care of fully in the derivation of the Reynolds number from the balance of inertia and viscous forces within the fluid. Thus, the Reynolds number is the basis upon which data are correlated for this case in which whatever accelerations are involved are certainly not due to gravitation.

Newton's law of similarity (Equation (1)) applies primarily to liquids with free surfaces, as Froude brought out for the wave-making resistance of ships, or the flow of liquids in open channels where the effect of the surface enters—including the case of weirs and Venturi flumes.

For steady flow in a closed conduit of fixed shape, including Venturi tubes and orifices, the Reynolds number is the adequate and correct basis for flow similarity and the correlation of data, as established beyond question by innumerable experiments. This criterion also holds for airplanes, dirigibles, submarines, and for air and water propellers, with an additional term to take care of the various slip ratios.

Fortunately, it is possible to subject fluid flow phenomena to an independent analysis which definitely establishes the variables and operators involved so that one may test the fitness of a generality, such as Newton's expression, to the case of fluid flow through a closed conduit of fixed shape. For this case the Reynolds number is found to be pertinent. In Fig. 2, the flow, Q, through a closed conduit is divided into two streams. One stream flows through a long capillary tube, M, in which the resistance to flow is due to viscous forces, and the other flows through a nozzle, N, or orifice, in which the resistance to flow is due to the inertia of the fluid.

In another paper[44] the writer has demonstrated that, from the known laws of flow, the ratio of flows (that is, "flow similarity") and the coefficient for the example given, depend only on the Reynolds number and not upon Froude's ratio (Newton's law involving the acceleration of gravity), as specified by Mr. Groat. Consequently, for closed conduits, gravity is shown to have no effect on the flow distribution. A familiar proof of this is that the direction of the axis of flow in a nozzle with respect to the vertical has no effect upon

---

[43] Hydr. Engr., Builders Iron Foundry, Providence, R. I.

[44] "Quantity-Rate Fluid Meters," by Ed. S. Smith, Jr., *Transactions*, Am. Soc. Mech. Engrs., Paper HYD-52-7b (Fig. 6).

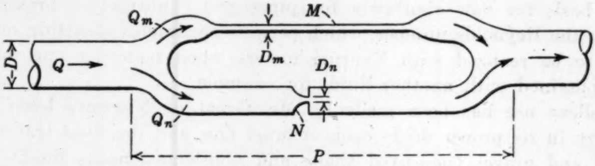


FIG. 2.

the performance of the nozzle. The Reynolds number is adequate and proper for use in the example cited herein and does not require the assistance of Newton's ratio for correlating data. The writer would appreciate Mr. Groat's confirmation, as an authority, that this is correct or, if he should fail to agree, then to demonstrate a solution of this problem in his closing discussion.

The method of similarity is a most valuable one in the field of fluid flow, but it must not be used blindly. In every case, the engineer must consider carefully which of the various physical quantities and phenomena is involved, and then select the proper dimensionless ratio as the basis for similarity. In the case of fluid flow in closed conduits this is the Reynolds number; for open channels subject to wave action it is Froude's ratio (Newton's ratio, using g as the acceleration of gravity); while, for expansible fluids, it is the acoustic velocity ratio. For example, in the case of airplane propellers (of which Mr. Groat treats in Appendix I) the tips frequently approach the velocity of sound and a correction for the compressibility of air is certainly required. The writer has proposed the use of the acoustic ratio, $\frac{x}{k}$, in which, x is the ratio of differential to absolute pressure and k is the ratio of specific heats, for fluid flow through quantity-rate meters. This acoustic ratio, $\frac{x}{k}$, may readily be modified so as to be applicable to the air-propeller problem when testing with another gas than air; this may prove to be more convenient in actual use than the acoustic velocity ratio.[45]

Mr. Groat's treatment of the case of air propellers (which does not involve Newton's, or Froude's, ratio) is inadequate and incorrect. It emphasizes the point that the various engineering fields require engineers actively engaged therein to be familiar with the use of the method of similarity so that they can make effective use of it. This is preferable to having "similarity engineers" attempting to use the method in fields with which they are unfamiliar.

---

[45] See "Similarity and Model Experiments," by A. H. Gibson, *Engineering*, March 21, 1924, p. 358, for correct "air screw" theory.

While the method of similarity has been highly useful in the field of fluid flow, this has been so largely because the pertinent variables have been sufficiently simple (density and viscosity) to be easily susceptible to attack by the method of dimensional analysis.

However, for structural similarity, the dimensional method may easily require considerable assistance before the results will be most useful, since hardness, toughness, resistance to shear, tearing, and plastic deformation seem scarcely to have been defined adequately in a dimensionally sound way. Furthermore, these bring in the factor of direction so that the problem introduces vector quantities which may or may not be covered by dimensional analysis in determining the proper basis for similarity—along this line is the familiar fact that work and torque have the same dimensional units: $F \times L$.

Complete use also must be made of established methods of detailed analysis wherever possible for checking the application of this general method of similarity to structures, or any new field. Otherwise, it will suffer temporary loss of confidence and, therefore, usefulness, due to improper use without adequate physical understanding. Wonderfully useful as this method is, it does not enable one to obtain something for nothing, being in fact a convenient basis on which to correlate data for most effective use.

The conclusions from the foregoing comments are:

1.—The present usage of models for flow in pipes, based on the Reynolds number without the encumbrance of Newton's ratio (actually Froude's), is entirely consistent with sound theory, being unquestionably established by a great mass of accurate data.

2.—For this case of flow in pipes, it is not necessary to synthesize fluids (to meet the joint requirements of the Reynolds number and Newton's ratio, since the latter does not apply).

3.—It is necessary to follow the specific dimensionless ratio which is pertinent to any given case, rather than to attempt to use a too broad (and, hence, generally incorrect) generalization, such as Newton's ratio, for all cases. It is a step backward to apply a universal cure-all to cases in which it obviously does not work.

These conclusions are completely in opposition to those of Mr. Groat.

DR. ING. F. J. EISNER[46] (by letter).—The writer agrees with the author in attributing to Newton the enunciation of the fundamental relation for comparability of forces, but it is not correct[47] to read into Newton's conception the so-called "law of corresponding velocities," as it is understood to-day, resulting from the special action of gravity on the surface of the earth. This law of corresponding velocities is a special form of Newton's fundamental relation and should be ascribed justly to Reech, Froude, and Helmholtz.

According to Newton's general statement, the basic relation between corresponding forces in similar systems holds rigidly for all kinds of forces;

---

[46] Head, Dept. of Applied Mathematics, Prussian Experiment Inst. for Hydr. Eng. and Shipbuilding, Berlin, Germany.

[47] See W. Herrmann, *Zeitschrift* d. Vereines Deutscher Ingenieure, 1931, pp. 611-616.

but, except for certain cases to be mentioned subsequently, this general statement does not in itself constitute a "law of transference" applicable to models of a definite linear scale-ratio to the prototype. A true "law of transference" stipulates that by choosing only this linear scale-ratio and the model media all transference rules are definitely and strictly determined solely by this single ratio, as are likewise all particular conditions for fulfilling the requirement of force-similarity under the free action of the forces actually produced by the physical properties of the chosen media and by the imposed movements.

These physical properties other than density are not taken into account in Newton's general statement. That statement is one of the most ingenious in mechanical science, but with present-day requirements for real and practical similarity, it is too general; since, in the most universal manner possible, it states only the fact that forces acting upon masses produce accelerations. Hence, the only physical property it involves is density, or mass distribution, $\rho = \frac{w}{g}$, which is always constant and independent of location in space since the specific weight, w, varies directly as the acceleration, g. To eliminate all ambiguity the author's notation for acceleration should be supplemented by differentiating between accelerations in general due to forces of any kind acting on masses and the acceleration due to gravity on the earth's surface. For the purposes of this discussion, the former will be designated by b and the latter by the particular value, g. Then Equation (1), which expresses Newton's general relation for comparability should read:

$$b = \frac{V^2}{D} \qquad \text{(119)}$$

This modification must be applied to all the author's deductions and discussions which deal with Newton's relation as distinguished from the Froude-Helmholtz law.

Newton's statement considers all forces without regard to their physical origin, and takes into consideration only the density or mass distribution, ρ. If only pure mass forces, or those of the type, $F \alpha \rho L^2 V^2$, are acting. Newton's requirement suffices for the establishment of a similarity law. In all other cases, true laws of similarity are deducible only when the special types of forces (other than density) produced by the special physical properties of the media are also taken into account. Then each particular property creates a new law of transference from model to prototype.

A true law of similarity expresses a definite relation between the three basic unit ratios, L, F, and T. If one of these is arbitrarily chosen (usually the length, or the linear-scale ratio, L), the other two are then fixed by the selected ratio (usually the scale ratio), together with the governing force-producing physical property (other than density) of the media in the model and prototype. In the particular case under consideration this property governs the phenomena and, therefore, pertains to this special similarity law. If, as in the case of Newton's general statement, the influence of the physical properties (other than density) is not considered, then, mathematically, F, L, and T may each be selected arbitrarily. In that event there is no true "law of similarity" and no certain practical possibility of producing model phenomena in reliable agreement with the prototype phenomena because every force (other than mass forces) which the experimenter brings into action on the model, must be caused by some physical property of the model media. This is the real reason why the author's Equations (11), (12), and (20), which when modified by the writer's notation become,

$$L = R \qquad (11)$$

$$\frac{V^2}{L} = \frac{L}{T^2} = \frac{B}{b} \qquad (120)$$

$$f_1(L, F, T) = \theta' (P') = \theta' (\rho) \qquad (121)$$

and,

$$\frac{F T^2}{L^4} = \rho \qquad (61)$$

are insufficient to establish a true model law. This is the fundamental reason also, why Newton's requirement should be regarded solely as a general statement and not as a law of similarity in any special sense (excepting, always, cases in which mass forces or those proportional to $\rho L^2 V^2$ act alone). This word "solely" is to be regarded as a limitation only as applied to practical model construction and interpretation of model results. Of course, the most important and the most brilliant accomplishment was the original conception of the general law (which includes all subsequent particularizations); but the immediate practical application of such a law is often impossible or unknown until particularization has been made. The great difficulty of performing such particularization and the fact that each particular law was developed by a different investigator so many generations after Newton first enunicated the general statement are shown clearly by the history of the science of similarity.

Equation (11) denotes geometric similarity; Equations (120) and (121) together denote general dynamic and kinematic similarity. Full mechanical similarity is ensured only if in a geometrically similar model similar paths are described in corresponding times (requirement for kinematic similarity) through the free action of the natural forces produced by the physical properties (special dynamic similarity). One or the other of Equations (120) and (121) may be replaced by an equation denoting similarity of "corresponding" times. In any event there are three independent equations in *L, F,* and *T,* the solutions of which give values of the three basic unit ratios, which are not expressible in terms of any single one of them as is necessary for the expression of a true law of similarity.

Such a law cannot be obtained until a fourth relation, Equation (21), involving some physical property other than density, is introduced. Each physical property, such as viscosity, elasticity, kinematic specific weight (that is, the effect of the earth's attraction), capillarity, etc., requires a particular form of Equation (21) resulting in a particular solution; that is, expressing a particular relation between *L, F,* and *T,* making possible the expression of any two of these unit ratios in terms of the third.

In order to obtain the clearest and most precise statements possible of the conditions for a mechanical law of similarity the physical properties (other than density) should be introduced solely in their kinematic form thus: $\nu = \frac{\mu}{\rho}$, kinematic viscosity; $g = \frac{w}{\rho}$, kinematic specific weight or earth gravitation; $\epsilon = {}^{E}$, kinematic modulus of elasticity, etc. The introduction of density according to Newton's general statement assumes in advance the requirement of general dynamic similarity. This is a new and a very important point of view.

The fact that each physical property results in a different relation between *L, F,* and *T* has an important consequence. If, as is usual, two or more physical properties other than density enter simultaneously into a problem, either the results obtained are entirely irreconcilable throughout (that is, the only possible model is the prototype itself), because the choice of model media is so limited in most cases that it is practically impossible to obtain a reasonable model-scale ratio, or the establishment of perfect similarity is not possible. This vital condition limiting the possibility of similarity has been designated by the writer in his lectures at the Technical University of Berlin as "Zusammenwirkungsgrenze" (literally: "together-working limit").

In the entire field of practical model science there are many other limitations. In the special field of laboratory practice for the solution of practical hydraulic engineering problems seven limits are recognized by the writer. The characteristics of these seven limits; the means of overcoming them whenever it may be possible; the cases of "incomplete" and "empiric" similarity; the cases of a predominating physical property; the very important question of obtainable accuracy and its influence on the scale ratio; and other aspects of the problem are all discussed in the writer's contribution to the "Handbuch der Experimentalphysik."[48] An abstract and some new viewpoints are to be found in the "Handbuch der Physikalischen und Technischen Mechanik".[49] There is also a brief sketch in the "Physikalisches Handwörterbuch".[50]

As emphasized in the foregoing remarks, Newton's general requirement embraces similarity in general without specific reference to particular physical properties, except density. The elaboration and extension of the general statement to specific physical properties and the resultant possibility of model construction (that is, of bringing into action on a model the actual forces due to the properties of the model media) must be credited to later investigators. An excellent summary of all these laws covering not only mechanics, but the entire field of physical science, is given by Weber[51] in two well-known papers read before the Schiffbautechnische Gesellschaft in 1919 and 1929, the first of which is referred to by the author. The second paper is more comprehensive in every respect. The present discussion covers very briefly only the following four laws of similarity, with which the practical hydraulic engineer is chiefly concerned. (1) Law of Gravitational Similarity; (2) Law of Viscous Similarity; (3) Law of Elastic Similarity; and (4) Law of Capillary Similarity.

---

[48] F. Eisner, "Offene Gerinne," Tome IV, Pt. 2, of the "Handbuch der Experimentalphysik," by Wien and Harms; Verlag der Akademischen Verlagsgesellschaft, Leipzig, 1931.

[49] P. Nemenyi, "Wasserbauliche Strömungslehre," Tome V of the "Handbuch der Physikalischen und Technischen Mechanik" by Auerbach and Hort, Ambrosius Barth, Leipzig, 1930–31.

[50] More than 400 articles and references to Hydraulics and Hydrodynamics, by F. Eisner, L. Hopf, S. Erk, and O. Schrenk, "Physikalisches Handwörterbuch," Julius Springer, Berlin, 1931. (Second Edition only.)

[51] Weber, "Die Grundlagen der Ahnlichkeitsmechanik und ihre Verwertung bei Modellversuchen," *Jahrbuch* der Schiffbautechnischen Gesellschaft 1919; "Das allgemeine Ahnlichkeitsprinzip der Physik und sein Zusammenhang mit der Dimensionslehre und der Modelwissenschaft," *Loc. cit.*, 1930, Julius Springer, Berlin.

(1).—Law of Gravitational Similarity.—This law, or to use its less exact name, the law of "corresponding velocities," was first enunciated by Reech in 1852, and independently by Froude in 1869, in the course of his famous ship-model experiments. It was also independently discoverd during the theoretical and practical investigations of Helmholtz in 1873 on the formation of waves at the surface of contact between two fluids of different densities (particularly for cloud formation in meteorology).

(2).—Law of Viscous Similarity.—The author ascribes this law to Stokes. It is true that Stokes considered forces caused by the action of fluid viscosity, but the mathematical enunciation of this action had already been accomplished by Newton. It is also true that Stokes worked with problems of similarity, involving the action of viscosity, but he did not carry his analysis through to the enunciation of a special law of viscous similarity. If the development of a correct expression for viscous tension were the determining factor, the author might with more justification ascribe the viscous law to Newton rather than to Stokes. As a matter of fact, neither Newton nor Stokes succeeded in establishing such a law. That accomplishment should be credited—as is commonly done—to O. Reynolds (1883). It is only by ascribing a misconception to Stokes' work that the author is able to give Stokes the credit, but Stokes has indeed never been guilty of such inaccuracy. The history of the development of the law of viscous similarity shows the great difficulties of particularizing Newton's general statement into a true practical and usable specific law of similarity.

(3).—Law of Elastic Similarity.—According to Weber[51] this law is to be attributed to Cauchy (1829).

(4).—Law of Capillary Similarity.—The writer (since 1920) has attributed this fourth law to Weber himself (1919). The designation of the special law of capillary similarity as "Weber's Law" has been fully accepted by European scientists and likewise has been adopted[52] by John R. Freeman, Past-President and Hon. M., Am. Soc. C. E.

Each special similarity law involves its own dimensionless number or characteristic expression based upon and containing the governing physical property. This number, as is known, must be of the same numerical value in the model as in the prototype, in order to produce similarity. In the European notation, the Froude-Helmholtz Number is designated by **F**; the Reynolds' Number by **R**; the Cauchy Number by **C**; and the Weber Number by **W**.

The author's Equations (16) and (17) involve the simultaneous action of gravity and viscosity. In this case, the possible values of the ratios,

---

[52] "Hydraulic Laboratory Practice," pub. in 1929 by the Am. Soc. Mech. Engrs.

L and T, lie within very narrow limits, because the variation of gravity obtainable by natural means on the earth is exceedingly small and the range of viscosity of the commonly available fluids is restricted by practical considerations, such as lack of transparency, evaporation, diffusion, etc. Thus, either the practical operation of these natural limitations dictates a model of a scale scarcely differing from that of the prototype or the "together-working-limit" it encountered and no model is possible except the prototype itself.

Exactly the same reasoning holds for the author's Example 1 which involves the simultaneous action of elasticity and gravity. His Equations (29), (30), and (31) are identical with Cauchy's law of elastic similarity. Equations (37) and (38) indicate the resulting "together-working-limit." Again, the only model possible is the prototype itself.

The author's Example 2 involves the simultaneous action of gravity, elasticity, and viscosity. Hence, the three dimensionless numbers of Froude, Cauchy, and Reynolds must each have the same magnitude for the model as for the prototype. The author's Equation (40) is a characteristic form that may be written as $\frac{\mathsf{R}^4\,\mathsf{C}^3}{\mathsf{F}^4}$. The numerical equality of this characteristic in prototype and model is not, as indicated by the author, in itself sufficient to satisfy the condition for mechanical similarity—namely, the equality of each of the three individual factors, F, C, and R, in model and prototype. The author's development of Example 2 is nevertheless instructive and very valuable in showing the tremendous difficulties of arriving at a practical solution of problems involving the "Zusammenwirkungsgrenze."

The writer does not share the author's optimism as to the possibility of finding suitable media or even of creating artificial media adaptable for model work. Exceptions may occur, but, in general, the physical properties of available materials are so well known and their possible range of application to model work are so limited, that there is little justification for the hope of establishing a new branch of engineering. The author's illustration of a ship model in mercury is not well chosen because, despite several theoretical advantages, the use of mercury has many practical disadvantages. The different nature of the adhesion phenomena between hull and water in the prototype and hull and mercury in the model tank would produce different wave and wave-resistance effects. The opaqueness of the mercury would prevent visual and photographic observation of important subsurface flow phenomena and, furthermore, the prohibitive cost of installing and operating a mercury-filled towing tank of adequate size is in itself enough to eliminate the practical use of mercury.

The author has included an historical treatment of the subject, and he deserves great credit for bringing the important and timely subject of model science to the attention of American engineers. In Europe, and particularly in Germany, several similar studies have been made, which are most ably presented in the two important papers by Weber previously mentioned. These articles summarize the knowledge of the theory of similarity and model science as well as its history, but they do not treat of the limitations in practical application. These practical limitations for the particular field of hydraulic engineering have been fully discussed by the writer.[a]

The writer wishes to express his thanks to H. Kramer, Lieut., Corps of Engrs., U. S. A., Assoc. M. Am. Soc. C. E., and Lt. H. W. Ehrgott, Corps of Engrs., U. S. A., for their willing co-operation and critical assistance in preparing the translation of this discussion.

Benjamin F. Groat,[53] M. Am. Soc. C. E. (by letter).—Engineers and designers are losing the advantages of Newtonian model engineering by neglecting those necessary features which convert models from indirect uncertain indicators to direct primary tools for accurate design.

This does not mean that one may not prefer and use the analytical method, when it is available. It does mean that there is a promising field for those who would cultivate and add to their professional equipment a valuable branch of research; a branch which enables one to make a theoretically correct solution of any dynamical or statical problem of design, even though no analytical theory may exist for such solution.

It also means that there is a great opportunity for engineering research of the most intelligent kind since it is necessary to formulate suitable technique in the various fields, to secure suitable materials and conditions for models, and especially to establish ways and means for synthesizing materials, or adjusting their properties, or both, so as to render them fit for useful service as constituent parts of models.

The research engineer who advances the part of the technique just mentioned will find plenty of glory in his successful achievements. The real prime foundation stone of any branch of engineering is an independent body of free-thinking capable engineers, who have no restrictions placed upon their activities, inventions, and writings, and who, therefore, may proceed to make of themselves real promoters of the advancement of engineering science.

No capable engineer should dismiss the idea of espousing Newtonian model engineering as an accurate technical system for design and research. Correct model engineering can supplant the most accurate analytical method. This new branch of engineering is constructed of the same elements of exactness as the analytical method, and it may be applied when the analytical method is deficient, impossible of application, or wholly wanting, all of which conditions actually exist. The only errors are those of observation and experiment, errors that are equally potent in all theoretically exact methods which involve physical relations. Thus it is, that Newtonian model engineering stands upon the same footing that supports the analytical method, namely, exact dynamical theory.

It should be most carefully observed that the writer does not criticize the great engineers of the past who have used models—he has only the most enthusiastic admiration for their famous achievements—but the successes of these engineers cannot in the least be ascribed to any inherent virtues of the models which they used. It would be far more nearly correct to state that the type of model engineering they used was rescued from total failure by the outstanding ability displayed by the engineers, two of the greatest of whom were W. Froude and R. E. Froude.

---

[53] Cons. Engr., Brookline, Mass.

The most careful scrutiny reveals that the advantages of true dynamic models have rarely, if ever, been secured. In every case it will be found that much redundant experimentation, unnecessary work, or pure speculation, was made to arrive at questionable relationships between a proposed structure and its model. This has been carried to such an extent that there was, in the end, no unique use for the model, the performance of the prototype having been independently determined by more or less reliable methods with which the engineer was familiar. Even the definition of the word "similarity" has been involved in obscurity. In all writings on that subject it will be found that, although the word has been used repeatedly, no adequate definition has been given with the exception of that in the writer's paper (see "Historic Background").

As some of the discussers fail entirely to grasp the ideas advanced by the writer, it will be justifiable to mention some of the original features of his paper, at the same time admitting that the foundation stone upon which he builds, was firmly laid by Newton in the theorem previously quoted. The whole point is that this theorem has never been understood except in a limited sense. This non-understanding is undoubtedly due to its extreme breadth and sweeping scope.

The three principal original features of the paper are:

(1) Association and solution of Equations (11), (12), (20), and (21), to develop the general specifications for any true dynamic model (see "Summary of Equations of Similitude").

(2) First discovery and publication[54] of the fact that true dynamical similarity depends only on the respective dynamical properties of the materials for the model and prototype, and on the general conditions under which they must work, but not at all upon the form or action of either of these devices. It is this fact which places true dynamic model engineering upon a theoretically exact foundation of its own.

(3) Explanation of the general nature of the technique to form materials and produce conditions by means of which Newtonian model engineering may be made effective.

The method is proposed to the designer as a direct, theoretically exact, systematic means for arriving at the proper construction and operation of all types of true models to serve as accurate primary tools for the design of engineering works generally, including, for example, steam turbines, hydraulic turbines, ships, airplanes, submarines, buildings, extensive water-power projects, etc. There is no limitation to the applicability of Newtonian models for such purposes, whatever may be the conditions and properties of materials and fluids to be suitably represented in the models by different materials and fluids, either real or virtual.

Such properties and conditions may include: Sliding friction, rolling friction, elasticity, fluid flow and viscosity, electricity and magnetism, cohesion, capillarity and surface tension, mass attraction, gravity at the earth's surface, acceleration generally, and any other dynamical characteristics, perhaps even including the dynamical features of chemical affinity and the modern atomistic theory, if one should be so enthusiastic as to delve into such interesting questions.[55]

---

[54] "Ice Diversion, Hydraulic Models, and Hydraulic Similarity." *Transactions*, Am. Soc. C. E., Vol. LXXXII (1918), p. 1138.

While such studies are delightful and highly instructive, it is clear that the world awaits a master generalization, before the nature and constitution of matter can be understood. The impression made upon the writer is that many authors seem anxious to assert the "limitations of classical mechanics," or, even its "failure" before they really have an explanation of their own difficulties and while they still seem compelled to use Newton's laws in the usual way for most of their work. Nothing illustrates better the force of fashion, even in science, than the present outcry against Newton's laws by those who continue to use them. The truth is that those who fall into the fashion, really do not understand the true situation, thereby making exaggerated and untrue statements.

(1) *Equations of Similitude.*—When the model has been constructed according to the principles explained in the paper, and is then properly set in operation, it will, in all essentials, be a theoretically exact reproduction of the proposed structure, or works, of which it is a true model. The systematic conditions may be made clear by reference to Table 3, which furnishes a general basis, or program, for the branch of design which the writer describes as "Newtonian model engineering," or the technical methods of constructing any model which is to perform with theoretically exact Newtonian similarity.

TABLE 3.—CHARACTER OF EQUATIONS OF SIMILITUDE.

Equation No.	Type of similarity.	Nature of the condition imposed by the equation.
(11)	Geometrical	Form and position only.
(12)	Kinematical	Similar motions in similar times.
(20)	Dynamical	Similar forces upon similar masses.
(21)	Physical	Dynamical properties imitated in suitable manner.

It is to be noted that Equation (11) is not independent, thus, the other three equations determine L, F, and T. Equation (21) may represent two or more equations involving as many physical constants, in which case the additional constants are to be made consistent by "adjusting" their values by the artifices of model engineering as illustrated in the paper, and as may be devised by the model engineer at different times to suit the occasion.

In Equation (21) (Table 3) the imitation need not be visual, or sensible, but merely virtual. Effective dynamical similarity is the essential requirement. It is well known, for example, that certain forms of rigid bodies are exactly equivalent in the dynamical sense without being geometrically similar.[56]

---

[55] See "Atomic Structure and Spectral Lines," by Arnold Sommerfeld, translated from the German by Henry L. Brose, about 1923; also, "Atoms, Molecules and Quanta," by Ruark and Urey, 1930.

[56] Routh's "Elementary Rigid Dynamics," Sixth Edition, Section 115.

Geometrically dissimilar rigid bodies which are dynamically equivalent in a vacuum may not, however, be dynamically equivalent when immersed in a fluid. One must be very careful about departing from geometrical similarity. The writer does not advise it except with the greatest caution.

(*2*) *Statistical Method.*—There is another feature of dynamic models which should be thoroughly understood. This applies especially to models of fluid flow. When the engineer uses the analytical method in arriving at a design, he usually deals with hypothetically stable conditions which have no existence in fact. Lines of flow are assumed to be fixed and the flow along them is assumed to be steady. In the case of a dynamic model, as a tool of design, the engineer is applying a "statistical" method; that is, the actual motion is, in one respect, like that postulated in the kinetic theory of gases, in which the actual configuration of the system is never twice the same at any two distinct instants, but in which the average condition, as deduced by Maxwell's law (application of the theory of probability), is perfectly stable, although any actual condition may never be the one indicated by the average—the system may never actually be in its average state. Theoretically, similarity can be made more exact for an irregular river bed than cut and fill can be determined for rough ground by well-planned engineering.

Hence, one concludes that if he can confine his model observations to measures which give the proper statistical averages, as to the pressure on the side of a containing reservoir, he need not invoke the kinetic theory, or the theory of probability, if the result is simply to be another average deducible by known means from such observations upon the model. It will be proper to explain that, in the paragraphs preceding Table 1, the writer did not mean that it would be of service to use the kinetic theory of liquids in the study of turbulence, but, rather, to study the rate of formation, and the distributions of sizes and rotations of eddies. Of course, this would not be as satisfactory as an exact determination of the mechanism of turbulence.

In what follows, some questions raised by the discussion will be treated in a more or less connected discourse based upon the writer's studies and experience, the intent being to illuminate the situation for the benefit of those who are interested in accurate dynamic models, especially as related to the science of hydrodynamics, which science has been involved with so much complexity, that one is very apt not to realize the true simplicity of its nature. An effort will be made to indicate the true status of these sciences of fluid dynamics so that one may have a grasp of their nature without having to unravel mathematical complexities.

The success of model engineering, like that of all other branches of engineering, depends upon the knowledge and judgment of the engineer in charge. He must be careful to see that all actions likely to have an appreciable influence have received his attention. In the gas problem (Appendix I, Table 2), it was as necessary to have regard to the elasticity (compressibility) of the fluid, as to the viscosity. In a problem of a different type, this may not be at all necessary, as, for example, when one experiments with a model conduit for the sole purpose of ascertaining the coefficient of fluid friction of the flow of a liquid which is practically incompressible. However, let the engineer in all such cases observe both the temperature and viscosity, for it is upon these that all hydraulic friction depends, as pointed out in the paragraphs in the paper preceding "Conclusions." Example 2 is a case in which the compressibility of a gas has been neglected for reasons explained in that example.

Whether or not it is necessary to take account of the elastic properties of a gas depends upon the extent to which such properties will be exercised in the model. Generalized, this is true of all properties of materials. The question of fluid elasticity (compressibility) has been stated clearly in Example 2, but it may be repeated that the elasticity of a gas may be provided for by proper control of the pressure, as was the case in Appendix I. The reason for this is that the pressure has the same dimensions as any elastic moduli. Furthermore, it is important to notice that the acceleration ratio (in a closed pipe system, for example) can be controlled by the pressure ratio, as it was in the first part of Example 3, where it was shown that the acceleration ratio, for closed pipes, could be made 125 by suitable pressure adjustments. This will be a most valuable fact to remember, when the question of the flow of a fluid in a closed pipe is involved. The principle will be very useful at times.

It may be further suggested that one need not be too punctilious about small unexplained variations in any of the coefficients and properties, because knowledge of them is limited. Moreover, as time passes, formulas will be subject to any changes which may show themselves necessary to ascending degrees of accuracy, but that has only favorable bearing on the principles of true model engineering.

(3) *Bertrand's Models.*—In 1848, M. J. Bertrand discussed Newton's theorem in a paper entitled: "Note sur la Similitude en Mécanique."[57] Of this, Routh states that the demonstration was "much improved."[58] The writer thinks the proof of the theorem was a secondary matter, and that the title of the paper shows clearly a fact of much greater importance, namely, that Bertrand had probably been the first really to read from Newton's theorem of similitude the possibility of actually constructing a dynamic model in certain cases, illustrating his ideas by proposing models of several machines, including a hydraulic turbine and a locomotive. Bertrand even mentions a substance which might create a proper coefficient of rolling friction for the wheels. Evidently, however, he did not seriously have the idea of using different materials, conditions, and artifices in the model, which would create exact similarity by suitably altering its physical properties and mechanism, because he clearly states that rolling friction requires a condition not at all realizable, that is, a coefficient reduced to one-fourth the value of the homologue. Bertrand anticipated Reech, Froude, and Helmholtz in stating Froude's law.

Bertrand's discussion of similitude, showing that dynamical equations of similar systems must be identically equivalent, seems to anticipate Stokes' discussion to the same effect in the paper on pendulums previously discussed

---

[57] *Journal de l'Ecole Polytechnique,* Tome XIX Cahier XXXII, p. 189.

[58] "Elementary Rigid Dynamics," Sixth Edition, p. 292.

(see "Analysis of Stokes' Remarks upon Similarity"), but Bertrand did not use any property equations, such as the writer's group represented by Equation (21), which are to be combined with Equations (11), (12), and (20). The writer has demonstrated by Equations (11), (12), (20), and (21) that similar systems can be constructed and set in similar motions by Newtonian model engineering, whatever may be the form and conditions of the prototype.

(4) Fundamental Corollary.[50]—The possibility of constructing a dynamically accurate model is proved by the solution of Equations (11), (12), (20), and (21) in view of the fact that Equation (21) represents as many equations as there are extra properties requiring the attention of the engineer. An important corollary may be read from the relations as follows:

> The advantages of constructing a Newtonian model to the specifications given by Equations (11), (12), (20), and (21), lie in the fact (proved by their solution) that the method depends only upon the given materials of which the structure (prototype) is composed and upon the given physical conditions (pressures, accelerations, etc.), under which the structure (prototype) operates, but not at all upon the particular type, form, or motion of the structure (prototype), or of any of its parts—the method makes the ratios of homologues independent of every physical constant and coefficient which can be associated dynamically with the structure, with its model, or with both.

This corollary is a generalization which includes, as special cases, the conditions described in the paragraphs following Equation (45) relative to the expressions "eliminating gravity" and "non-accelerated motion." In fact, the paragraphs referred to might be taken as including a statement of the corollary.

Therefore, the designer needs no correction factors for his experiments upon the model, other than the "conversion ratios," all of which may be derived immediately from the ratios L, F, and T, as found by Equations (22), (23), and (24), and as mentioned in the paragraph preceding Equation (2). Even these will be unnecessary if he uses model "weights, measures, and watches," in making his observations, as described in Example 2, and in the reference there given.[14] His model is a theoretically perfect representative of the prototype and needs no correction.

The problem thus reduces itself to the selection, or making, of suitable corresponding materials for the model, or of devising representative artifices, and to the arranging of suitable conditions under which the model must operate. These will be the principal matters requiring the intelligent attention of the engineer. The action of the proposed structure will then be determined directly by tests of the Newtonian model, instead of by unnecessary and doubtful corrections to be applied to the tests of a model of other type. For two fluids separated by gravity the corollary shows that this has no effect on similarity in a true dynamic model constructed to the specifications of the equations of similitude.

(5) Classification of Coefficients.—Coefficients cannot be understood until they are classified. For this purpose two important distinctions must be made by the model engineer:

---

[50] Also a corollary of theorem and discussion stated in *Transactions*, Am. Soc. C. E., Vol. LXXXII (1918), pp. 1187–1188.

First.—The distinction between (Class a), dimensioned coefficients, and (Class b), dimensionless, or self-cancelling, coefficients.

Second.—The distinction between (Class 1), "property coefficients," or "physical constants," or "general co-ordinates," or parameters, of a system, and (Class 2), "type coefficients," or "non-property" coefficients. (Limit coefficients are treated in Article (9).)

The importance of the first distinction lies in the fact expressed generally by the fundamental corollary previously stated, that when a coefficient is purely dimensionless (Class b), its values in similar systems will be identical. Hence, in similar systems (model and prototype), dimensionless coefficients are self-cancelling from all ratios of homologues which contain them. The model engineer, therefore, may disregard them, as having any material bearing on the construction and testing of the model. There is just one caution—care must be taken by the engineer to see that any particular coefficient in question really is effectively dimensionless.

As an example, the coefficient of sliding friction is usually considered to be dimensionless; that is, as the ratio of two forces, or as the tangent of the "angle of friction," but it is a dimensionless "property coefficient" (Class b_1). If the kind or character of either, or both, the mutually sliding materials is changed, the value of the coefficient will change correspondingly. Thus, the ratio between the "friction" and the "pressure" is determined by the value of the coefficient, which latter, however, is a joint property of the sliding materials, and, therefore, cannot be regarded as being dependent upon the ratio of friction to pressure. Hence, a theorem is established:

Theorem I.—As related to a dimensionless property coefficient (Class b_1), a material acts as a "dimension" and cannot be changed in a model without requiring a compensatory adjustment of some kind, in accord with the principles of Newtonian model engineering.

It is the duty of the engineer to see that dimensionless property coefficients have appropriate values in model and prototype.

Theorem I raises, therefore, the second broad distinction between coefficients. The important characteristic of the "type" or "non-property," coefficient, as distinguished from all others, is that its value depends only upon the type, form, and action of the particular system (either the model, or the prototype, for example) with which it is associated. It depends only upon the ratios of like kinds of numbers, including not only the ratios of like geometrical magnitudes, but also the ratios of like property magnitudes. For example, a type, or non-property, coefficient, might depend upon the ratio of two physical constants, or properties, of like kind associated with different parts of the same system (specifically, for example, the ratio of a modulus of elasticity of the steel reinforcement of a concrete beam, to the same kind of modulus of elasticity of the concrete). Hence, a type coefficient is the antithesis of the dimensionless property coefficient, in that the value of the type coefficient (necessarily dimensionless) depends upon the distribution of certain like features in a given system; whereas, the dimensionless property coefficient determines the distribution of certain like features in a given system. It appears that all type coefficients (Class 2) are dimensionless

(Class b), but that some dimensionless coefficients may be identified with property coefficients and physical constants (Class 1).

It will be very much worth while to study further the nature of type coefficients, that is, purely dimensionless coefficients (Class b_3). By the foregoing, such coefficient must be the ratio of an action (or characteristic feature) at one part of a system, to the same kind of action (or characteristic feature) at another part of the system. For example, the thermal efficiency of a steam engine is a coefficient of Class b_3, since it is the ratio of the energy delivered by the engine to the energy supplied in the fuel. In this relation, it is important to observe that this coefficient is calculable, because the dynamical and thermodynamical equations of steam engineering are very accurately known. If these equations were not known, or even only partly known, the efficiency of the engine would become a purely empirical coefficient, which could be found only by experiment, either upon a Newtonian model or upon the engine itself, or its equivalent, another engine of the same size and type operated under the same general conditions, as by use of the same kind and grade of fuel, etc.

It is important to fix clearly in the mind just why a purely dimensionless coefficient is calculable when the theory involved is complete. The reason is that the complete theory represents by equations all things of the same type, so that all the parts, motions, and performance of an engine, for example, can be determined by the equations of the associated science; that is, the output and the input, and, therefore, the ratio, are calculable. If any part, or action, of the mechanism of a given system is not correctly and completely represented by the equations of the science involved, then the theory is incomplete and the equations must fail to give a solution; in which case, some, or all, of the dimensionless coefficients of the system cannot be calculated.

Let it now be observed that when the theory of a particular system is complete and correct, all the type coefficients of the system become wholly unnecessary to the solution of any problem of the system. In other words, it is a theorem that:

Theorem II.—The necessary use of a purely dimensionless coefficient is an infallible mathematical criterion that the complete mechanism of the dynamical system is not represented by the dynamical equations in which that dimensionless coefficient must be introduced in order to effect an empirical solution.

The failure to obtain a rational mathematical solution is not due to the nature of the problem, but to the nature of the equations—they do not represent the system. The cause of every empiricism in engineering is lack of the knowledge of the corresponding dynamical mechanism necessary to furnish a sufficient number of satisfactory equations.

It will be understood, of course, that the writer does not refer to mathematical difficulties of reduction. A problem is here considered solved if the conditions of the problem can be introduced into dynamical equations (including experimentally constructed curves, or tables), which, when solved by any process (say, by trial and error) must necessarily give a theoretically exact result.

(6) *Class 1.—Property Coefficients, or Physical Constants.*—Examples of the "constants" are, the "constant of mass attraction" and the acceleration of gravity at a fixed position on the earth's surface. Examples of variables are, specific gravity, specific weight, and density—the density of a gas depends upon temperature and pressure. Among the property constants as ordinarily assumed are: (*a*) The "bi-constant" systems of isotropic elastic coefficients now in general use;[30] (*b*) the elastic moduli, pliabilities, and strains, which are functions of Coefficients (*a*); (*c*) the coefficient of viscosity; (*d*) dynamical electric and magnetic constants; and (*e*) any other fixed properties of materials, or conditions under which similar structures must work. Capillarity and surface tension belong together. Observe that Poisson's ratio, which is a dimensionless number, classed as b_1, may be derived from any two independent elastic constants, or moduli of elasticity.

Coefficients of Class 1 measure the dynamic physical properties with which the model engineer is principally concerned. An improper value of any coefficient of this class in a model of a proposed, or existing, structure would destroy the similarity in a measure dependent upon the discrepancy. It is by proportioning correctly each of the coefficients (Class 1) in the model, that the engineer imposes the conditions of true similarity. In this sense, the scale-ratio may be treated either as a physical constant to be suitably "adjusted," or as an arbitrary constant against which all other affected property constants must be matched in proper relation. By allowing the scale-ratio to be somewhat dependent, it may render the use of certain materials, or other conditions, permissible, when that is desirable. All such adjustments and requirements are properly indicated by Equations (11), (12), (20), and (21). Any dynamical science depends upon properly determined (hypothesized, or experimentally established) Class 1 coefficients which are entirely independent of size, form, and action, of the particular apparatus involved. They are postulates of the science, as, for example, the coefficient unity in Newton's formula, "force equals mass times acceleration" (that is, $F = kma$; $k = 1$); or the coefficient of viscosity as hypothesized by Newton, which solves all problems in pure viscous flow, but fails completely in problems of turbulence, for want of knowledge of its complete functionality.

(7) *Class 2.—Type Coefficients, or Non-Property Coefficients.*—All these coefficients are dimensionless, and may be better illustrated under Class *b*. (See remarks under "Classification of Coefficients.") Confusing the idea of the two different values of the same type coefficient which generally result from two different adjustments of general conditions of operation of the same mechanism, with the idea of two values of the same type coefficient for two distinct but similar mechanisms under similar general conditions of operation, has been the most fruitful source of misconceptions of true Newtonian models. In the former case, the two values are generally different. In

---

[30] One can find no better assemblage and elucidation of the principles of elasticity than that given by Professor Bateman under the title, "Elasticity," in the Fourteenth Edition of the Encyclopaedia Britannica; another source of first importance, is Pearson's edit of Todhunter's "History of Elasticity;" Appendix *b*, on notation, gives a foundation.

the latter case, they are necessarily the same. These two conditions occur simultaneously in Newtonian similarity.

Hence, in considering the theory of models, no attention need be given to the variations of type coefficients in one and the same system, since no conclusion as to the accuracy of models can be reached thereby. This results most clearly from the equations of similitude and their fundamental corollary, as previously stated. Every model engineer must understand this matter most fully, or he is certain to waste an immense amount of time in redundant labor and effort. For example, it avails naught to think of the size, form, or service, of a reinforced concrete beam as having any bearing on the specifications for the construction of a true dynamic model because a Newtonian model depends only upon materials, physical constants, and the general conditions for the service.

(*8*) *Class b.—Dimensionless, or Self-Cancelling, Coefficients.*—The case of a dimensionless property coefficient has been illustrated under Article (5), "Classification of Coefficients," which includes a cautionary remark calling attention to the importance of knowing that such coefficients had been made effectively dimensionless. As typical of purely dimensionless coefficients, Poisson's ratio in elasticity, and the coefficient of hydraulic "skin friction" in the presence of turbulent flow, may be mentioned. Poisson's ratio is classed here because it is a type coefficient of a material, thus becoming a type coefficient of any system containing the material.

The fact that Poisson's ratio is calculable by the elastic formulas, whereas, the coefficient of skin friction for turbulent flow cannot be calculated by the formulas of hydrodynamics and aerodynamics, is a positive mathematical criterion that the mechanism of elasticity is well represented by the elastic equations, but that the mechanism of turbulent flow of fluids is not represented by the present equations of fluid dynamics. When the mechanism of turbulence and eddying motion is properly introduced into hydrodynamical and aerodynamical equations, the dimensionless coefficients of hydraulic skin friction will become calculable in either science, just as Poisson's ratio is now calculable in elastic problems when the values of the two necessary independent elastic property coefficients are given.

Let it be observed that no dimensionless coefficient of hydraulic skin friction is necessary in the case of pure viscous flow, regardless of the nature of the conduit. This is the rigorous mathematical criterion showing that the mechanism of pure viscous flow is well represented by the fluid dynamics arising from Newton's definition of the coefficient of viscosity.[61] That one definition expanded the theory of hydrodynamics (aerodynamics included) so as to cover completely the field of pure viscous flow. Any difficulty of solving a problem in pure viscous flow is solely one of mathematical reduction, and not one of incomplete dynamical equations. Thus, the question suggests itself, may not some simple definition, or hypothesis, now carry the theories of fluid dynamics throughout the domain of turbulent flow? (See Article 14.)

Poisson's ratio and the coefficient of hydraulic skin friction are both dimensionless, and, therefore, both are self-eliminating from problems of true model construction. The model engineer need have no regard for them at all when he designs his model according to the requirements of Equations (11), (12), (20), and (21). When the property coefficients of the materials and conditions of the model are properly adjusted, by means of these equations, all purely dimensionless coefficients become at once automatically adjusted in their proper ratios, expressing what Professor Bateman would undoubtedly term "proportions of the system," calculable from simpler component proportions. As these dimensionless coefficients depend only upon the types of their respective systems, and as these systems have been made dynamically similar in the Newtonian sense by the model engineer, the values of any two corresponding, or homologous, dimensionless coefficients must be identical. Thus, the model engineer makes no tacit assumption of similarity. He simply creates similarity by the engineering control of all acting dynamical conditions. These dynamical conditions are very much limited in number in fluid dynamics, just as the generalized co-ordinates of problems in rigid dynamics are very much limited in number by the Lagrangian method. Even if R (scale) is counted as a property, or condition, the greatest number of such constants which the writer has encountered in the problems given, is only six. Finally, therefore, these dimensionless coefficients cancel from all ratios of homologues which contain them. They disappear from all relations between the model and its prototype. Hence, all efforts to treat such coefficients individually are wholly redundant in Newtonian model engineering.

---

[61] "Principia" (1687), Hypothesis, Lib. II, Sec. 9.

One more suggestion may be appropriate. Some physical coefficients have not been properly determined. It may have been imagined necessary to place them in a term the dimensions of which were such that the whole term gave the measure of the effect which it was desired to represent. The failure of the experimenter to do this with effect may mean that he is doing it with a single coefficient when more than one is needed.

(9) *Limit Coefficients, Arbitrary, or Natural.*—This type, as an arbitrary limit coefficient, is well represented by an arbitrary limiting safe stress in a material, while the natural limit coefficient would be represented by a modulus of rupture. If a model, for example, is to have a correct modulus of rupture, the material of the model must have the property of rupturing under the proper "model stress." In the usual method of the past this condition has frequently been produced by an artificial loading, with use of the same materials in the model and structure; but the writer believes this results merely by following precedents, and that true models will be found much more satisfactory for many purposes.

After all, Newtonian model engineering differs in no essential respect from any other branch of engineering—it advances along its own special path. In many cases, of course, the character of the research will not be concerned with limit coefficients, except to see that stresses are confined to proper values.

(10) *Newtonian Cancellation of All Coefficients.*—In the broadest view, the type coefficients, or "proportions," of a system must include every kind of measurable quantity which can be associated with different parts of the same system. Any dimensionless term, or factor, containing a dimensioned coefficient is nothing but the ratio of the coefficient to its own dimensions,

and, therefore, may be looked upon as a type coefficient, or proportion, of the system.

If the dimensioned coefficient is not divided by its own dimensions, it may be both multiplied and divided by them so as to produce a term or factor containing it. Reynolds' number is a good example. It represents the ratio of length and velocity to kinematic viscosity. Thus, the engineer, in framing his particular Reynolds' number, selects the product of a certain length and velocity to compare with the virtual product of length and velocity, as expressed by the kinematic coefficient. This is truly a type factor of the system, corresponding exactly to the purely dimensionless self-cancelling Type *b* coefficients previously described. Thus, these corresponding type factors, or proportions, of the respective similar systems (just typified by Reynolds' number) must have identical values in systems which have been made dynamically similar by the model engineer. Hence, they must cancel, and with them the dimensioned coefficients which they contain, when comparing a model with its prototype. Therefore, since it has already been shown that all dimensionless coefficients cancel, in Newtonian dynamic model engineering, it follows that:

Theorem III.—All coefficients of every type vanish from the ratios necessary to connect experiments upon the model with corresponding experiments upon the prototype. And, a curve giving the value of an experimental dimensionless coefficient for the model plotted against the value of a corresponding dimensionless number appurtenant to the model, will coincide with the corresponding curve plotted against the same dimensionless number for the prototype.

This, of course, is just what is proved generally by Equations (11), (12), (20), and (21).

An excellent illustration of the foreging proposition is the following reasoning taken partly from letters of instruction written by the writer in July, 1928: In the case of turbulent flow in a pipe or open channel, the friction gradient may be written:[62]

$$\frac{H}{l} = \left(f' \frac{4\nu}{rv} + f \right) \frac{v^2}{2rg} = \psi \ldots\ldots\ldots\ldots\ldots (122)$$

in which,

r = hydraulic mean depth;
l = length of, or reach of, the pipe or channel;
v = actual mean velocity in the cross-section;
f = dimensionless coefficient of loss due to turbulence;
f' = dimensionless friction factor due to viscous resistance;
ν = kinematic coefficient of viscosity; and,
H = friction head lost in the length, l.

Equation (122) exposes the fundamental nature of the whole theory of similitude, when Newton's theory of similarity for systems of particles in space which re-act upon one another only during the moments of collision, is extended so as to include the possibility of viscous reactions during a finite time interval of contact.

---

[62] *Transactions*, Am. Soc. C. E., Vol. 94 (1930), p. 840, Equation (10).

Let it be required to examine Equation (122) to see what conditions are necessary to secure both geometric and dynamic similarity of model to prototype: First, by definition, geometric similarity requires that the longitudinal slope, $\frac{H}{l}$, in prototype, and $\frac{H_m}{l_m}$, in model, at homologous points, shall be the same. In fact, it is a condition imposed in the problem, that all pairs of homologous linear dimensions, whether they are longitudinal or otherwise, shall be in one fixed ratio, namely, the scale ratio of the homology; and, second, dynamical similarity requires that all pairs of homologous forces shall be in a fixed ratio, be similarly situated, and be similarly directed. This is the force ratio of the homology which was first described by Newton; for example, any pair of homologous centripetal forces acting upon a pair of homologous masses so as to cause them to move in geometrically similar curves. Since the intensities of these forces are measured by the respective centripetal accelerations produced by them, and since the accelerations are, respectively, $\frac{v^2}{r}$ and $\frac{v^2_m}{r_m}$, it must follow that the forces are as $\frac{mv^2}{r}$ to $\frac{m_m v^2_m}{r_m}$, in which, m and m_m are the homologous masses involved. Hence, if $\frac{F}{F_m}$ is the force ratio:

$$\frac{F}{F_m} = \frac{m\frac{v^2}{r}}{m_m\frac{v^2_m}{r_m}} = \frac{mg}{m_m g_m}$$

In the particular case when $g = g_m$, as when both model and prototype, or homologous parts of them, are exposed to accelerations due directly to the earth's gravity (the pipe is not full of water under pressure, or is there a water-fall in the river), it is at once apparent from the foregoing equations, that $\frac{v^2}{r} = \frac{v^2_m}{r_m}$, or that all homologous accelerations must be equal. However, this is only a special case of acceleration. In the general case, as in similar pipes running full of water under pressure, there is still required an acceleration ratio, in order that dynamical similarity may exist, as in the first part of Example 3, where it is shown that the acceleration ratio must be 125 to create similarity in the horizontal discharge; and as it was shown in Appendix I that just such an acceleration must exist under pressure when it is required to make the same fluid represent itself as a model of itself.

It is thus clear that a necessary condition for similitude is that ψ shall have the same value at any pair of homologous points, the respective dimensions introduced being also correspondingly homologous. Of course, ψ will not necessarily have the same value at all points in a natural stream, nor in a model of a stream. It is also very clear that this condition can be secured by imposing in ψ the conditions that, $\frac{rv}{\nu} = \frac{r_m v_m}{\nu_m}$ and $\frac{rg}{v^2} = \frac{r_m g_m}{v^2_m}$.

Hence there are two equations for the two quantities, $\frac{r}{r_m}$ and $\frac{\nu}{\nu_m}$, from which Equations (16) and (17), may easily be derived expressing the necessary relation between L, G, and ν to be satisfied for conditions of similarity. In this reduction the hydraulic radius, r, evidently has the dimension of length, so that $\frac{r}{r_m}$ is the linear scale ratio.

It is interesting to notice that the condition expresses the equality of homologous values of both Reynolds' (Stokes') number, and Froude's (Newton's) number, in that most interesting relation which the writer has pointed out as expressing the necessary condition that the flow in a viscous fluid may not be such as to destroy Newtonian similarity. In other words, to make the illustration complete, equal homologous values of ψ express Equation (11) and equal homologous Froude's (Newton's) numbers express Equation (12). Equation (20) has been obscured by making the problem kinematical, as remarked under the heading, "Properties of Materials," in the paper, the implication being that $\frac{F}{\rho} = \nu^2$, since, in this case, all forces are to be measured per unit density; equal Reynolds' (Stokes') numbers express Equation (21), or the condition that the dynamical physical property, viscosity, has been properly imitated in the model. This may be verified by confirming references to Equations (11), (12), (20), and (21), under, "Summary of Equations of Similitude," Table 3, as to the character of the equations, and Table 1, after dividing the line containing F throughout by ρ.

(11) Type Factors and Form Factors.—When like magnitudes are merely geometrical, as lines, areas, and volumes, pure dimensionless coefficients (Type *b*), are commonly known as "form factors" ("shape factor" of a helical inductance coil); but when such proportions of the system include, as in the writer's paper, the ratios of all kinds of denominate numbers, velocities, masses, forces, physical constants, etc., the name, "type factor," may be sufficiently broad to describe the ratio; for a ratio such factor must be, since it is the product of two or more other ratios, or merely itself a ratio of simple dimensions.

A most important example of a type factor is the recognized permanency, as to time and place, of the ratio of a "statistical," or average velocity, at a point in any stream, conduit, or pipe, as compared to the mean velocity as to time and position in any given cross-section of the flow, which permanency occurs under permanent fixed conditions which govern the flow, even though the resulting flow may be turbulent and, therefore, unsteady at every point of the channel. Under these conditions, the time-average mean-velocity throughout any given cross-section becomes a kind of "generalized coordinate" (Lagrangian method) of the flow in the channel, thus reducing enormously the number of variables involved. The permanence of values of the "statistical" type-factors of a given system, or mechanism, under fixed general conditions, may be referred to as the "principle of fixed average type-factors." The values of homologues are the same in two similar systems.

That these variations of mass-motion can be treated by the laws of probability, much as Maxwell applied his law to the distribution of molecular velocities in the kinetic theory of gases, would appear to be a certainty. The average mass-motion at a point, as thus defined by implication, might be compared with what Jeans calls "mass-motion,"[63] as distinguished from molecular motion, while the variable component of the mass-motion (as here defined) would find its analogue in Jeans' molecular motions. The difference between the mechanism of a continuous fluid and that of an aggregation of individual particles must not be disregarded, however, although the mechanism of the kinetic theory is, as Sir James states (page 10), so "extremely fine-grained" that it is practically indistinguishable from that of a continuous medium.

(*12*) *No Tacit Assumptions in Newtonian Model Engineering.*—In extending the ideas from a simple form-factor to a type-factor, as just defined, it must be carefully observed that the values of any two homologous type-factors, as between two geometrically similar systems, will not be identical unless the systems are similar in the Newtonian dynamical sense. Newton's theorem postulates initial similarity; therefore, it is as much the engineer's duty to see that similarity exists initially as it is to construct a model which is potentially similar to its prototype in the dynamical sense. When this requirement is satisfied, the systems will be similar and will move with similar motions, if the model engineer has properly applied Equations (11), (12), (20), and (21), to determine the essential characteristics of the model.

The establishment of initial similarity by the model engineer is by far the least of his difficulties. When he is designing a structure of any kind, the motions and actions of the structure are completely in his control. He can make them whatever he chooses, just as the entire architectural design of a structure is at his disposal. He simply constructs and tests his ideas in miniature, by means of his model, perhaps re-designing his model until it and its operation are satisfactory. The re-design would not necessarily be difficult, since, by the fundamental corollary, one determination of materials, properties, and conditions for the model is sufficient for all forms of the same general type. Thus, the materials for a former model will serve in the next of the same type. The problem then reduces itself to building the structure, and starting it under conditions of similarity. By the equations of similitude (Equations (11), (12), (20), and (21)), or their corollary, the resulting motions, however complex, must be similar to homologues in the model. This has been most clearly illustrated in Example 2, as in the explanation of the method of starting similar weights suspended by similar springs. If one desires an experimental demonstration, no better one can be found than Stanton and Pannell's experiments, previously referred to so many times.[64] In such experiments, Newtonian similarity surely establishes itself when conditions are properly adjusted by the engineer.

There are also to be found, among about one thousand papers on viscosity and related action, which have appeared during the fifteen years (1916–1931), a number of other verifications, precisely to the same effect. Among all these, however, Stanton and Pannell's paper still stands as the outstanding example.

---

[63] "Dynamical Theory of Gases," Chapter VII, by Sir James Hopwood Jeans.

[64] Collected Researches, National Physical Laboratory (British).

In this connection there is one caution to be heeded. Changing the shape, including roughness, of the boundary will change coefficients, notwithstanding the fact that the coefficients of similar systems must be equal; that is, must change equally, when changes occur. It is exactly here that so many have failed to make the distinction between the change in value of a coefficient in a particular system, and the continued equality of two homologously changing coefficients in different, but similar, systems.

It is scarcely necessary to give an example relative to the establishment of initial similarity between rigid systems, since the proof for fluid systems must apply with greater force.

(13) Generality of the Newtonian Method.—It appears clearly that the model of the tank discharging mercury (Example 3) differs in no essential manner from the model of a ship, since, by the fundamental corollary, the form of the model does not affect the specifications for the materials of the model, nor the general nature of the conditions for testing the model. Of course, it will be understood that different materials may be equivalent, as model materials. Hence, there is considerable latitude for the use of judgment and expediency, even after the general conditions for the model have been fixed upon. This is an aid rather than a hindrance.

For another instructive illustration, one might imagine the model tank with mercury, and its prototype tank with water, to be converted into corresponding ships (tugboat and its model, say) by actual wrecking and reconstruction in a series of "similar" operations. No change at all would be required in materials, fluids, and general controlling conditions. The only necessary and sufficient condition is that the model retain its Newtonian character of dynamic similarity, which it must do, since the engineer has provided the proper materials and conditions of operation.

(14) The Nature of Turbulent Flow.—The writer desires again to refer to his discussion[65] of the paper by Messrs. Nagler and Davis, which is to be considered as part of this closure. In this discussion the general nature of pure viscous flow, and also of turbulent flow, is explained.

The prime cause of every eddy is viscosity, because, if the fluid were not viscous, it would be frictionless, and no eddy could be formed. The prime cause of the ultimate stilling of every eddy is viscosity, because if the fluid were not viscous it would be frictionless, and by Newton's first law of motion, there would be nothing to stop the rotation. There can be no other kind of loss of energy, since it was clearly pointed out that there can be no such thing as loss due to impact, either of fluid upon fluid, or of fluid upon a solid, or the reverse.

It does not seem that these facts have been consistently digested, even by the foremost scientists, because the conclusion is irresistible that all hydraulic friction of every possible type depends primarily and ultimately upon viscosity, whereas it has been asserted by the most profound authorities that turbulent friction is independent of viscosity (see paragraphs and footnotes immediately preceding "Conclusions").

---

[65] *Transactions*, Am. Soc. C. E., Vol. 94 (1930), p. 842.

If all frictional loss is due to viscosity, then there must be an actual mechanism involving viscosity, which, when revealed, will furnish a perfectly rational system of dynamical equations sufficient to solve any problem of actual fluid flow without the necessary use of empirical type coefficients, as discussed under "Dimensionless Coefficients (Type *b*)."

While the writer has not yet attempted to set up this rational system of equations, he does believe he can state the general nature of the postulates which must be made to furnish the footing of the complete science of dynamical viscosity.

When one is concerned with the physical properties and constitution of matter, and when it is desired to set up their characteristics upon an ultimate rational basis, the question of the divisibility of matter is among the very first to arise. Perhaps the next question is as to the divisibility of energy. Chemists, physicists, and mathematical savants have long since satisfied the scientific world that matter is discontinuous, and that its ultimate form is divided progressively into molecules, atoms, and electrons; although it must be admitted that there is yet much obscurity surrounding the ultimate nature of matter, and energy too, as will be apparent after consulting the references previously made. Let it be observed that, as a consequence, the postulation of the divisibility of matter into equal elements must carry with it the elementary subdivisions of energy; for energy must exist only as associated with the ultimate elements of matter in their mechanical relations.

The question of procedure arises. Shall it be by the dynamics of continuous, or discontinuous, matter? Moreover, is there any particular mathematical form which will have an advantage over all others? What place has the kinetic theory of gases? In particular, shall Newtonian dynamics, which has furnished, and still furnishes, the foundation for all studies in mass-motion (even for studies by those who seem to think Newton's laws no longer have any applicability) be discarded?

The writer answers these questions by saying that he thinks there is no reason at all why the engineer should discard Newtonian dynamics, nor any reason why the kinetic theory of gases should enter into dynamical problems of engineering, unless the constitution of matter should enter into the problem in some vitally important manner. The reason for this conclusion is that both the constitution of matter and the kinetic theory of gases are, as pointed out for the kinetic theory by Jeans (see Article (11), "Type Factors and Form Factors"), so very "fine-grained" that, for practically all engineering problems, matter may be treated as being absolutely continuous, but, of course, this must be carefully borne in mind as a fundamental hypothesis of modern engineering. It is too frequently the case that a fundamental hypothesis fails to be understood, because the familiar use of it without thought is so frequent that one might even find difficulty in explaining his equations fully, if asked to do so. The nature of the kinetic theory of gases in relation to viscosity will be discussed subsequently. Until then, the discourse will proceed on the Newtonian basis, and upon the hypothesis of continuous matter and continuous energy. Even then, it will be shown that Newtonian dynamics is pre-eminently the base of the kinetic theory of viscosity.

Transposing l, Equation (122) is the writer's general formula for the flow of any viscous fluid in any type of channel, or pipe, open or closed.

The factors, f', f, and the entire quantity in the parenthesis are each dimensionless, Type *b*, coefficients of friction. The acceleration, g, need not necessarily be that due to gravity, but may be the actual effective acceleration controlling the flow, as in the case of a fluid in a closed pipe running full under pressure. One must regard the fact that an acceleration must control every case of fluid motion. When that acceleration was not gravity, and when the flow was supposed to be uniform, it has been a frequent mistake to lose sight of the necessary acceleration to introduce the flow, maintain it, and especially to generate and maintain turbulent conditions. Equation (122) is perfectly general. It covers every case of fluid flow. It represents the resistance on the side of a ship, or on the sides of one of Froude's boards.

It is true that a coefficient had been proposed by D'Aubisson in the form, $a + \frac{b}{v}$, but in that coefficient no regard was had for viscosity, nor were the component coefficients introduced in the same manner used, and for the same reasons given, by the writer when forming the coefficient for Equation (122).

To get a little nearer to a practical understanding of the writer's coefficient (F, say) within the parenthesis of Equation (122), and to make those who deal with pipes more at home, let R_1 be the resistance per unit area of wet pipe skin, and d, the diameter of the pipe, so that $d = 4r$. Then, the following reduction is easily traced:

$$R_1 = \frac{\rho g H (\pi \frac{1}{4} d^2)}{\pi d l} = \rho g \frac{1}{4} \frac{d}{l} \left(F \frac{l}{\frac{1}{4} d} \frac{v^2}{2g} \right) = \tfrac{1}{2} F \rho v^2$$

or,

$$\frac{R_1}{\rho v^2} = \tfrac{1}{2} F = \tfrac{1}{2} \left(f' \frac{16\nu}{dv} + f \right) = \tfrac{1}{2} \left(f' \frac{4\nu}{rv} + f \right)$$

which shows that the familiar dimensionless coefficient of hydraulic skin friction, represented by F, is precisely double the usual coefficient of skin friction in the familiar formula, $R' = f_2 \rho A_p v^2$, in which, A_p is the area of the skin, and f_2, the coefficient. It is also important to notice that $\frac{1}{2} F \ (= f_2)$ is the ordinate used by Stanton and Pannell in the plottings of their experiments previously, and hereafter, mentioned.

It is most important to notice carefully that Equation (122) is a perfectly rational formula, complete in every respect but one; and that the final perfection and corresponding alteration of its form would in no sense invalidate it as an exact theoretical representative of the flow in the pipe, or other channel. The only difference would consist in the substitution for f' and f of terms containing what the writer will call, and what Lagrange perhaps would have called, "generalized co-ordinates of turbulence." These co-ordinates would include all those dimensionless type-factors of the system which in any way affect the delivery of the kinetic energy of the mass-motion to the "energy-sinks" of viscosity as it operates in turbulent motion.

The words, "source" and "sink," have usually been employed in hydrodynamics to indicate points, lines, areas, and volumes, which may be considered as productive or destructive of the fluid, to suit the assumptions of the problem. The words are used herein to indicate the transformation of energy from one form to another. In particular, it should be stated that a "viscous-sink" would refer to the loss of energy by reason of the conversion of kinetic energy of flow into heat by viscous friction. Treatises on fluid dynamics give formulas for the dissipation of energy by viscous flow. In reality, all the energy is conserved by being converted from kinetic energy of flow to kinetic energy of molecular motions; but, as a factor in the dynamics of flow, such viscous friction must be regarded as an "energy-sink." Any treatment of viscosity by the kinetic theory of gases which does not consider the increase in molecular motion due to the transformation of mass-motion into molecular motion must be decidedly incomplete.

The reasons why Equation (122) is perfectly rational in form have been fairly well explained elsewhere,[66] but it is very difficult to explain a generalization which involves a wide variety of conditions. Perhaps it will be possible to illuminate the subject somewhat more by explaining the nature of the principal experimental facts which can be cited in support of the formula. The first of these has already been explained as a "most important example" in Article (11), "Type Factors and Form Factors." This is nothing but an application of the well-established fact that the average velocity of a fluid at any point in the cross-section of a steadily flowing turbulent stream, bears a fixed ratio to the mean velocity in the cross-section, when the times of observing velocities have been of sufficient duration to furnish reliable averages.

This same permanency of ratio must hold for the time-average velocity at any point of an average eddy which exists at any point in the cross-section of the stream. The eddies move with translation, of course, but there is a "statistical" eddy at any given point. The detailed dynamics includes the translatory motion of the eddy, but it is not necessary to consider this here, if one considers average conditions rather than details.

Perhaps the most important experimental facts to consider, relative to the generation of an eddy, are: Viscous friction depends upon the rate of shear, or the transverse velocity gradient; this rate may be increased up to a certain critical limit (determined by Reynolds' critical velocity, and the distribution of the previously existing pure viscous flow); beyond this limit the flow breaks into turbulent, or eddying, motion.

In view of these established facts, one might think that some kind of bond, or elastic stress, is suddenly broken when the rate of shear (strain) exceeds the critical limit; but it is well known in hydrodynamics that pure viscous, or laminar, motion can be resolved into three components:[67] Translation, rate of shearing distortion, and rotation; as long as these three components exist unchanged, laminar motion results. Furthermore, in laminar flow, as a simple example, the distortion means a rate of stretch along an axis, and a rate of contraction along another axis perpendicular to the first, these axes being at an angle of 45° with the direction of laminar motion. Therefore, calling to mind Maxwell's definition of the "rate of relaxation" of elastic stress,[68] the writer concludes that, instead of the breaking of an elastic bond by exceeding the critical rate of strain, the contrary is the fact, and that it is the excess of the rate of strain over the rate of relaxation that causes the elastic bonds to hold, and, hence, to suppress, or reduce, the rate of deformation, thereby permitting the component of rotation to prevail in producing a rotation of the element as a whole. Poisson, it appears, also had similar ideas, deducing equations of viscous flow from equations of elasticity by assuming strains to relax as fast as they were generated.[69] The same equations were also deduced by Navier in 1822.[70]

---

[66] *Transactions*, Am. Soc. C. E., Vol. 94 (1930), pp. 834-841.

[67] "Hydrodynamics," by Lamb, Article 30.

In illustration of this action of a viscous fluid, one may experiment upon a very soft piece of candy (say, 3 or 4 lb.) on a marble slab. If the candy is picked up by the hand very slowly, the relaxation of the candy will cause it to stretch and pull apart. If it is picked up with sufficient celerity, the candy will hold together as a more or less rigid solid, and it may be transferred to a plate with very little distortion.

Thus, it is seen that the component rotation of the viscous fluid is left to produce rotation when the rate of shearing strain exceeds the rate of relaxation, and that these persisting elemental rotations will combine to form eddies and turbulence in place of the previously existing laminar flow. Of course, the principle is precisely the same, whatever the nature of the initial purely viscous flow, whether laminar, or even existing initially in a previously formed eddy. Eddies and vortices are usually screw motions.

Whether the motion is laminar, or rotary, viscous friction can occur only through the mechanism of shearing distortion, and in no other way, excepting some other possible action of cohesion, but the writer is inclined to believe that this will not be found to be effective.

The entire question of hydraulic friction thus reduces to the study of pure viscous friction, as it operates in directly resisting the flow; as it operates in the formation of eddies; and as it exists in the eddies, after they are formed. There can be no doubt that the whole eddy system in any turbulent flow is produced primarily by viscous action.

Of course, the pipe, or other envelope, will have a great bearing upon the distributions of viscous action. The shape of the envelope, for example, will have a great influence upon the form of the flow, and, therefore, upon the relative motion which must occur between neighboring filaments of the fluid. In other words, the coefficient of viscosity will be combined in some determinate manner with all the type-factors of the channel (including all dynamic type-factors), to form the two dimensionless coefficients, f' and f.

---

[68] "Collected Papers," Maxwell, Vol. II, pp. 31, 39, *et seq;* also ***Philosophical Transactions,*** Vol. CLVII, 1866.

[69] L'Ecole Polytech. (1829), tome XIII, p. 139.

[70] ***Memoires*** de l'Acad. des Sciences, tome VI, p. 389.

This will be entirely independent of that factor in F which contains the kinematic coefficient of viscosity in the numerator. There will be independent necessity for f' and f, since one relates to pure viscous action directed against the flow, as in the boundary film, while the other relates to pure viscous friction (the only kind there can be) within the eddies.

The pure viscous action directed against the flow must vary as the rate of distortion. Hence, because of the fixed relations of all average velocities at given points to the mean velocity in the cross-section, previously mentioned, the resulting resistance from this cause will be in proportion to the mean velocity in the cross-section directly.

The rate of eddy generation is produced and maintained by the action of the combined mechanism in converting mass-motion into turbulence. As in the preceding case, the rate of conversion must be proportional to the velocity at the point of conversion, and, therefore, to the mean velocity in the cross-section; but, in this case, the entire velocity changed to eddy motion must finally be dissipated by the viscous action in the eddy, so that the resulting loss of energy is proportional to the square of the velocity at the point of diversion, and also, therefore, to the square of the mean velocity in the cross-section.

Hence, there is but one natural mechanism for the conversion of fluid motion into heat; but this mechanism operates under two regimens, one of which is pure viscous resistance in "pure viscous" flow, and the other is pure viscous resistance within an eddy. Consequently, turbulent motion is only an intermediate stage of motion occurring temporarily in fluid flow, as between effective mass-motion and the ultimate molecular heat motions of the fluid. It is the result of one of those "irreversible" processes of Nature. It is merely one progressive stage in the universal degradation of energy enunciated by Carnot in the second law of thermodynamics, which was first understood and built into the foundation of the science of energetics by Lord Kelvin. Had it not been for the combined greatness of intellect and magnanimity of the greatest of philosophers it is a question whether Carnot's theory would be known to-day by his name.

Because there are only two regimens of viscous friction, there can be but two terms in Equation (122), one of which must clearly be proportional to the first power of the mean velocity when there is no turbulence ($f = 0$), and the type-factor, f', is constant (unity) by reason of the permanence of relative conditions in pure viscous flow (that is, the theory of pure viscous flow is perfect, and complete); the other of which must as clearly be jointly proportional to the square of the mean velocity and the whole type-factor, F. As between similar systems, F is constant. For changing conditions in a given system, F is variable. If the conditions in two systems are changed, but in such manner that the two systems remain similar, then the two coefficients, F, change equally and remain equal. Wave-making is distinct.

It must not be forgotten, however, that Equation (122) cannot be considered a complete formula of hydrodynamics, or of aerodynamics, until the type-factors, f' and f, have been replaced by effective generalized co-ordinates. Hence, in closing this section, it cannot be emphasized too strongly, that engineers who would make discoveries by means of Newtonian models, should begin the experimental study of turbulent flow at the very earliest opportunity. Success in this respect will enable the engineer to fill the great lacuna of fluid dynamics, which is treated in the following section.

(*15*) *The Great Lacuna of Hydrodynamics and Aerodynamics.*—When the general nature of the particular mechanism involved in a particular apparatus (mechanism covering a particular class of machines and motions of parts and fluids) is correctly postulated, the general dynamical equations may be made to represent the mechanism by introducing the terms, or equations containing the new property coefficients (Class 1) which have been defined by the postulates. This creates a new science, or branch of engineering analysis, which thenceforth becomes capable of solving all problems that involve nothing beyond the general nature of the mechanism defined.

Hence, the equations established by the postulations of the science will suffice to determine all the dimensionless type-coefficients of any particular apparatus of the class defined, thus rendering the use of purely dimensionless coefficients, as in empirical formulas, wholly redundant, since the solutions may proceed without explicit evaluation of the unnecessary coefficients.

This was illustrated by previous reference to Newton's creation of the dynamical science of pure viscous flow, solely by the definition of the coefficient of viscosity (Class 1), and by nothing else except his own laws of motion (see Class *b*, Coefficients).

It is now to be shown that no such science of turbulent fluid motion has yet been created, and that, in this respect, all the present sciences of fluid dynamics are wholly barren. To this end it will be highly instructive to make an exploration into these barren domains of dynamics to gain an exact view of the nature and extent of this great lacuna, at least as it relates to turbulent fluid motion:

First.—Solve a problem in "pure viscous flow;" say, that of Article 318, Article 319, or Article 320, Third Edition of Lamb's "Hydrodynamics" (the writer's solution of the pipe problem, Lamb's Article 319, is given in very simple terms elsewhere.[n] The other two problems may be solved with corresponding informality). Theoretically correct results are immediately and very easily deduced in each case in terms of a few working conditions (time included), boundary conditions, limits of integrations, and property coefficients (viscosity, density, etc.), results which are consequently independent of any system of co-ordinate axes. These given data become ultimate true "generalized co-ordinates" of the motions of the millions of particles involved, provided that care has been taken to see that these "co-ordinates" are independent of each other. Each such "co-ordinate" represents a "degree of freedom," which is nothing more than a degree of ability to change the form, or type, of the system and its motions. By means of them (or an equivalent system), a calculation may be made which will determine the instantaneous position of any given particle, or its path.

Observe that among these "co-ordinates" may be found the property-coefficient of viscosity, μ; another is the physical-constant, ρ, or density.

---

[n] *Transactions*, Am. Soc. C. E., Vol. 94 (1930), p. 835.

No non-property coefficients (dimensionless type-coefficients) are explicitly involved, as necessary to the solutions. If they were, the successes of the solutions could not possibly be ascribed to the accuracy of the science of hydrodynamics, for the solutions would have been wholly empirical as a consequence, to the extent that the hydrodynamic formulas would have been of no theoretical service.

Second.— Let one then attempt to solve a simple problem involving turbulent flow; say, the problem of turbulent flow in a pipe as discussed by Lamb.[72] A result is even more immediate than before, but only by use of a purely non-property, dimensionless, type-coefficient of "skin friction" (the usual empirical coefficient), of which the value cannot be found by any known hydrodynamic method, or formula.

It is this last fact which suddenly discloses the nature and extent of the great lacuna of hydrodynamics and aerodynamics, as a gaping void which the writer believes has not been fully appreciated hitherto, especially by hydraulic engineers. This great lacuna can now be bridged with theoretical accuracy only by Newtonian model engineering, or by constructing the full-sized structure, and then testing it in place. Under "Turbulent Motion,"[73] Lamb states:

> "It remains to call attention to the chief outstanding difficulty of our subject. * * * It must unfortunately be confessed that the theoretical explanation of the instability of linear flow under the conditions stated, and of the manner in which eddies are maintained against viscosity, is still somewhat obscure. * * *"

These words are too mild. There is a complete failure of hydrodynamics and aerodynamics, for want of a correct theory of the mechanism and functions of viscosity, and perhaps, also, of another property, say, cohesion, in producing turbulent motion. It may be remarked, that determination of the rate of eddy generation will be sufficient to determine all the loss due to pure turbulence, since the kinetic energy of the eddies is all rapidly dissipated in the form of heat motions. By the "principle of fixed average type-factors" (see Article (11), "Type Factors and Form Factors"), the loss of head due to pure turbulence will appear in a term by itself which is proportional to the exact square of the mean velocity in the cross-section, or its equivalent. Any direct loss due to viscous action, as in the boundary film, will be directly proportional to the mean velocity, and must also appear in a term by itself, when the dynamical equations are finally perfected.

In Article (14), "The Nature of Turbulent Flow," the writer has endeavored to give his views as to the nature of the mechanism which must be expressed mathematically, in order to establish rational turbulent fluid dynamics. It should be mentioned that correct fluid dynamics cannot be created by use of the present hydrodynamical and aerodynamical equations. They will be barren for this purpose, until emended on the basis of enabling postulates, perhaps creative of new property coefficients.

(*16*) *Confusing Laws.*—It seems that the first positive step toward founding Newtonian model engineering was taken by M. J. Bertrand in his paper previously quoted and cited. After mention of Newton's similitude Bertrand refers to one of Galileo's dialogues in which the latter called attention (probably for the first time) to the fact that a large machine is not relatively so capable of resisting destructive loads as a small one.[74] (Stress is proportional jointly to linear scales and specific weights. This covers such quantities as moduli of elasticity, etc.). This very true remark has been repeated by numerous writers without reference to Galileo.

---

[72] "Hydrodynamics," Article 344, p. 591.

[73] *Loc. cit.*, Articles 344 and 345

The criticism however, is based wholly upon the assumption that strengths of materials and the stresses to which materials are subjected must act without any application of the arts of the engineer to improve or preserve the proper relative strength in the model. If correct model engineering is to be applied, the argument would have no theoretical force; there would simply be a practical difficulty to be overcome by the engineer. To imagine that the criticism blocked the use of true dynamic models would be confusing the dynamical properties of materials with the laws of similarity.

The following discussion not only shows that confusion of ideas persists, but it will illustrate the truth of the fundamental corollary; show the characters of the two classes of coefficients, *a* and *b*; and give further suggestions as to the construction of dynamic models.

Bertrand states, and Routh repeats, that the ratios of homologous sliding frictions are correct since the friction is proportional to the pressure, but rolling frictions are not in correct ratios, since they are inversely proportional to the diameters of the homologous wheels. If resistances of air are proportional to areas and squares of speeds, the air resistance to the model will be correct. (The writer's paper[75] on "Ice Diversion," etc., gave this also).

It would be unfair to say of these authors that they really had the thought that similarity would be impossible in the case of rolling friction, and in the case of air resistances which do not vary as the square of the velocity; but they neither say, nor show, that it is possible, thus leading a hasty reader to assume that it is impossible. Other writers have made similar statements.[76] A caution against making an error of this kind is clearly stated under "Historic Background" in the writer's paper. It must be remembered that similitude is distinct from every other physical law. The only point of contact is when one of these physical laws happens, in its limited sense, to express the same relations stated or implied in Newton's Theorem of Similitude, thus causing the concomitant property-coefficients to appear in dimensionless type-factors, and, therefore, to be self-eliminating from relations between similar systems, as explained under Article (10), "Newtonian Cancellation of All Coefficients" (see Article (26), Conclusion No. 10).

Notice this feature in the following treatment of sliding and rolling friction. Consider a problem referred to by Routh:[77]

---

[74] *Transactions,* Am. Soc. C. E., Vol. LXXXII (1918), p. 1188; also, p. 288, Equation (26) of the present paper.

[75] *Transactions,* Am. Soc. C. E., Vol. LXXXII (1918), p. 1138.

[76] Units, Dimensions of, Eleventh Edition, Encyclopaedia Britannica, v. 27, p. 738, by J. Larmor.

[77] "Elementary Rigid Dynamics," Article 165, p. 138.

"A carriage (R. R., or horse) on n pairs of wheels is dragged on a level horizontal plane by a horizontal force, $2\ P$, with uniform speed. Find the magnitude of P."

By the fundamental corollary, no regard at all need be given to the fact that the machine is an old-fashioned carriage. The problem would be precisely the same for an automobile, or a locomotive. Of course, the actual design and construction of a model might be far more difficult in one case than in another. In the case of the carriage it may not be necessary to have regard to many properties, whereas, in the case of the locomotive, the model engineer would find plenty of use for his ability. Overcoming difficulties is one of the chief delights of a good engineer; to run in a rut is by no means so glorious.

To spot the physical properties requiring attention, one may glance at Equations (11), (12), (20), and (21), treating the first three as was done in the writer's paper. Equation (21) becomes simply, $L = R = f$, if f is the coefficient of rolling friction, of which the dimensions are merely length. The symbols refer to ratios of homologues. Since the coefficient of sliding friction is dimensionless, it is self-eliminating from ratios of homologues, so long as the rubbing surfaces are of the same materials, in which case it requires no further attention. There may be several different coefficients. Ratio G need not refer to gravitations. Suppose, for example, that (neglecting effects of gravity on parts like connecting rods) gravity is "eliminated" in the sense mentioned immediately following Equation (45), as by running the carriage on a level plane, not necessarily in a straight line. Then a proper ratio, G, must be preserved in Equation (12), or one could not start or stop the model consistently or operate it on curves.

Proper treatment of the equations of similitude gives for Equations (22), (23), and (24): $L = R = f$; $F = \rho f^3 G$; and $T = \sqrt{\frac{f}{G}}$.

Characteristics are, $\mu = 1$; $\frac{\text{weight}}{\rho f^3 G} = 1$, the symbols, of course, relating to ratios of homologues. The values, L, F, and T, thus found, in terms of known property ratios, give all other ratios of homologues, by use of properly dimensioned forms. Air resistance has been neglected, but may easily be included by the methods presented, if necessary.

If the same materials compose the rubbing surfaces in model and prototype, the density ratio of these will be inconsistent if the density ratio is not to be unity. Whether this is serious is a matter for engineering judgment. The difficulty may be avoided by: (*a*) Using the same or equivalent materials when $\rho = 1$; (*b*) using different combinations of materials giving equal coefficients, μ, when $\rho \neq 1$; and (*c*) by any artifice which gives proper values, as by change of lubrication, or even by use of a special bearing in the model, etc.

Thus, the solution is effected without knowing the exact nature of the prototype. Let the definite problem proposed by Routh now be solved. Suppose the carriage to be pulled along an asphalt pavement, for which, $f = 0.012$.[78]

---

[78] Kent's Handbook (1923), p. 1705.

Then, since $R = f$, a much better roadway must be selected for the model if it is to be small. The coefficient for a steel roadway may be $f = 0.001$, thus giving $R = 12$, the scale ratio. If the model is to be tested on a hill, gravity must act, thus making $G = 1$. As there seems here to be no important reason for change of materials in making the model, the density ratio may be unity. Therefore, $L = R = f = 12$; $F = 1\,728$; and $T = \sqrt{12} = 3.46$; from which, for example, the velocity ratio and power ratio are, respectively, $V = \frac{L}{T} = \frac{12}{3.46} = 3.46$; and $P = FV = 6\,010$, so that the power requirement for the model is very small. (Note that P varies as the density ratio.)

Turning to Routh's solution, it is now easy to show that the pull on the model is in the proper relation to the pull on the carriage, and that it is because the coefficients, μ and f, have been allowed to have, or made to have, proper values in the model by application of proper technique.

It is highly instructive to notice how the coefficients eliminate themselves, when taking the ratio of homologous members of similar equations, using any of Routh's three forms for P. This illustrates perfectly the theorem of the equality of homologous type-factors (Articles 4, 10, and 11). In this respect, the theories of fluids and rigid bodies are identical.

Nothing has been said of the properties of the roadways, beyond the values of f, since there would seem to be no great importance in this, unless elasticity and variations of f with velocity are to be considered. If necessary, elasticity may be treated as in the paper. If such matters are to be considered, it may be recalled that rolling friction offers a great field for the research engineer, since this form of friction is not thoroughly understood, while use of 20 000 000 automobiles in the United States indicates that much use could be made of a correct theory.

(17) *D'Alembert's Paradox.*—Professor Bateman has described this "paradox" in a most excellent article on "Hydromechanics and Hydrodynamics" in the Fourteenth Edition of the Encyclopædia Britannica. The writer thinks it is really not a paradox, because such phenomena as a stationary float, "dead water," log in a pool below a dam, and even a float moving up stream, are matters of common experience. The stationary sphere in D'Alembert's paradox (or stationary cylinder, in two dimensions) is secured by creating a system of forces symmetrically placed, proportioned, and directed, about the center of symmetry. This dynamical system, therefore, must hold the sphere in equilibrium when it rests at the center of symmetry.

Any one of the stream-line surfaces surrounding the sphere in infinite liquid may be chosen as the shape of a pipe, or conduit, which envelope, as a study of the figures presented by Lamb[79] shows, must swell out about the sphere, or cylinder, according with the shape of the body at the center of symmetry. The body will remain at rest in frictionless fluid only when at rest at the center of symmetry of this envelope. If the body (sphere or cylinder) is now displaced up stream somewhat, while the flow of liquid is throttled properly at some down-stream point to maintain down-stream pressure, the body will continue to move up stream with further closing of the throttle, until the flow is completely shut off or materially reduced by the body of the sphere (or cylinder) obstructing the inlet end. On the other hand, if the body is displaced down stream it will continue to move down stream until it cuts off the flow by choking the outlet end, or is itself ejected. Dynamically, there is no difference between the actions in the up-stream and down-stream ends, so long as the fluid remains frictionless.

---

[79] "Hydrodynamics," pp. 74 and 232.

It makes no material difference in the experiment if the fluid is viscous, say, water. The body will remain at rest if it is located at a proper point up stream from the original center of symmetry of the envelope, while the flow is properly throttled at the valve down stream. This results from the fact that the pressure is greater on the down-stream face of the body than on the up-stream face, by an amount just equal to, or greater than, the friction, or viscous drag, on the sides of the body. The experiment is most simply made by letting water enter horizontally through the depressed lip of a small bowl or hemi-spherical vessel. A bowl, or cup, of somewhat smaller size, partly filled with water to give it depth, when floated near the entering stream, will move toward and against the entering stream until it is stopped by the side of the vessel, supposing that the crest of the opposite side of the vessel is at an elevation slightly higher than the crest of the lip, but low enough to permit the water to spill over the side at that point. If the entering water has a downward component entering at the lip the effect is increased. Of course, a better apparatus can be specially constructed for laboratory purposes, but it will not be more instructive. The principle was used by the writer in planning ice diversion works at Massena, N. Y., to cause ice to tend to drift up stream from the divergent channel and then on down the main stream. The combined effect of this and the ridges accomplishes the object, and the ice drift is actually up stream when the draft of water is not in excess of the constructed capacity; even in the case of over-loading the intake, there is no consequential amount of ice entering the intake, because the power-house now operates all year, whereas, before, it was compelled to shut down in winter.[80] (See Footnote 5, p. 276.)

Returning to the case of a frictionless fluid flowing past a sphere, or a sphere moving in such fluid, Newton was quite correct in stating that a cavity would form if the pressure were too small for the velocity, or the velocity too great for the pressure, a fact seldom mentioned by writers, but which is noted by Lamb.[81] It is perfectly clear that, whether fluid is frictionless or viscous, the inertia of the fluid will cause a cavity if the velocity and curvature of the stream lines are great enough; and this is why there must be different accelerations in model and prototype under different pressures. (For effective areas of impact, see Article (18), "Momentum.")

(*18*) *Momentum.*—The coefficient of viscosity and the kinetic theory of gases have been brought into the discussion by several references. Neither of these subjects can be understood without a correct conception of the nature of momentum, the actual misuse of which concept is an error of no infrequent occurrence, especially in hydraulics. Routh's "Elementary Rigid Dynamics" contains two excellent chapters on momentum and *vis viva*.

---

[80] *Transactions*, Am. Soc. C. E., Vol. LXXXII (1918), p. 1138.

[81] "Hydrodynamics," p. 98, under "Discontinuous Motion."

No formula for friction loss in fluid flow can be safely based upon a momentum change as an action isolated from the circumstances and details of that change. Without refined definitions and explanations the important relation for present purposes is:

Theorem IV.—A definite system of forces acting upon a definite mass (particle) free to move, produces a definite change in the momentum of the particle; but the converse is not true—a definite change of the momentum of the particle does not require any definite system of forces, for there are an infinite number of force-time relations which will produce the same change of momentum during the time of the action.

It is a most important consequence, that the time rate of change of momentum is not necessarily, and not usually, any definite general function of the forces existing at the beginning and end of the change. On the other hand, it must be understood that, although a definite change of momentum does not demand any definite system of forces, yet all systems of forces which produce that definite change must be equivalent to each other in the sense that the effective "whole force" is the same in all. In substance, Routh states that the resolved component in the direction, z, of the "whole force" expended in the time, $t_1 - t_0$, is defined by $\int_{t_0}^{t_1} m\,Z\,dt$, Z being the instantaneous component of the accelerating force acting upon the mass particle, m, in the direction, z.

Consider the case of a vessel discharging water at a velocity, v, through a horizontal pipe under the head, h. It is well known that the effective whole force causing that discharge through an orifice in place of the horizontal pipe, would be equal to a reaction on the apparatus due to twice the head, h, acting on the effective area of the orifice. This reaction must remain precisely the same for the case of the horizontal discharge, but the actual head on the discharge may be any whatever, depending upon the length and friction of the pipe. Hence, it is clear that there is no fixed relation between observed head (pressure) and change of the water's momentum. One must have positive knowledge of the circumstances of the change of momentum to be in a position to connect that change with pressures and potentials.

It is clearly incorrect to state (as usual) that the reaction of the discharge is "due to twice the acting head," for there is no possibility of a double head. The error lies in the customary statement which, properly, should read to the effect that it is the area upon which the head acts that must be doubled. This will be clear when one observes that in a reservoir the water must be accelerated from rest to a velocity of discharge, v; that is, the mechanism of the usual reservoir can provide an acceleration averaging only half ($\frac{1}{2}v$) sufficient. Therefore, the area of approach must be double the effective orifice (*vena contracta*) area to supply water enough. That area is provided by automatic reduction of the pressure on the side of the vessel in the vicinity of the orifice—thus admitting flow to the orifice from all directions—and it is here very interesting to observe that the convex area of a hemisphere is exactly twice the area of its base. If double pressure could be made effective in the reservoir, without alteration of the head, no drop in pressure around the orifice would occur; but that is clearly impossible in the absence of extra mechanism. Statistical analysis would develop an infinite number of equivalent methods of approach to the orifice, all giving the same reaction.

Another illustration of loose uses of the principles of momentum is furnished by the formula, $\frac{(v-v')^2}{2g}$, for the friction loss due to abrupt change of pipe section. This formula appears in many of the older textbooks. That it still persists is shown by the fact that Professor Gibson[82] uses it, and without explanation, leaving the reader to assume that wherever there is abrupt increase in section the entire change of momentum is lost, instead of being conserved to a greater or less extent, depending upon the circumstances of the change. William C. Unwin, Hon. M. Am. Soc. C. E., was careful, in giving the original deduction of the formula[83] to explain that it was based upon an assumption as to the pressure at the beginning and end of the action, and that it was equivalent to saying that the change of momentum is wholly lost. Necessity for this assumption and its explanation, so carefully made by Professor Unwin, is the criterion of Theorem II which shows that the problem of loss due to sudden expansion, and its formula, $\frac{(v-v')^2}{2g}$, fall within the great lacuna of fluid dynamics previously disclosed. This problem cannot be solved solely by the principles of momentum (insufficient dynamics) until the physical mechanism of turbulence has been properly introduced into dynamics. Meanwhile, tabulations of values for the empirical coefficients to be applied directly to the velocity head are to be preferred.

It may be emphasized again in other words, that no problem in correct dynamical physics can be solved without use of the dynamical physical constants defined by the theory of the particular branch of science to which the particular problem belongs. To this extent Newton's laws of motion are insufficient, *per se*, to the solution of any problem. On the other hand, they are necessary to such solution. The postulates of a science defining physical constants merely describe the mechanism which governs the application of Newton's laws by showing how they are to be applied. It is in this way that Newton's laws form the footings for all branches of mathematical dynamics. Stokes' failure to notice this in the case of viscosity was discussed in connection with Equation (10) of the writer's paper. This insufficient but necessary application of Newton's laws may be the cause of so much misunderstanding as to the status of pure dynamics.

That the formula, $\frac{(v-v')^2}{2g}$, really has no place in fluid dynamics is shown by the well-known fact that an "open," or "free," jet of water entering a large reservoir by abrupt expansion of section at entrance is capable of supporting a very material head of water in the reservoir. This head, therefore, must be conserved from the kinetic head existing prior to the change of momentum; thus, a corresponding part of the momentum is also conserved.

---

[82] "Hydraulics," Encyclopaedia Britannica, Fourteenth Edition, Vol. 11, p. 972.

[83] Encyclopaedia Britannica, Eleventh Edition, Vol. 14, "Hydromechanics," pp. 45-46.

A clearer understanding of fluid dynamics will be cultivated, if the criticized type of analysis is abandoned; especially if, at the same time, the terms, "shock" and "impact," are changed to "rate of delivery of energy to the sinks of viscosity." This latter term, or a descriptive abbreviation, will be a constant reminder of the true nature of all energy losses in hydraulics, and will point to the right path for the research engineer to follow, in formulating the dynamical science of eddying motion. The latter is all that is needed to complete dynamical science as to viscosity. When that is accomplished, there will be no further need for empirical coefficients of hydraulic friction, since the perfected science will determine all friction losses, just as the present science of pure viscous flow now solves all its problems without necessary recourse to special coefficients for special problems.

(*19*) *Momentum Transfer.*—The foregoing remarks on momentum will be a great help relative to the questions of momentum transfer, as raised in the discussion. The misuse of this concept has resulted in a most serious fundamental error in the deduction of the formula connecting the molecular free paths with the coefficient of viscosity of a gas. Before disclosing and correcting this error, however, it will be necessary to explain the nature of momentum transfer by several homely but perfect illustrations.

Suppose two railroad trains traveling at different speeds in the same direction along two parallel tracks, as a passenger train moving at 60 ft. per sec. on one track, and passing a freight train moving at 40 ft. per sec. on the other track. If a ball is thrown laterally (perpendicularly, horizontally) from the freight train so as to impinge on the smooth flat front of a passenger coach, the ball will "transfer" 40 ft. per sec. of "freight" momentum, which will be increased by $60 - 40 = 20$ ft. per sec. by the impact (necessarily supposed inelastic), which is exactly sufficient to increase "freight" momentum to "passenger" momentum, by the mutual impulse between the ball and passenger train. Observe most carefully that there is no mutual impulse, or action, in the direction of train motion, between the ball and freight train, because the ball is thrown "laterally;" it is the passenger train which meets the resisting impulse. The lateral component motion of the ball will not be altered, as there is supposed to be no friction between ball and front of cars, except that of inelastic impact in the direction of train motion. It must also be carefully noticed that the "lateral" impulse of throwing the ball has no effect at all upon the freight train in the direction of train motion. It is most important of all, in the question about to be considered, to remember that the mutual impulse of two bodies necessary to bring them to the same motion in any given direction must necessarily be wholly inelastic in that direction. It is about to be shown how a virtually inelastic impulse can be generated by collisions of perfectly elastic bodies.

If, now, the ball is thrown laterally from the passenger train to the freight train, so as to impinge on the smooth flat rear of a freight car, the ball will transfer 60 ft. per sec. of "passenger" momentum, which will be decreased 20 ft. per sec. by the inelastic impact necessary to decrease passenger momentum to freight momentum. There is no mutual impulse, or action, in the direction of train motion, between the ball and passenger train, because the ball is thrown "laterally;" it is the freight train which feels the accelerating impulse.

If the trains were moving at the same speed in the same direction, the lateral transfer of a ball carrying the momentum of either train to the other would have no impulsive effect on either train in the direction of train motion. The two trains may be of equal weight. The freight train may consist of an engine drawing a "string of flats," one flat for each passenger coach engine and tender. Each flat may be equipped with a powerful crane. If then, the cranes should load the passenger train on to the "flats," the total momentum of the freight train will be doubled, but no impulse will occur in the direction of train motion, because of the equal speeds of the two trains during the loading. Another illustration is provided by imagining each molecule arriving in any stream of molecules to be piloted by one of Maxwell's "demons" in such a way that it does not collide with any molecules of the stream. No resisting impulse to the flow of the stream can occur.

The conclusion is that mere transfer of momentum from one position to another, as by motion of a particle carrying momentum from one system to another, has no impulsive effect, by itself, and that it is only when the particle arriving in a system has its velocity changed thereby, that any impulse can occur by reason of the transfer:

Theorem V.—The mere removal of a mass from, or its addition to, a system of particles moving with a common velocity, cannot have any impulsive effect upon the system when that mass has initially the common velocity of the system, or when it does not collide with any molecule of the system.

It is failure to recognize this fundamental truth which has caused the serious error in the kinetic theory of gases as mentioned in the foregoing remarks.

(20) *Kinetic Theory of Gases.*—Before explaining the nature of the error in the kinetic theory, it will be well to review a few of the principles of that science, especially in its relation to hydrodynamics and aerodynamics; for engineers will wish to be familiar with every science which is alleged to be useful in model theory, whether the allegations can be supported or otherwise. If the allegations are true, it will be all the more important to learn that the kinetic theory cannot be applied with accuracy in any way until the existing errors are corrected, not only in formulas, but also in the numerous constants which have been computed by them. Moreover, there is still another error which affects the kinetic theory in its relation to hydrodynamics, and this, too, will be made clear by reference to the writer's remarks under Article (15), "The Great Lacuna."

In adding this necessary corrective contribution to kinetic theory the writer asks merely that engineers base conclusions upon analysis of what is said, rather than rely upon supposed authority. The lack of such careful constructive analysis and consideration is clearly evident in some statements made in the discussion. The writer attributes this to no prejudice, but, rather, to insufficient attention to his presentation, and to the consequent failure to confine those statements of discussion to actual facts. By ascribing false ideas and remarks to the writer, one is certain to arrive at false conclusions, however apparently correct the logic may seem.

The great lacuna of hydrodynamics was disclosed under its appropriate heading. It follows at once, that any theory which connects relations between existing hydrodynamics and the kinetic theory of fluids must be either wholly invalid, or at least open to the gravest suspicion, as to every relation which involves turbulent motion of the fluid. This conclusion is of particular force relative to the calculation of the coefficient of viscosity from assumed conditions which involve nothing more than a state of pure viscous motion, when the coefficient is intended for use under conditions of turbulence. In other words, the great lacuna spreads itself over into the domain of the kinetic theory, which, consequently, must be quite as barren in the region of turbulence as hydrodynamics itself. Therefore, both these sciences are unfit, in their present state, to solve the simplest problem in their respective fields, when turbulence and eddy motion are involved. Neither science has yet been equipped with the complete mechanism of viscosity, the prime cause of all fluid friction, as previously shown for hydrodynamics, and, hence, for all its relations to other sciences.[84]

Space here cannot be devoted to any study of kinetic theory, beyond that necessary to answer questions arising from the discussion. Should the engineer be interested in deeper studies, reference should be made to the best works upon the subject, of which there are surprisingly few in the English language. A treatise by Sir James Hopwood Jeans includes the statistical theory and quantum dynamics.[85] From the chemical point of view, the work of Oskar Emil Meyer describes the kinetic theory at the time of its most rapid primary development.[86] It was first published in German in 1877 and was later translated into English by Robert E. Baynes. The only recent publication in the United States is the work of Leonard B. Loeb,[87] which describes the most recent experiments. All other treatises on this subject are referred to in these books with one exception which, unfortunately, has received little or no notice. That is the work of Severinus J. Corrigan,[88] which the writer believes is really based on the first idea of the nature of the "electron" and "quantum."

While some preliminary "excavation" had been made, the first three "foundation stones" of the theory were: (1) Experimental discovery by the botanist, R. Brown (1827)[89] of the microscopic "Brownian Movement" among small particles of matter suspended in a liquid, motions now known to result from "bombardment" by molecular motions of the liquid. Not until 1911 was there any extension of experimental knowledge of such motions among gas molecules. At this time, the experiments of Professor R. A. Millikan,[90] of the University of Chicago, showed that minute oil drops floating in air were certainly subjected to visible bombardment by gas molecules, movements which could be measured and plotted into a diagram for statistical analysis; (2) Joule's[91] experimental determination of the mechanical equivalent of heat, and his application (1848) of the kinetic theory showing that the temperature, internal energy, and pressure of the gas, were associated with the kinetic energy of molecular motion. The mechanical equivalent connected external work and internal energy with the specific heats. He also showed that the total molecular kinetic energy in a gas volume must be distributed equally among any three rectangular directions chosen at will; thus, one-third the molecules in the volume may be considered to move parallel to each of any three rectangular co-ordinate axes, which gave at once an approximation to the mean molecular velocity (square root of mean square); and (3) in 1859, Maxwell[92] published his first paper on the "Dynamical Theory of Gases," in which he promulgated the idea of applying the formal theory of probabilities as treated in the method of least squares, to the random velocities of the molecules, which he assumed had equal possibilities in all directions, the magnitudes of the velocities being distributed among the molecules according to the law of error, which, as applied in the kinetic theory, is called "Maxwell's Law." Many other great names are associated with the kinetic theory. They must be sought in the references to gain a correct notion of the labors of these men. The quantum theory of Planck will also be found. The work of the first three mentioned is that most closely associated with the questions now to be discussed.

---

[84] *Transactions*, Am. Soc. C. E., Vol. 94 (1930), pp. 834-842.

[85] "The Dynamical Theory of Gases," by Sir James Hopwood Jeans.

[86] "The Kinetic Theory of Gases," by Oskar Emil Meyer.

[87] "Kinetic Theory of Gases," by Leonard B. Loeb, 1927.

[88] "The Constitution and Functions of Gases, the Nature of Radiance, and the Law of Radiation," by Severinus J. Corrigan, Pioneer Press Co.. St. Paul, Minn. (1895).

To gain some idea of the absolute magnitudes of the kinetic theory, one may consult Jeans' treatise (page 8), or Loeb's (Appendices I and III), both of which have been previously mentioned. Some of these magnitudes are:

Number of molecules per cubic centimeter, Avogadro's number	2.7×10^{19}
Number of molecules per gramme molecule, Loschmidt's number	6.06×10^{23}
Electronic charge in electromagnetic units	1.59×10^{-20}
Molecular diameter of hydrogen, in centimeters	2.32×10^{-8}
Velocity of hydrogen molecule, in centimeter seconds	18.39×10^{4}
Hydrogen mean free path, in centimeters	1.83×10^{-5}

This exhibit certainly warrants Jeans' conclusion, or statement,[93] that the kinetic theory is so "fine-grained" that "the phenomena exhibited by a gas constituted in the way described will be indistinguishable, so far as experiment and observation go, from those of a continuous medium."

---

[89] *Philosophical Magazine,* Vol. 4, 1828, p. 162, *et seq.;* also, 1829, 1830.

[90] "The Electron," Univ. of Chicago Press, 1917.

[91] Report, British Assoc. for the Advancement of Science, 1848, Pt. II, p. 21.

[92] "Collected Papers," Maxwell, "Dynamical Theory of Gases," Vol. 1, p. 377.

[93] "The Dynamical Theory of Gases," p. 10.

(*21*) *Kinetic Theory Unnecessary in Model Engineering.*—This follows from what has just been said, but the writer has shown the same thing in quite another way, by means of Equations (10), (11), (20), and (21) and their fundamental corollary, previously stated. The necessary and sufficient conditions for similarity are expressed by only the first three equations. If any two systems whatever, satisfy these three equations, the two systems will be similar, regardless of their nature and form. If it is then required to introduce into the systems some new dynamic physical action, it can be done by adjusting the appropriate physical constants by means of Equation (21), which equation may call upon the engineer in charge for certain assisting technique. It follows that, if the technique is required only to adjust the physical constants of continuous matter, or of discontinuous matter found by experiment to satisfy the assumption that it is continuous, there will be no necessary use for any theory of discontinuous matter, such as the kinetic theory; and, as experiments without number have shown gases to act as continuous substances for the ordinary problems of engineering, there should be no urgent need for the use of the kinetic theory in model engineering.

On the other hand, the writer sees no reason at all why the kinetic theory could not be applied, had there been defined and determined the proper physical constants necessary to set up a model gas, or even a model atomistic atmosphere, for example. When the atomistic theory is completed, and experiment has been applied to the corresponding mechanisms, it will almost surely be found that Nature has already provided many atomistically similar systems. In another generation as remarkable as the past, as it was with radio, engineering science may enable every school boy to construct atomistic models by means of "stock parts" on sale at the nearest variety store.

The conclusion is that gases, as well as liquids, may still be treated as continuous, and that the kinetic theory has found no place in model engineering, especially since it was shown in Article (15) that the great lacuna of fluid dynamics also extends to the kinetic theory, as it relates to problems in turbulent flow.

(*22*) *Limitations of a Science not Necessarily Limitations on Newtonian Model Engineering.*—This question is involved in Professor Bateman's reference to Professor Dr. Th. v. Kármán's work. The very next, and more extensive, article in the same journal, which is also by Kármán, is even more interesting than the former.

Nothing illustrates the truth of the proposition better than the facts disclosed in the writer's paper, and this closure, that, although hydrodynamics is a barren science in the domain of turbulent motion, a true model of a hydraulic system may be constructed so as to perform with theoretical exactness, because the only physical constant upon which turbulence depends is known with practical accuracy—the coefficient of viscosity.

All that is needed to complete the theory of hydrodynamics is to know the complete functionality of the coefficient of viscosity, that is, the complete method of its operation. By the fundamental theorem of similarity the construction of a true dynamic model is entirely independent of any method of operation or mechanism. It depends only upon Newton's law of similarity, and the engineer's ability to match physical properties, materials, and general conditions against that law, as indicated by Equations (10), (11), (20), and (21). In other words, the great lacuna of hydrodynamics does not extend into the domains of Newtonian model engineering.

Properties, materials, and general conditions being made by the engineer to satisfy the requirements of Newton's law of similarity, the model will be similar to the prototype regardless of the functions of the physical constants. This is the great advantage of a Newtonian model, which becomes a veritable calculating machine for the solution of the most difficult problems of mechanics.

It is concluded, that a model in the region of the Brownian motion (a model particle suspended in water, say), as suggested in Professor Kármán's papers, does not depend for its accuracy upon the motion, but only upon the compatibility of the physical constants which govern the Brownian motion, properties of molecules, etc. A Newtonian model constructed on these lines must be exact, even in the Brownian region. In the same way, in passing to the region of the velocity of sound, the engineer is not at all concerned with that fact, but only with the physical constants which control that velocity; that is, with the modulus of elasticity and viscosity of the gas. If these have been properly treated, the model will perform with accuracy with respect to the propagation of sound. No attention need be given to the detailed mechanics of the propagation. This is another question answered in its relation to Professor Kármán's papers, as referred to by Professor Bateman.

The velocity of a projectile is another such case. Although the laws of resistance are not at all known theoretically, yet, if the physical constants (viscosity and elasticity, certainly) are known, there is nothing to obstruct a Newtonian model, other than the difficulties of engineering technique. The celebrated tests of Bashforth on projectiles are described under the title "Ballistics" in several editions of the Encyclopaedia Britannica. It seems to the writer that viscosity and elasticity should explain all, and that it is more than likely that similarity would obtain in a Newtonian model of air and bullet. It could not obtain in air as a model of itself, unless the pressure and viscosity are properly adjusted, as in the writer's Appendix I.

(*23*) *Coefficient of Viscosity—Fundamental Error in Theory.*—The questions raised in the discussion relative to viscosity led the writer into investigation yielding prodigious results. The most consequential of these is the discovery of an error made by Maxwell in the use of momentum in the original deduction (1859)[94] of his formula, $\mu = \frac{1}{3}\rho \bar{c} l$, connecting the coefficient of viscosity of a gas with the density, molecular mean velocity, and molecular mean free path for the same gas and its molecules. This disclosure shows that the values of the mean free path which have since been calculated by

---

[94] *Philosophical Magazine*, June and July, 1860; "Collected Papers," Maxwell, Vol. I, p. 377.

Maxwell's formula from experimental observations of the coefficient of viscosity, must all be doubled, in order to effect the necessary correction. The last statement is founded also upon another disclosure of the investigation, showing that the error mentioned has never before been corrected. The full consequences of doubling the values of all the free paths hitherto calculated by the erroneous formula, and its more recent modifications, will be appreciated when it is recalled that these calculated values, of which there are a great many, should give the most reliable values of the sizes of the molecules of the various gases, since the diameters are known functions of the free paths, by a formula first deduced by Clausius and almost immediately afterward corrected, as to numerical coefficient, by Maxwell (1859).

The writer so concludes from the statements of Jeans[95] and Loeb. The latter states:[96] "The measurements of μ afford the best values of l obtainable."

Maxwell's law applied to Clausius' free path gives the relation, $l = \frac{1}{\sqrt{2}\pi\sigma^2 N}$, connecting free path, l, with molecular "diameter," σ, N being the number of molecules per unit volume ("molecular density"). Hence, it is clear that all previous values of σ calculated from the erroneous values of l must be reduced in the ratio of $\sqrt{\frac{1}{2}}:1$, since σ^2 must be changed inversely as l is changed, in order to effect the correction. All other values calculated from the erroneous free path will also be in error by corresponding amounts. Textbooks should be revised as to relations between the coefficients of viscosity, conduction, and diffusion.

This error, universally committed in the deduction of the formula for the coefficient of viscosity in terms of the free path, consists in the misuse of the concept of momentum and its transfer as previously discussed. Newton's law states that "change of motion is proportional to force applied, and takes place along the straight line in which the force acts," but it is not stated that force must result from every change of momentum. When Newton said "change of motion," he referred to motion, not momentum. For example, one may increase, or decrease, the momentum of a system by adding to, or abstracting from, that system amounts of mass without changing their velocities, as in the manner described in Article (19), "Momentum Transfer." No force, or impulse, results from such a change of momentum, that is, a change of mass with no change of motion.

Therefore, having careful regard for Newton's laws of motion, the coefficient of viscosity of a gas may be calculated as follows: Assume that the gas flows horizontally in the positive direction of the conformably chosen x-axis of right-hand rectangular co-ordinates, $x\ y\ z$, positive direction of z being upward. Suppose, furthermore, that the gas is limited by two horizontal planes, $z = z_1$ and $z = z_2$. The lower of the two planes, $z = z_2$, is considered at rest, the motion of the gas being created by the positive motion of the upper plane, $z = z_1$, on the theory that there can be no slip along a boundary surface as between that surface and the gas. Thus, molecules of gas in contact with the upper plane (face of a belt) must be dragged along with the plane until their "molecular motion" (heat motion) causes them to spring away with downward component velocity averaging perpendicular relatively to the upper plane for a distance equal to the "mean free path" before they can collide with another molecule. It is also a supposition of the theory, that each molecule traveling over a free path carries with it the horizontal motion ("mass-motion," as distinct from molecular or heat motion) appropriate to the point of beginning of the free path. Therefore, a particle springing on to the lower face of the upper plane will reach it with a deficient velocity, since the mass-motion is horizontal and must gradually decrease to zero at the lower plane, $z = z_2$.

---

[95] "The Dynamical Theory of Gases," p. 326.

[96] "Kinetic Theory of Gases," p. 188.

Under these circumstances, the transverse velocity gradient, $\frac{du}{dz}$, of the mass-motion of velocity, u, parallel to the planes and the x-axis, becomes constant from one plane to the other, and may be represented by a straight line sloping forward and upward in the positive direction of mass-motion. Molecules arriving in the lower plane, $z = z_2$, will have only the slight component mass-motion, $l\frac{du}{dz}$, due to an average elevation of one mean free path above the plane, while those in equilibrium in the plane, $z = z_2$, and also those traveling upward in the first free path, l, above that plane, will have no mass-motion, since the plane, $z = z_2$, is supposed to be at rest.

It is also necessary to understand why the velocity gradient, $\frac{du}{dz}$, will be constant under the circumstances supposed. (The mechanism is an ingenious device of Maxwell to simplify the analysis.) It is not only because the planes, $z = z_1$ and $z = z_2$, are infinite in their horizontal dimensions, but (which is much more important), because the gas pressure is tacitly assumed to be uniform in each horizontal plane; there is no pressure drop along the direction of flow. If there were, $\frac{du}{dz}$ could not be constant, for, otherwise, the shears on the upper and lower surfaces of a horizontal layer of the gas would be different, which cannot be the case for non-accelerated mass-motion. In other words, the only source of energy to keep the gas in motion is that supplied to the molecules of the gas which reach the upper plane (lower face of a moving belt) with deficient mass-motion, and which, therefore, must be accelerated by the contact with the upper plane. The mere assumption of a steady state of motion is not sufficient, by itself, to establish a constant transverse velocity gradient (the steady flow in a capillary tube is a practical illustration—the velocity gradient is a parabola).

By Joule's law of energy distribution, one-third the molecules of the gas, and, therefore, one-third its mass, may be considered to move parallel to each of the three rectangular co-ordinate axes, and with the mean molecular velocity, c; one-sixth the molecules, or mass, positively, and one-sixth, negatively. Moreover, each molecule, on the average, moves over one mean free path, l, between any two of its consecutive collisions. Hence, each time an upward moving molecule collides, it will, on the average, have its velocity increased by $l\frac{du}{dz}$ units. Correspondingly, a downward moving molecule will have its velocity decreased by $l\frac{du}{dz}$ units. By the principles pointed out under Article (19), "Momentum Transfer," no molecule, or molecules, can experience any change of mass-motion when any molecule leaves its position in any horizontal plane to pass to another parallel plane, for it is supposed to have acquired the mass-motion of the plane which it leaves, before taking its departure.

Now, to find the effective "shear" on any plane parallel to $x\ y$, one may proceed exactly as in finding the shear on any section of a rigid structure, as by the "free-body method." Hence, for a gas, this may consist in finding the "shearing stress" per unit area on the lower face of the portion of gas above the plane of shear, which may be at any elevation, $z = z_0$.

The mass passing up through the unit area of the plane, $z = z_0$, of shear must (by Joule) be $\frac{1}{6}\rho\bar{c}$ per unit time (ρ being the density), while its change of momentum in the x direction must be, by the ensuing action, that due to the increase, $l\frac{du}{dz}$, of velocity in passing over the distance, l, in the positive z direction. This change of momentum demands a "whole force" on the mass, $\frac{1}{6}\rho\bar{c}$, in the positive x direction of $\frac{1}{6}\rho\bar{c}\,l\frac{du}{dz}$, which, consequently, is the required negative effective "shearing stress" per unit area, exerted on the plane, $z = z_0$, and by similar reasoning, on any other parallel plane in Maxwell's particular mechanism herein used.

Therefore, considering the well-known formula, $S = \mu\frac{du}{dz}$, for the shearing stress on a plane in a viscous liquid, it is concluded that the coefficient of viscosity of a gas is, $\mu = \frac{1}{6}\rho\bar{c}\,l$, in which, the numerical coefficient is exactly one-half the value found by Maxwell and all other analysts since his time.

By similar reasoning, the shear on the same unit of area, considered as part of the upper face of the portion of gas lying below the horizontal plane of shear, will be found to be the same as before, but in the opposite direction, just as with rigid bodies.

The error which Maxwell and subsequent analysts have made, consists in adding the changes of momentum which are experienced by both the upward and downward moving molecules, just as though a molecule upon leaving the upper volume should require a change of motion, $l\frac{du}{dz}$, before it could depart; but this is evidently not true, for it is supposed by Maxwell that each molecule starting its free path has only the mass-motion appropriate to the starting point, just as a ball thrown laterally from a railway train has no mutual action and reaction with the train in the direction of train motion, as explained under Article (19), "Momentum Transfer." Therefore, it is an error to integrate the molecular impulses throughout the whole volume of the gas above and below the plane on which the shearing stress is required, since it is perfectly clear, by Newton's laws, that the integration should extend only throughout the volume on one side of the plane of shear, in order to find the shear on that plane.

In other words, as confirmed by the principles of statics, and as made clear by the free-body method (a most valuable aid to the elastician and structural engineer), the shear on the plane is in equilibrium with the forces on only one side of that plane. The integration in the volume above the plane gives the shear on the plane considered as the lower face of the free body above the plane; the integration in the volume below the plane gives the equal and opposite reactive shear on the plane considered as the upper face of the free body below the plane. It is simply an application of D'Alembert's principle enabling one to treat a problem in dynamics by the method of statics.[97]

This is a very simple matter, but very deceptive, owing to the fact that transfer of momentum, by itself, without reference to the circumstances of that transfer, is not associated with any definite impulse.

In pointing out the nature and amount of the error just disclosed, it was not at all necessary or desirable to introduce the detailed ideas connected with the distributions of random velocities, directions, and free paths, since the error consists merely in doubling the result of an integration involving such distributions, and does not consist in any error of statistical analysis, other than the errors in one set of limits of integration due to the erroneous doubling.

Of course, the foregoing does not relate in the least to those numerous corrections of the numerical coefficient of $\rho \bar{c} l$, which from time to time have been proposed or adopted by reason of changes in the method of averaging. The error herein disclosed is quite distinct from any of these others.

As mentioned in Article (19), "Momentum Transfer," the only way in which two or more bodies can be brought to a common velocity by their mutual impacts, is that the impacts be wholly inelastic, or have the precise effect of such inelastic impacts. Hence, it remains to be explained how it can be, that the supposedly "perfectly elastic" mutual actions of the molecules of the kinetic theory can become effectively inelastic when changes of mass-motion require consideration. A little reflection will soon convince one that the very existence of the kinetic theory depends upon a satisfactory explanation of this possibility. Otherwise, the kinetic theory must fall, because it would be impossible for molecules to be both elastic and inelastic at the same time. The explanation follows.

(24) *Independent Verification of the Writer's Calculation.*—Since the velocity of a given body cannot be brought to the common velocity of a stream of other bodies by their mutual actions and reactions, except by an impact, or aggregation of impacts, of resulting inelastic nature, it is neces-

---

[97] See Maurer's "Mechanics," First Edition, Article 146.

sary to explain the possibility stated in the proposition in Article (23), or the kinetic theory which connects mass-motion with molecular motions must collapse. This follows from the accepted fact that the mutual actions of the kinetic theory are "perfectly elastic," or, more properly described, wholly "conservative"; and, further, from the axiom that the action of a molecule cannot be conservative and non-conservative simultaneously.

It will appear that the proposition is true because of the presence of two mutually convertible types of motion—molecular (heat) motion and mass-motion. If, in a series of mutual actions between molecules, some of the mass-motion is converted into molecular motion, the energy of the latter may be regarded as having been dissipated, in so far as mass-motion is concerned. The convertibility of mass-motion into heat motion is the necessary mechanism which supplies to elastic molecules the property of effective inelasticity; the "dissipated" mass-motion thus becomes the energy "lost" by "inelastic impact."

The general problem of impact is usually treated in books on dynamics (see Routh's treatise on momentum,[98] previously cited), but the writer doubts that the problem here to be treated has received much attention; that is, the effect upon a molecule which is suddenly placed in a stream of other molecules. Detailed analysis agrees with the abbreviated method that follows.

When two bodies bring themselves to the same velocity in the same straight line by an impact (as, for example, during the "axial" collision of two perfectly elastic spheres at the instant when "restitution" begins), it is doubtless well understood that there must be dissipated, or stored, an amount of energy exactly equivalent to that exchanged between the two bodies in order to accelerate one and retard the other sufficiently to equalize their velocities. In the perfectly elastic axial impact of two equal spheres, therefore, the numerically equal variable acceleration of one body and variable retardation of the other, must continue after restitution begins, so that, owing to mechanical symmetry and the equality of action and reaction, the restoration of resilient energy must occur by a continuance of the respective variable acceleration and equal variable retardation, until the two equal spheres have exactly exchanged their original velocities which existed before the axial collision. At this instant the resilient energy has been fully restored to kinetic form. If the spheres are not equal, the statement is precisely the same if, instead of the word, "motion," or "velocity," the word, "momentum," be inserted, thus harmonizing the ideas of elastic collisions and "conservation of momentum." In using the word, "momentum," in this way, however, the greatest care must be taken not to violate Newton's laws of motion, as clearly explained under Article (18), "Momentum," and Article (19), "Momentum Transfer." Otherwise, the theorem of conservation of momentum does not apply at all. It is the eccentric collisions which cause dissipation.

In Maxwell's mechanism, described in Article (23), the number of molecules and the total kinetic energy at any level continue the same, since the density and motion of the gas are supposed not to change. Therefore, for every molecule which enters a level another must leave that level; and for every amount of energy, $\frac{1}{2}\,ma^2 = \frac{1}{2}\,m\left(l\frac{du}{dz}\right)^2$, deposited (or withdrawn) in any level, an equal amount must be rejected (or received) by that level. If, on the whole, Maxwell's mechanism supplies an excess energy to any or all levels, that energy must be rejected as mass-motion to re-appear as molecular motion.

---

[98] "Elementary Rigid Dynamics."

The upper plane (moving) delivers kinetic energy to the molecules impinging upon it from below. These molecules then spring downward, and, on the way down, have an excess velocity at every elevation they reach, owing to the fact that the excess velocity, a, is always dropped (Maxwell's hypothesis) during the passage of the molecule over each vertical mean free path, l, on the way down, or at the rate, $\frac{a}{l}$, per unit length of the vertical.

The molecule itself may not travel to the lowest plane on account of its frequent collisions with other molecules, but, since it transfers only its excess energy to other molecules, the amount retained by the molecule is always exactly appropriate to any level in which the molecule may linger.

Maxwell's mechanism clearly requires that, for every molecule which arrives at any level bringing with it the energy, $\frac{1}{2}\,m\,(v+a)^2$, from above, another molecule must arrive at that level bringing with it the energy, $\frac{1}{2}\,m\,(v-a)^2$ from below, and that, simultaneously, two other molecules, or these same two molecules, must leave the level carrying away with them the energy, mv^2. Therefore, for every molecule which reaches any given level from either above or below, but not from both directions combined, there will be delivered over every vertical mean free path an excess energy of $\frac{1}{2}m\,[(v+a)^2 + (v-a)^2 - 2v^2] = ma^2$, which, accordingly, must be dissipated in molecular (heat) motions. This dissipation is at the rate of a^2 per vertical mean free path per unit mass passing down, or up, but not both.

By Joule's rule, the rate of molecular flow of mass, in only one direction of the vertical, must be $\frac{1}{6}\,\rho\,\bar{c}$ per unit horizontal area per sec. Hence, the rate of dissipation, or power loss, is $\frac{1}{6}\,\rho\,\bar{c}\,a^2$ per vertical mean free path, l, or $\frac{\frac{1}{6}\rho\bar{c}a^2}{l}$ per unit volume per sec. Substituting the value of a, the time-rate of dissipation of energy per unit volume of Maxwell's device is, $\Phi = \frac{1}{6}\,\rho\,\bar{c}\,l\left(\frac{du}{dz}\right)^2$, in which, Φ is the dissipation as expressed by Stokes (1850).[90]

By substituting the proper values of the differential coefficients, appropriate to this analysis, in Lamb's Equation (8) (or in Stokes' expression, which is the same, when the density is made constant), to get the value of Φ, remembering that, in Maxwell's device, u is the velocity parallel to x, $2g = \left(\frac{du}{dz}\right)$, and $a = b = c = f = h = v = w = 0$ (see Lamb's Articles 30 and

---

[90] "Hydrodynamics," Lamb, Third Edition, p. 541, Equation (8), and footnotes. See, also, Stokes, "On the Effect of the Internal Friction of Fluids on the Motion of Pendulums," *Cambridge Transactions*, Vol. 9, p. 58, last expression.

317), involving different notation; that is, $a = (\frac{du}{dx})$, one easily finds $\Phi = \mu \left(\frac{du}{dz}\right)^2$, in which, μ is the coefficient of viscosity of the fluid of constant density.

Therefore, after comparing the two values of Φ, it is concluded that the coefficient of viscosity of a gas is, $\mu = \frac{1}{6} \rho \bar{c} l$, which agrees exactly with the value deduced previously by the correct use of properties of momentum.

It is permissible to use the hydrodynamical equations as being rigorously exact in the foregoing method of deducing the formula for the coefficient of viscosity of a gas, since the problem treated does not lie within the region of the great lacuna of hydrodynamics.

If one does not wish to use hydrodynamics, the verification of the writer's calculation of the correct value and form for the coefficient of viscosity may be very easily effected by simple mechanical considerations. It follows from the simplest axioms of dynamics and statics that the shear on the face of the belt (upper plane, moving) is precisely the same as on any other horizontal plane within the fluid, and that the transverse velocity gradient must also be the same as at any level in the fluid. Therefore, the shearing stress, or friction, per unit area of the face of the belt is $\mu \frac{du}{dz}$, and, as the velocity of the belt is $Z \frac{du}{dz}$, in which, Z is the height of Maxwell's mechanism, the power supplied to the whole device, of length, X, and breadth, Y, is $\mu\, Z \left(\frac{du}{dz}\right)^2 \times XY$. Hence, since the whole volume is XYZ, and since the whole of the energy must be dissipated into molecular motion on account of the steady state of the mass-motion of the fluid, the time-rate of dissipation of energy per unit volume of Maxwell's device must be, $\Phi = \mu \left(\frac{du}{dz}\right)^2$.

Therefore, as before, $\mu = \frac{1}{6} \rho \bar{c} l$, which is another independent verification of the formula for the coefficient of viscosity of a gas, in terms of the density, molecular velocity, and mean free path. Both these verifications of the writer's calculations are entirely independent of any explicit considerations of momentum.

Such are the results to be expected by considering the effects of the collisions of spheres, or any other type of bodies of perfectly elastic quality. It is not stated by the writer that the collisions of elastic spheres are attended with an actual dissipation of energy. The only dissipation which occurs, is that of mass-motion when it is transformed into molecular, or heat, motion, and as that mass-motion is being replenished by the motion received from the upper plane, at a rate equal to that of its transformation into heat, the state of the gas will not become steady until the loss of heat by conduction and radiation balances the energy supply, so as to bring the temperature of the gas to a limit consistent with surrounding conditions.

It will be of interest to predict the result to be expected after correcting values which have been computed by the erroneous formulas connecting viscosity and free path. Jeans gives[100] the upper limit values for the molecular radii of sixteen different substances which the correct values should not exceed. In another table he gives corresponding lower limits. Of the sixteen supposedly correct values calculated by Jeans with the uncorrected formulas, eight exceed their upper limits.

Jeans states:

"* * * the circumstance that some of the values of $\frac{1}{2}\sigma$ (molecular radius) given in the last column are greater than their upper limits is one that demands explanation. * * * If the molecules were known to be hard spheres the circumstances just mentioned would be perfectly incomprehensible."

After correcting the sixteen supposedly correct values by means of the factor, $\sqrt{\frac{1}{2}}$, previously deduced, corresponding to a reduction of about 30%, it will be found that all the corrected values fall between their upper and lower limits, as they should.

This, therefore, indicates that by applying the writer's correction (halving) to the numerical factors of the various modifications of Maxwell's formula, which correction is in accord with correct uses of momentum and Newton's laws of motion, there should be greater harmony between the kinetic theory and experiment than there has been in the past.

Lamb extends[101] Bernoulli's equation for an incompressible fluid to one for a gas by mere addition to the energy supply of the expansion work, $\int p\,dv$. If other energy supplies and friction, which were not included by Lamb, are disregarded, a number of well-known flow formulas may be deduced from Lamb's formula; but, to consider all kinetic and potential possibilities, the writer framed the following equation:

$$p_1v_1 + I_1 + h_1 + L_1 + \frac{u_1^2}{2g} + JH = pv + I + h + L + \frac{u^2}{2g} + F \ldots\ldots (123)$$

in which, I is any internal potential; h and L are sensible and latent heats; JH is the work equivalent of any heat which enters the gas through the pipe skin, or otherwise; and F includes all losses due to friction and other agencies. The ideal gas formula is $pv = Rt$, while u represents velocity. Since v is a specific volume, each term is a mere length, thus relating to 1 lb. of gas.

Equation (123) is perfectly general, taking account of all stores of energy, and of all heat transfers between the gas and its surroundings, including the three standard Bernoullian stores. Relative to the conservative relation between mass- and molecular-motions discussed previously, Professor Unwin has made a most cogent hypothesis for the flow of a gas by supposing all friction immediately returned to the gas as molecular-motion. Application of Unwin's hypothesis to Equation (123) leads to a rigorous solution of that equation for the flow of a gas in any conduit, which solution, however, contains a baffling integral, $\frac{dy}{dx} = \frac{xy + ax^2 + b}{xy - mx^2 + n}$. The particular case of isothermal flow, however, is readily soluble when the external and internal temperatures, and internal pressures at pipe ends, are observed. For a permanent gas, since *pv, I, h,* and *L,* become constant for isothermal flow,

---

[100] "Dynamical Theory of Gases," p. 330.

[101] "Hydrodynamics," Article 22.

$$JH = \frac{u^2 - u_1^2}{2g} + z - z_1 + F$$

in which, by Unwin's supposition, F may be cancelled, since there must be inserted on the left an increase of molecular-motion equivalent to F, and both must be dropped. Hence, the only extra energy supply necessary is that transferred through the pipe skin. Making the proper substitutions, and reducing,

$$Q^3 - \frac{2g(z - z_1)}{v^2 - v_1^2} Q - \frac{1}{900} \frac{gJ}{(v^2 - v_1^2)} \frac{\log(T - t)}{r^2 \sum \frac{1}{k} \log_e \frac{r+1}{r}} = 0 \ldots\ldots\ldots (124)$$

Equation (124) is a cubic, to be used for determining the weight-rate of flow, Q, per unit area of pipe section, in which, r equals the internal radius of pipe, while the well-known summation relates to the three consecutive layers: Gas film; pipe skin; and outer film. Coefficients of additional pipe coverings may be included in the summation. When the pipe is level, $z - z_1 = 0$, the central term vanishes, and the solution becomes a cube root. Cubics of this form are easily solved by means of tables[102]

(*25*) *Individual Discussions.*—All the important questions raised in the discussions may be reduced to two: Fluid dynamics; coefficients and viscosity and kinetic theory. The former class arises in most of the discussions, while the latter arises mainly from the remarks and reference by Professor Bateman and Lieut. Roop, questions which have seldom or never been connected with models, but which are herein explained in this connection.

By reference to Articles (4) to (12), (16), and (22), it will be seen that in Newtonian model engineering "dominance" is immaterial to the engineer, except as to economy. Referring to Articles (4) and (18) to (24), one should not be led astray by ill-defined, little used sciences, possibly involving serious fundamental errors. The aero-difficulties raised by Professor Bateman, can all be avoided by either of the artifices of Appendix I, which the writer suggested in 1918.[103] This suggestion has actually been used with great success in testing airplanes.[104] Mr. B. Melvill Jones has stated:

> "The highly practical importance of some of these critical flows has led to great efforts being made to devise laboratory experiments in which the Reynold's number will be the same as that of the full-scale flow to be represented. One way in which this has been successfully accomplished, though at great cost, is to enclose the whole wind tunnel in an air-tight chamber pumped up to a pressure of some 20 atmospheres; this increases ρ some 20 times * * *."

---

[102] See "Recueil de Formulas et de Tables Numériques," par J. Hoüel, 1901, p. XXXII; also, Smithsonian Mathematical Tables, Hyperbolic Functions, by Becker and Van Orstrand, p. XXVI, Washington, 1909; Des Ingenieurs Taschenbuch der Hütte, Berlin, Eighteenth Edition. See Kennison's formula, *Transactions,* Am. Soc. C. E., Vol. LXXX (1916), p. 344.

[103] *Transactions,* Am. Soc. C. E., Vol. LXXXII (1918), see footnotes, pp. 1179 and 1187.

[104] Encyclopaedia Britannica, Vol. 1, p. 236.

Evidently, this refers to the tests at Langley Field, but the writer does not know that his suggestion has ever been mentioned by any of these other writers. It would surely not be correct routine to enclose the whole wind tunnel, as mentioned, thereby creating excessive cost. This points to the probable fact that the original designers had no thought of such an artifice as similarity control by pressure in designing the wind tunnel.

Dr. Th. v. Kármán might easily have discovered Maxwell's error in deducing the coefficient of viscosity of a gas by the kinetic theory, had he recognized the consequences of his method of comparing $\rho \bar{c} l$ with μ, in the article cited by Professor Bateman. However, the result would have been obtained by means of two compensating errors: (1) Use of momentum transfer ("impulstransport") regardless of Newton's laws of motion, as pointed out in Articles (18) to (24); and (2) use of one-half the mean free path instead of the whole.

The writer did not say: "Nothing has ever been added to any of Newton's theories." He referred only to specific theories, and re-affirms his statement. Relative to Lieut. Roop's assumption of Newton's ignorance of viscous friction, the best answer is given by O. E. Meyer in his "Kinetic Theory of Gases" (previously cited).[105] Meyer states:

> "Newton's theory will therefore be proved to be true if different measures of the internal friction, especially if made by different methods, give the same value for this constant. Such an agreement of measures has really been found, both for liquids and for gases, as will be described later in fuller detail. In gases a perfect agreement with experiment has also been found for those properties which, on the kinetic theory of gases, are connected with the coefficient of viscosity."

Newton's error in "Principia," when first applying his theory (as pointed out by Stokes[106] in 1845), as well as Maxwell's error, relates to viscosity.

It is idle to talk about reading a meaning into Newton's words before one has acquired the ability to get from those words the meaning really expressed, especially when read in the Latin. Lieut. Roop's remarks to the effect that a sphere would necessarily meet no resistance in an infinite frictionless fluid are most certainly wrong. If the velocity is high enough a cavity must form (just as Newton stated)[107] and resistance must occur, the dissipated energy resulting in accumulating kinetic disturbances in the fluid, which, however, may never be abated. (See Article (17).) Concerning frictional drag of a solid surface, the writer cautioned against making the error charged against him by Lieut. Roop (See "Historical Background" in the writer's paper). Relative to Lieut. Roop's remarks referring to Froude (see Articles (14) and (16)), there can be no greater mistake than to refer viscous resistances to an exponential power of the velocity. By so doing the two theoretically exact components of the coefficient of resistance in Article (14) are completely confused, thus blockading the development of the exact science of hydrodynamics as pointed out in Articles (4) to (11), and (14) and (15). Lieut. Roop is on the wrong "track" and will surely fail to secure results of permanent general value.

---

[105] In the section entitled, "Viscosity: Newton's Fundamental Law of Internal Friction," pp. 172-174.

[106] *Cambridge Transactions*, t. VIII, p. 303.

[107] *Transactions*, Am. Soc. C. E., Vol. 94 (1930), p. 836.

The writer was correct in stating that, if the over-all exponent of skin friction is 2, the scale of the model is immaterial. This fact was discovered many years ago by that most able analyst, M. J. Bertrand (see Articles (3) and (16)). If the exponent is not 2, a dynamic model cannot exist unless the scale is properly related to the constants involved with the model. It was to remove this inaccuracy from model technique that the writer adduced Equations (11), (12), (20), and (21), as fully explained in the paper (see particularly, Articles (4) to (13)). Lieut. Roop's models are not true dynamic models. Froude never claimed that they were anything but geometrical models. It is not the model method which excels, but Froude's ingenious method of correcting defective geometrical models by means of his tables of resistances applied to planks towed edgewise. The power of ships can be estimated nearly as well without such models, as Froude and others were amply able to do. In a true dynamic model it is never necessary to adjust the kinematic coefficient of viscosity, or any other coefficient, for various speeds, as that is properly cared for in the construction of the model to the specifications of the equations of similitude, Equations (11), (12), (20), and (21). (See Articles (4) to (13) as well as the main paper.) Lieut. Roop is wholly wrong in his idea that viscosity is not a dynamic phenomenon (see Articles (18) to (24)).

Mr. Karpov's methods seem to be similar to that of Eisner rather than to the new method which has been previously described by the writer.[108] The entire matter of elastic models arises very simply out of the equations of similitude, Equations (11), (12), (20), and (21). All one has to do is to see that any two independent elastic coefficients are properly installed in the materials of the model (synthesized materials) according to the specifications written from these equations, and he may then rest assured that every other elastic coefficient (excluding limit coefficients, which must be treated separately) will perform correctly. In the case of reinforced concrete, special materials must be found, or synthesized, so as to have these same two coefficients in the proper ratio to the other two in the concrete. When that is done, the entire reinforced structure will be in proper elastic regimen, assuming that the limit coefficients have also received proper attention throughout. The writer invites the attention of Mr. Karpov and other structural engineers, not only to his paper, but especially to the sections of this closure inclusive, from Articles (1) to (13), which will be found to give a generalization concerning coefficients and their classification.

In reply to Professor Draffin it may be stated that an immense amount of river flow and river-control engineering is done in the United States. The work on the Mississippi alone will compare favorably with all the rest of such work in the world during the past hundred years. Besides the work of the U. S. Army Engineers in general, only one name need be mentioned, that of the late James B. Eads, M. Am. Soc. C. E.

---

[108] *Transactions*, Am. Soc. C. E., Vol. LXXXII (1918), p. 1186–1188.

Relative to the last two lines of Professor Draffin's discussion, the writer states confidently, that, in respect to similitude, the United States is actually in advance of the rest of the world. Since all hydraulic frictional resistance of every kind is primarily due to viscosity, it is the most important constant in hydraulic models of all sizes (see Article (14)).

Neither g nor G relates to the acceleration of gravity in problems of similitude. They relate merely to resultant accelerations at particular points in all similar systems, and frequently may each be zero at particular similar points, regardless of what the gravitational force may be. Mr. Smith confuses the law of similarity with the laws of gravitation, which error has been treated in Article (16). The idea that there is a material difference between the flow in open and closed channels is wholly wrong. They are identical, in so far as dynamical relations are concerned, the same general equations applying to both, indiscriminately. Newtonian similitude is much more easily applied to closed channels than to open ones. This results from the fact that the acceleration ratio, $\frac{G}{g}$, can be controlled in the case of closed channels, whereas, in the case of open channels, it is frequently necessary to make that ratio unity, thus limiting testing conditions considerably. The acceleration ratio in closed pipes may be controlled simply by adjusting terminal pressures, just as they were controlled, or observed, in Stanton and Pannell's experiments, and many others since. The case for gases is similar to that for liquids (see Example 2 and Appendix I, as well as Articles (2), (10), and (14)).

Reynold's number is just as important in open channels as in their "equivalent" closed forms. The acoustic ratio is self-eliminating from true dynamic models, since it depends solely upon well-known physical constants which may be properly treated by the theory presented, so that no attention whatever need be paid to it (see Article (10)). Appendix I, which involves all equations of similitude, corrects permanently for combined compressibility and viscosity, without requiring calculations, thus providing automatically for velocity of sound and resistances, as successfully applied in the airplane tests mentioned by B. Melvill Jones. Far from using the dimensional method of analysis, as inferred by Mr. Smith, the writer uses only elementary algebra, as, for example, Equations (11), (12), (20), and (21). Of course, dimensions of terms occur (and, alas, should appear) in all dynamical equations. Isotropic elasticity, as used in structural engineering, is a perfect science (see Article (1) to (15)) for safe conditions of loading. All the constants depend upon only two, with a few limit coefficients (see Articles (5) to (11)). Therefore, all the elastic properties mentioned by Mr. Smith, and some others also, are easily determinable. There is less trouble than with fluids. Mr. Smith mentions vectors in connection with elasticity, but not with fluids. As a matter of fact, the fundamental equations of elasticity and of fluid motion are mutually convertible (see Article (14)), one into the other, so that there is no fundamental difference between them. Vector analysis finds its best foundation in parts of hydrodynamics, as, for example, in Stokes' solutions of important vector problems in the papers cited, some of which problems are famous. E. B. Wilson's presentation of Gibbs' "Vector Analysis" includes some of Stokes' problems and theorems reduced to modern vector notation. Other good works are: Mason and Weaver's "Electromagnetic Field"; Abraham's Theorie der Electrizität"; and, of course, Maxwell's "Electricity." These citations, including Stokes' papers, will give a real view of vector analysis and its development. Dynamic models are independent of vectors (see "Corollary" in Article (4)). Reference to Rankine's "Applied Mechanics" and the treatises on elasticity previously mentioned, will clear up all questions in elasticity.

By referring to Mr. Smith's citation to a correct "air screw" theory, it will be found[109] that Professor Gibson has given the substance of the writer's demonstration of the accuracy of Newtonian models of viscous flow as described in his paper on "Ice Diversion, Hydraulic Models, and Hydraulic Similarity" previously cited. The resulting theorem was stated by the writer[110] as follows:

> "What we have done is to show that the conditions of geometric and dynamical similarity can be imposed simultaneously in a hydraulic model so that it will reproduce the prototype both in form and action, and that this possibility is in full accord with the generally accepted laws of turbulent stream flow, including viscosity resistances."

Dr. Eisner's discussion is very interesting, but discloses the fact that he is inadvertently confusing Newton's theorem of similitude with Newton's laws of motion. Of Newton's theorem of similitude, he says: "* * * it states only the fact that forces acting upon masses produce accelerations." This is merely a qualitative statement of the second law. Furthermore, Dr. Eisner states: "The introduction of density according to Newton's general statement assumes in advance the requirement of general dynamic similarity." This is another result of Dr. Eisner's confusion. Moreover, although he mentions Newton's implied reference to corresponding forces, he neglects to mention Newton's expressed statement concerning corresponding accelerations, corresponding velocities, and corresponding linear dimensions, which he later calls "Froude's law." Similitude cannot be understood until Newton's definitions and theorem concerning it are understood (see Article (16)).

The elementary practical essence of Newton's theorem of similitude is the adjustment by capable engineers of corresponding accelerations (in modern words, "accelerative force" = acceleration), corresponding velocities, and corresponding linear dimensions, in such manner that Equations (1) (or Equation (12)) shall be satisfied simultaneously with the other adjusted specified similar conditions. Independently of any materials contained by the systems, the theorem clearly states that if these adjustments are made by the engineer, who then starts the bodies with similar motions, the dynamical and kinematical similarity will continue. There is no tacit assumption in the theorem concerning initial similarity (see Article (12)).

---

[109] "Similarity and Model Experiments," by A. H. Gibson, *Engineering*, March 21, 1924, p. 327.

[110] *Transactions*, Am. Soc. C. E., Vol. LXXXII (1918), p. 1188.

It makes no difference how the adjustments are effected (see Articles (1) to (16), and Article (26), Conclusion 10.)

Concerning Equations (11), (12), (20), and (21), the first is not to be independent. It is merely a specification of scale ratio (see Article (1)). The other three are solved for L, F, and T, using the physical constants of such materials as the engineer deems most appropriate. If resulting scales are not satisfactory, materials must be changed, treated, or synthesized, as now practised in the United States by the writer and others, so as to produce satisfactory testing conditions. The writer is advised by James W. Rickey, M. Am. Soc. C. E., that A. J. Ackerman, Assoc. M. Am. Soc. C. E., has put into execution this general idea of synthesizing concrete of proper ultimate strength (see Article (9), "Limit Coefficients") for the model of the obelisk at Chute à Caron.[111] This is "a step in the right direction." It is hoped that Mr. Ackerman may extend his control of materials so as to create proper values for all physical constants of his models by means of the equations of similitude (see Equations (11), (12), (20), and (21)).

Equations (20) and (21) represent as many equations as there are property constants to be introduced. When these property equations and Equation (12) exceed three in number, each additional property (in excess of two) must be "adjusted" by proper selection, treatment, or synthesis of the material. The problem is one of Newtonian model engineering. It is immaterial how adjustments are effected.

Dr. Eisner expresses Equation (40) in terms of "numbers," R, C, and F, which, indeed, can be done, the correct value in Example 2 being $\frac{\mathsf{R}^4\mathsf{C}^3}{\mathsf{F}}$. It is better to express characteristics by the constants, since they must be determined or given definitely, whereas, the "numbers" are not. If there are n property equations, there will be $n-2$ characteristics, each containing property ratios. After using the first four properties, including the ratio, G, each new constant ratio is matched to its own dimensional ratio in terms of the previously found L, F, and T, to get its value. In solving the first four equations, the process may be inverted so as to find values of three property ratios instead of L, F, and T.

It is really a mathematical fact that Equation (40) (eliminant), or its inverted form, sets up the condition that any four property ratios (scales, inverted form), including G, determine consistent scales (properties) for a model, based upon the corresponding solution of Equations (12) and (20) and the two Equations (21). Consequently, it is easily shown that the individual factors satisfy $\mathsf{F}_1 = \mathsf{F}_2$, $\mathsf{R}_1 = \mathsf{R}_2$, $\mathsf{C}_1 = \mathsf{C}_2$ (etc., *ad infinitum,* in general). It is much simpler by ratios; for example, $\mathsf{C} = \frac{k}{L\rho v^2} = 1$, since $E = \frac{k}{L}$. Thus, $\mathsf{C}_1 = \mathsf{C}_2$ etc. Although this alone is a necessary condition that elasticity may enter a model, it does not alone necessarily establish similarity, unless Newton's theorem of similitude is also satisfied, which may not be the case, as in the first part of Example 3. The same is true of properties in general. The writer's treatment of history covers about all there is of Newtonian models. The problem of providing suitable materials is not the main difficulty of Newtonian model engineering. The real difficulty is that caused by the novelty of the methods, and the scarcity of engineers who know anything about it. No criticism of past methods is intended, as Newtonian model engineering has played no part in them. Article (3), "Bertrand's Models," contains a discussion of Froude's law; the writer has treated the subject of accelerations in his reply to Mr. Smith; and the subject of properties is treated in Article (4), "Corollary, Fundamental."

---

[111] *Civil Engineering,* December, 1930.

Ratios L and T are not necessarily confined to narrow limits (see Article (2) and Example 3, in which, $G = 125$, in the part where similarity exists, by a very simple artifice; see, also, all other remarks on pressure). The writer uses mercury in small models with satisfaction. As stated in Article (4), Newtonian models are independent of functionality of physical properties, so that adhesion can be eliminated from relations between similar systems as treated by the writer. As a matter of fact, adhesion is probably a mere phenomenon of viscosity. Besides pressure, acceleration may be controlled as explained in a paragraph preceding Equation (44). It seems to the writer that Dr. Eisner's models are used empirically. They really do not constitute dynamic models. Newtonian models are exact, thus avoiding empiricism.

The object of deducing Equations (29), (30), and (31), was to show that the writer's equations of similitude, Equations (11), (12), (20), and (21), were capable of producing Newton's form for the velocity of sound in any medium, Equation (31). Laplace's correction of this formula was not, in reality, a correction of the formula, but an insertion of the correct value of the elasticity, when the formula was used to determine the velocity of sound in a gaseous medium. Dr. Eisner's method, especially by making transferability depend solely on linear scale, leads to difficulties such as mentioned in connection with the section on "Reynolds' Similarity" in the main paper. Thus, if a model of an airplane with a 75-ft. wing spread and a 1 000-h.p. motor, capable of attaining a speed of 200 miles per hour, is to be built to one-twenty-fifth of the full size, the model would need to have a wing spread of 3 ft. and its power plant would have to generate 25 000 h.p. so as to permit traveling 5 000 miles per hour. Newtonian model engineering avoids such impractical requirements because of its flexibility. It gives an engineer opportunities to use his technical skill.

Recently, there have been some very interesting experiments to determine internal elastic stresses by means of a condenser internally inserted, the effects being amplified by a radio tube. There have also been some tests by means of a magnetic needle. The measurement by polarized light is well known.

(*26*) *Conclusions.*—The following will summarize this paper and its discussion:

1.—All former conclusions are re-affirmed.

2.—The principle of momentum transfer, as it has been applied in the kinetic theory of gases, is invalid.

3.—The mean free paths which have been calculated by the erroneous formulas based upon momentum transfer, are all of just one-half their correct magnitude. It will be necessary to correct all these values by doubling them.

4.—Other magnitudes of the kinetic theory which have been calculated by means of the erroneous free paths, of which magnitudes there are many, will require correction by suitable alterations of formulas and values.

5.—All textbooks and handbooks, and all books of chemical and physical tables, should be carefully examined and revised, with reference to the errors mentioned. Such necessity for revision will be found to extend to tables, sections, and chapters on viscosity, heat conduction, and diffusion, if not elsewhere.

6.—Use of the kinetic theory of gases is not necessary, or desirable, in ordinary Newtonian model engineering.

7.—The first paragraph of "Conclusions" in the paper, is general as to physical properties and characteristic numbers, when appropriate interchanges of properties, numbers, dimensions, and notation are made. Similarity exists only in Newtonian form, regardless of physical properties, which must all be adjusted to the system in which they occur, as required by Newton's theorem of similitude.

8.—Froude's law is merely a corollary[112] of Newton's theorem of similitude, which corollary results by making the ratios of corresponding accelerations all equal to unity. Correctly stated, it is: "As between model and prototype, squares of corresponding velocities must vary directly as corresponding linear dimensions." M. J. Bertrand (1848) anticipated Reech, Froude, and Helmholz in stating it. This law holds rigorously only in true Newtonian similitude.

9.—Stanton and Pannell's experiment indicates most satisfactorily the truth of the hypothesis which the writer now makes, namely, that the great lacuna of hydrodynamics, including aerodynamics, is due wholly to the fact that hydrodynamics is not yet fully equipped with the complete mechanism and functionality of viscosity.

10.—Newton's theorem of similitude is one of adjustment of ratios of homologues. It is independent of dynamical laws of all kinds, with which the theorem has nothing to do directly. That is to say, any of the dynamical physical laws, including Newton's laws of motion, may be changed at will without destroying the similarity, provided the acceleration ratio is kept in adjustment, or is continuously re-adjusted, as required by the theorem. If materials and general conditions are unchanged, the original adjustment is permanent, even if the form and action of the model is changed.

11.—By Conclusions 9 and 10 and Articles (4) and (22), Newtonian model engineering stands proved as the only existing method which will yield a

---

[112] "History of Mathematics," Cajori, p. 380.

theoretically correct solution of every problem in rigid dynamics, hydrodynamics, aerodynamics, elasticity, and capillarity, either separately, or in any combination, without an actual construction and test in place of the full-sized prototype.

12.—By far the greater portion of the writer's closure results from his equations of similitude, Equations (11), (12), (20), and (21). These equations express the entire theory of similarity, and represent any kind of true dynamic model that can be devised, because they are the mathematical expression of Newton's theorem of similitude.

In a paper[113] entitled "Misuse of Postulations in Theory of Viscosity," the writer has elaborated this subject further and has treated aspects of it not included herein.

---

[113] *Bulletin*, Am. Math. Soc., Vol. XXXVIII, No. 5, May, 1932, p. 353.

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852

TRANSACTIONS

Paper No. 1797

TESTS OF BROAD-CRESTED WEIRS[1]

BY JAMES G. WOODBURN,[2] ASSOC. M. AM. SOC. C. E.

WITH DISCUSSION BY H. W. KING, SHERMAN M. WOODWARD, DAVID L. YARNELL, BORIS A. BAKHMETEFF, D. D. CURTIS, ERIK G. W. LINDQUIST, RALPH L. PARSHALL, AND JAMES G. WOODBURN.

SYNOPSIS

This paper gives the results of 305 tests made during 1928-29 at the University of Michigan, Ann Arbor, Mich., on broad-crested weirs of various designs in a rectangular wooden flume 2 ft. wide. The crests varied in breadth from 10 to 15.5 ft., and in slopes and combinations of slopes from level to 0.085. The range of head was from 0.5 ft. to 1.5 ft. and of volume of flow from 2.0 to 11.0 cu. ft. per sec.

The purpose of the tests was to obtain experimental data in regard to certain advantages suggested in favor of the broad-crested weir over the sharp-crested weir as a device for measuring flowing water. One advantage established by the tests is that the tail-water can be backed up to an elevation only slightly lower than the water surface at the down-stream end of the weir without changing appreciably the relation between head-water elevation and discharge. Moreover, if the slope of the weir is such that a hydraulic jump can be made to occur on the crest, nearly all the initial head can be recovered, still without changing the head-water-discharge relation.

This ability to operate when submerged adapts the broad-crested weir for use under conditions where head is at a premium, as, for example, in the measurement of irrigation water. The broad-crested weir, moreover, even when operating under a considerable degree of submergence, requires the measurement of head at only one point and is thus preferable to the submerged sharp-crested weir and the Venturi flume, which require the measurements at two points.

Another advantage is the possibility of causing flow to occur at critical depth at some predetermined point on the crest at which the depth can be readily measured. The discharge is then a simple function of the depth, independent of the confusing effects of velocity of approach.

Previous discussions of this subject had indicated that critical depth might be expected to occur at the change in grade between slopes of channel, one of

---

[1] Published in September, 1930, *Proceedings*.

[2] Associate Prof. of Hydr. Eng., State Coll. of Washington, Pullman, Wash.

which would maintain flow above, and the other below, the critical depth. To investigate this theory tests were made of weirs formed of 10 ft. of level crest followed by 3½ ft. of apron with a small down-stream slope, the entrance to the weir being rounded.

These tests showed that the location of critical depth is affected by waves on the crest. The conformation of these waves is, in turn, affected by the volume of flow. With flows up to about 8 cu. ft. per sec., critical depth in a number of tests occurred at the change in grade, provided the slope of the apron was not too steep; but in other tests the waves caused the position of critical depth to move short distances up or down stream. When the flow was increased to more than 8 cu. ft. per sec. the position of critical depth moved from 1 to 3 ft. up stream from the change in grade.

Waves occurred at certain volumes of flow with every weir model tested in which part or all of the crest was level. If the crest was given a slope sufficient to maintain flow without waves, critical depth was found to occur, as a rule, only at some point of the initial drop at the entrance to the weir, at which the water surface sloped steeply and a small error in selection of the point of measurement would cause considerable error in the measured depth.

The tests showed that there was no single location of critical depth for all volumes of flow. Some interesting relations, however, were brought out between the design of the weir, the number of waves on the crest, and the location of critical depth.

Much information was obtained in regard to weir coefficients and to the factors affecting their variation. Coefficients as high as 3.08 were reached with a weir with rounded entrance followed by a down-stream slope of 0.026. This slope of crest permitted the formation of the hydraulic jump and the recovery of nearly all the initial head.

INTRODUCTION

Few previous tests have been made of broad-crested weirs of the designs described in this paper. Experiments have been made by Bazin, Blackwell, Fteley and Stearns, the U. S. Geological Survey, and others, on many designs of dams and weirs not sharp-crested, but only a small part of their investigations has dealt with weirs with broad level crests or with broad crests having a small down-stream slope.

However, recent discussions and tests on the subject of the critical depth and hydraulic jump[3] have suggested certain advantages which the broad-crested weir may have over the sharp-crested weir as a device for measuring flowing water. These suggested advantages are:

---

[3] "The Hydraulic Jump in Open-Channel Flow at High Velocity," by K. R. Kennison, M. Am. Soc. C. E., *Transactions*, Am. Soc. C. E., Vol. LXXX (1916), p. 338; "Theory of Hydraulic Jump and Back-Water Curves," by S. M. Woodward, M. Am. Soc. C. E.; also "The Hydraulic Jump as a Means of Dissipating Energy," by R. M. Riegel and J. C. Beebe, The Miami Conservancy Dist., Tech. Repts., Pt. III, 1917; "The Hydraulic Jump and Critical Depth in the Design of Hydraulic Structures," by Julian Hinds, M. Am. Soc. C. E., *Engineering News-Record,* November 25, 1920; "The Improved Venturi Flume," by Ralph L. Parshall, Assoc. M. Am. Soc. C. E., *Transactions,* Am. Soc. C. E., Vol. 89 (1926), particularly discussions by Julian Hinds and J. C. Stevens, Members, Am. Soc. C. E., p. 841 *et seq.*; "Berechnung der Wasserspiegellage," by P. Boess, *Karlsruhe Forschungsheft No. 284,* Verein Deutscher Ingenieure, Berlin, 1927; "The Hydraulic Design of Flume and Siphon Transitions," by Julian Hinds, M. Am. Soc. C. E., *Transactions,* Am. Soc. C. E., Vol. 92 (1928), p. 1423.

1.—Its adaptability to the use of the hydraulic jump to reduce to a minimum the head lost in the weir; and

2.—The possibility of producing flow at critical depth with a resulting simple relation between depth and discharge which is independent of the confusing effects of velocity of approach.

Purpose and Scope of the Tests

The purpose of the tests was to obtain experimental data in regard to these two suggested advantages of the broad-crested weir. The breadth of crest of the weirs tested varied from 10 to 15.5 ft., with various down-stream slopes and combinations of slopes from level to 0.085. The range of heads was from 0.5 ft. to 1.5 ft. and of volume of flow from 2 to 11 cu. ft. per sec. These and other details of the tests will be given in the summaries of the experiments.

Coefficients of discharge of various weir models were determined both with free over-fall and with the weir submerged. The data thus obtained permitted a study of the effect of submergence, of the effect on the coefficient of various factors in the design of the weir, and of the variation of the coefficient with the head. Profiles of the water surface were made with different weir models to permit a study of the factors affecting the position of critical depth, with a view to suggesting if possible a design of weir which would produce flow at critical depth at the same point on the crest for all volumes of flow.

Notation

The following notation is used in the paper:

H = head on the weir, in feet.
H_a = total head, in feet, at any point in a flowing stream.
h_v = head due to velocity of approach, in feet.
D = depth, in feet.
D_a = depth in channel of approach, in feet.
D_c = critical depth, in feet.
V = velocity, in feet.
v_a = velocity of approach, in feet.
Q = discharge, in cubic feet per second.
Q_1 = discharge, in cubic feet per second, per foot of width.
C = weir coefficient.
g = acceleration due to gravity, = 32.16 cu. ft. per sec. per sec.
α = velocity-head coefficient, correcting for unequal distribution of velocities in cross-section; here considered = 1.0.

Review of Critical Depth Theory

The total head in any point of a flowing stream is:

$$H_a = D + \alpha \frac{V^2}{2\,g}$$

Considering $\alpha = 1.0$.

$$V = \sqrt{2\,g\,(H_a - D)}$$

Then,

$$Q_1 = D\,\sqrt{2\,g\,(H_a - D)}$$

Critical depth, by definition, is the depth at which, for any given total head, the discharge is a maximum. Differentiating Q_1 with respect to D and equating to zero,

$$H_a = \frac{3}{2} D$$

Substituting,

$$Q_1 = \sqrt{g}\ (D_c)^{\frac{3}{2}}$$

The discharge is, therefore, a simple function of the critical depth; the velocity of approach is not involved. Thus, if the critical depth can be made to occur at any known point where it can be measured accurately, a simple method of determining the discharge is available.

The effect of error in measurement of D_c may be shown by differentiating Q_1 with respect to D_c and dividing; that is,

$$d\,Q_1 = \frac{3}{2} \sqrt{g}\ (D_c)^{\frac{1}{2}}\ d\,D_c$$

$$\frac{d\,Q_1}{Q_1} = \frac{3}{2}\,\frac{d\,D_c}{D_c}$$

Thus, for any percentage, p, of error in D_c, the computed Q is in error 1.5 p per cent. This degree of accuracy is the same as that of the usual equations for rectangular weirs (aside from velocity of approach) and is greater than that of the equations for V-notch weirs, where an error of $p\%$ in measuring the head results in an error of $2.5p\%$ in the computed Q.

THE LABORATORY

Details of the hydraulic laboratory are shown in Figs. 1 and 3. The quantity of water flowing in the system was controlled by the gate valve at the upper end of the 15-in. pipe line.

The 90° V-notch weir by which the discharge was measured, was calibrated by means of the weighing tanks. The methods and results of this calibration are described under "Measurement of Discharge".

Weir.—At a point 32 ft. from the head end of the flume, the bottom was stepped up 1.75 ft. (Fig. 1), and carried for 10 ft. at this elevation. This formed a weir crest 10 ft. broad, with square-cornered entrance. Series *A* includes the tests of this weir. Rounded entrances of various curvatures were obtained by means of laminated wooden blocks set on bases of proper height and thickness against the up-stream end of the weir.

Variation in slope was obtained by a false bottom 12 ft. long. This bottom was given the desired slope by laying it on shims on the 10-ft. level section. Series *D D* and *D E* give the results obtained with two different slopes of crest.

Combinations of level and sloping crests were obtained by means of a platform 3½ ft. long, used with either the permanent level section or the sloping false bottom. With the former the platform was used as a down-stream apron with three different slopes (Series *DA*, *DB*, and *DC*) while with the sloping bottom the platform was placed ahead to form a level section of crest (Series *DF* and *DG*).

The flume and weir were built in August, 1928. Just prior to the tests which began in November, the crest of the permanent weir was planed to a true level surface, and the sides of the flume, as well as the weir crest, were scraped and sand-papered to give as smooth a surface as possible. The surfaces of the false bottom and short platform were also made smooth.

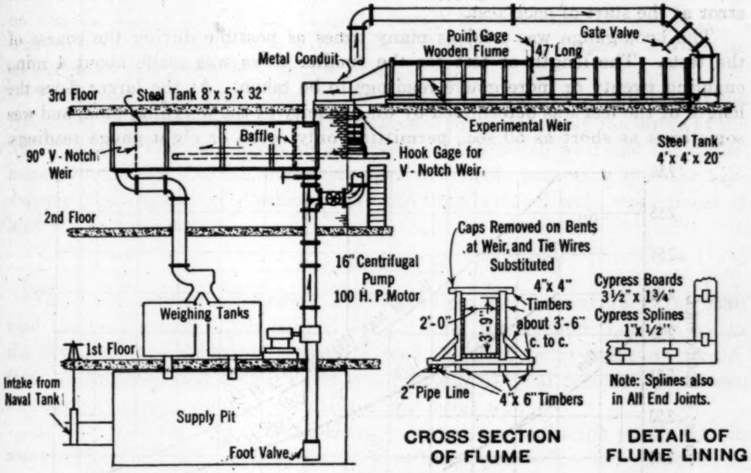

Supply Pipe 15" Dia.
Point Gage
Wooden Flume
47' Long
Gate Valve
Metal Conduit
3rd Floor
Steel Tank 8' x 5' x 32"
Experimental Weir
Baffle
Steel Tank 4' x 4' x 20"
90° V-Notch Weir
Hook Gage for V-Notch Weir
2nd Floor
Caps Removed on Bents at Weir, and Tie Wires Substituted
16" Centrifugal Pump 100 H. P. Motor
4" x 4" Timbers about 3'-6" c. to c.
2'-0"
3'-9"
Cypress Boards 3½" x 1¾"
Cypress Splines 1" x ½"
Weighing Tanks
1st Floor
Intake from Naval Tank
Supply Pit
Foot Valve
2" Pipe Line
4" x 6" Timbers
Note: Splines also in All End Joints.
CROSS SECTION OF FLUME
DETAIL OF FLUME LINING

FIG. 1.—DETAILS OF EXPERIMENTAL FLUME.

Aeration.—Five 1-in. holes to provide aeration of the nappe were bored in the wooden bulkhead forming the down-stream end of the permanent weir section. These holes were in a horizontal line 5 in. below the weir crest. Wooden plugs were used to stop the holes when required. As a general rule, the plugs were removed and the nappe was aerated in tests in which the back-water from the fall did not rise high enough to cause leakage through the holes. The nappe was thus aerated for flows up to about 7 cu. ft. per sec.

Measuring Elevation of Water Surface.—The elevation of the head water was measured by a point-gauge mounted in the center of the flume on the cross-cap of one of the bents (Figs. 1 and 3). The distance from the up-stream edge of the weir to the gauge was 7 ft. When the weir was extended up stream by the addition of the 3½-ft. platform, the gauge was moved up to the next bent. Profiles of the water surface over the weir were taken with another point-gauge mounted on a light channel iron. This movable gauge was supported by parallel channels attached to the tops of the posts.

Measurement of Discharge.—The discharge over the broad-crested weir was measured by means of a 90° V-notch weir which was calibrated gravimetrically just previous to the tests. The methods and results of this calibration will be discussed here briefly.

The flying-start-and-stop method was used in measuring the water. With the flow steady and the water discharging through the weighing tank, the

weight on the scale-beam was set to a small initial reading and the outlet valves of the weighing tank were closed. At the instant the beam rose to the cross-bar the two watches were started. The weight was then set ahead to the desired final quantity, and the watches were stopped when the beam rose to the bar again. The average of the watch readings, corrected for initial error, was used in computing the coefficient. The watches were checked for initial error at the start of each test.

The hook-gauge was read as many times as possible during the course of the tests. The length of test for the smaller flows was made about 4 min., enabling twenty or more gauge readings to be taken. At the larger flows the length of the test was determined by the capacity of the weighing tank, and was sometimes as short as 50 sec., permitting only seven or eight gauge readings.

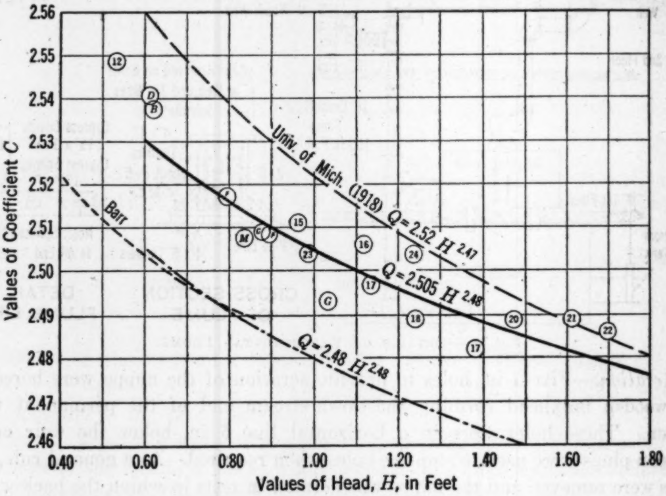


FIG. 2.—CALIBRATION TESTS OF 90° V-NOTCH WEIR: VARIATION OF C WITH H IN $Q = CH^{2.5}$ FOR THE EQUATIONS SHOWN.

The volume of flow, in cubic feet per second, was obtained by using 62.40 lb. as the weight of 1 cu. ft. of water. The computed volume of flow and the observed head on the weir corrected for zero reading of the gauge, were used to compute the coefficient, C, in the equation, $Q = CH^{\frac{5}{2}}$. Values thus obtained for C are plotted against values of H in Fig. 2, which shows the usual decrease in coefficient with increase in head. The equation, $Q = CH^{\frac{5}{2}}$, is awkward to use because it involves two variables. By making a slight change in the exponent of H, a suitable equation can be derived in which C is constant. The equation best fitting the points on the graph was adopted for use, namely, $Q = 2.505\,H^{2.48}$. The curve representing this equation (Fig. 2) was obtained by plotting values of C against H. C is computed from the equation, $C = \frac{Q}{H^{\frac{5}{2}}}$,

Q having been computed from the equation, $Q = 2.505\ H^{2.48}$. Thus, the curve and the test results are comparable graphically.

Curves of two other series of experiments with 90° V-notch weirs are also shown on Fig. 2. The equation, $Q = 2.52\ H^{2.47}$, was obtained as the result of experiments performed at the University of Michigan in 1918. In Scotland, some years previously, Barr made some experiments with results as also shown on Fig. 2. Barr's coefficients fit quite closely the equation, $Q = 2.48\ H^{2.48}$. The curve corresponding to this equation is drawn on the graph to serve as an extension of Barr's results to higher heads.

The writer's equation is seen to lie between the Barr experiments and the University of Michigan equation, but approaching more nearly the latter at the higher heads. The care taken in calibrating the V-notch weir and the agreement with former V-notch weir tests offer reasonable assurance that the subsequent measurement of discharge over the broad-crested weir was correct at least within 1 per cent.

TEST PROCEDURE

The usual method of making a test required four observers. One man read the hook-gauge which showed the head on the V-notch weir by which the discharge was measured. Another read the fixed point-gauge up stream from the broad-crested weir. These two gauges were read regularly at 30-sec. intervals. Twenty readings constituted the usual test.

A third observer operated the movable gauge to obtain the profile of the water surface (Fig. 3). Cross-sections were taken, proceeding down stream, at as many points as were required to permit drawing an accurate profile. The usual cross-section consisted of three readings, one at the center and one 3 in. from each side. The average of these readings was used in plotting the profile.

The fourth man kept notes for the second and third observers. The man reading the hook-gauge recorded his own notes.

A number of tests were made merely to determine coefficients of discharge without taking profiles of the water surface. These tests required an observer on the hook-gauge, one on the fixed point-gauge, and a recorder.

METHOD OF COMPUTING WEIR COEFFICIENTS

The weir coefficient for each test was computed by the method outlined by Horton in his treatise on "Weir Experiments, Coefficients, and Formulas."[4] The steps in the computation will be given briefly.

The average reading of the hook-gauge, corrected for the zero reading of the gauge, was used to find the discharge, Q, from a table prepared by the writer, based on the formula already stated, $Q = 2.505\ H^{2.48}$.

The average reading of the fixed point-gauge, corrected for the zero reading, was used as the observed head, H, on the weir.

The depth of water in the channel of approach is,

$$D_a = H + \text{the height of the weir}$$

---

[4] *Water Supply and Irrigation Paper No. 200*, U. S. Geological Survey, pp. 66-67.

Then,

$$v_a = \frac{Q}{2\,D_a}$$

$$h_v = \frac{v_a^{\,2}}{2\,g}$$

$$H_a = H + h_v$$

$$C = \frac{Q}{2\,H_a^{\frac{3}{2}}}$$

$$D_c = \left(\frac{Q_1}{\sqrt{g}}\right)^{\frac{2}{3}} = \left(\frac{Q}{2\sqrt{g}}\right)^{\frac{2}{3}} = 0.1982\;Q^{\frac{2}{3}} \text{ (see page 390).}$$

D_c was found from a table prepared by the writer for a rectangular flume, 2 ft. wide, giving critical depths for values of Q from 0 to 12 cu. ft. per sec.

Outline of Tests

An outline of the 305 tests of broad-crested weirs, with the series designations, in the chronological order of their performance, is as follows:

Tests of weirs with level crest, 10 ft. broad, with varying curvature of entrance:

Square cornered entrance	37 tests	Series *A*
Rounded entrance, 2-in. radius	13 tests	Series *B*
Rounded entrance, 3-in. radius	11 tests	Series *C*
Rounded entrance, 6-in. radius	26 tests	Series *D*
Rounded entrance, 3 by 6-in. ellipse, long axis horizontal	10 tests	Series *E*
Rounded entrance, 8-in. radius	12 tests	Series *F*
Rounded entrance, 4 by 8-in. ellipse, long axis horizontal	12 tests	Series *G*

Tests of weirs from 12 to 15.5 ft. broad, with various slopes and combinations of slopes. Entrance rounded on 6-in. radius:

10-ft. level, followed by 3.5 ft. of 0.015 slope	25 tests	Series *DA*
10-ft. level, followed by 3.5 ft. of 0.004 slope	36 tests	Series *DB*
10-ft. level, followed by 3.5 ft. of 0.085 slope	8 tests	Series *DC*
12 ft. of 0.004 slope	33 tests	Series *DD*
12 ft. of 0.026 slope	36 tests	Series *DE*
3.5-ft. level, followed by 12 ft. of 0.026 slope	33 tests	Series *DF*
3.5-ft. level, followed by 12 ft. of 0.004 slope	13 tests	Series *DG*
Total tests	305	

Analysis of Results

The results will be discussed under three heads:

1.—Effect of submergence on the weir coefficient.

2.—Location of critical depth.

3.—Variation of coefficient with head for different weir models.

Effect of Submergence on Coefficient.—The results of fifty-four pairs of tests which show the effect of submergence on the weir coefficient, are sum-

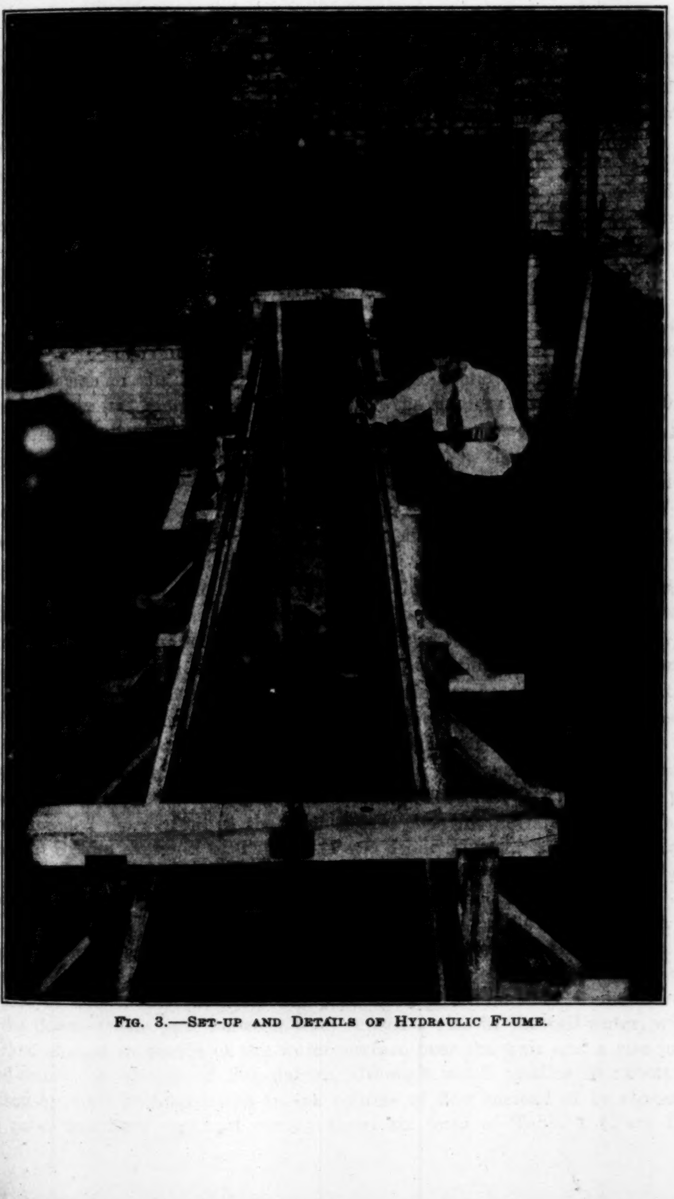


FIG. 3.—SET-UP AND DETAILS OF HYDRAULIC FLUME.

marized in Table 1. These tests were made on eight different weir models and covered the entire range of head from 0.5 ft. to 1.5 ft. Each pair, in nearly every case, was performed consecutively with no change between tests in the quantity of water flowing.

Either with or without the hydraulic jump, the average difference in the coefficients of the weirs submerged and with free over-fall was found to be inappreciable. In thirty-two pairs of tests in Table 1 (Part I), in which the jump could not be produced when the weir was submerged, the coefficient for the submerged weirs was, on the average, 0.219% less than for the weirs with free over-fall; while in twenty-two pairs of tests in Table 1 (Part II), in which the jump was formed on the crest when the weir was submerged, the coefficient for the submerged weirs was, on the average, 0.081% less than for the weirs with free over-fall. In the fifty-four pairs of tests in both parts, the average coefficient for the submerged weirs was 0.163% less than for the weirs with free over-fall.

The depths of submergence are shown in the profiles of water surface in Fig. 4. In the tests in which the slope of the crest was steep enough to permit the formation of the hydraulic jump (Series *DE* and *DF*); the loss of head over the weir was limited to about 0.2 ft. at the smallest flow and about 0.3 ft. at the largest flow. In the tests of weirs with slopes so flat that the jump could not be formed (Series *DA*, *DD*, and *DG*), the tail-water could be raised only to an elevation somewhat below the water surface at the down-stream end of the weir without noticeably affecting the conditions of flow over the weir, and the loss of head was from about 0.3 ft. for the smallest flow to about 0.6 ft. for the largest flow.

The attempt was made at the start of each test with the weir submerged to back up the tail-water to an elevation just under the maximum which would leave the weir coefficient unchanged. If at the start of a test, however, the tail-water was very close to the permissible elevation, slight fluctuations in volumes of flow might have caused this elevation to be exceeded momentarily with a resulting change in the conditions of flow over the weir and a decrease in the weir coefficient. The permissible elevation of the tail-water was purposely approached more closely in the tests in which the jump could not be formed than in those in which it could be formed, and as a result was probably exceeded more often in the former case than in the latter. This fact is reflected by the averages of Table 1 (Parts I and II) which show a difference in coefficients of weirs submerged and with free over-fall somewhat larger when the jump could not be formed than when it could be formed.

A typical example of the change in conformation of the water surface over a submerged weir without the jump when the tail-water rises through this permissible elevation is shown by Tests 4 and 5 of Series *DD* (Fig. 4). The volume of flow in the two tests was practically the same; but a small closure of the down-stream gate between tests caused a rise in the tail-water, with a marked change in profile of the water surface over the weir and a rise in the head-water. A change of this nature, although much smaller in extent and caused by a slight fluctuation in the volume of flow instead of by closure of the gate, doubtless occurred during those six tests of Table 1 (Part I) in

TABLE 1.—TESTS OF BROAD-CRESTED WEIRS: COMPARISON OF COEFFICIENTS OF SUBMERGED WEIRS AND WEIRS WITH FREE OVER-FALL.

(Length of weir, $L = 2$ ft.; entrance rounded on 6-in. radius.)

Test No.	Discharge, in cubic feet per second, Q.	Total head = observed head plus velocity head, H_a.	Coefficient in equation, $Q = C\,L\,H_a^{3/2}$, C.	Percentage by which C submerged is greater or less than with free over-fall.
PART I: WEIRS WITH SLOPE OF CREST SO SMALL THAT HYDRAULIC JUMP WILL NOT FORM.				
SERIES *DA*.				
1	2.603	0.6062	2.757	−0.62
2*	2.582	0.6054	2.740	
9	3.291	0.7056	2.776	−0.76
8*	3 278	0.7074	2 755	
10	4.135	0.8201	2.784	−0.22
11	4.118	0.8189	2.778	
12	5.649	1.0074	2.794	−0.21
13*	5.634	1.0070	2.788	
14	10.108	1.4758	2.817	−0.53
15*	10.040	1.4752	2.802	
16	8.170	1.2910	2.786	+0.32
17*	8 184	1.2882	2.795	
18	6.770	1.326	2.808	−0.46
19*	6 775	1.369	2.795	
20	4 922	0.9150	2.812	−0.71
21*	4 913	0.9183	2.792	
22	7 577	1.2241	2.7 8	+0.07
23*	7.483	1.2132	2.800	
24	9.189	1.3 64	2.785	0.00
25*	9.212	1.3 86	2 785	
Average for Series *DA*				−0.31
SERIES *DB*.				
1	2.628	0.6057	2.787	−1.04
2*	2.622	0.60 2	2.758	
3	4.036	0.8046	2.796	−0.11
4*	4.055	0.8077	2.793	
5	5.654	1.0041	2.810	−0.18
6*	5.640	1.0035	2.805	
7	7.601	1 2230	2.810	−0.43
8*	7.585	1.2249	2.798	
9	9.478	1.4189	2.804	+0.18
10*	9.503	1.4196	2.809	
Average for Series *DB*				−0.32

* Submergence.

TABLE 1.—(*Continued.*)

Test No.	Discharge, in cubic feet per second, Q.	Total head = observed head plus velocity head, H_a.	Coefficient in equation, $Q = C L H_a^{3/2}$, C.	Percentage by which C submerged is greater or less than with free over-fall.
		SERIES *DD*.		
1	2.670	0.5903	2.943	−1.02
2*	2.671	0.5946	2.913	
3	4.005	0.7755	2.932	−0.27
4*	3.993	0.7753	2.924	
7	5.656	0.9826	2.904	−0.45
8*	5.621	0.9813	2.891	
9	7.429	1.1855	2.878	−0.07
10*	7.369	1.1797	2.876	
11	8.879	1.3377	2.869	+0.24
12*	8.864	1.3343	2.876	
13	2.061	0.4959	2.951	+0.37
14*	2.064	0.4951	2.962	
15	3.423	0.6980	2.935	−0.03
16*	3.415	0.6970	2.934	
17	4.892	0.8884	2.921	−0.14
18*	4.898	0.8900	2.917	
22	2.396	0.5483	2.951	−0.03
24*	2.410	0.5505	2.950	
25	4.396	0.8277	2.919	+0.03
26*	4.427	0.8314	2.920	
27	7.137	1.1529	2.883	−0.21
28*	7.120	1.1527	2.877	
29	8.097	1.2570	2.873	−0.03
30*	8.076	1.2552	2.872	
31	9.318	1.3830	2.865	−0.10
32*	9.321	1.3840	2.862	
Average for Series *DD*				−0.13
		SERIES *DG*.		
4	2.758	0.6103	2.893	−0.21
5*	2.746	0.6094	2.887	
8	6.781	1.1208	2.857	−0.07
9*	6.810	1.1249	2.855	
12	9.711	1.4308	2.837	−0.28
13*	9.713	1.4336	2.829	
Average for Series *DG*				−0.19

*** Submergence.**

TABLE 1.—(*Continued.*)

Test No.	Discharge, in cubic feet per second, Q.	Total head = observed head plus velocity head, H_a.	Coefficient in equation, $Q = C\,L\,H_a^{3/2}$, C.	Percentage by which C submerged is greater or less than with free over-fall.
SERIES *D*.				
17	5.144	0.9412	2.817	−0.04
18*	5.258	0.9552	2.816	
Average for thirty-two pairs of tests in Part I	....	....	....	−0.219
PART II: WEIRS WITH THE HYDRAULIC JUMP FORMED ON THE CREST BY SUBMERGENCE.				
SERIES *DE*.				
1	4.361	0.8027	3.032	−0.03
2*	4.361	0.8032	3.031	
3	5.835	0.9780	3.016	+0.03
4*	5.845	0.9791	3.017	
5	7.385	1.1473	3.005	−0.33
6*	7.378	1.1492	2.995	
7	8.860	1.3024	2.981	+0.17
8*	8.856	1.3005	2.986	
10	2.822	0.5983	3.049	−0.39
11*	2.792	0.5956	3.037	
16	2.513	0.5502	3.079	−0.19
17*	2.529	0.5533	3.073	
18	4.012	0.7555	3.055	−0.20
19*	3.998	0.7548	3.049	
20	6.605	1.0600	3.026	−0.36
21*	6.558	1.0577	3.015	
22	5.229	0.9054	3.035	+0.16
25*	5.225	0.9039	3.040	
23	5.233	0.9051	3.039	0.00
26*	5.224	0.9039	3.039	
24	5.228	0.9035	3.044	−0.23
27*	5.239	0.9061	3.037	
28	7.546	1.1582	3.027	−0.36
29*	7.528	1.1592	3.016	
30	8.548	1.2669	2.997	+0.13
31*	8.547	1.2658	3.001	
32	9.772	1.3830	3.004	−0.20
33*	9.747	1.3825	2.998	
35	3.122	0.6385	3.060	+0.26
36*	3.146	0.6406	3.068	
Average for Series *DE*	....	....	....	−0.10

* Submergence.

TABLE 1.—(*Continued.*)

Test No.	Discharge, in cubic feet per second, Q.	Total head = observed head plus velocity head, H_a.	Coefficient in equation, $Q = C L H_a^{3/2}$, C.	Percentage by which C submerged is greater or less than with free over-fall.
SERIES *DF*.				
9	5.025	0.9059	2.914	−0.38
10*	5.007	0.9060	2.903	
11	6.995	1.1363	2.887	+0.10
12*	7.011	1.1375	2.890	
13	8.825	1.3274	2.885	+0.31
14*	8.833	1.3255	2.894	
16	7.642	1.2051	2.888	+0.10
17*	7.659	1.2060	2.891	
18	2.104	0.5050	2.931	−0.07
19*	2.092	0.5034	2.929	
20	3.416	0.6987	2.925	−0.10
21*	3.411	0.6984	2.922	
22	5.947	1.0192	2.890	−0.21
23*	5.956	1.0217	2.884	
Average for Series *DF*				−0.04
Average for twenty-two pairs of tests in Part II				−0.081
Grand average of fifty-four pairs of tests				−0.163

* Submergence.

which the coefficient submerged was more than 0.5% less than the coefficient with free over-fall. If these six pairs of tests are disregarded, the remaining twenty-six pairs give coefficients for the submerged weirs only 0.09% less on the average than for the weirs with free over-fall.

With the jump formed on the crest, the largest difference in coefficients submerged and with free over-fall was 0.39 per cent. Various positions of the jump were tried, but none closer to the entrance of the weir than about 5 ft., at which distance a slight fluctuation in volume of flow merely caused a small movement of the jump up or down stream with no other visible effect on the conditions of flow.

LOCATION OF CRITICAL DEPTH

Profiles of the water surface over the weir for each weir model tested, both submerged and with free over-fall, and with a range of head from 0.5 ft. to 1.5 ft., are shown in the series of graphs in Fig. 4; also the points at which the measured depth equals the computed critical depth for each flow.

The graphs show that none of the weir models tested produced flow at critical depth at any fixed location throughout the entire range of heads. At points where previous discussions had indicated that critical depth might be expected to occur, as, for instance, at the change from level crest to a small down-stream slope (Series *DA*, *DB*, *DF*, and *DG*), the depth of flow was found to be affected by waves on the crest of the weir. On the other hand, with crest slopes sufficient to maintain flow free of waves (Series *DE*), critical depth occurred, as a rule, only on the initial drop in water surface at the entrance to the weir. One series of tests in which the crest had a down-stream slope of 0.004 (Series *D D*) showed for the smaller flows a depth very near the critical through a distance of 3 or 4 ft. at about the center of the crest; but for flows larger than about 6 cu. ft. per sec., the depth was distinctly less than the critical at all points except on the initial entrance drop.

Although for the models tested no one location was indicated at which critical depth may be expected to occur at any flow, nevertheless, the profiles show some interesting facts in regard to the manner in which it varies with the number of waves on the crest; and also the manner in which the width and slope of the crest, the rounding of entrance, and the volume of flow affect the number and position of the waves.

Waves occurred at certain flows with every weir model tested in which part or all of the crest was level. The profiles of Series *B* to *G* show typical conditions of flow over a weir with a level crest, 10 ft. wide, with rounded entrance. The locus of the points of critical depth in the profiles of this group is a series of disconnected straight lines, apparently with about the same slope, each line containing the points of critical depth accompanying a particular number of waves on the crest. Thus, all profiles showing only one wave on the crest have their points of critical depth on Line *BB*; profiles with two waves have their points on Line *CC*; those with three waves, on Line *DD*; with four waves, on Line *EE*; and with five waves, on Line *FF*. Test No. 6 in Series *B* appears to be a border-line case between one wave and two waves, since its profile shows points of critical depth on both Line *BB* and Line *CC*. Line *AB*, containing critical depth points for flows of about 11 cu. ft. per sec. and greater, appears, from the few tests made at those flows, to slope up to the left instead of up to the right like the other loci.

The same arrangement of the points of critical depth is apparent also in the profiles in Series *DA* and *DB* for free over-all. A majority of the points of critical depth for the lower flows in these two series lie at, or very close to, the change in slope. However, even with these smaller flows, the effect of the waves is apparent in the variations of some of the points from the change in slope. At the larger flows, the points of critical depth all occur from 1 ft. to 3 ft. up stream from the change in slope, and their locus is similar to that of the points on the profiles with one wave in Series *B* to *G*.

The profiles show the variation of the number of waves on the crest with the volume of flow and with the breadth and slope of the crest. At the smallest flows tested (Series *B* to *G*, and *DA*), five distinct waves appear. With increasing flow, the number of waves decreases consecutively until at the largest flows the profiles present a smooth curve free from waves. Moreover, it is apparent that at least within the limits of these tests, the waves on a level crest continue to occur until smoothed out by sufficient increase in the slope

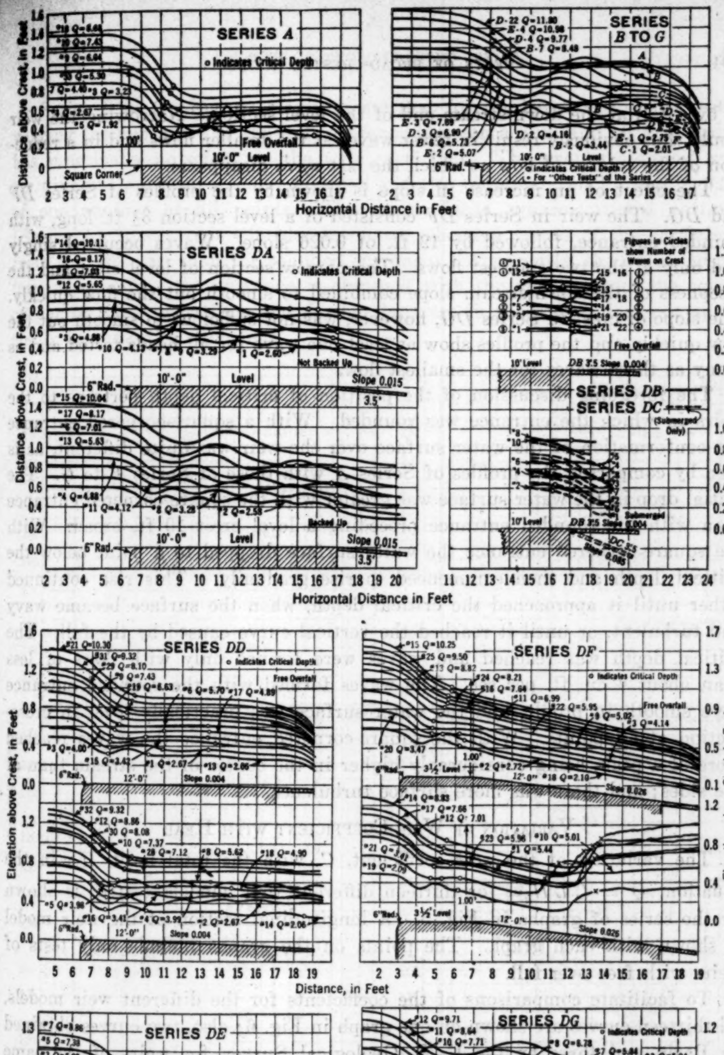


FIG. 4.—PROFILES OF WATER SURFACE AND LOCI OF CRITICAL DEPTH.

or by a fall at the down-stream end of the level section. Narrowing the weir would thus obviously result in fewer waves at the smaller flows and in a reduction of the volume of flow at which the last wave disappears.

The effect of an increase in slope is shown by the profiles of Series *DF* and *DG*. The weir in Series *DF* consisted of a level section 3½ ft. long, with rounded entrance, followed by 12 ft. of 0.026 slope. Waves occurred singly and only at the two smallest flows. The narrow section of level crest and the steepness of the down-stream slope combined to smooth out the flow quickly. The slope of crest in Series *DG*, however, was not sufficient to smooth out the flow quickly and the profiles show at least one wave for all heads tested and as many as three waves for the smallest flow.

The foregoing discussion of the position of critical depth pertains to the weirs in which the entrance was rounded. With a square-cornered entrance the conformation of the water surface over the weir was quite different, as is seen by comparing the profiles of Series *A* with those of Series *B* to *G*. The initial drop in the water surface was greater with the square-cornered entrance than with the rounded entrance preceding a level crest 10 ft. broad. With the square-cornered entrance the water surface dropped to a point below the critical depth and then commenced to rise gradually. This rise continued either until it approached the critical depth, when the surface became wavy and turbulent, or until it reached the vertical curve caused by the fall. The critical depth was reached and waves were formed only with flows of less than about 4 cu. ft. per sec. The waves formed with the rounded entrance were smooth in profile, and the water surface was practically level in cross-section at all points. With the square-cornered entrance the waves reached more of a peak, being considerably higher in the center of the stream than at the sides; and there was more surface turbulence.

VARIATION OF WEIR COEFFICIENT WITH HEAD

The variation of the weir coefficient, C, with the total head, H_a, in the equation, $Q = C L H_a^{\frac{3}{2}}$, for thirteen different weir models tested, is shown by the series of graphs in Fig. 5. A longitudinal section of the weir model is shown with each graph. The points on the graphs include only tests of weirs with free over-fall.

To facilitate comparisons of the coefficients for the different weir models, all thirteen curves are shown on one graph in Fig. 6; also, two curves obtained by Bazin and three by the U. S. Geological Survey for weirs of the same general type as those reported in this paper.

One important comparison (Fig. 6) is that for the weirs with the broader part of the crest level, with rounded entrance, and either with or without a narrow sloping down-stream apron, the coefficient increases with the head up to values of H_a of about 1.10, after which it is approximately constant (Series *B* to *G*, *DA*, and *DB*); while for weirs with the broader part of the crest sloping down stream, with rounded entrance, and either with or without a narrow level platform preceding the sloping section, the coefficient decreases with the head and shows a tendency to become constant at values of H_a of about 1.30 (Series *DD*, *DE*, *DF*, and *DG*).

Series *B* to *G* (Fig. 6) also indicate that for the breadth of weirs tested, varying the curvature of the weir entrance between a 2-in. and an 8-in. radius has practically no effect on the weir coefficient. The graphs obtained with six different curvatures—four circular and two elliptical—all lie within 0.5% of one another at all points. The graph for a square-cornered entrance (Series *A*), however, is considerably lower, showing values of the coefficients from 6 to 8% less than for rounded entrances. Apparently, the entire advantage of rounding the entrance is obtained with the change from the square corner to the 2-in. radius, and further increase in the radius does not appreciably increase the coefficient.

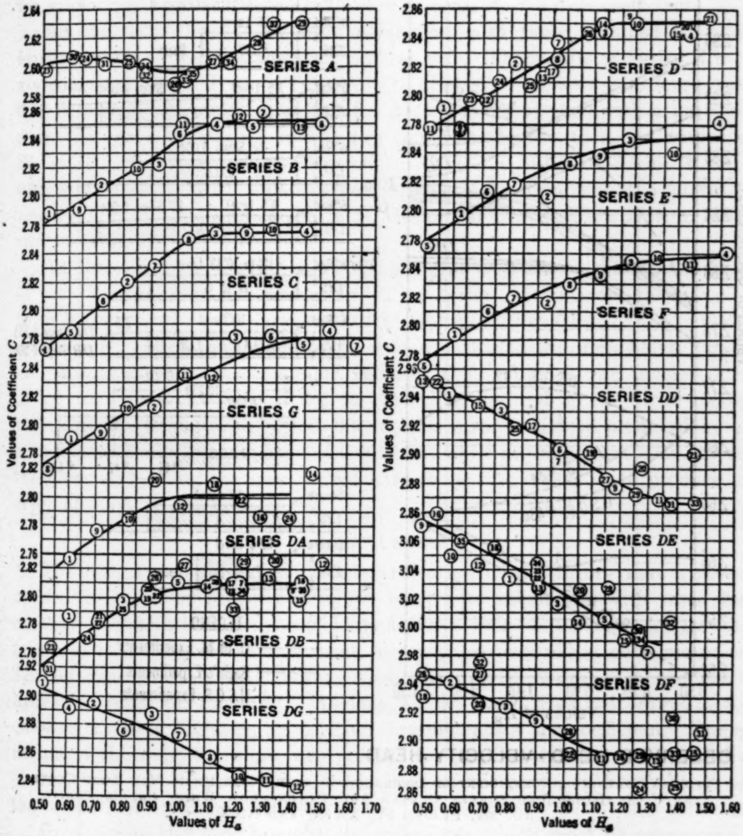


FIG. 5—VARIATION OF COEFFICIENT OF DISCHARGE, *C*, WITH TOTAL HEAD, H_a, IN FORMULA, $Q = C L H_a^{3/2}$. H_a = OBSERVED HEAD + VELOCITY HEAD; L = 2.0 FEET.

A definite increase in coefficient with increase in slope of the crest is also shown. A comparison of the curves of Series *DE* and *DD* for the same width of crest, but with different slopes, shows that the coefficients for the weir with a crest slope of 0.026 are about 4% higher throughout the range of heads tested than the coefficients for the weir with a crest slope of only 0.004. Tests in Series *DF* and *DG*, in which a 3½-ft. level section preceded the sloping section, also show higher coefficients for the weir with steeper slope. The difference is not so great, however, as in the narrower weirs of Series *DE* and *DD*, indicating that a greater width tends to reduce the effect of a change in slope.

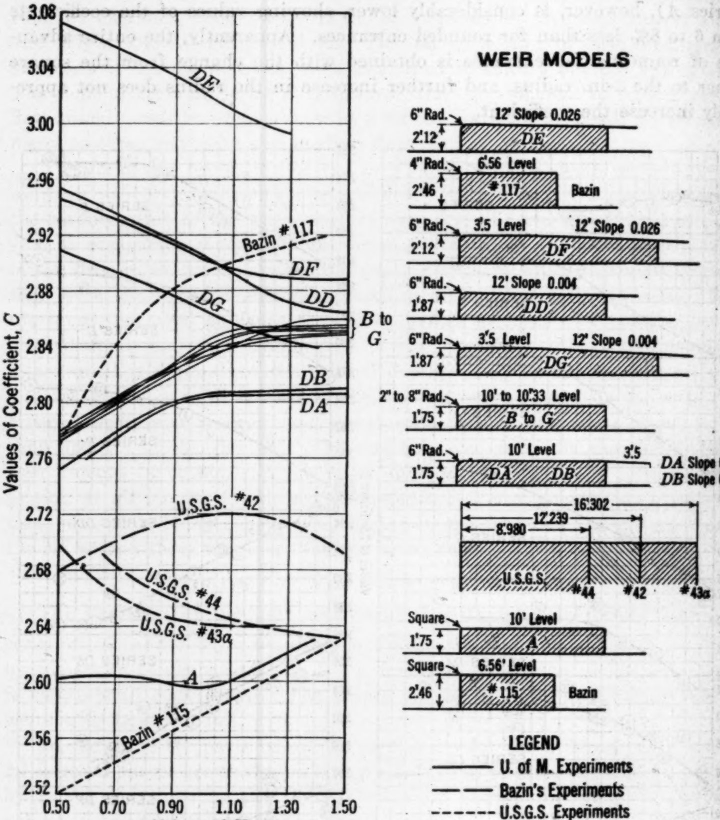


FIG. 6.—VARIATION OF COEFFICIENT OF DISCHARGE, *C*. WITH TOTAL HEAD. H_a BAZIN CURVE AND U. S. GEOLOGICAL SURVEY CURVES ARE TAKEN FROM *Water Supply Paper No. 200*, PLATES IV, XXIX, AND XXX.

The tests show a reduction in coefficient with an increase in the width of the weir. Comparison of Series *DA* and *DB* with Series *B* to *G* shows that a decrease of about 1% in the coefficient resulted when a 3½-ft. downstream apron with a slight slope was added to the weir with level crest. Moreover, comparison of Curves *DE*, *DF*, and *DD* shows that the coefficients were reduced about as much by the addition of 3½ ft. of level section ahead of the sloping section as by reducing the slope from 0.026 to 0.004. The height of these weirs varied from 1.75 to 2.12 ft., but there is little probability that the conclusions are appreciably in error on this account, especially since the variation in height was largely compensated for by including the correction for velocity of approach in the head used to compute the weir coefficients.

All the tests show fairly good agreement with those obtained by other experimenters on weirs of the same general type. The results of Series *B* to *G* are comparable with Bazin's Test No. 117 (Fig. 6) on a weir with level crest, 6.56 ft. broad, with rounded entrance. As might be expected with a narrower weir, the curve of Bazin's results lies somewhat higher than the curves of Series *B* to *G*. For a sharp-cornered entrance, the curve of Series *A* for a weir with a crest 10 ft. broad agrees in general location with the curves of Bazin's Test No. 115 and of the U. S. Geological Survey Tests Nos. 42, 43A, and 44 (Fig. 6).

An inspection of the individual graphs of Fig. 5 shows that in 201 tests of the 13 weirs, only one point, No. 21 in Series *DD*, is more than 1% off from its curve, while the large majority are well within 0.5% of the curve.

Since H_a enters the broad-crested weir equation to the three-halves power, an error of 0.5% in the computed value of C is caused by an inverse error of approximately one-third of 1% in observing the head on the broad-crested weir. At the lowest head tested, 0.5 ft., this error would be produced by an actual error in reading the head of 0.0017 ft.; while at the highest head, 1.5 ft., the error in reading the head would have to be only 0.005 ft. to produce an error of 0.5% in the coefficient.

CONCLUSIONS

The tests furnished experimental verification of the theory that a broad-crested weir can be properly designed to operate submerged with small loss of head, and with the same coefficients as for a weir of the same design with free over-fall. Measurement of head, moreover, is required at only one point. The minimum loss of head occurs when the weir is designed to produce a hydraulic jump on the crest. As long as the jump does not approach too close to the weir entrance, its position does not affect the coefficient. Proper design of the weir, therefore, can provide for considerable fluctuation in discharge, with resulting variation in position of the jump, without change in the head-water-discharge relation.

In addition, the tests brought out difficulties which must be overcome in producing flow at critical depth at some predetermined point on the crest of the weir. The chief difficulty is the formation of waves on the crest which affect the location of critical depth. Tests of other types of entrance with different alignment of the sides of the channel may lead to a design of weir which will maintain the flow at the desired depths without these disturbing waves. The measurement of flowing water with a broad-crested weir of this type would require only a determination of the depth at the predetermined point and would not involve velocity of approach, which has been a continued source of uncertainty in measurements with sharp-crested weirs.

PERSONNEL

The tests were performed by the writer under the leadership and guidance of H. W. King, M. Am. Soc. C. E., and with the assistance of C. O. Wisler, Assoc. M. Am. Soc. C. E., and Messrs. H. A. Seaman, J. G. Jobes, C. L. Palmer, A. J. Paddock, P. Singh, and C. Li, graduate students and seniors at the University of Michigan. Most of the drawings were by Mr. Seaman.

APPENDIX

SUPPLEMENTAL TESTS—WEIRS WITH APRONS INCLINED UP STREAM AND DOWN STREAM

By ALEX. R. WEBB,[5] M. AM. SOC. C. E.

SCOPE

This Appendix describes experiments conducted in continuation of Mr. Woodburn's work on broad-crested weirs. The object was to determine if possible a form of weir crest which would cause the locus of critical depths to lie in a vertical line. The general dimensions of the weir used are illustrated in Fig. 7. The apex of the two sloping aprons forms the highest section of the weir crest.

Results are given for 235 separate experiments. The down-stream apron was set at slopes of 1 in 15.4, 1 in 24.8, 1 in 25.0, 1 in 25.5, and 1 in 46.4. The slope of the up-stream apron was varied between 1 in 7.5 and 1 in 98.5. The heads ranged from 0.12 to 1.33 ft. and the discharges from 0.22 to 9.33 cu. ft. per sec.

The experiments show that for the flatter slopes waves formed on the up-stream apron and that the location of critical depth changed with the position of the wave. This condition was less marked for the smaller discharges. Except for these very flat slopes it was found that for each slope of the down-stream apron there was a corresponding slope of the up-stream apron, which caused critical depths for all discharges to lie in a vertical straight line near, but not necessarily above, the apex of the crest.

The effect of submergence on the relation of head to discharge was investigated by making parallel runs both with and without submergence. In all, fourteen pairs of comparative tests were made. The value of the coefficient, C, in the weir formula, $Q = C L H_a^{\frac{3}{2}}$, was also investigated. In addition, a series of experiments was conducted to check the formula for the V-notch weir which was used to measure discharges.

---

[5] Prof., Civ. Eng., Ohio Northern Univ., Ada, Ohio.

Laboratory

With slight modification the equipment used is the same as that described in the paper. The horizontal broad-crested weir was modified by building a small bulkhead at the up-stream end and attaching the sloping aprons, shown in Fig. 7. The down-stream apron, the false bottom referred to by Mr. Woodburn, was fixed for each set of experiments. The up-stream apron was attached by hinges to the bulkhead so that the slope could be easily changed.

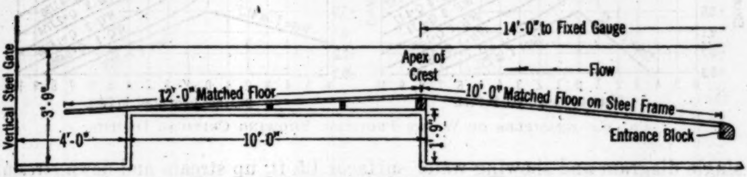


Fig. 7.—Longitudinal Section of Weir.

During the first experiments, Series *WA* to *WH*, uplift pressure caused the floor to pull away from the joists, forming a slightly convex surface instead of a plane. Thereafter, matched flooring attached to a welded steel frame ensured a plane surface under all conditions.

For all experiments except Series *WA*, *WB*, and *WC*, a 3 by 6-in. elliptical laminated block was bolted to the up-stream apron to give a rounded entrance.

Test Procedure

An outline of conditions to which the experiments conform is given in Table 2. In making each run, the profile of water surface over the weir was determined from observations 2 ft. or less apart. At the time of taking readings along the center line of the channel, simultaneous readings were made for the fixed gauge and the V-notch weir gauge. Six readings were taken across the apex of the weir while simultaneous readings were being taken at the fixed gauge. While these observations were being made, as many readings as possible of the V-notch weir gauge were taken. (See Table 3.) One reader attended each of the three gauges, and one man acted as recorder for the fixed gauge and the movable gauge.

Analysis of Results

The results of the investigation will be discussed under several heads.

Location of Critical Depth.—A profile of water surface was plotted for each run. Usually 6 to 8 runs (constituting a series of experiments) were conducted for each setting of the aprons. In all there were 37 series. The point of critical depth was determined from the formula, $Q = \sqrt{g}\ L\ D_c^{\frac{3}{2}}$, or, for length of weir, $L = 2$, $D_c = 0.1981\ Q^{\frac{2}{3}}$. This depth was plotted on each profile.

Profiles for each series of experiments were plotted on a single sheet. Typical examples are shown in Fig. 8. Five sets of experiments containing, respectively, 8, 10, 9, 6, and 4 series were performed. In each set the slope of the down-stream apron was kept constant and that of the up-stream apron was varied. Profiles of each set of experiments plotted to a larger scale on a

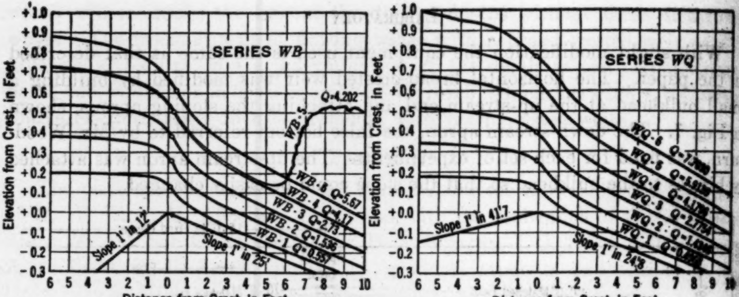


FIG. 8.—EXAMPLES OF WATER PROFILES, SHOWING CRITICAL DEPTHS.

single diagram and showing water surfaces 1.5 ft. up stream and down stream from the apex (Fig. 9), indicate that while there are individual variations, in general the points of critical depth lie on well-defined curves. These curves plotted to a greatly distorted scale are shown in Fig. 10.

TABLE 2.—OUTLINE OF EXPERIMENTS.

(Series *WA*, *WB*, and *WC* were made without the use of the block for rounded entrance. The wooden frame of the up-stream apron was changed to a welded steel frame after Series *WH*.)

Series.	Number of runs.	Up-stream slope.	Down-stream slope.
W A	6	1 in 7.4	1 in 25
W B	6	1 in 12	1 in 25
W C	3	1 in 46.8	1 in 25
W D	6	1 in 46.8	1 in 25
W E	13	1 in 31.5	1 in 25
W F	12	1 in 26.2	1 in 25
W G	3	1 in 22.2	1 in 25
W H	5	1 in 30 to 32	1 in 25
W I	15	1 in 25.6	1 in 24.8
W J	16	1 in 24.0	1 in 24.8
W K	7	1 in 24.45	1 in 24.8
W L	9	1 in 27.1	1 in 24.8
W M	5	1 in 30.3	1 in 24.8
W N	6	1 in 33	1 in 24.8
W O	5	1 in 34.8	1 in 24.8
W P	5	1 in 39.8	1 in 24.8
W Q	6	1 in 41.7	1 in 24.8
W R	6	1 in 50.2	1 in 24.8
W A A	8	1 in 40.3	1 in 25.5
W A B	5	1 in 37.6	1 in 25.5
W A C	6	1 in 42.3	1 in 25.5
W A D	6	1 in 47.1	1 in 25.5
W A E	6	1 in 34.1	1 in 25.5
W A F	6	1 in 29.8	1 in 25.5
W A G	6	1 in 40.0	1 in 25.5
W A H	6	1 in 41.3	1 in 25.5
W A I	6	1 in 40.5	1 in 25.5
B A	5	1 in 40.9	1 in 46.4
B B	6	1 in 50.0	1 in 46.4
B C	5	1 in 65.1	1 in 46.4
B D	4	1 in 80.5	1 in 46.4
B E	6	1 in 89.55	1 in 46.4
B F	6	1 in 98.5	1 in 46.4
C A	4	1 in 40.2	1 in 15.37
C B	8	1 in 30.15	1 in 15.37
C C	5	1 in 19.95	1 in 15.37
C D	4	1 in 24.99	1 in 15.37

TABLE 3.—SERIES *WP-5*: GAUGE READINGS AND HEADS FOR DETERMINING DISCHARGE, CRITICAL DEPTH, AND PROFILE OF WATER SURFACE.

V-NOTCH WEIR.		FIXED GAUGE.		Point on weir.	MOVABLE GAUGE.				Elevation above crest, in feet.
Observed readings, in feet.	Head, in feet.	Observed readings, in feet.	Head, in feet.		Observed Readings, in Feet.				
					Left.	Center.	Right.	Average.	
1.157	1.102	3.796	0.650	10	......	0.762	......	......	—0.172
1.157	1.102	3.797	0.651	8	......	0.856	......	......	—0.078
1.157	1.102	3.796	0.650	6	......	0.956	......	......	+0.022
1.156	1.101	3.796	0.650	4	......	1.041	......	......	0.107
1.158	1.103	3.796	0.650	3	......	1.096	......	......	0.162
1.158	1.103	3.797	0.651	2	1.152	1.154	1.158	1.155	0.221
1.159	1.104	3.798	0.652	1.5	1.189	1.189	1.198	1.192	0.258
1.158	1.103	3.798	0.652	1.0	1.239	1.235	1.242	1.239	0.305
1.158	1.103	3.797	0.651	0.5	1.294	1.293	1.296	1.294	0.360
1.1581*	1.1036	3.7972	0.6517	0.0	1.353	1.354	1.352	1.3527	0.4183
				0.0	1.353	1.352	1.352		
1.158	1.103	3.796	0.650	0.5	1.393	1.395	1.394	1.394	0.460
1.158	1.103	3.797	0.651	1.0	1.416	1.417	1.421	1.418	0.484
1.156	1.101	3.795	0.649	1.5	1.435	1.435	1.440	1.437	0.503
1.156	1.101	3.795	0.649	2	1.448	1.447	1.450	1.448	0.514
1.155	1.100	3.796	0.650	3	......	1.468	......	......	0.534
1.154	1.099	3.796	0.650	4	......	1.482	......	......	0.548
1.155	1.100	3.795	0.649	5	......	1.494	......	......	0.560
1.156	1.101	3.797	0.651	6	......	1.505	......	......	0.571
Average ..	1.1025	......	0.6509	....	......	......	......	......	......

*Average of 12 readings.

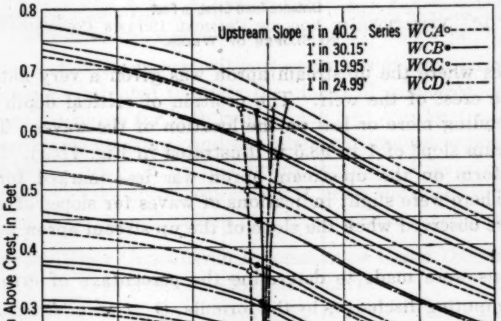


FIG. 9.—ENLARGED PROFILES, SHOWING CRITICAL DEPTHS.

In one series of experiments, where the slope of the down-stream apron was 1 in 25.5 and that of the up-stream apron, 1 in 40.3, the points of critical depth for all discharges fell on a vertical straight line over the apex. This condition was not duplicated for any other combination of slopes. With down-stream and up-stream slopes, respectively, 1 in 24.8 and 1 in 39.8, critical depths fell in a vertical line, 0.08 ft. up stream from the apex. Experiments on a much steeper down-stream slope (1 in 15.4) indicate that critical depths would lie in a vertical line about 0.04 ft. up stream from the apex when the up-stream slope was about 1 in 28.

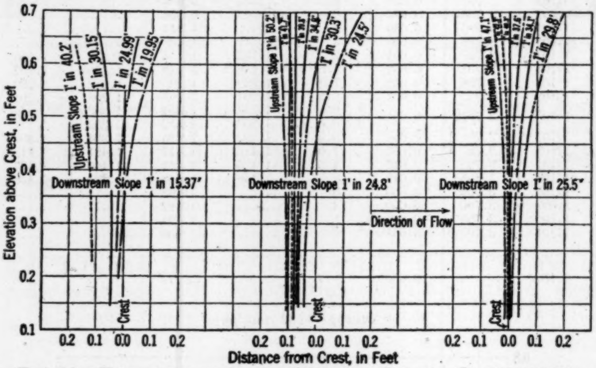


FIG. 10.—VARIATIONS IN LOCI OF CRITICAL DEPTHS CORRESPONDING TO SLOPES OF WEIRS.

In all cases where the up-stream apron was given a very flat slope, waves formed on the crest of the weir. The position of critical depth then became variable, depending more or less on the location of the waves. The condition for an up-stream slope of 1 in 98.5 is illustrated in Fig. 11(a). The tendency of waves to form on the up-stream apron was less marked for the smaller discharges. There were slight indications of waves for slopes of about 1 in 50. No waves were observed when the slope of the up-stream apron was 1 in 40, or less.

Calculations were made to determine the percentage of error that would result from computing discharges by the formula, $Q = \sqrt{g}\ L\ D_c^{\frac{3}{2}}$, substituting the measured depth over the apex D_c. The results of the computations for Series *WAA* and *WP*, in which the critical depths were approximately in verticals, over the apex and 0.08 ft. up stream from the apex, respectively, are shown in Table 4. In the first case the percentages of error that would result from using the measured depth over the apex as the critical depth, lie between the limits, — 0.7% and + 0.5 per cent. In the second case the corresponding percentages range from — 2.1 to — 7.4. These results show clearly the sensitive character of this type of structure.

Effect of Submergence on Head-Discharge Relation.—One of the advantages of weirs with broad crests is that the down-stream water surface may be at an elevation materially higher than the crest of the weir without affecting the relation between head and discharge. Weirs having this characteristic per-

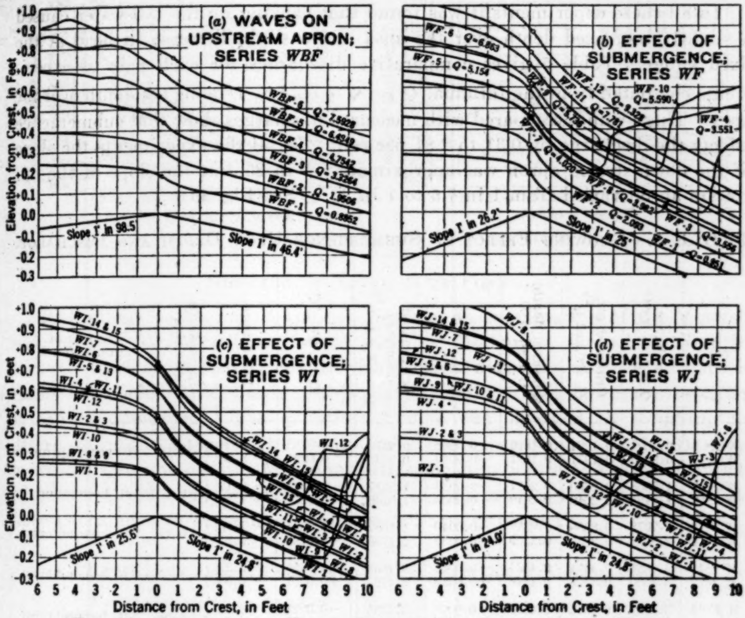


FIG. 11.—EXAMPLES OF WATER PROFILES AND INFLUENCE OF WAVES.

mit the measurement of discharges without great loss of head. Fourteen pairs of experiments were conducted to study the effects of submergence. In so far as practicable the discharge for both runs of any pair was kept constant. One run was made with free discharge and the other with the water backed up so as to cause a hydraulic jump on or just below the down-stream apron. Since the down-stream end of the apron was only 2 ft. from the outlet of the flume, it was not possible to determine the effect of a jump farther down stream.

TABLE 4.—EFFECT UPON CALCULATED DISCHARGE OF USING DEPTH OVER APEX INSTEAD OF CRITICAL DEPTH.

Series No.	Discharge, $Q = 2.505\, H^{2.48}$.	Critical depth, $D_c = 0.1981\, Q^{2/3}$.	Measured depth at crest = D.	$D - D_c$.	Calculated discharge, $Q' = \sqrt{g}\, L\, D^{3/2}$.	Error, $Q' - Q$.	Percentage.
W A A-3	0.4984	0.1246	0.1250	+0.0004	0.5006	+0.0022	+0.44
W A A-4	1.4377	0.2523	0.2515	−0.0008	1.4300	−0.0077	−0.53
W A A-5	2.7604	0.3898	0.3880	−0.0018	2.7410	−0.0194	−0.70
W A A-1	3.1153	0.4226	0.4231	+0.0005	3.1211	+0.0065	+0.21
W A A-6	4.0544	0.5038	0.5055	+0.0017	4.0750	+0.0206	+0.51
W A A-7	5.8270	0.6416	0.6405	−0.0010	5.8129	−0.0141	−0.24
W A A-2	6.1185	0.6628	0.6626	−0.0002	6.1157	−0.0028	−0.05
W A A-8	8.0123	0.7933	0.7933	0.0000	8.0123	0.0000	0.00
W P-5	0.6040	0.1416	0.1346	−0.0070	0.5594	−0.0446	−7.40
W P-4	1.8595	0.2995	0.2906	−0.0089	1.7764	−0.0831	−4.47
W P-3	3.1982	0.4300	0.4183	−0.0117	3.0678	−0.1304	−4.09
W P-2	5.1040	0.5873	0.5784	−0.0089	4.9875	−0.1165	−2.28
W P-1	7.5400	0.7619	0.7509	−0.0110	7.3788	−0.1612	−2.14

In all these experiments it was found that backing up the tail-water caused a slightly increased depth over the apex and a slight increase in head at the fixed gauge. Table 5 gives comparative discharges for each pair of experiments computed from the formula, $Q = \sqrt{g}\ L\ D^{\frac{3}{2}}$, D being the depth over the apex. These values compared with measured discharges show that submergence affects the discharge by 0.31 to 2.81 per cent. In all the experiments the slope of the down-stream apron was approximately 1 in 25, but the slope of the up-stream apron varied from 1 in 7.5 to 1 in 30. (See Fig. 11.)

TABLE 5.—Showing Effect of Submergence Upon Depth and Discharge.

Series No.	Discharge. $Q = 2.505\ H^{2.48}$.	Critical depth. $D_c = 0.1981\ Q^{\frac{2}{3}}$.	Measured depth, at crest = D.	$D - D_c$.	Calculated discharge. $Q' = \sqrt{g}\ L\ D^{3/2}$	Error, $Q1 - Q$.	Percentage.	Increase, in percentage.	Extent of submergence.
W I- 8	0.9657	0.1934	0.1896	−0 0038	0.9372	− 0.0285	− 2.96	−0.12	Beyond apron.
W I- 9	0.9633	0.1931	0.1891	−0.0040	0.9336	− 0.0297	− 3.08		
W J- 2	1.5558	0 2660	0.2621	−0.0039	1.5217	− 0.0341	− 2.19	0.47	4 ft. beyond crest.
W J- 3	1.5465	0.2649	0.2619	−0.0030	1.5200	− 0.0265	− 1.72		
W I- 2	2.2078	0.3359	0.3319	−0.0040	2.1682	−0 0396	− 1.80	0.45	7 ft. beyond crest.
W I- 3	2.2078	0.3359	0.3329	−0.0030	2.1780	−0.0298	− 1.35		
W I-11	3.5088	0.4574	0.4534	−0.0040	3.4633	−0.0455	− 1.29	1.87	8 ft. beyond crest.
W I-12	3.4566	0.4528	0.4546	+0.0018	3.4767	+0.0201	+ 0.58		
W F- 3	3.5556	0.4615	0.4597	−0.0018	3.5335	−0.0221	− 0.62	1.86	3 ft. beyond crest.
W F- 4	3.5508	0.4611	0.4649	+0.0038	3.5944	+0.0436	+ 1.24		
W F- 7	4.0195	0.5009	0.5014	+0.0005	4.0244	+0.0049	+ 0.12	1.85	8 ft. beyond crest.
W F- 8	3.9820	0.4978	0.5035	+0.0057	4.0511	+0.0691	+ 1.73		
W A- 4	3.9266	0.4931	0.5480	+0.0549	4.6000	+0 6734	+17.14	1.58	4 ft. beyond crest.
W A- 5	4.0102	0.5001	0.5607	+0.0606	4.7613	+0.7513	+18.72		
W B- 4	4.1699	0.5134	0.5422	+0.0288	4.5263	+0.3564	+ 8.55	1.28	7 ft. beyond crest.
W B- 5	4.2021	0.5160	0.5492	+0.0332	4.6150	+0.4129	+ 9.83		
W J-10	4.2434	0.5194	0.5121	−0.0073	4.1550	−0.0884	− 2.08	0.99	Beyond apron.
W J-11	4.2224	0.5177	0.5139	−0.0048	4.1763	−0.0461	− 1.09		
W J- 5	5.0350	0.5821	0.5771	−0.0050	4.9713	−0.0637	− 1.27	1.54	9 ft. beyond crest.
W J- 6	5.0340	0.5820	0.5831	+0.0011	5.0475	+0.0135	+ 0.27		
W I- 5	5.2280	0.5967	0.5961	−0.0006	5.2200	−0.0080	− 0.15	0 62	10 ft. beyond crest.
W I- 6	5.2690	0.5999	0.6018	+0.0019	5.2938	+0.0248	+ 0 47		
W F- 9	5.7360	0.6349	0.6330	−0.0019	5.7113	−0.0247	− 0.43	2.81	6 ft. beyond crest.
W F-10	5.5900	0.6241	0.6339	+0.0091	5.7229	+0.1329	+ 2.38		
W J-14	6.9138	0.7190	0.7156	−0.0034	6.8643	−0.0495	− 0.72	0.31	Beyond apron.
W J-15	6.9842	0.7238	0.7219	−0.0019	6.9557	−0.0285	− 0.41		
W I-14	7.0235	0.7266	0.7204	−0.0062	6.9343	−0.0892	− 1.27	1.48	Beyond apron.
W I-15	7.0168	0.7261	0.7271	+0.0010	7.0314	+0.0146	+ 0.21		

THE VARIATION OF WEIR COEFFICIENT WITH HEAD AND SLOPE

Values of the coefficient, C, in the weir formula, $Q = C\ L\ H_a^{\frac{3}{2}}$, show a wide variation for comparatively small changes in discharges and in slopes of aprons. This variation is indicated quite clearly in Fig. 12. In determining Q and H, gauge readings for a complete run were averaged.

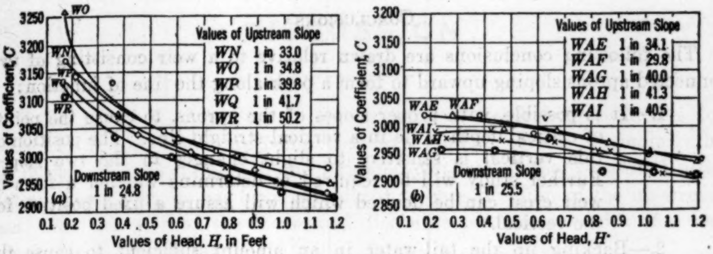


FIG. 12.—VARIATION IN VALUES OF COEFFICIENT, *C*.

CHECKING V-NOTCH WEIR FORMULA

Preliminary to beginning the experiments, an examination of the V-notch weir used by Mr. Woodburn for determining discharges, showed the face of the weir to be quite heavily coated with mineral deposits. A series of experiments was made with the weir in this coated condition and another series after it had been cleaned. The quantity of flow was obtained by weighing as described in the paper. The results of these experiments (Table 6) show average discrepancies with Mr. Woodburn's formula before and after cleaning, of 0.44 and 0.33%, respectively. In view of this close agreement Mr. Woodburn's formula was accepted as being substantially correct.

TABLE 6.—EFFECT OF DEPOSITS ON FACE OF V-NOTCH WEIR.

Head on V-notch weir, in feet.	Discharge obtained by weighing.	Discharge computed by formula, $Q = 2.505\ H^{2.48}$.	Percentage in error.
	BEFORE CLEANING.		
0.83156	1.5664	1.5862	+1.26
0.93995	2.1256	2.1488	+1.10
1.05436	2.8309	2.8570	+0.92
1.15504	3.5636	3.5808	+0.48
1.15995	3.6118	3.6197	+0.22
1.16639	3.6546	3.6693	+0.40
1.26225	4.4590	4.4634	+0.10
1.39642	5.7543	5.7342	−0.35
1.39914	5.7743	5.7617	−0.21
Average..........	....	....	+0.44
	AFTER CLEANING.		
1.15604	3.5688	3.5893	+0.54
1.16309	3.6197	3.6426	+0.64
1.23025	4.1533	4.1892	+0.87
1.22749	4.1545	4.1657	+0.15
1.29310	4.7165	4.7390	+0.48
1.29335	4.7234	4.7411	+0.38
1.39090	5.6864	5.6779	−0.15
1.39220	5.7041	5.6910	−0.24
Average..........	....	....	+0.33

CONCLUSIONS

The following conclusions are drawn relative to a weir consisting of two connected aprons sloping upward to form a peak along the line of junction:

1.—It is possible, with proper slopes of the aprons, to cause the points of critical depth to lie in a vertical straight line. The position of this vertical is sensitive to slight changes in the two slopes. Further study will be required to determine whether a form of weir crest can be devised which will assure a fixed position for the vertical.

2.—Backing up the tail-water in an amount sufficient to cause the hydraulic jump to occur upon the apron increases the depth of water on the weir.

3.—Values of the coefficient, C, in the weir formula, $Q = C L H_a^{\frac{3}{2}}$, are very sensitive to small changes in the slopes of the aprons.

ACKNOWLEDGMENTS

The experiments were performed under the direction of H. W. King, M. Am. Soc. C. E. Harry Larson, Assoc. M. Am. Soc. C. E., collaborated in conducting the experiments and in computations. The following students in the College of Engineering, University of Michigan, assisted in the laboratory, Messrs. E. V. Schulhauser, V. K. Ipe, M. Mortenson, D. W. Richardson, I. Curtis, F. F. Smalla, and C. O. Bloom.

During the past year (1929-30) the Society's Special Committee on Irrigation Hydraulics has co-operated in the experiments. In particular, D. C. Henny, and J. C. Stevens, Members, Am. Soc. C. E., Chairman and Secretary, respectively, of the Committee have made valuable suggestions.

DISCUSSION

H. W. King,[*] **M. Am. Soc. C. E.** (by letter).—Critical-depth devices for measuring flowing water are being used to a limited extent, particularly by the Bureau of Reclamation in the irrigated districts of the Western United States. The discharge has been considered to be a function of the up-stream head. For this method the theoretical discharge per foot of width of a rectangular cross-section is:

$$Q_1 = 3.087\, H_a^{\frac{3}{2}} \qquad (1)$$

Since H_a is the sum of the static head and velocity head the problem of correcting for velocity of approach is the same as for the weir. Moreover, all the tests in the paper and in the Appendix show that the coefficient varies considerably from the theoretical value of 3.087. Used in this manner the critical-depth device, therefore, does not provide a precise means of measuring flowing water. It is useful, however, for approximate measurements such as are satisfactory for apportioning irrigation water.

If a structure could be designed so that it would be known in advance that flow at critical depth would occur at a definite cross-section for all discharges, the discharge could be determined by measuring the critical depth, D_c, at this section and then making proper substitution in a simple rational formula. For a rectangular cross-section the discharge per foot of width is:

$$Q_1 = \sqrt{g}\, D_c^{\frac{3}{2}} \qquad (2)$$

With such a device there would be no coefficient to apply for velocity of approach, nor any other corrective factor to consider, and the measurement of discharge would be reduced to its simplest terms. Another advantage is that it provides for measuring discharges with a minimum loss of head, as the tail-water can be nearly as high as the head-water without changing the position of critical depth.

If a channel is sufficiently constricted either by elevating the bottom, or by drawing in the sides, or by combining the two so as to form a short reach which has a gently sloping bottom, flow at critical depth will occur near the upper end of this slope. The position of critical depth can be predicted approximately, but not with the accuracy required if discharges are to be determined from measurements of this depth. Investigations of several types of structures by the Society's Special Committee on Irrigation Hydraulics, and by others, have failed to produce a design for which the position of critical depth could be predicted accurately.

Professor Woodburn began an investigation of this problem in the fall of 1928. His experiments on a broad-crested weir with rounded entrance and gently sloping crest showed that critical depths for the different discharges lie in quite regular lines down stream from the upper edge of the weir. The law affecting their position could not be determined. Since critical depths on this structure, as well as on other structures with which the writer is familiar, occur where the water surface has a comparatively steep slope, it is necessary that their positions should be known within very narrow limits. An error of ½ in. up stream or down stream in the point of measuring critical depth results in an error of several per cent. in the discharge determined by Equation (2).

---

[*] Prof. of Hydr. Eng., Univ. of Michigan, Ann Arbor, Mich.

An unsuccessful attempt was made to flatten the water surface near the point of critical depth by attaching an apron with a gentle slope to the up-stream end of the weir. Too flat a slope of the apron, by producing flow in the unstable condition existing near critical depth, causes waves to form on the weir crest with one or more points of critical depth occurring most irregularly. It appeared, therefore, that to assure the occurrence of critical depth at a fixed section very flat slopes on the weir crest should be eliminated.

Since Professor Woodburn's experiments with a single sloping crest showed critical depth to occur down stream from the upper edge of the weir, while with a level apron attached to the up-stream edge critical depth moved up stream and over the level apron, it appeared that the point of critical depth could doubtless be moved up stream or down stream by changing properly the respective slopes of the two aprons. Professor Webb's experiments were undertaken for the purpose of investigating this principle. (See Appendix I.)

He was successful in finding slopes for the two aprons which caused the critical depths for all discharges to lie very close to vertical lines. It was found, however, that slight changes in the slope of either apron would displace points materially from these verticals and also that the position of the vertical when it was obtained was not precisely the same for the different combinations of slopes. One series of experiments in the Appendix shows the vertical to be over the apex; another shows it to be 0.08 ft. up stream from the apex; while a third indicates a position about midway between the other two. In view of the steep slope of water surface that obtains, if a device of this type is to be useful for precise measurement of water the position of critical depth must be known between much narrower limits than these.

The investigations of Professors Woodburn and Webb give much additional and useful information regarding flow at critical depth. They have not completely accomplished the original purpose of devising a structure which will produce critical depth at a predetermined cross-section. They have made progress, however, and by bringing to light conditions which affect the stability of flow as well as the position of critical depth, they have paved the way for intensive investigations in the future. In the writer's opinion it is quite probable that a design satisfactory for measuring flowing water by this method may yet be devised.

SHERMAN M. WOODWARD,[7] M. AM. SOC. C. E. (by letter).—Whenever flow is near the critical depth and critical velocity, the depth is in a condition of peculiar unsteadiness, leading easily to the formation of standing waves by some very slight disturbing factor. On this account, it will probably be difficult to devise any arrangement of the channel which will insure that at some prescribed point the depth shall be exactly the critical depth. This state of uncertainty does not extend, however, to the quantity of flow, since for a given total head it is at a maximum value during critical flow, and, therefore, independent of slight changes in depth. The result is that the very same conditions which are most unfavorable for an experimental measurement of critical depth are most favorable for the prediction of quantity of flow.

---

[7] Prof. of Mechanics and Hydraulics, State Univ. of Iowa, Iowa City, Iowa.

Compared with many other hydraulic phenomena, the flow over weirs is nearer stream-line flow and is, therefore, relatively more stable and less subject to accidental and erratic disturbance. Through a careful exploration of the velocity distribution throughout the sheet flowing over the weir crest it may be expected that the effect of each separate factor influencing the flow can be determined. After such an analysis it may be possible to know how the flow over the weir is influenced by the shape and angles of the approach channel, by the dimensions and curvature of the crest, by the friction on the crest, and by the nature of the exit conditions from the crest. The phenomena are particularly well adapted to study by the laws of similarity.

The ideal should be to accumulate experimental data sufficiently comprehensive and precise so that the exact flow over any full-sized overflow weir can be predicted in advance of its construction. The author and Professor Webb have made a most commendable contribution toward the realization of this ideal.

The paper opens up a series of fascinating questions, and it is to be hoped that it will be followed by further studies. This field appears to be particularly favorable for intensive studies designed to show the fundamental nature of stream flow.

David L. Yarnell,[8] M. Am. Soc. C. E. (by letter).—This paper presents interesting data on various types of broad-crested weirs both in regard to coefficients of discharge and the location of the critical depth.

During 1929 the U. S. Bureau of Public Roads, in co-operation with the State University of Iowa, conducted a series of experiments on the flow of water over full-sized sections of railway and highway embankments for the purpose of developing formulas for use in calculating the quantity of flood water flowing over such embankments.

Full-sized sections of single and double-track railway embankments, 10 ft. in length, were built in the laboratory testing canal. With the rails removed the embankments resembled somewhat the weir for Test Series *B* to *G*, Fig. 4.

The single-track railway test embankment with rails removed is shown in Fig. 13 (*a*). Profiles of the water surface are shown for free flow for quantities of from 12 to 78 cu. ft. per sec. The position of critical depth (indicated by circles) has been computed for each quantity of flow. It will be noted that, with certain exceptions, the locus of critical depth is approximately a vertical line. Fig. 13 (*b*) shows a check series of tests run on the same embankment for quantities of flow of from 15 to 74 cu. ft. per sec. The locus of critical depth was again found to be practically a vertical, as is also the case in Fig.

---

[8] Senior Drainage Engr., Bureau of Public Roads, U. S. Dept. of Agriculture, Iowa City, Iowa.

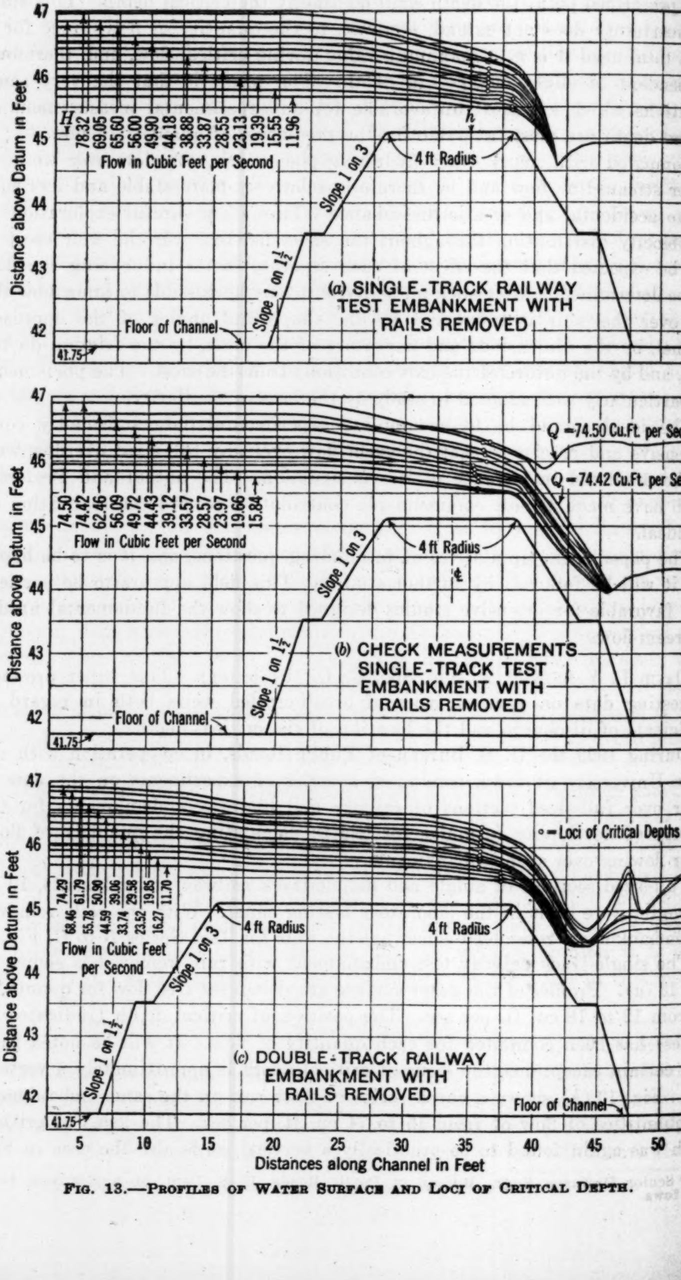


FIG. 13.—PROFILES OF WATER SURFACE AND LOCI OF CRITICAL DEPTH.

13 (*c*) which shows a series of tests on a double-track railway embankment (with rails removed) for quantities of flow of from 11 to 74 cu. ft. per sec.

The water-surface profiles were obtained from a series of staff gauges placed along both walls of the testing canal. Due to the continual pulsation of the water surface at the various gauges, the surface profile may be a few hundredths of a foot in error. This error is of great importance in locating exactly the position of critical depth. Hence, it is believed that for Fig. 13, all the loci of critical depths in each case would lie in verticals if the surface profiles could have been obtained more accurately.

Attention is called to the position of the tail-water for a flow of 74.50 cu. ft. per sec. in Fig. 13 (*b*). For this experiment the hydraulic jump occurred at Station 41, while the point of critical depth lies practically in the same vertical as in the case of the tests with free flow.

In connection with some tests on the hydraulic jump on sloping floors the writer had an opportunity to make a series of experiments on a weir with aprons inclined up stream and down stream. These experiments were made in a glass-walled channel, 2.5 ft. wide. The quantities of flow ranged from 3 to 8 cu. ft. per sec. The surface profiles and the loci of critical depths are shown in Fig. 14. In this case the locus of critical depth is in a vertical approximately at the crest of the weir.

Apparently, the author made no velocity traverses at the critical section. This would have added much value to the experimental work. The critical velocity as obtained from the formula, $V_c = \sqrt[3]{g\,Q}$ per unit of width, is the mean value for the critical section. Vertical velocity curves for the two largest discharges shown, Fig. 14, indicate that the greatest velocity is next to the crown of the weir and that the velocity decreases toward the surface of the water. This velocity distribution is characteristic of that existing in bends of pipes.

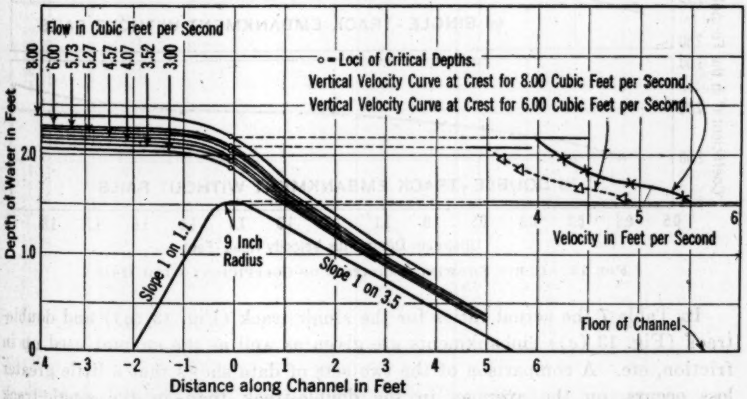


FIG. 14.—PROFILES OF WATER SURFACE AND LOCI OF CRITICAL DEPTH, AND VERTICAL VELOCITY CURVES FOR TWO DISCHARGES.

In Fig. 15 the coefficients obtained for the single and double-track railway embankments (without rails) are shown for the tests with free flow.[9] The standard American Railway Engineering Association railway embankments may be considered to have aprons inclined up stream and down stream, the slope being joined to the crown by a curve with a 4-ft. radius. Comparison of these coefficients with those given in Fig. 6 shows some differences, due no doubt to the fact that none of the weir models shown in Fig. 6 approaches the shape of a railway embankment. The coefficients for the weir with the rounded crest are given in Fig. 16.

If h (Fig. 13), represents the critical depth, and H, the head on the embankment, then $h = \frac{2}{3} H$. In an actual test, however, this relation does not always exist, the constant generally being less than two-thirds. The difference between the actual ratio and the theoretical, 0.666, is used up in overcoming friction on the up-stream face and crown of the embankment and may also be partly due to losses within the mass of moving water, such as eddies.

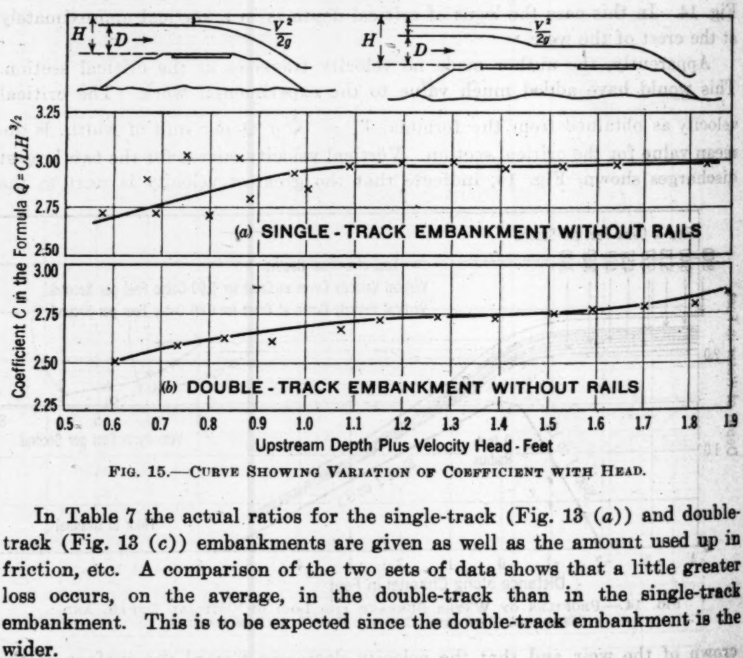


FIG. 15.—CURVE SHOWING VARIATION OF COEFFICIENT WITH HEAD.

In Table 7 the actual ratios for the single-track (Fig. 13 (a)) and double-track (Fig. 13 (c)) embankments are given as well as the amount used up in friction, etc. A comparison of the two sets of data shows that a little greater loss occurs, on the average, in the double-track than in the single-track embankment. This is to be expected since the double-track embankment is the wider.

---

[9] These figures are taken from the report on the Bureau investigation printed in *Public Roads*, Vol. 11, No. 2, April, 1930.

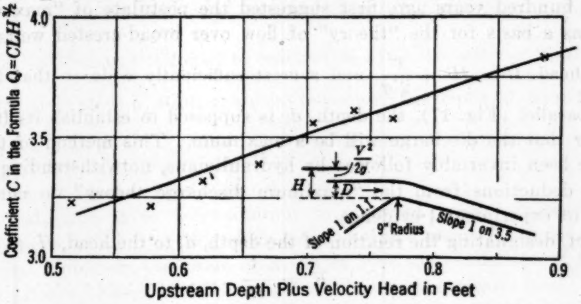


FIG. 16.—CURVE SHOWING VARIATION OF COEFFICIENT WITH HEAD.

TABLE 7.—COMPUTATION OF FRICTION LOSSES.

Discharge, in cubic feet per second. (1)	Head on embankment, H, in feet. (2)	Critical depth, h, in feet. (3)	Ratio, $\frac{h}{H}$. (4)	Critical velocity, V_c, in feet per second. (5)	$\frac{V_c^2}{2g}$. (6)	$h+\frac{V_c^2}{2g}$. (7)	$H+\frac{V_1^2}{2g}$. (8)	Loss due to friction Column (8, minus Column (7), in feet. (9)
SINGLE-TRACK EMBANKMENT.								
11.96	0.576	0.355	0.616	3.37	0.177	0.532	0.577	0.045
15.55	0.686	0.422	0.615	3.68	0.211	0.633	0.688	0.055
19.39	0.796	0.489	0.614	3.96	0.244	0.733	0.799	0.066
23.13	0.876	0.550	0.628	4.20	0.274	0.824	0.881	0.057
28.55	0.996	0.634	0.637	4.51	0.316	0.950	1.003	0.053
33.87	1.116	0.710	0.636	4.78	0.355	1.065	1.125	0.060
38.88	1.216	0.778	0.640	5.00	0.389	1.167	1.227	0.060
44.18	1.306	0.847	0.648	5.22	0.424	1.271	1.320	0.049
49.90	1.416	0.918	0.648	5.43	0.458	1.376	1.433	0.057
56.00	1.516	0.992	0.654	5.65	0.496	1.488	1.537	0.049
65.60	1.666	1.101	0.661	5.95	0.550	1.651	1.692	0.041
69.05	1.736	1.142	0.658	6.06	0.571	1.713	1.765	0.052
78.32	1.876	1.24	0.661	6.32	0.621	1.861	1.911	0.050
DOUBLE-TRACK EMBANKMENT.								
11.70	0.596	0.350	0.587	3.35	0.175	0.525	0.602	0.077
16.27	0.726	0.436	0.601	3.74	0.218	0.654	0.735	0.081
19.85	0.816	0.499	0.612	3.99	0.248	0.747	0.829	0.082
23.52	0.916	0.558	0.609	4.23	0.278	0.836	0.933	0.097
29.58	1.046	0.648	0.619	4.56	0.323	0.971	1.069	0.098
33.74	1.126	0.708	0.629	4.76	0.352	1.060	1.155	0.095
39.06	1.236	0.781	0.632	5.01	0.390	1.171	1.271	0.100
44.59	1.346	0.854	0.634	5.23	0.425	1.279	1.388	0.109
50.90	1.456	0.931	0.639	5.46	0.464	1.395	1.507	0.112
55.78	1.536	0.991	0.645	5.64	0.495	1.486	1.594	0.108
61.79	1.616	1.059	0.655	5.83	0.528	1.587	1.683	0.096
68.46	1.736	1.135	0.654	6.04	0.567	1.702	1.812	0.110
74.29	1.806	1.194	0.662	6.21	0.600	1.794	1.892	0.098

BORIS A. BAKHMETEFF,[10] M. AM. SOC. C. E. (by letter).—The remarks that follow are offered with the sole purpose of further elucidating the phenomenon described in Professor Woodburn's valuable paper.

"Theory" of Flow.—The author defines the critical depth as one "at which, for any given total head, the discharge is a maximum". It was Bélanger who,

---

[10] Prof. of Civ. Eng., Columbia Univ., New York, N. Y.

nearly a hundred years ago, first suggested the postulate of "maximum discharge" as a basis for the "theory" of flow over broad-crested weirs. Given the total head, $H = H' + \frac{v^2}{2\,g}$, and a crest sufficiently wide so that flow may **become parallel (Fig. 17), the depth, *d*, is supposed to establish itself in such a manner that the discharge will be a maximum. This method of treatment has since been invariably followed by hydraulicians, notwithstanding the fact that the deductions from the "maximum discharge theory" do not hold in** the face of experimental evidence.

In fact, designating the relation of the depth, *d*, to the head, *H*, as

$$K = \frac{d}{H} \quad \text{..............................(3)}$$

and introducing a "velocity coefficient", ϕ, to take account of the hydraulic losses which occur between Sections *A* and *B*, the velocity, *v*, and the discharge, *q*, per unit width in Section *B* will be:

$$v = \phi \sqrt{2\,g\,(H - d)} = \phi \sqrt{2\,g\,H} \sqrt{1 - \frac{d}{H}} = \phi \sqrt{1 - K} \sqrt{2\,g\,H} \ldots (4)$$

and,

$$q = d\,v = K\,H\,v = \phi\,K \sqrt{1 - K}\,H^{\frac{3}{2}} \quad \text{..............(5)}$$

Designating the "discharge coefficients" in the usual weir formula by *m* and $M = m \sqrt{2\,g}$,

$$q = m \sqrt{2\,g}\,H^{\frac{3}{2}} = M\,H^{\frac{3}{2}} \quad \text{..................(6)}$$

by comparison with Equation (5),

$$m = \phi\,K \sqrt{1 - K} \quad \text{..........................(7)}$$

and

$$M = \phi\,K \sqrt{1 - K} \sqrt{2\,g} \quad \text{...................(8)}$$

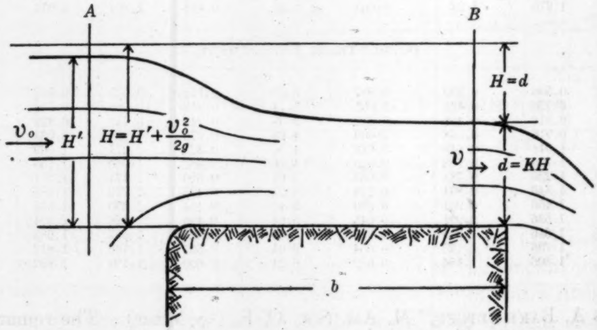


FIG. 17.—PARALLEL STREAM FILAMENTS IN UNIFORM FLOW.

Obviously, the "maximum discharge" is reached whenever *m* in Equations (6) and (7) is a maximum. For a given form of the entrance edge, ϕ may be assumed to be unchangeable. By writing,

$$\frac{dm}{dK} = \phi\sqrt{1-K} - \frac{1}{2}\,\frac{\phi K}{\sqrt{1-K}} = 0$$

the maximum discharge is found to occur at $K = \frac{2}{3}$ irrespective of ϕ. In other words, if the maximum discharge were to govern, the depth, d, near the end of the crest would be $d = \frac{2}{3}H$ for any type of crest, irrespective of its length, roughness, or the form of the entrance edge.

This deduction is in contradiction to observed facts. Professor Woodburn's profiles (Fig. 4), and other experiments,[11] show K to be less than two-thirds. Moreover, the value of K varies with the conditions of flow. For rounded entrance edges, under average conditions, K is nearly 0.63. For sharp entrance edges and relatively long crest, the value of K is nearly 0.6 and eventually less.

Cognizant of the discrepancy between the traditional theory and experiment, the writer suggested some years ago that investigators should approach the problem from a different angle. "Energetic" considerations were to be applied. For a given total head, H, the depth, d, and the resulting discharge were assumed to be such as to make the contents of energy in Section B the smallest possible minimum.[12]

If a discharge, Q, flows with a uniform depth and with parallel stream filaments in a canal of given form, such as Fig. 18, and if the depth of flow, d, varies, there is a certain "specific energy of flow" that corresponds to each depth of flow:

$$\varepsilon = d + \frac{v^2}{2\,g} = d + \frac{Q^2}{2\,g\,a^2} \ldots\ldots\ldots\ldots\ldots\ldots\ldots (9)$$

which is the energy head (energy per unit weight) referred to the bottom line of the cross-section. The factor, a, in Equation (9) is the cross-sectional area at the depth, d. The graph of ε, which features the contents of specific energy as it changes under varying depth, has the shape illustrated in Fig. 19. The energy curve is tangent to the horizontal axis and to a straight line, drawn through 0 at 45%, and passes through a minimum point at C.

The ordinate, d_{cr}, at Point C is the critical depth. From this point of view critical flow acquires a definite physical meaning, namely, that at which the discharge in the canal flows with the smallest possible minimum content of energy per unit of weight. The energy curve, as shown in Fig. 19, and which has proved to be of incalculable service in considering different problems of varied flow, has been discovered and presented in different countries by different hydraulicians seemingly independent of each other. In the United States a lucid exposition has been given[13] by Julian Hinds, M. Am. Soc. C. E. In Germany, Professor Rehbock alludes[14] to a similar curve. In Russia, the energy curve was introduced by the writer[12] in 1912 and, as far as he is aware, that was its first appearance. For a canal of any form, the critical depth is determined by the equation, $\left(\frac{a^3}{b}\right)_{cr} = \frac{Q^2}{g}$, in which, a is the cross-sectional area, and b, the width of the surface at the depth, d_{cr}. For a rectangular canal, the critical depth is related to the discharge per unit width by the well-known formula used by Professor Woodburn, $d^3_{cr} = \frac{q^2}{g}$.

---

[11] See, for example, *Water Supply Paper 200*, U. S. Geological Survey.

[12] "On Varied Flow of Liquid in Open Channels," (in Russian), St. Petersburg, 1912; reprinted, 1928; also, "Hydraulics," Lectures at the Polytechnical Inst., St. Petersburg, 1913.

[13] *Engineering News-Record*, 1920.

[14] "Betrachtungen über Abfluss, Stau, und Walzenbildung," Berlin, 1917; see, also, Böss, No. 284, Forschungsarbeiten, V. D. I, 1927.

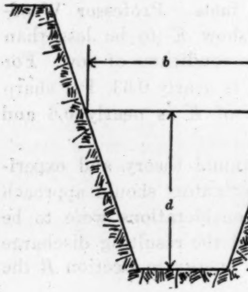


FIG. 18.

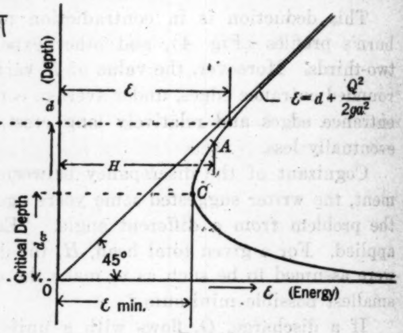


FIG. 19.

On the energy diagram, Fig. 19, flow over a broad-crested weir is shown by the movement from an initial point, *A*, corresponding to the energy content, *H*, down along the energy curve toward *C*. Obviously, in its natural tendency to become lower with a corresponding loss of energy, the level on the crest may not fall below Point *C*, as a further lowering of the depth could take place only through an increase, an addition of energy from outside, which is obviously impossible.

Thus, the critical depth (that at which flow takes place with a possible minimum content of energy), appears to be the natural limit, to which the surface level may lower, and which will establish itself at the end of the crest. The analytical relations between the different parameters of flow may be determined, as follows: The discharge in Section *B*, in accordance with Equation (5) and Equation (6), is:

$$q^2 = m^2\,2\,g\,H^3 = \phi^2 K\,(1 - K)\,2\,g\,H^3 \ldots\ldots\ldots\ldots\text{(10)}$$

The condition of minimum energy (critical depth) is expressed, as follows:

$$q^2 = g\,d^3 = g\,K^3\,H^3 \ldots\ldots\ldots\ldots\ldots\ldots\text{(11)}$$

Hence,

$$m^2\,2\,g\,H^3 = g\,K^3\,H^3 \ldots\ldots\ldots\ldots\ldots\ldots\text{(12)}$$

and,

$$\phi^2 K^2\,(1 - K)\,2\,g\,H^3 = g\,K^3\,H^3 \ldots\ldots\ldots\ldots\ldots\text{(13)}$$

From Equation (12), $m = \sqrt{\frac{K^3}{2}}$; $K = \sqrt[3]{2\,m^2}$; and,

$$M = \sqrt{2\,g\,m} = \sqrt{g\,K^3} = 5.67\,K^{\frac{3}{2}} \ldots\ldots\ldots\ldots\ldots (14)$$

On the other hand, the relation between K and the velocity coefficient may be expressed in the form:

$$\frac{2\,\phi^2\,(1 - K)}{K} = 1 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (15)$$

which gives, $K = \frac{2\,\phi^2}{2\,\phi^2 + 1}$, and $\phi = \sqrt{\frac{K}{2\,(1 - K)}}$.

It should be noted that this analysis applies to crests that are horizontal or inclined at a slope not exceeding the "critical slope." For slopes beyond the critical, the very nature of the phenomenon changes, as outlined subsequently. Curves for various numerical values of Equations (14) and (15) are plotted in Fig. 20.

The factor, 3, is a "resistance coefficient", obtainable from the relation,

$$\frac{v^2}{2\,g}\,(1 + \zeta) = H - d \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (16)$$

which, in view of Equation (3), gives, $\zeta = \frac{1}{\phi^2} - 1$.

In contradistinction to the "maximum discharge" theory, K is no longer constant. In the "ideal" case, with $\phi = 1$ (no losses), K is two-thirds. Under actual conditions, with $\phi < 1$, K is less than two-thirds. The greater the losses the smaller the relative value of K. Preliminary experiments, carried out during 1913 in the Hydraulical Laboratory of the Polytechnical Institute of St. Petersburg, gave results, which on the whole appeared consonant with the foregoing theory. The principal difficulty in comparing theoretical deductions with observations, lies in the fact that flow over a weir, unless the relative length of the crest, $\left(\frac{b}{H}\right)$, is considerable, is seldom "parallel" in the sense of the paragraph to follow. A large part of the surface is covered with waves. Moreover, the surface at the end of the weir is usually marked by pronounced curvature.

Effect of Curvature on the Critical Depth.—Professor Woodburn and Professor Webb both seek to determine the locus of the critical depth as a means of establishing a simple and reliable method of measuring the volume of flow. This locus is assumed by them to lie in a section, such that the depth of flow corresponds with the relation,

$$d'_{cr} = \sqrt[3]{\frac{q^2}{g}} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (17)$$

in which, d'_{cr} is the "apparent" critical depth.

The main point to be taken into account in this connection is that the critical depth as defined by Equation (17) is determined under the assumption of parallel flow; that is, flow in which the stream filaments are practically

parallel, having "no appreciable curvature nor divergence". These requirements for flow being "parallel", which underlie the whole theory of varied flow, were formulated by Bélanger as early as 1828. In this case, and only in such a case, the potential energy, $y + \frac{P}{\Delta}$, in the Bernoulli equation is identical for all and every point of the cross-section and is everywhere equal to the depth, *d*, which accounts for the simple expression of specific energy of flow as given in Equation (9). The physical characteristic of parallel flow is, that the pressures within a cross-section are distributed according to hydrostatic law, the pressure diagram being the hydrostatic triangle *abc* (Fig. 21).

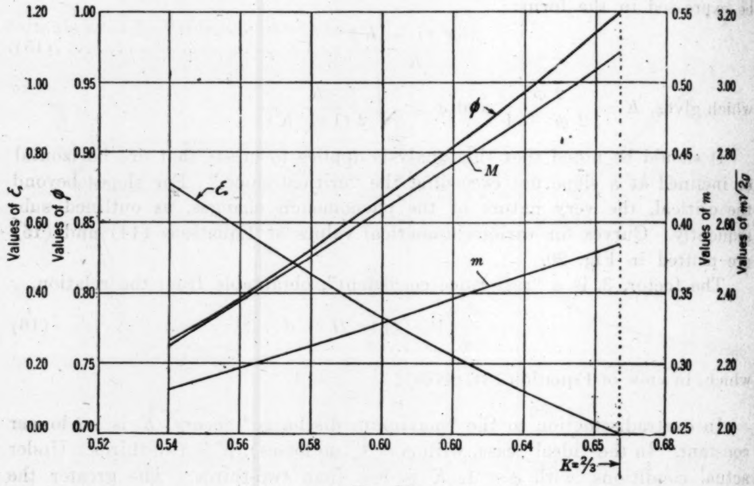


FIG. 20.

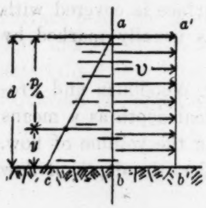


FIG. 21.

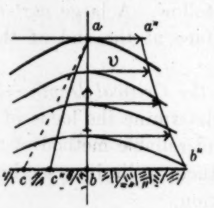


FIG. 22.

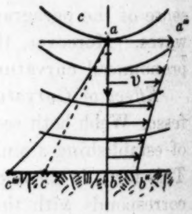


FIG. 23.

Now, if the surface and the stream filaments are curved, the pressure diagram is no longer a triangle. Centrifugal forces come into action, which for concave filaments (Fig. 22) bear downward, thus adding to the hydrostatic pressure and changing the triangle into *abc″′*. For convex filaments (Fig. 23), the centrifugal forces are directed upward; the hydrostatic pressure is

diminished and follows the curve, abc'''. If the velocities and the curvature are substantial enough, the pressure at the bottom may change into a vacuum.

In general, additional pressure caused by curvature results in a lowering of the velocities and *vice versa*. Therefore, the "ideal" rectilinear velocity diagram of Fig. 21 is modified into $a''\ b''$ and $a'''\ b'''$ in Figs. 22 and 23.

The specific energy varies from point to point. The average contents per unit weight will be:

$$\varepsilon = \frac{\int_0^d \left(\frac{P}{\Delta} + y + \frac{v^2}{2\,g}\right) v\,dy}{\int_0^d v\,dy} \quad \text{.................}(18)$$

To determine the energy, one must know the distribution of pressures and velocities over a cross-section, a problem usually requiring recourse to methods of hydrodynamics.

To get a concrete appreciation of the effect of curvature, consider the elementary case of a canal, the bottom of which is laid on a circle of radius, R_0 (Fig. 24), under the further assumption that in the vicinity of the vertical section, *Oab*, the filaments form concentric circles with a common center of curvature in *O;* and, moreover, that the motion occurs in conformity with the law of the so-called "free vortex". In this case, as is well known, the velocity is inversely proportioned to the radii, so that $v\,R$ is a constant. In the case of concave streaming (Fig. 24 (*a*)), designating the surface velocity by v_a, the formula, $v\ (R_0 - y) = v_a\ (R_0 - d)$ is obtained, or,

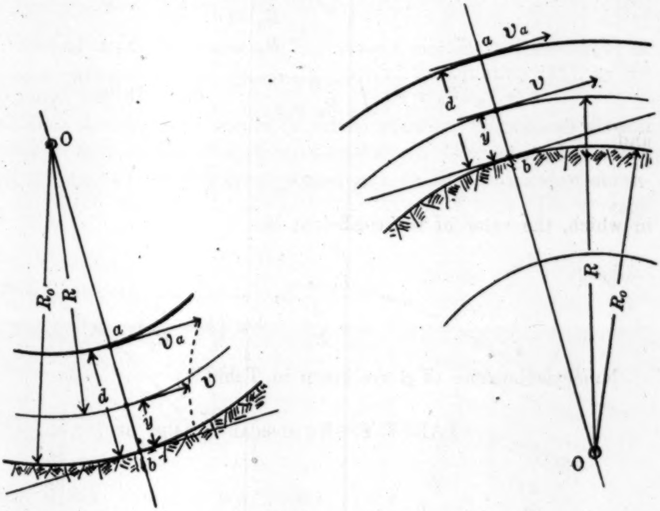


FIG. 24.

$$v = v_a \frac{R_0 - d}{R_0 - y} \quad \text{........................}(19)$$

The discharge is:

$$q = \int_0^d v\, dy = \int_0^d v_a \frac{R_0 - d}{R_0 - y}\, dy = v_a\, (R_0 - d) \log \frac{R_0}{R_0 - d} \ldots (20)$$

Furthermore, designating the average velocity over the cross-section by v_m so that $q = v_m d$,

$$v_m\, d = v_a\, (R_0 - d) \log \frac{R_0}{R_0 - d} \ldots\ldots\ldots\ldots (21)$$

or,

$$v_a = \beta\, v_m = \beta \frac{q}{d} \ldots\ldots\ldots\ldots (22)$$

in which, the coefficient, β, is equal to,

$$\beta = \frac{1}{1 - \frac{d}{R_0}} \times \frac{1}{\log \frac{1 - d}{R_0}} \ldots\ldots\ldots\ldots (23)$$

In the case of convex streaming (Fig. 24 (*b*)), analogous reasoning gives:

$$v = v_a \frac{R_0 + d}{R_0 + y} \ldots\ldots\ldots\ldots (24)$$

$$q = v_a\, (R_0 + d) \int_0^d \frac{d\, y}{R_0 + y} = v_a\, (R_0 + d) \log \frac{R_0 + d}{R_0} \ldots\ldots (25)$$

and,

$$v_a = \beta\, v_m = \beta \frac{q}{d} \ldots\ldots\ldots\ldots (26)$$

in which, the value of the coefficient is,

$$\beta = \frac{\frac{d}{R_0}}{1 + \frac{d}{R_0}} \times \frac{1}{\log \left(1 + \frac{d}{R_0}\right)} \ldots\ldots\ldots\ldots (27)$$

Numerical values of β are given in Table 8.

TABLE 8.—NUMERICAL VALUES OF β

$\frac{d}{R_0}$	0.10	0.20	0.30	0.40	0.50	0.60	0.75
β for concave streaming.......	1.063	1.12	1.20	1.303	1.44	1.64	2.16
β for convex streaming........	0.953	0.925	0.88	0.848	0.82	0.795	0.77

The specific energy of the surface filament (at *a*) is $d + \frac{v^2_a}{2\, g}$. For any point within the stream, the energy is $y + \frac{P}{\Delta} + \frac{v^2}{2\, g}$. Now, free vortex is irrotational flow with a velocity potential. In such case the Bernoulli constant, $y + \frac{P}{\Delta} + \frac{v^2}{2g}$, has an identical value for the entire body of moving liquid. Hence, in the section, *Oba* (Fig. 24(*b*)), the specific energy at all points is the same, and thus is equal to the energy of the surface stream line. The specific energy, therefore, is:

$$\varepsilon = d + \frac{v^2_a}{2g} = d + \beta^2 \frac{q^2}{2gd^2} \dots\dots\dots\dots\dots\dots (28)$$

The critical depth may now be determined as the depth at which ε is a minimum. It is found by making

$$\frac{\partial \varepsilon}{\partial d} = 1 - \beta^2 \frac{q^2}{g d^3} + 2\beta \frac{q^2}{2gd^2} \frac{\partial \beta}{\partial d} = 0 \dots\dots\dots\dots (29)$$

The value of $\frac{\partial \beta}{\partial d}$ being comparatively small, one may, in a first approximation, neglect the third member in Equation (29), with the result that,

$$d_{cr} = \sqrt[3]{\beta^2 \frac{q^2}{g}} = \beta^{\frac{2}{3}} \sqrt[3]{\frac{q^2}{g}} = \beta^{\frac{2}{3}} d_{cr} \dots\dots\dots\dots\dots (30)$$

The actual critical depth in concentric curvilinear motion is thus larger or smaller than the apparent critical depth, d'_{cr}, defined in Equation (17), by the value of the coefficient, $\beta^{\frac{2}{3}}$.

In curvilinear motion, as it occurs in actual practice, the various stream filaments usually have different centers of curvature. For example, at the extremity of the weir in Fig. 25, the surface point, *C*, has the greatest curvature while the bottom filament is parallel to the floor at $R = r$. The average energy of flow and the critical depth may still be represented by an expression similar to Equations (28) and (30), but the value of β will be different from that of Equation (23) or Equation (27), and will obviously lie between the value of β, corresponding to the extreme curvature of the surface filaments, and $\beta = 1$, corresponding to the bottom point. Finding the exact value of β presents in most cases a complicated and thankless venture. The foregoing analysis, however, clearly shows, that whenever the surface is curved, the real

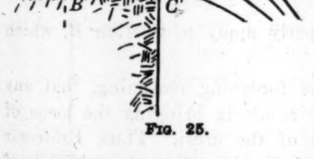


FIG. 25.

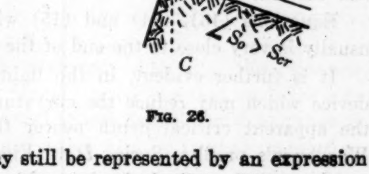


FIG. 26.

value of the critical depth is substantially different from the "apparent" depth corresponding to parallel movement and that such apparent value may not be used in hydraulic reasoning without due regard to the actual circumstances of flow. Combining Equations (28) and (30), the result with $d = d_{cr}$ is the value of $\varepsilon_{\text{min.}}$, the minimum possible contents of energy, which a discharge, q, carries in critical flow in curvilinear motion; in fact,

$$\varepsilon_{\text{min.}} = d_{cr} \left(1 + \beta^2 \frac{q^2}{2\, g\, d^3_{cr}}\right) = 1.5\, \beta^{\frac{2}{3}} \sqrt[3]{\frac{q^2}{g}} \ldots\ldots\ldots\ldots (31)$$

The minimum energy in parallel flow, $\varepsilon'_{\text{min.}} = 1.5 \sqrt[3]{\frac{q^2}{g}}$, is derived from Equation (31), assuming $\beta = 1$. Equation (31) shows that, in curvilinear motion, $\varepsilon_{\text{min.}}$ is different from $\varepsilon'_{\text{min.}}$ in parallel flow. It is greater with $\beta > 1$, when the streaming is concave, and less with $\beta < 1$ when the elementary filaments are convex.

With reference to the energy diagram, Fig. 19, this means, for example, that for convex flow the location of the point, C' (determined by d_{cr} and $\varepsilon_{\text{min.}}$), will be to the left and below Point C corresponding to parallel flow. Because of convex streaming, critical flow will occur with a smaller depth and with a smaller minimum contents of energy.

In this light, one may reconcile the apparent contradiction between the theory, according to which the minimum contents of energy were to be reached at the very end of the crest, and the observed fact, shown, for example, in Professor Woodburn's profiles, Series *A-B-C*, (Fig. 5), namely, that the critical depth, $d'_{cr} = \sqrt[3]{\frac{q^2}{g}}$, is found to lie at some distance from the end of the crest. Referring to Fig. 25, it is seen that $d'_{cr} = \sqrt[3]{\frac{q^2}{g}}$ will be properly reached in Section *B*, where the parallel flow is supposed to end, but that flow may continue with a further loss of energy between Sections *B* and *C*, at the expense of a further reduction of energy, made possible by the fact that the surface becomes convex, with the curvature increasing as it approaches the end section, *C*.

Equations (13), (14) and (15) will properly apply to Section *B*, which usually is very close to the end of the crest.

It is further evident, in the light of the foregoing reasoning, that any device which may reduce the curvature, will result in bringing the locus of the apparent critical depth nearer the end of the crest. Thus, Professor Woodburn's profiles, Series *D A* (Figs. 4 and 5), show how the addition of an apron, and particularly the backing up of tail-water, has the effect stated in the foregoing remarks.

Another instance, which may be usefully considered, is the fact well known from the theory of varied flow, namely, that whenever the surface passes the critical depth, there is an inversion of curvature. For example, in Fig. 26, *C-C* divides the upper convex from the lower concave part of the surface curve. At the point, C', where the inversion of surface curvature takes place, the radius of curvature is infinite. Assuming that the inversion of curvature inside the stream takes place in close proximity to the section, C'-C'', the movement at the intersection of the stream lines with C'-C'' will be close to parallel, and β for such section will approach $\beta = 1$. This explains, to some extent, the location of the critical depth in Fig. 8.

Waves on the Surface Over the Crest.—Under the caption, "Location of Critical Depths", Professor Woodburn records interesting data relating to the formation of waves on the surface over the weir. The theory of varied flow gives a sufficiently definite answer to this question. In general, it may be stated, that waves will form or will not be present, depending on whether the slope of the crest is smaller or larger than the so-called critical slope:

$$S_{cr} = \frac{g}{c^2} \times \frac{P}{b} \qquad (32)$$

Equation (32) determines S_{cr} as dependent on the depth, d, P being the wetted parameter, b, the top width, and c, the Chézy coefficient corresponding to a given depth of flow. For the rectangular flume with $b = 2$ ft., experimented upon, with C computed by means of the Ganguillet-Kutter formula with $n = 0.011$, the critical slope for the different depths is, as follows:

d, in feet.	S_{cr}.
0.1	0.0055
0.2	0.0044
0.4	0.0039
0.6	0.0039
0.8	0.0040
1.0	0.0043

The rule, as stated, follows from the study of the forms, which a surface assumes in the process of establishing a flow, given by Boussinesq.[15] With reference to the particular case under consideration, the deductions to be drawn from Boussinesq's most ingenious analysis, which are known to accord well with observation, would be as follows:

1.—When S is less than S_{cr} so that the movement which establishes itself is "tranquil", meaning a flow which, with the slope as given, would lead to a uniform movement with a depth greater than the critical, then the surface in the transition section will be undulated.

2.—When on the other hand, with S greater than S_{cr}, the movement which is in process of establishment is "rapid" (meaning that with the slope as given, the depth in uniform movement will be less than the critical), then the transition section will feature one inflected curve with no further undulations.

In Fig. 4, the slope in Series *D E*, is $S = 0.026$, far in excess of S_{cr}, as determined in the foregoing tabulation. Accordingly, there are no undulations.

On the other hand, Series *B G*, with a horizontal floor ($S = 0 < S_{cr}$), and partly Series *D D* and *D G*, register undulations. By coincidence the slope,

---

[15] "Essai sur la théorie des eaux courantes," Paris, 1877; also, "Théorie de l'écoulement tourbillonant," etc., 1897.

0.004, used in Series *D D* and *D G* is practically identical with the critical slope as given in the foregoing list. Therefore, the movement, which is in process of establishment, is a uniform movement with a depth equal to the critical. That is the reason why in these series the surface curve pendulates around the critical depth, and seems to appear repeatedly in the same profile in different locations.

D. D. Curtis,[18] Assoc. M. Am. Soc. C. E. (by letter).—The importance of various forms of weirs as means of measuring discharge is manifest. In particular, broad-crested weirs—because of several advantages, and perhaps more because of the comparative scarcity of data available—furnish especially interesting topics for investigation. This paper, therefore, is timely.

The writer was engaged in an experimental study of a round-crested weir, operated chiefly under condition of submergence, in the Hydraulic Laboratory of the University of Iowa during the summer of 1930. Although the work was undertaken for purposes different from those of Professors Woodburn and Webb, a comparison may prove of interest.

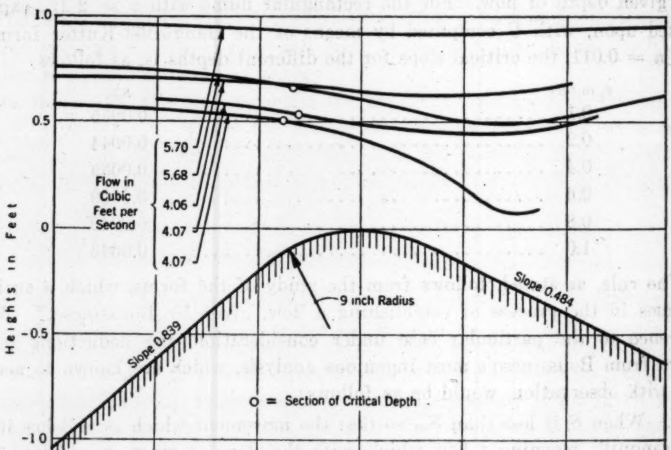


Fig. 27.—Water-Surface Profiles for One Set of Runs on Round-Crested Weir, with Location of Critical Depth.

A weir, 18 in. high with a circular crest of 9-in. radius, was installed in a glass channel 2½ ft. wide. The up-stream face had a slope of 40°, and the down-stream face was a movable apron, set at angles varying from 10 to 45 degrees. The quantity of water was determined from readings of the head on a sharp-crested rectangular weir which had been previously calibrated volumetrically by Mr. M. E. Nelson, of the U. S. Army Engineer Office, at

---

[18] Prof. of Mechanics, Clemson Agri. Coll., Clemson College, S. C.

St. Paul, Minn. Velocity readings were taken with a Pitot tube designed and made in the laboratory. Water surface elevations were read with a movable point-gauge.

Fig. 27 shows the water surface profiles for a typical series of runs. In this set, two runs show depths greater than the critical and three were adjusted so that flow occurred at the critical depth in each. Sections at which critical depth existed are indicated by small circles.

With the steepness of the aprons used in these experiments, it is indicated that the error in discharge, due to assuming critical depth to exist at the mean of the actual positions, might range up to ± 6 per cent. This, of course, is too inaccurate to be of value except where the most approximate results would be sufficient, and it demonstrates that this type of weir is not suitable for the determination of discharge from measurement of depth in one vertical plane if this depth is assumed to be the critical one.

Manipulation of tail-water height and quantity led the writer to believe that a condition of flow might persist through a greater degree of change on rising stage than on falling stage. For example, the unstable conditions accompanying critical flow might not become apparent so quickly when the water stage was rising as when it was falling, apparently indicating greater stability under rising than under receding waters. If this is true it is necessary for the experimenter to exercise some caution in adjusting the flow to meet any definite condition he desires, particularly if what he is attempting is a stage bordering on the critical.

ERIK G. W. LINDQUIST,[17] M. AM. SOC. C. E. (by letter).—If the measurements of water surface profiles in this paper had been accompanied by simultaneous measurements of pressure on the deck of the weir and within the flowing masses, as well as simultaneous measurements of the velocity distribution up stream from the weir, the paper would have been a notable contribution to the study of the flow of water over dams. It would have been possible then to analyze and discuss every conclusion from a theoretical point of view. Nevertheless, the author is entitled to the appreciation of the profession for his laborious work, which will certainly direct the way to future experiments.

If measurements of pressure had been made, the author would have found that the ordinary formula for computing the critical depth is not always applicable to a dam. This statement is made on the basis of the following analysis. In addition to the notation given by Professor Woodburn, the following must be introduced:

Let B = length of the crest, perpendicular to the direction of flow.
z = distance above some horizontal plane of reference.
p = internal pressure.
k = velocity head.
h_f = loss of head due to friction.
r = radius of curvature.
w = specific weight.

---

[17] Capt., Royal Swedish Corps of Civ. Engrs.; Office Engr., Stockholm City Water Supply, Stockholm, Sweden.

Consider a vertical section at the crest of a dam with the form illustrated in Fig. 28. The discharge is expressed by,

$$Q = B \int_0^D v_z \, d\, z \quad \text{.......} (33)$$

For a section, *a-a*, Bernoulli's equation of energy is,

$$H = z_0 + h_0 + \alpha\, k_0' \quad \text{.......} (34)$$

and for a section, *b-b* (assuming that the stream lines, or filaments, are perpendicular to the sections),

$$H = z_0 + z + p_z + k_z + h_e \quad \text{.......} (35)$$

Hence, since $k_z = \frac{v_z^{\,2}}{2\,g}$, Equations (34) and (35) may be solved simultaneously to produce,

$$v_z = \sqrt{2\,g\,(h_0 + \alpha\,k_0) - z - p_z - h_e} \quad \text{.......} (36)$$

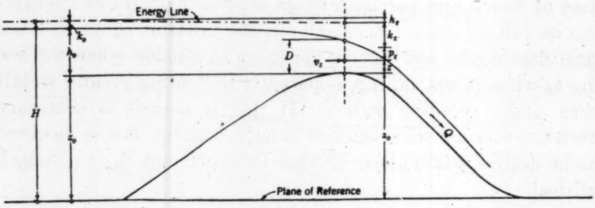


FIG. 28.

The discharge then (from Equation (33)) is equal to,

$$Q = \psi\, B \sqrt{2\,g} \int_0^D \sqrt{(h_0 + \alpha\,k_0) - z - p_z}\, dz \quad \text{.......} (37)$$

in which, the coefficient, ψ (less than 1), has been inserted instead of h_e to account for friction. A pressure, p_z, is a function of the curvature of the stream line; that is, its value depends on the height of the dam and on the shape of its cross-section as well as on the head.

Assuming, first, that $p_z = D - z$ (see Fig. 28), and integrating Equation (37),

$$Q = \psi\, B\, D \sqrt{2\,g} \sqrt{(h_0 + \alpha\,k_0) - D} \quad \text{.......} (38)$$

In 1848, Belanger introduced the principle that the critical depth is that at which the discharge is a maximum for any given head. In other words, for $\frac{\partial Q}{\partial D} = 0$,

$$D_c = \frac{2}{3}\,(h_0 + \alpha\,k_0) \quad \text{.......} (39)$$

and,

$$Q = \frac{2}{3}\,\frac{\psi}{\sqrt{3}}\,B \sqrt{2\,g}\,(h_0 + \alpha\,k_0)^{\frac{3}{2}} \quad \text{.......} (40)$$

Next, assume that $p_z = 0$, and again integrate Equation (37):

$$Q = \frac{2}{3}\ \psi\, B \sqrt{2\,g} \left\{ (h_0 + \alpha\, k_0)^{\frac{3}{2}} - (h_0 + \alpha\, k_0 - D)^{\frac{3}{2}} \right\} \ldots\ldots (41)$$

Applying Belanger's principle, as before, $D_c = h_0 + \alpha\, k_0$, and:

$$Q = \frac{2}{3}\ \psi\, B \sqrt{2\,g}\,(h_0 + \alpha\, k_0)^{\frac{3}{2}} \ldots\ldots\ldots\ldots\ldots (42)$$

Thus, it is demonstrated that the discharge depends upon the pressure in the overfalling sheet. For equal values of energy head, the discharge is lower when the pressure has a hydrostatic distribution from the surface to the crest, that is, when $p_z = D - z$. This condition is fulfilled only on a broad-crested weir with a very smooth entrance end.

Assuming, finally, that $p_z = m\,(D - z)$, and again integrating Equation (37),

$$Q = \frac{2}{3}\ \frac{\psi\, B}{1 - m} \sqrt{2\,g} \left\{ (h_0 + \alpha\, k_0 - m\,D)^{\frac{3}{2}} - (h_0 + \alpha\, k_0 - D)^{\frac{3}{2}} \right\} . (43)$$

Once more Belanger's principle indicates that, for $\dfrac{\partial\, Q}{\partial\, D} = 0$,

$$\frac{3}{2}\, m\,(h_0 + \alpha\, k_0 - m\,D)^{\frac{1}{2}} + \frac{3}{2}\,(h_0 + \alpha\, k_0 - D)^{\frac{1}{2}} = 0 \ldots\ldots\ldots (44)$$

which reduces to the form,

$$D_c = \frac{1 + m}{1 + m + m^2}\,(h_0 + \alpha\, k_0) \ldots\ldots\ldots\ldots\ldots\ldots (45)$$

For D, in Equation (43), insert the value of D_c, given in Equation (45) and simplify. The result is,

$$Q = \frac{2}{3}\,\psi\, B\, \sqrt{2\,g}\,(h_0 + \alpha\, k_0)^{\frac{3}{2}}\,\frac{1}{\sqrt{1 + m + m^2}} \ldots\ldots\ \ldots (46)$$

The ordinary discharge formula is,

$$Q = \frac{2}{3}\,\mu\, B \sqrt{2\,g}\,(h_0 + \alpha\, k_0)^{\frac{3}{2}} \ldots\ldots\ldots\ldots\ldots\ldots (47)$$

Combining Equations (46) and (47), for purposes of comparison,

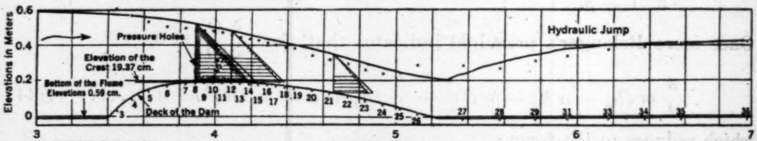
$$\mu = \frac{\psi}{\sqrt{1 + m + m^2}} \ldots\ldots\ldots\ldots\ldots\ldots\ldots (48)$$

in which, ψ is a coefficient the value of which depends upon the loss of head due to friction. Assuming that $\psi = 1$, the values of μ in Table 9 are compared with the values of D_c for the range from $m = +1$ to $m = -1$. Only when the stream filaments are parallel and straight can m equal $+1$, and m can equal zero only in a freely falling sheet of water, in which the stream filaments are concentric and in which the contours of the nappe are exposed to the air or to some other surrounding medium of constant pressure. When the contours of the escaping stream begin to converge, the internal pressure becomes higher than the pressure in the air or other medium surrounding the nappe.

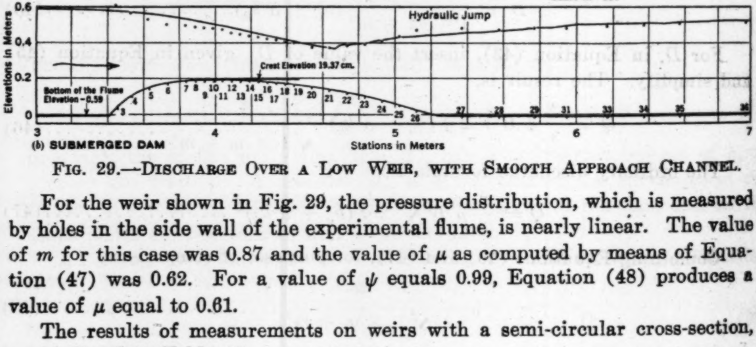
TABLE 9.—COMPARISON OF CRITICAL DEPTHS

m	$\frac{D_c}{h_o + a\,k_o}$ (Equation (45))	μ (Equation (48))	m	D_c	μ
+1.0	0.667	0.578	0.0	1.0	1.0
+0.9	0.702	0.610	−0.1	0.999	1.049
+0.8	0.738	0.640	−0.2	0.953	1.091
+0.7	0.777	0.677	−0.3	0.886	1.125
+0.6	0.817	0.715	−0.4	0.790	1.147
+0.5	0.858	0.754	−0.5	0.667	1.155
+0.4	0.898	0.802	−0.6	0.527	1.147
+0.3	0.936	0.849	−0.7	0.328	1.125
+0.2	0.968	0.898	−0.8	0.238	1.091
+0.1	0.992	0.950	−0.9	0.110	1.049
+0.0	1.0	1.0	−1.0	0.0	1.0

The linear pressure distribution given by the expression, $p_z = m\,(D - z)$, is only an assumption. However, it has proved itself applicable in some cases as will be explained subsequently.



(a) OVERFLOW DAM



(b) SUBMERGED DAM

FIG. 29.—DISCHARGE OVER A LOW WEIR, WITH SMOOTH APPROACH CHANNEL.

For the weir shown in Fig. 29, the pressure distribution, which is measured by holes in the side wall of the experimental flume, is nearly linear. The value of m for this case was 0.87 and the value of μ as computed by means of Equation (47) was 0.62. For a value of ψ equals 0.99, Equation (48) produces a value of μ equal to 0.61.

The results of measurements on weirs with a semi-circular cross-section, are as listed in Table 10.

TABLE 10.—EXPERIMENTS ON A SEMI-CIRCULAR DAM, 800 MILLIMETERS HIGH

Q, in liters per second. (1)	Total head, m. (2)	Observed pressure at the crest, m. (3)	m, observed. (4)	μ, experimental Equation (47). (5)	μ, computed by Equation (48). (6)	m, computed by Equation (53) (7)
253.4	0.361	0.172	0.69	0.648	0.666	0.70
151.7	0.262	0.139	0.78	0.625	0.627	0.78
78.7	0.159	0.092	0.87	0.606	0.607	0.87
26.2	0.085	0.052	0.93	0.594	0.598	0.93

The values of μ, computed by Equation (48) (see Column (6), Table 10) in which, ψ equals 0.98, agree very closely with those determined by Equation (47) (see Column (5) Table 10).

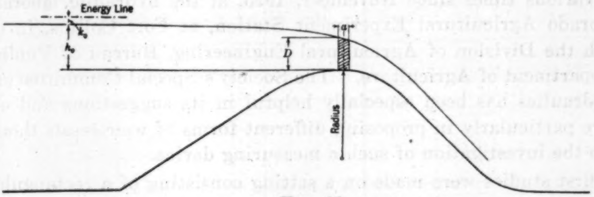


FIG. 30.

The internal pressure close to the deck at the crest can be computed approximately as follows: In Fig. 30, consider the mass element, $a\,D\,B$, in motion. The vertical forces acting are the weight of the element, $w\,a\,D\,B$, the pressure on the deck, $-\,w\,p\,a\,B$, and the centrifugal reaction, $-\,a\,D\,B\,\frac{w\,v^2}{g\,r}$. Hence,

$$w\,a\,D\,B - w\,p\,a\,B - a\,D\,B\,\frac{w\,v^2}{g\,r} = 0 \ldots\ldots\ldots\ldots\ldots (49)$$

or,

$$p = D\left(1 - \frac{v^2}{g\,r}\right) \ldots\ldots\ldots\ldots\ldots\ldots\ldots (50)$$

Since $m = \frac{p}{D}$,

$$m = 1 - \frac{v^2}{g\,r} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (51)$$

The mean velocity above the crest is approximately,

$$v = \sqrt{2\,g\,\frac{(h_0 + \alpha\,k_0)}{3}} \ldots\ldots\ldots\ldots\ldots\ldots (52)$$

For v, in Equation (51), insert the value of v in Equation (52) and,

$$m = 1 - \frac{2}{3}\left(\frac{h_0 + \alpha\,k_0}{r}\right) \ldots\ldots\ldots\ldots\ldots\ldots (53)$$

The values of m, in Column (7), Table 10, were computed by means of Equation (53). Although the method used is very approximate the observed and computed values are very close; but this does not prove that the method used is always applicable. As soon as the path of the particles flowing near the deck does not agree with the contour of the dam, the problem becomes very much complicated. This is the condition that exists for the dams tested by Professor Woodburn. Cushions or eddies will be found on the crest near the entrance end.

Fig. 29(*b*) illustrates that position of the hydraulic jump when the dam is submerged. As soon as this condition occurs, small waves are formed on the surface and are propagated up stream, with the result that the up-stream level is raised.

Ralph L. Parshall,[18] Assoc. M. Am. Soc. C. E. (by letter).—The study of the possibilities of measuring the rate of flow over broad-crested weirs of different types, based on the principle of critical depth, has been given attention at various times since November, 1925, at the hydraulic laboratories of the Colorado Agricultural Experiment Station, at Fort Collins, in co-operation with the Division of Agricultural Engineering, Bureau of Public Roads, U. S. Department of Agriculture. The Society's Special Committee on Irrigation Hydraulics has been especially helpful in its suggestions and criticism, and more particularly in proposing different forms of weir crests thought best suited to the investigation of such a measuring device.

The first studies were made on a setting consisting of a rectangular flume, 2 ft. wide, 2½ ft. deep, and about 10 ft. long. Near the up-stream end, on the floor of this flume, a metal convex-shaped hump was placed so that the up-stream and down-stream faces curved in an easy transition to the crest line, which was level and normal to the axis of the flume. This crest, or hump, was 10 in. high. A secondary humped crest of similar design, but only 4 in. high, was placed at a distance of 4 ft. down stream. It was soon discovered, however, that this secondary crest was of no effect. The principal feature involved was the application of the critical depth in the formula,

$$Q = L \sqrt{g}\ H_c^{1.5} \quad \text{.......................(54)}$$

in which, Q is the rate of flow, in cubic feet per second; L, the length of crest, in

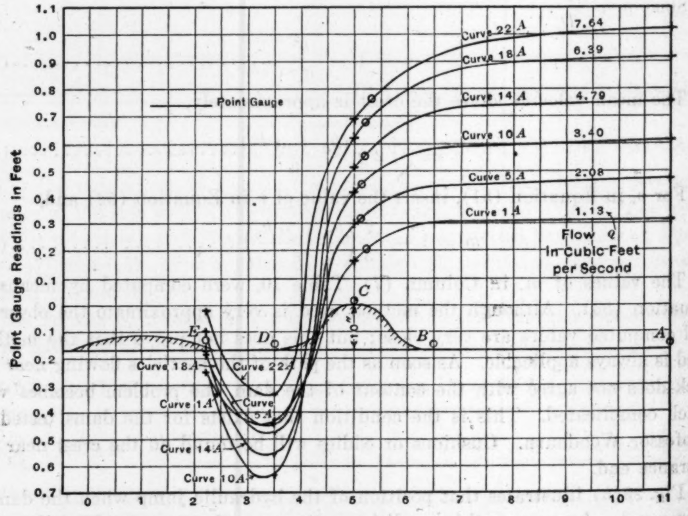


Fig. 31.—Profiles of Water Surface with 10-Inch Crest and Free Flow, Fort Collins Laboratory, November, 1925.

---

[18] Irrig. Engr., U. S. Dept. of Agriculture, Fort Collins, Colo.

feet; g, the acceleration of gravity; and H_c, the critical depth, in feet. From a theoretical standpoint, this critical depth should occur at the crown or crest of the hump, but the preliminary investigation of this setting indicated that the location of this particular depth was not over a fixed point.

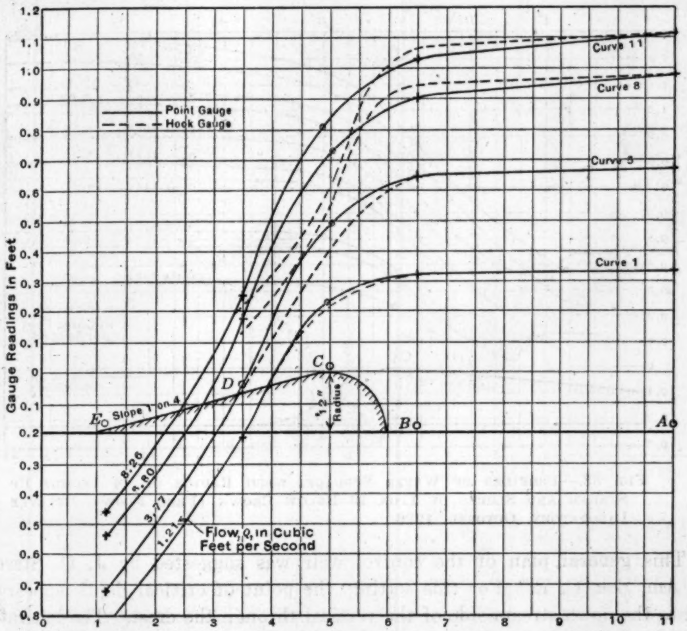


FIG. 32.—PROFILES OF WATER SURFACE, WITH RADIUS OF 12 INCHES UP STREAM AND SLOPES OF 1 ON 4 BELOW CROWN (FREE FLOW), FORT COLLINS LABORATORY, JUNE, 1926.

The profiles of the water surface along the axis of the flume with the double hump (Fig. 31) show, for a limited number of tests, that this point of critical depth shifts up stream from the vertical through the crest as the rate of discharge increases. This arrangement was changed and a single crest substituted, having a radius of 12 in. up stream, with a slope of 1 on 4 inclined downward from the crown. It will be noted in Fig. 32 that the actual location of the point of critical depth substantially agrees with the vertical through the crest.

In October, 1926, experimental apparatus consisting of a flume similar to that just described, having a single crest, 13 in. high, placed about 5 ft. down stream from the upper end of the channel, was installed at the laboratory in Bellvue, Colo. The up-stream face of this weir was of an ogee shape, in section, curved from the floor line with a radius of 31 in., with a transition from a concave to a convex surface, having a radius of 24 in. Just down stream from the crest line it was made tangent to a plane surface, inclined downward with a slope of 1 on 10. For this setting, under a free-flow discharge, the point of critical depth shifted down stream as the rate of discharge increased. The profiles of the water surface along the axis of the flume and the location of the point of critical depth are shown in Fig. 33. This setting was altered later, to the arrangement shown in Fig. 34. The height of the crest is 13 in.

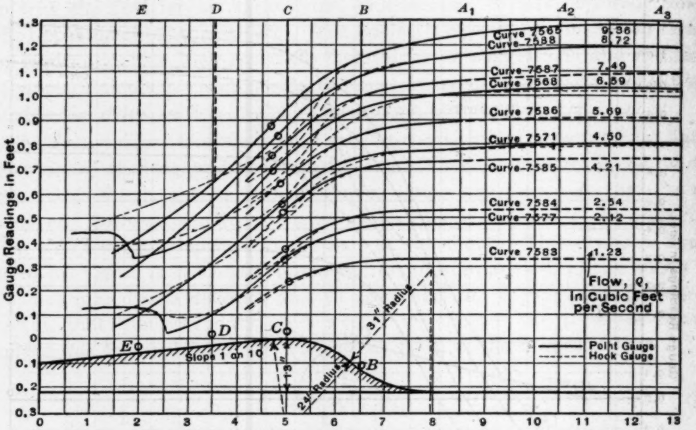


FIG. 33.—PROFILES OF WATER SURFACE, WITH RADIUS OF 24 INCHES UP STREAM AND SLOPES OF 1 ON 10 BELOW CROWN (FREE FLOW), BELLVUE LABORATORY, OCTOBER, 1926.

This general plan of the control weir was suggested by J. C. Stevens, M. Am. Soc. C. E.[19] For this setting, the point of critical depth appears to fall to the down-stream side of the vertical through the crest. The laboratory setting of this arrangement is shown in Fig. 35, with a discharge of 7.68 sec-ft.

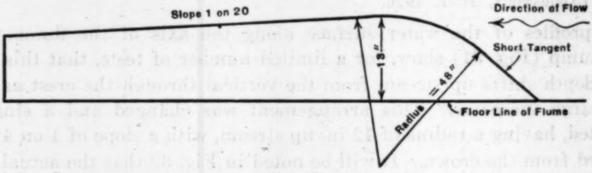


FIG. 34.

Fig. 36 is the laboratory setting for the double hump previously described (see Fig. 31). The head, as measured by the hook-gauge at the crest (where a piezometer tube to the stilling-well was set flush with the inside face of the flume wall) was consistently less than the actual depth as determined by a point-gauge. This variation in the two gauges, intended to give the head or depth over the crest, is believed to be due to the effect of cavitation or negative pressure existing just down stream from the crown of the weir. Profiles of the water surface over this type of weir are shown in Fig. 37.

---

[19] *Transactions*, Am. Soc. C. E., Vol. 89 (1926), p. 874.

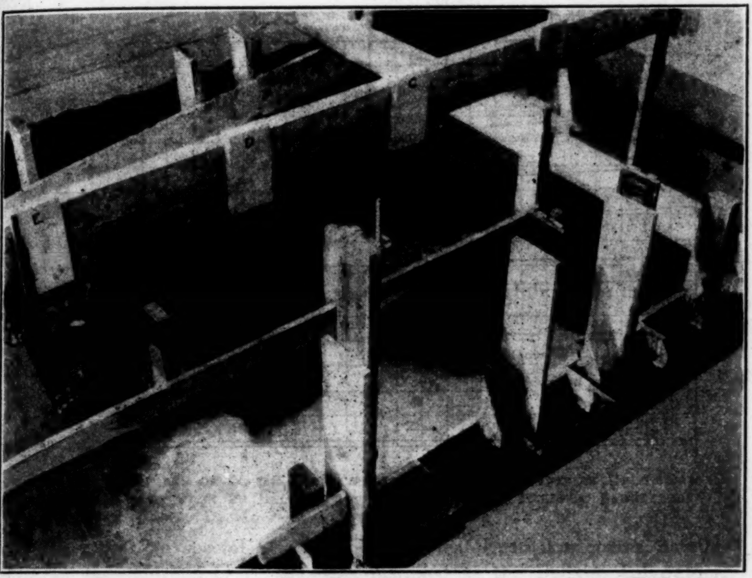


Fig. 35.—Flow of 7.68 Second-Feet Over Crest with Convex Curved Up-Stream Face (see Fig. 34)

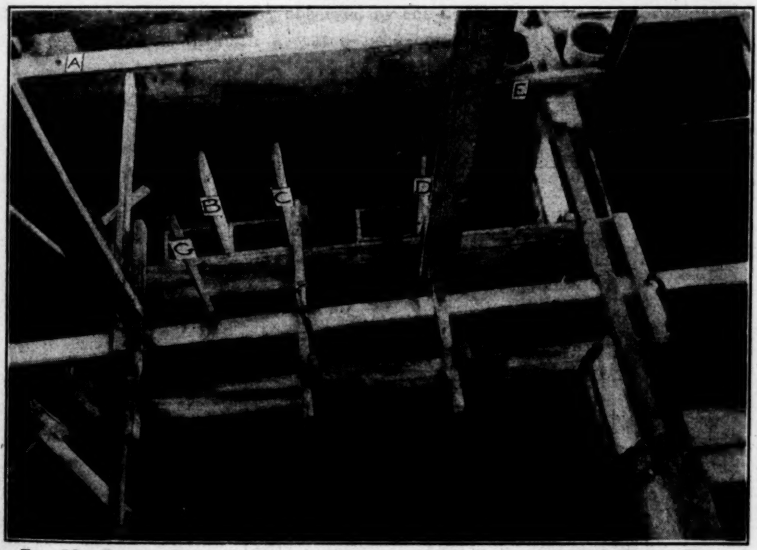


Fig. 36.—Double-Crest Setting of Control Weir, Discharge 2.08 Second-Feet. Point-Gauge *G* Locating Water Surface at Section *B*.

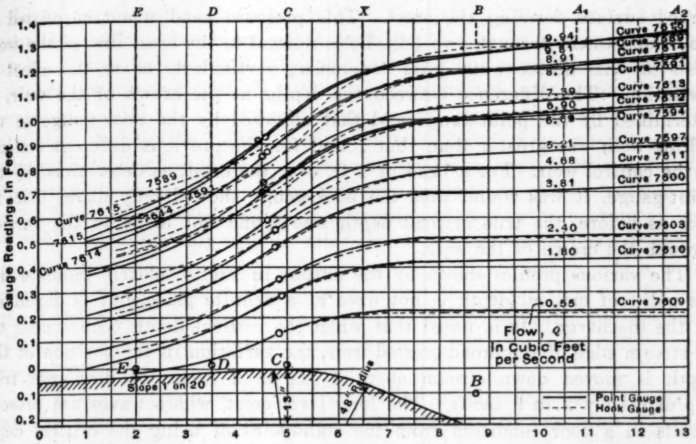


FIG. 37.—PROFILES OF WATER SURFACE WITH RADIUS OF 48 INCHES UP STREAM AND SLOPE OF 1 ON 20 BELOW CROWN (FREE FLOW), BELLVUE LABORATORY, OCTOBER, 1926.

In June, 1927, a special setting of the critical-depth flow meter, as suggested by Mr. Stevens (Fig. 38), was installed at the Bellvue Laboratory. It is essentially the same as the Herschel hollow-crest weir.[19] In this setting the pressure head on the crest, observed by the hook-gauge, was communicated to the stilling-well through a water-tight compartment placed beneath the

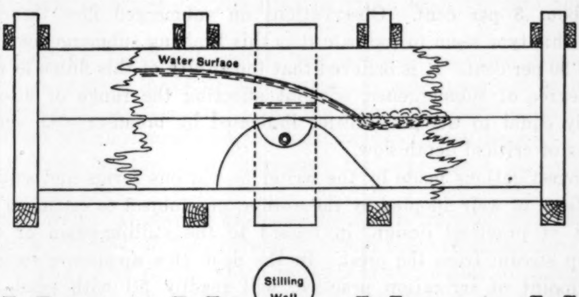


FIG. 38.—CRITICAL-DEPTH FLOW METER.

curved surface forming the crest. This pressure head was transmitted to the compartment through two ¼-in. holes normal to the crest line of the weir. Tests on this device were quite conclusive, particularly as to the effect of cavitation. The difference between the depths at the crown of the weir, as determined by the point-gauge and that observed by the hook-gauge in the stilling-well, was greater than that found for the previous different settings of the control weir. For a head of 1 ft. over the crest, as determined by the point-gauge, it was found that the hook-gauge showed only about 0.35 ft., and, as before, the true critical depth was found not to be in the vertical through the crown of the weir.

The various profiles shown by the author in Fig. 4, clearly indicate that the point of critical depth is not fixed at a definite place, but is dependent on the discharge. It is noted that when the critical depth occurs near the up-stream edge of the broad-crested weir, the variation in the position of this depth is moved down stream about 1 ft., in a range of discharge from approximately 2 to 9 sec-ft. The long level crest, where waves are present, results in a poor condition from the standpoint of using the critical depth as a criterion of the rate of flow.

Unfortunately, the data, as given in Table 1, do not show the degree of submergence for the tests thus indicated. By inspection of Fig. 4, it is estimated that for Series *D E* and *D F*, where the hydraulic jump occurs, this submergence may reach 75 to 80% as a maximum, with no material change in the coefficient, *C*. It is found that for the 2-ft. Parshall measuring flume for the head, H_a, ranging from 0.8 ft. to 1.4 ft., and with the submergence varying between 74 and 77%, the observed and computed free-flow discharges agree within about 3 per cent. Observations on submerged flow through large flumes of this type seem to indicate that this limiting submergence may equal or exceed 80 per cent. It is believed that the ability of this flume to withstand a high degree of submergence, without affecting the range of discharge, is practically equal to the possibilities indicated by broad-crested weirs under conditions of critical depth flow.

The investigations made by the writer on various forms and settings, and also the form of weir adopted by the author, are subject to criticism from the viewpoint of practical design, in regard to the stilling-basin or depressed section up stream from the crest. In the field, this up-stream section, from the standpoint of irrigation practice, will readily fill with sand, silt, and possibly gravel, to a level approaching the elevation of the weir crest. This rise in the bottom of the channel will decrease correspondingly the entrance drop at the up-stream edge of the weir, which, at the limit, will entirely eliminate the control at this point. This condition may best be obviated by providing a lateral constriction, or converging section, similar to that in the Parshall measuring flume, which provides a control. Observations relative to the point of critical depth near the crest of a 4-ft., Parshall measuring flume (free-flow), show this point to be about 0.8 ft. up stream from the crest for discharges of 17 to 34 sec-ft. When the submergence is about 77%, this point is found about 0.3 ft. up stream for like flows.

James G. Woodburn,[20] **Assoc. M. Am. Soc. C. E.** (by letter).—The able discussions which have been called forth by this paper are greatly appreciated. These discussions emphasize the varied and complex nature of the phenomena connected with flow over a broad-crested weir. As Professor Woodward points out, this type of weir presents many fascinating problems for study. Among the infinite variety of crest forms there must be certain designs which best combine a simple and constant head-discharge relation with minimum obstruction to flow and with ruggedness of construction and durability of crest. The ability of a broad-crested weir to operate at least 50% submerged (possibly 80% in some cases, as mentioned by Mr. Parshall) with practically no change in coefficient, gives it an advantage over a sharp-crested weir where head is at a premium; and the fact that it can be built without sharp edges to wear round and thus materially alter the discharge coefficient, recommends it for use in measuring the discharge of débris-carrying streams.

As a closing discussion of this first part of an extended study, the writer offers the following summary of the characteristics of flow over a broad-crested weir and of the outstanding points which require further study and explanation.

Since any weir not sharp-crested is, in effect, a short open channel, most of the phenomena of flow in open channels can also be observed in flow over broad-crested weirs. The chief cause of complications in broad-crested weir studies, as in any open-channel studies, is that the simple conditions of uniform, parallel flow are almost always absent. The problem is usually one of accelerated or retarded flow, of back-water or down-drop curves, and of wave formation, a complete explanation of which leads far into physics and hydrodynamics.

In considering the broad-crested weir as a short open channel, it is apparent that friction losses in flow across the weir must not be neglected. These losses reduce the discharge below the theoretical, frictionless maximum by an amount which depends on the location of critical depth.

Critical depth, or depth at maximum discharge for a given height of energy gradient, always occurs at some point in non-submerged flow over a broad-crested weir. It may be expressed both as a fraction of the total head and also as a function of the discharge. From these relations the discharge at the point of critical depth, which is the largest possible discharge for the channel in question, can be expressed as a function of the total head. It should be noted that the head referred to in the case is the total head at the section of critical depth.

The location of critical depth in flow over a broad-crested weir is usually not easy to determine. Two cases arise for consideration.

1.—Parallel Stream Lines.—In this case the relations between depth, discharge per unit width of channel, and total head, are: $D_c = \frac{2}{3} H_a$, $D_c = \sqrt[3]{\frac{Q_1^2}{g}}$, and $Q_1 = 3.087\, H_a^{\frac{3}{2}}$ (English units).

---

[20] Associate Prof. of Hydr. Eng., State Coll. of Washington, Pullman, Wash.

The total head is represented by the elevation of the energy gradient above the stream bed, and with parallel flow it is obtained by adding the velocity head, $a \frac{V^2}{2g}$, to the elevation of the water surface, since the pressure head at any point in the cross-section of the stream equals the height of the column of water above the point.

2.—Vertically Curving Stream Lines.—Since there is seldom any point at which flow over a broad-crested weir is parallel, it is necessary to consider the effect of vertical curvature of the stream lines, and to modify the equations for discharge and critical depth to take care of the change in pressure caused by centrifugal forces. The effect of vertical curvature of the stream lines has been treated fully in the discussions by Professor Bahkmeteff and Dr. Lindquist. Another treatment of this subject, applied particularly to down-drop curves, was published in 1929 by Dr. P. Boess, in reporting on some experimental studies in the Karlsruhe Laboratory.[21]

The curvature of the stream lines may be either downward or upward with respect to the stream bed. If they curve down toward the stream bed, centrifugal action reduces the pressure at all points in the cross-section below true hydrostatic pressure. Dr. Boess found by tests that it could usually be assumed with sufficient accuracy that this reduction in pressure is in straight-line variation with the depth. Since friction losses in accelerated flow are usually small, the energy gradient is very nearly horizontal. Thus, a reduction in pressure head causes an increase in velocity head, with resulting increase in velocity, in discharge, and in critical depth, for a given height of energy gradient.

With downward curving stream lines, therefore, D_c is larger than $\frac{2}{3} H_a$, and Q_1 (max.) is larger than $3.087 \, H_a^{\frac{3}{2}}$. Conversely, also, with upward curving stream lines, D_c is less than $\frac{2}{3} H_a$ and Q_1 (max.) is less than $3.087 \, H_a^{\frac{3}{2}}$.

The application of these relations to a study of the coefficients of discharge of broad-crested weirs may be made through the equation for maximum unit discharge,

$$Q_1 \,(\text{max.}) = C \, H_a^{\frac{3}{2}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (55)$$

The largest possible value of C in this equation is equal to, greater than, or less than, 3.087, depending on whether the stream lines are parallel, downward curving, or upward curving with respect to the stream bed. Furthermore, H_a in Equation (55) is the total head at the section of critical depth. Because of the continual loss of energy in a flowing stream, if the head at any point up stream from the point of critical depth is used in the equation, $C = \frac{Q_1}{H_a^{\frac{3}{2}}}$, the value obtained for C is less than the maximum value by an amount depending on the loss of head between the two points. This loss of head varies with the distance, with approximately the square of the velocity of flow, and probably also with the number of waves, between the two points.

---

[21] Berchnung der Abflussmengen und der Wasserspiegellage bei Abstürzen und Schwellen," *Wasserkraft und Wasserwirtschaft*, 1929, No. 2–3. The writer's translation of this work has been filed for reference in Engineering Societies Library, 33 West 39th Street, New York, N. Y.

Herein lies an explanation of why the coefficient of discharge of some of the weirs tested increased as the head increased, while that of other weirs decreased. In the former class are all the weirs with level crest. Critical depth occurred well down the crest at or below a point of approximately parallel flow, signifying a limiting value of C of about 3.087. Movement of critical depth up stream and decrease in the number of waves with the increase in head tend to reduce the losses between the point of measurement on the weir and the location of critical depth. Increasing velocity, on the other hand, tends to increase the loss. The increase in the coefficient with increasing head indicates that the resultant of the three factors was a reduction in the loss of head.

The writer's weirs with crests sloping slightly down stream, and Professor Webb's weirs which sloped slightly both ways from a crest line, show the coefficient decreasing as the head increases. Critical depth occurred usually near the entrance or highest point, and was either nearly constant in location or moved down stream as the head increased. This increase in distance and the increasing loss of head due to increasing velocity combine to cause a decrease in the coefficient.

Of course, the limiting value of C is not necessarily constant for a given weir, but may change with the head. This change becomes more pronounced as the down-stream slope of the crest increases. Thus, the pressure on the round-crested weir described by Mr. Yarnell (Figs. 14 and 16) is evidently reduced further below true hydrostatic pressure as the head increases, with resulting increase in C.

This reasoning with regard to the weir coefficient suggests the possibility of designing a broad-crested weir having a constant coefficient. The general requirements apparently are that there be approximately parallel flow at some point on the weir so that the coefficient shall have a definite limiting value; and that the resultant loss between the point of measurement of head and the location of critical depth shall be constant. It appears to the writer that the design of a weir of constant coefficient presents fewer difficulties than the design for which critical depth occurs at the same location at all heads.

Further progress in the study of flow over broad-crested weirs requires first an accurate experimental determination of the energy gradient. The height of the energy gradient at any point can be computed accurately only when the distribution of pressures and velocities over the cross-section is known. The accumulation of data regarding pressures and velocities on many different forms of weir crest is to be desired.

With the energy gradient drawn, and the pressure distribution known, the position of true critical depth can be located and the variation of this position can be studied with varying head and with different shapes of crest.

Further study is needed of the coefficient of discharge of broad-crested weirs of many different designs. An interesting question is to determine the boundary point in design between weirs which show a coefficient increasing as the head increases and weirs which show a coefficient decreasing as the head increases. Another point to be studied is the effect on the coefficient of small changes in the slope of the weir crest. There are also few data available regarding the effect of various degrees of rounding of the entrance to the weir.

The writer is continuing the experimental study of the flow over broad-crested weirs in the Hydraulics Laboratory of the State College of Washington. Following certain lines of investigation suggested in the discussion, and from other sources, preliminary tests are being made of a number of different weir crests in a glass-walled flume, 1 ft. wide. The first three weirs tested have been made of wood, with level crest 2 ft. broad, and 4 in., 8 in., and 12 in. in height, with rounded entrance. Comparison will be made with the results of the tests described in the writer's paper in order to study the application of the laws of similarity. A number of other crest designs will then be tested to study in a general way the coefficient of discharge and profile of the water surface so that certain designs may be selected for more accurate construction and study.

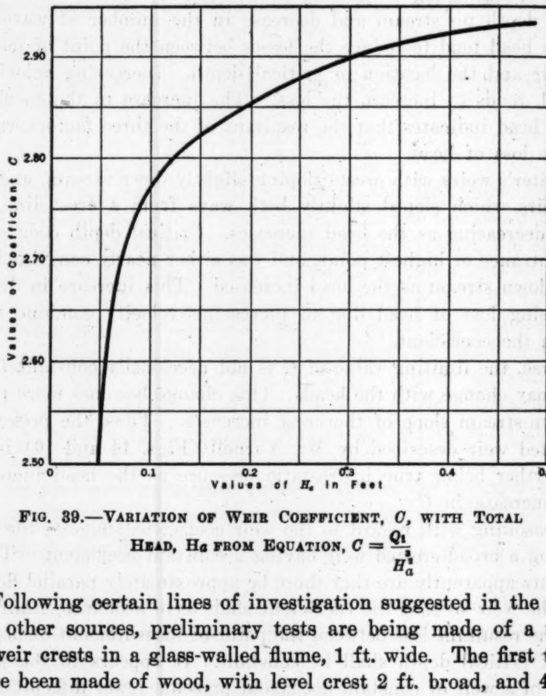
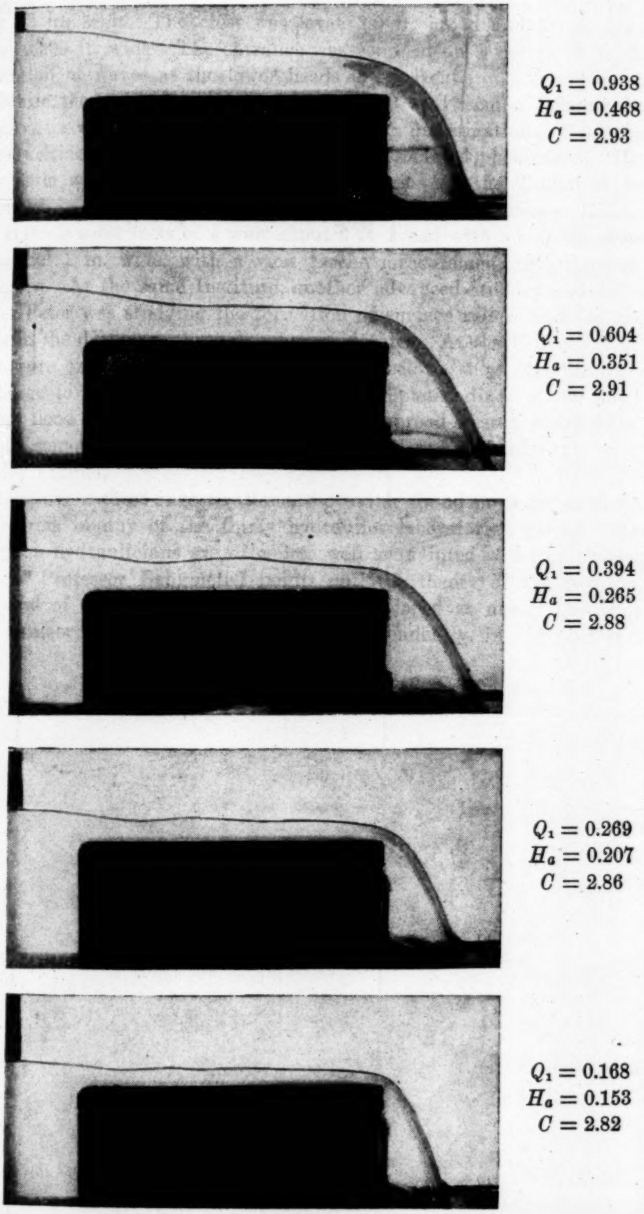


FIG. 39.—VARIATION OF WEIR COEFFICIENT, C, WITH TOTAL HEAD, H_a FROM EQUATION, $C = \frac{Q_1}{H_a^{\frac{3}{2}}}$

The tests of the three weirs (2 ft. broad and 4, 8, and 12 in. high, respectively) with level crest show that for heads up to 0.5 ft. and discharges up to 1 cu. ft. per sec., the weir coefficient increases with the head. (See Fig. 39.) The point of measurement of the head was 2.0 ft. up stream from the entrance to the weir. Practically the same curve has been obtained for each of the three heights of weir. Fig. 40 shows conditions of flow over the



$Q_1 = 0.938$
$H_a = 0.468$
$C = 2.93$

$Q_1 = 0.604$
$H_a = 0.351$
$C = 2.91$

$Q_1 = 0.394$
$H_a = 0.265$
$C = 2.88$

$Q_1 = 0.269$
$H_a = 0.207$
$C = 2.86$

$Q_1 = 0.168$
$H_a = 0.153$
$C = 2.82$

FIG. 40.—FLOW OF WATER OVER BROAD-CRESTED WEIR

weir 12 in. high. The crest was level, 2.0 ft. broad and 1.0 ft. high, in a flume 0.999 ft. wide. The entrance was rounded on a radius of 0.1 ft. The formation of waves at the lower heads is apparent.

While the writer was in Europe in 1930 as Freeman Scholar from the Society, he was on the alert particularly for investigations of broad-crested weirs, critical depth, hydraulic jump, and associated phenomena. He found very little work being done along these lines. At the Technical Institute of Zurich, a candidate for a doctor's degree under Professor Robert Dubs was making some tests of a weir about 6 ft. broad with a sloping entrance in a channel 1 m. wide, with a view to the measurement of irrigation water in Egypt. At the same Institute, another advanced student under Professor Meyer-Peter was studying the formation of surface rollers and the hydraulic jump in the discharge through submerged gates. At the University of Delft, tests were in progress under Mr. J. Thijsse to show the nature of the discharge to be expected over the partly completed dikes of the Zuyder Zee during flood tides. These dikes are really broad-crested weirs of a cross-section somewhat similar to that of the railroad embankments mentioned by Mr. Yarnell.

Aside from these investigations, the writer found no tests on this subject in progress in any of the thirty hydraulics laboratories visited. However, European hydraulicians are of course well acquainted with the problem. In fact, as Professor Bahkmeteff points out, the theory of the two stages of flow, and of flow at critical depth, was developed at nearly the same time by a number of investigators, working independently, in different countries.

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
TRANSACTIONS

Paper No. 1798

HIGHWAY LOCATION: GENERAL CONSIDERATIONS[1]

BY GIBB GILCHRIST,[2] M. AM. SOC. C. E.

WITH DISCUSSION BY MESSRS. C. S. MULLEN, H. L. THACKWELL, B. P. HARRISON, E. P. ARNESON, AND GIBB GILCHRIST.

SYNOPSIS

Highway location problems are treated very broadly in this paper. Basically, the subject has been divided into two parts. The first part, dealing with physical aspects, emphasizes the dangers attendant upon locating new highways so that they follow old routes. The advantages of aerial photography are mentioned, and aerial survey methods adopted by the Texas State Highway Department are described briefly.

In the second part, the paper describes the non-physical aspects of highway location, such as safety, economy, and utility of operation. This section is concerned with the influences of local and through traffic demands on the location problem. The paper also contains a brief comment on financing.

INTRODUCTION

When the highway location problem is solved, the highway problem itself is well on its way to solution. Early in 1929 one of the influential and best informed members of the Texas Legislature sent a questionnaire to the State Highway Department, requesting information on various phases of its program. One of the questions was as follows: "Give the average life of roads built of the various approved types of construction, such as reinforced concrete, concrete with an asphalt surface, macadam, gravel, etc." The answer given was that the life of any particular type of road depended, first, on its location and, second, on the width of its right of way, and that the type of surface was of secondary consideration. Of course, in this reply the writer had in mind the utility of a highway as applied to its use as an important trafficway. He had no reference to the stability of any particular type of surfacing, but wished to bring forcibly to the attention of those engaged in enacting highway legislation that the life of a highway should be considered as the length of time during which it would or could be used as an artery of traffic. He wanted, furthermore, to emphasize the fact that necessity and public opinion would demand that such a highway, either now or in the future, be located with a fair degree of economy.

---

[1] Presented at the Joint Meeting of the Highway and Construction Divisions, Dallas, Tex., April 25, 1929.

[2] State Highway Engr., Austin, Tex.

PHYSICAL ASPECT OF HIGHWAY LOCATION

In laying out a State highway the engineer has been too prone to follow what has been generally and popularly termed the "line of least resistance." For example, he has followed the right of way of a railroad or interurban, or if there has been no such easy route, he has chosen to locate one along section lines or property lines without any particular regard for the terrain which he traverses or for the suitability of such location for possible future heavy traffic. This policy of the engineer may be partly justified by the explanation that a majority of the locations for highways has not been made by engineers. Too often they have been determined at mass meetings or conferences of citizens who were thoroughly eager and sincere in their determination of routings but who failed to calculate, or to have the economy of either road location or construction determined by experts. This is not stated as criticism because without such action by those interested in road improvement in the various States, the Nation would be much less advanced than it is to-day. Furthermore, the writer knows of very few cases in which the location or construction of any type of road or highway has not at least paid for its investment over the period in which it has been used. How much greater return, however, could have been obtained from the use of highways if, as time went on, the cost of traveling a much greater distance could have been saved; and with how much more safety the public could have reached its goal!

Almost every important railroad is rather closely paralleled by a State highway. This, of course, is largely due to the fact that the railroads have developed the population centers and the demand for highways has been along the same general line. However, with the location of State highways parallel to the railroads, the engineers have been too ready to accept the railroad location as being along the best ground. This is usually true for a steam road, but problems of highway location are considerably different from those confronting the railroad locating engineer, and much distance can be saved to highway travel in country where railroads have considerable trouble making 1% and 1.25% grades and have to make distance for gradient.

The writer was once in charge of a railroad location in which the limiting grade was 1.25 per cent. The final located route between two points measured 14.5 miles, and more than 75 miles of preliminary lines were run and much adjustment was necessary to obtain that. Between these same two points a State highway is now being planned, with a maximum 3% or 3.5% grade, and the total distance will be shortened to 11.5 miles. Instances of this kind are very common in the State of Texas.

Aerial Photography.—Any discussion of highway location by one familiar with the situation in Texas would be incomplete without a reference to the use of aerial photography. In reconnaissance the Texas Highway Department is using such service to a great extent and has found it very valuable. No other improvement in the field of reconnaissance or location has contributed so largely to the efficiency of engineering.

The usefulness of an aerial survey begins at the first inception of the work and continues until the last shovelful of dirt is in place. It puts in the hands of the engineer many of the details that are to be found on the ground and in addition it gives him a broad view of the field of operation that has never been available in any other way. He may make locations directly on aerial mosaics; he may study minutely the locations of drainage structures and other proposed improvements; he may calculate stream flow and plat drainage areas without the use of field operations; and throughout the progress of his work the aerial survey stands by as a constant check upon the more exact field calculations.

The Texas State Highway Department has secured some very excellent results by making a location on the ground after having laid it out on an aerial map. The aerial photographer furnishes an approximate scale of each map, and in most cases these determinations are found to be very accurate. The aerial maps are especially useful in crossing streams and determining routes through cities and towns. Where little information is available from previous surveys or maps it is always economical to have an aerial map made. Such maps on an approximate scale of 700 ft. to an inch cover a strip of terrain about 6 000 ft. wide for single-flight, or 9 000 to 10 000 ft. wide for double-flight, maps. The cost is from $14.00 to $17.50 per lin. mile for a single-flight and approximately twice these amounts for double-flight maps.

After the completion of reconnaissance either by aerial photography or by preliminary lines, the best plan is to locate the final line on the ground, making such shifts of tangent or curve as may better fit the terrain from the actual field measurements. Very little difficulty has been experienced in laying out locations from aerial reconnaissance after walking over the proposed line once or twice, noting any physical conditions which might affect the location and then taking them into account. After the line has been laid out on the ground to complete what might be termed the locating problem, the engineer has only to prepare his alignment and right-of-way map, showing thereon such land as may be necessary to enable field notes to be prepared and giving the name and abstract reference to each tract of land. These plats are usually made on a 400-ft. scale which has been found to be the most adaptable to the use to which the map will be put.

In general, with regard to the location of a State highway, it is of prime importance to select the most direct line between two objectives. The line should be so located as to provide a minimum of waterways and the easiest grades and curves possible for the terrain. An important consideration is to select a line that will give not less than a 500-ft. sight distance between the eye of the driver and the top of an approaching automobile. Easy curves are not objectionable in order to increase sight distance where they do not materially lengthen the route. Such locations should be made and grades prepared with the primary object in mind so that, regardless of the development of the country through which the highway might pass, any surfacing placed on the prepared grade would be only a stage of improvement, the ultimate life of which would depend upon traffic requirements of the highway. This was what the writer had in mind in answering the question by the Texas State legislator. In other words, a highway properly located with a grade line of sufficient flatness to meet the exacting demands of increased traffic, and with curves in easy stages, will have its ultimate life more permanently assured as years go by and its condition should be better at the end of fifty years than at the end of five. With these considerations in mind the locating engineer, in considering his route between objectives, should disregard "the line of least resistance" unless he is prepared to see the day come within his own lifetime—or better still, within a decade—when the route he lays out will be relegated to the status of a lateral or community road, or be entirely abandoned. When such time comes, very likely it will be forgotten that the original location was perhaps determined by considerations purely local, and the necessity for relocation or abandonment, may, without much fear of contradiction, be explained as the correction of a mistake in engineering.

The problem of locating a highway with respect to topography, drainage, grade line, sight distance, and all those phases which must be studied in their entirety, is one that must command the expert attention of the locating engineer and the construction engineer. Nevertheless, its difficulty of solution is secondary to that of the economy of location with regard to traffic, the distance that must be traveled, and safety.

LOCATION WITH RESPECT TO SAFETY, ECONOMY, AND UTILITY OF OPERATION

The writer believes in the democratic principle that the voice of the majority should rule. He has never known it to fail, that where the people of any State or Nation were acquainted with all the facts with regard to any problem, they did not eventually (by their majority) vote to solve this problem in a way that the thinking people of such a community or commonwealth might determine that it should be solved. The people should have what they want, even if it costs more, that is, when they have decided with all facts before them.

The building of State highways is a public enterprise and the solution of the problems with regard to such a public enterprise should be entered into with the idea that it would be inadequate if it did not meet and satisfy the demands of the thinking public and if it were not in line with sound public policies.

Highway engineers are very often concerned with the problem of routes leading into cities and towns. This problem has been uppermost in the minds of the people for some time and public sentiment is now undergoing a rapid transformation in regard to highway location in population centers, large and small. In the State of Texas much prominence has been given to certain isolated cases of location where the State Highway Department has, for safety purposes, undertaken to re-route certain highways. Charges have been made that the officials and engineers of the State Department have in mind the laying out of a highway system that would isolate the cities and towns. In order to combat this erroneous impression the Highway Department has gathered some statistics and has made some calculations. The State now (1929) has a highway system of approximately 20 000 miles as measured along present routes. When this system has been economically located or relocated the mileage will be about 19 000 miles and not only will all the control points be reached, but this system will reach directly, without material loss of distance, 97% of all towns, the population of which exceeds 200. Included in this percentage are all towns that have a prospective main-line highway passing through their corporate limits or, in case of no corporation, within four blocks of the main business street.

Through Traffic.—The general argument for the route of a State highway to pass through the business district of a town or along its most important street is that business is brought to the town by reason of the traffic on that highway. There is no doubt that certain classes of business establishments do profit to a certain degree by transient trade, but it would seem that a town depending to any degree on such trade would lack the foundation necessary for stable and continuous growth; that is, an established and a certain rural trade territory. The highway or road problem for any community will not be solved until every village and town is either provided with a highway or with a durable connection to the highway. The farmer or community trader will then have access at all times, not only to the State highway but to the main business center of his trading point, and the residents of the community will have a means of leaving and entering. It seems contrary to sound economic judgment and good public policy to deflect the location of an important State highway, carrying 2 000, or perhaps 5 000, vehicles per day, 2 or 3 miles out of the general course in order to pass through the main business district of a community which would perhaps not originate more than 100 to 200 vehicles per day. The problem should require and demand the careful thought of the people of any State, and they should consider the relative advantages and disadvantages to the traveling public in districts and cities. It has been the policy of the Texas Highway Department to link the various towns with State highways and to provide for first-class construction along these link routes. In many cases, it has been possible without materially lengthening the line, to serve, directly, the vast majority of towns of more than 200 population and a great many villages of smaller population. However, in serving towns, it is the purpose to provide a direct line, without right angles if possible, and along a street of sufficient width to care for traffic. It is, or should be, the purpose of the engineer in selecting a route through a town to relieve instead of to create congestion. A paved highway connecting with an unimproved street in a city or town or a provision for such connection should satisfy all conditions.

Locating Roads for the Tourist.—It would be quite a different problem if a special frontage tax or assessment were paid by business men dealing in commodities for which there is an incidental demand from the traveling public. Neither the stability nor the growth of a community can depend upon the trade that will come from those who would ordinarily not stop at a place of business unless the highway happened to lead by the door. It would seem that the authorities in charge of the streets in a small town would prefer to reserve such streets for the citizenship of the community to use for parking their trucks or for stopping their produce wagons. In one Texas town, the main street, which was of considerable width, had been paved and largely paid for by the merchants of the street. A complaint was registered with the Mayor that the people who lived in the town found the street almost blocked with cotton wagons and other traders. The Mayor's reply was that the street was paved for that purpose and that they should adjust their shopping periods to the needs of the community at large, and not try to use the street at the busy period when they could just as well select another time. This is a somewhat radical viewpoint, but the thought is worthy of consideration. In another instance, a filling station operator complained to one of the Texas Highway Commissioners that the highway was being moved from in front of his business to a street two blocks away. The operator exclaimed: "That road has been where it is for fifty years, to my own knowledge," to which the Commissioner replied: "Do you not think it is about time that your neighbor had it for a while?"

One of the chief arguments in registering complaints against the change of location for safety or other reasons within or adjacent to the towns is to say that the State is not building the road for the local people who pay the taxes, but for the tourist. Considerable misinformation has been spread under the name of "tourist trade and traffic." Recently, a Central Texas newspaper contained an editorial stating: "Do not build the road for the tourist but let the taxpayer benefit by having the road." The next editorial a few days after the first said: "Let the traffic pay for the road and relieve the property owner." Within a few days a slogan was published in the same paper: "Make our city the tourist headquarters of the South." These varying comments appeared within a period of seven days.

Webster's dictionary defines a tourist as "one who travels from place to place for pleasure or culture." A citizen of Texas, for example, might leave his home on the morning of a certain day and travel as a tourist along a State highway through many cities and towns; and then, overnight, his status might be changed so that on his return home he would belong to whatever classification might be given him, depending on his rights as a traveler or the contributions he might make for the construction and maintenance of State highways. Such a traveler in Texas might, and very likely would, pay an *ad valorem* tax to build State highways or lateral roads in his home county. He would also, or rather should, pay an *ad valorem* tax on his automobile. He will pay a license fee distributed between the State and County for the maintenance of State highways and, finally, a gasoline tax. When such a traveler leaves his home county for pleasure or culture and travels across his own State, or any other, should he have no consideration given him in that his journey might be made as safe as possible and as direct? Consider, also, for example, a traveler from some other State who might enter the State of Texas and travel for pleasure or culture. In a material way this visitor is required to pay a gasoline tax and, if for no other reason, he is entitled to consideration. State highways should be bought and paid for by the traffic using them and any local funds used in construction should be refunded. Consequently the maximum consideration should be given the traveler in making locations. He should have the option of proceeding on his journey along the most practical route, and if he desires to visit the business districts of any of the towns along the highway a way should be provided for him in case the route does not lead directly through. No particular distinction except for statistical purposes should be made in the classification of traffic. The motorist is not a class and does not pay taxes with a class for the reason that the vast majority of the Nation's tax-payers in all lines use the Nation's highways.

The Influence of Financing on Location.—The day of the *ad valorem* tax for highway construction is passing. In 1929 the Texas Highway Department recommended to the State Legislature that provision be made for adjustment between the various counties and reimbursement of expenditures for State highways, and this issue failed of submission by a very small margin. There is no doubt that State highways should be built from sources of revenue other than property taxes, and this principle has already been put in effect by a number of progressive States. A study of the accomplishments of such States shows that they are well in the lead of others in the status of their road program. The method of financing State highways should and does have considerable bearing on the location, but even with a portion of the construction cost being paid from an *ad valorem* tax in a road district or in a county, no property owner should claim any special rights to the traffic along a certain road since he pays no special privilege for such traffic.

It is only fair that when a hard surfaced highway has once been constructed along a certain street in a town or into a larger town as far as the laws will permit, that such highway or road should be continued and maintained by some agency; but the owners of business houses along such a route have no right to say that the general traveling public cannot, with finances belonging to the general public, construct such additional highways or by-passes as will enable it to proceed with safety.

The writer agrees with a recent editorial entitled "Getting Away with Antiquated Ideas."[3] The author deplored the conservatism of many highway executives in clinging to conceptions of road building that were in vogue during the age of horse-drawn vehicles. To salvage the old roads is often not worth while because it is costly in terms of human life.

CONCLUSION

State highways should be located for present and anticipated traffic requirements so that—whether he be tourist (if there is such a classification), business man, or farmer—the user of the highway can proceed along his route and accomplish his purpose or his duty or receive his pleasure or culture with ease, directness, safety, and economy. Any plan or system of location which does not take these features into account is running contrary to sound public policy. There can be no denying the fact that such public policy, in laying out a great system of highways and lateral roads, must necessarily prescribe a practical directness for the mass of traffic. It must provide more conveniences for traffic originating locally or bound for local destinations. With the adoption of a plan whereby the revenue for State highways comes from State sources, the solution of this problem becomes easier and the "dog will begin to wag the tail" instead of the contrary, which has been the case in many highway locations.

---

[3] *Highway Engineer and Contractor,* Vol. 20, April, 1929, p. 28.

Highway engineers should take every opportunity to acquaint the public with facts regarding the economy of highway location and to lay before it for inspection the ledger entries on the debit and credit side regarding the operation of a State highway. They should always have an eye for the future and if it is not possible to lay out a plan for economical routing when first construction is obtained, the plan should be so broad as to anticipate future demands.

DISCUSSION

C. S. MULLEN,[4] ESQ. (by letter).—The author has covered his subject so thoroughly that a discussion is practically limited to an approval or disapproval of his statements.

Greater emphasis might be placed on the fundamental assertion that the life of a highway is not a function of surface, but of location. Thousands of examples all over the United States could be cited, where road surfaces entirely adequate to carry existing traffic for many years are being abandoned, because the location on which they were built is no longer safe or economical.

The use of aerial photography as an aid to highway location is comparatively new, but in many cases will prove very economical and at times invaluable.

The suggestion as to introduction of horizontal curves to secure increased sight distance is, in the writer's opinion, questionable. As the added sight distance is desirable for greater safety it is suggested that this safety feature could be better secured by widening the surface at summits, rather than by breaking up the alignment and introducing a curve which is a constant potential danger point.

Mr. Gilchrist's belief in the democratic principle that the majority should rule, is not a safe guide in highway location. It has been demonstrated a number of times that where this principle has been followed, serious mistakes have been made which later were condemned by that selfsame majority.

The question as to locating a highway so as to pass through all cities and small towns on its approximate route, is one of the most puzzling that confronts the highway engineer. It is believed that in such cases the question of traffic congestion should receive most serious consideration and the location made, if possible, to avoid this condition.

H. L. THACKWELL,[5] ASSOC. M. AM. SOC. C. E. (by letter).—The answer given to Mr. Gilchrist's questionnaire, that "the life of any particular type of road depends, first, on its location and, second, on the width of its right of way, and that the type of surface was of secondary consideration", is certainly true in part. There are other factors, however, that play an important rôle, namely, climate, attention to maintenance, drainage, etc. Highway maintenance funds should vary with the topography, traffic, and climate.

State highways in Texas, as located in 1930, certainly have been improved in alignment, and distances have been shortened by curve elimination. However, grades are still prominent by their frequent reversals. The rigid policy of exactly proportioning cut and fill, and not allowing waste fills or excessive borrows, is not in accord with the vision of to-morrow.

---

[4] Chf. Engr., Dept. of Highways, Richmond, Va.

[5] Cons. Engr., Jacksonville, Tex.

Many highways from 5 to 10 years old are being abandoned or their locations are being changed radically; such change of mind or policy would indicate that still further changes or eliminations may be expected in the near future on newer locations.

A city of several thousand people in Texas surely has an expectancy of life equal to 1 000 years. It is also reasonable to conclude, in spite of air travel, that wheeled traffic will still be in vogue 1 000 years hence. If these two assumptions are correct, the roadbed (earth foundation which has an exceedingly long life) should be designed and located so that there would be no likelihood of changes in grade or curvature for many decades in the future. Hard surfacing which may have an expected life of from 2 to 30 years (depending on the type, location, and traffic), may be changed in a few years without violating fundamentally economic laws. Roadbeds, with right of way and drainage structures, often cost 50% of the total in places where a standard concrete pavement is used; hence, a mile of roadbed costing $20 000, that has an ultimate life of 1 000 years, would capitalize without interest at $20 per mile per year. The concrete surface might also cost $20 000 ($800 per mile per year), and have a life of 25 years.

The roadbed is frequently altered or abandoned by change of policy, administration, or vision, while it still may retain 99% of its expected life, but not its economic utility. The surface, however, is never abandoned or changed, until its useful life has been largely exhausted.

It does not take great imagination to visualize railroads replacing steel and ties with concrete surfacing and using huge wheeled vans drawn by tractors or individually propelled cars. They may run on their own roadbed in the country districts and, traversing cities, run on city paving and unload the cargo and freight at warehouses wherever they may be situated.

State highways of the future will abandon the present system of numerous short reverse grades and deep drainage ditches, and will incorporate storm sewers, curb, and gutters, with widened pavement, somewhat similar to city pavements, thus taking care of the ever-increasing traffic. Such a highway will provide relief from congestion, will increase safety, and will utilize the acquired rights of way economically.

The writer is of the opinion that aerial highway location should be followed by plane-table topography, the latter being conspicuous by its absence in Texas highway surveys.

Formerly, the improved highway locations for King County, Washington, were always projected on topographic maps which were sufficiently accurate to make possible an economic paper location that would conform to any preconceived rule as to alignment and grade economies.

The dissimilarity between highway location and city street location is marked most strongly by the tendency for highway grades to be above the adjoining property so that free drainage may be adequately maintained. In city paving, on the contrary, the grades are lower than abutting property, and storm water falls over the curbs and is ejected through the storm sewers. This cardinal difference in tendency has caused many cases of objectionable grade transition on roads located through small cities, by highway engineers, and of paving construction carried out by city engineers at a later date.

In conclusion, it is the writer's commentary, but not criticism, that it would be better to have fewer reversed grades, fewer deep ditches, greater use made of storm sewers and gutters, and a gasoline tax so proportioned for maintenance that the hilly country with high precipitation would have more maintenance funds per mile of road than the flat dry country of equal traffic weight. No State aid for cities should be permitted and State highways should begin and end abruptly at city limits. Cities should be required by State law to keep up an economic paved roadway connecting the State highway through the city, that is financed by refund of a proportion of the gas tax to the municipality. State highways should be financed, owned, and operated by the State and Federal Governments and these groups should not expect aid from counties or local communities. The so-called co-operation of funds is largely responsible for frequent criticism by local communities against the removal of existing highways, and has naturally culminated in selfish interference, through politics, with economic engineering locations. The engineer's call of distress is well justified, but the cure will be at hand only when the financing policies have been changed.

The grand success of the Appian Way and its magnificent permanence are due more to the splendid Roman administration and policy than to the engineering of the highway itself. The original location is unchanged and the foundation still endures, while gravestones crumble and empires fall.

B. P. HARRISON,[*] M. AM. SOC. C. E. (by letter).—Too little emphasis is given in this paper to the part that safety should have in the location of highways.

Should not the engineer feel some responsibility for the lives lost due to a faulty location? The roads are crowded with poor drivers and the toll of human life is rapidly increasing. The tendency of the day is for increased speed. Some States have increased the speed limit, while others have even done away with the speed limit entirely. Every one agrees that the loss of life is far too great and that something should be done to lessen the mortality.

Is the engineer doing his part by insisting upon proper locations? In days gone by, with slow traffic, it was probably economical and proper that bridges should be built at right angles to the streams. The writer has seen many examples in which locations have been badly distorted in order to accomplish this purpose. Bridge engineers sometimes fight a location because it will require a bridge to be built crossing the stream on a skew. The time has come when the highway engineer must insist upon the proper location, whether it will require a bridge to be built on a tangent, a curve, or a skew. The hazard is too great to permit the old right-angle crossing with its dangerous curves at or near each end of the bridge. Generally speaking, engineers must build the bridge to fit the location and not make the location to fit a bridge site.

---

[*] Associate Highway Engr., U. S. Bureau of Public Roads, Washington, D. C.

Often there are restrictions, as to sight distance, or curvature that does not require super-elevation. If the sight distance required in a specific case is 500 ft., how seldom does the motorist have any more? The tendency is to do no more than is necessary "to get by". If it should be required that all curves of more than 6° be superelevated, the curves will be made 5° 58′, to avoid superelevating. Why should there be the tendency to do no more than is absolutely necessary to "get by", with any location and plan? Economy has played too great a part in past locations. The engineer responsible for highway locations should spend a little more money, make highways "fool" proof, cut out grade crossings, and thus save many lives. The locating engineer has a wonderful opportunity. Will he take advantage of it?

E. P. Arneson,[7] M. Am. Soc. C. E. (by letter).—The author's views are refreshing and stimulating, dealing as they do with the human element in highway location. His conviction, that the people want the highways located for the interest of the many and not for the few, may well be taken by the locating engineers of America as a tenet of faith. All rules of highway location flow as corollaries from that proposition.

The author makes another sententious and challenging statement—that the life of a road is not so much dependent on the type of its surface as on its location. There is an implication here as to the importance of the time element, which the writer would like to discuss.

Referring to the original interest to be served—that of the many as opposed to the few—one knows that this interest may change with the passing of the years. Therefore, the influence of the time element should always be weighed when one is about to fix the position of a proposed highway. The need for the road may be greatly lessened, or indeed it may virtually disappear, in a number of years, by reason of changes in the habits of the people served. The point is, as the author states, that not only does the road wear out, but its very location may have a limited life.

Because of the influence of the time element one may be justified in giving a road an impermanent location, if the ascertained facts warrant. The writer remembers a case in point, where two highways were to be located on general routes which converged on a city with an angle of about 45 degrees. The Northeast Highway, being the more important, was run on a line as nearly direct as the topography permitted. For the East Highway there was the choice of an independent and direct location into the city, or of a combined location by joining the Northeast Highway six miles outside the city. In the latter alternative about $360 000 in first cost could be saved at the expense of some lost distance for the East Highway traffic.

The solution of this problem resulted about as follows: A reasonable estimate of the future traffic on both highways indicated that the joint track would be safe from congestion for twenty years. Moreover, the comparison of the costs and benefits of the independent route for the East Highway indicated that it would take about twenty years for the benefits to pay for the extra investment of $360 000 and its interest. The decision was to combine the two highways, reasoning that it was unwise to build the more direct and independent location of the East Highway, when it would apparently take as long as twenty years to pay for itself, provided the future developed as predicted by the engineer. Nevertheless, the time may come when the East Highway can properly be given an independent and direct entry to the city. Then the road, as theretofore located, will be one more route cast into the limbo of impermanent locations. Permanency of location is a relative matter.

---

[7] Civ. Engr., San Antonio, Tex.

GIBB GILCHRIST,[8] M. AM. SOC. C. E. (by letter).—That serious mistakes have been made by following the principle of the majority rule is the opinion expressed by Mr. Mullen, who also states that these have later been condemned by the selfsame majority. This theory would be entirely correct if an engineer should build a structure of inferior and inadequate design, such as an important river bridge which might fail under increased loads, but in the case of highway location the engineer's duty should be to acquaint the governing body—whether it be the board of directors or the entire electorate —with all the facts and arguments for proper location, so that a principle of location may be established along which he may exercise the best technical knowledge for the most economical construction. He will find in a great majority of cases that, when the facts are presented by an engineer impartially and correctly, the decision as to policy will be along the proper line.

Mr. Harrison believes that too little emphasis is given to safety and he discusses the tendency of engineers to build a right-angle crossing of a stream with too much curvature at each end. Just as much criticism may be given to a plan in which alignment only is considered, and the bridge is constructed on too much of a skew. Latitude in this case should be given each way as easy curvature is not particularly objectionable and a compromise can easily be found.

The writer cannot agree with Mr. Harrison that the tendency is to do no more than is necessary "to get by." It is believed that engineers are giving more and more attention to location in a common-sense way and that radicalism in one direction is perhaps as bad as carelessness or vacillation in another. He cannot agree, either, with Mr. Harrison that more money should be spent to make highways "fool-proof"; this is an impossibility as far as location or construction is concerned. The only way to make highways "fool-proof" is to keep the "fools" off the highways, and the people riding in automobiles with "fools" as drivers will simply have to take their chances. Every consideration, of course, should be given for the safety of highways along reasonable lines for the protection of the traveling public.

Public sentiment has changed wonderfully on highway location, and it is still changing. It had evidently taken a step backward in the not too distant past, if evidence[9] given by Mr. James Dean, Land Agent and Civil Engineer, before a Committee on Highways of the House of Commons, on April 1, 1819, can be given credence. This might have been written in 1932:

---

[8] State Highway Engr., Austin, Tex.

[9] "Road Making," by McAdam, p. 183.

"The first and most obvious improvement is to shorten distances; but even that must be governed by circumstances often of a local nature; a sound foundation, and the contiguity of good stone or gravel to a road, should not be overlooked in choosing a new line, or departing from an old one."

Reference was made in the original paper to a principle of democracy. Two letters written by President Thomas Jefferson under date of August 6, 1808, indicate that problems have not changed greatly in the meantime. They also show that the attitude of those in authority was not much different 125 years ago.

Writing to Kerr, Moore, and Williams, Commissioners of the "Western Road," Jefferson called attention[10] to a petition by the citizens of the Town of Washington (then in Pennsylvania) for a relocation of a through highway so that it would pass through their town. The argument was advanced that such a detour would be only a mile longer, that it would pass through better ground, and would cost less. While the principle purpose of the road was a communication directly westward, President Jefferson decided that if minor deflections would benefit particular places and accommodate travelers better, these circumstances should be considered. The letter ends by charging the Commissioners with the duty of reporting material facts and their own opinions, for the President's consideration.

On the same day President Jefferson wrote[11] to the Secretary of the Treasury, Albert Gallatin, and explained some of the location problems behind the letter to the Commissioners. It developed that this was a move made after a first concession had been granted to another town; a third petition was being prepared at still another town.

"I should have listened to these solicitations with more patience," wrote Jefferson, "had it not been for the unworthy motives presented to influence me by some of those interested. Sometimes an opposition by force was held up, sometimes electioneering effects, as if I were to barter away, on such motives, a public trust committed to me for a different object."

While many States are still using county aid obtained from the *ad valorem* tax for the construction of State highways, the tendency is not only for the construction funds to be obtained from transportation, but also for transportation to bear the assumption or refund of county funds spent on highway construction. For this and other reasons highway location should be determined according to the needs of the traveling public and the safety and convenience with which a traveler, or a group of transportation units, may reach their destinations with the minimum of travel and the maximum of safety, decisions being at all times governed by common sense for which a suitable substitute has never been found.

---

[10] "The Writings of Thomas Jefferson," Vol. XII, p. 117.

[11] *Loc. cit.*, pp. 118–119.

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
TRANSACTIONS

Paper No. 1799

HIGHWAY LOCATION: PRACTICAL CONSIDERATIONS[1]

By A. R. Losh,[2] Assoc. M. Am. Soc. C. E.

With Discussion by Messrs. H. K. Bishop, J. C. Carpenter, H. J. Spelman, H. S. Kerr, H. W. Giffin, and A. R. Losh

Synopsis

The purpose of this paper is to present some of the major problems of highway location. There are administrative as well as physical limitations which determine in a large measure the work of the engineer on location. It is fundamental that the ultimate purpose of the road be kept in view and the location made to accomplish that end. Highway location does not include merely lines, grade, and construction costs, but traffic as related to towns and cities, other highways, and the general needs of the public. Too much location "for present needs" has been followed in the past with resulting loss, under relocation, to the public and to the individual citizen along the highway as well. Insufficient right of way for the protection of the highway is a serious handicap to its future usefulness.

Introduction

It is recognized that the detail problems of location depend on the peculiar conditions of the individual project. It is equally true that there are general problems on roads of the same class, but these may be decidedly different for roads dissimilar in their uses. Thus, different governing conditions apply in the location of roads for sectional and interstate traffic than for local roads serving small agricultural communities. Roads located primarily for parkways, pleasure drives, and for recreational purposes where scenic considerations are of major importance, will naturally impose conditions differing from those of the other classes. There is, however, one fundamental principle which should be the controlling feature in any road location. Briefly, this is that the resulting location should be adequate for present and prospective future needs of the highway. Present needs are readily determined; future needs require the exercise of judgment and foresight.

---

[1] Presented at the Joint Meeting of the Highway and Construction Divisions, Dallas, Tex., April 25, 1929.

[2] Mgr., Asphalt and Road Oil Div., Anderson-Prichard Corp., Oklahoma City, Okla.

The reconstruction and improvement of roadway surfaces and structures can be readily justified from prolonged usage or for needed increased facilities in the same way that the rebuilding and remodeling of homes, office buildings, mills, and other structures, but the relocation of a large portion or all of the road does not present a similar case. This change of location involves a substantial loss of the right of way, roadbed, and structures, as well as the expense of their reproduction on the new line. It is difficult to find suitable explanation or justification for the building, by the present generation, of many of its main highways three times and on three different locations.

It is not to be understood that the mistakes in location in the past rest solely upon the highway engineer; this is not the case. Many of the earlier improved roads were not located; they merely followed land lines or existing rights of way of older roads. The engineer was given little or no opportunity to develop a suitable location. Road funds were provided by local governmental units and the roads were placed so as to be acceptable to the governing boards or commissions. Most of the present lines of travel were originally only local roads, and the subsequent traffic development was not anticipated. This condition no longer exists except for the local roads. The main highways are financed by the road user at large, by the State, and by the Federal Government. The engineer should approach the location problem with the future uses of the road in mind and should endeavor to provide a location which will be suitable for this purpose. This will include—in addition to alignment, grades, stream crossings, and similar engineering work—provision for future development of the highway and its protection from encroachment by industries and other utilities. Briefly, the mere located line without adequate right of way and roadside protection will not be sufficient.

It will be the writer's purpose to discuss the major problems of locating State and Federal Aid highways. The engineer has received very little assistance on highway location from engineering textbooks. The subject is discussed only briefly if at all, and then usually on the assumption that the existing road will be followed with only minor changes in alignment. The location of highways purely as transportation lines is not the usual practice. This is not a condition readily attainable, nor is it strictly desirable on public highways. The service of the project in the highway plan as a whole is the consideration which should be followed.

DESIGNING FOR PRESENT OR FUTURE CONDITIONS

There are still some who would consider present highway building as primarily of local importance and for the immediate local needs, it being proposed at some future time to build a system of through highways on more direct lines. Such a line of reasoning has been applied to a considerable extent in the past with the following results:

(*a*) The location of important through highways has been materially affected by purely local considerations.

(*b*) Frequently, local and through traffic were combined where a separation would be preferable.

(*c*) When reconstruction for the through traffic is undertaken it is exceedingly difficult to make adequate relocation.

(*d*) When the indirect line is used a number of years subsequent development results in increased property values which may be seriously impaired by substantial changes of location.

(*e*) Right-of-way changes resulting from relocations are usually more expensive than if made under the original construction, due to increased property values and damages to improvements.

(*f*) The value of the old road in terms of right of way, roadbed, surfacing, and structures, is a loss unless the road can be used advantageously for local purposes. Even then, it must be considered that 1 mile of main highway costs as much as several miles of serviceable local roads and there is a loss in changing a main road to a local one.

(*g*) The roadbed structure consolidated and compacted by years of use is lost for reconstruction purposes.

It is this line of reasoning which would make the work of the engineer follow the development of the country rather than to lead and aid materially in that development. It is "the line of least resistance", of temporary expediency, and of future changes with attendant losses. The aim of the engineer should be to provide a location which will remain as a major line of highway transportation and which is so planned as to meet the requirements of the future. A location of this character approaches the "permanent road feature" as nearly as is humanly possible.

RESTRICTIONS ON LOCATION

On all highway projects there are administrative controls or restrictions which determine at least the general location of the project, or the route of which it is a part. Thus, the termini of the project are predetermined and the intermediate location may or may not be fixed. These restrictions will occur under the following classifications:

1.—Major control points of the Federal Aid System determined by the Secretary of Agriculture and the State Highway Departments and subject to modification by joint action.

2.—State highways and control points designated by State Highway Commissions under legislative authority.

3.—Control points for State highways fixed by legislation.

4.—County or road district bond issues used for a part of the construction cost may require a definite location to be followed.

Under Conditions 1 and 2 there are opportunities for a general engineering study before the controls are specified and also for a revision of controls based on further engineering investigations. Condition 3 may or may not be based on engineering investigation, or it may permit some departure from the established route at the discretion of the State Highway Commission. Condition 4 presents the greatest difficulty because frequently, where local funds are contributed, the detailed location is fixed primarily to serve local wishes. The writer has known of cases in which the increased cost of these features placed by local influences was greater than the local funds provided. For example, in Jefferson County, Oklahoma, county funds were provided to aid in the construction of U. S. Highway 81 on a location determined by the county officials. This location involved three more railroad intersections, one additional bridge, and some increase in distance over a location selected by the State Highway Commission and the United States Bureau of Public Roads.

This system of financing has been responsible for most of the unsatisfactory locations in the Southwest during the fifteen years since 1914. Where it is necessary to continue this system of financing in fairness to all parties concerned, the location should be determined in advance of the local arrangements for financing.

Even under Conditions 1, 2, and 3, there is too great a tendency to follow existing lines of improvement with relatively unimportant control points established; under Condition 4 this tendency often reaches the point of affecting the detail of location. Regardless of how the conditions are imposed they may in effect restrict the work of the locating engineer to mere details of design. Administratively, controls are necessary and should be general in character and limited to the following conditions:

(*a*) A town or city on or near the general line of the route between adjacent control points of the route.

(*b*) A town or city of sufficient importance to be a terminal point for the traffic passing between the adjacent controls.

(*c*) A junction point, preferably near a town or city, of several highways for the distribution of traffic.

(*d*) Topographical features based on engineering investigations.

Within the limitations established as controls the engineer should determine, as far as practicable, upon an economic location, considering at their relative weights both through and local traffic. This selection will be the line which will provide the long-run least cost, including construction, maintenance, and traffic operations. The shortest line will usually be the economic location for the reason that practically all items of cost except earthwork and structures vary directly with the length of the line. In a country of broken topography the cost of earthwork and structures may be much less on the longer line, but the other items of construction, and of maintenance and operation, will usually determine the location. Local traffic can generally be served more cheaply by lateral spurs from the main line than by attempting to give direct service.

One of the usual problems before the engineer is to determine whether or not to swing the main line from its direct course in order to give highway service to an intermediate point, such as a small town or community. Theoretically, the problem can be solved by estimating the cost of construction, maintenance, and operation for each of the two possible layouts; that is, the cost of the main line direct with a lateral spur and cost of the main line on indirect location through the intermediate point.

A similar problem is that of combining two or three highways for some distance out from the city in order to reduce construction mileage. While economically desirable, frequently this involves the questionable practice of encouraging traffic congestion at junctions and terminals.

Studies of State highways show a substantial volume of traffic outside the purely local class. Thus, the California studies of 1922 show the average length of truck hauling to be 31 miles and that of passenger buses, 41 miles. The Pennsylvania studies show that 43.8% of the State highway traffic travels 50 miles or more, and for the Ohio study this figure is 30.3 per cent. Similar data are not available for the State highways of the Southwest, but it is believed that there is a larger percentage of long-distance movement.

Traffic of this character is benefited more by the possible economic operation over the whole line than by the detail location through small communities with which they may have little or no contact. This provides the basis for considering the needs of local and through traffic separately. It is recognized that grades, character of road surface, alignment, traffic hazards, and conditions causing loss of momentum, use of brakes, and extra wear on the machine are all factors affecting adversely the cost of traffic operation, but the data are not available in a form to be readily used.

EFFECTS OF TOPOGRAPHY AND GRADES

In working out the detail location further economies can be made by taking advantage of topographic conditions, soils, available local material, or railroad facilities for importing materials. Approximately 4 000 tons of materials are required per mile of 18-ft. concrete pavement. An increased dead haul of 1 mile will increase the cost of the road about $1 000 per mile. In addition, there will also be the cost of moving construction equipment on and off the job as well as the cost of hauling structural materials. The use of long radius curves reduces the total length of line and the amount of surface widening. For a change in direction of 90°, a curve of 1 000-ft. radius gives a line 214 ft. shorter than a 500-ft. radius. To widen the curve of 500 ft. radius would require about 938 sq. yd. of additional pavement.

Topographical conditions affect the details of location and, in some instances, are major controlling factors. All soils which are unsuitable for subgrades, such as shifting blow sands and rock classifications, are to be avoided if practicable. Overflow sections require high embankments with costly structures, and, for this reason, valley locations are frequently undesirable. Stream crossings must be selected with the view of reasonable permanency of the banks, suitable foundations, and required span length, and they must be within the general location. Grades and curves adjacent to bridges should be reduced to the minimum. It is desirable to have a tangent of at least 150 ft. and a light or flat grade at each end of a major structure. These conditions permit the driver to adjust his machine into the bridge alignment. In the past, grades have probably received more consideration than other features. Considering the motor vehicles now in use some sacrifice of grade can be made to improve alignment. Short grades as steep as 8% can be climbed readily by modern motor vehicles, but continuous grades of more than 6% are objectionable. Tangent profile grades are not particularly advantageous, due to the headlight glare, and do not justify increased expense. A rolling profile fitting the topography is less objectionable for night driving.

Grade separations at railroad intersections are desirable, but not always possible financially. When grade crossings are adopted the line should be located so that future separation can be developed, and in all cases approach curves and grades should not restrict sight distance.

Restricted areas in and near cities are frequently obstructions to direct highway location. This includes parks, golf courses, institutional reservations, cemeteries, railroad yards, and air ports. Generally, it is impracticable to pass through these areas and the location must be carried around them. The location should give access to railroads and air ports, but should be sufficiently removed to permit full operation of all facilities without congestion. A location directly adjacent to the railroad may be congested by trucks loading and unloading at the station and also by industrial spur tracks being constructed across the highway. Location along the railroad is usually adopted because good alignment is obtainable in this way and because the right of way is not difficult to secure. It has the additional disadvantage of permitting development on only one side of the highway whereas if removed ¼ mile or more from the railroad, property on both sides of the highway would develop.

One of the major problems of the engineer is to provide suitable locations into, through, and around towns and cities. It is necessary to provide suitable connections into the street system of the city. It is highly desirable also to have a direct through connection, even if it may be congested by city traffic, in order to take care of through traffic making a temporary stop in the city. A belt line, or by-pass location, is equally desirable for the through traffic to avoid the city congestion in reaching its destination and to permit the sorting or segregation of traffic for different locations.

The desirable condition for the urban approach would be connections which would permit the distribution of traffic to:

(*a*) The retail section.
(*b*) The wholesale and industrial sections.
(*c*) Railroads and air ports.
(*d*) The several residential sections; and
(*e*) To other highways by direct connections, outside the city proper.

Many town and city officials believe that the highway should be considered as a continuation of "Main Street" out into the country and that it should be carried through the principal thoroughfares of the city. This is especially true of cities of less than 10 000 population and the smaller the municipality the more insistent it is as a rule that the highway pass through the main streets. The larger cities have their local traffic problems and as a rule are glad to secure any relief possible by routing through traffic over outlying streets or through the suburbs. In a great many cities, however, highways are taken directly through retail sections where the streets are congested with electric street cars, motor vehicles, and pedestrians. There are numerous street intersections, sharp curves, grade crossings, congestion due to parking, and other hazards of city traffic. The through traffic is delayed and local traffic congestion is increased with possibilities of accidents multiplied. Too many towns and cities are placing restrictions on through traffic by means of speed ordinances, stop signs, signals, and other regulations. These measures in most cases are necessary for the local traffic, but the through traffic can operate better without them and outside the city. Table 1 gives information on several highways through cities of the Southwest.

The development of proper approaches to cities with arterial routes for traffic to and from the city and belt lines for traffic distribution requires the co-operation of State, county, and municipal authorities. It also requires a careful traffic study and analysis, upon which to base present and future plans for improvements. The Plan of Highway Improvement in the Cleveland Regional Area is an outstanding example of this character of work. As a result of this survey the City of Cleveland and its contiguous territory have developed a plan of major road and street improvements for ten years ahead and may proceed with individual projects, knowing their final position in the broader plan.

TABLE 1.—Data on State Highways in Cities.

City and highway.	Miles in city.	Sharp Curves.		Railways (All Classes).		Street Intersections.		Legal speed, in miles per hour.
		Number.	Total degrees.	Number of tracks crossed.	Route used by street railroad, percentage.	Control signs, signals, police.	Total number.	
Dallas, Tex.:								
U. S. 80	8.5	12	735	19	12	15	89	10 to 20
U. S. 75	7.6	8	870	26	16	16	119	10 to 20
Fort Worth, Tex.:								
U. S. 80	10.9	6	390	9	51	12	123	12 and 18
U. S. 81	9.5	11	1 150	27	77	7	111	12 and 18
Houston, Tex.:								
U. S. 90	8.5	14	780	9	4	13	110	8 to 20
Little Rock, Ark.:								
U. S. 65	4.5	4	525	7	19	4	40	15 to 35
U. S. 167	3.3	2	180	7	6	4	35	15 to 35
U. S. 67	5.0	6	515	7	15	4	56	15 to 35
New Braunfels, Tex.:						....		
U. S. 81	3.2	9	635	1	0		30	20
Oklahoma City, Okla.:						6		
U. S. 77	6.2	6	510	21	9		70	25
U. S. 66	4.15	3	330	4	40	4	42	25
Purcell, Okla.:								
U. S. 77	1.2	4	360	4	0	1	13	....
San Antonio, Tex.:								
U. S. 81	10.0	14*	1 200*	8	28	13	84	10 and 20
U. S. 90	8.6	9	845	23	55	12	95	10 and 20
San Marcos, Tex.:								
U. S. 81	2.2	6	390	0	0	....	32	20
Tulsa, Okla.:								
U. S. 66	9.1	8	591	22	4	7	97	8 to 25
Waco, Tex.:								
U. S. 81	4.2	3	360	25	33	7	39	15 and 20

* Other routes give less curvature, but more traffic congestion.

Increase in most of the speed limits now in force (1929) appear inevitable. The engineer should anticipate this with provisions in his location to meet the conditions. Greater sight distance on horizontal and vertical curves is beneficial; 600 ft. is not excessive. Where this cannot be obtained extra width of surfacing is desirable. In the level and rolling country of the Southwest, engineers are providing horizontal curves of 4° and less. This makes extra width unnecessary, very little superelevation is required, and line length is reduced. The older practice of placing changes in alignment at the top of hills has been discontinued. It is considered a hazard to traffic. Other conditions being equal, curves should be placed where they may be seen readily by the approaching driver.

RIGHT-OF-WAY FEATURES

To a great extent engineers have allowed right-of-way considerations to influence location. This is especially true in regions where the section-line road system prevails. It is seldom that on rural highways more than 80 ft. of width is secured, which amounts to approximately 9½ acres per mile. Considering the cost of State highways at about $30 000 per mile there could be a justified expense of $3 000 per acre for a proportional reduction in length of construction. This does not take into consideration any savings in maintenance and operation costs.

The width of right of way needed depends on the future requirements of the road and also the means for its protection from encroachment and exploitation. Few States have adequate laws regarding right-of-way acquisition and roadside protection. A Texas statute permits the State to obtain 100 ft. for a State highway, which ordinarily is ample. Oklahoma may obtain an easement for any width "for highway purposes." There is nothing, however, to prevent the abutting property owners from building garages, service stations, and other roadside establishments on the property line, and utilizing a part of the highway for the transaction of their business.

Until such time as adequate legislation is enacted placing roadside development under the supervision of highway authorities, the engineer should use such means as are available to protect and preserve the highway for its complete utilization. To this end the writer offers five suggestions:

(1) Acquire sufficient right of way for future development of the highway.

(2) Obtain extra widths at all road intersections and other strategic points where it is likely that roadside enterprises would be established.

(3) In making acquisitions it will usually be possible to retain in the right of way the narrow strips, gores, and other small parcels of land cut off from the original tract. These parcels may be utilized for highway purposes in the future or for roadside planting and beautification. In any event they are not available as sites for billboards and roadside establishments.

(4) Through areas of natural beauty either obtain in fee or protect with long-term leases, several hundred feet extra width on each side of the right of way.

(5) The interest of civic organizations and public-spirited citizens should be encouraged and their aid secured to obtain the desired ends.

PUBLIC CONSIDERATIONS

It is apparent to every one that, aside from the commercial features of highways, there are social and political considerations which cannot and should not be disregarded. Roads are for public usage and must be located, constructed, and maintained with that end in view. Whether for business or for pleasure the public travels to definite points of interest and attraction. Schools, churches, and Courts are the mainstays of the Commonwealth. The public must have access to these institutions by highways. Rural mail and express deliveries, transportation of farm products, and rural community life, while usually served directly by local roads, are affected by their connection with the main highways. There are patriotic shrines in every section of the country which should be made accessible from the highways. National and State parks and forest preserves are visited annually by thousands of citizens and, with improvements in the roads, these properties will be used by many more for recreational purposes.

Every governmental unit should be connected finally with the country at large. The writer has always believed that the larger unit of government has this obligation to its constituent parts: The Federal to the State, the State to the county, and the county to its towns and communities.

These considerations of schools, churches, Courts, and other conditions vitally affecting the citizenship of the Nation cannot be weighed in terms of economics or engineering, but must be taken under the "General Welfare" clause of the fundamental law.

DISCUSSION

H. K. Bishop,[3] M. Am. Soc. C. E. (by letter).—The primary use of a modern highway, generally speaking, is to provide means for vehicular transportation of the various classes of travel and commercial haulage from one common center to another, either local, State, or National. They should be established over the most direct route possible with the least gradient. The most direct route may be defined as a road with alignment free from curves that have dangerously short sight-distances based on the importance of the road and the speed limit involved; and the least gradient should be such that it will not retard materially the reasonable speed of present-day vehicles.

The most direct route and least gradient, however, may be dispensed with advantageously, to some extent, in certain areas of heavy snowfall for reasons of economy in snow control. These factors may be ignored also in cases of scenic development, when locating forest and park roads or other lanes of traffic promoted solely for pleasant landscape effect, recreation, or educational travel.

For the winter of 1929-30 the States in the heavy snowfall area of the country reported that about 185 000 miles of main rural roads were kept clear of snow at an approximate cost of $8 250 000, and statistics show that more than 779 000 private automobiles entered, and more than 2 270 000 people visited the National parks during the 1930 season. Also, more than 28 760 000 people visited the various National forests by automobile during 1929.

The cost of clearing roads of snow in areas presenting certain topographical features can be greatly reduced by appropriate study of the proposed locations, and consideration should be given to the many patrons of the great out-of-door sections by creating alignment for the roads built, that will unfold, to the best advantage, the scenic wonders of the region visited.

Location to Prevent Snow Drifting.—Removal of deep snow is likely to create considerable expense in regions subject to heavy snowfall, and anything that can be accomplished to prevent drifting is of great value, not only from the viewpoint of economy, but also for the convenience of winter traffic. Snow drifting is often prevented or greatly reduced by proper road location. For this reason a number of highway departments give the snow problem careful consideration in planning their new or relocated roads, and, consequently, they experience fewer blockades on highways recently constructed than on the older ones. For drift prevention, proper location is often of primary importance and is influenced by efforts to keep the roadbed away from northern slopes and to eliminate various other conditions that tend to cause drifting. The locating engineer for roads of this nature should be familiar not only with the territory in which he is engaged in connection with ordinary features, but he also should have a knowledge of snowfall and prevailing wind conditions. In some instances, the sacrifice of alignment and grade, to a reasonable extent, may be necessary to secure drift-free roads. In open areas the highway should follow ridges where possible or should be placed on slopes against the prevailing winter winds. Long shallow cuts should be avoided, but where this is impracticable the location and construction should be such that they will not cause drifting.

---

[3] Chf., Div. of Constr., U. S. Bureau of Public Roads, Washington, D. C.

In low flat country snow drifting has been lessened and often eliminated over a location where the new grade can be elevated 2 or 3 ft. above the surrounding land and with the construction of 1 on 4 slopes. In instances of this kind it would also be better to widen the cuts on either side to the boundary lines of the road. In mountainous or sparsely settled areas where the winter traffic is not sufficient to justify the expense of keeping the roads open during the entire winter, proper exposure is highly essential in order that the snow may melt and free the road for traffic as early in the spring as possible.

In States with heavy snowfall a winter reconnaissance should be made, or at least an inspection, during the snow season and prior to the completion of the plans in order that all snowfall, wind, and drifting conditions may be given careful consideration and data submitted for the use of the locating engineer in the final determination of his alignment. A number of these and other methods have been followed by various States and, in some instances, have caused the roads to remain fairly free of drifts through the winter months.

Tourist Parkways.—In the location of roads proposed to carry ordinary traffic, examination of the country through which it is to pass, with reference to the topographical features encountered, is necessary. Furthermore, consideration must be given to the nature and extent of traffic. On the other hand, for sight-seeing roads, other questions may arise regarding the proposed lines relative to greater comfort and convenience of travel and also regarding the pleasure and knowledge to be derived from use of the roads proposed. Often esthetic and historical features are present that suggest modifications of alignment that would not appear as good engineering practice in choosing locations for common roads.

In locating State and county roads, existing traveled ways are often utilized with necessary changes here and there to improve the alignment and grades; but for park and forest roads it is at times desirable to avoid all traveled thoroughfares, both in regard to following their alignment and in crossing them. Recreational roads are built principally for sight-seeing traffic and should be located with the convenience of the traveler in mind regardless of reasonable extra expense. Where intersections of existing roads are unavoidable, under or over-passes should be provided with ramps constructed where necessary for feeders to the parkway.

For ordinary roads the most suitable locations are those that reduce the yearly expense for transportation to the minimum, reduce the annual maintenance cost for the roadbed, and keep the original cost of construction as low as possible consistent with good engineering practice. These elementary problems are also applicable to tourist roads in greater or less extent, especially relative to the cost of construction. This first cost depends largely upon adjustment of the grade to the existing surface in order that minimum quantities may be involved in excavation and embankment and the location determined to avoid in the largest degree both major and minor drainage structures. For all types of roads great care must be exercised in selecting locations away from unstable foundations, such as swamps and soft ground, where construction is expensive and drainage difficult.

In locating tourist roads existing historical sites must be skirted to provide interest for the traveling public, but they must not be mutilated in any way by proximity or intersection. While wooded routes are generally desirable, large or rare trees must be spared and saved from the axe. In building parkways deep cuts are apt to cause scars in the natural landscape and, therefore, are not allowed. In places where embankments are essential, instead of balancing quantities in the usual manner, a reasonable cut is made with the remaining material provided by borrow from pits out of sight of the traveling public. When rock cuts and fills are unavoidable their new surfaces are re-covered with top soil for vegetation growth. Consequently, in locating roads of this character, the availability of suitable loam to mask the scars inflicted must be considered.

Forest and park roads usually traverse one or the other of these playgrounds, form a parkway between them, or act as lanes connecting historical points. Their routes vary in topography from flat and rolling country at tidewater levels to rough and rugged mountainous regions 10 000 ft. and more above sea level. Such proposed roads, however, usually extend through scenic country primarily for recreational purposes. From their nature they follow wooded and wild territory where location is difficult. At times, in fact, the degrees of roughness encountered are such that tunneling is considered more economical and desirable. For these classes of roads, as well as for most others, study of the proposed alignment should begin with the reconnaissance and should continue until the improvement is advertised or construction actually started.

For wooded sections or regions with irregular configuration, the benefit of aerial photography can not be over-estimated. This is especially true considering the fact that a reconnaissance should embrace a territory and should not merely follow a line. From these photographs a strip of land for location can be approximated between control points. In a general way the proposed location can be narrowed thus within reasonable limits. With this accomplished, topographical maps in one form or another are very useful. Drawn to a scale of 1 : 62 000 such maps are available for most sections of the country. While these are helpful in determining a line, maps covering the same territory greatly enlarged are procurable from the various Federal agencies at least at reasonable cost.

With maps covering the territory selected from the aerial photographs enlarged to about 400 ft. per in., the best and most economical alignment can be determined with surprising accuracy. Another method followed by the Government is to use maps for approximate alignment data compiled from so-called strip, topography surveys. The work is performed by the U. S. Geological Survey at a cost per square mile depending upon the scale desired. These strip, topography maps are prepared in about the same manner and carry data similar to those found on the ordinary U. S. Geological Survey maps, but the area is confined to the strip selected—probably $\frac{1}{8}$ or $\frac{1}{4}$ mile wide—with the salient points located, if possible, with greater care.

With Geological Survey maps at hand, especially those enlarged, or others plotted from strip, photography surveys with accurate contours, a trained and competent location engineer will note the direction of flow of streams; the positions of thick or light wooded sections; the height and formation of the various slopes; the position of ridges that may be followed; and the location and elevation of the various passes through which it might be possible to carry the line; and from them he will adopt the most suitable alignment.

Scenic routes generally are proposed for country composed of numerous valleys enclosed by ridges of varying heights, all branching in like direction with the main watercourse of the territory. With a careful study of these principal features from the map used, the location may be determined with a fair degree of accuracy, leaving only the minor details to be settled for final location and for staking out the center line.

J. C. Carpenter,[4] M. Am. Soc. C. E. (by letter).—The paper by Mr. Losh brings out the fact that highway location in the past has followed the path of least resistance. This is obvious to any one who studies the existing system. Until a broader policy is adopted by the officials in control of this important feature of engineering the same mistakes will be made as are now evident in past practice.

The first State highways were located adjacent to the larger centers of population and the location was not a matter of serious thought, the object being to improve the "old road". With the passage of the Federal Aid Road Act, in 1916, the States were required to map out a system of highways, on which Federal Aid could be expended. The systems thus laid out were not limited to mileage, and while an attempt was made to provide a system of transportation that would properly serve the State, the result was not at all satisfactory. The projects built on this system were more or less development roads and some of them are now being eliminated as parts of the system. Short sections, radiating in all directions from the centers of population, were the rule, rather than the exception, under the procedure in vogue at that period of the highway development program.

The "Federal Highway Act" of 1921 added some desirable features which were introduced with the intention of developing a system of properly located, connected routes. A new system of primary and secondary highways was adopted, and the mileage was limited to 7% of the total public highway mileage of the State. This system was approved by routes with official control points tied in to a description. The procedure in approving projects requires that the routing between official control points be determined and approved before any part of the route is improved. This was undoubtedly a step forward and has served to locate the improvements on a system of main highways, "interstate in character". There have been numerous instances, however, of poor locations due to the necessity of tying in the old work that was completed under the first system. The plan also results in "center-to-center" locations which lead the traffic to the congested parts of the larger centers of population.

---

[4] Fort Worth, Tex.

Under the foregoing procedure the highways of the United States have been developed in such a way that it is possible to travel to all sections on some type of improved road. Traffic trends are becoming stabilized. It is an opportune time to "take stock" and do some real planning that will serve to provide some permanent locations. With the present systems as a basis, a revised system of highways can be laid out in such a manner as better to serve local and through traffic for the entire country. It is important that such a system be developed at an early date. Based on present and probable traffic trends, on the development of the industrial and agricultural sections of the country, the growth of cities, and the controlling topographic features, a careful study should be made in each State and a major system laid out and filed for use on new work. It will be necessary carefully to consider entirely new locations, extending across the State, in some instances. The principles discussed in the paper should be applied in developing this ideal system, as, for instance, the development of locations through cities, locations at suitable distances from railroads, etc. While it will be impossible to change from the present system to such an ideal system at once, the data should be compiled and the system developed and placed on file with the expectation of utilizing it as funds become available. With such a plan in reserve, a State Highway Department is prepared to justify a change from the present routing and any short section constructed on the major plan eventually will become a part of the larger plan. Under the present policy, large outlays of money are required to reconstruct old projects on narrow rights of way with undesirable alignment, and traffic congestion is increased rather than diminished. With the same funds a new highway can often be built on good alignment, with adequate width of right of way for future development and with much better planned surfacing, leaving the old road to serve local traffic with a much smaller annual maintenance cost for the two routes. The new location develops and serves an additional local community and, in addition, provides ample room, at low ground rental, for commercial activities necessary for highway traffic, such as garages and hotels, without interfering with established stores and other services, essential to the community, on the old road.

In determining the location through small centers it is sometimes advisable to route short sections of the highway on an undesirable location, as a temporary measure. The location should be so developed outside the village that, when traffic increases to a point such that the local congestion becomes unbearable, the road can be projected past the town on a direct location, leaving a detour through the village. All highway departments are more or less dependent on local sentiment for their popularity. They cannot carry out a large program, even where purely State funds are available, unless they have the active support of the people. Opposition of a small community may be serious. In the early development of the highway system for any locality, due to the small amount of through traffic, there is no very serious operation expense involved in routing the highway along a longer line to carry it through the center of trade for the unit under consideration. If the line is on a through highway the foreign traffic will increase in a short time, to such an extent that the same interests that were insistent upon the "Main Street" location will be clamoring to have this outside traffic taken off their city streets to save maintenance expense and relieve the traffic congestion. Local people will then "discover" the short direct line outside the city and, as has sometimes happened, they may provide the funds to build the by-pass. If the traffic does not develop in a measure that will induce this action, the road is not of enough importance to be classed as a through highway. Local traffic will still be the portion to be accommodated and the line should follow the main street.

H. J. SPELMAN,[5] M. AM. SOC. C. E. (by letter).—Unquestionably, too many of the roads built during the life of the present generation have become obsolete too early; long before the expiration of the "reasonable life" anticipated by the engineer who located and designed them. In the future, highway engineers must locate more boldly than ever, lest the next generation find locations of the present day as obsolete as those built in 1915 or 1920 have become in 1931.

In 1919, and prior to that time, the average motor vehicle traveled at a speed of 20 to 25 miles per hour. The average motor vehicle now travels at a speed of from 40 to 50 miles per hour. Who knows but that the average speed of a generation from now, on many roads, may not be from 60 to 75 miles per hour, with 100 and 125-mile speeds at least as common as the 65 and 70-mile speeds at present? It is the increases in speeds that have caused the older road locations to become obsolete, because these higher speeds require gentler curvatures, longer sight distances, and greater widths of paving, roadway, and right of way.

In making present-day locations, highway engineers should not only locate with a view to traffic conditions as they are to-day, but should endeavor to anticipate those of the future. The future will undoubtedly bring higher speeds which will create a need for still wider sections and gentler curvatures.

As Mr. Losh has stated, property values increase very rapidly adjacent to a road after it is improved, and, hence, re-location and widening of a road once built, are particularly expensive. This again stresses the need for location and width of right of way adequate for the future. It is very much a question if any highway department to-day is obtaining rights of way of adequate width for the future on very many of the roads they are building.

In many cases a road has been located under low standards of alignment, and with narrow section and right of way, on the theory that such a road was a secondary or tertiary road that would never carry much traffic. With the improvement of the road, traffic flowed to it, and it really became a primary road from a traffic standpoint. Consequently, it soon become inadequate and required reconstruction. In order to avoid such occurrences the only remedy would appear to be a study of the traffic needs of the entire community or district, and a comprehensive road plan for such community or district, in the same way that city planning is now being done in many of the larger urban centers.

---

[5] Washington, D. C.

This paper seems particularly valuable in that it is an excellent summary of just what the highway engineer-executive must consider in making highway locations. It is valuable, too, in pointing out the mistakes of location in the past; and it is to be hoped that a study of these mistakes will enable the avoidance of similar ones in future locations.

H. S. Kerr,[*] M. Am. Soc. C. E. (by letter).—The major problems of highway location as presented in Mr. Losh's paper have been particularly well handled. The writer is generally in accord with the views therein set forth and can consistently approve the conclusions reached.

The prime consideration in highway location, with reference to topographic conditions, is, of course, to find the man who is a competent locator. In all the State highway departments the country over there are an extremely limited number of men gifted with that "sixth sense" which is the peculiar distinction of the high-class locator, and when such men are found, their worth is beyond price. In the old days of active railroad expansion these "sixth sense" locators were always in great demand, even if in some instances they might be decidedly below the average in handling engineering work aside from locating. The policy of the State of Utah is to confine difficult locations to the few men in the State Highway organization who have this qualification, and to apply railway methods. No better method has been evolved than an adherence to the long established railway practice of projecting the location in difficult sections, based on a carefully determined preliminary line, established after thorough consideration has been given to all alternate routes possible. Aerial photography is a help in finding the satisfactory location, and it is to be regretted that this method was not available several years before the problems (at any rate, as affecting interstate highways in Utah) were practically solved. Even though aerial photography came into vogue after the difficult locations in Utah were made, the final results are eminently satisfactory. The writer knows of no revisions that merit consideration in the case of any highway locations in this State during the eight years, 1924 to 1931, inclusive.

While agreeing that short sections of road with grades as steep as 8% are, in some instances, allowable, there are only a few such cases in Utah, although it is a mountainous State, where such gradient applies. In these cases a future reduction to 6%, in most cases without re-alignment, is reasonably practicable when traffic increase justifies the change.

In Utah the counties and the Road Commission have the power to protect the right of way from encroachments and prohibit advertising signs thereon, and there is necessity for constantly combatting the tendency, noted in the paper, of infringing upon the public right in this respect. A number of earlier projects, including some introduced subsequent to the enactment of the Federal Aid Act, have a right-of-way width of 66 ft. which it is proposed to increase, where feasible, to 100 ft., the general standard. Even wider rights of way are now being considered for roads across the public land areas.

---

[*] Chf. Engr., State Road Comm., Salt Lake City, Utah.

With reference to the conflicting interests between local and through traffic, the conditions cited are those that have formerly obtained in Utah, and, in many instances, it would not have been practicable to have attempted to work out an ideal location, particularly between 1915 and 1925, during which period local requirements were largely controlling factors in location. However, now that the towns and cities have generally been taken care of, to the maximum extent possible, between control points, consideration may properly be given to the best location for accommodating through traffic, utilizing to this end, portions of existing secondary highways, in by-passing congested areas and in reducing mileage. This would not have been practicable between 1915 and 1925 because the need of good roads was then so urgent in connecting centers of population. There are only a few cases in Utah, however, where such revisions are necessary on this account, or because construction standards are so much higher now (1931); this latter consideration is due to the fact that the first requirements in Utah were for roads in the valley sections where construction costs were relatively light and grading and structure costs per mile were not such as to cause public opinion to interfere with suitable location. In later years highway improvements have been extended from the more populous districts, to and through the mountain and desert sections, to the State borders. In so doing it has been possible to finance projects which average, in some instances, far in excess of $30 000 per mile, an accomplishment which would not have been feasible some years earlier. Now, however, it can be readily effected on account of the more recent trend of public opinion with reference to highway costs. The result has been that location involving heavy work during several years past has been entirely satisfactory.

With reference to location the following factors should be given due consideration, namely, annual interest, maintenance, and operation costs. Only the first item can be computed accurately, the remaining two, at best, can be but closely approximated, the result depending upon the ability and judgment of the locator in properly considering all items appertaining to each and every project.

Railroad grade crossings have been and are being eliminated by the Utah State Highway Department as rapidly as possible, the majority by relocation rather than by structures. In regard to co-operative railroad costs, the practice in Utah has been, unless conditions justified a variation, to divide the costs of structures on a "50-50" basis, and to arrange for the most satisfactory highway alignment, resulting generally in skew crossings. The most recent grade separation divided the costs equally for the overhead structure and the approach fills. Elimination of grade crossings by revision, in the case of low-cost construction, is handled on a "50-50" basis also, but where heavy work is involved, a contribution on the part of the railroad benefited, is arranged on the basis of $9 000 per double-track, main-line crossing and $6 000 for a single-track crossing. The construction or the vacating of railroad crossings is, in Utah, subject to the approval of the Public Utilities Commission. However, with respect to grade crossings on State highways, the

Commission requests that the Highway Department present, in advance of hearing, the details and stipulations tentatively agreed upon by that Department and the utility concerned.

Improvements are under way in the State on U. S. Highway No. 50. From a connection with U. S. Highway No. 91 this route extends across the mountains and the desert a distance of 223 miles, on the original route, to the Colorado line. The problems of location are, in the main, purely of an engineering character; they are entirely so from Green River eastward to the State line, a distance of 85 miles. The original route included twenty-eight railroad grade crossings, numerous right-angle turns, steep gradients, and sharp curves. Elevations varied from 4 080 to 7 450 ft. In the re-location survey of this highway it was possible to obtain results eminently satisfactory from an engineering standpoint, there being only a few restrictions applying that prevented an ideal location. The re-alignment effects a saving on the total distance of 14 miles. The major portion is now (1931) completed or under construction. All double-track crossings will have been eliminated by the programmed construction. A few single-track and relatively unimportant crossings will be temporarily continued although the plans call for the ultimate elimination of these crossings also. Little value is attached to the old highway between Price and Colorado, 155 miles, which lacked even numerous necessary bridges. The new 223-mile line, when entirely completed, will have less than one-half the total degrees of curvature of the trans-continental railway which it approximately parallels. Minimum surfacing requirements call for 18-ft. gravel surfacing of 6-in. depth, in two courses, 100% of which must pass a 1-in. screen, round opening. From this, types range to 12 miles of completed concrete pavement.

The total cost, including 1931 construction, is estimated at $3 047 548, with an estimate of $838 000 additional, to complete, or an average cost of $18 800 per mile. Considerably less than this average develops on the desert section of Eastern Utah, while on the mountain section, costs, in 1930, on some projects ran as high as $60 000 per mile for grading and structures only. Of the gravel mileage about one-half will be oil-treated within a short time and some miscellaneous paving will be done, which costs are not included in the foregoing figures.

H. W. Giffin,[7] Assoc. M. Am. Soc. C. E. (by letter).—Highway location is not altogether an engineering problem. The construction of highways is dependent on public support and the locating engineer or highway planner must constantly keep in mind that his handiwork must be approved by the present generation, irrespective of how future generations may regard it. Perhaps in the future the highways of to-day will be judged as the highways of a former day are now regarded. The principal reason for highway construction to-day is the love of the human race for swift, smooth motion without effort. Probably no highway bond issue was ever floated because of a desire to increase the economy of transportation. The policies of the highway planner in determining location are the result of his estimate of public approval and it is useless to try to get too far ahead of that. This means some waste as the years go by and locations become obsolete with changing conditions and the greater demand for locations approaching more closely to the ideal.

---

[7] Field Engr., State Highway Dept., Trenton, N. J.

The author states that "curves should be placed where they may be seen readily by the approaching driver". It would be well to go further and say that every condition of the highway, whether it is a change of direction, an intersection, a railroad crossing, or anything that requires the driver's attention, should be placed, or otherwise brought to his notice, so as to give him time to take appropriate action. It is suggested that some advance designation, such as a number denoting the sharpness of the curve be used on signs which now denote curves and their direction. Although some motorists would not know what a stated degree of curve meant, they would soon learn to associate the numbers with the relative sharpness of the curves. To-day, the same sign that frequently is used to warn the motorist of a sharp curve is also used for flat ones.

Because of the adoption of some degree of curve as a maximum, locating surveyors do not always feel that every effort should be made to secure flatter curves. Long flat curves mean additional work for the survey party, but such curves should be as flat as 30′, cost permitting. Where necessary to have a curve near a summit it should be located if possible so as to extend for some distance on both sides. Where vertical curves are coincident with horizontal curves on summits, pleasing alignment is attained. It is not easy to define the conditions which produce an alignment that displeases the eye, but it is suggested that the location that contains, within one view, three tangents and two curves, or two complete changes of direction, whether horizontal or vertical, is not pleasing.

The sight distance desired along the highway is governed by the prevailing speed of the traffic at the particular location. Highways laid out for fast traffic (that is, those which are wide and reasonably straight) in the country should have sight distances of 800 to 1 000 ft. if obtainable at reasonable cost. City conditions do not require more than 300 ft. Where difficulty is experienced in obtaining satisfactory sight distances the roadway, under some circumstances, may be separated into one-way roads, thus permitting the required sight distance to be cut in half.

Because of the relative high cost per unit of length in comparison with the remainder of the highway, bridges have exerted a great influence on location. Despite this high cost, the location engineer should remember that the ordinary bridge is only a part of a highway, and is often unnoticed by the motorist unless the alignment impresses it upon his mind. An otherwise satisfactory alignment is sometimes spoiled because the requirement of securing a less costly bridge is exaggerated.

The segregation of through and local traffic is desirable and advantageous to both classes, but it is seldom reasonable or economical until each reaches sufficient volume to justify separate roadways. The construction of the by-pass is the common method of accomplishing this purpose. This may be termed second-stage planning and has not been as readily understood and approved by the public as the first stage, which may be defined as the extension of the main street of one town to that of another.

Near metropolitan centers the subject of intersections is an important, and often neglected, phase of highway location. The capacity of a highway is limited by its intersections and, obviously, the combined capacity of two intersecting highways is limited to that of the larger unless some special provision is made to increase the intersection capacity. Where new highways are planned, that cross a network of other important highways, it is sometimes necessary for the locating engineer to study first the available locations for the type of intersection suitable to the requirements.

It is not necessary to go far back in one's memory to prevailing speeds of 25 to 30 miles per hour. Prevailing speeds to-day (1931) are at least 20 miles per hour faster. Must the highway planner anticipate speeds of 70 miles per hour as the ordinary condition 10 or 15 years hence?

While it seems wise in laying out new highways to obtain sufficient right of way for widening within a few years, too much planning for the future should not be attempted. How many could have appreciated to-day's highway problems ten or fifteen years ago? Any attempt, other than widening, to plan for to-day's highways fifteen years ago would probably have been unappreciated at that time and would probably prove unsatisfactory now.

Mr. Losh's excellent paper has focused attention on some important problems of highway location.

A. R. Losh,[8] Assoc. M. Am. Soc. C. E. (by letter).—The writer appreciates the favorable comments his paper has received from members of the Society who have had extensive experience in National and State highway programs. The information and suggestions contributed have added materially to the discussion.

The importance of conditions which will reduce or eliminate the difficulties of snow drifting was stressed by Mr. Bishop. This is worthy of careful attention not only from the viewpoint of reducing the cost of snow removal, but, as Mr. Bishop states, to keep the roads open to the public. The cost of snow removal during the winter of 1929-30 as given by Mr. Bishop was $8 250 000 for 185 000 miles of road, or an average of $44.50 per mile. Considering the severe conditions of that season this is not high. The cost of snow removal on Oklahoma highways for the same season was an average of $30.00 per mile. The loss to the State in gasoline taxes alone was more than three times the cost of snow removal, due to the fact that extreme winds created drifts along the roads three times after they had been broken open. The real loss to the traveling public is, of course, unknown. In the plains country fine dry snow carried by a winter gale will drift into every cut and depression. Until the snow has settled and partly packed, it is difficult to keep an east and west road open. Freedom from snow hazard is an important location consideration in places subject to heavy snowfall.

---

[8] Mgr., Asphalt and Road Oil Div., Anderson-Prichard Corp., Oklahoma City, Okla.

Referring to tourist roads, parkways, scenic highways, and places of historic or recreational interest, Mr. Bishop points out the necessity of departing from the engineering phases of grades and alignment in location studies and giving due weight to those features which determine the general nature of the road. Conditions such as these are unusual in highway location, but their importance has been properly stressed by Mr. Bishop.

The advancement of highway location standards resulting from the Federal Aid acts is well brought out by Mr. Carpenter. He also makes the point that public sentiment too often must be given consideration on detail location. Of particular value is his suggestion that a major location plan be worked out for future improvement, and that this be utilized on individual sections as the opportunity arises. His solution of the "main street" location for the small city is a practical one frequently used to advantage.

The improvement of the motor vehicle and the higher permissible speeds of recent years have brought about the necessity of better location, as stated by Mr. Spelman. Unless, as he suggests, highway engineers locate more boldly and with a freer hand than in the past, 1931 locations will be obsolete a few years hence.

Mr. Kerr's discussion referring to the work of the Utah State Highway Department brings out a number of general conditions to be met in highway location. First is the necessity of solving the political and economic problems as was done in the earlier locations of that State when the towns and cities were provided with service along somewhat local lines. Having met this local pressing need the Department could work with a freer hand in developing the transcontinental routes. The trend of public opinion was for the cross State routes once the immediate needs of the populous sections had been met. This later work permitted the exercise of engineering skill to the fullest extent. Fortunately, the more expensive construction came at a time when the location was possible on an engineering basis.

Public support is a necessity in any highway program. This statement by Mr. Giffin is borne out in Mr. Carpenter's discussion, and also by Mr. Kerr. While it is highly desirable to locate for the future, present problems must be met. The public can be led to a certain extent, but the leader must not get too far ahead of the procession. The writer agrees fully with Mr. Giffin in that every condition on the highway which requires special action on the part of the operator should be brought to his attention in ample time for him to control the vehicle; and, as he further states, speed is the important factor to consider. A curve of 50-ft. radius was no hazard for vehicles moving at 8 miles per hour; but a 500-ft. radius is a serious condition for speeds greater than 35 miles per hour. In answer to Mr. Giffin's query, the writer feels that anticipated speeds of 70 miles per hour are not improbable ten years hence. Traffic is moving at nearly this speed in the open country now (1932), and at too great a hazard from intersecting highways at grade, narrow structures, embankments of insufficient width, and other conditions which the engineer must eliminate in the future.

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
TRANSACTIONS

Paper No. 1800

STRESSES IN GRAVITY DAMS BY PRINCIPLE OF LEAST WORK[1]

By B. F. Jakobsen,[2] Esq.

With Discussion by Messrs. William Cain, William Gore, L. J. Mensch, Lars R. Jorgensen, C. A. P. Turner, Donald P. Barnes, Anthony Hoadley, Fred W. Ely, M. H. Gerry, Jr., Fredrik Vogt, Eugene Kalman, Fernand Campus, A. Floris, Robert E. Glover, P. Wilhelm Werner, and B. F. Jakobsen.

Synopsis

In order to determine the stresses in gravity dams, it has been customary to assume a linear stress distribution on horizontal planes. An examination of English tests on model dams shows a non-linear stress distribution. A theoretical investigation, in which the Principle of Least Work was substituted for the usual assumption of a linear stress distribution, leads to a stress curve similar to that found in the English tests.

The bending stresses, that is, the vertical normal stresses due to the water load, found by the usual method, are equal at the up-stream and down-stream faces, while the proposed theory gives a considerably higher bending stress at the up-stream face and a correspondingly lower stress at the down-stream face. The maximum stress is found to be less than by the usual method, but this is of no particular advantage, since the compressive stress in the up-stream face with reservoir full should exceed a definite minimum, and this will generally be the determining factor in the design.

The writer believes that the stress distribution based on the Principle of Least Work will more nearly approach the true load stresses than the results obtained by the usual method.

I.—Introduction

The section of the straight gravity dam to be considered is triangular with a vertical up-stream face. It is assumed, that the concrete is homogeneous and isotropic in every respect, that is, that each unit of volume weighs exactly as much as any other; that its temperature, moisture content, and general conditions do not vary from point to point; and that the modulus of elasticity

---

[1] Published in September, 1930, *Proceedings*.

[2] Cons. Engr., Los Angeles, Calif.

is the same for compression as for tension and is the same at all points of the dam. It is also assumed that there is no uplift and no residual stress. The stresses to be considered are those due to the load imposed by the weight of the structure itself and by the horizontal thrust of the water acting on the up-stream face. It is further assumed, that the deformations are elastic and infinitesimal, so that the shape of the structure is not altered when the load is applied. These are the usual assumptions made when dealing with engineering structures.

Evidently none of these assumptions is rigorously true. Some of them, as for example, those regarding uplift and residual stress, may be so far wrong that for purposes of engineering design they must be separately considered in order to obtain the actual stresses. Föppl has shown,[3] that the load stresses are independent of the residual stresses, when the usual assumptions are made. Regarding plastic deformations (time effect), the tests made by R. E. Davis, M. Am. Soc. C. E., indicate[4] that these increase linearly with the stress and therefore are not likely to influence the stress distribution. They may be taken into account by assuming a lower modulus of elasticity.

II.—Usual Theory—Stresses

Referring to Fig. 1, let:

n_y and n_x = vertical and horizontal normal stresses at any point;
t = shear or tangential stress in the same point;
w = weight of concrete, in pounds per cubic foot;
g = weight of 1 cu. ft. of water, in pounds;
M = moment, in foot-pounds, about a point on the Y-axis, y ft. below the crest;
W = weight, in pounds, of a 1-ft. thick slice of dam, y ft. high;
b = base width, in feet, y ft. below the crest;
ϕ = angle at crest between up-stream and down-stream faces; and,

$$k = \tan \phi = \frac{b}{y}.$$

Then,

$$W = 0.5\, w \cdot b\, y = 0.5\, w\, k\, y^2 \;\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

and,

$$M = 0.5\, k\, y^2 \frac{b}{3} + \frac{g}{6}\, y^3 = (g + w\, k^2)\, \frac{y^3}{6} \;\ldots\ldots\ldots\ldots (2)$$

Referring to Fig. 2, take tension as positive and the sign for shear such that on the plane on which the positive direction of a normal stress is in the positive direction of the axis to which it is parallel, the positive direction of shear will also be in the positive direction of the axis to which it is parallel; thus, the horizontal shear in a gravity dam will have the negative sign.

It is assumed that there are no stresses acting at right angles to the section, that is, normal to the plane of the page in Fig. 1. The only stresses which will be considered are n_y, n_x, and t. This implies, that the vertical limiting planes of the section are free to deform in accordance with these stresses. (See Appendix III.)

---

[3] "Drang und Zwang," Vol. 2, Article 91 (1920 Edition).

[4] *Proceedings*, Am. Soc. C. E., May, 1928, Pt. 3, Table 28, p. 213. When plotted, these tests give a straight line between deformation and stress.

The statical conditions require that,

$$\int_0^b n_y \, d\, x = - W = - 0.5\, w\, k\, y^2 \ldots\ldots\ldots\ldots\ldots\ldots\ldots (3)$$

$$\int_0^b n_y \, x\, d\, x = - M = - (g + w\, k^2)\, \frac{y^3}{6} \ldots\ldots\ldots\ldots\ldots\ldots (4)$$

and, also, that,

$$\int_0^b t\, d\, x = - S = - 0.5\, g\, y^2 \ldots\ldots\ldots\ldots\ldots\ldots\ldots (5)$$

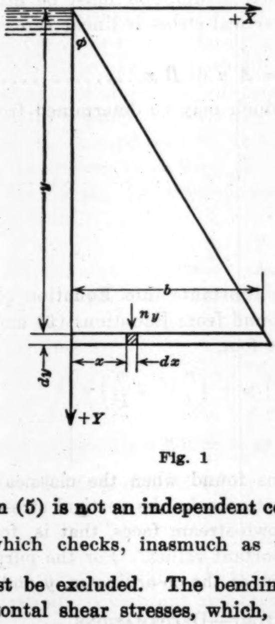


Fig. 1

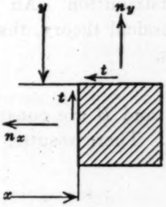


Fig. 2

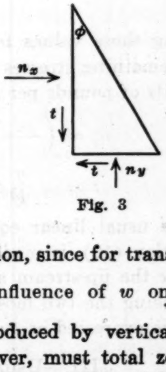


Fig. 3

Equation (5) is not an independent condition, since for transverse loading, $S = \frac{d\, M}{d\, y}$, which checks, inasmuch as the influence of w on the bending moment must be excluded. The bending produced by vertical forces gives rise to horizontal shear stresses, which, however, must total zero across the entire section. The equilibrium equations for any small particle, $d\, y$, $d\, x$, $d\, z$, in a point (y, x), are:

$$\frac{d\, n_y}{d\, y} + \frac{d\, t}{d\, x} + w = 0 \ldots\ldots\ldots\ldots\ldots\ldots\ldots (6)$$

$$\frac{d\, n_x}{d\, x} + \frac{d\, t}{d\, y} = 0 \ldots\ldots\ldots\ldots\ldots\ldots\ldots (7)$$

in which, $\frac{d\, n_y}{d\, y}$, etc., are the partial differential coefficients, usually written $\frac{\partial\, n_y}{\partial\, y}$. The boundary conditions are, at the up-stream face, $t = 0$ and $n_x = - g\, y$. At the down-stream face (Fig. 3), it must be,

$$t = n_y \tan \phi \ldots\ldots\ldots\ldots\ldots\ldots\ldots (8)$$

and,

so that,

$$\left.\begin{aligned} n_x &= t \tan\phi = n_y \tan^2\phi \\ n_y\, n_x &= t^2 \end{aligned}\right\} \qquad (9)$$

Equations (8) and (9) follow from the fact that the forces acting on the small triangle must be in equilibrium. The weight of the triangle is not involved, since this is infinitely small compared to the forces.

These conditions, that is, Equations (3) to (7), inclusive, and the stress relations at the up-stream and down-stream faces, do not suffice to determine the stress distribution. An additional assumption must be made; and this is, in the classical theory, that the vertical stress is linear across a horizontal joint, that is,

$$n_y = A\, y + B\, x \qquad (10)$$

in which, A and B are constants, which may be determined from Equations (3) and (4). There results,

$$A = -w + \frac{g}{k^2} \qquad (11)$$

and,

$$B = \frac{w}{k} - 2\,\frac{g}{k^3} \qquad (12)$$

Introducing these values for the constants into Equation (10) gives n_y; and the two remaining stresses are found from Equations (6) and (7). There results, in units of pounds per square foot,

$$\left.\begin{aligned} n_y &= \left(-w + \frac{g}{k^2}\right) y + \left(\frac{w}{k} - 2\,\frac{g}{k^3}\right) x \\ t &= -\frac{g}{k^2}\, x \text{ and } n_x = -g\, y \end{aligned}\right\} \qquad (13)$$

These are the usual linear equations found when the classical assumption, that is, Equation (10), is made. In textbooks the vertical stress is generally given only for the up-stream and down-stream faces, that is, for $x = 0$ and $x = b$, these being the two most important values.[5] For the purpose in hand, the stresses are expressed as functions of the co-ordinates, y and x.

III.—USUAL THEORY—DEFORMATIONS

The usual assumption, that n_y is a linear function of x, is equivalent to stating that horizontal planes remain plane, provided the modulus of elasticity is constant and that the shear deformation is neglected. Fredrik Vogt, Assoc. M. Am. Soc. C. E., has shown,[6] that the bending moment produces an **S**-shaped curving of the foundation; therefore, the classical assumption is incorrect, at least for sections near the foundation. The tests made at the Stevenson Creek Test Dam show a remarkably close agreement between the actual displacements of the foundation and those computed from Dr. Vogt's formulas.[7]

---

[5] See "Notes on Arched Gravity Dams," *Transactions*, Am. Soc. C. E., Vol. 92 (1928), Equation (14), p. 771. Note that for a straight dam, $R_u = \infty$.

[6] "Ueber die Berechnung der Fundamentdeformation," Det Norske Videnskaps Akademi, Oslo, 1925.

[7] *Proceedings*, Am. Soc. C. E., May, 1928, Pt. 3, p. 127, and Fig. 82.

That the usual assumption was probably incorrect near the foundation has been accepted for many years by authorities on dam design.[8]

IV.—STRESS DISTRIBUTION BY THE PRINCIPLE OF LEAST WORK

This principle was first applied to engineering structures by Castigliano in his work[9] published in 1879. It was introduced into American technical literature by the late William Cain, M. Am. Soc. C. E., in a paper entitled "Determination of the Stresses in Elastic Systems by the Method of Least Work"[10] and it was applied by him to obtain a correct and elegant solution of the problem of stresses in arches of dams in his well-known paper, "The Circular Arch under Normal Loads."[11]

The writer has shown[12] that the arch stresses found from the tests on the Stevenson Creek Dam agree remarkably well with the Cain formulas, when the deformations and displacements of the abutments are included in the calculations in the manner suggested by Dr. Vogt. (See Article III.) These tests are, therefore, an experimental verification of the Principle of Least Work as applied to the stresses in an arch dam.

In lieu of the classical assumption, that n_y is a linear function of x, assume that the stress distribution is such as will make the internal work the least possible.[13] It is clear that this assumption is less restrictive than the classical assumption and includes the latter as a specific case. If, for example, the shear stress, t, and the horizontal normal stress, n_x, are assumed as negligible, the principle of least work requires that the stress, n_y, must be linear.

The internal work is,[14]

$$L = \frac{1}{2\,G} \int\!\!\int\!\!\int \left[\frac{n_y^2 + n_x^2}{2} - \frac{(n_y + n_x)^2}{2\,(m + 1)} + t^2 \right] dy\, dx\, dz \ldots\ldots(14)$$

in which, G is the shear modulus and $\frac{1}{m}$ is Poisson's ratio.

The shear modulus is defined by,

$$G = \frac{m\,E}{2\,(m + 1)}$$

in which, E is Young's modulus of elasticity. Equation (14) implies that the modulus of elasticity is constant and the same for tension as for compression, which is believed to be true for ordinary working stresses.[15]

Instead of the simple Equation (10) for n_y, the condition that Equation (14) must be a minimum is now introduced. This is a problem in variational calculus, but instead of treating it as such, which would be very complicated, a method of approximation may be resorted to, by assuming n_y as a function of y and x. This method, due to Ritz,[16] transforms a problem in variational calculus into a simple problem of differentiation. The procedure is as follows: The stress, n_y, is assumed with more constants than are needed to satisfy Equations (3) and (4); the equilibrium equations, (6) and (7), serve to determine n_x and t as functions of y and x and of the constants in the equation for n_y. The condition that Equation (14) must be a minimum then serves to determine the free constants in the equation for n_y. Assume,

---

[8] Professor Cain calls attention to this in the opening sentence of his paper, "Stresses in Masonry Dams," *Transactions*, Am. Soc. C. E., Vol. LXIV (1909), p. 208; and Professor Résal in his paper, "Formes et Dimensions des grands barrages en maçonnerie" in *Annales* des Ponts et Chaussées, II, 1919, deals at some length with this subject.

[9] "Théorie de l'equilibre des systèmes élastiques"; this book is out of print, but there is an English translation available.

[10] *Transactions*, Am. Soc. C. E., Vol. XXIV (1891), p. 265.

[11] *Loc. cit.*, Vol. LXXXV (1922), p. 233.

[12] *Proceedings*, Am. Soc. C. E., May, 1929, Papers and Discussions, p. 1241.

[13] See Föppl, "Drang und Zwang," Vol. I, 1924, Paragraphs 13, 14, and 55.

[14] *Loc. cit.*, p. 336.

[15] *Transactions*, Am. Soc. C. E., Vol. 95 (1931), p. 679.

$$n_y = A\,y + B\,x + C\,\frac{x^2}{y} + F\,\frac{x^3}{y^2} \quad \text{.............} (15)$$

in which, A, B, C, and F are four constants, any two of which, for example, A and B, may be expressed as functions of C and F by means of Equations (3) and (4), by carrying out the simple integrations indicated. There results,

$$A = -\,w + \frac{g}{k^2} + C\,\frac{k^2}{6} + F\,\frac{k^3}{5} \quad \text{.............} (16)$$

and,

$$B = \frac{w}{k} - 2\,\frac{g}{k^3} - C\,k - 0.9\,F\,k^2 \quad \text{.............} (17)$$

The two independent constants, C and F, must now be determined so that the internal work is a minimum, or,

$$\frac{d\,L}{d\,C} = 0; \text{ and } \frac{d\,L}{d\,F} = 0 \quad \text{....................} (18)^{17}$$

It will simplify the calculations to differentiate under the integral sign, instead of first integrating and then differentiating. There results,

$$\frac{1}{2\,G}\int_0^1 d\,z \int_0^H d\,y \int_0^b d\,x \left[\frac{m}{m+1}\left(n_y\,\frac{d\,n_y}{d\,C} + n_x\,\frac{d\,n_x}{d\,C}\right)\right.$$
$$\left. -\,\frac{1}{m+1}\left(n_y\,\frac{d\,n_x}{d\,C} + n_x\,\frac{d\,n_y}{d\,C}\right) + 2\,t\,\frac{d\,t}{d\,C}\right] = 0 \quad \text{........} (19)$$

in which, H is the total height of the section considered.

Likewise,

$$\frac{1}{2\,G}\int_0^1 d\,z \int_0^H d\,y \int_0^b d\,x \left[\frac{m}{m+1}\left(n_y\,\frac{d\,n_y}{d\,F} + n_x\,\frac{d\,n_x}{d\,F}\right)\right.$$
$$\left. -\,\frac{1}{m+1}\left(n_y\,\frac{d\,n_x}{d\,F} + n_x\,\frac{d\,n_y}{d\,F}\right) + 2\,t\,\frac{d\,t}{d\,F}\right] = 0 \quad \text{........} (20)$$

It is now necessary to express t and n_x as functions of the co-ordinates, y and x, and of the constants, A, B, C, and F. Differentiating Equation (15) toward y and introducing the result into Equation (6),

$$A - C\,\frac{x^2}{y^2} - 2\,F\,\frac{x^3}{y^3} + \frac{d\,t}{d\,x} + w = 0$$

---

[16] *Transactions,* Am. Soc. C. E., Vol. LXXXVII (1924), p. 325. The writer used this method to determine the stresses in the buttress of a multiple-arch dam.

[17] $\frac{d\,L}{d\,C}$ and $\frac{d\,L}{d\,F}$ indicate the partial differential coefficients usually written $\frac{\partial\,L}{\partial\,C}$ and $\frac{\partial\,L}{\partial\,F}$. In this paper, d has been used for ∂ as there is no danger of misunderstanding.

Integrating and bearing in mind that, for $x = 0$ (that is, at the up-stream face), $t = 0$,

$$t = -A\,x + \frac{C}{3}\,\frac{x^3}{y^2} + \frac{F}{2}\,\frac{x^4}{y^3} - w\,x \ldots\ldots\ldots\ldots\ldots (21)$$

Differentiating Equation (21) toward y and introducing this into Equation (7),

$$\frac{d\,n_x}{d\,x} = -\left(-\frac{2}{3}\,C\,\frac{x^3}{y^3} - \frac{3}{2}\,F\,\frac{x^4}{y^4}\right)$$

Integrating toward x, remembering that the constant of integration, $f(y)$, is a function of y, but not of x, and that for $x = 0$, $n_x = -g\,y$, or $f(y) = -g\,y$, there results,

$$n_x = \frac{C}{6}\,\frac{x^4}{y^3} + 0.3\,F\,\frac{x^5}{y^4} - g\,y \ldots\ldots\ldots\ldots\ldots (22)$$

The equations for t and n_x must also satisfy Equations (8) and (9) at the down-stream face; that is, for $x = b = k\,y$, $t = k\,n_y$, and $n_x = k\,t$.

By introducing the values of n_y, t, and n_x, from Equations (15), (21), and (22), into Equations (8) and (9), and eliminating the two constants, A and B, by means of Equations (16) and (17), the desired check is obtained. Equations (15), (21), and (22) for the stresses, n_y, t, and n_x, therefore, satisfy the statical requirements (Equations (3) and (4)), the equilibrium equations, (6) and (7), the boundary conditions at the up-stream face (that is, $t = 0$ and $n_x = -g\,y$), and also the boundary conditions at the down-stream face (Equations (8) and (9)). This means that Equations (15), (21), and (22) represent a possible stress distribution, no matter what arbitrary finite values, including zero, are assigned to the two constants, C and F.

The differentials required for Equations (19) and (20) may now be formed, bearing in mind that the constants, A and B, are functions of C and F as given by Equations (16) and (17), so that,

$$\frac{d\,A}{d\,C} = \frac{k^2}{6};\ \frac{d\,B}{d\,C} = -\,k;\ \frac{d\,A}{d\,F} = \frac{k^3}{5};\ \frac{d\,B}{d\,F} = -\,0.9\,k^2$$

Then,

$$\frac{d\,n_y}{d\,C} = \frac{k^2}{6}\,y - k\,x + \frac{x^2}{y}$$

$$\frac{d\,n_x}{d\,C} = \frac{x^4}{6\,y^3}$$

$$\frac{d\,t}{d\,C} = -\,k^2\,\frac{x}{6} + \frac{x^3}{3\,y^2} \ldots\ldots\ldots\ldots\ldots (23)$$

Likewise,

$$\frac{d\,n_y}{d\,F} = \frac{k^3}{5}\,y - 0.9\,k^2\,x + \frac{x^3}{y^2}$$

$$\frac{d\,n_x}{d\,F} = 0.3\,\frac{x^5}{y^4}$$

$$\frac{d\,t}{d\,F} = -\,\frac{k^3}{5}\,x + 0.5\,\frac{x^4}{y^3} \ldots\ldots\ldots\ldots\ldots (24)$$

Substituting these values in Equations (19) and (20), integrating from $x = 0$ to $x = b = k\,y$, and arranging, the final Equations (25) and (26) are obtained. The calculations are somewhat extensive, but involve nothing but multiplications and integrations of simple algebraic expressions. Equation (25) has been divided by k^5 and Equation (26) by k^6. The final results are,

$$\frac{1}{2\,G}\int_0^1 d\,z \int_0^H y^3\,d\,y\left[\frac{m}{m+1}\left(\frac{C}{180}+\frac{F}{120}\,k+\frac{C}{324}\,k^4\right.\right.$$
$$\left.+\frac{F}{200}\,k^5-\frac{g}{30}\right)-\frac{1}{m+1}\left(\frac{A}{30}+\frac{B}{36}\,k+\frac{8}{315}\,C\,k^2\right.$$
$$\left.\left.+\frac{F}{42}\,k^3\right)-\frac{A}{45}+\frac{C}{105}\,k^2+\frac{F}{72}\,k^3-\frac{w}{45}\right]=0\ldots\ldots\ldots\ldots(25)$$

$$\frac{1}{2\,G}\int_0^1 d\,z \int_0^H y^3\,d\,y\left[\frac{m}{m+1}\left(\frac{C}{120}+\frac{9}{700}\,F\,k+\frac{C}{200}\,k^4\right.\right.$$
$$\left.+\frac{9\,k^5}{1\,100}\,F-\frac{g}{20}\right)-\frac{1}{m+1}\left(\frac{A}{20}+\frac{3}{70}\,B\,k+\frac{C}{25}\,k^2\right.$$
$$\left.\left.+\frac{4}{105}\,F\,k^3\right)-\frac{A}{30}+\frac{3}{200}\,C\,k^2+\frac{F}{45}\,k^3-\frac{w}{30}\right]=0\ldots\ldots\ldots(26)$$

These two equations, combined with Equations (16) and (17), suffice to determine the four constants, *A*, *B*, *C*, and *F*. It is apparent that the values of the constants are independent of the height, *H*, of the dam, since in Equations (25) and (26) the expressions in the square brackets must be zero.

An examination of these equations shows, that if the shear stress, *t*, and the horizontal stress, n_x, are both equal to zero, Equations (25) and (26) reduce to,

$$\frac{C}{180}+\frac{F}{120}\,k=0;\ \text{and}\ \frac{C}{120}+\frac{9}{700}\,F\,k=0$$

Both these equations cannot be correct, except when $C = F = 0$. Then *A* and *B* take on the values given in Equations (11) and (12) and all the stresses are linear.

V.—Stress Distribution—Numerical Example

In order to investigate the stress distribution found by the Principle of Least Work and to compare it with the usual theory, assume that $k = \tan\phi = \frac{b}{y} = 0.645$, $g = 62.5$, and $m = 8$. Let the weight of the concrete be chosen so that the stress in the up-stream face is zero for reservoir full, when the ordinary theory is used. This requires that the coefficient for *y* in Equation (10) must be zero, or from Equation (11), that $w = \frac{g}{k^2}$, or $w = 150.23$ lb. per cu. ft.[13]

When these values are introduced into Equations (16) and (17), there results,

$$A = 0.069337\,C + 0.05367\,F\ldots\ldots\ldots\ldots\ldots\ldots(16a)$$
$$B = -\,232.92 - 0.645\,C - 0.37442\,F\ \ldots\ldots\ldots\ldots(17a)$$

---

[13] All computations have actually been carried out several decimal places beyond the recorded results.

From Equations (25) and (26), when multiplied by 1 000,

$$8.2013\,C + 8.2909\,F - 25.926\,A - 1.9907\,B = 5\,190.3 \ldots\ldots (25a)$$

and,

$$12.568\,C + 13.011\,F - 38.889\,A - 3.0714\,B = 7\,785.5 \ldots\ldots\ldots (26a)$$

The solution of these four equations gives the values of the constants, when the stresses are expressed in pounds per square foot. Dividing these values by 144, there results, $A' = 0.3765$; $B' = -5.777$; $C' = 9.50$; and $F' = -5.263$. The stresses are in pounds per square inch,

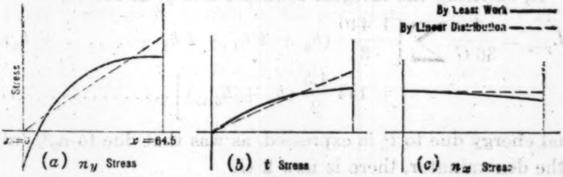
$$\left.\begin{aligned} n_y &= 0.3765\,y - 5.777\,x + 9.504\,\frac{x^2}{y} - 5.263\,\frac{x^3}{y^2} \\ t &= -1.420\,x + 3.168\,\frac{x^3}{y^2} - 2.632\,\frac{x^4}{y^3} \\ n_x &= -0.4340\,y + 1.584\,\frac{x^4}{y^3} - 1.579\,\frac{x^5}{y^4} \end{aligned}\right\} \ldots\ldots\ldots\ldots (27)$$

TABLE 1.—STRESSES IN NO-UPLIFT SECTION (k = 0.645), 100 FEET BELOW CREST ($y = 100$), WHEN RESERVOIR IS FULL.

(Stresses, in pounds per square inch.)

x.	n_y.	t.	n_x.
0	+37.650	0.000	−43.408
3	+21.161	....	....
6	+ 5.274	....	....
10	−11.140	−13.908	−43.389
20	−44.082	−26.238	−43.200
30	−64.332	−36.172	−42.503
40	−75.048	−43.254	−40.962
50	−79.389	−47.839	−38.438
60	−80.512	−50.868	−35.201
64.5	−80.809	−52.119	−33.616

For a horizontal section, 100 ft. below the crest (y = 100 ft.), and with reservoir full, the stresses are as given in Table 1. At the down-stream face, $n_y\,n_x = t^2$; this acts as a check on the calculations.



(a) n_y Stress (b) t Stress (c) n_x Stress

FIG. 4.—COMPARATIVE STRESS DISTRIBUTIONS. UP-STREAM FACE TO THE LEFT.

The stresses are plotted to scale in Fig. 4. On the assumption of a linear stress distribution, Equations (11) and (12) give, when divided by 144, $A'' = 0$ and $B'' = -1.6175$ and the stresses are by Equation (13), $n_y = -1.6175\,x$; $t = -1.0433\,x$; and $n_x = -0.43403\,y$. These stresses are also plotted in Fig. 4, for y = 100 ft. For the down-stream face, x = 64.5 ft., and, $n_y = -104.33$; $t = -67.291$; and $n_x = -43.403$.

A comparison of the vertical normal stresses shows, that according to the new stress distribution, there is considerable tension in the up-stream face and the angle between the n_y-stress and the horizontal decreases as x increases, while the usual method gives zero stress at the up-stream face and a constant angle. Furthermore, the tension in the up-stream face increases directly with y.

VI.—Calculation of Internal Work—Numerical Example

As a check on the calculations, from Equation (3),

$$\int_0^b n_y\,d\,x = A\,y\,b + B\,\frac{b^2}{2} + \frac{C}{3}\,\frac{b^3}{y} + \frac{F}{4}\,\frac{b^4}{y^2} \quad \text{.........(28)}$$

in which, $b = 64.5$ and $y = 100$. This gives 484 791 lb., while 0.5 $w\,k\,y^2$ gives 484 497 lb., an error of about 6 in 10 000. Equations (4) and (5) were likewise found to check closely. The stresses at the down-stream face are related, as shown by Equations (8) and (9). Since $n_y = -\,80.809$, t should be $-\,52.122$, and n_x should be $-\,33.619$, whereas in Table 1, $t = -\,52.119$ and $n_x = -\,33.616$. Also, from Table 1, $n_y\,n_x = 2\,716$ and $t^2 = 2\,716$. That these figures check, proves merely that the calculations are correct as far as Equations (16) and (17) and Equations (6) and (7) are concerned; it proves nothing with respect to the correct values of the two free constants, C and F.

When the stresses are expressed in pounds per square inch, the internal work, in foot-pounds, is, from Equation (14),

$$L = \frac{144}{2\,G}\int_0^1 d\,z \int_0^H d\,y \int_0^b d\,x\left[\frac{n_y^{\,2} + n_x^{\,2}}{2} - \frac{(n_y + n_x)^2}{18} + t^2\right] \text{..(29)}$$

In order to obtain the internal work, Simpson's rule was used. The internal work due to n_y^2 is,

$$L_y = \frac{1}{4\,G}\sum\left[\frac{1\,440}{3}\,(h_0 + 4h_1 + 2h_2 + \ldots. + h_6) + 144\,\frac{4.5}{2}\,(h_6 + h_{6.45})\right] \text{..(30)}$$

h_0, h_1, h_2, etc., represent the ordinates, n_y^2, for $x = 0$, 10, 20, etc., as given in Table 2. The same formula applies to n_x^2. The internal work due to $(n_y + n_x)^2$ is, omitting the integral toward z and y, as before,

$$L_{y+x} = \frac{1}{36\,G}\sum\left[\frac{1\,440}{3}\,(h_0 + 4\,h_1 + 2\,h_2 + \ldots\ldots + h_6) + 144\,\frac{4.5}{2}\,(h_6 + h_{6.45})\right] \text{.......................(31)}$$

The internal energy due to t^2 is expressed, as was that due to n_y^2, only, instead of 4 G in the denominator, there is now 2 G.

The internal work, when the usual assumption is maintained, was found by integration. It was $n_y = -\,1.6175\,x$; $n_x = -\,0.43403\,y$; and $t = -\,1.0433\,x$, so that,

$$L' = \frac{144}{2\,G}\int_0^1 d\,z \int_0^H d\,y \int_0^b d\,x\left[\frac{1.6175^2}{2}\,x^2 + \frac{0.043403^2}{2}\,y^2 - \frac{(1.6175\,x + 0.43403\,y)^2}{18} + 1.0433^2\,x^2\right] \text{..............(32)}$$

TABLE 2.—VALUES OF STRESSES FOR CALCULATION OF INTERNAL WORK.
(Stresses, in pounds per square inch.)

x.	n_y.	n_y^2.	n_x.	n_x^2.	t.	t^2.	$(n_y + n_x)$.	$(n_y + n_x)^2$.
0	+37.65	1 417.5	−43.40	1 883.6	0.00	0.0	− 5.75	33.1
10	−11.14	124.1	−43.39	1 882.7	−13 91	193.5	− 54.79	3 002.
20	−44.08	1 943.0	−43.20	1 886.2	− 26.28	690.6	− 87.28	7 618.
30	−64.33	4 138.3	−42.50	1 806.3	−36.17	1 308.3	−106.83	11 413.
40	−75.05	5 632.5	−40.96	1 677.7	−43.25	1 870.6	−116.01	13 458.
50	−79.39	6 302.8	−38.44	1 477.6	−47.84	2 288.7	−117.83	13 884.
60	−80.51	6 481.9	−35.20	1 239.0	−50.87	2 587.8	−115.71	13 389.
64.5	−80.81	6 530.3	−33.62	1 130.3	−52.12	2 716.5	−114.43	13 094.

When the calculations are carried out, as indicated, and the integrations toward z and y, are omitted, the results given in Table 3 are obtained. The difference between the two theories amounts to 564 480, or about 3.26 per cent.

TABLE 3.—COMPARISON OF INTERNAL WORK BY USUAL AND PROPOSED THEORY. INTERNAL WORK × G FOR A STRIP, b, $d\,z$, $d\,y$.

Due to.	By usual theory.	By proposed theory.
(n_y^2)……………………………	8 424 000	8 891 000
(n_x^2)……………………………	4 374 000	3 902 000
$(n_y + n_x)^2$……………………………	−2 590 000	−2 489 000
t^2……………………………	7 010 000	6 349 000
Total energy……………	17 218 000	16 653 000

There was some doubt in the writer's mind regarding the accuracy of the term depending upon $(n_y)^2$ because this becomes zero between $x = 0$ and $x = 10$. Applying Simpson's rule to the first three values of n_y, that is, for $x = 0, 10$, and 20, the internal work is,

$$\frac{1}{4\,G}\,144\,\frac{10}{3}\,(h_0 + 4\,h_1 + h_2) = \frac{462\,800}{G}$$

If the n_y curve is assumed as straight, the energy represented by the first three values is,

$$\frac{1}{4\,G}\,1\,440\left(\frac{1\,417.5 + 1\,943.0}{2} + 124.1\right) = \frac{604\,900}{G}$$

There is a difference of $\frac{142\,100}{G}$, equivalent to about 40% of the difference between the total energy of the usual and the proposed theory. The correct value, therefore, was determined by integration for $y = 100$,

$$\frac{144}{4\,G}\int_0^b d\,x\left(0.3765\,y - 5.777\,x + 9.504\,\frac{x^2}{y} - 5.263\,\frac{x^3}{y^2}\right)^2 \dots (33)$$

This gives 8 875 000, which is 16 000, or 0.18%, less than is found by Simpson's rule. The internal work by the new theory is, therefore, slightly less than that given in Table 3. It is interesting to note that if t and n_x are neglected (see

Table 3) the usual theory gives the smaller value for the internal work. The terms inside the square bracket in Equations (14) and (29) then reduce to

$$\frac{m}{2\,(m+1)}\,n_y^{\,2} = \frac{4}{9}\,n_y^{\,2}.$$

VII.—RESERVOIR EMPTY ($g = 0$)

When, in Equations (16), (17), (25), and (26), g is put equal to zero, there results, in pounds per square inch, after dividing by 144,

$$\left.\begin{aligned} n_y &= -1.0579\,y + 1.8251\,x - 0.6558\,\frac{x^2}{y} + 0.5752\,\frac{x^3}{y^2} \\ \text{and,}\qquad t &= +0.0146\,x - 0.2186\,\frac{x^3}{y^2} + 0.2876\,\frac{x^4}{y^3} \end{aligned}\right\}\quad\ldots\ldots(34)$$

TABLE 4.—STRESSES IN NO-UPLIFT SECTION ($k = 0.645$), 100 FEET BELOW CREST ($y = 100$), WHEN RESERVOIR IS EMPTY.

(Stresses, in pounds per square inch.)

x.	n_y.	t.
0	−105.79	0.0000
10	− 88.137	+0.127
20	− 71.450	+0.163
30	− 55.385	+0.0807
40	− 39.596	−0.0788
50	− 23.738	−0.2050
60	− 7.466	−0.1182
64.5	+ 0.0837	+0.0536
55		+0.8203

For $y = 100$ and $b = 64.5$ the values are given in Table 4 and the shear stress is plotted in Fig. 5. The value of n_y checks Equation (3) and t checks Equation (5); the total horizontal shear is zero. The n_y stress distribution is practically a straight line, the same as that found by the usual method. This was to be expected, because the shear stresses are small. There is a slight tension at the toe, while the usual method gives $n_y = 0$ at the toe. Equations (11) and (12) give for $y = 100$ ft., $A = -w$ and $B = \frac{w}{k}$, so that, $n_y = -w\,y + \frac{w}{k}\,x$, which, for $x = 0$ and $x = 64.5$, gives $n_y = -104.327$ and $n_y = 0$. The shear is zero throughout as is also n_x.

VIII.—BENDING STRESSES ($g = 62.5$ AND $w = 0$).

With the usual assumption, the stresses for a full reservoir may be obtained by superimposing the stresses produced by the water on those obtained when the reservoir is empty. The stresses for $g = 0$ and $w = 150.23$ have been determined in Article VII. If, now, $w = 0$ and $g = 62.5$ are introduced into the equations determining the constants, A, B, C, and F, there is obtained, when the stresses are expressed in pounds per square inch,

$$n_y' = 1.4344\,y - 7.6016\,x + 10.159\,\frac{x^2}{y} - 5.8380\,\frac{x^3}{y^2}\ldots\ldots(35)$$

When this is added to the stress found for $g = 0$ in Article VII, there results,

$$n_y'' = 0.3765\,y - 5.7765\,x + 9.503\,\frac{x^2}{y} - 5.263\,\frac{x^3}{y^2} \quad \ldots\ldots\ldots(36)$$

This is identical with the stress found directly in Article V (see Equation (27)). Since E is assumed constant, the two results should be equal. The fact that the two results check is merely evidence of correct numerical calculations.

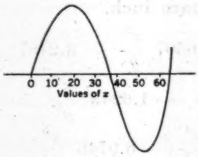


FIG. 5.—VARIATION IN SHEAR STRESS, $g = 0$

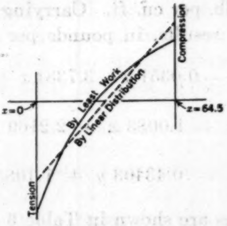


FIG. 6.—VARIATION IN BENDING STRESS, $w = 0$.

It may be noted that the bending stress found by the usual method is uniform and gives the same value at the up-stream face as at the down-stream face. In this manner the bending stresses for a triangular section are the same as for a rectangular one. This does not seem plausible, and the resulting principal stresses are not alike. By the usual method, the bending stress is, for $y = 100$,

$$n_y' = \frac{62.5}{0.41602}\,\frac{100}{144} = 104.33 \text{ lb. per sq. in.}$$

This may also be obtained by the usual formula,[19]

$$n_y' = \frac{M}{I}\,y = 6\,\frac{M}{b^2} = \frac{1\,000\,000 \times 0.43403}{4\,160} = 104.33$$

TABLE 5.—BENDING STRESSES IN NO-UPLIFT SECTION ($k = 0.645$), 100 FEET BELOW CREST ($y = 100$); RESERVOIR FULL AND WEIGHT OF DAM NEGLECTED.

(Stresses, in pounds per square inch.)

x.	n_y.	x.	n_y.
0	+143.41	40	− 35.45
10	76.99	50	− 55.65
20	27.37	60	− 73.04
30	− 8.94	64.5	− 80.88

The bending stress found by the Principle of Least Work is listed in Table 5 and both curves are shown in Fig. 6. The distribution of the bending stress could readily be tested, since it does not involve w.

---

[19] "Notes on Arched Gravity Dams," *Transactions*, Am. Soc. C. E., Vol. 92 (1928), p. 766.

IX.—The Section of Maurice Lévy

Lévy laid down the rule that when the normal vertical stresses are calculated by the usual method, the compression in the up-stream face with reservoir full should be qual to, or greater than, the hydrostatic pressure of the water.[20] Assume $g = 62.5$ and $k = 0.844$, then from Equation (13), for $x = 0$,

$$n_y = -62.5\ y = \left(-w + \frac{62.5}{0.844^2}\right) y$$

or, $w = 150.24$ lb. per cu. ft. Carrying out the calculations for the constants, as before, there results, in pounds per square inch,

$$\left.\begin{aligned} n_y &= -0.0351\, y - 3.7381\, x + 6.6507 \frac{x^2}{y} - 3.2487 \frac{x^3}{y^2} \\ t &= -1.0083\, x + 2.2169 \frac{x^3}{y^2} - 1.6243 \frac{x^4}{y^3} \\ n_x &= -0.43403\, y + 1.1085 \frac{x^4}{y^3} - 0.9746 \frac{x^5}{y^4} \end{aligned}\right\} \ldots\ldots (37)$$

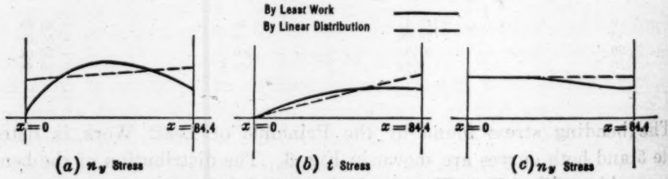
These stresses are shown in Table 6 for $y = 100$ and are plotted in Fig. 7; they were checked by planimeter to see that they conform to Equations (3) and (5).

TABLE 6.—Stresses in Full Uplift Section ($k = 0.844$), 100 Feet Below Crest ($y = 100$); Reservoir Full.

(Stresses, in pounds per square inch.)

x.	n_y.	t.	n_x.
0	− 3.507	0.000	−43.403
10	−34.562	− 9.877	−43.393
20	−54.265	−18.652	−43.257
30	−64.566	−25.578	−42.850
40	−67.412	−30.301	−41.563
50	−64.754	−32.854	−39.521
60	−58.540	−33.662	−36.616
70	−50.721	−33.539	−33.169
80	−43.244	−33.688	−29.937
84.4	−40.546	−34.237	−28.896

When the usual theory is employed, the stresses at the up-stream face, for $y = 100$ ft., are $n_y = -43.403$ and $t = 0$, and at the down-stream face, $n_y = -60.930$ and $t = -51.425$. The proposed theory gives compression



(a) n_y Stress (b) t Stress (c) n_y Stress

FIG. 7.—Comparative Stress Distribution for Full Uplift Section.

---

[20] "Quelques considérations sur la construction des grands barrages," *Comptes Rendus*, Vol. 121 (1895).

in the up-stream face, which is materially smaller than that found by the usual theory.

X.—MAXIMUM STRESS

The two principal stresses* are,

$$\left.\begin{aligned} s_1 &= \frac{n_y + n_x}{2} - \frac{1}{2}\sqrt{(n_y - n_x)^2 + 4\,t^2} \\ s_2 &= \frac{n_y + n_x}{2} + \frac{1}{2}\sqrt{(n_y - n_x)^2 + 4\,t^2} \end{aligned}\right\}\ldots\ldots\ldots\ldots(38)^{[21]}$$

The maximum stress occurring in a horizontal cross-section is the greatest value of the principal stress. With the usual assumption, all three stresses increase with x and reach their greatest value for $x = b$, so that the maximum stress occurs at the down-stream face and is, $s_{\text{max.}} = \frac{n_y}{\cos^2 \phi}$. An inspection of Table 1 shows that the same is at least approximately true for the section there investigated. Table 6 shows that it does not hold for the Maurice Lévy section. The principal stresses for this section are given in Table 7. The maximum stress is 87.4, while on the usual assumption it is $\frac{60.93}{0.584} = 104.33$ lb. per sq. in. As a check, $n_y + n_x$ should equal $s_1 + s_2$.

TABLE 7.—PRINCIPAL STRESSES IN FULL-UPLIFT SECTION ($k = 0.844$), 100 FEET BELOW CREST ($y = 100$); RESERVOIR FULL.

(Stresses, in pounds per square inch.)

x.	s_1.	s_2.
0	−43.4	− 3.5
10	−49.8	−28.2
20	−68.2	−29.3
30	−81.5	−25.9
40	−87.4	−21.6
50	−87.3	−16.9
60	−83.0	−12.2
70	−76.6	− 7.3
80	−70.9	− 2.2
84.4	−69.4	0.0

At the up-stream face, s_1 is horizontal and at the down-stream face, s_1 is parallel to the face of the dam. The angle between the vertical and s_1 should be,

$$\tan(2\,\phi) = \frac{2\,t}{n_y - n_x}\ldots\ldots\ldots\ldots\ldots\ldots\ldots(39)$$

This gives $\phi = 40° \, 10'$, for $x = 84.4$ ft., which checks $\tan \phi = k = 0.844$, as it should.

In engineering literature too much emphasis has been placed on the necessity of limiting the maximum stress and too little on the importance of not designing with too low a value of the least principal stress at the up-stream face, or rather that stress which is parallel to the up-stream face and which is generally the least principal stress. The writer has called attention to this before,[22] but it needs to be insisted upon, quite irrespective of whether or not the proposed stress distribution is the correct one. The stress in the up-stream face with reservoir full, is obtained by subtracting one stress from another, and the result will always be subject to considerable uncertainty, especially when the residual stresses are also considered, as they should be. Professor Cain has also called attention to this point.[23]

---

[21] *Transactions*, Am. Soc. C. E., Vol. LXXXVII (1924), p. 294. A more comprehensive explanation is given by Professor Cain in his paper, "Stresses in Masonry Dams," *Transactions*, Am. Soc. C. E., Vol. LXIV (1909), Appendix, p. 220. In Fig. 5 of that paper, the direction of the greatest principal stress is shown for a number of points of a horizontal section.

The writer has never heard of the failure of a triangular-shaped, gravity dam due to excessive maximum stress, but a number have failed because they were designed with too little compression in the up-stream face, which, by uplift or other action, resulted in tension.[24]

XI.—The Influence of Poisson's Ratio, $\frac{1}{m}$

There is considerable variation between the values of m for concrete given by various investigators, and it is admittedly most difficult to obtain reliable values. It may also be that m varies, as does the modulus of elasticity, with the water-cement ratio, the aggregates, the age of the concrete, and the length of time during which the load is applied.

Consider again the section, $k = 0.645$, for which the stresses were determined in Article V on the assumption that $m = 8$, and calculate the constants, A, B, C, and F, on the assumptions that $m = \infty$ and $m = 2$. The latter value is applicable to rubber; the English tests, to which a more detailed reference will be given, were made on rubber having a value of m approximately equal to 2. In each case, the calculations are exactly the same as before, and the results are given in Tables 8 and 9.

TABLE 8.—Value of Constants for Various Values of m.
($k = 0.645$; $g = 62.5$; and $w = 150.2$.)

	$m = 8.$	$m = \infty.$	$m = 2.$
$A' =$	0.3765	0.3832	0.3581
$B' =$	−5.7768	−5.8061	−5.7047
$C' =$	9.5037	9.3941	9.8553
$F' =$	−5.2633	−4.9958	−6.0611

There is no great difference between the three sets, the tension is smallest for $m = 2$. When $m = \infty$, C and F are independent of w, the weight of the concrete, and the constant, B, does not enter into Equations (25) and (26). The value of w determines the slope, B, and also affects the value of A, which is the stress at the up-stream face.

---

[22] *Transactions*, Am. Soc. C. E., Vol. 89 (1926), p. 763.

[23] *Loc. cit.*, Vol. 92 (1928), p. 796.

[24] Report of Committee of the Board of Direction on Essential Facts Concerning the Failure of the St. Francis Dam, *Proceedings*, Am. Soc. C. E., January, 1930, p. 163, and April, 1930, p. 827.

TABLE 9.—VALUES OF n_y FOR VARIOUS VALUES OF m.
(y = 100 ft.; k = 0.645; stresses, in pounds per square inch.)

x.	m = 8.	m = ∞.	m = 2.
0	+37.650	+38.325	+35.806
10	−11.140	−10.842	−11.992
20	−44.082	−44.217	−43.716
30	−64.332	−64.800	−62.003
40	−75.048	−75.587	−73.488
50	−79.389	−79.576	−78.812
60	−80.512	−79.765	−82.607
64.5	−80.809	−79.408	−84.785

XII.—DISCUSSION OF THE ENGLISH TESTS

Some tests on gravity dam models were made by Mr. L. W. Atcherly and Professor Karl Pearson in 1904.[25] These models were built up of thin strips of wood, arranged horizontally in one set of tests and vertically in another. It was claimed that these tests indicated the presence of considerable tension across vertical planes near the down-stream toe. In order to throw additional light on this important subject, some additional tests were made.[26]

One set of tests was made with a model of plasticine, a kind of modeling clay. An inspection shows that the results near the up-stream face are too erratic to permit of any conclusions being drawn. The curving of the n_y-stress seems established and also the unlikelihood of tension near the toe. The model was constructed so that the resultant fell in the down-stream middle-third point when the reservoir was full. Another set of tests was made on rubber models. The weight of the dam was increased by attaching weights acting vertically downward, and the water loading was imitated by weights acting horizontally. It is not quite clear what the corresponding water load was, but it seems to have been intended that the resultant should fall at the down-stream middle-third point.

These tests have been generally taken to demonstrate the correctness of the ordinary assumption of linear stress, although, as stated in the conclusions,[27] tensile stresses were observed in every case at the up-stream heel, that is, where the concrete meets the foundation. Moreover, all the tests show the curving of the n_y-stress curves, as found by the Principle of Least Work. Fig. 8 (which is Fig. 11 in the original paper) shows the strains as measured, and Fig. 9 (which is Fig. 19 in the original paper) shows plainly the curving of the n_y-stress for the nearly triangular section, *C*. It shows no tension, except at the heel. The curves at Elevations 3-3 and 4-4 show the curving in the opposite direction to those at Elevations 5-5 and 6-6.

More details than those given are necessary to estimate the reliability of the results. The tests are not wholly satisfactory on account of the possibility

---

[25] "On Some Disregarded Points in the Stability of Masonry Dams," *Drapers' Company Research Memoir*, Lond., 1904; abstract in *Minutes of Proceedings*, Inst. C. E., Vol. CLXII, p. 456.

[26] "Experimental Investigations of the Stresses in Masonry Dams Subjected to Water Pressure," by Sir John W. Ottley and A. W. Brightmore and by J. S. Wilson and William Gore, excerpt from *Minutes of Proceedings*, Inst. C. E., Vol. CLXXII, Session 1907–08, Pt. II, Lond., 1908.

[27] *Loc. cit.*, p. 40.

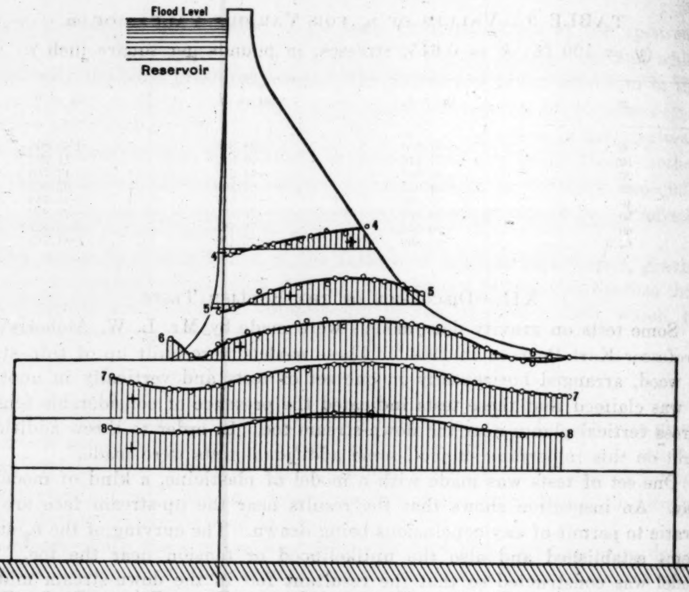

Flood Level
Reservoir
4
4
+
5
5
+
6
+
7
+
7
8
+
8

FIG. 8.—ENGLISH TESTS: VERTICAL STRAINS AS MEASURED. ENDS OF FOUNDATIONS NOT SUPPORTED.

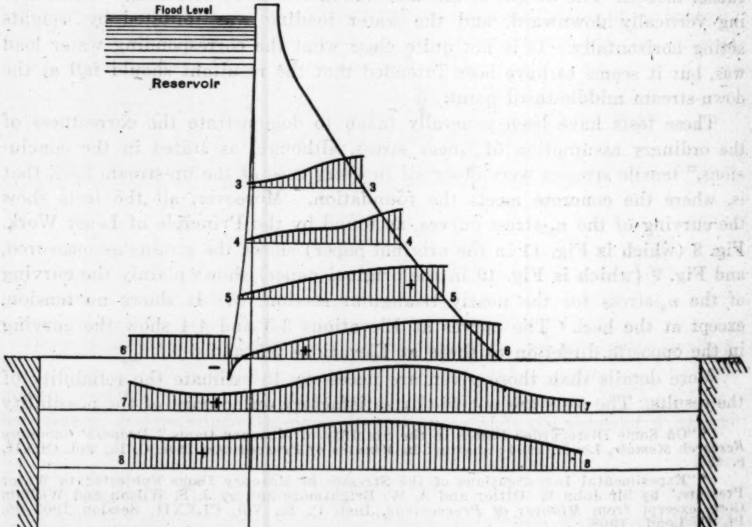

Flood Level
Reservoir
3
3
4
4
5
+
5
6
+
6
−
7
+
7
8
+
8

FIG. 9.—ENGLISH TESTS: VERTICAL PRESSURES ON HORIZONTAL PLANES; ENDS OF FOUNDATIONS SUPPORTED.

of considerable local strains, where the weights are attached, and the uncertainty as to the elastic behavior of the rubber, the time effect, etc. There is need of carefully made tests on models, which would represent a high gravity dam. The foundation should be sufficiently deep, and the method of attaching the dam to it might present a problem.

XIII.—GENERAL REMARKS

For some years the writer has doubted the correctness of the assumption of a linear stress distribution and has suggested that some caution might be justified.[28] He has called attention to the fact that horizontal sections are not normal sections, as the theory of bending requires, and that the assumption of a linear stress distribution on horizontal sections is not a conservative assumption when dealing with multiple-arch dams.[29] This question was also discussed by Professor Cain[30] and had been discussed by Résal in his paper to which reference has already been given. The writer found, as did Résal, that for gravity dams the usual horizontal sections gave practically the same result as normal sections, when the usual assumption of linear stress distribution was made.[31]

The writer has used the Principle of Virtual Velocities to determine the stress distribution in thick arches of dams[32] and Professor Cain showed that the same result was obtainable by the Principle of Least Work.[33] After Dr. Vogt's work in connection with the deformation of the foundations[34] had become known to the writer, he made an attempt to determine the stress distribution at the abutments of the Pacoima Dam by the Principle of Least Work. Due to the fact that the abutments were not radial, it was evident, that the usual formulas were not exact. Meantime, further studies had convinced him, that the assumption of linear stress distribution in gravity dams was probably incorrect, entirely aside from the effect of internal stresses. It was believed,[35] that if the assumption was in error, this would be likely to be aggravated in very high dams. The result of the investigation set forth in this paper seems to justify this apprehension.

If the proposed theory had given values for the stresses, which were less conservative than those found by the usual theory, the new theory should be disregarded until it could be firmly established by carefully made tests. As it is, the new theory gives stresses which are less advantageous, and dams designed by the usual method may not be safe when tested by the proposed theory. It will also be recognized, that the Principle of Least Work is more fundamental than the arbitrary assumption that n_y is linear. Tests only can determine which theory is the correct one.

---

[28] *Transactions*, Am. Soc. C. E., Vol. 91 (1927), p. 253.

[29] *Loc. cit.*, Vol. LXXXVII (1924), p. 300, Table 1.

[30] *Loc. cit.*, p. 315.

[31] *Loc. cit.*, p. 294.

[32] *Loc. cit.*, Vol. 90 (June, 1927), p. 475.

[33] *Loc. cit.*, p. 522.

[34] *Loc. cit.*, p. 554.

[35] Rept. by the writer to the Colorado River Comm. of Arizona, sent to the Colorado River Board of Engrs. (care, U. S. Reclamation Service), November, 1928.

The proposed solution is only an approximation, limited by the equation assumed for n_y, which has two free constants. If a more complicated equation is assumed for n_y, one having more free constants, a greater degree of approximation should be attained. This would involve much more work.

By putting $F = 0$ in Equation (15), a solution is obtained with only one free constant. There results, $A' = 0.296$; $B' = -4.37$; and $C' = 4.27$; and, for $y = 100$ ft., the corresponding values of n_y are shown in Table 10. These values agree quite closely with those found in Table 1, indicating that the true stress distribution is probably not far from that obtained when using two free constants. (See, remarks relative to Equation (36) in Article VIII.)

TABLE 10.—ONE FREE CONSTANT ONLY.
($y = 100$; $k = 0.645$; and $m = 8$.)

x.	n_y.	n_y from Table 1.
0	+29.6 (tension)	+37.7
10	− 9.87	−11.1
20	−40.8	−44.1
30	−63.2	−64.3
40	−77.0	−75.0
50	−82.4	−79.4
60	−79.2	−80.5
64.5	−75.0	−80.8

The writer has made an attempt to apply the theory to arched gravity dams. The mathematical complications were found to be considerable, although only one free constant was used in the expression for n_y. Equations (3) and (4) become more complicated in that W contains both y^2 and y^3 and M contains y^3 and y^4. Also, the equilibrium equations, (6) and (7), are more complicated. The solution found thus far agrees with the formulas presented in this paper when the radius becomes infinitely large, but it involves calculations with infinite series, which greatly augment the labor. This difficulty may be inherent in the problem, but it may also be that an expression can be selected for n_y which will avoid these difficulties. If the theory proves valid by tests on gravity dams the question arises as to how the stress distribution in multiple-arch dams is affected, when the principle is substituted for the assumption of linear stress across normal sections.

A few historical remarks may be of interest. In 1747, Maupertuis, President of the Berlin Academy under Frederick the Great, enunciated a principle, which he called "le principe de la moindre quantité d'action."[36] His paper was in three parts, the first being, "an examination of the proofs of the existence of God", which according to the second part must be sought in the general laws of motion. He declared the principle to be one which eminently accorded with the wisdom of the Creator and applied it to the equilibrium conditions of the lever and to the motion of light. Maupertuis was not a skilled mathematician and his exposition was confused and obscure.[37]

---

[36] "The Principle of Least Action," by Philip E. Jourdain, The Open Court Pub. Co., Chicago, Ill.

[37] "The Science of Mechanics," by Dr. Ernst Mach, late Professor at the University of Vienna, Open Court Pub. Co. (1919), pp. 364 and 454.

Mach suggests[37] that the principle would soon have been forgotten, had not Euler taken up the suggestion, magnanimously leaving Maupertuis the glory of the invention, and converted it into something serviceable. The theological point of view was retained by Euler who declared that since the universe is the handiwork of an All-Wise Maker, nothing can be met with in which some maximal or minimal property is not displayed. These were the studies which culminated in Euler's work in the calculus of variations and with the isoperimetrical problems; and the equation for the solution of these is still known as Euler's equation.[38]

That Nature performs in such a manner as to involve the least expenditure of energy, is an assumption which appeals strongly to the intuition; but, whether or not it applies in a specific case, or how it is to be interpreted, only actual tests can determine. For, as Mach says, it cannot be proved by mathematics that Nature must be exactly what it is.

ACKNOWLEDGMENT

The first draft of this paper was submitted for criticism to Professor Cain and to Dr. Vogt, to whom the writer's sincere thanks are due for calling attention to some errors of calculation and encouraging the writer to continue his studies of this subject.

APPENDIX I

From the tests on the plasticine model (having a base of 26 in.), referred to in Article XII, Sir John W. Ottley and Mr. A. W. Brightmore derived a stress distribution[39] which at the heel is zero (the same as found by the linear stress distribution) and at the toe is about 80% of the linear stress. Calling the stress at the toe for linear distribution, 100%, the stress distribution is as given in Table 11 and as plotted in Fig. 10.

TABLE 11.—STRESS DISTRIBUTION FOUND FROM TESTS ON PLASTICINE MODEL, IN TERMS OF PERCENTAGE OF STRESS AT TOE.

x, in inches.	Stress.	n_y.
(Heel) 0	0	0
2	7.7	5.2
4	15.4	11.8
6	23.1	18.8
8	30.8	27.4
10	38.5	36.8
12	46.1	48.4
14	53.9	60.8
16	61.5	72.6
18	69.2	80.6
20	77.0	83.6
22	84.6	83.6
24	92.4	82.2
(Toe) 26	100.0	80.0

---

[38] "Advanced Calculus," William F. Osgood, Macmillan Co. (1925), Chapter XVII.

[39] *Minutes of Proceedings*, Inst. C. E., Vol. CLXXII (Session 1907–08), Pt. II, Lond., 1908, Fig. 4, Pl. 3.

The question now arises whether Equation (15) can represent a curve similar to Fig. 10, so that in case this is the correct stress distribution, the constants could be determined so as to give a curve similar to Fig. 10.

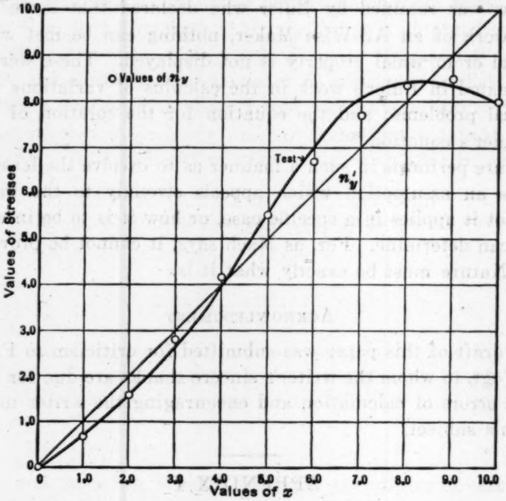


FIG. 10.—COMPARISON OF TEST AND EQUATION (15a), FOR $F = -0.02$.

Let $n_y' = a\,x$ be the linear stress distribution. From Equation (15), since y is a constant, $n_y = A + B\,x + C\,x^2 + F\,x^3$. Since, for $x = 0$, n_y is zero, it follows that $A = 0$.

From Equations (3) and (4),

$$\int_0^b n_y\,d\,x = \frac{B}{2}\,b^2 + \frac{C}{3}\,b^3 + \frac{F}{4}\,b^4 = \int_0^b n_y'\,d\,x = \frac{a}{2}\,b^2$$

and,

$$\int_0^b n_y\,x\,d\,x = \frac{B}{3}\,b^3 + \frac{C}{4}\,b^4 + \frac{F}{5}\,b^5 = \int_0^b n_y'\,x\,d\,x = \frac{a}{3}\,b^3$$

For the down-stream toe, $b = 10$ (Fig. 10), there results, $C = -12\,F$ and $B = 30\,F + a$; so that for $a = 1$, $n_y' = x$, and,

$$n_y = (30\,x - 12\,x^2 + x^3)\,F + x \ldots\ldots\ldots\ldots\ldots (15a)$$

For $x = 0$, $n_y = n_y' = 0$. If $F = 0$, then $C = 0$ and $n_y = n_y' = x$. **If $F \neq 0$, the points in which n_y and n'_y are equal are found from, $30 - 12\,x + x^2 = 0$, or $x = 3.55$ and 8.45, no matter what F is. If F is less than zero, the resulting curve is similar to the one found in the English tests. If F is greater than zero, the curve is reversed with respect to the straight line, n_y', that is, the part nearest to the up-stream face lies above n_y', etc.**

In Equation (15a), for n_y, F could be determined by the method of least squares, so as to fit the test curve in the best possible manner, but a quite close fit is obtained by assuming that $F = -0.02$, as may be seen by comparing

Tables 11 and 12, and from Fig. 10, in which the small circles are values of n_y determined from Equation (15a), when $A = 0$ and $F = -0.02$.

TABLE 12.—VALUES OF n_y FOR $A = 0$ AND $F = -0.02$.

x.	n_y.	x.	n_y.
0	0	60	6.72
10	0.62	70	7.70
20	1.60	80	8.32
30	2.82	90	8.46
40	4.16	100	8.00
50	5.50		

APPENDIX II

An Airy's stress function, F, may be determined to satisfy the linear stresses given by Equations (13). This stress function is defined by:

$$n_y = \frac{d^2\,F}{d\,x^2};\; n_x = \frac{d^2\,F}{d\,y^2};\; t = -\,\frac{d^2\,F}{d\,x\,d\,y} - w\,x \ldots\ldots\ldots\ldots(40)$$

in which, n_y, n_x, and t are given by Equation (13). This gives,

$$F = \left(-\,w + \frac{g}{k^2}\right)\frac{y\,x^2}{2} + \left(\frac{w}{k} - 2\,\frac{g}{k^3}\right)\frac{x^3}{6} - g\,\frac{y^3}{6} \ldots\ldots\ldots(41)$$

Equation (41) satisfies Equations (13) and, therefore, satisfies the equilibrium equations, (6) and (7), the statical conditions, Equations (3) and (4), and the boundary conditions, which lead to Equations (13) for the stresses. According to Föppl, the compatibility equations, which guarantee the geometrical continuity of the structure, are satisfied when:[40]

$$\frac{d^4\,F}{d\;x^4} + 2\,\frac{d^4\,F}{d\,x^2\,d\,y^2} + \frac{d^4\,F}{d\,y^4} = 0 \ldots\ldots\ldots\ldots\ldots\ldots(42)$$

Equation (41) satisfies this condition. Of course, a function, F, may also be determined from Equation (27), and Equation (42) may be written,

$$\frac{d^2\,(n_y + n_x)}{d\,y^2} + \frac{d^2\,(n_y + n_x)}{d\,x^2} = 0 \ldots\ldots\ldots\ldots\ldots\ldots(43)$$

Equation (27) does not satisfy Equation (43); and, according to Föppl, it should not.[41]

APPENDIX III

The plane stress distribution considered in this investigation, assumes that $n_z = t_{zy} = t_{zx} = 0$ (Fig. 11). The quantity, n_z, is the normal stress acting parallel to the Z-axis and t_{zy} and t_{zx} are the shear stresses acting in the X-Y

---

[40] "Drang und Zwang," Vol. I, p. 248 (1924 Edition).

[41] *Loc. cit.*, Article 13, p. 97.

plane. When $n_z = 0$, the displacement, u_z, in the direction of the *Z*-axis, is determined by:

$$\frac{d\,u_z}{d\,z} = \frac{1}{2\,(m+1)\,G}\,(n_y + n_x) \ldots\ldots\ldots\ldots\ldots\ldots (44)$$

Since $n_y + n_x \neq 0$ (Table 2), it follows that the plane limiting surface, *ABC*, in the *X-Y* plane of Fig. 11, is transformed into a warped surface by the load stresses, n_y, n_x, and t. Consequently, t_{zy} and t_{zx} are zero only in the boundary surfaces, such as *ABC* and in the middle plane (on account of symmetry); and n_z, in general at least, is zero only in the boundary surfaces, *ABC* and *A'B'C'*, of Fig. 11. A plane stress condition, therefore, in general,

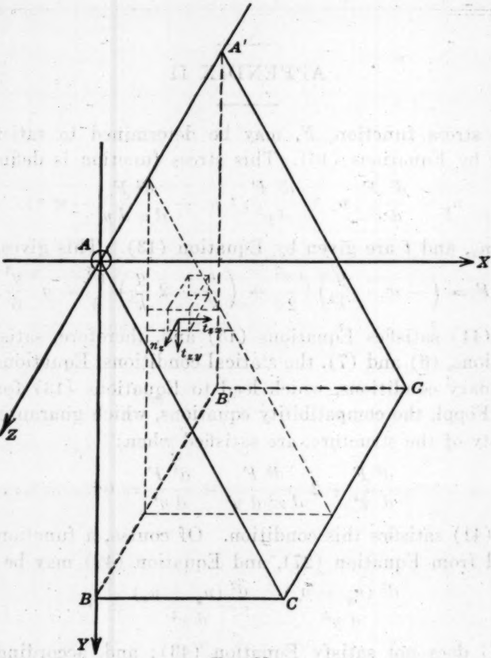


FIG. 11.

must lead to contradictions, if both the equilibrium and compatibility equations are used.[42] None the less Föppl advances the idea that if the exact stress function is not known and if F_a is an approximate function, containing free constants, these may be determined from:

$$\int \left(\frac{d^4\,F_a}{d\,y^4} + 2\,\frac{d^4\,F_a}{d\,y^2\,d\,x^2} + \frac{d^4\,F_a}{d\,x^4} \right)^2 d\,A = \text{Minimum} \ldots\ldots (45)$$

in which, dA is an element of area and the constants in F_a are to be determined by the method of least squares.[43] This solution, according to Föppl, has the same right to be considered a valid approximation as the result obtained by the Principle of Least Work. Equation (45), however, must lead to Equation (41), which is equivalent to Equations (13), and represents a linear stress distribution, which does not agree with the result obtained by the Principle of Least Work.

---

[42] "Teknisk Elasticitetslaere," by A. Ostenfeld, Article 20, p. 105 (1916 Edition); see, also, Föppl, "Drang und Zwang," Vol. I (1924), p. 129.

It is clear that Equation (14) for the internal work is inaccurate, in that it assumes $n_z = t_{zy} = t_{zx} = 0$, for the entire volume under consideration. If $n_z = \frac{1}{m}(n_y + n_x)$, that is, if n_z is just sufficient to prevent displacements in the direction of the *Z*-axis, then t_{zy} and t_{zx} are both zero, and while there is no displacement in the direction of n_z, this latter, since it varies with the load stresses, influences the displacements in the direction of n_y and n_x. In general, nothing definite is known about the value of n_z in gravity dams. It is affected by shrinkage, temperature changes, etc., and no doubt varies from point to point and from time to time.

When $m = \infty$ it may be assumed, without error, that $n_z = t_{zy} = t_{zx} = 0$; all the compatibility equations are satisfied, and Equation (14) is exact. It has been shown in Article XI that there is no great difference between the stresses obtained for $m = 8$ and for $m = \infty$ and the assumption that $m = \infty$ leads to considerable simplification of the calculations. It is admitted that the assumption, $n_z = t_{zy} = t_{zx} = 0$, is not exact, except when $m = \infty$, but it is believed that this inaccuracy will not materially influence the stress distribution obtained by the Principle of Least Work, while the use of the compatibility equations in connection with the equilibrium equations may lead to contradictions due to the incorrect assumption.

The present investigation does not take account of the irregularities introduced by the foundation, where the section is suddenly changed. Whether or not the stress distribution furnished by the Principle of Least Work applies at the foundation more nearly than the ordinary assumption, only tests can decide. The ordinary assumption is incorrect at the foundation, since as Dr. Vogt has shown, the foundation does not remain plane when the stresses are linear. This is also shown by the English tests (Article XII), which also indicate that the stress distribution obtained by the Principle of Least Work may at least approximate the actual stress distribution at the foundation.

---

[43] "Drang und Zwang," Vol. I, p. 336 (1924 Edition).

DISCUSSION

WILLIAM CAIN,[44] M. AM. SOC. C. E. (by letter).—In designing straight gravity dams it has been customary to limit the position of the resultant of the water pressure and the weight of the dam to the middle third of any horizontal section. This rule held whether the uplift due to water penetrating the sections was considered, or otherwise. Furthermore, for the given resultant, the vertical unit stresses on a horizontal plane were supposed to be computed as if the vertical section of the dam was rectangular, on the hypothesis that plane sections remained plane sections after stress. This led to the so-called "trapezoid law" of variation of stress, or a uniformly increasing stress from the water face to the down-stream face. It followed that, if the resultant cut the corresponding horizontal section at one-third its width from the outer toe, the unit stress at the inner toe was zero and the stress increased uniformly from the inner to the outer toe where it was a maximum. There was consequently no tension exerted anywhere on the horizontal section.

It has long been the belief of some engineers that such conclusions or hypotheses were untenable; but it remained for Mr. Jakobsen to prove it theoretically[45] and for English engineers by experiments on plasticene and India-rubber models of dams[46] to establish it experimentally. In the case of the plasticene model dam, water was applied to the face of the model through a thin rectangular India-rubber bag made to fit the frame. The results were only qualitative, but were sufficient to indicate the variation of stress on horizontal sections and to realize the fear that tension might be exerted at the inner toe and that cracks might be formed at the water face even when the resultant cut the base at one-third its width from the outer toe.

In the case of the India-rubber model, water was not applied, but its exact equivalent was substituted in the form of weights, acting through strings over pulleys, the strings being given directions at right angles to the water surface where they were attached to small plates bearing, through thin rubber pads, directly on the model. It was found necessary virtually to increase the density of the rubber by attaching other weights to small pins passed through holes in the rubber cut through centers of gravity of certain imaginary sections. Such weights were adjusted to correspond with the volume of rubber they were made to represent. It was not found that the small holes appreciably affected the physical constants of the rubber. Wooden wedges were placed at the upper and lower ends of the large foundation portion of the dam to give it full support. The entire apparatus was most ingenious, as was likewise the method of measuring strains.

The unit stresses at numerous points in the dam and foundation were determined from the displacements by usual formulas and are represented by the ellipses of stress shown in Fig. 12. The principal stresses, or those acting normally to their planes, are represented in amount and direction by the semi-axes of the ellipse. These are observed to act parallel to the inclined down-stream face near that face, but at the up-stream face where the dam is subjected to water pressure the direction of the principal stress is nearly normal to that face to counteract the large normal pressure of the water. The variation of stress along horizontal planes above the foundation is of especial interest in relation to the author's main problem, and such variations for the vertical pressure on horizontal planes, the horizontal pressure on vertical planes, and the shear on horizontal planes have all been computed and diagrams made to show the variation.

---

[44] Prof. Emeritus, Univ. of North Carolina, Chapel Hill, N. C. Professor Cain died December 7, 1930.

[45] *Transactions*, Am. Soc. C. E., Vol. LXXXVII (1924), p. 276.

[46] Excerpt, *Minutes of Proceedings*, Inst. C. E., Vol. CLXXII, Session 1907–08, Pt. II.

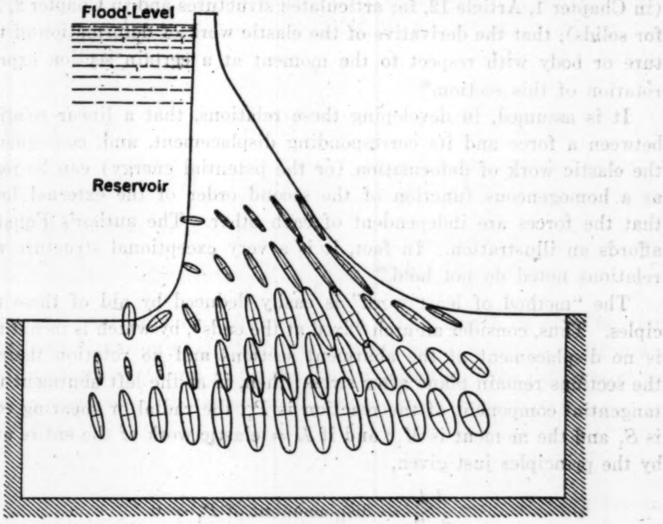


FIG. 12.—ELLIPSES OF STRESS, ENDS OF FOUNDATION SUPPORTED.

The author has endeavored to find this stress distribution theoretically by the method of least work and he has succeeded remarkably well in effecting it. To this end, after writing certain fundamental relations, he assumes that the vertical unit stress on a horizontal plane at (x, y) is given by an expression of the form of Equation (15),

$$n_y = A\,y + B\,x + C\,\frac{x^2}{y} + F\,\frac{x^3}{y^2},$$

in which, A, B, C, and F, are constants; and in which, by antecedent equations, A and B can be expressed as functions of C and F, as given by Equations (16) and (17). The elastic internal work, L, is expressed by Equation (14), as given by Föppl, which equation is likewise given in another form, easily reduced to this, by Molitor.[47]

---

[47] "Kinetic Theory of Structures," p. 37.

It will be convenient now to refer to Castigliano's "Systèmes Elastiques"[48] for a fundamental principle (given in Chapter 1, Article 11, for an articulated system of bars, and in Chapter 2, Article 4, for a solid body), that if the elastic work of deformation is expressed as a function of the exterior forces (including the reactions as exterior forces), then its partial derivative with respect to any one of these forces will give the relative displacement of its point of application in the direction of the force. Similarly, it is proved (in Chapter 1, Article 12, for articulated structures and in Chapter 2, Article 5, for solids), that the derivative of the elastic work of deformation of the structure or body with respect to the moment at a certain section expresses the rotation of this section.[49]

It is assumed, in developing these relations, that a linear relation exists between a force and its corresponding displacement, and, consequently, that the elastic work of deformation (or the potential energy) can be represented as a homogeneous function of the second order of the external forces, and that the forces are independent of each other. The author's Equation (14) affords an illustration. In fact, it is a very exceptional structure where the relations noted do not hold.[50]

The "method of least work" is easily deduced by aid of these two principles. Thus, consider an arch "fixed at the ends", by which is meant that there is no displacement at the abutment sections and no rotation there, so that the sections remain plane after stress; then, if at the left abutment (say), the tangential component of the reaction is P_1, the radial or shearing component is S_1 and the moment is M_1; and if L = elastic work of the entire arch, then, by the principles just given,

$$\frac{d\,L}{d\,P_1} = \text{displacement of } P_1 = 0$$

$$\frac{d\,L}{d\,S_1} = \text{displacement of } S_1 = 0$$

and,

$$\frac{d\,L}{d\,M_1} = \text{rotation of abutment section} = 0$$

These partial derivatives being equal to zero express the fact that the elastic forces after deformation are such as to render the elastic work a minimum. Thus is proved for this case (and similarly for other structures) the "Method of Least Work".

Castigliano's treatment of the change of the unknown quantities[51] is very informing. It proves that, if the work of deformation can be expressed in terms of an equal number of new unknowns, the latter can be found directly by placing the partial derivative of the elastic work with respect to each of them equal to zero.[52] This treatment suggests the following, referring to Mr. Jakobsen's main problem. Note that from Equations (16) and (17), it is seen that A and B are functions of C and F. For convenience, the values of n_y, n_x, and t, are grouped together. Thus, in Equation (15),

---

[48] An English translation, with the title, "Elastic Stresses in Structures," is pub. by Scott Greenwood and Son, 8 Broadway, Ludgate, E. C. 4, London, England.

[49] All the fundamental principles of Castigliano's theory are given by the writer, by aid of the theory of "virtual velocities," in "Determination of the Stresses in Elastic Systems by the Method of Least Work," *Transactions*, Am. Soc. C. E., Vol. XXIV (April, 1891), p. 265.

[50] This subject is very clearly put in Timoshenko and Lessells' "Applied Elasticity," pp. 117-126.

[51] "Systèmes Elastiques," Chapter 10, Article 2.

$$n_y = A\,y + B\,x + C\,\frac{x^2}{y} + F\,\frac{x^3}{y^2}$$

In Equation (22),

$$n_x = \frac{C}{6}\,\frac{x^4}{y^3} + \frac{3}{10}\,F\,\frac{x^5}{y^4} - g\,y$$

and in Equation (21),

$$t = -A\,x + \frac{C}{3}\,\frac{x^3}{y^2} + \frac{F}{2}\,\frac{x^4}{y^3} - w\,x$$

The partial derivatives as to C are:

$$\frac{d\,n_y}{d\,C} = \frac{x^2}{y};\ \frac{d\,n_x}{d\,C} = \frac{x^4}{6\,y_3};\ \frac{d\,t}{d\,C} = \frac{x^3}{3\,y^2}$$

If L is the total work of deformation for a 1-ft. length of dam, as given by the author's Equation (14), then L is a direct function of n_y, n_x, t; and as these are functions of C and F, by the known formula of the differential calculus (the derivatives with respect to C, being partial derivatives),

$$\frac{d\,L}{d\,C} = \frac{d\,L}{d\,n_y}\,\frac{d\,n_y}{d\,C} + \frac{d\,L}{d\,n_x}\,\frac{d\,n_x}{d\,C} + \frac{d\,L}{d\,t}\,\frac{d\,t}{d\,C} = \frac{d\,L}{d\,n_y}\,\frac{x^2}{y^2} + \frac{d\,L}{d\,n_x}\,\frac{x^4}{6\,y^3} + \frac{d\,L}{d\,t}\,\frac{x^3}{3\,y^2}$$

Now by Castigliano's theorems stated on the derivatives of work for any point (x, y) of a horizontal section of the dam,

$$\frac{d\,L}{d\,n_y} = \text{vertical displacement}$$

$$\frac{d\,L}{d\,n_x} = \text{horizontal displacement}$$

and,

$$\frac{d\,L}{d\,t} = \text{displacement due to shear}$$

If at the base, each of these displacements is zero, the equation reduces to,

$$\frac{d\,L}{d\,C} = 0$$

Similarly, it can be found that,

$$\frac{d\,L}{d\,F} = 0$$

These two equations show that the internal work is a minimum, as stated by the author. This demonstration assumes the base of the dam unyielding.

---

[52] This treatment of the change of unknown quantities is likewise given in full in the paper cited, *Transactions*, Am. Soc. C. E., Vol. XXIV (April, 1891), p. 292.

Fredrik Vogt, Assoc. M. Am. Soc. C. E., in a very interesting discussion[52] regards the base, whether the foundation of the dam or the abutment of an arch, as yielding for a certain small distance, below which it is absolutely unyielding and plane sections remain plane.[52a] If this hypothesis is adopted, it is perhaps only necessary to assume approximately the base of the dam removed to a parallel position, presumed to be unyielding, for which the supposed plane section remains plane and the method of least work is seen to apply to the revised position or to a slightly increased size of dam. On this supposition the original foundation will no longer remain plane, but will become a slightly warped surface.

Recurring now to the original problem, where the base of the dam (or foundation) is supposed to be unyielding, the writer is glad to testify to the complete scientific accuracy of the author's analysis, of which he has checked Equations (1) to (25), inclusive. For the stress distribution, for the numerical illustration of Article V, the values of A', B', C', F', are readily found, and the formulas for the stresses at (x, y) are given by Equation (27), and their numerical values at the base, $y = 100$ ft., are recorded in Table 1.

The author has noted the important fact that the constants, A, B, C, F, are independent of the height, H, of the dam, "since in Equations (25) and (26) the expressions in the square brackets must be zero." It is thus only necessary in Equation (27) to substitute for (x, y), their numerical values to compute the stresses at any point (x, y) in the dam. This dam is designed so that the stress in the up-stream face is zero when the usual theory of (linear) stress distribution is used, straight lines limiting the unit stresses as shown in Fig. 4. Likewise, Fig. 4 gives the stress distribution as determined by the method of least work and it is seen that by this method there is appreciable tension at the up-stream face, and, further, that the angle between the curve of the n_y stress and the horizontal decreases as x increases. The important fact is the occurrence of tension at the up-stream face; and it is evident that the only way to avoid it is to increase the base or the value of $k = \frac{b}{y}$. This would alter slightly Equations (16a), (17a), (25a), and (26a), and lead to new values of A, B, C, F, and new formulas for n_y, t, n_x.

In place of assuming k and computing w, assume $w = 145$ (say), for the weight of 1 cu. ft. of concrete and compute, $k^2 = \frac{g}{w} = \frac{62.5}{145} = 0.431$; whence $k = \frac{b}{y} = 0.656$; or for $y = 100$ ft., $b = 65.6$ ft., thus giving the base for reservoir full for stress zero at the up-stream face when the usual linear stress distribution is assumed. Now b must be increased to avoid tension at $x = 0$ when the method of least work is used. As a first trial, suppose b increased by 5 ft., so that $b = 70.6$, whence for the same $w = 145$, $k = 0.706$, and the new values of A and B in terms of C and F are derived from Equations (16) and (17).

[52a] *Transactions*, Am. Soc. C. E., Vol. 90 (June, 1927), p. 554.

The solution proceeds by the use of Equations (25) and (26) exactly as the author indicates, so that eventually the formulas for n_y, t, n_x, can be written. If n_y at $x = 0$ should turn out to be either zero or a small negative value (compression), the solution may suffice. Otherwise, another small change in b will have to be assumed, and a new formula for n_y must be derived and tested as before. It should not require more than three trials before a satisfactory formula for n_y can be written.

As all the formulas refer to a triangular dam, they are approximate near the top when the usual additions to the section are made there; but for a high dam, they are sufficiently accurate for the lower sections. They furnish practical formulas for distribution of stress for most straight dams.

Hitherto, uplift (from water penetrating the base) has been neglected. If the uplift at the heel is assumed as the greatest possible (= 62.5 y lb. per sq. ft.), and as decreasing linearly to zero at the toe, then the total uplift is,

$$\tfrac{1}{2}\, 62.5\, y\, b = \tfrac{1}{2}\, 62.5\, k\, y^2$$

which is equivalent to the weight of a triangular mass of water acting upward on the base.

For 1 ft. in length of the triangular dam, its weight less the uplift is $\frac{1}{2}\, k\, y^2\, (w - 62.5)$.

It can now be easily verified that, for the triangular dam considered, all the general Equations (1) to (26) are valid when uplift is considered, by simply replacing w by $w' = (w - 62.5)$, which simplification is entirely due to the assumption of a triangular dam.

In Article VII, Mr. Jakobsen has derived the results for reservoir empty (when, of course, there is no uplift to consider) by the method of least work, and finds a small amount of tension at the toe. This would probably disappear when the base is slightly increased as previously suggested.

Partly as a check, the author has computed the internal work by the usual and the proposed theories and has subsequently computed the bending stresses for the same cases, and illustrated the results in Fig. 6. To render the solution more complete, the principal stresses have been computed and the influence of Poisson's ratio examined.

The labor involved in developing all the formulas and computing numerical results has been very great, and the author deserves the thanks of the profession for his excellent and convincing paper. The English experiments first proved the ordinary "inner-third rule" inadequate, and now Mr. Jakobsen confirms it by a close theoretical analysis. It will not suffice that the resultant of the water pressure and the weight of the dam should be required to cut the base simply at one-third its width from the outer toe; it must be at a greater distance from this toe. In other words, to avoid tension at the heel, the resultant should be confined to narrower limits than the middle third. It is a difficult problem to find such narrower limits, but, at least, it has been shown that the method of least work can materially aid in establishing them.

William Gore,[53] M. Am. Soc. C. E. (by letter).—Since the English tests of nearly a quarter of a century ago were published the general problem of the stresses in gravity dams has not received much attention. At that time this problem appeared to have been solved, but Mr. Jakobsen now raises it again from a new angle. The English tests indicated that, except at or near the foundations of a dam, the classical assumption of the trapezium or straight-line law, although not correct, was not so far wrong as to cause any anxiety concerning the safety of dams designed in accordance therewith, and all subsequent designs have followed these assumptions.

This situation has been a source of gratification to Mr. J. S. Wilson and the writer, who conducted most of the English tests. These tests had a direct bearing on the design of the first raising of the Assuan Dam, and their accuracy is a matter of considerable moment in discussing the author's paper. Mr. Jakobsen draws special attention to this, but he states that "the tests are not wholly satisfactory on account of the possibility of considerable local strains where the weights are attached, and the uncertainty as to the elastic behavior of the rubber, the time effect, etc." All tests, of course, are subject to possible inaccuracies such as those indicated by the author. They are also subject to inaccuracies due to the fact that the general assumptions and arrangements of the experimenters may not have been in agreement with the practical case they are intended to represent; but in the case of the English tests every effort was made to keep such inaccuracies as small as possible.

The work on gravity dams extended over a period of about four years and involved a considerable amount of labor, most of which was not seen in the published results. Everything possible was done to make the results classical. The exaggerated virtual weights of 6 000 lb. per cu. ft. for the masonry, and 2 500 lb. per cu. ft. for the water, were adopted in order that the strains produced could be large enough to be measured with reasonable accuracy while maintaining the same relative densities of concrete to water. The dam sections unloaded were distorted in form and it was only when fully loaded that the correct profiles were secured.

In Fig. 13, the broken lines show the model section and the principal lines in the unstrained position and the full lines show the model section and principal lines when fully strained with both "gravity" and "water weights". Test samples, cut from the same slabs as the sections, similarly treated and prepared, were subjected to various loadings under the same conditions as the sections, and were found to give an approximately straight-line relation between stress and strain within the limits of the tests. Owing to the time effect of stress upon rubber the models and similarly treated rubber sample tests were loaded for 24 hours, then rested for 48 hours, and the cycle repeated again and again until a permanent régime had been established. The measurement photographs of the loaded sections were taken after the loads had been applied 24 hours, and the unloaded sections, 48 hours after the loads had been taken off. Some time after loading, the pads conveying the water pressures were gently rolled sidewise to allow the soft rubber under them to creep to freedom. The models and fixed camera were in a room in which none beside the experimenters was allowed to enter, and great care was taken that the precise distance relationship between the camera and object was undisturbed.

---

[53] Cons. Civ. Engr. (Gore, Nasmith & Storrie), Toronto, Ont., Canada.

The later measurements were made upon large photographic negatives, especially prepared on plate glass, from 1½ to 2 times the actual size, by means of especially designed micrometer gauges. The gauge lines on the glass were opaque; they were projected by a lens in a vertical optical bench on the negatives and adjusted with a low-power microscope so that the black images of the gauge lines of the micrometer were caused to register upon the transparent lines or circles on the negatives. Length measurements to 0.001 in. were measured accurately and lengths of 0.0001 in. could be estimated in most cases. Angles of shear were read in the same manner accurately to tangents of 0.002.[54] All principal measurements were taken along lines midway between the load points, and the effects of the concentration of the loads are hardly discernible as the published measurements show. This is supported by theoretical considerations. In interpreting the stresses from the strains the calibration of every through line was adopted.

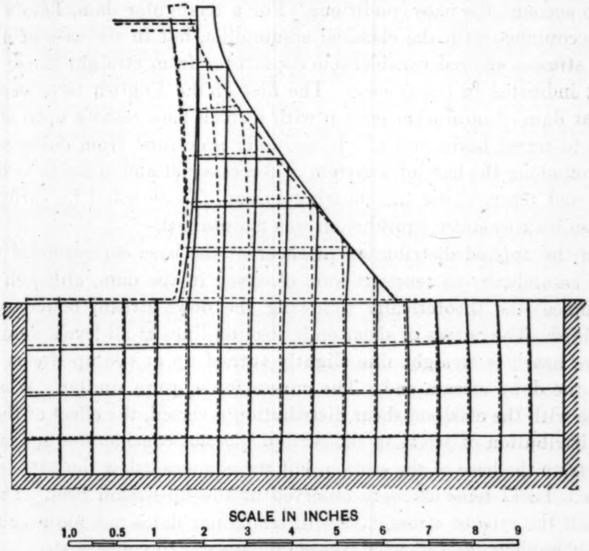


FIG. 13.—ENGLISH TESTS: MODEL SECTION CORRESPONDING TO FIG. 9.

It is possible that the elastic constants of a material such as rubber, even of the very best quality such as that adopted, do vary from point to point to some extent, but observations made during the English tests did not show these variations to be of great importance. The two sections (Figs. 8 and 9) constructed at different times compare very favorably; but in any practical case—

---

[54] *Engineering*, May 22, 1908.

whether by this or any other method—it is desirable that two or more models exactly alike be used and the results averaged. The results of the English tests have a high degree of accuracy and may reasonably be compared with the author's equations for n_y, n_x, and t.

At the time the English tests were made, it was held that a correct mathematical solution was not possible owing to the unknown and possibly arbitrary conditions at the base of the dam. The French engineer, Maurice Lévy, had attempted the solution by means of differential equations, but these did not take into account the base conditions. For a triangular dam, Lévy's solution was in accordance with the classical assumption, but in the case of a vertical wall the stresses showed considerable departures from straight lines; tensions were not indicated in either case. The first of the English tests were with a triangular dam of minimum section with a small base resting upon steel balls and free to travel horizontally.[55] It was only prevented from doing so by the application along the base of a system of forces equal and opposite to the water pressure and representing the shearing forces. These could be varied in any manner so long as static equilibrium was maintained.

When the applied distribution of shear at the base corresponded with the classical assumption no tensions were observed in the dam, although the line of resistance was theoretically following the down-stream boundary of the middle third. The curves of shear on horizontal lines at all levels were similar; they were nearly a straight line slightly turned up at the up-stream end and down at the down-stream end. The curves for n_y were similar. When a test was made with the classical shear distribution reversed, the effect of the change on the distribution of stress in the section was not considerable for more than 40% up from the base of the section, but the changes below the 40% level were extensive. Local tensions were observed at the up-stream heel. These tests established the type of stress curves in triangular dams not too near the base, and all subsequent tests of whatever source appear to confirm it.

The sections, Figs. 8 and 9, were developed later to deal in a natural and practical way with the lower part of the dam and the bed-rock foundations in the vicinity, including a considerable portion under the reservoir. For this purpose it was assumed that the rock and masonry were homogeneous and isotropic, and that they had the same elastic constants. The measuring photographic negatives did not include the higher parts of the sections although these were treated at a later date and disclosed n_y-curves similar to Line 3-3, Fig. 9. These tests did not indicate tensions in the upper parts of the dam, and Mr. Wilson and the writer were led to the conclusion that the tensions at the up-stream heel observed were due to the influence of the restricted connection to the great mass of the rock under the bed of the reservoir and the water pressures upon it. The tensions observed were quite local. These general results, although of considerable importance in dam design, except above the 40% level, appear to be beyond the problem dealt with by Mr. Jakobsen, which relates to a simple triangular section in contact with a rigid base at infinity.

---

[55] *Engineering*, August 4, 1905.

In interpreting his results, Mr. Jakobsen has introduced in addition to the conditions of static equilibrium, the requirement that the internal work done by the applied water and the masonry loadings shall be a minimum and this condition replaces the assumption of straight-line stress. The work has involved considerable analysis and is a very valuable contribution to this most important problem. Professor Cain has checked the author's work, and it may be regarded as accurate within the assumptions adopted.

An examination of the method, in conjunction with the English tests, indicates that the results should apply to the upper parts of a dam, not too near the rectangular top. The results, however, appear more nearly to fit the base conditions only as shown by the English tests, but the dropping of the n_y-curve at the up-stream heel is not nearly so local. This result is to be expected owing to the restrictions at the bases which, although local in the models, appear to be carried up the entire structure in Mr. Jakobsen's calculations. Thus, it must be admitted that there are serious differences between the author's analysis and the English tests, and as Mr. Jakobsen's method is an exceedingly attractive one, it would be interesting to find the cause of the differences—whether they are fundamental or due to some elusive further necessary consideration, or to over-restriction. His close association with the models leads the writer to the presumption that the conditions (both internal and external) applied to them do not nearly correspond with those assumed by the author in his calculations and, therefore, the results must be different.

The most noticeable feature of Mr. Jakobsen's treatment is his assumption of a rigid base and possibly an equivalent at higher levels. In all probability this is the cause of the excessive tension indicated and would be very serious indeed, if true, in actual gravity dams. In all the model sections, lines originally horizontal and straight move and tip down stream and also sag, and the change is such as to relieve the possible tension and to reduce maxima of either kind. Fig. 13 shows the strain picture of the English tests traced from photographs corresponding to the author's Fig. 9. The application of a straight-edge to the horizontal lines will show the sag mentioned, and it will be noticed that considerable external work must have been done by the section, which somehow must come into any calculation of least work under the model conditions. Thus, the old conclusion is reached, namely, that when the conditions at the base are unknown both as to stress, strain, and work, no calculation is satisfactory, and the assumption of a rigid base or horizontal plane is not much nearer satisfaction than the classical assumption of the straight-line law and may lead to greater errors.

In any actual dam the supports go down into the body of the earth and extraordinary large displacements, including settlements, are only prevented by the unstrained banks at the ends of the dam. Except in very narrow gorges, where gravity sections are now seldom or ever adopted, the elastic bases may be more free than is provided in the models.

The author's reference to Maupertuis is interesting. The application of the principle of least work in this case so far seems to lead in the opposite direction to that indicated by Maupertuis. Its application to gravity dams leads not to a beneficient providential situation, but to destructive tensions in structures unable to resist them. It causes them to fail, releasing great bodies of water and spreading death and destruction over extensive territories. In this case, however, it is more providential to know the truth and then meet it than to carry on without the knowledge.

It seems in all probability that the principle of least work does actually apply to this most interesting problem and that the author has secured a solution for one over-restricted case; but the actual application of it in full to the general problem is beset with difficulties which may prove insuperable. It might be that the author could re-state his problem by assuming the strains at the base of the structure to be proportional to the stresses determined by his present results regarded as a first approximation, and then re-calculate the amount of least work, and so on, successively, until a permanent form had been established. Of course, any assumption that straight lines in the dam before loading remain straight lines after loading is not permissible. Fig. 14 shows some further results from the English tests on the model in Fig. 9; it relates only to the base line, 6-6. It should be noted that the departures from the straight line of the n_y-curves for reservoir empty and reservoir full are approximately reversed.

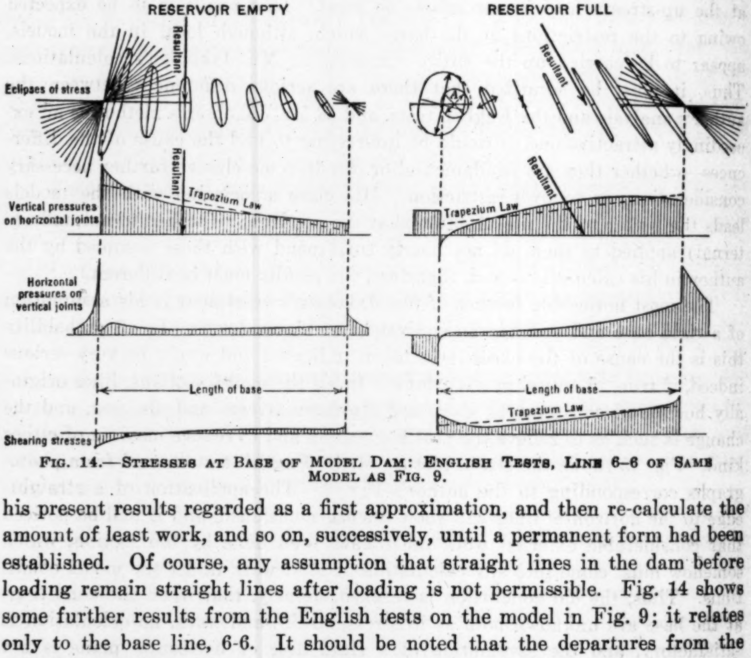

RESERVOIR EMPTY
RESERVOIR FULL
Resultant
Resultant
Resultant
Resultant
Eclipses of stress
Vertical pressures on horizontal joints
Trapezium Law
Trapezium Law
Horizontal pressures on vertical joints
Length of base
Length of base
Trapezium Law
Shearing stresses

FIG. 14.—STRESSES AT BASE OF MODEL DAM: ENGLISH TESTS, LINE 6–6 OF SAME MODEL AS FIG. 9.

L. J. MENSCH,[56] M. AM. SOC. C. E. (by letter).—The title of this paper is a misnomer. The author confounds the Theory of Least Work of "Elasticity" with the metaphysical conception of the philosophers of the Eighteenth Century. Castigliano's theory is only a kind of shorthand notation of the general laws of an elastic body in equilibrium, and the equation of work on which it is based is practically identical with the equations expressing the principle of virtual work, the principle of potential energy, Maxwell's equation, or the many other methods which have been given names in the Nineteenth Century for the solution of elastic problems. This fact was emphasized by the writer on a previous occasion.[57] At that time the writer pointed out that tests to destruction do not agree with the result of the mathematical theory of elasticity, often not within 50%, because the latter is based on the assumption that Hooke's law holds and neglects the inelastic actions and internal frictions. The formula for the work of strain, as given by Equation (14), is of a highly speculative nature, and really satisfactory results (within 25 to 50% of tests to destruction) have been obtained only where stresses are low, or where they are of preponderant influence in one direction.

---

[56] Gen. Contr., Chicago, Ill.

Consider, for example, the case of a column under concentric axial load. The work performed by the load equals load times shortening of the column and this must be equal to the sum of the work performed by all interior particles, which are assumed to be uniformly loaded and strained, during their shortenings. The modulus of elasticity in compression is derived from this point of view; but according to the mathematical theory of elasticity there are a great many other stresses in such a simple structure. All textbooks in common use show that shearing stresses are acting in such a column in sections inclined to the axis and that these shearing stresses, disregarding internal friction, are a maximum for an inclination of 45°, when they are equal to one-half the axial stresses. The simple method of computing the internal work is incorrect, and the actual work of the axial stresses is only a part of the entire internal work. No way has been found, since the inception of the mathematical theory of elasticity, to verify experimentally the assumed distribution of work performed on each particle in the various directions of strain. For a proper solution it will probably be necessary to wait until the ultimate constitution of matter has been determined.

The novelty introduced by the author is a non-linear stress distribution in a horizontal section of a dam, the distribution following a curve of the fourth order, given by Equation (15). He has not tried to show that this distribution of stresses will really produce the least amount of work compatible with the loading conditions; he has only calculated the coefficients of this particular curve to satisfy the hypothetical Equation (14). Other writers have assumed a parabolic stress distribution; and the writer has tried a cubic parabola[58] to which he was led by comparing the results of tests to destruction on a large number of eccentrically loaded columns.

The writer finds it strange that the author uses the stresses in a horizontal section, which is inclined to the neutral axis of the dam, in such a delicate operation as the solution of Equation (14). From experiments with wedge-shaped beams of cast iron and reinforced concrete the writer knows that the line of failure follows somewhat the line, *AO′B*, in Fig. 15, which for convenience of calculation may be replaced by the dotted line, *A′ B′*. An investigation of this section by Mr. Jakobsen with a stress distribution similar to Equation (15) would have shown the point of zero stress a great deal farther down stream than for a horizontal section, namely, at N instead of n in Fig. 15.

---

[57] "On a New Principle in the Theory of Structures," by the late George F. Swain, Past-President, and Hon. M. Am. Soc. C. E., *Transactions*, Am. Soc. C. E., Vol. LXXXIII (1919-20), p. 654.

[58] *Transactions*, Am. Soc. C. E., Vol. LXXXIII (1919-20), p. 1667; also, Vol. 93 (1929), p. 1250.

The author finds tension at the up-stream face and does not mention that such tension cannot exist in the joints of a non-monolithic structure. Sometimes it is permitted to assume tension in the design of brick masonry, but this is done for the following reason: When brick smokestacks were first designed on scientific principles the calculations required an undue thickness of wall to prevent tension, which compared unfavorably with the work of ordinary masonry contractors who had built satisfactory stacks for a long time. In order not to be in contradiction with good practice, the custom of allowing tension in the statical calculation was developed, although the originators of this practice knew that such tensions are only imaginary. The straight-line theory was at fault.

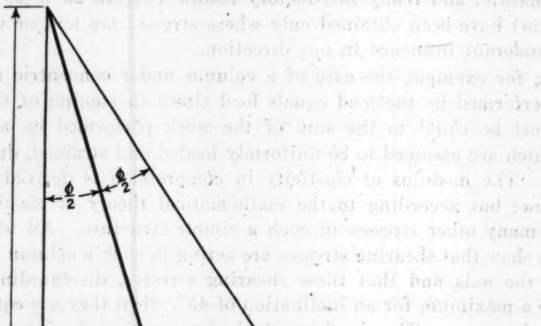


FIG. 15.

The author has endeavored to correct the errors of the straight-line theory, but still persists in the error of assuming imaginary tensions. Tension in a joint indicates the opening of a crack which will have a width at the face of about 0.01 in. when the joints are about 10 ft. apart vertically. What will happen to the dam when the intruding water exerts a pressure in the joint? The great dams constructed by the New York Board of Water Supply have just such a section, and a discovery such as that claimed by the author might cause many sleepless nights to the engineers in charge. While the writer is quite convinced that such water pressure is exerted in the joints of many dams, he quite admires the judgment of the New York engineers who adopted this profile. It is entirely safe although it has been criticized lately by a number of engineers. Water pressure in joints opened up by the deformation of properly constructed dams is not dangerous because the joint just above it also exerts a downward pressure which is only slightly less than the one below it and the difference of pressure does not produce a great change in the stresses. The writer has shown[58] that a stress distribution according to a cubic parabola leads to reliable and easily computed results.

Fig. 16 shows the properties of such a parabola and G denotes the center of gravity of all stresses. In a horizontal section such as that chosen by the author, the point, G', in Fig. 16, must be at the intersection of the resultant of all forces with the line, $A\,C$, of Fig. 15. The distance of the point of zero stress from the down-stream face will be $2.5 \times \frac{b}{3} = 0.833\, b$, provided it is at all permitted to assume such a stress distribution in a horizontal section. Since the vertical load at G' in Fig. 15 is $\frac{1}{2}\, w\, y\, b$, the maximum vertical stress, in pounds per square foot, is, $\dfrac{\frac{1}{2}\, w\, y\, b}{\frac{3}{4} \times (0.833\, b)} = 0.8\, w\, y$. By a very laborious calculation the author obtains $0.775\, w\, y$; the straight-line theory gives $w\, y$ as maximum vertical stress. An investigation of the section, $AO'B$ (Fig. 15), shows that the stresses parallel to the down-stream face are about $1.17\, w\, y$, in which, y is the depth below the top at Point B; but the point of zero stress is now at N. The failure of the Bouzey Dam, in France, in 1895 shows a line of failure following the line, AND, in Fig. 15. The writer conceives this failure as follows: After tension, in a section, $AO'B$, developed to a point, N, the shear in the line, $N\,D$, became greater than the leached-out mortar of the masonry could stand and the dam sheared off. The shear is a maximum along a line inclined nearly 45° with the principal stress parallel with the down-stream face, and thus the line, $N\,D$, was located. Hence, shear failures are very dangerous to masonry dams as commonly built, or to concrete dams with improper horizontal and vertical joints. These shearing stresses are probably the dangerous ones in high dams of non-uplift section. For dams 700 ft. high they might reach values of 500 lb. per sq. in.

When a dam is sheared off at its foundation, or at higher elevations, it will have to move quite a distance before it topples over; consequently, an arched gravity dam must be safer than a straight dam. Most engineers deny the help given to a gravity dam by arch action, because the deflection of the dam as a cantilever is too small to allow of arch action. If, however, through some error, dangerous shearing stresses should develop, the movement will be large enough to close vertical cracks and open expansion joints, and arch action will prevent complete failure. The danger to a dam without an uplift section is in the foundation. When uplift can be exerted at the base of the dam, that is much greater than the pressure in the joints above it, due to the rock being weakened by the intruding water, as in the St. Francis Dam, or by fissures in the rock below the dam, then failure is likely to occur sooner or later, and for such doubtful foundation conditions a much larger section of dam must be adopted or resort must be had to deep anchorage into the rock near the up-stream face.

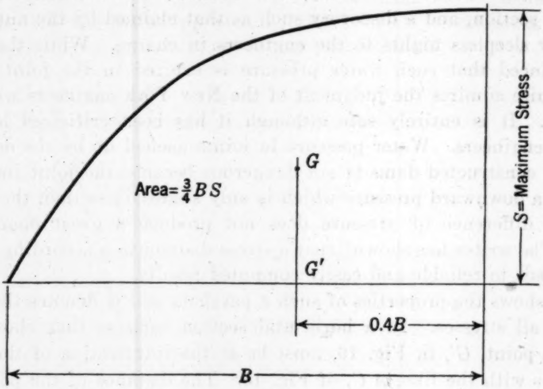


FIG. 16.

LARS R. JORGENSEN,[59] M. AM. SOC. C. E. (by letter).—That the stress distribution in the cross-section of a gravity dam is such as will make the internal work the least possible, is undoubtedly the best assumption. Mr. Jakobsen has made a solution of the problem which is a decided step forward in dam engineering. This paper takes the same position for gravity dams as the work of the late Professor Cain does for arch dams.

The curved shape of the stress distribution diagram appeals to the instinct as being more correct than the linear. The inside of a great body should exert greater resistance than the edges. The curved shape of the stress distribution made by the author checks the English tests; there is considerable satisfaction in this. The material in the English tests evidently had a constant value of E whether wet or dry.

---

[59] Cons. Hydro-Elec. Engr., Berkeley, Calif.

The writer has come to the conclusion that for tests, the model ought to be of the same material as the dam itself. Even then it is difficult to reproduce equality. One reason is that the modulus of elasticity for concrete is extremely variable, between the wet and the dry faces, for sustained load. On a straight gravity dam this fact may not have much influence on the stress distribution, but it will have great influence on the deflection in a down-stream direction.

Mr. Jakobsen refers to tests made by R. E. Davis, M. Am. Soc. C. E., in connection with the concrete in the Stevenson Creek Dam. The writer regards these tests as most valuable, since they are made with dam construction in view. Professor Davis finds the instantaneous deformation and the plastic flow after, say, 300 days of loading, the two deformations are added together and give an expression for the working modulus of elasticity of the material for sustained load. The interesting and valuable part of the tests is that it is shown that E for sustained load is from 60 to 63% higher for wet than for dry concrete for stresses between 600 and 900 lb. per sq. in.

If the dam body is either entirely dry or entirely water-soaked, the working value of E will be constant to all intents and purposes for the entire dam body, and the stress distribution on the horizontal plane will be, as found by Mr. Jakobsen, in accordance with the theory on the principle of least internal work.

For arch dams the worst case and the most plausible one (and, therefore, the one to reckon with) is when the concrete is wet on the up-stream side and dry on the down-stream side, with a more or less uniform distribution between the two faces, then the value of E of the material is approximately 60% higher along the up-stream face than along the down-stream face.

This large variation in the working value of the modulus of elasticity between the wet and the dry faces will materially affect the stress distribution. It will cause the arch stresses to move in an up-stream direction, so to speak, in the horizontal arch section, relieving the maximum compressive stress at the abutments, where the compression is greatest. The water-soaking effect will relieve the tensile stresses, if any, at the abutments, due to the expansion of the up-stream face relative to the down-stream face.

The change in stress distribution does not alter the arch deflection appreciably, whereas on a straight gravity dam a variable working value of E may not change the stress distribution appreciably, but will materially alter the deflection of the dam in a down-stream direction.

As indicated previously, the assumption of a more or less uniform distribution of the water-soaking effect between the up-stream and down-stream faces is most plausible. This is in line with the ordinary assumption made for the action of uplift.

When computing maximum arch stresses in accordance with Professor Cain's formulas, it seems best to neglect any help as to better distribution obtainable from the water-soaking effect and the variation in the value of E between the two faces for sustained load, since their values can not be definitely measured; but if that is done, there is no reason why the maximum compression could not be run up to, say, 1 000 lb. per sq. in., provided the engineer insisted upon a 28-day strength of the concrete of 3 000 lb. per sq. in. The 1-

year strength of this concrete would be about 5 000 lb. per sq. in. and, therefore, the load stress would be carried with a margin of five.

It is not possible to design really high dams economically using maximum stresses as low as, or lower than, 600 lb. per sq. in., and since the maximum stresses only exist in parts of the dam, it should be entirely practical to make these parts of a better grade of concrete. Such parts would be near the abutments and near the bottom of high dams. Since E is not constant anyway and is least on the down-stream side, it seems perfectly logical to introduce a higher value where this is mostly effective in the form of richer concrete. With good material and good workmanship a 28-day strength of 3 000 lb. per sq. in. can be obtained in most cases, with but little more than 1 bbl. of cement per yd. of concrete.

The concrete in the Diablo Dam, on the Skagit River, in Washington, has a 28-day strength of 4 200 lb. per sq. in. This strength was insisted on, and it took 5 sacks of cement per cu. yd. of concrete to produce it. In the writer's opinion this strength was unnecessarily high since the maximum stresses in the dam were moderate, but the engineers for the City of Seattle decided it was worth while. Concrete with a 28-day strength of 3 000 lb. per sq. in. can be depended upon for good lasting qualities; high dams should be built of such material only. Such dams would have an ample safety factor even with an apparent compression of 1 000 lb. per sq. in. in some of their parts.

With Mr. Jakobsen's assumptions of $k = 0.645$ at 100 ft. below the crest and no uplift, it is apparent from Table 1 and Fig. 4, that in order to eliminate tension in the up-stream face entirely, k would have to be increased to about 0.70. This is of rather great importance. Since uplift is an unknown factor and since the most positive way of taking care of it is by adding to the weight of the dam, to the base width, or to both, a further change of k would be necessary.

The writer feels that nothing less than $k = 0.75$ should be adopted for a triangular section of a gravity dam. For very high dams, and for dams in which it is necessary to assume considerable uplift pressure, k will be still greater, but a minimum of $k = 0.75$ ought to be established by the Engineering Profession, for dams of moderate height on good foundations. Mr. Jakobsen has shown that for full uplift it is necessary to make $k = 0.844$.

C. A. P. Turner,[60] M. Am. Soc. C. E. (by letter).—Mr. Jakobsen's effort to analyze the stresses in gravity dams seems predicated upon fundamental misconceptions of the ordinary theory of flexure. For example, he refers to the ordinary theory of flexure as "classical". The customary lack of precision in stating the assumptions on which this theory is based combined with a total disregard of elementary principles by textbook writers in its elucidation, seems to predicate an antiquity of thousands of years, although its origin in fact dates back less than two and one-half centuries.

The earliest effort to investigate the resistance of beams is found in Galileo's "Discoursi e Dimostrazioni Mathematiche" published in 1638. Erroneously Galileo assumed stresses of one kind only, with an axis of rotation at the bottom of the cantilever beam considered. In 1686, Mariotte published results of tests showing that Galileo's theory was not in accord with experiment. He noted that while part of the fibers are stretched, others are compressed, and assumed without proof ("on peut concevoir") that one-half the fibers are stretched and one-half are compressed. Discussing the tests of Mariotte, Leibnitz concluded that the resistance of the fibers was in proportion to their extension.

---

[60] Cons. Engr., Minneapolis, Minn.

The further development of this theory embodying the hypothesis that the extension of the fibers is in proportion to the bending moment and to their distance from the center of gravity of the section as well is the Bernoulli-Eulerian theory or the common but inaccurate theory of flexure taught in engineering schools to-day. The assumptions involved for a rectangular section are:

1.—As many fibers are stretched as compressed.
2.—The neutral plane is the gravity axis of the section.
3.—A plane section normal to the gravity axis before bending remains normal to this axis after bending.
4.—A plane section normal to the gravity axis before bending remains a plane section after bending.
5.—The material of the beam obeys Hooke's law that strain is proportional to stress.

Internal horizontal shearing stress is correctly treated as the reaction to the summation of the horizontal fiber stresses. The center of action of the fiber stresses is 0.7071 times the half depth above the gravity axis and that of the half parabolic shear stress area is approximately 0.348 times the half depth above the same axis. Hence, the summation of a triangular fiber stress area results in a parabolic shear stress area and the neutral plane must be lowered 0.179 times the half depth below the gravity axis at the end of the beam, so that the reaction of the shearing stresses may be opposite the action of the fiber stresses, as required by Newton's third law.

Therefore, there are, in fact, more fibers compressed than are stretched in a simple beam, and the linear law of fiber stress intensity in the tension zone differs from that in the compression zone except at sections of zero internal shear strain. Since the positive shear in the compression zone at the neutral plane must equal the negative shear in the tension zone, the rate of increase of the fiber stress must be greater in the tension zone than in the compression zone except at that normal section at which the true neutral plane is tangent to the gravity axis of the beam, namely, the center of a symmetrically loaded simple beam and at the support of a cantilever.

Weisbach in his "Mechanics of Engineering", appears to have been the first to note that equilibrium of the shearing stresses must be taken into account in the problem of determining the position of the neutral plane, but he failed to solve the problem.

Assumptions (3) and (4) of the ordinary theory are inconsistent with and contradictory of Assumption (5). They present the grotesque presumption that the shearing modulus may be treated as infinitely great in lieu of its normal finite value of $\frac{3}{8}E$ for homogeneous materials.

Assumption (4) is largely innocuous; thus a normal plane before bending remains substantially plane after bending in the web of a wide flanged I-beam, yet the anti-clockwise rotation of the plane from the normal renders the error in computing the elastic deflection by the ordinary theory equal to from 15% for ordinary spans to 50%, dependent upon the relative length to depth of the beam considered.[61] By disregarding shear strain the deflections of simple beams are computed as less, and those of continuous beams as greater, than actually occurs.

For the development of the exact theory of flexure it is necessary only to assume that the material obeys Hooke's law. The linear law of fiber stress distribution upon the normal central cross-section of a symmetrically loaded simple beam and a like section at the supports of a cantilever follows inevitably from this hypothesis.

For equilibrium under internal shear the sum of the compressive fiber stresses parallel to the neutral plane must equal the sum of the tensile stresses parallel to the same plane acting upon any section normal to the neutral plane along x. Because these positive and negative forces form a couple, the moment of which is the same about any point in its plane of action, the hypothesis of a neutral plane at the gravity axis is unnecessary in developing exact summation equations of flexure.

Let the tensions and compressions parallel to the neutral plane be defined as moment stresses. Then, for a simply supported uniformly loaded beam, if w equals the uniform load per unit of length; S, the external shear force at any section; M, the applied moment; a', the slope; and D, the deflection caused by moment stresses: $S = \Sigma\, w$; $M = \Sigma\, S$; $a' = \sum \frac{M}{E\,I}$; and $D = \Sigma\, a$. The moments, slopes, and deflections of the ordinary theory are then determined at once by simple arithmetic, noting the geometrical relation that a parabolic moment area is two-thirds of its rectangle and that the third degree slope area is five-eighths of its rectangle in the computation.

It appears therefore that the ordinary theory of flexure is an exact analysis of the slopes and deflections accruing from moment-stresses, that is, those parallel to the true neutral plane. Such stresses in a horizontal beam cause a clockwise rotation with respect to the vertical of every plane section normal to the true neutral axis when looking toward the left side of the simply supported beam. Shear strain, on the contrary, gives rise to anti-clockwise rotation tending to relieve the moment stresses; hence increased deflection follows until static equilibrium is restored at mid-span. Therefore, a, the true slope, is greater than $\sum \frac{M}{E\,I}$ in the simple beam or cantilever.

Because shear strain at the neutral plane is the reaction to the moment stresses and because the anti-clockwise rotation results therefrom and is limited by the relative cross-section and elastic properties of the material, the following relations may be written: True slope, a, equals moment slope, $a' \times 1 + g$; and true deflection, Δ, equals moment deflection, $D \times (1 + g)$, which constitute the only modification that need be made in the ordinary summation equations of beam theory to reduce them to values in accord with exact analysis.

---

[61] These ratios apply to steel I-beams with broad flanges.

The factor, g,[62] is a fraction that is positive for simple and cantilever beams, but negative for continuous and restrained beams. It is a constant for a given beam regardless of kind or character of loading, but varies for different beams dependent upon relative length and cross-sections of flange and web.

Shearing strains, by exact analysis, differ from those determined under the arbitrary assumptions of the common theory far more widely than the moment stresses. In the common theory of flexure the assumption that internal shear strain may be disregarded is false, because shear failure occurs in short beams and must be accounted for somehow. Accepting this assumption, textbook writers and building code authors again resort to the erroneous procedure of disregarding Newton's third law.

Although recognizing that horizontal shear strains furnish the necessary stress reactions to the positive and negative moment stresses, in order to permit their joint action as a couple, they assume, erroneously, that internal shearing stress is the reaction to external shear force because rotational strain (pure shear) presents the same rotational resistance through 360°, as shown by Cauchy; that is, horizontal shear = vertical shear at the neutral plane.

Because the horizontal shears are reactions equal to the summation of the moment stresses they cannot be equal also to the vertical external shear force since the moments under the accepted equations are equal to the summation of the shears and, therefore, cannot in general equal the external shear force at a section. To state the criticism with Euclidian finality, "two things which are not equal to the same thing cannot be equal to each other".

A graphic diagram of an unsymmetrically loaded beam best illustrates this divergence. With a concentrated load at P, in Fig. 17, distant $0.3\,L$ from the left support, the external shear passes through zero and changes sign under the load, P, and there the moment is a maximum; but at AA, a plane which divides the moment area in equal parts, internal shear strain must pass through zero where the bending moment is not a maximum and the external shear force is not zero.

Because the moment area to the left of AA equals the moment area to the right, the summation of bending to the right and left of AA must be equal as action and reaction and the shear stress parallel to the neutral plane is zero at AA and sums to the right and left toward the end of the beams. Likewise in a horizontal cantilever the summation of the horizontal shear stresses commences from zero at the support, increasing toward the free end in a manner analogous to the summation from the mid-span toward the end of the simply supported beam.

In a cantilever, horizontal shear strain passes through zero at the neutral surface at the support and the external shear force is a maximum at this point.

---

[62] The computation of g is given in Section II, "Elasticity," pp. 16 to 21.

On the contrary, in a simply supported beam the internal shear strain at the support is a maximum and the external shear force is also a maximum. Therefore, the simply supported beam is capable of resisting much smaller unit shear forces per unit of cross-section area than the cantilever, the continuous, or the restrained beam of the same cross-section. In the case of the cantilever or a continuous beam, where the internal shear at the neutral axis is zero, the external shear force must be resisted by the components of the principal stresses and the diagonal tensions and compressions due to the sharpness of curvature.

Mr. Jakobsen's painstaking investigation proceeds on the common but erroneous practice of confusing external shear force with internal shear strain.

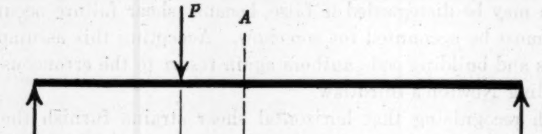


FIG. 17.

Unfortunately, therefore, it is founded on the theoretical "quicksand" of the differential calculus of assumptions and the integral calculus of presumptions rather than on the bed-rock of fundamental principles and the sound logic of the "Principia".

The linear law of fiber stress intensity at a section normal to the neutral plane is an inescapable consequence of the assumption of homogeneous elastic material subject to Hooke's law. Any supposed proof to the contrary indicates error. In the case of Mr. Jakobsen the error is embodied in the misapplication of both the common theory and exact theory of flexure to the triangular dam. The linear law applies to a section normal to the neutral plane which, at the base, coincides with the gravity center of the mass. By erroneously assuming a horizontal plane diagonal to the gravity axis the designer concludes that fiber stresses of flexure are much less than their maximum intensity; and the linear law of fiber stress intensity does not apply, and is not presumed to apply, to a section diagonal to the normal under the ordinary theory.

From the preceding discussion it appears that whether the exact theory of flexure or the common approximate theory is applied, the maximum bending stresses in the triangular dam will be determined to be of like magnitude.

The first step is to locate the critical section. The gravity axis of the triangular mass makes an angle, $\frac{1}{2}\phi$, with the vertical. The critical section inclines upward from the bed at the base of the dam at an angle of $\frac{1}{2}\phi$ to the horizontal. Obviously, this resisting section is smaller in area than the horizontal section considered by Mr. Jakobsen; the lever arm of the dead weight is less and the maximum stresses will be greater than those erroneously computed on the assumption that the critical section is horizontal and diagonal to the gravity axis. The variation in stress must follow the linear law on the critical section if the material is elastic and obeys Hooke's law. Its rate of variation on the diagonal section is an academic matter without practical interest to the busy constructor.

In attempting to point out some of the misconceptions and erroneous assumptions presented in textbook theory, it will be well to call attention to the general misunderstanding of the exact nature of Poisson's ratio.

The lateral contraction of elastic material under longitudinal strain is brought about by the inclination of the planes of greatest shear to the axial direction of the stress. This inclination is a constant at 45° in all cases of homogeneous strain, but it differs from 45° in all cases of heterogeneous strain. Therefore, $\frac{1}{m}$ for a case of heterogeneous strain is different and distinct from $\frac{1}{m}$ in the case of homogeneous strain, a fundamental relation which is lost sight of in Mr. Jakobsen's discussion.

The determinate structure is one in which the reactions are determined by the law of gravitation and the principle of the simple lever; hence the deflection and work of deformation are the greatest that the resistance of the material permits under the principle of the lever.

The indeterminate structure, on the contrary, is governed by the principle of rigidities, which asserts that where there are different paths by which the load may travel to the support, it divides itself between these several paths strictly in proportion to the respective rigidities, and when so divided the work of deformation will be less than with any other or different division. Thus arises the so-called "law of least work."

The old preparatory school algebras taught that if a function of the two variables is plotted when the value of Y along X is a maximum, the tangent to the curve becomes parallel to the axis of abscissas; that is, its angle thereto equals zero and its tangent, $\frac{dy}{dx}$, is also zero. In like manner where the value of Y along X is a minimum the tangent to the curve is likewise parallel to the axis of abscissas and its angle therewith passes through zero.

These relations comprise the old-fashioned criterion for maximum and minimum values. Men of the modernistic school and textbook writers insist upon treating this old and familiar principle in the theory of work as the derivative of the work function and render it so mysterious that maximum values are confused with minimum, and the precise mathematical distinction between the determinate and indeterminate structure is lost sight of, as appears in Mr. Jakobsen's discussion.

Thus, at the outset, Mr. Jakobsen correctly classifies the section of the gravity dam as determinate under the law of gravitation and principle of the lever. Then he proceeds to derive equations of resistance upon this basis and, finally, assumes that they may be inserted in his work equations and solved by the "principle of least work", thus confusing the diverse characteristics of the determinate and indeterminate structure.

If gravity dams are to be built, the maximum stresses at the critical section should be computed with a reasonable or practical degree of accuracy. This cannot be done if sections other than the critical are selected. Taking a horizontal section of the dam, when the normal to the neutral plane makes an angle thereto, results in a moment of inertia far greater than at the critical section; so that there is a large error on the unsafe side in the computation so made; and this, according to Mr. Jakobsen, is the ordinary but inaccurate procedure of hydraulic engineers in general.

The writer's opinion of the gravity dam is that it may be regarded as reasonably economical where the head of water to be impounded does not much exceed 15 or 20 ft. For a higher dam the gravity type which has been handed down from an ancient period is highly uneconomical. It lacks the fundamental characteristics that are desirable from the standpoint of safety. Thus, with the gravity type, as the head increases the over-turning resistance decreases, but with a properly designed reinforced concrete dam having an inclined up-stream face, on the contrary, as the pressure increases the overturning resistance by that pressure increases in like ratio, and the rational elements of stability are combined with greater economy. Why hold on to that which is obsolete instead of that which is up to date and modern? This is a question to which hydraulic engineers should give consideration in the light of the wastefulness and reduced safety in their customary manner of building.

DONALD P. BARNES,[63] JUN. AM. SOC. C. E. (by letter).—The accumulated evidence that the assumption of linear stress distribution is in error may be regarded as conclusive. Mr. Jakobsen's analysis is theoretical proof for the case of a dam on a rigid foundation, and the various experimental results cited by him are factual evidence of the inaccuracy of the old theory. That this inaccuracy is appreciable, and that the real maximum stresses are materially greater than indicated by the straight-line theory are likewise justifiable inferences. There remains, however, considerable uncertainty as to the closeness with which the results obtained by Mr. Jakobsen's method approximate the true stresses.

---

[63] Instr., Civ. Eng. Dept., Missouri School of Mines and Metallurgy, Univ. of Missouri, Rolla, Mo.

This uncertainty arises, perhaps primarily from the assumption of a rigid foundation. The author has himself observed in the closing paragraph of Appendix III that no account is taken of irregularities of the base, and also that "* * * as Dr. Vogt has shown, the foundation does not remain plane when the stresses are linear." The philosophy of the principle of least work is probably entirely acceptable, but the statement of its application should be the condition that the sum of the internal work of the dam and of the foundation shall be a minimum. It does not follow that the work of either, alone, would be minimum.

The inclusion of the internal work of the foundation may be an insurmountable mathematical obstacle. Its neglect, however, would seem to render the utility of Mr. Jakobsen's results largely dependent upon experimental confirmation. In this connection, the results of a physical investigation recently completed on a celluloid model may be of interest.[64]

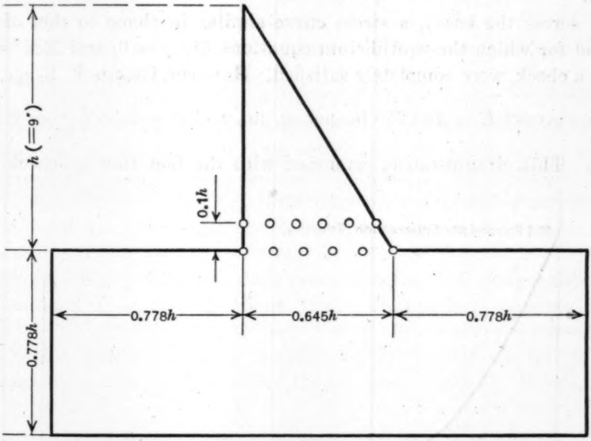


FIG. 18.—SHAPE OF MODEL AND POSITION OF POINTS THE DISPLACEMENTS OF WHICH WERE MEASURED.

The model was cut in the proportions shown in Fig. 18, the dam portion being exactly the same shape as the first profile analyzed by Mr. Jakobsen, but cut in one piece with the base, in order to include the effect of the elasticity of the foundation as nearly as practicable. Loads graduated to simulate water pressure were applied to the the up-stream face. The weight of the structure itself (gravity load) was neglected, as was also the vertical water pressure on that part of the foundation under the reservoir. The condition was that discussed by Mr. Jakobsen in Article VIII.

---

[64] "On Stresses in Gravity Dams," Thesis presented at California Inst. of Technology, June, 1930.

The vertical components of the displacements of the points shown were measured with micrometer microscopes. These distances were plotted to an exaggerated vertical scale and curves drawn through the points. The curve of the upper row of points was extrapolated at the down-stream end. Differences between ordinates to points lying in the same vertical line on the two curves, divided by the interval between the rows, were assumed to equal the unit vertical deformations, δ, of corresponding points on the intermediate section (one-twentieth of the height from the base).

From the application of Hooke's law, considering Poisson's ratio,

$$\delta = \frac{n_y}{E} - \frac{n_x}{E}\,\frac{1}{m} \quad \dots\dots\dots\dots\dots\dots\dots\dots (46)$$

in which, n_y, n_x, E, and $\frac{1}{m}$ are the vertical and horizontal unit stresses, the modulus of elasticity, and Poisson's ratio, respectively, for celluloid. From Equation (46) and the measured values of δ, assuming $E = 295\,000$ lb. per sq. in., $\frac{1}{m} = 0.32$, and $n_x = 58$ lb. per sq. in. (approximate average, assumed constant across the base), a stress curve similar in shape to that of Fig. 19 was found for which the equilibrium equations, $\Sigma\, n_y = 0$, and $\Sigma\, M = 0$, constituting a check, were completely satisfied. However, George E. Beggs, M. Am. Soc. C. E., gives[65] $E = 263\,750$ lb. per sq. in., and $m = 2.38$ $\left(\frac{1}{m} = 0.42\right)$ for celluloid. This circumstance, combined with the fact that no check of these values was made for the model in question, detracts somewhat from the value of the check noted.

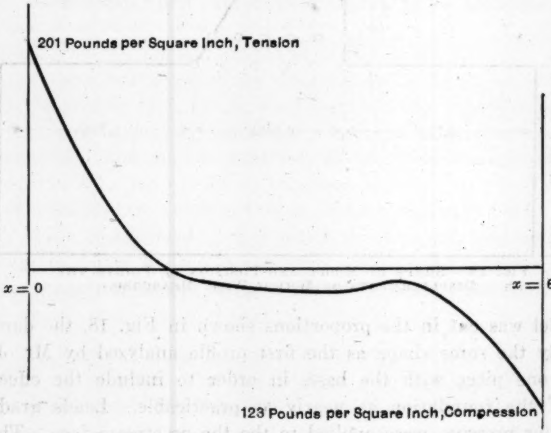


FIG. 19.—BENDING STRESSES AS EXPERIMENTALLY DETERMINED.

---

[65] Report on Arch Dam Investigation, *Proceedings*, Am. Soc. C. E., Pt. 3, May, 1928, p. 219.

The stresses in the hypothetical concrete dam were obtained from the stresses in the celluloid by applying the constant ratio, 0.651, obtained from the relations of the dimensions and loads of the dam and model. Fig. 19 is the curve of these stresses.

For this case of horizontal loads only, it will be seen that the experiment referred to, indicates a maximum tensile stress about twice as great as that found by the straight-line theory and a compressive stress about 25% greater. Mr. Jakobsen's analysis gives a tensile stress about 43% greater and a compressive stress about 20% less than those found by the linear theory.

It does not appear that the experimental confirmation of Mr. Jakobsen's method at present available is sufficient to warrant its use in practical design, particularly inasmuch as its fundamental assumption, that of the principle of least work, is not so applied as to include the effect of the foundation. His paper is, however, a strong argument for the use of much greater conservatism in the selection of a profile than is the present practice.

ANTHONY HOADLEY,[66] ASSOC. M. AM. SOC. C. E. (by letter).—The author has thrown considerable doubt on the validity of the "middle-third" rule which has been widely used in the design of dams. The results of his theoretical investigations form another piece of evidence, in addition to several others which have been brought sharply to the engineer's attention in the last few years, calling for a conservative viewpoint in the design of gravity dams. In order to obtain a workable set of equations it has been necessary to set up a formidable array of restrictions. These restrictions are analogous to those used in developing the theory of stress distribution in reinforced concrete. Although the actual stress distribution in either dam or reinforced concrete member will differ from the theoretical distribution, it seems to the writer that the theory is still a useful tool to the designer.

The theory developed by Mr. Jakobsen indicates that many existing dams must be in tension at the up-stream face. That this is possible has been indicated[67] by Searcy B. Slack, M. Am. Soc. C. E., in reporting strain measurements which indicate stresses as high as 250 lb. per sq. in. in tension. In commenting on the Yadkin River Bridge tests, Clyde T. Morris, M. Am. Soc. C. E., states,[68]

> "2.—The calculated compressive stress at any section of the rib checks the observed stress more closely when it is assumed that the concrete takes tension, than when it is assumed that the concrete is not active in resisting tensile stresses, even when high tensile stresses have caused cracks in the concrete."

How much stress a horizontal construction joint (which destroys the homogeneity of the mass, and, consequently, limits the applicability of Mr. Jakobsen's equations) will carry, is another question.

The use of a factor of safety of one in the design of dams seems to the writer to be unsafe. He is glad to see a theory presented, which calls for increasing the base width. In 1928, the writer was doing some experimental work in which a silk line (50-lb. test) was used to carry the load. The consequences of a break in the line were not severe. It meant merely an half hour spent in putting in a new piece of line plus the possibility of a cut or bruised hand; but even at that, a very uncomfortable feeling was produced each time the load approached the 50-lb. mark, which would give a "factor of safety" of one. Surely the designer of a dam should protect himself from that "uncomfortable feeling"—or worse—by using a factor of safety materially greater than one. It may be argued that the loads acting on a dam are static and definitely known, but this does not seem to be borne out by evidence produced in two papers published by the Society.[69]

---

[66] Asst. Prof., Civ. Eng., Union Coll., Schenectady, N. Y.

[67] "Behavior of an Arch During Construction", *Proceedings*, Am. Soc. C. E., November, 1929, Papers and Discussions, pp. 2291–2293.

[68] *Proceedings*, Am. Soc. C. E., March, 1929, Papers and Discussions, p. 611.

It is very interesting to notice the slight change in stress-distribution (Table 9) caused by assuming widely different values for Poisson's ratio. The writer wonders what the effect would be if Poisson's ratio varied from one section of the dam to another.

The writer found that, according to the theory developed by Mr. Jakobsen, the width of the base of a dam of triangular section should be 0.82 times the height if the tension is to be avoided at the heel; or, expressed in another way, the resultant force should not fall on the down-stream side of the middle thirteenth of the base. This section closely approximates that suggested by Maurice Levy. Equations (16), (17), (25), and (26) were solved for the constants A', B', C', and F', with the results shown in Table 13.

TABLE 13.—VALUES OF CONSTANTS FOR VARIOUS VALUES OF k.

($g = 62.5$; $w = 150.2$; and $m = 8$)

Constants.	$k = 0{\cdot}70$.	$k = 0{\cdot}75$.	$k = 0{\cdot}80$.
A'	0·22617	0.11718	0.02993
B'	−4.9998	−4.4528	−4.0436
C'	8.5316	7.7501	7.1513
D'	−4.5640	−4.0027	−3·5916

Values of n_y at the up-stream face of the dam were then obtained for $k = 0.70$, 0.75, and 0.80. These values, together with those given by Mr. Jakobsen for $k = 0.645$ and 0.844, were plotted against k. This curve gave $k = 0.82$ for $n_y = 0$. The distribution of the n_y-stress was then investigated. The results, together with those obtained by Mr. Jakobsen are shown in Fig. 20.

Comparing the n_x-curve for $k = 0.645$ (Fig. 4), with the n_x-curve for $k = 0.844$ (Fig. 7), it is seen that the general shape is very closely the same. This is also true of the corresponding t stress-distribution curves. For this reason the writer has computed values of n_x and t at the toe only. These values, including those given by Mr. Jakobsen for $k = 0.645$

---

[69] "Hydrostatic Uplift in Pervious Soils", by H. de B. Parsons, M. Am. Soc. C. E., *Transactions*, Am. Soc. C. E., Vol. 93 (1929), p. 1317; and "Upward Pressure Under Dams: Experiments by the United States Bureau of Reclamation", by Julian Hinds, M. Am. Soc. C. E., *Loc. cit.*, p. 1527.

and $k = 0.844$, together with data showing the relationship between n_x n_y and t^2 are shown in Table 14.

The n_y-stresses have been obtained for a dam of triangular section which would be modified in construction by placing additional material at the top. The stress distribution in this modified section could be determined in the following manner, which is based on the fact that Mr. Jakobsen has shown

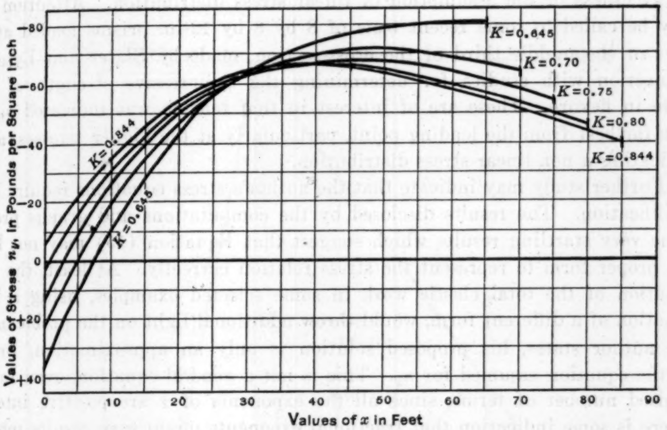


FIG. 20.—DISTRIBUTION OF STRESS, n_y, FOR SECTIONS WITH VARYING BASE WIDTH.

the stresses due to the weight of the dam alone as computed by the theory of least work to be nearly the same as those computed on the assumption of a linear stress distribution. Compute n_y by the author's method for the triangular section. The stresses due to the added mass of concrete are then computed on the straight-line, stress-distribution theory. The resultant vertical stress will be very closely the algebraic sum of these two.

TABLE 14.—STRESSES AT TOE OF DAM FOR DIFFERENT WIDTHS OF BASE.

($y = 100$ ft.; $b = 100\ k$; $g = 62.5$; $w = 150.2$; $m = 8$; and stress is in pounds per square inch.)

k.	n_y.	n_x.	t.	$n_y\ n_x$.	t^2.
0.645.........	−80.81	−33.62	−52.12	2716.0	2716.0
0.70.	−65.87	− 32.27	−46.10	2125.6	2125.2
0'75...........	−55.16	−31.03	−41.37	1711.6	1711.5
0.80..........	−46.70	−29.89	−37.35	1395.8	1395.0
0.844.........	−40.55	−28.90	−34.24	1171.9	1172.4

Mr. Jakobsen is to be congratulated on having made a valuable addition to present knowledge of the stress distribution in gravity dams.

Fred W. Ely,[70] M. Am. Soc. C. E. (by letter).—The author is to be commended for re-opening the discussion of stresses in gravity dams. By attacking the problem from a new angle he apparently has proved that the vertical pressures on a horizontal plane of a gravity dam do not vary in a linear relation. The popularity of the arch dam has withheld from the gravity dam the consideration that it merits, particularly as more than twenty years have passed since in 1909 the first English model tests suggested the incorrectness of the assumption of linear stress distribution. Attention also may be called to some recent tests of 8 by 8 by 12-in. prisms loaded at the edge of the middle third of the cross-section, made by Slater and Lyse[71] in connection with studies for determining the compressive strength of concrete in flexure. These are of interest in that tension was indicated on the face farthest from the loading point, particularly at the higher stresses, again indicating a non-linear stress distribution.

Further study may indicate that the author's stress equations require some modification. The results disclosed by the computations and graphs present some very startling results which suggest that Equation (15) may not be of the proper form to represent the stress relation correctly. At least, the computation of the total elastic work in some selected examples, using a basic equation of a different form, would throw additional light on the problem. As the author states, his proposed solution is only an approximation, limited by the equation assumed for n_y. This is not a general equation, even for the limited number of terms, since all the exponents of x are positive integers. There is some indication that fractional exponents might give results more in line with the results of theory and experiment in the flexure of beams.

Referring to Fig. 6 the shape of the curve for bending stress with $w = 0$ appears to be somewhat inconsistent with the assumption of similar elastic properties in tension and compression. This assumption would logically presuppose a point of inflection at the place where the stress passes through zero, the two ends of the curve having similar curvature with respect to the X-axis. This would be in line with the stresses in a beam after the material has reached the elastic limit[72] and also with the **S**-shaped foundation deformation mentioned by Vogt.[73] A tendency in this direction is also indicated in the plotting of the results of the English tests, as shown on Section 5-5 of Fig. 8. The apparent inconsistency of the shape of this curve as well as that disclosed between Equations (27) and (36) might easily be due to the fact that Equation (15) cannot represent the stress relation properly.

The writer has worked through several numerical examples by the author's method, which indicate that, for a triangular section of an average specific weight, a base width of approximately 81% of the height would be required to prevent tension at the heel, even disregarding uplift. If this stress distribution is correct, practically no existing gravity dam is free from tensile stresses and, in the majority of gravity dams more than 150 ft. high, the tendency toward tension would be so great that the horizontal joints must have opened at some elevation, thereby assuring uplift which would tend to extend the cracks. The freedom from excessive leakage through the drainage wells which have been placed in many high dams, casts further doubt on the correctness of this stress distribution.

---

[70] Chf. Designing Engr., Hydr. Dept., Aluminum Co. of America, Pittsburgh, Pa.

[71] "Compressive Strength of Concrete in Flexure as Determined from Tests of Reinforced Beams," by the late Willis A. Slater, M. Am. Soc. C. E., and Inge Lyse, Assoc. M. Am. Soc. C. E., *Journal*, Am. Concrete Inst., June, 1930, p. 831.

[72] See Föppl, "Drang und Zwang," Second Edition, Vol. 2, p. 297; also Timoshenko and Lessells, "Applied Elasticity," p. 54.

[73] "Ueber die Berechnung der Fundamentdeformation," Det Norske Videnkaps Akademi, Oslo, 1925; see, also, Love, "A Treatise on the Mathematical Theory of Elasticity," Fourth Edition, p. 140.

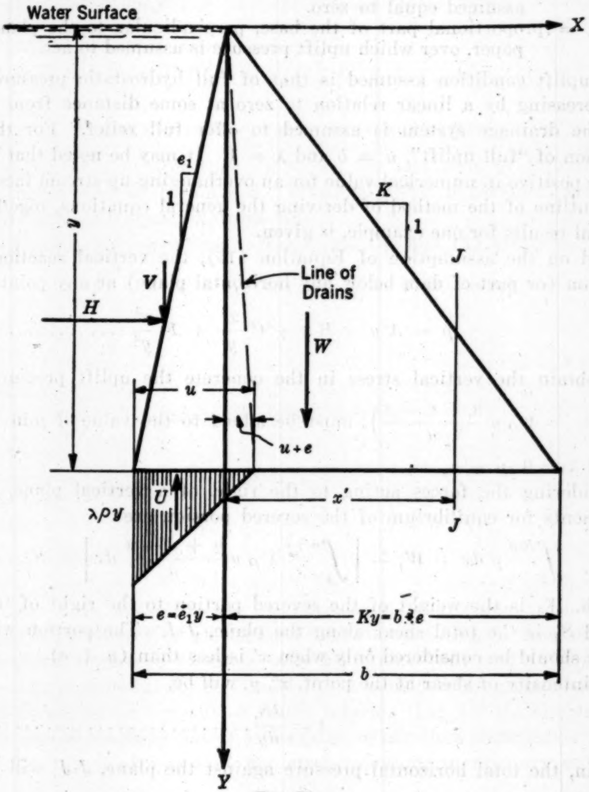


Fig. 21.

In order to obtain a more comprehensive insight into the stress distribution under the author's assumptions when applied to less usual types of section, the writer extended the method to include dams with a sloping up-stream face and has included in the general equations the effect of uplift. The additional nomenclature required is as follows (see Fig. 21):

e = minimum value of x for any given value of y (negative for the section shown in Fig. 21).

e_1 = batter of up-stream face $= \frac{e}{y}$.

k = batter of down-stream face $\left(\text{not equal to } \frac{b}{y} \text{ unless } e = 0\right)$.

ρ = unit water pressure in masonry units $= \frac{g}{w}$.

u = distance from up-stream face to point where uplift pressure is assumed equal to zero.

λ = proportional part of the base, perpendicular to the plane of the paper, over which uplift pressure is assumed to act.

The uplift condition assumed is that of full hydrostatic pressure at the heel, decreasing by a linear relation to zero at some distance from the heel where the drainage system is assumed to offer full relief. For the usual assumption of "full uplift", $u = b$ and $\lambda = 1$. It may be noted that e and e_1 would be positive in numerical value for an overhanging up-stream face.

An outline of the method of deriving the general equations, together with numerical results for one example, is given.

Based on the assumption of Equation (15), the vertical reaction of the foundation (or part of dam below any horizontal plane) at any point, x, y, is

$$p = A\,y + B\,x + C\,\frac{x^2}{y} + F\,\frac{x^3}{y^2} \ldots\ldots\ldots\ldots\ldots\ldots (47)$$

To obtain the vertical stress in the concrete the uplift pressure at the point, $\left(= -\,\lambda\,\rho\,y\,\frac{u + e - x}{u}\right)$, must be added to the value of p in Equation (47). If $\lambda = 0$, $p = n_y$.

Considering the forces acting to the right of a vertical plane, *J-J*, the requirements for equilibrium of the severed portion are:

$$\int_{x'}^{K\,y} p\,dx + W_1 - \left[\int_{x'}^{u+e} \lambda\,\rho\,y\,\frac{u + e - x}{u}\,dx\right] = S_1 \ldots\ldots (48)$$

in which, W_1 is the weight of the severed portion to the right of the plane, *J-J*, and S_1 is the total shear along the plane, *J-J*. The portion within the brackets should be considered only when x' is less than $(u + e)$.

The intensity of shear at the point, x', y, will be,

$$t = \frac{dS_1}{dy} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (49)$$

Again, the total horizontal pressure against the plane, *J-J*, will be represented by,

$$N_1 = \int_{x'}^{K\,y} t\,dx \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (50)$$

and the intensity of horizontal pressure at the point, x', y, will be,

$$n_x = \frac{dN_1}{dy} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (51)$$

A solution of Equations (47), (48), (49), (50), and (51), gives the following stress relations in masonry units for the general case:

$$n_y = A\,y + B\,x + C\,\frac{x^2}{y} + F\,\frac{x^3}{y^2} - \lambda\,\rho\,y\,\frac{u + e - x}{u} \ldots\ldots (52)$$

$$n_x = A\,(3\,K^2\,y - 2\,K\,x) + B\,(2\,K^3\,y - K^2\,x)$$
$$+ C\left(\frac{3}{2}\,K^4\,y - \frac{2}{3}\,K^3 x + \frac{x^4}{6\,y^3}\right) + F\left(\frac{6}{5}\,K^5\,y - \frac{K^4\,x}{2} + \frac{3}{10}\,\frac{x^5}{y^4}\right)$$
$$+ K^2\,y - K\,x - \lambda\,\rho\,\frac{(u + e)^2}{u\,y}\,(u + e - x) \ldots\ldots (53)$$

and,

$$t = A\,(2\,K\,y - x) + B\,K^2\,y + C\left(\frac{2}{3}\,K^3\,y + \frac{x^3}{3\,y^2}\right) + F\left(\frac{K^4\,y}{2} + \frac{x^4}{2\,y^3}\right)$$
$$+ K\,y - x - \lambda\,\rho\,\frac{u + e}{u}\,(u + e - x) \ldots\ldots (54)$$

For values of x greater than $(u + e)$, the last term of Equations (52) to (54), inclusive, must be omitted. When no uplift is assumed, $\lambda = 0$, and the last term of each equation drops out.

The equations corresponding to Equations (3) and (4) are:

$$\int_{e_1 y}^{K y} p\,dx = \frac{y^2}{2}\,[e_1\,(\rho + 1) - K + \lambda\,\rho\,u_1] \ldots\ldots (55)$$

and,

$$\int_{e_1 y}^{K y} p\,x\,dx = -\frac{y^3}{6}\,[\rho + K^2 - e_1^{\,2} - 2\,\rho\,e_1^{\,2} - \lambda\,\rho\,u_1\,(u_1 + 3\,e_1)] \ldots (56)$$

in which, $u_1 = \frac{u}{y}$. The boundary conditions are:

At the toe,

$$t^2 = n_y\,n_x \ldots\ldots (57)$$

At the heel,

$$n_y\,e_1 + \rho\,y\,e_1\,(1 - \lambda) - t = 0 \ldots\ldots (58)$$

and,

$$n_x + \rho\,y - t\,e_1 = 0 \ldots\ldots (59)$$

Equations (19) and (20) are modified only by changing the limits of integration with respect to x. These become $e_1\,y$ to $k\,y$ instead of 0 to b. These modified equations together with Equations (55) and (56) suffice to determine the parameters A, B, C, and F. A considerably greater amount of numerical computation is required in the solution than for the simple case of a vertical up-stream face, but it is lengthy rather than difficult.

The following equations show the stress relations in masonry units for a specific case, in which, $\lambda = 0$; $\frac{b}{y} = 0.80$; $k = 0.60$; $e_1 = -0.20$; $\rho = 0.416025$; and $m = 6$; thus,

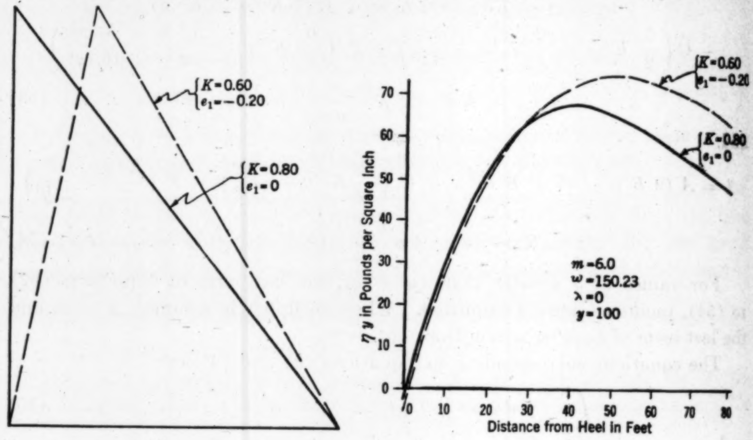
$$n_y = -0.47905\,y - 1.7814\,x + 4.0734\,\frac{x^2}{y} - 2.4214\,\frac{x^3}{y^2} \ldots\ldots (60)$$

$$n_x = -0.36102\,y + 0.18650\,x + 0.67890\,\frac{x^4}{y^3} - 0.72643\,\frac{x^5}{y^4} \ldots\ldots (61)$$

and,

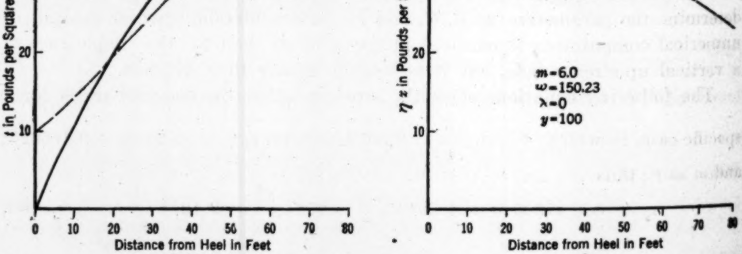
$$t = -\,0.18650\ y - 0.52095\ x + 1.3578\ \frac{x^3}{y^2} - 1.2107\ \frac{x^4}{y^3} \ldots\ldots (62)$$

The stresses from Equation (60) for $y = 100$, reduced to pounds per square inch, are plotted to scale in Fig. 22, which also shows the n_y-curve for a section of identical base width, but with a vertical up-stream face. A comparison of the two sections is shown in the upper part of the diagram. Fig. 23 shows a plotting of the horizontal stress and shearing stress for the two sections.



(a) (b)

FIG. 22.



(a) (b)

FIG. 23.

It is interesting to note the close correspondence of the two vertical stress curves in Fig. 22(*b*), throughout the up-stream 30 ft. of base width. The conditions at the heel are more favorable for the inclined up-stream face as compared to a vertical face than would be the case with linear stress distribution.

If the stress distribution assumed in Equation (15) is confirmed by further theoretical analysis and experiment, gravity dams with an appreciable upstream slope would appear worthy of more consideration because of their greater resistance to sliding on the foundation.

M. H. Gerry, Jr.,[74] M. Am. Soc. C. E. (by letter).—This paper is important to engineering science, because it directs attention to certain fundamental principles underlying the transmission of stress through a mass. It does not of itself, however, offer a solution of stress distribution in actual dams, but is rather a mathematical discussion based on an assumed state of unknown facts. Nevertheless, it is of value because it points the way to further progress and calls to notice some common errors.

The manner in which stress acts on and through bodies of various kinds is unknown in any exact sense and no general solution is possible until far more is learned of the molecular state of matter. Moreover, stress acts in three dimensions and not only in two, as assumed for convenience of computation in this paper. The mathematical difficulties of the problem are very great, but by no means insuperable. The crux of the situation is the insufficiency of knowledge concerning physical facts and this can be overcome only by long and extensive laboratory research; there is no short cut.

The theory of "least work" is apparently a sound and scientific method of approach, but more than mere assumptions must be available before general conclusions of value can be determined from its application.

The present methods of treating stress distribution in structures are well known to be empirical and approximate only, and it is most important that this fact be fully and constantly recognized by designing engineers.

The so-called elastic theory is such a case and it should never be applied outside the known limits of actual experiment. A definite "modulus of elasticity" is a mathematical fiction in so far as it represents a positive condition of a material. Over any considerable range it is quite commonly a variable and at best the modulus is merely a constant of experiment, its correctness being limited to the special physical conditions and dimensions at which it was determined.

Concrete is elastic in only a very limited way. It is not a distinct material, but rather an artificial conglomerate of which little is known in a scientific sense. It has no true elastic limit, and it flows and yields inelastically under moderate loadings. At all ages, concrete when passing from a dry to a wet state expands and its strength and elasticity materially change, while time, temperature, stress, and vibration further alter its character. Thus, it is idle to attempt an exact general analysis of stress distribution in an uncertain mass, such as a dam, as long as the profession remains intensely ignorant of the basic facts concerning stress in highly uniform and far more elastic materials.

---

[74] Cons. Engr., San Francisco, Calif.

At present, the design of gravity dams is purely empirical; attempts to cloak certain phases in a mathematical fog have only resulted in serious errors, causing many failures. Unfortunately, there has been a tendency to follow a fetish and to place total reliance on a margin of safety in one particular only, while overlooking entirely, weakness in other directions. All modern gravity dams of concrete, having triangular sections and bases of more than two-thirds the height, provide good margins of safety as to stress, but all too frequently they are close to the danger line in other respects. The proper limiting of stress in gravity dams is important, and, in future very high dams, it may control; but for present-day structures, it does not enter to the same degree as other important conditions for safety.

FREDRIK VOGT,[75] ASSOC. M. AM. SOC. C. E. (by letter).—Gravity dams are now designed to heights which surpass those of all previous dams, and hence stresses are approaching the allowable limit. For such large dams an eventual saving by reducing the ratio, base width to height, is so great that it pays to improve the analysis in order to find the lower limit of this ratio which can be used without danger, and to ascertain the actual value of the safety factor. It is fortunate, therefore, that the author has chosen for discussion the stress problems for gravity dams and has thus broadened the knowledge of engineers on this subject.

The principal question is whether or not the classical theory, which supposes a linear distribution of the vertical normal stresses, is on the safe side. The tensile stresses are the most dangerous, particularly those at the up-stream face, since the tensile strength of concrete is not to be trusted, and since horizontal tensile cracks at the up-stream face will eventually cause uplift forces from water penetrating into such cracks. The greatest emphasis, therefore, should be laid on a correct computation of the stresses at the up-stream face, and the design should be executed so that all possibilities of vertical tensile stresses at the up-stream face are eliminated as far as possible for all dams in which stability depends mostly on gravity action.

According to the classical theory, the triangular-shaped dam in Fig. 1 will be exposed to maximum principal stresses, $n_1 = - g\,y \left(1 + \frac{1}{k^2}\right)$, at the down-stream face when loaded, and $n_2 = - w\,y$, at the up-stream face when unloaded.

Smaller ratios of base width to height, than $k = \sqrt{\frac{g}{w}}$ are never used. With $g = 62.5$ and $w = 150$ lb. per cu. ft., this gives $k = 0.645$, and a maximum compression of 600 lb. per sq. in. is then obtained for $y = 407$ ft. Even if the classical theory is not accurate, this will suffice to prove that the magnitude of the compressive stresses in gravity dams has no direct significance in the design before the height is approaching or surpassing 400 ft., assuming that first-class concrete and rock foundation are obtained.

---

[75] Prof. of Mechanics, Norges Tekniske Höiskole, Trondhjem, Norway.

It seems desirable to study in detail the solution by the classical theory for comparison with the author's method. Using the notations from Fig. 1 the author gives the stresses according to the classical theory in Equations (13), and these stresses satisfy the equilibrium equations and the boundary conditions both at the up-stream and the down-stream faces. Let e_x and e_y be the displacements of a point, x, y. For the two-dimensioned system the compatibility equations are, therefore,

$$E \frac{\delta e_x}{\delta x} = n_x - \frac{n_y}{m} \qquad \text{(63)}$$

$$E \frac{\delta e_y}{\delta y} = n_y - \frac{n_x}{m} \qquad \text{(64)}$$

and,

$$t = G \left(\frac{\delta e_x}{\delta y} + \frac{\delta e_y}{\delta x} \right) \qquad \text{(65)}$$

in which, $m E = 2 (m + 1) G$.

The displacements, e_x and e_y, can be eliminated after derivation of Equation (63) for e_x twice with respect to y, of Equation (64) twice with respect to x, and of Equation (65) twice with respect to x and y. By introducing the equilibrium Equations (6) and (7), t and the derivative of n_x with regard to x can also be eliminated, and the following compatibility equation is obtained:

$$\frac{\delta^2 n_y}{\delta x^2} + 2 \frac{\delta^2 n_y}{\delta y^2} + \frac{\delta^2 n_x}{\delta y^2} = 0 \qquad \text{(66)}$$

By a repeated double derivation n_x also can be eliminated, but the form of Equation (66) is convenient for the purpose in question.

The stresses, Equations (13), satisfy the compatibility equation, and the displacements, e_x and e_y, can be found, therefore, by integration:

$$e_x = \frac{1}{E} \left[\left(\frac{g}{k^3} - \frac{w}{2 k} \right) y^2 - \left(g + \frac{g}{m k^2} - \frac{w}{m} \right) x y + \left(\frac{g}{m k^3} - \frac{w}{2 m k} \right) x^2 - a y + c \right] \qquad \text{(67)}$$

and,

$$e_y = \frac{1}{E} \left[\left(\frac{g}{2 m} + \frac{g}{2 k^2} - \frac{w}{2} \right) y^2 - \left(\frac{2 g}{k^3} - \frac{w}{k} \right) x y - \left(\frac{g}{k^2} + \frac{g}{2 m k^2} - \frac{g}{2} + \frac{w}{2 m} \right) x^2 + a x + d \right] \qquad \text{(68)}$$

The coefficients, a, c, and d, give only the position of the deformed section relative to the co-ordinate system, but have no influence on the stresses or on the deformation of the section.

The dead load is placed successively during the construction period, and part of the dead load deformation, therefore, is obtained before the section is completed. Equation (68) is, therefore, not valid for the dead load. The fresh concrete also will be partly plastic and not elastic; and the modulus of elasticity is lower for fresh than for aged concrete. Hence, it seems better to consider the stresses and deformations due to water load only, which are found by making w equal to zero in the equations. The author's criticism of the classical theory also mainly concerns the stresses due to water load only.

For the foundation line of the section, y $(= h)$ is a constant, and the vertical displacements are then given by a parabolic curve convex downward since the equation for e_y contains a term, x^2, with a negative coefficient. Assuming that $m = 6$ and $k = 0.645$, the rise of the curve is found to be,

$$\frac{1}{4}\left(1 + \frac{1}{2\,m} - \frac{k^2}{2}\right)\frac{g\,h^2}{E} = 0.22\,\frac{g\,h^2}{E} \text{ .... } (69)$$

At the same time, the width of the base line is reduced with $k\,\dfrac{g\,h^2}{E} = 0.645\,\dfrac{g\,h^2}{E}$.

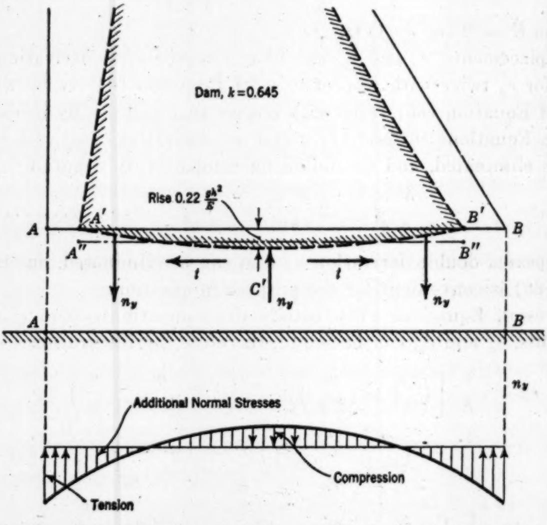


FIG. 24.—DEFORMATION OF DAM SECTION DUE TO WATER LOAD ONLY, ACCORDING TO CLASSICAL THEORY, AND ADDITIONAL STRESSES TO BE APPLIED IN ORDER TO MAKE DAM SECTION FIT AN UNYIELDING FOUNDATION

If the foundation is unyielding, additional stresses would have to be applied, as shown in Fig. 24, in order to straighten the line, $A'\,C'\,B'$, and to lengthen it to fit the line, $A\,B$, of the foundation. Shearing stresses directed from the center toward A and B are necessary in order to increase the length, but these acting alone would increase the rise. Hence, the additional normal stresses must be tensile stresses near both faces and compressive stresses at the center; that is, the corrected normal stresses due to water load only must be tensile stresses, greater than those found by the classical theory near the up-stream face, and compressive stresses, less than those found by this theory at the down-stream face. The tendency of these corrections is the same as that found by the author (Fig. 6).

The stresses represented by Equations (13) resulting from the water load only, will lengthen the height, h, of the up-stream face with $\frac{g\,h^2}{2\,k^2\,E} = 1.2\,\frac{g\,h^2}{E}$. To straighten the line, $A'\ C'\ B'$, so that it coincides with the line, $A''\ B''$, when the distance, $A'\ A''$, equals $\frac{2}{3}$ times $0.22\,\frac{g\,h^2}{E}$, an average change in stresses at both faces equal to $\frac{2}{3}$ times $\frac{0.22}{1.2}$, or 12%, approximately, is required. Because of uneven distribution of these additional stresses over the height, and because of additional curving due to shearing stresses, the corrections to be added may be somewhat larger at the base.

The foregoing remarks are based on the assumption that the foundation is unyielding, but it will always yield elastically, and the deformations can be computed for any given load on a plane foundation, as shown in a paper by the writer.[76] It is convenient to consider three different types of loads on the foundation, namely, the direct pressure, $g\,h$, per unit area on the foundation at the left of A, the shearing stresses from the dam between A and B, and the bending stresses from the dam between A and B.

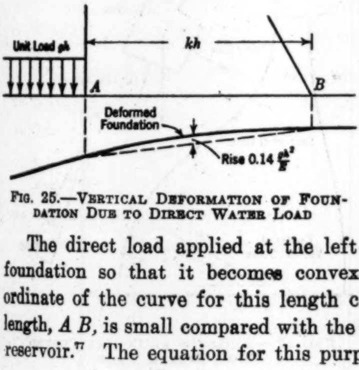


FIG. 25.—VERTICAL DEFORMATION OF FOUNDATION DUE TO DIRECT WATER LOAD

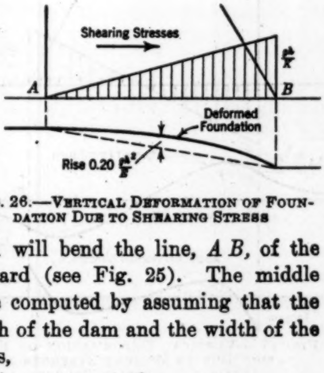


FIG. 26.—VERTICAL DEFORMATION OF FOUNDATION DUE TO SHEARING STRESS

The direct load applied at the left of A will bend the line, $A\,B$, of the foundation so that it becomes convex upward (see Fig. 25). The middle ordinate of the curve for this length can be computed by assuming that the length, $A\,B$, is small compared with the length of the dam and the width of the reservoir.[77] The equation for this purpose is,

$$\frac{m^2 - 1}{m^2} \times \frac{1\,n\,2}{\pi} \times k\,\frac{g\,h^2}{E_r} = 0.14\,\frac{g\,h^2}{E_r} \;\ldots\ldots\ldots\ldots (70)$$

in which, E_r is the modulus of elasticity for the rock, and m and k are introduced with the values used before.

The shearing stresses, with distribution, as given by Equations (13), will also curve the foundation line, $A\,B$, the same way (see Fig. 26). The middle ordinate in this case is computed by means of the general formula:[78]

$$\frac{(m - 2)\,(m + 1)}{4\,m^2}\,\frac{g\,h^2}{E_r} = 0.20\,\frac{g\,h^2}{E_r} \;\ldots\ldots\ldots\ldots (71)$$

---

[76] "Ueber die Berechnung der Fundamentdeformation," Det Norske Videnskaps Akademi, Oslo, 1925.

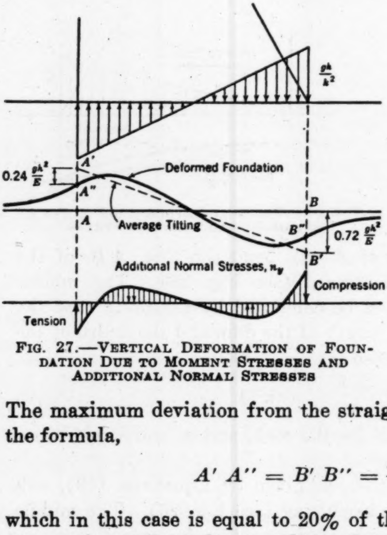
[77] *Loc. cit.*, p. 7.

[78] *Loc. cit.*, p. 26.

Both these curvings of the foundation line increase the disagreements already shown in Fig. 24. The total rise of the different curves is (assuming that $E_r = E$), $(0.22 + 0.14 + 0.20)\,\frac{g\,h^2}{E} = 0.56\,\frac{g\,h^2}{E}$. This must be compensated by stresses, as shown in Fig. 24, which will deform both dam and foundation. Since the foundation is assumed to yield, the method of estimating used in connection with Fig. 24 cannot be applied directly to this case. The writer believes that the stresses due to water load may be changed as much as 25 to 30% because of this curving. According to Table 5 the author found an increase by his method of +37% and −23% at the two faces, which checks very well with the writer's estimate.

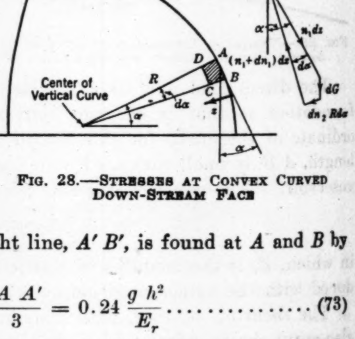
The bending stresses on the foundation due to water load on the dam are, according to the classical theory, $\pm\,\frac{g\,h}{k^2}$ at both faces (see Fig. 27). These stresses will deform the foundation to an S-shape, as shown in Fig. 27. The average tilting is represented by a line, $A'\,B'$, or,

$$A\,A' = B\,B' = \frac{m^2-1}{m^2}\,\frac{3}{2\,\pi\,k}\,\frac{g\,h^2}{E_r} = 0.72\,\frac{g\,h^2}{E_r}\,\ldots\ldots\ldots\ldots(72)$$



$\frac{gh}{k^2}$
A'
$0.24\,\frac{gh^2}{E}$
Deformed Foundation
A"
B
A
Average Tilting
B"
$0.72\,\frac{gh^2}{E}$
B'
Additional Normal Stresses, n_y
Compression
Tension

FIG. 27.—VERTICAL DEFORMATION OF FOUNDATION DUE TO MOMENT STRESSES AND ADDITIONAL NORMAL STRESSES



Detail of Element
$n_1\,dz$
dz
$R\,d\alpha$
$dn_2\,R\,d\alpha$
$(n_1+dn_1)\,dz$
$dG = w\,dz\,R\,d\alpha$
Section of Dam
α
$n_1\,dz$
A
$(n_1+dn_1)\,dz$
$d\alpha$
D
B
R
dG
Center of Vertical Curve
C
$d\alpha$
$dn_2\,R\,d\alpha$
z
α
α

FIG. 28.—STRESSES AT CONVEX CURVED DOWN-STREAM FACE

The maximum deviation from the straight line, $A'\,B'$, is found at A and B by the formula,

$$A'\,A'' = B'\,B'' = \frac{A\,A'}{3} = 0.24\,\frac{g\,h^2}{E_r}\,\ldots\ldots\ldots\ldots\ldots(73)$$

which in this case is equal to 20% of the total tensile deformation at the up-stream face. In order to adjust the dam and foundation so that they fit together, additional stresses must be applied as shown in Fig. 27. It is difficult to estimate the magnitude of these stresses. However, since they change from maximum tension to maximum compression in a much shorter length than the additional stresses shown in Fig. 24, it is obvious that they must also decrease to zero at depths which are correspondingly smaller. In order to obtain a large deformation in a short distance, the additional stresses (Fig. 27), must be rather large, but the effect will be negligible at a comparatively small distance from the foundation joint.

The base width, *A B,* will be changed by the deformation of the foundation, which again may cause some new redistribution of the stresses.

All the foregoing computations concern the stresses due to water load only. If the dead load stresses were applied after the construction was completed, the total difference between the deformed dam and foundation was reduced to about one-half. Furthermore, the additional stresses due to these disagreements then would be reduced to one-half. However, for reasons previously discussed, such computations for the dead load deformations cannot be considered to be correct.

The author's method is based on the assumption that the stresses, n_y, are distributed in the same manner for two joints at different elevations and that the magnitude at two points similarly located in these joints is proportional to y. Two points are similarly located in the horizontal joints if the ratio, $z = \frac{x}{y}$, is the same for both points, and this assumption, therefore, can be expressed by the formula,

$$n_y = y\, f\,(z) \qquad \text{(74)}$$

in which, $z = \frac{x}{y}$.

Equation (15) can be found from Equation (74) by developing $f\,(z)$ in series, neglecting all terms after the fourth. By introducing n_y, as given in Equation (74), in the equilibrium Equations (6) and (7), these can be integrated. For convenience, let $F\,(z)$ be a function of $z = \frac{x}{y}$, and let the derivatives thereof with regard to z be $F'\,(z)$, $F''\,(z)$, etc., and let this function be defined so that $F''\,(z) = f\,(z)$. Then, by integration,

$$n_y = y\, f\,(z) = y\, F''\,(z) \qquad \text{(75)}$$

$$n_x = y\,[6\,F\,(z) - 4\,z\,F'\,(z) + z^2\,F''\,(z)] + h\,(y) - x\,g'\,(y) \qquad \text{(76)}$$

and,

$$t = y\,[-\,w\,z - 2\,F'\,(z) + z\,F''\,(z)] + g\,(y) \qquad \text{(77)}$$

in which, $g\,(y)$ and $h\,(y)$ are two independent functions of y. The correctness of Equations (75), (76), and (77) can easily be checked by introducing in Equations (6) and (7), which are satisfied since $\frac{\delta\, z}{\delta\, x} = \frac{1}{y}$ and $\frac{\delta\, z}{\delta\, y} = -\frac{z}{y}$.

At the down-stream face, $z = k$, and for this value of z Equations (75) to (77) must satisfy the boundary conditions defined by Equations (8) and (9), whereby the functions, $g\,(y)$ and $h\,(y)$, are restricted to the forms,

$$g\,(y) = a\,y \qquad \text{(78)}$$

and,

$$h\,(y) = b\,y \qquad \text{(79)}$$

in which, a and b are constants. After introducing these functions in Equations (75) to (77), both t and n_x can be expressed by y times the functions of z only; that is, by equations of the type given in Equation (74).

The stresses must satisfy the compatibility Equation (66), and by introducing the stresses defined by Equations (75) to (79) in Equation (66), the latter is reduced to,

$$\frac{(z^2 + 1)^2 \, F'''' (z)}{y} = 0 \ldots\ldots\ldots\ldots\ldots\ldots\ldots (80)$$

Since the squared polynom cannot become zero, the compatibility equation is reduced to,

$$F'''' (z) = 0 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (81)$$

which, integrated twice, gives

$$F'' (z) = A + B\,z \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (82)$$

in which, A and B are constants, and, hence,

$$n_y = y\,F'' (z) = y\left(A + B\,\frac{x}{y}\right) = A\,y + B\,x \ldots\ldots\ldots (83)$$

This equation is identical with Equation (10) representing the classical theory.

In this manner it has been proved that no stress functions exist, except those given by the classical theory, which satisfy the condition of equilibrium, the compatibility equations, and the boundary conditions, and which at the same time give stresses proportional to the depth, y, at the joint considered. The writer has proved that the classical theory does not satisfy the compatibility equation at the foundation line. Hence, no exact solution can be found at all by means of stress functions of the type of Equation (74), or by Equation (15), which is only a special case of Equation (74).

The classical theory gives stresses which are greatly in disagreement with the compatibility equation at the foundation joint, and several large corrections are required. For non-triangular sections the classical theory does not satisfy the compatibility equation even in the dam proper.

The author's theory gives stresses which include approximately a number of necessary corrections, namely, all except those due to the S-shaped deformation of the foundation. It has been proved that the method can be only approximate since the compatibility equation is not satisfied. An exact solution can be obtained only by stress functions of a more complicated type. However, even an approximate solution means a great step forward.

The deformation of the foundation is the key of the correct solution since a change in this deformation will also change the stresses in the dam proper. Exact solutions including this point are difficult to find, particularly because of the corners at the heel and the toe of the dam, which represent singular points.

The best check on the theory can be obtained by model tests. The English tests indicate large tensile stresses at the heel of the dam in agreement with the author's theory and with the additional stresses shown in Fig. 27, but in disagreement with the classical theory. Furthermore, at some distance from the base, these tests give a slight curving of the diagram for the vertical stresses in agreement with the author's theory. The writer has been connected with tests on models of arch dams, and such a curving of the stress diagrams was found probable. However, these tests were conducted with other problems in view, and the results regarding this particular problem are rather indirect and should not be taken as a definite proof. It is to be hoped that the model tests demanded by the Consulting Board for the Hoover Dam may give final solutions of some of these problems.

Tensile stresses may also be produced by volumetric changes in the concrete, not only in longitudinal, but also in vertical, directions. Horizontal cracks have developed in a number of dams,[79] and such cracks have been observed even at the up-stream face of the Barberine Dam, in Switzerland, the Saint-Marc Dam, in France, and the small Tömmerholt Dam, in Norway. These cracks are obviously caused by non-uniformity in shrinkage and temperature in the mass. Such non-uniformities, therefore, have a significance comparable with that of the tensile stresses found by the author.

Another danger of tensile stresses is introduced by a convex curving of the down-stream face in a vertical plane, as shown in Fig. 28. If principal compressive stresses, n_1, are acting parallel to the face, the convex curving will eventually be followed by tensile stresses, n_2, normal to the face. On the surface of the hatched element, *A B C D* (Fig. 28), no shearing stresses are acting, and the equilibrium demands that,

$$dn_2 \, R \, d\alpha + w \, dz \, R \, d\alpha \sin \alpha = n_1 \, dz \, d\alpha$$

or,

$$\frac{dn_2}{dz} = \frac{n_1}{R} - w \sin \alpha \ldots\ldots\ldots\ldots\ldots\ldots\ldots (84)$$

That is, if n_1 is larger than $R \, w \sin \alpha$, $\frac{dn_2}{dz}$ will be positive, and tensile stresses, n_2, will be developed normal to the face, increasing from zero at the very face to a maximum some place in the interior. This maximum value depends on the direction and the magnitude of the first principal stress, n_1, in the interior. The greatest danger of such tensile stresses is obtained for dams with a sharply curved steep down-stream face, that is, for small values of R and $\sin \alpha$.

Such curving of the down-stream face has been suggested in order to reduce the maximum compression in high dams, but a reduction of the compressive stresses accompanied by development of tensile stresses is no better than going from "the frying pan into the fire". Furthermore, the English tests (Fig. 8), indcate that the compressive stresses can be reduced by curving in the opposite direction, and this opposite curving shown on Fig. 8 will not introduce any tensile stresses. The suggestion to curve the face, as shown in Fig. 28, is based on the assumption that the classical theory is correct not only for a triangular section, but for all sections regardless of shape, which is certainly not true.

---

[79] "Shrinkage and Cracks in Concrete of Dams," by Fredrik Vogt, Det Kgl. Norske Videnskabers Selskab, Nidaros, Norway, 1930, No. 3.

EUGENE KALMAN,[80] M. AM. SOC. C. E. (by letter).—In this paper Mr. Jakobsen introduces a new method for studying stress distribution in gravity dams, which like any new method must be judged from two points of view, as expressed in the questions: What is the intrinsic value of the method as applied to problems of structures in general, and of dams in particular; and, what results was the author able to derive in making use of it?

The so-called Ritz method has been applied very successfully in the solution of many problems in modern engineering. Some time ago, this method, which is an adaptation of the Principle of Least Work, was first used in dam theory by the writer[81] as a means of determining radial shear acting between arch slices, or between the cut faces of sections in cantilevers of arch dams.

The writer applied the same method to the analysis of stresses in curved or straight gravity dams at least as early as 1926. Mr. Jakobsen's presentation is the first that has appeared in published form and hence the credit for its introduction into the literature of gravity dams is due him.

As to the results of the paper, there are two distinct fundamental defects in the author's use of the Ritz method, which prevent him from deriving all the value that may be obtained by its proper application.

The principle of least work is applied when the unknown stress distribution is sought in a given structure. Many times, the probable or possible stress distribution can be guessed. Sometimes, two or more different stress distributions appear to be probable or possible. The most correct among them must then be selected in some way. Since a selection is involved, the assumed group of different possible stress distributions may be termed (in accordance with current usage in problems of variation) a "competing" group. The various, assumed stress distributions may be termed, "competing individuals". Making use of this terminology, the principle of least work states that among all individuals of a given competing group, in which the true stress distribution is also included, this true stress distribution yields a smaller value for the work of deformation than any other individual of this group. This formulation of the principle of least work presumes that the true system of stress is included in the competing group, and this condition is generally satisfied in the more elementary applications of the principle. As a simple example consider a beam with two equal spans acted upon by a uniform load. Any numerical value whatever chosen for the reaction at the middle support will be statically consistent with the given system of loads if the corresponding values of the end reactions are selected accordingly.

In other words if the three reactions are $p\,l - \frac{1}{2}X$, X, and $p\,l - \frac{1}{2}X$, respectively (in which, X may vary), the result will always be consistent with the given load. The competing values for the reaction, X, of the middle support include any number whatever between $-\infty$ and $+\infty$. Each of these numbers is an indvidual of the competing group. The corresponding end reactions are defined by the expression, $p\,l - \frac{1}{2}X$. If $X = 0$, for instance, the end reactions would equal $p\,l$, etc.

---

[80] Visiting Prof., Civ. Eng., California Inst. of Technology, Pasadena, Calif.

[81] "Sulle piastre curve," R. Rendiconti, Inst. Lomb. Scienze, Serie II, Vol. LXII, Fasc. VI, x.

To obtain the true reaction of the middle support, the principle of least work may be used and thus the reactions are found to be 3 $p\,l:8$, 5 $p\,l:4$, 3 $p\,l:8$, respectively, a result which may be verified by other means. Hence, the principle of least work permits an absolutely exact selection of reactions because the true one is present in the competitive group of sets.

An entirely different condition arises when the given competitive group of stress systems does not actually contain the true stress system. Then, the principle of least work must be modified so as to state that, of all competing systems, the one that gives the smaller value for the work of deformation than any other stress system in the group under consideration, will constitute the best possible approximation to the real working stress system, unless another group of supplementary stress systems is considered.

In the ordinary problems of elementary statics, in which application of the so-called theorem of Castigliano is involved, there is always a definite number of unknowns present, such as the reactions. Any of the unknowns may vary from $-\infty$ to $+\infty$, as X in the foregoing example. Since the parameter may assume any value whatever it is obvious that this range of possibilities must include the correct one, which is to be determined.

On the other hand, in all problems of advanced statics or elasticity, in which the so-called Ritz method of the principle of least work is applicable, an infinite number of tentatively assumed values in an infinite number of points must be taken into consideration. For instance, in the case cited herein, parameters should be introduced for the values of the unknown stresses at all points of the triangular profile which represents a two-dimensional region containing a number of points designated ∞^2. It is clearly impossible to deal with so many parameters. Hence, Ritz proposed the method of introducing a limited number of parameters or unknowns which will yield an approximation to the true stress system, the exact representation of which would require an infinite number of parameters. The underlying principle is the same as in the case of the approximation of a function by the first n terms of a Fourier or power series.

It is easy to see, therefore, that there is a basic difference between the two. In one case, the designer is sure that the individual selected by application of the method represents the correct solution. In the other case, he is sure that the individual selected does not represent the correct solution; it represents merely an approximation.

This fact raises the question as to how to estimate the degree of approximation and how to secure satisfactory results in all cases involving the application of the Ritz method.

Consider a beam fixed at one end and acted upon by a single load, P, at the free end, as shown in Fig. 29. Assume that the longitudinal stresses are distributed in accordance with the following equation:

$$\delta_x = \left(a \sin \frac{11\,\pi}{h}\,y + b \sin \frac{9\,\pi}{h}\,y\right) x \ldots\ldots\ldots\ldots\ldots (85)$$

The parameters, a and b. may be assumed to be any value whatever consistent with the condition of equilibrium for the given load. The most satisfactory combination may be found by applying the principle of least work.

The integral of work will reduce to the simple expression,

$$\int\int \delta_x{}^2\, dx\, dy = \int_{-\frac{h}{2}}^{+\frac{h}{2}} \int_0^c \left(a \sin \frac{11\,\pi}{h}\, y + b \sin \frac{9\,\pi}{h}\, y\right)^2 x^2\, dx\, dy$$

$$= \int_0^c x^2\, dx \int_{-\frac{h}{2}}^{+\frac{h}{2}} \left(a^2 \sin^2 \frac{11\,\pi}{h}\, y + b^2 \sin^2 \frac{9\,\pi}{h}\, y + 2\, a\, b \sin \frac{11\,\pi}{h}\, y \sin \frac{9\,\pi}{h}\, y\right) dy. \text{(86)}$$

The form of Equation (85) indicates that the stresses are in equilibrium. In addition, they must balance the bending moment caused by the load, and this requires that,

$$\int_{-\frac{h}{2}}^{+\frac{h}{2}} x \left(a \sin \frac{11\,\pi}{h}\, y + b \sin \frac{9\,\pi}{h}\, y\right) y\, dy = P\, x \ldots\ldots\ldots \text{(87)}$$

Equation (86) integrated and simplified becomes,

$$\frac{1}{2}\, h\, a^2 + \frac{1}{2}\, h\, b^2 = P \ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(88)}$$

and Equation (87) becomes,

$$-\frac{2\, a\, h^2}{11^2\, \pi^2} + \frac{2\, b\, h^2}{9^2\, \pi^2} = P \ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(89)}$$

In other words, the most satisfactory combination of a and b is that in which $a + b$ is a minimum, and $9^2\, a^2 - 11^2\, b^2 = \dfrac{9^2\, 11^2\, \pi^2}{2\, h^2}$. Therefore, the proper values are, respectively,

$$a = \frac{9^2}{9^4 + 11^4}\, \frac{\pi^2}{2\, h^2}\, P$$

and,

$$b = -\frac{11^2}{9^4 + 11^4}\, \frac{\pi^2}{2\, h^2}\, P$$

Accordingly, the most logical approximation of the true stress system is expressed by the following equation:

$$\delta_x = \frac{1}{9^4 + 11^4}\, \frac{\pi^2}{2\, h^2}\, P \left(9^2 \sin \frac{11\,\pi}{h}\, y - 11^2 \sin \frac{9\,\pi}{h}\, y\right) x \ldots\ldots \text{(90)}$$

The curve of this equation as plotted in Fig. 30 is a very complex one. No engineer would ever attempt to apply this kind of distribution as a substitute for Navier's linear law. Nevertheless, of all competing stress systems represented by Equation (85), this is the best approximation to the true stress system.

From this example it follows that even the correct application of the principle of least work may yield the most absurd results. In other words, prudence is required when the principle of least work is used in the form known as the Ritz method. The result thus obtained will be only a more or less satisfactory approximation of the actual conditions. This is in contrast to the result secured by means of the Castigliano theorem.

The principle of least work operates in a manner similar to an automatic machine. It will select the most acceptable item from a group of items put before it; but the quality of the selection will depend on the material offered. It is clear from what has been explained in the foregoing remarks that the results obtained by the author will depend upon the skill used in constructing his competing group of stress systems.

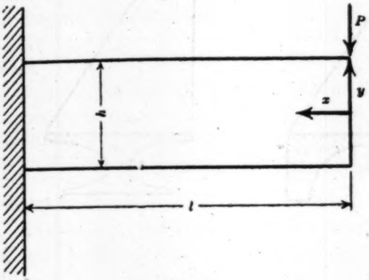


FIG. 29

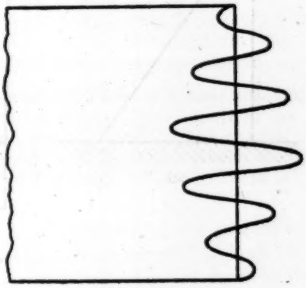


FIG. 30

Mr. Jakobsen's choice of a stress system is represented by Equations (15), (21), and (22). These equations do not define elastic stress systems, because they do not satisfy Equation (43), as given by the author. *Vice versa,* as a matter of fact the stress systems that actually apply in the case of a dam will be those of elasticity. This difference in the characters of the two stress distributions, which may be large quantitatively, constitutes the first fundamental defect, or imperfection in the author's analysis.

A more correct way would have been to assume all competing stresses as elastic systems acting in the profile of the dam. Of course, these should be chosen so as to be statically consistent with the boundary stresses defined in accordance with Equations (15), (21), and (22), by n_y and t along the base of the dam. Each stress system defined by n_y and t on the base, determines one elastic stress system in the profile of the dam and of all these elastic stress systems the most convenient should have been determined by means of the principle of least work.

Of course, there was an extremely difficult problem to be solved, namely, how to determine in all those elastic stress systems the stresses acting in the interior of the dam, thus making possible their substitution in the integral of work and the evaluation of that integral. The author did not attempt to attack this problem, and, consequently, he was forced to assume a certain stress system in the dam, consistent with conditions of stress along the base. That, however, is an arbitrary assumption which might differ radically from actual conditions.

After having selected his competing group of equations, the author computes the work of deformation in the profile of the dam and obtains the stress system which furnishes the smallest value of the work. He then states that this is the most probable stress distribution in the dam. The writer feels, however, that the result obtained, without considering at all the effect of the foundation, will not represent even a reasonable approximation of the true stress distribution in the dam.

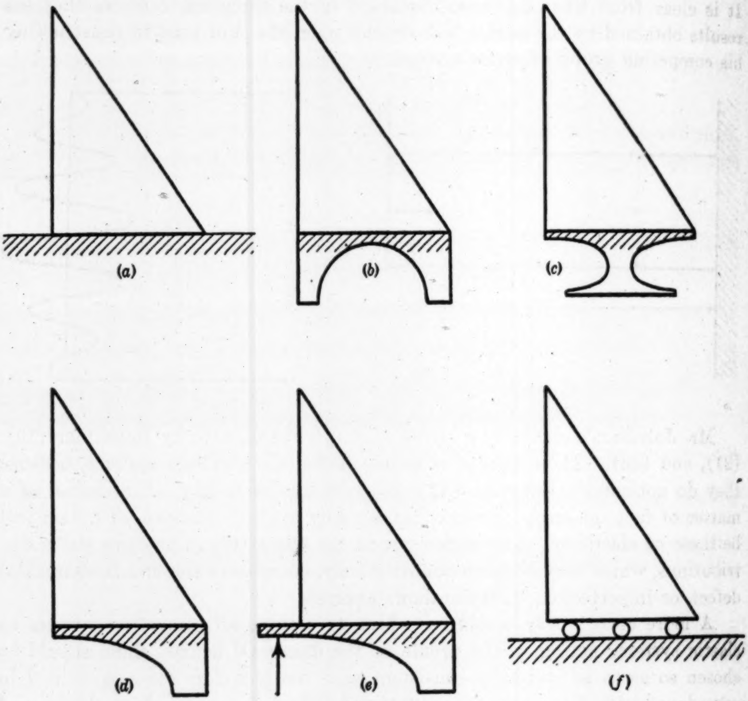


FIG. 31

Evidently, stress conditions in the triangular cross-section of a dam will be entirely different if the dam is considered as a homogeneous solid block with different types of foundation supports as shown, for example, in Fig. 31. Not only will the different structural shape of the supports have its influence on the resulting stress system in the combined or integral section of dam and foundation, but the presence of incidental loads acting directly on the foundation, as shown in Fig. 31, will also have its effect. The author does not consider this phase of the problem. According to him the principle of least work applied to the dam separated from the foundation, furnishes the best stress distribution in the dam, in any case, for any shape of foundation, and for any system of independent load acting directly on the foundation. It is obvious from Fig. 31 that in actual cases the stress distributions will differ completely from each other and from that found by Mr. Jakobsen. This constitutes the second defect in the use of Ritz's method by the author.

The author's aim was to furnish a stress distribution that would be an improvement over that of Lévy, which he claims to be unsatisfactory because of the deflection conditions in the foundation. He states that Lévy's stress distribution is not acceptable because it implies a sine-shaped curving of the foundation. Will the author's distribution cause the deflection to appear less curved? Is he satisfied that his result is an improvement over that of Lévy in this respect? If so, why?

After this preliminary criticism of the author's general principles, some remarks should be made referring to a few particular statements contained in the paper under consideration.

Under Article IV, entitled "Stress Distribution by the Principle of Least Work", Mr. Jakobsen states,

> "The writer has shown that the arch stresses found from the tests on the Stevenson Creek Dam agree remarkably well with the Cain formulas, when the deformation and displacements of the abutments are included in the calculations in the manner suggested by Dr. Vogt. (See Article III.) These tests are, therefore, an experimental verification of the Principle of Least Work as applied to the stresses in an arch dam."

The writer cannot agree with this statement. By means of the principle of least work, certain formulas or relations between two quantities may be derived, as, for example, between load and stresses, or between load and deflections, etc. For example, the Cain formulas express a relation between load and stresses. When the load is given, the stresses may be determined by means of those formulas, or, conversely, the load may be determined from the stresses given.

How can the exactness of a formula be checked? Obviously, by comparing the values of the right and the left-hand members obtained by independent measurements. For example, the load and the stresses may be measured directly; then the two parts of the Cain formula may be compared; and if they agree, one could declare that the formula is experimentally checked.

In the case under consideration, Mr. Jakobsen takes the stresses found experimentally for the arch at Elevation 30 of the Stevenson Creek Dam and compares them with a certain load (depending on several parameters) which he determines by using the Cain formulas. It follows then that the load data at his disposal do not represent measured quantities. They represent values derived by reasoning from the stress data. In the writer's opinion this does not constitute an experimental check of the Cain formula. Mr. Jakobsen has merely applied the Cain formula to evaluate the load.

Suppose, furthermore, that the experiments had revealed variable horizontal load. This might have occurred in the case of an arch dam with central angles of the magnitude of those normally used; that is, central angles considerably larger than those which unfortunately were chosen in the Stevenson Creek Dam.

In that case, the author would follow the same procedure as outlined in his paper and would find no acceptable agreement between the measured stresses and those obtained by Cain's formula which assumes a constant load. Logically, he would then have to state that the principle of least work does not hold in the case of arch dams. On the contrary, however, the principle does hold also in this case as it does in all phenomena thus far encountered in the field of mechanics. The correct conclusion to be drawn, therefore, would be that the load distribution is different from that implied by the Cain formula.

In Appendix II, Mr. Jakobsen states that "Equation (27) does not satisfy Equation (43); and, according to Föppl, it should not".

Now, he is dealing with stress systems of a competing group without requiring from them anything except that they be consistent with the given system of load, that is, water pressure and dead load. An infinite number of elastic stress systems may be found that satisfy this prerequisite, all of which would satisfy Equation (43). Consequently, the foregoing statement should be revised to read, "and according to Föppl it need not".

In Appendix III, the author places considerable emphasis on the fact that in the interior of a thin plate which is loaded by lateral forces and is free from any load on the two parallel faces, there may be, or will be, stresses acting normally to the plane of that elementary plate.

The writer believes that it is of no use to attempt to provide for those stresses at the actual beginning stages of a research. Consider, for example, two horizontal sections in the profile of a dam, and assign to them the stresses indicated in Equations (15), (21), and (22), respectively. The two horizontal sections may be a considerable distance apart. It is clear that the nature of the elastic stresses may change infinitely more between these two sections that are relatively far apart than between the two limiting parallel faces of the thin plate mentioned that are infinitely close together. On the latter the normal stresses are equal to zero and this implies an exceedingly small margin for the development of stresses in the interior of the plate normal to the faces.

Nevertheless, the author neglects the possibly wide differences between his system and the true system in the region between those two horizontal sections, while he devotes considerable space to the treatment of the normal stresses in the plate of elementary thickness. The effect is similar to that of making refinements based on possible variation in the fifth decimal place of a number when the numeral in the first decimal place is open to question.

The author states that horizontal sections are not normal sections, and that, therefore, in his opinion, they could not be used as a basis for the application of the trapezoidal rule. He also states that according to his analysis of gravity dams, however, the customary horizontal sections give "practically" the same result as normal sections.

While the concept of a section and the application of Navier's linear distribution law, are entirely satisfactory for beams of limited height, it has no meaning in the analysis of dams. If the stress distribution on the base of a triangular section is assumed to be linear, then the entire elastic system or the condition of stresses in the interior of the section is linear, as stated by Lévy many years ago.[82] Hence, in that case there is no difference at all between normal, horizontal, or inclined sections. The trapezoidal rule applied to any of these sections will furnish the same result exactly, and not only "practically", as the author says.

If conversely the distribution of stresses on the base is assumed not to be linear, then it will not be linear on any section whatever in the interior of the entire profile. The conceptions of the terms "section", "resistance against overturning", etc., are incompatible with modern stress analysis. Mr. Jakobsen deserves credit for having abandoned in the present paper the conception of the section, on which his paper quoted previously was based, introducing instead the stress system defined along every point in the profile by means of Equations (15), (21). and (22).

---

[82] The writer's exposition in *Beton und Eisen*, 1930, Heft 9, p. 174.

In Appendix I, the author states:

> "The question now arises whether Equation (15) can represent a curve similar to Fig. 10 [which is the stress distribution of Ottley and Brightmore], so that in case this is the correct stress distribution, the constants could be determined so as to give a curve similar to Fig. 10."

Then the author proceeds to determine the constants for that case.

The writer cannot understand the significance of that statement. Since for $y = H$ the author's stress system represents a parabola of the third degree in x, and since every curve whatever may be approximated by such a parabola when the parameters are conveniently selected, it is self-evident that such an approximation may also be made for the Ottley-Brightmore stress curve. Therefore, the writer is unable to understand the reasons for and the aim of such computations in connection with the problem under consideration.

From Equations (25) and (26), the author deduces the fact that the four parameters are independent of the height, H. This fact can be recognized without the use of these equations since it simply follows from the fact that the competing stress system is homogeneous. Using polar co-ordinates instead of Cartesian co-ordinates, all stresses appear to be functions of r and β, in the form of $r\ f\ (\beta)$. In other words, the parameters have nothing to do with r, and will be independent of the longitudinal dimensions of the dam.

The homogeneity of the competing stress system represents a weak point in the author's choice. It has the apparent advantage that it simplifies the computations, but on examination it proves to be one of the less favorable among all possible selections. In fact, since all stresses are of the form, $r\ f\ (\beta)$, they correspond to the type of elastic stress distribution defined by Lévy. This is fundamentally in opposition to the conception of the integral profile composed of the dam and the foundation. It involves simply repeating the stresses in the foundation at an ever-increasing rate, due to the presence of an ever-increasing r, while it is evident that the stresses will decrease and disappear toward the interior of the foundation. Stress conditions in the foundations as encountered by the author's system are shown schematically in Fig. 32.

Finally, in his closing remarks Mr. Jakobsen makes the statement that, according to Mach, only actual tests can determine whether or not the principle of least work applies in any specific case.

The writer differs with this opinion in that he believes speculative research is as important as experimental research. They both help to advance science. The author's findings may be compared conveniently with results of tests made by Mr. E. Barnes at the California Institute of Technology in 1929 and 1930. In the writer's opinion, all such tests supply unreliable information as to the actual stress distribution in dams due to the influence of an infinitely extensive foundation and of the direct water pressure on the rock with its indirect effect on the stresses in the dam itself. These factors are and must be neglected in all these tests.

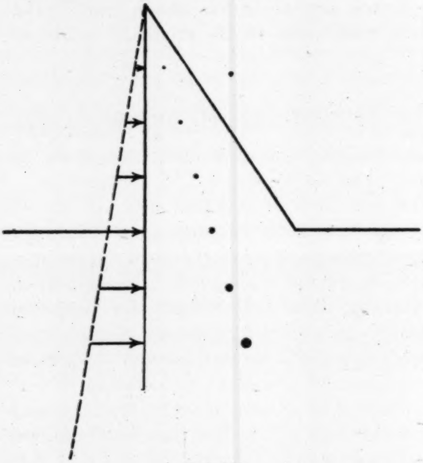


FIG. 32

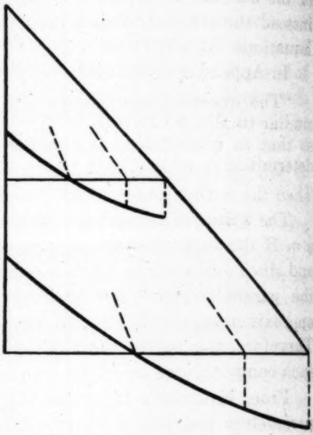


FIG. 33

This opinion is based on rigorous theoretical considerations contained in a previous paper by the writer, entitled "The Validity of Lévy's Elastic Regimen in Gravity Dams",[83] in which paper the deflection curves of the foundation (considered as unlimited), were determined and plotted both with and without the influence of direct water pressure at the bottom of the reservoir. Since the time of its presentation European authorities have shared this opinion fully. This was evident, for example, in a paper on dams presented by Professor Probst at the World Power Conference.[84]

The foregoing general objections refer similarly to the experiments by Mr. Barnes on a model having a moderately wide base, rectangular in shape, and clamped along three sides. The water pressure was represented by a relatively small number of concentrated loads applied normally to the up-stream face.

Mr. Barnes' stress diagram was a kind of regular curve of third degree with a point of contraflexure, whereas Fig. 6, as presented by Mr. Jakobsen, has the general form of a second-degree curve. Both curves agree, inasmuch as they involve an increase of stress at the up-stream face. On the down-stream face, however, Mr. Barnes found an increase of stress, while the author's diagram shows a decrease when compared with the linear distribution. This is a basic difference. Furthermore, the magnitude of the maximum stresses on the up-stream face is quite different in the two diagrams. The author's diagram involves an increase of stress of 50%, while Mr. Barnes' diagram indicates the increase as being equal to 200 per cent.

---

[83] *Energia Elettrica,* March and April, 1927.

[84] *Zeitschrift* des Vereins Deutscher Ingenieure, 1930, Nos. 13 and 15.

Evidently, one of the two diagrams is entirely in error. The difference in the character of the diagrams is peculiarly striking, because the author disposes of four parameters, that is, of a virtual third-degree parabola, which fact would have permitted a selection of a diagram of the type given by Mr. Barnes.

This difference cannot be explained by stating that Mr. Jakobsen's Fig. 6 represents conditions at the base of the profile studied by Mr. Barnes, while Mr. Barnes' diagram refers to a horizontal section of the profile at the level of one-twentieth of the height. In fact, owing to the special structure of the stress system, introduced by Mr. Jakobsen, $\sigma = r\, f\,(\beta)$, his diagram retains its character of a second-degree curve in all horizontal sections of the profile, and thus, also, in the section considered by Mr. Barnes, as shown in Fig. 33.

A paper by Wolf, published in 1914,[85] introduces a method which gives results that are considerably superior to those obtained by Mr. Jakobsen. Wolf deals with a perfectly elastic stress system, and also takes into consideration, at least to a certain extent, the interaction between the dam and its foundation.

After having pointed out the imperfections of the author's analysis, the writer wishes to explain, briefly, how the same method of Ritz may yield satisfactory results if properly applied.

The effect of interaction between the dam itself and a limitless foundation must be taken into consideration. Furthermore, the water pressure acting directly on the bottom of the reservoir will have considerable influence on the deformation of the foundation and, consequently, it will effect the stress distribution in the dam, especially in the more important lower elevations. Hence, the principle of least work must be applied to the entire structure as a unit, including not only the dam, but also the limitless foundation upon which it stands. This phase of the problem must be considered in evaluating the integral of work:

$$A = \frac{1}{2\,G} \int\!\!\int \left[\frac{1}{2}\,(\delta_x{}^2 + \delta_y{}^2) - \frac{1}{2\,(m+1)}\,(\delta_x + \delta_y)^2 + \tau_{xy}{}^2 \right] dx\, dy \ldots (91)$$

The principle of least work provides for the selection of a stress system from a given group which selection represents the best approximation to the true stresses. In other words, the average of the differences between the true stresses and the selected ones in all points of the dam, is smaller than the average of the differences between the true stresses and those determined by any other stress system of the given group.

It is possible, however, that in certain isolated regions of the profile such differences between the true and the selected stresses may be considerably greater than those between the true stresses and the stresses determined by some other unselected system of the given group. The stress system selected by the principle of least work will supply the best average approximation to the actual stresses; but it may also supply results that represent the actual stresses much less correctly in certain parts of the profile (as, for example, along the base) than some or many others of the systems that were not so selected.

---

[85] "Zur Integration der Gleichung $\nabla \nabla 0 = 0$ durch Polynome im Falle des Staumauer Problems," *Sitzungsberichte* der Kaiserlichen Akademie der Wissenschaften in Wien, 1914, v. 123, No. 2.

Relatively, designers are not so much interested in conditions at the higher elevations, but rather in the lower parts of the dam, especially along the base. Therefore, the writer proposes to use parameters referring to the points of the base only. The tentative stress system will be applied only on the base. In this way, the source of error previously mentioned is practically eliminated. Besides, the possibility of another group of errors is greatly reduced as may be seen from the following: The author prescribes at every point of the profile the stresses which, in reality, should be determined, while the writer prescribes them only on the base.

The assumption of tentative stresses along the base alone has still another advantage, namely, that the actual elastic stress system corresponding to a certain load and boundary stress is absolutely fixed; hence, there are no indeterminate stresses to be encountered in the interior of the section. On the other hand, a tentative stress distribution with four or five parameters may be expected to furnish results that agree quite closely with the actual stress condition at the base.

The only remaining problem is to determine, according to the principles of elasticity, the stresses in the interior if the load and the stresses along the base are known, since these values must be substituted in their proper place in Equation (91), in order to compute the work of deformation. This is a very difficult problem, even when boundary conditions are most simple. To solve it analytically for the more complicated systems of normal and shearing stresses prescribed along the base would appear to be almost hopeless. Furthermore, the design of gravity dams is not limited to triangular sections. Other shapes, including some that cannot be treated analytically, should be analyzed by some method, and there are no analytical means available for performing such a formidable task.

In his research work the writer reached this stage at the beginning of 1927, and reported on it to J. L. Savage, M. Am. Soc. C. E., of the U. S. Bureau of Reclamation, more than two years ago He determined upon a solution somewhat later, describing it fully to Franklin Thomas, M. Am. Soc. C. E., in July, 1929. The new concept of the exterior work of deformation was substituted for the usual concept of an interior work of deformation. Suppose, for example, that a system of loading is tentatively applied to the dam along the base line. This load will cause stresses in the interior of the section and the work of the deformation will be as expressed in Equation (88). However, the work performed by the interior stresses is equal to that performed by the load. On the other hand, the work performed by the load (that is, by a group of forces), is measured by the sum of products formed by the single forces and the displacement of their points of application in their various directions.

These deflections may be determined experimentally by the use of an elastic model, and thus the work performed by a trial set of forces may be evaluated without the necessity of knowing all the stresses in the interior.

Computing the work performed by every individual unit of a competing stress system, both in the dam itself and in the limitless half plane that represents the foundation, the particular stress system that involves the least work will be selected.

FERNAND CAMPUS,[86] ESQ. (by letter).—Since the paper is purely theoretical, the writer will first consider the question from that point of view. He believes that the author does not make correct use of the principle of least work. This principle gives rise to great difficulties in interpretation and application. Föppl expressly warns against the errors it may induce.[87] The author gives a faulty interpretation of Föppl's text in the final reference in Appendix II.

The theorem of least work is only applicable in the case of elasticity. This point is duly established in the works of Clapeyron, Lamé, Castigliano, Menabrea, Boussinesq, Maurice Lévy, Bertrand de Fontviolant,[88] Föppl,[89] and others. The principle of least work and the principle of virtual velocities are not, as the author seems to consider them, essentially different; they are identical or, more correctly, they are only slightly different expressions for one and the same principle or assumption—the hypothesis of elasticity. This is demonstrated clearly by de Fontviolant and especially by Föppl's treatise.[90] Föppl states the principle of virtual velocities in the form of the law of least work of deformation, and he proves beyond any possible doubt that these two principles are only valid when the hypothesis of elasticity is assumed.

If so, the stresses must satisfy the author's Equation (43)[91] which has not the same meaning as Föppl's compatibility equations;[92] or Airy's stress function may be determined, which must satisfy Equation (42). The author improperly identifies this formula with the compatibility equation in plane elasticity. As a matter of fact, Equations (42) and (43) are identical, but quite different from the compatibility equations.

In the last statement of Appendix II, the author has misinterpreted Föppl's text. Föppl states[93] explicity that the stresses, n_x, n_y, n_z, t_{xy}, t_{yz}, and t_{zx}, satisfy the hypothesis of elasticity, thus, also, the compatibility equations, and he mentions very small variations, δn_x, δn_y, δn_z, δt_{xy}, etc., which do not satisfy them, but satisfy the equilibrium equations. He proves then that the corresponding value of δL is zero; therefore, the work done by the stresses, n_x, n_y, n_z, t_{xy}, etc. is a minimum. This relates only to the stresses satisfying the elasticity equations, and not to the other internal forces. It is true[94] that the stresses computed by the Ritz method do not satisfy the elasticity equations; but this method is only approximate; it cannot exactly satisfy the laws of elasticity and all the formulas and principles derived from these laws.

---

[86] Prof., Liége Univ., Liége, Belgium.

[87] "Drang und Zwang," 1920, Article 9, p. 68.

[88] "Les méthodes modernes de la resistance des matériaux," Paris, 1920.

[89] "Drang und Zwang," 1920, Article 13, p. 99.

[90] *Loc. cit.*, Article 9, pp. 61–66, inclusive.

[91] "Résistance des matériaux et elasticité," G. Pigeaud, p. 736, Paris, 1920.

[92] "Drang und Zwang," 1920, Articles 7 and 40.

[93] *Loc. cit.*, 1920, Article 13.

[94] *Loc. cit.*, 1920, Article 55, p. 325.

In other words, the basic principle is that of elasticity. The elasticity equations, the author's Equation (43), the Airy function (the author's Equation (42)), the principle of virtual velocities, the principle of least work (which states that the total differential of the work is zero and not certain partial differentials as in the Ritz method)—all these formulas and principles are but slightly different forms for the expression of one and the same fundamental hypothesis—elasticity. The principle of the superposition of effects depends also upon this fundamental hypothesis. This principle is necessarily true in this case, when elasticity is assumed to apply. The writer will now prove that the author did not properly consider these fundamental facts in his paper.

Arbitrarily, Mr. Jakobsen assumes the law,

$$n_y = A_y + B_x + C\frac{x^2}{y} + F\frac{x^3}{y^2}$$

which is homogeneous. In order to determine *A, B, C,* and *F,* he uses the equations of static equilibrium and to these he joins, by means of the Ritz method, the relations, $\frac{dL}{dC} = 0$ and $\frac{dL}{dF} = 0$.

There is nothing to justify him in considering this the principle of least work; it is only an approximation commonly designated as the Ritz method. The author arrives at formulas for n_y, n_x, and t, and he ascertains that these quantities do not satisfy the compatibility equations, which means exactly that they do not harmonize with the hypothesis of elasticity. As a matter of fact, they should not, because they were obtained by the Ritz method. Consequently, the formulas and principles derived from elasticity are not applicable to these stresses, although the author assumes that elastic deformations occur. For instance, the work of 16 653 000 ft-lb. in Table 3, is not a minimum. The minimum is 17 218 000 ft-lb., as computed by means of the linear solution. If the work calculated, using the author's stresses, is 3.26% less than the minimum, this is due entirely to the inaccuracy of the computation. Equations (14), (29), (30), and (31), for the work, only have a bearing upon stresses that satisfy the assumption of elasticity. The internal forces as given by Mr. Jakobsen do not satisfy this condition. This also explains the noticeable differences between the stresses given in Articles VII and VIII.

If they satisfy, actually, the principle of superposition, it results only from the fact that Equations (16), (17), (25), and (26) (which determine the constants, *A, B, C,* and *F*) are linear and homogeneous functions of *w* and *g*.

Now, examine the differences between the principle of elasticity and the author's formulas. It can be proved by accepting the author's assumptions, that the exact solution is linear (ignoring all lateral deformations, in accordance with Appendix III, which is applicable to every condition).

The writer has pointed out that the law chosen by the author (Equation (15)) is homogeneous. In the case of a straight line, $x = ky$, in the interior of the dam:

$$n_y\, k = y\,(A + Bk + Ck^2 + Fk^3) = Ky \quad \text{.............(92)}$$

The validity of this statement is not limited to the up-stream face as given by the author in Article VI; it is general. Furthermore, $n_x k = K' y$ and $t k = K'' y$. This variety of stress distribution is designated homothetic, that is, similar and similarly placed.

The summit of the triangular section of the dam is the homothetic center. In cases of homothetic stress distribution it is easy to prove that the solution satisfying the hypothesis of elasticity and all its consequences—for example, least work—is the linear solution. A very simple demonstration is given in the work by Pigeaud.[95]

When the author assumes his homogeneous law, he implicitly assumes a homothetic condition. In this case the exact solution satisfying elasticity is the linear law; and this is the only solution that verifies the elasticity equations and the formula of work, and the principles of least work and of superposition. The stresses found by the author by means of the Ritz method show great divergence from the exact solution. This demonstrates the degree of error that the Ritz method may involve. It is sufficiently illustrated by Figs. 4, 5, 6, and 7, of the paper.

In Article II, Mr. Jakobsen seems to consider the assumption of linear stress distribution as arbitrary; and, in Article III, he maintains that it is equivalent to stating that horizontal planes remain plane. It is possible that this opinion is based on a memoir by Lévy[96] which is now becoming obsolete. Incidentally, the author mentions the consideration of normal instead of horizontal sections, thus referring to the work of Résal.

These points of view have been displaced by a more precise conception of the linear law. Consider a triangular dam of infinite height, infinite resistance, and infinite elasticity; the stress distribution is a plane and a homothetic one. Each unit of volume weighs exactly as much as any other; the pressure is hydrostatic and the water level is at exactly the same height as the ridge of the dam. Under these conditions the general equations of internal equilibrium and of elasticity lead to a linear stress distribution which is identical to that of Lévy. The demonstration of this theorem is to be found in the works of many authors. The writer needs only to mention Pigeaud's treatise,[97] in which the starting point is the equation,

$$\frac{d^2\,(n_x + n_y)}{dx^2} + \frac{d^2\,(n_x + n_y)}{dy^2} = 0 \ldots\ldots\ldots\ldots\ldots\ldots (93)$$

Mr. S.-D. Carothers, of London, England, has given a new and very elegant demonstration of this theorem. He has applied the Airy function and polar co-ordinates in accordance with Love's "Treatise on Elasticity" (4th Edition).[98]

It should be noted that this theorem does not imply any assumption as to the deformations or the conservation of plane horizontal sections. On the contrary, the writer has proved,[99] in a most general way, that assuming the linear stress distribution, plane horizontal sections actually become curved and are changed into cylinders with conic sections.

---

[95] "Resistance des matériaux et elasticité," Paris, 1920, pp. 736 and 737.

[96] Comptes rendus de l'Academie des Sciences, Paris, 1895.

[97] "Resistance des matériaux et elasticité," Paris, 1920.

[98] *Proceedings*, First International Cong. for Concrete and Reinforced Concrete, Liége, 1930.

Therefore, the correct solution for the equations of stress, corresponding to the author's assumptions (elasticity and homogeneity, implying an homothetic condition), is the linear solution, which satisfies the principles of least work and of superposition of the effects of forces. The question of whether to consider horizontal or normal sections is unimportant; they do not influence the equations. The writer does not mean to say that the linear solution is consistent with actual conditions; but neither does the correct solution depend on the principle of least work. It depends, as the author states in the final sentence of Article III, upon the influence of the foundation. All authors agree on this point, Résal, Mesnager, Pigeaud, Haegelen, etc.[100]

The linear solution is based on the assumption of an infinitely large dam in order to avoid the difficulties resulting from the foundation. There is only one inaccuracy in the linear solution, namely, dams are actually finite and they have a foundation. Therefore, to the linear stress distribution, must be superposed the secondary stresses arising from the contact between the base of the dam and the foundation soil. A mathematical solution for this problem is impossible with the present limitations of science. It is clear, however, that these stresses are most important near the base, and that at a certain distance above, the linear law is only slightly altered. This is demonstrated in photo-elastic tests by Mesnager, according to Haegelen's opinions.[101]

The writer believes, furthermore, that this is proved (within the limits of experimental errors) by Fig. 9, referring to experiments by Ottley and Brightmore. In this diagram, however, no allowance is made for conditions at the two extremities of Section 6-6. It indicates continuity between the dam and the base, representing the soil and producing singular stresses in the angles. Nothing of that kind exists in a real foundation. Sections 5-5, 4-4, and 3-3 (Fig. 9) are less influenced by these unreal conditions, and the writer's remarks are more plausible for such places, although conditions at these sections are influenced by the fact that the dam is not triangular and also by corresponding stresses originating at the crest (especially in Section 3-3). Mathematical proof for linear stress is only possible for strictly triangular dams. The author mentions (not without reason) the numerous facts which may influence the reliability of the test results. The approximations made in reading the instruments may be added to the list. Hence, extreme caution is necessary in interpreting tests relating to dams. The interpretation based on the English investigation in favor of the linear law is quite reasonable.

In any case a non-linear stress distribution based on the theory of elasticity and taking into account the complications resulting from the foundations, in order to have a more approximate image of the real stress law in a dam, should not have as its starting point a homogenous formula implying homothetic stress distribution. The foundation suppresses homothetic distribution; this is undoubtedly the point which the author ignored.

---

[99] *Bulletin* de la Classe des Sciences de l'Academie Royale de Belgique; and *Le Genie Civil,* January 3, 1931.

[100] See, also, F. Campus, "Conditions de stabilité des barrages à gravité en béton," Paris et Liége, pp. 37 *et seq.*

[101] Congrès de l'Union Internationale des producteurs et distributeurs d'energie électrique, Paris, 1928.

Briefly, Mr. Jakobsen's paper is instructive, not only because it represents a meritorious effort, but also because of the objections that arise in discussing it. Theoretically, the method contains a radical error; practically, the writer does not think it is superior to the ordinary method and he believes it will meet with no success. It is more complicated even for the case of a vertical up-stream face which is the only one discussed by the author. In an actual dam this face is inclined; but this would have occasioned great analytical difficulties. Hence, the author's method is not as general as the linear law.

The writer agrees with Mr. Jakobsen that it is less important to limit the maximum compression at the down-stream face than to require minimum compression at the up-stream face when the reservoir is filled. However, the writer believes that the formulas presented by Lévy are satisfactory for application in this case. The advantage of imposing more severe conditions, in accordance with the author's remarks in Articles IX and X, seems quite illusive considering that many dam constructors are so severely opposed to the section proposed by Maurice Lévy.

The writer has been somewhat loath to discuss this paper, especially since he was forced to criticize the contribution of an author whose valuable work in the study of dams he holds in great esteem.

A FLORIS,[102] ESQ. (by letter).—The author questions the correctness of basing the analysis of gravity dams on a linear distribution of the vertical normal stress, and proposes a new law of stress distribution that differs widely from that determined by the rule of the trapezoid. His investigations are interesting because they lead to results that are not in accord with the findings of other writers. The possibility of disclosing the reason for the discrepancy between the classical theory and that presented by the author is very attractive to any one of an inquiring turn of mind. On the other hand, to those who are concerned merely with the application of the theory to practical cases, such differences are bewildering. For this reason, any discussion on this able paper must necessarily attempt to reconcile these important differences.

Mr. Jakobsen assumes a triangular profile of unlimited height acted upon by the water pressure and the weight of the dam; that is, he neglects the influence of the foundation on the stress distribution in the dam. Exactly the same assumptions are made in the classical theory, developed by Maurice Lévy,[103] Paul Fillunger,[104] E. Selényi,[105] and others. It follows, therefore, that although the problem is the same in both cases, the results are quite different. The writer will explain the reason for this difference in some detail.

---

[102] Civ. Engr., Los Angeles, Calif.

[103] "Sur l'équilibre élastique d'un barrage en maçonnerie à section triangulaire," par Maurice Lévy, *Comptes rendus* des Séances de l'Académie des Sciences de Paris, Vol. 127, 1898, p. 10.

[104] "Drei wichtige Spannungszustände des keilförmigen Körpers," von. Dr. Ing. Paul Fillunger, *Zeitschrift für Mathematik und Physik,* Vol. 60, 1912, p. 273.

[105] "Sul regime elastico nelle dighe di tipo gravità," by E. Kalman, *L'Energia Elettrica,* February, 1927.

Fig. 34 illustrates a dam with a triangular profile. It has a vertical up-stream face and unlimited height, and a thickness equal to unity. The water surface is assumed to be level with the top of the dam which is also the origin of the co-ordinates, x and y. The equations for the up-stream and down-stream boundaries are, respectively, $x = 0$ and $x = \tan \alpha\, y = \kappa\, y$. Furthermore, assume that the stress distribution in the interior follows the linear law as expressed by the formulas,

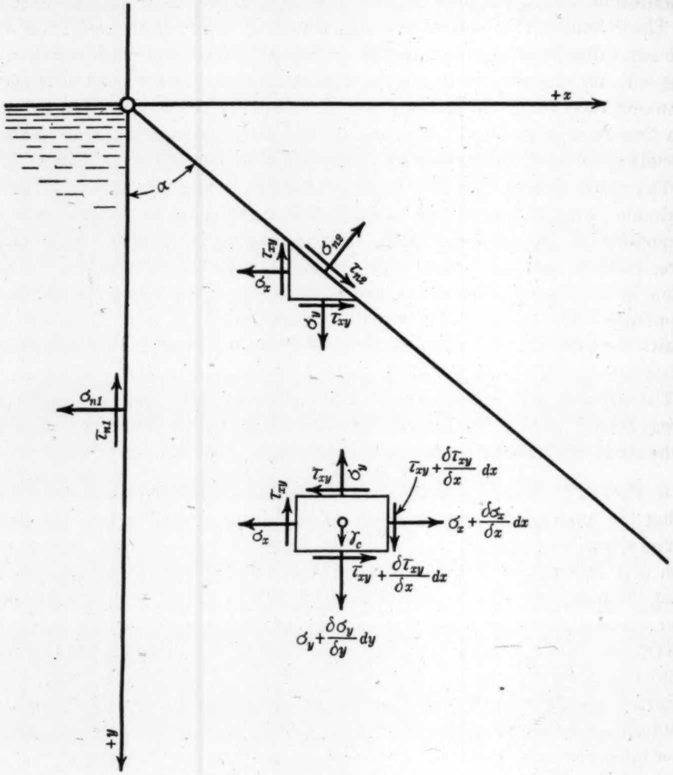

$+x$
α
τ_{xy}
σ_x
σ_{n2}
τ_{n2}
σ_y
τ_{xy}
σ_{n1}
τ_{n1}
τ_{xy}
τ_{xy}
σ_y
$\tau_{xy} + \frac{\delta \tau_{xy}}{\delta x}\, dx$
σ_x
γ_c
$\sigma_x + \frac{\delta \sigma_x}{\delta x}\, dx$
$\tau_{xy} + \frac{\delta \tau_{xy}}{\delta x}\, dx$
$\sigma_y + \frac{\delta \sigma_y}{\delta y}\, dy$
$+y$

FIG. 34

$$\sigma_x = a\,x + b\,y \qquad (94)$$

$$\sigma_y = c\,x + f\,y \qquad (95)$$

and,

$$\tau_{xy} = e\,x + h\,y \qquad (96)$$

Consider a rectangular element, as shown in Fig. 34, in the interior of the dam. From the condition that the horizontal and vertical forces acting on this element must be in equilibrium, the following equations may be written:

$$\frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} = 0 \qquad (97)$$

$$\frac{\partial \sigma_y}{\partial y} + \frac{\partial \tau_{xy}}{\partial x} + \gamma_c = 0 \qquad (98)$$

The boundary condition will be expressed by the equations:

$$\sigma_{n1} = \sigma_x = -\gamma_w y \qquad (99)$$

$$\sigma_{n2} (x, \kappa x) = 0 \qquad (100)$$

$$\tau_{n1} = \tau_{xy} = 0 \qquad (101)$$

and,

$$\tau_{n2} (x, \kappa x) = 0 \qquad (102)$$

In these equations, γ_c and γ_w are the specific weights of the dam and of the water, respectively. If the stresses, σ_x, σ_y, and τ_{xy}, in two mutually perpendicular planes are known, then the stresses in a plane forming an angle, α, with the vertical will be given by the following equations:

$$\sigma_n = \sigma_x \cos^2 \alpha + \sigma_y \sin^2 \alpha - 2 \tau_{xy} \cos \alpha \sin \alpha \qquad (103)$$

and,

$$\tau_n = (\sigma_x - \sigma_y) \sin \alpha \cos \alpha + \tau_{xy} (\cos^2 \alpha - \sin^2 \alpha) \qquad (104)$$

For the down-stream boundary:

$$\sin \alpha = \frac{\kappa}{\sqrt{1 + \kappa^2}} \qquad (105)$$

and,

$$\cos \alpha = \frac{1}{\sqrt{1 + \kappa^2}} \qquad (106)$$

in which, $\kappa = \tan \alpha$, as before.

Introducing in Equations (103) and (104) the values of Equations (94), (95), (96), (105), and (106), Equations (107) and (108) are obtained, for the down-stream boundary, as follows:

$$\sigma_{n2} = [a \kappa + b + c \kappa^3 + f \kappa^2 - 2 e \kappa^2 - 2 h \kappa] y = 0 \qquad (107)$$

and,

$$\tau_{n2} = [a \kappa^2 + b \kappa - c \kappa^2 - f \kappa + e (1 - \kappa^2) \kappa + h (1 - \kappa^2)] y = 0 \qquad (108)$$

Because these expressions are valid for all values of y, it follows that:

$$a \kappa + b + c \kappa^3 + f \kappa^2 - 2 e \kappa^2 - 2 h \kappa = 0 \qquad (109)$$

and,

$$a \kappa^2 + b \kappa - c \kappa^2 - f \kappa + e (1 - \kappa^2) \kappa + h (1 - \kappa^2) = 0 \qquad (110)$$

The same expressions in a somewhat modified form may be derived more easily by making use of the following theorem which holds true for all points of a boundary free from loads in any arbitrary two-dimensional domain:[106]

$$\sigma_x = \kappa \tau_{xy} = \kappa^2 \sigma_y \qquad (111)$$

Equation (111) may be written as two equations:

$$\sigma_x - \kappa \tau_{xy} = 0 \qquad (112)$$

and,

$$\tau_{xy} - \kappa \sigma_y = 0 \qquad (113)$$

---

[106] "Sulla validità dell' ipotesi Navier nelle mensole," by E. Kálmán, Reale Instituto Lombardo di Scienze e Lettere, Estratto dai Rendiconti, Vol. LXI, Fasc. XVI–XX, 1928.

As before, substitute Equations (94), (95), and (96), into Equations (112) and 113. The result will be:

$$a\,\kappa + b - e\,\kappa^2 - h\,\kappa = 0 \dots\dots (114)$$

and,

$$-c\,\kappa^2 + e\,\kappa - f\,\kappa + h = 0 \dots\dots (115)$$

Combining Equations (94), (96), (99) and (101):

$$\sigma_x = -\gamma_w\,y = -(a\,\kappa + b\,y)_{\kappa=0} = -b\,y \dots\dots (116)$$

and,

$$\tau_{xy} = 0 = (e\,\kappa + h\,y)_{\kappa=0} = h\,y \dots\dots (117)$$

from which, $b = -\gamma_w$ and $h = 0$.

Differentiating Equations (94), (95), and (96) with respect to x and y and introducing the resulting values into Equations (97) and (98), the following expressions are derived:

$$a + h = 0 \dots\dots (118)$$

and,

$$f + e + \gamma_c = 0 \dots\dots (119)$$

Hence, $a = 0$. By introducing the values, $b = -\gamma_w$, $h = 0$, and $a = 0$, into Equations (109) and (110):

$$-\gamma_w + c\,\kappa^3 + f\,\kappa^2 - 2\,e\,\kappa^2 = 0 \dots\dots (120)$$

and,

$$-\gamma_w\,\kappa - c\,\kappa^2 - f\,\kappa + e\,\kappa\,(1 - \kappa^2) = 0 \dots\dots (121)$$

Solving for e:

$$e = -\frac{\gamma_w}{\kappa^2} \dots\dots (122)$$

Substituting this value of e in Equation (119),

$$f = -\left(\gamma_c - \frac{\gamma_w}{\kappa^2}\right) \dots\dots (123)$$

Introducing Equations (122) and (123) into Equation (120):

$$c = \left(\gamma_c - 2\,\frac{\gamma_w}{\kappa^2}\right)\frac{1}{\kappa} \dots\dots (124)$$

Making use of the parameter, $b = -\gamma_w$, $h = 0$, and $a = 0$, as well as those expressed by Equations (122), (123), and (124), the general expressions in Equations (94), (95), and (96), respectively, will take the final form,

$$\sigma_x = -\gamma_w\,y \dots\dots (125)$$

$$\sigma_y = \left(\gamma_c - 2\,\frac{\gamma_w}{\kappa^2}\right)\frac{x}{\kappa} - \left(\gamma_c - \frac{\gamma_w}{\kappa^2}\right)y \dots\dots (126)$$

and,

$$\tau_{xy} = -\frac{\gamma_w}{\kappa^2}\,x \dots\dots (127)$$

Thus, it is proved that the tentative linear stress distribution, as defined by Equations (94), (95), and (96), can be fitted to the boundary conditions of the problem by choosing its six parameters properly. For the special case, $\alpha = 45°$, $\gamma_w = 1$ and $\gamma_c = 2$, Equations (125), (126), and (127), respectively, become $\sigma_x = - y$; $\sigma_y = - y$; and $\tau_{xy} = - x$.

The linear expressions with respect to x and y given by Equations (125), (126), and (127) define the stresses at any point of the dam regardless of whether horizontal or inclined sections are used.[107] These equations express Lévy's well-known stress regimen.

The foregoing analysis shows in an unequivocal manner that, under the influence of water pressure and the weight of the dam, the stresses in a triangular profile must be linearly distributed if an additional straight-line section (either horizontal or inclined) is considered as a third boundary. The author prescribes a parabolic distribution of stress for this third boundary without attempting to prove that such a distribution will hold equally true in the interior of the profile. The up-stream and down-stream boundary conditions in both the author's and Lévy's cases are the same.

In applying the method introduced by W. Ritz, it is not merely sufficient to minimize the work by choosing arbitrarily a function belonging to a competitive system; it is also necessary to take care that this system is correctly chosen from the infinite number of possible choices, any one of which might supply a minimum function by a proper choice of the parameters. For example, among n competitive systems, there are n solutions which satisfy the condition of least work. In all problems of this nature it is necessary to give the characteristics of the functions that are permitted to compete. For this purpose the shape of the curve that represents these functions is assumed only approximately and upon this choice the correct solution of the problem depends to a large extent. Otherwise, in order to obtain the correct solution, it would be necessary to take into consideration all competitive systems that contain a sufficient number of functions in series, each series having a sufficient number of terms.

For the reasons explained in the foregoing remarks, the writer is forced to conclude that the results of the author's theory are, unfortunately, misleading. The Ritz method of least work has been applied successfully to the design of circular tanks[108] and arch dams,[109] but the writer considers that stress analysis of triangular gravity dam profiles of unlimited height is not possible by this method.

To prove the correctness of his theory, the author refers to the well-known English tests on models. It is questionable whether the analysis of these two divergent cases can be made to agree. The stress distribution in the models was influenced considerably by the foundation conditions at the dam. This influence was disregarded entirely by Mr. Jakobsen. In the upper parts of the models the stresses seem to follow a linear rather than a parabolic law. This is in agreement with theory because, according to Saint-Venant's principle, the stresses in these models cannot be influenced materially at elevations considerably above the base. The stresses will follow the linear law quite closely, so that Lévy's regimen will be approximately correct at such elevations.

---

[107] "Ueber den Spannungszustand in Staumauern", von E. Kálmán, *Beton und Eisen*, May 5, 1930, p. 174.

[108] "Die Berechnung der Spannungen in zylindrischen Behälterwänden mit veränderlichem Querschnitt," von Theodor Pöschl, *Sitzungsberichte* der Kaiserl. Akademie der Wissenschaften in Wien, June, 1912. See, also, "Berechnung von Behältern nach neueren analytischen und graphischen Methoden," von Dr. Th. Pöschl und Dr. K. von Terzaghi, Berlin, 1926, p. 127.

[109] "Sulle piastre curve", Nota dell' Ing. Prof. Kálmán, Estratto dai Rendiconti del R. Instituto, Lombardo di scienze e lettere, Adunanza dell 11 Aprile, 1929, Serie II, Vol. LXII, Fasc. VI–X.

The writer doubts very much whether progress can be made in the analysis of a triangular dam profile when the influence of the foundation upon the dam is neglected. The important problem that still remains to be solved is that of considering the dam and its unlimited rock foundation acting together as an elastic body. This is impossible at present, however, due to the great mathematical difficulties involved. Therefore, it is necessary to make certain assumptions regarding the deformation of the rock or to be guided by the results of experiment.

As is well known, the stress components in any two-dimensional domain can be expressed by means of the Airy stress function satisfying a partial differential equation of the fourth order under given boundary conditions. (See the author's Equations (40) and (42).) Among the particular solutions of this differential equation are also those that can be expressed in the form of polynomials.

Professor Karl Wolf has expressed the normal loads (water pressure) at the up-stream boundary of a triangular dam profile by an arbitrary polynomial of the nth degree with respect to y.[110] It is evident in this case that the tangential loads are zero. On the other hand, the down-stream boundary of the dam is free from loads while the rigid foundation represents the third boundary of the triangular profile.

The coefficients can be determined in such a way that the polynomial will express with sufficient accuracy not only the water pressure within the limits, $y = 0$ (at the top) and $y = h$ (at the bottom), but, also (because of the assumed rigidity of the foundation), the vertical displacements for $x = h$ shall be a minimum. This can be accomplished in the following manner. The height and the base of the profile are each divided into n equal parts. For each of the n intervals of the height of the profile, the integral of the polynomial is made approximately equal to the corresponding water pressure (by the method of least squares). For each n interval of the base ($y = h$), the integral of the vertical displacements is made equal to zero, in order to satisfy the condition imposed by assuming that the base is rigid. This procedure supplies n linear equations for the determination of n unknown coefficients in the assumed polynomial.

Consider a polynomial of the fifth degree with respect to y, in which the down-stream face of the dam makes an angle of 45° with the up-stream face, and the assumed height of the profile as well as the specific weight of water are assumed equal to unity. Let the specific weight of the dam equal 2. For this case, Professor Wolf determines the following expression for the horizontal normal stress at the up-stream face:

---

[110] "Zur Integration der Gleichung $\Delta \Delta F = 0$ durch Polynome im Falle des Staumauern-problems", von Karl Wolf, *Sitzungsberichte* der Kaiserl. Akademie der Wissenschaften in Wien, February, 1914.

$$(\sigma_x)_{x=0} = -y + 0.116\,y - 1.101\,y^2 + 3.325\,y^3 - 4.000\,y^4 + 1.668\,y^5 \quad (128)$$

At the base of the dam, that is, for $y = h$, Equation (128) gives a value for σ_x which differs from the water pressure by only 7 per cent. The first term of this equation corresponds to the stress given by Equation (125) for the special case, namely, $\sigma_x = -y$.

The remaining stresses at the base and at the mid-height of the dam are, respectively,

$$(\sigma_y)_{y=1} = -0.836 - 0.939\,x + 0.829\,x^2 + 0.904\,x^3 - 2.452\,x^4 + 1.766\,x^5 \quad (129)$$

$$(\tau_{xy})_{y=1} = -1.288\,x + 1.677\,x^2 + 3.495\,x^3 + 1.898\,x^4 + 0.490\,x^5 \quad (130)$$

and,

$$(\tau_{xy})_{y=0.5} = -1.123\,x + 0.321\,x + 0.459\,x^3 - 1.057\,x^4 + 0.490\,x^5 \quad (131)$$

Equation (129) corresponds to the rule of the trapezoid in Lévy's regimen. For different values of x in Equation (129), corresponding values of σ_y are, as follows:

x	$(\sigma_y)_{y=1}$
0	0.836
0.1	0.930
0.3	1.035
0.5	1.084
0.7	1.080
0.9	0.903
1.0	0.730

When plotted, these values appear in the form of a parabolic curve concave upward. At the up-stream or down-stream faces the stress is smaller than that resulting from Equation (126) ($\sigma_y = -y$), while at the middle of the section the stress is somewhat greater. In Equation (129) y does not appear because the expression is valid only for $y = 1$. For $x = 0$, in Equation (129), $\sigma_y = -0.836$, while for $y = 1$ in the special case of Equation (126), $\sigma_y = -1$. It is important to note that the shearing stress toward the top of the dam rapidly approaches that determined by Lévy's linear law. Even at the base the distribution of the shearing stress does not differ radically from that given by Lévy's regimen. For example, in both cases, the shearing stresses vanish at the up-stream face (for $x = 0$) throughout the height of the dam.

It is true that Professor Wolf's solution may be open to certain objections. However, it represents an elastic state which is correct while Mr. Jakobsen's solution is not. Furthermore, the results obtained by this method are not extreme so that they cannot be rejected as useless. Unfortunately, the same reasoning cannot be applied to the author's findings. Before accepting a new theory in which there are substantial differences when compared with methods in current use, it is necessary that such theory be proved to be correct beyond doubt.

Further advance can be made in the theory of stress distribution in triangular gravity dams by modifying Lévy's stress regimen at the base to take care of the conditions actually existing at that place.

In analyzing the stress condition in triangular dam profiles near the foundation, Professor Kálmán has proved that the water pressure at the bottom of the reservoir has an appreciable influence on the stresses in the dam itself.[111] This is a fact that has been overlooked in the English tests and in all theoretical investigations on stresses in dams. Furthermore, Professor Kálmán shows by a vigorous analysis that the stresses at the bottom of a triangular profile cannot be linear. By cutting the dam just above its foundation and applying Lévy's stresses at the cut surfaces as loads, Professor Kálmán finds that both surfaces do not remain congruent after deformation; and this fact proves conclusively that the stresses in that part of the dam cannot be linear. If they were linear, the separated surfaces must remain congruent after deformation. In reality, of course, the dam is not cut at this place, and both surfaces must remain congruent before and after deformation. Therefore, the stresses developed therein must necessarily differ from those given by Lévy's regimen.

Basing his thesis on the theory that the two separated surfaces must be congruent, Professor Kálmán investigates the application of corrective stresses on these surfaces. This should be done in such a way that the added stresses will assist in improving the conditions of congruence as stated. Obviously, this method of attack, which will correct Lévy's stress regimen, is a marked advance in the design of gravity dams. It is an outstanding contribution to the design of these structures which, perhaps, will yield results of far-reaching consequence.

Robert E. Glover,[112] **Esq.** (by letter).—In Part IV[113] of his paper Mr. Jakobsen states that,

> "Instead of the simple Equation (10) for n_y, the condition that Equation (14) must be a minimum is now introduced. This is a problem in variational calculus, but instead of treating it as such, which would be very complicated, a method of approximation may be resorted to, * * *."

The writer has observed that a certain amount of perplexity exists among engineers as to what is to be expected of the variational calculus. This is not surprising since the subject is not included among the courses of study usually prescribed for students in engineering colleges.

It is believed, therefore, that it would be helpful to solve the variational problem presented, in order to have the results available for comparison with those obtained by other methods.

---

[111] "Sulla validità del regime Lévy nelle dighe del tipo di gravità," by E. Kálmán, *L'Energia Elettrica*, March and April, 1927.

[112] Engr., U. S. Bureau of Reclamation, Denver, Colo.

[113] A statement of the theorem of minimum energy may be found in Paragraph 119 of Love's "Treatise on the Mathematical Theory of Elasticity," Fourth Edition.

Equation (14) is the expression for the potential energy of deformation. If z = unity, it reduces to,

$$L = \int\int\left[\frac{n_y^2 + n_x^2}{2} - \frac{(n_y + n_x)^2}{2\,(m+1)} + t^2\right] dy\, dx \ldots\ldots\ldots (132)$$

It will be convenient to express the stresses in terms of an Airy's function, F, which is connected with the stresses by the relations:

$$n_y = \frac{\partial^2 F}{\partial x^2} - w\,y\;;\; n_x = \frac{\partial^2 F}{\partial y^2}\;;\;\text{and } t = -\frac{\partial^2 F}{\partial x\, \partial y}$$

This form of expression will be permissible, providing it is consistent with the author's equilibrium Equations (6) and (7). This may be ascertained by direct substitution, as follows:

$$\frac{\partial n_y}{\partial y} = \frac{\partial^3 F}{\partial x^2\, \partial y} - w\;;\; \frac{\partial n_x}{\partial x} = \frac{\partial^3 F}{\partial x\, \partial y^2}\;;\; \frac{\partial t}{\partial x} = -\frac{\partial^3 F}{\partial x^2\, \partial y}\;;$$

$$\text{and } \frac{\partial t}{\partial y} = -\frac{\partial^3 F}{\partial x\, \partial y^2}$$

Then,

$$\frac{\partial n_y}{\partial y} + \frac{\partial t}{\partial x} + w = \frac{\partial^3 F}{\partial x^2\, \partial y} - w - \frac{\partial^3 F}{\partial x^2\, \partial y} + w = 0 \ldots\ldots (133)$$

$$\frac{\partial n_x}{\partial x} + \frac{\partial t}{\partial y} = \frac{\partial^3 F}{\partial x\, \partial y^2} - \frac{\partial^3 F}{\partial x\, \partial y^2} = 0 \ldots\ldots\ldots\ldots (134)$$

and,

$$L = \int\int\left[\frac{\left(\frac{\partial^2 F}{\partial x^2} - w\,y\right)^2 + \left(\frac{\partial^2 F}{\partial y^2}\right)^2}{2} - \frac{\left(\frac{\partial^2 F}{\partial x^2} - w\,y + \frac{\partial^2 F}{\partial y^2}\right)^2}{2\,(m+1)} + \left(\frac{\partial^2 F}{\partial x\, \partial y}\right)^2\right] dx\, dy \ldots\ldots\ldots (135)$$

The writer's computations indicate that if,

$$L = \int\int \phi\left[F, x, y, \frac{\partial F}{\partial x}, \frac{\partial F}{\partial y}, \frac{\partial^2 F}{\partial x^2}, \frac{\partial^2 F}{\partial y^2}, \frac{\partial^2 F}{\partial x\, \partial y}\right] dx\, dy$$

the condition for a minimum is,

$$\frac{\partial \phi}{\partial F} - \frac{d}{d\,x}\left(\frac{\partial \phi}{\partial p}\right) - \frac{d}{d\,y}\left(\frac{\partial \phi}{\partial q}\right) + \frac{d^2}{d\,x^2}\left(\frac{\partial \phi}{\partial r}\right) + \frac{d^2}{d\,y^2}\left(\frac{\partial \phi}{\partial s}\right) + \frac{d^2}{d\,x\,d\,y}\left(\frac{\partial \phi}{\partial t}\right) = 0 \ldots\ldots\ldots (136)$$

in which, $p = \frac{\partial F}{\partial x}$; $q = \frac{\partial F}{\partial y}$; $r = \frac{\partial^2 F}{\partial x^2}$; $s = \frac{\partial^2 F}{\partial y^2}$; $t = \frac{\partial^2 F}{\partial x\, \partial y}$; and ϕ is a symbol used to denote that the integrand is a function of the variables in the brackets. In the present case, F, x, $\frac{\partial F}{\partial x}$, and $\frac{\partial F}{\partial y}$, are absent, and, therefore,

$$\phi\left[F, x, y, \frac{\partial F}{\partial x}, \frac{\partial F}{\partial y}, \frac{\partial^2 F}{\partial x^2}, \frac{\partial^2 F}{\partial y^2}, \frac{\partial^2 F}{\partial x\, \partial y}\right]$$

$$= \frac{\left(\frac{\partial^2 F}{\partial x^2} - w\,y\right)^2 + \left(\frac{\partial^2 F}{\partial y^2}\right)^2}{2} - \frac{\left(\frac{\partial^2 F}{\partial x^2} - w\,y + \frac{\partial^2 F}{\partial y^2}\right)^2}{2\,(m+1)} + \left(\frac{\partial^2 F}{\partial x\, \partial y}\right)^2$$

Then,

$$\frac{\partial \phi}{\partial r} = \frac{\partial^2 F}{\partial x^2} - w\,y - \frac{\left(\dfrac{\partial^2 F}{\partial x^2} - w\,y + \dfrac{\partial^2 F}{\partial y^2}\right)}{(m+1)} \quad \text{.......(137)}$$

$$\frac{\partial \phi}{\partial s} = \frac{\partial^2 F}{\partial y^2} - \frac{\left(\dfrac{\partial^2 F}{\partial x^2} - w\,y + \dfrac{\partial^2 F}{\partial y^2}\right)}{(m+1)} \quad \text{..........(138)}$$

$$\frac{\partial \phi}{\partial t} = 2\,\frac{\partial^2 F}{\partial x\,\partial y} \quad \text{..........(139)}$$

and,

$$\frac{d}{d\,x^2}\left(\frac{\partial \phi}{\partial r}\right) = \frac{\partial^4 F}{\partial x^4} - \frac{\left(\dfrac{\partial^4 F}{\partial x^4} + \dfrac{\partial^4 F}{\partial x^2\,\partial y^2}\right)}{(m+1)} \quad \text{........(140)}$$

$$\frac{d^2}{d\,y^2}\left(\frac{\partial \phi}{\partial s}\right) = \frac{\partial^4 F}{\partial y^4} - \frac{\left(\dfrac{\partial^4 F}{\partial x^2\,\partial y^2} + \dfrac{\partial^4 F}{\partial y^4}\right)}{(m+1)} \quad \text{.........(141)}$$

$$\frac{d^2}{d\,x\,d\,y}\left(\frac{\partial \phi}{\partial t}\right) = 2\,\frac{\partial^4 F}{\partial x^2\,\partial y^2} \quad \text{................(142)}$$

The condition for a minimum states that the sum of Equations (140), (141), and (142), must equal zero; therefore, the condition for a minimum of potential energy of deformation is,

$$\frac{\partial^4 F}{\partial x^4} + 2\,\frac{\partial^4 F}{\partial x^2\,\partial y^2} + \frac{\partial^4 F}{\partial y^4} = 0 \quad \text{..............(143)}$$

Note that the result of the solution of the variational problem is a condition, expressed in terms of a differential equation. This is generally the case.

The problem may be approached from another angle; namely, by basing the argument on the ideas of equilibrium and continuity. As in the previous case, it will be assumed that Hooke's law applies. Mr. Jakobsen has supplied the equations of equilibrium required, Equations (6) and (7). The equation of continuity, or of compatibility, expresses the condition that if the x, y surface in its unstressed state is marked off into squares by closely spaced lines parallel to the x and y-axes, each of the squares so defined will maintain contact with its neighbors on all four sides after deformation has taken place. The equation of continuity may be arrived at by expressing the strains in terms of the displacements and eliminating the displacements from the equations. To accomplish this let,

u = the displacement of points in the x-direction.
v = the displacement of points in the y-direction.
ϵ_x = the unit strain in the x-direction.
ϵ_y = the unit strain in the y-direction.
γ_{xy} = the unit shearing strain.

Then,

$$\varepsilon_x = \frac{\partial u}{\partial x}\,;\ \varepsilon_y = \frac{\partial v}{\partial y}\,;\ \text{and } \gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}$$

Differentiate as follows, and add:

$$\frac{\partial^2 \varepsilon_x}{\partial y^2} = \frac{\partial^3 u}{\partial x \, \partial y^2}$$

$$\frac{\partial^2 \varepsilon_y}{\partial x^2} = \frac{\partial^3 v}{\partial x^2 \, \partial y}$$

$$-\frac{\partial^2 \gamma_{xy}}{\partial x \, \partial y} = -\frac{\partial^3 u}{\partial x \, \partial y^2} - \frac{\partial^3 v}{\partial x^2 \, \partial y}$$

then,

$$\frac{\partial^2 \varepsilon_y}{\partial x^2} - \frac{\partial^2 \gamma_{xy}}{\partial x \, \partial y} + \frac{\partial^2 \varepsilon_x}{\partial y^2} = 0 \ldots\ldots\ldots\ldots (144)$$

Equation (144) is the required equation.[114]

Introduce Hooke's law in terms of the ordinates to Airy's surface, as follows:

$$\varepsilon_x = \frac{1}{E} (n_x - \mu \, n_y) = \frac{1}{E} \left(\frac{\partial^2 F}{\partial y^2} - \mu \frac{\partial^2 F}{\partial x^2} + \mu \, w \, y \right) \ldots .(145)$$

$$\varepsilon_y = \frac{1}{E} (n_y - \mu \, n_x) = \frac{1}{E} \left(\frac{\partial^2 F}{\partial x^2} - w \, y - \mu \frac{\partial^2 F}{\partial y^2} \right) \ldots\ldots .(146)$$

$$\gamma_{xy} = \frac{2\,(1 + \mu)\, t}{E} = \frac{2\,(1 + \mu)}{E} \left(-\frac{\partial^2 F}{\partial x \, \partial y} \right) \ldots\ldots\ldots .(147)$$

Note that, in Equations (145), (146), and (147), $\mu = \frac{1}{m}$. If these expressions are substituted into the equation of continuity (Equation (144)):

$$\frac{\partial^4 F}{\partial x^4} + 2 \frac{\partial^4 F}{\partial x^2 \, \partial y^2} + \frac{\partial^4 F}{\partial y^4} = 0 \ldots\ldots\ldots\ldots (148)$$

As a result of this work it may be seen that, not only does Equation (148) imply that the equations of compatibility, as stated by the author in Appendix II, are satisfied, but it also implies the satisfaction of the equations of equilibrium. Furthermore, the variational calculus indicates that if this equation is satisfied the potential energy of deformation will be less than that of any other distribution whatever, which will hold the forces in equilibrium.

A specification may be written, therefore, for a solution of the problem of finding the stress distribution in a two-dimensional elastic structure. The specification is, solve Equation (148) subject to the boundary conditions. This test may be applied to the solutions in the author's paper. Equations (13) satisfy Equation (148). That Equations (27) do not, is not conclusive in this case, since the author does not pretend that these equations represent an exact solution. Neither Equations (13) nor (27), however, recognize explicitly any elastic action of the foundation and, therefore, it must be concluded that neither is entitled to acceptance as a true solution of the problem. The objection may be stated specifically in the case of Equations (27). The limits given for the integrals in Equations (19) and (20) show that the integration was carried over the section of the dam only. This being the case, one may conclude that Mr. Jakobsen has succeeded in solving, approximately, the problem:

---

[114] Compare Love's "Mathematical Theory of Elasticity," Paragraph 17, Fourth Edition.

To find, among all possible stress distributions which satisfy the boundary conditions at the up-stream and down-stream faces of a triangular profile, that distribution which shall make the potential energy of deformation within the section of the dam a minimum.

The problem to be solved in case the law of least work is to be complied with, is:

To find among all possible stress distributions which satisfy the boundary conditions at the surface of the dam and foundation, that distribution which shall make the total potential energy of deformation in the dam and foundation, a minimum.

P. Wilhelm Werner,[115] Assoc. M. Am. Soc. C. E. (by letter).—The author's contribution to the development of the theory of gravity dams is interesting. Especially in view of the tendency toward ever-increasing height of dams, it is important that the methods on which the design is based should be reviewed. It is well known that the theory now generally used is not strictly correct from the standpoint of the general theory of elasticity.

In discussing the paper, the writer wishes to call attention to an aspect of the problem that has special relation to one of the boundary conditions at the base of the dam.

Under the usual assumption that,

$$n_z = t_{zy} = t_{zx} = 0 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (149)$$

and that the material in the dam is homogeneous and isotropic and follows Hooke's law, the exact solution of the problem involves finding a stress function, *F*, which satisfies the homogeneous partial differential equation:

$$\frac{\partial^4 F}{\partial x^4} + 2\,\frac{\partial^4 F}{\partial x^2\,\partial y^2} + \frac{\partial^4 F}{\partial y^4} = 0 \ldots\ldots\ldots\ldots (150)$$

and which, furthermore, satisfies all the boundary conditions, when the coordinates of the edges of the cross-section are introduced into the following general expressions for the stresses, n, and the displacements, u:

$$n_x = \frac{\partial^2 F}{\partial y^2} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (151)$$

$$n_y = \frac{\partial^2 F}{\partial x^2} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (152)$$

$$t = -\frac{\partial^2 F}{\partial x\,\partial y} - w\,x \ldots\ldots\ldots\ldots\ldots\ldots (153)$$

$$\frac{\partial u_x}{\partial x} = \frac{1}{E}\left(n_x - \frac{1}{m}\,n_y\right) \ldots\ldots\ldots\ldots\ldots (154)$$

$$\frac{\partial u_y}{\partial y} = \frac{1}{E}\left(n_y - \frac{1}{m}\,n_x\right) \ldots\ldots\ldots\ldots\ldots (155)$$

---

[115] AB. Vattenbyggnadsbyran, Stockholm, Sweden.

As pointed out by the author, the solution given in Equations (27) does not satisfy the general differential equation, Equation (150). As regards the boundary conditions, it is evident that neither Equations (27) nor Equations (13) fulfill the conditions for the displacements at the base of the dam. The author discusses the condition for u_y, that is, the displacement in the direction of the y-axis, but he does not discuss the condition for u_x, that is the displacement in the direction of the x-axis. In the writer's opinion, however, the boundary condition for u_x at the base is also of rather great importance.

The first conception one has regarding the boundary condition for u_x at the base is that,

$$\left.\frac{\partial u_x}{\partial x}\right]_{y=H} = 0 \dots\dots\dots\dots\dots\dots\dots\dots (156)$$

that is, that all the points of the base, under loaded conditions, retain their positions in relation to each other in the direction of the x-axis. The extent to which this is actually true, is, of course, impossible to determine. A certain foundation deformation may exist even in the direction of the x-axis. Such deformation (if it does exist), may be influenced by irregularities in the foundation, etc. However, the strain condition expressed by Equation (156) should be found more nearly to correspond to the actual conditions at the base of the dam than any arbitrarily chosen distribution of the shearing stress, and, in any case, Equation (156) must be considered as a possible limiting case.

It is easy to verify the fact that neither Equations (13) nor Equations (27) satisfy the boundary condition expressed by Equation (156). In spite of this, both theories give (in accordance with St. Venant's principle) very nearly correct values for the stresses in points at some distance from the base of the dam. Near the base, however, there might be considerable discrepancy between the actual stress conditions and the stresses computed according to either theory, as long as the boundary conditions along the base are not fulfilled. Unfortunately, the inaccuracy of the methods applies to the most vulnerable part of the dam, that is, the base.

It is interesting to compare the values of the strain, $\varepsilon = \frac{\partial u_x}{\partial x}$, at the base, computed on the basis of Equations (13) and Equations (27). In a no-uplift section and under the assumption that $m = \infty$, the following values are determined by substitution in Equation (154):

(1) By the straight-line theory:

$$\varepsilon_1 = \frac{1}{E}\, g\, H = -\,0.4340\,\frac{H}{E} \dots\dots\dots\dots\dots\dots (157)$$

(2) By the author's least work theory:

$$\varepsilon_2 = -\,0.4340\,\frac{H}{E} + k^4 z^4 \,(1.584 - 1.579\,k\,z)\,\frac{H}{E} \dots\dots\dots (158)$$

in which, $z = \frac{x}{b}$.

Evidently, for $k < \frac{1.584}{1.579}$ — that is, if k is less than about unity — the numerical value of ϵ_2 is always less than that of ϵ_1, except at the up-stream face ($z = x = 0$), where the two values are exactly equal, or,

$$\varepsilon_1 = \varepsilon_2 = -0.4340\,\frac{H}{E} \qquad \text{(159)}$$

At the down-stream face of the dam ($x = b$, $z = 1$), the difference between ϵ_2 and ϵ_1 is greatest, amounting to about 22% for $k = 0.645$.

In the writer's opinion a revision of the theory for gravity dams, and for buttresses of hollow dams as well, must aim at the fulfillment of the boundary condition for u_x at the base. The author's trial function, as expressed by Equations (27), leads to considerably more dangerous stresses than the trial function according to the old theory, as expressed by Equations (13). However, the writer has shown in the foregoing remarks that the author's least work theory affects the boundary condition for u_x at the base only to a very moderate extent. If the same proportion for a complete fulfillment of the boundary condition for u_x at the base exists, it would seem to indicate disastrous stresses in the dam. The writer is fully aware that this same proportion might not exist, but the question certainly seems worth while investigating.

In general, it is very difficult, if not impossible, to find, in a given case, the solution of the general differential Equation (150) which at the same time satisfies all the boundary conditions. As a rule the engineer must be content with an approximate solution, such as that given by the Equations (27). Such approximate solutions must be carefully viewed and discussed, especially in regard to the boundary conditions.

In this special case the writer would suggest that the method outlined in the paper be verified in such a way that it can be applied to another problem of similar nature the exact solution of which, from the standpoint of the general theory of elasticity, is known beforehand.

B. F. Jakobsen,[116] **Esq.** (by letter).—Before taking up the discussions, the writer wishes to restate his problem. This seems advisable, because several contributors, notably Professor Campus, have misundestood the writer's reference to the classical theory. The writer had the ordinary engineering theory in mind, while Professor Campus had the classical theory of elasticity in mind. The writer did not properly distinguish between these two theories, and this led to the misunderstanding. An excellent and critical résumé of the classical and engineering theories has been given by Professor Campus in another paper.[117]

The Classical Theory of Elasticity.—This assumes a homogeneous, elastic, triangular section of indefinite length. It implies, as Professor Vogt has shown in his discussion,[118] that sections horizontal before loading are curved, (concave side upward), due to the loading. When this theory is applied to ordinary dams, which are necessarily of limited height, it implies as an assumption, that the foundation is thus curved by the loading, but this implied assumption is no part of the classical theory, which does not consider any foundation.

---

[116] Cons. Engr., Los Angeles, Calif.

[117] "Conditions de Stabilité des Barrages à Gravité en Béton," Extrait du *Bulletin* de la Société Belge des Ingénieurs et des Industriels 1930.

[118] See, also, Professor Campus, "Note sur la Déformabilité des Barrages à Gravité" Extrait des *Bulletins* de l'Académie royale de Belgique (Classe des Sciences) Séance du 6 décembre 1930 (n. 12, pp. 1384–1395) (1931), and *Le Génie Civil*, January 3, 1931, p. 9.

The writer has shown that the linear stress distribution satisfies the statical conditions, Equations (3) and (4), the equilibrium Equations (6) and (7), and the boundary conditions, Equations (8) and (9). That these stresses also satisfy the continuity equations was shown in Appendix II.

The Usual Engineering Theory.—This assumes linear stresses, which satisfy the statical conditions, Equations (3) and (4). Since the bending formula is applied to horizontal sections in order to derive the magnitude of the linear stresses, it implies that these sections remain plane, when shear deformations are neglected. As already stated, this assumption is in error.

Theory of the Rigid Base.—This assumes that the foundation is unyielding in every direction. This is implied in the fact that Equations (25) and (26) were integrated from zero to *H*, which excludes deformations in the foundation, and, in Appendix III, the writer stated that no account had been taken of the irregularities introduced by the foundation. The late Professor Cain stated this explicitly in his discussion.

General Remarks.—The classical theory of elasticity is exact with the assumptions made. The ordinary engineering theory, which deals with a dam on a foundation, both of them elastic, is incorrect. As Dr. Vogt has shown, linear stresses curve the horizontal sections of the dam, concavity upward, while these same linear stresses curve the foundation in a complicated fashion, but in the main with the concavity downward. As stated by Dr. Vogt:

> "The classical theory gives stresses which are greatly in disagreement with the compatibility equation at the foundation joint, and several large corrections are required."

Dr. Vogt has calculated approximately the additional stresses necessary to bring about similar deformations of the dam and its foundation, and checks the writer's results on the assumption that deformation due to water load only is involved. If the weight of the dam is included, he gets a smaller additional tension at the heel. He also calls attention to the possible effect of the dam being built by degrees and of the lower part partaking in the stresses due to weight along with the foundation. This brings up another complicated problem which may bear considerably upon the actual stresses in the foundation.

The late Professor Cain derives the conditions for minimum of internal work: $\frac{dL}{dC} = 0$, and $\frac{dL}{dF} = 0$, by means of Castigliano's theory, and states:

> "These two equations show that the internal work is a minimum as stated by the author. This demonstration assumes the base of the dam unyielding."

He concludes:

> "The English experiments first proved the ordinary 'inner-third rule' inadequate, and now Mr. Jakobsen confirms it by a close theoretical analysis."

In his discussion, Mr. Gore gives additional information regarding the English tests. Fig. 14 shows the piling up of stress at the heel of the dam, when the reservoir is empty. This was to be expected, but is probably somewhat too large in the model tests, due to the overhang when the reservoir is empty, shown in Fig. 13. In that respect, the writer's assumption of a rigid base is in error, because it gives nearly linear stress (see Article VII).

Mr. Gore, in collaboration with Mr. J. S. Wilson, conducted these tests. It was not the writer's intention to detract in any way from these very skilfully made tests, which to this day have not been surpassed or equalled. The writer's interest centered largely on the stresses at and near the foundation, where the English tests show tension at the heel. Mr. Gore has correctly understood that the writer assumed a rigid base and he concludes that this is the reason for the excessive tension found by the writer. Mr. Gore further concludes that the writer's assumption of a rigid base is not much nearer satisfaction than the straight-line law. After careful study of the discussions, the writer has concluded that the truth regarding tension at the heel is probably somewhere between the straight-line law and his own results. There is, however, this important distinction, that the writer's error is on the side of safety, while the straight-line law unquestionably errs on the side of danger. This is a matter of considerable importance, because all are agreed on the uncertainties of the stress distribution at the base and yet this is the deciding factor in determining the width of the base and, therefore, the design of a gravity dam.

Mr. Gore's conclusion in favor of the linear stress distribution disagrees with his own and other tests, which led Professor Love to question the correctness of the classical theory.[119]

Mr. Mensch refers to Equation (14) for the internal work as a hypothetical equation, thus leaving it to be inferred that something is amiss with this equation. It is, however, the usual and accepted form for the work equation and is to be found in all standard works on elasticity (see, also, Professor Cain's discussion, in which this matter and the method of determining the least work, is carefully analyzed).

To replace the writer's Equation (15) Mr. Mensch offers a cubic parabola, but he makes no effort to show that this can be made to satisfy the equilibrium equations and the boundary conditions. His proposed curve (Fig. 16) shows a maximum at the down-stream face, no matter how wide the base, which is in contradiction to all tests of which the writer has any knowledge, and is otherwise improbable. Mr. Mensch takes the writer to task for finding tension in the up-stream face and for not mentioning that it cannot exist. The writer believes tension can exist and no doubt does exist in some dams, but he is agreed that the design should be made so that it will not occur. The writer agrees with Mr. Mensch regarding the danger of designing gravity dams with no allowance for uplift, such as was the case with the St. Francis Dam.

Mr. Jorgensen calls attention to the influence of water-soaking, the large variation in the modulus of elasticity, and to the influence of uplift, all of which are factors that make for uncertainties in any stress determination. They warn, or should warn, the designer that he must provide a liberal factor of safety to include a reasonable factor of ignorance.

---

[119] "The Mathematical Theory of Elasticity," Third Edition, p. 211.

Mr. Turner gives an interesting historical account of the development of the theory of bending. His statement that the writer has confused shear force with shear strain will be found incorrect upon further study. His assertion that the linear law at a section normal to the neutral axis follows from Hooke's law, is incorrect, as a reference to any text on elasticity will show. An attempt to satisfy the equilibrium equations and the boundary conditions for an ordinary beam stressed in bending will also convince Mr. Turner of his error. Furthermore, Mr. Turner will find that the linear law applies to any section, horizontal or otherwise, of a triangular, homogeneous dam of indefinite length, and not only to the normal section. The writer noted this in his paper, "Stresses in Multiple Arch Dams",[120] and Professor Résal, in France, noted it in his paper[121] published in 1919. A few simple calculations will convince Mr. Turner of the correctness of this statement.

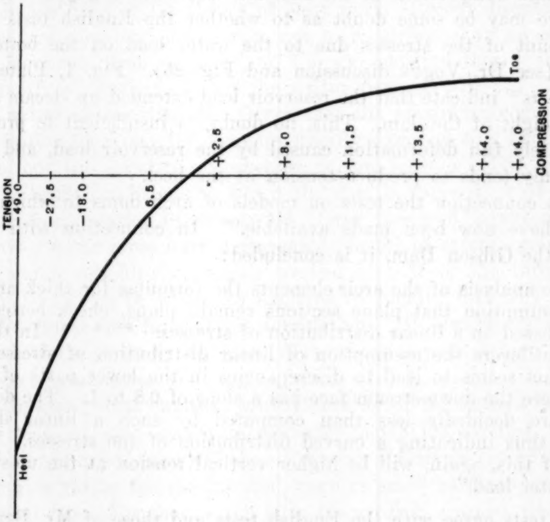


FIG. 35.

The tests by Mr. Barnes are of considerable interest. The writer has had an opportunity to check over the test data and his results are in substantial agreement with those of Mr. Barnes, except possibly at and near the down-stream toe where no deformation measurements are available. Mr. Barnes did not measure the horizontal deformations, but the assumption which he makes regarding n_x is no doubt quite close to the truth. It is also evident (see Equation (46)) that, for large values of n_y, an error in n_x is of little consequence.

---

[120] *Transactions*, Am. Soc. C. E., Vol. LXXXVII (1924), p. 294.

[121] "Formes et Dimensions des Grands Barrages en Maconnerie," *Annales* des Ponts et Chaussées, II, 1919.

From Mr. Gore's data in Fig. 14, a stress curve for bending may be obtained, by subtracting the "reservoir-empty" stresses from the "reservoir-full" stresses. The result is shown in Fig. 35. The tension area checks the compression area within about 10 per cent. No doubt, the discrepancy is due to the small scale of Fig. 14 from which the stresses were measured, and does not reflect upon the accuracy of Mr. Gore's tests. The up-stream tension is 3.5 times the down-stream compression. The curve in Fig. 35 is of the same general shape as the writer's Fig. 6, in which the ratio between the maximum tension and the maximum compression is only 1.78. The curve in Fig. 35 is steeper in the up-stream parts of the dam and flatter in the down-stream portion than the writer's Fig. 6. The probable influence of the overhang (see Fig. 13), which is to increase the heel stresses for "reservoir empty," has already been mentioned. Both tests are in accord as to high tension at the heel near the base. Some of this tension will be compensated by high compression at the heel for "reservoir empty." On the other hand, there may be some doubt as to whether the English tests took sufficient account of the stresses due to the water load on the bottom of the reservoir (see Dr. Vogt's discussion and Fig. 25). Fig. 1, Plate 4, of the English tests[122] indicate that the reservoir load extended up stream about one-half the height of the dam. This, no doubt, is insufficient to produce even approximately full deformation caused by the reservoir load, and this loading certainly tends to produce tension at the heel.

In this connection the tests on models of arch dams to which Dr. Vogt referred, have now been made available.[123] In connection with tests on a model of the Gibson Dam, it is concluded:

> "In the analysis of the arch elements the formulas for thick arches based on the assumption that plane sections remain plane, check better than the formulas based on a linear distribution of stresses. * * * In the analysis of the cantilevers the assumption of linear distribution of stresses over the cross-section seems to lead to discrepancies in the lower parts of the cantilevers, where the down-stream face has a slope of 0.8 to 1. The down-stream stresses are decidedly less than computed by such a linear distribution of stress, thus indicating a curved distribution of the stresses. The consequences of this, again, will be higher vertical tension at the up-stream face due to water load."

These tests agree with the English tests and those of Mr. Barnes. It is interesting to note that neither the arch stresses nor the cantilever stresses are linear, but are more nearly those found by assuming that plane sections remain plane.

Professor Hoadley calls attention to the uncomfortable feeling associated with work when the factor of safety approaches one, although little is at stake, and he approves of a more conservative viewpoint in gravity dam design. There is no doubt in the writer's mind that some gravity dams are in tension at the up-stream face and especially at the heel, namely, all those which have insufficient or no allowance for uplift. A number of such dams have already failed. The writer is well aware that the leaders in the profession never have countenanced such design methods, as the discussions published by the Society show.

---

[122] *Minutes of Proceedings*, Inst. C. E., Vol. CLXXII, Session 1907–08, Pt. II.

[123] Engineering Foundation, Committee on Arch Dam Investigation, Sub-Committee on Model Tests, J. L. Savage, M. Am. Soc. C. E., *Chairman*, June 1, 1931, p. 271, of multigraph copy.

Mr. Ely brings up a number of interesting points. Referring to Fig. 6, it seems that the fact that the section is not symmetrical would account for the different stress distributions in the up-stream and down-stream halves of the dam. At the up-stream face the principal stress is vertical, while at the down-stream face it is parallel to that face.

Mr. Ely is no doubt correct in concluding that Equation (15) can be improved, possibly by introducing fractional exponents, as he suggests; perhaps also by writing Equation (15) so that the shape of the stress curves may vary with y.

Mr. Ely discussed uplift, and so did Professor Cain. The writer frankly admits that he is somewhat at sea as to how uplift stresses should be treated and just what effect they will have upon the stress distribution when considered in the light of the theory of least work. Recent discussions indicate that this is a complicated question, not to be answered in an off-hand manner.[124] However, dams designed on the straight-line theory should have a liberal uplift allowance; of that the writer feels certain.

The writer agrees with Mr. Gerry that the maximum compressive stress is no criterion of safety for ordinary gravity dams up to a height of 500 ft., or more, but the question of whether or not tension exists, or may exist, under the worst conditions to which the dam is exposed, is of great importance and is a question of stress. Mr. Gerry does well, however, in insisting that gravity dams must have a reasonable factor of safety against sliding.

The writer has already largely availed himself of the information supplied by Dr. Vogt, in which he explains so lucidly just why and to what extent the linear law fails at the foundation. He also shows that an equation of the form given by Equation (74), which is more general and includes the writer's assumption for n_y (Equation (15)), cannot satisfy all conditions. He also points out that convex curving of the down-stream face may produce tension; such curvature evidently should be avoided.

As a simple calculation will show Professor Kálmán's Equation (85) cannot possibly give values for the internal work as small as those found from the usual linear law. For that reason it is disqualified, *a priori*, and it is useless to determine the constants, a and b. Obviously, the writer's Equation (15) can equal Equation (10), the linear law, and, in Article VI, it was shown that the internal work found was less than that obtained from Equation (10). Professor Kálmán's repeated references to the true system leads the writer to remark that the Ritz method is of use only when the true system is not known. Fig. 31 is inapplicable, because the writer assumed a rigid base and it is immaterial how that is secured.

---

[124] See articles in *Le Génie Civil*, by E. Baticle, March 24, 1928; by G. Pigeaud, March 31, 1928; by A. and Jacques Mesnager, April 7, 1928; and by G. Pigeaud, April 21, 1928. See, also, discussion by A. Floris, in *Transactions*, Am. Soc. C. E., Vol. 95 (1931), p. 200.

Professor Campus has apparently overlooked the basic assumption made by the writer, namely, that the base is unyielding while, as he himself has shown, the linear theory implies a curving of the base. This is the reason why Equation (45) leads to Equation (41) which is the linear solution, while Equation (14) does not. In other words, when no assumption is made regarding the deformations of horizontal sections of an indefinitely high dam, the principle of least work must, and does, lead to the linear law.

Mr. Floris states that the writer has made exactly the same assumptions as were made in the classical theory of elasticity. He also has overlooked the fact that the writer assumed the base as unyielding, while the classical theory implies curvature, as shown by Dr. Vogt in Fig. 24.

Mr. Glover determines, by variational calculus, Equation (148), which is the same as the writer's Equation (42), and shows that if this equation is satisfied the internal work is a minimum. He has correctly understood that the writer assumed an unyielding base. The complete solution, of course, would have to include foundation deformations as well.

Referring to Mr. Werner's discussion, the boundary condition given by Equation (156) does not apply to the linear law, Equations (13), as both Professor Campus and Dr. Vogt have shown. It does apply to Equations (27), but since this does not satisfy Equation (150), which is the same as the writer's Equation (42), given and discussed in Appendix II, Equations (154) and (155) cannot be applied to Equation (27).

In practical application of the theory of elasticity, it is in general not possible to find an exact solution, which satisfies all the conditions of equilibrium and the geometrical continuity. One may then either determine the stresses from the equilibrium equations, in which case the geometrical continuity is not respected, and Equations (154) and (155) do not hold. If they are applied they must lead to contradictions, unless one accidentally hits upon the correct solution, or, one may find the stresses from Equations (154) and (155) and the geometrical boundary conditions, in which case the equilibrium equations are not likely to be exactly satisfied and cannot be applied.[125]

The writer has made an effort to reply to the various objections raised, but it is, of course, impossible adequately to cover the field gone over by so many able men. The discussions show a general agreement that the linear law errs on the side of danger and that there is more tension or less compression in the dam at the up-stream edge of the foundation than this law indicates. On the other hand, the writer's assumption and solution probably amounts to an over-correction, which errs on the side of safety. This is after all not such a bad feature, where so many uncertainties regarding uplift, internal tension due to shrinkage, etc., are involved, when the lives and property of so many people are at stake, and in view of the number of failures of gravity dams, which have occurred in the last decade. The assumption of a rigid base may lead to more nearly correct results by amending Equation (15) so that it allows for a change in shape of the stress curves with y, or it may be possible to include the foundation deformations in Equation (14) for the internal work. In any case, there is great need of carefully made tests which should include the water load on the reservoir and the consequent deformations at the heel.

---

[125] See Föppl's "Drang und Zwang," Vol. I, Second Edition (1924), p. 128.

In conclusion, the writer wishes to thank the many contributors for their assistance and interest and hopes that this paper will prove a contribution toward safer design, in order that the gravity dam designed so that it has insufficient or no uplift allowance, will be outlawed as it should be.

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
TRANSACTIONS

Paper No. 1801

FORMULAS FOR RAINFALL INTENSITIES OF LONG DURATION[1]

By Merrill M. Bernard,[2] M. Am. Soc. C. E.

With Discussion by Messrs. W. B. Gregory, W. W. Horner, Adolph F. Meyer, R. W. Powell, C. E. Grunsky, Charles W. Sherman, and Merrill M. Bernard.

Synopsis

The object of this paper is to make available for any locality within the limits of the study, rainfall intensity formulas for frequencies of 5, 10, 15, 25, 50, and 100 years, applicable to rainfall duration periods of 120 to 6 000 min.

The need for some means of obtaining for a specific locality, a knowledge of rainfall intensities, durations, and frequencies, is a vital one. This is especially true of those outlying communities which do not have the advantage of long, local rainfall records.

Introduction

The following treatment of rainfall intensities has resulted from the writer's experience in applying the rational method of flood-flow analysis, so successfully utilized in storm-sewer design, to the broader field of small stream flow, particularly agricultural drains. Its scope, therefore, will be limited to that section of the United States lying between the 101st and the 81st meridians, wherein land drainage is a matter of great interest. The writer has found that while some thought[2a] has been given to this problem, much remains to be done by way of engineering research.

The rational expression of flood discharge is:

$$Q = C\,i\,A \qquad (1)$$

in which,

Q = discharge, in cubic feet per second.
C = a coefficient expressing the ratio of run-off to rainfall.
i = rainfall intensity, in inches per hour.
A = area of water-shed, in acres.

---

[1] Published in October, 1930, *Proceedings*.

[2] Cons. Civ. Engr., Crowley, La.

[2a] "Maximum Stream Flow", by C. E. Grunsky, Past-President, Am. Soc. C. E., *Public Roads*, June, 1926; "Run-Off from Small Agricultural Areas", by C. E. Ramser, M. Am. Soc. C. E., *Journal of Agricultural Research*, Vol. 34, No. 9; "Determining Run-Off from Local Rainfall Records," by Merrill M. Bernard, M. Am. Soc. C. E., *Proceedings*, Louisiana Eng. Soc., Vol. LX, No. 5.

Two marked distinctions between the use of Equation (1) in the field of storm sewers and that of agricultural drains are to be noted: First, the run-off coefficient, *C*, has an entirely different range of values; and, second, concentration periods, and therefore, duration periods, are much longer.

It is with the rainfall intensities of storm periods longer than 120 min. that this paper deals.

RAINFALL INTENSITY, DURATION, AND FREQUENCY

With the longer concentration periods encountered in agricultural drainage areas comes the need of a more complete knowledge of rainfall intensities for duration periods longer than 120 min. One, two, three, and four-day storm periods are successions of highly intense downpours, separated by intervals of light, or of no, rainfall. In the case of sewers, each storm phase may be considered as an individual occurrence. Concentration periods are usually less than 2 hours, and the drainage system has the opportunity to clear itself before the effect of the next period of intense rainfall is felt. The higher degree of saturation throughout the remainder of the storm period is the only relating factor.

This condition does not prevail in the larger, more pervious water-sheds. The concentration time, comparable with the inlet time in storm-sewer design, will range from 50 to 1 200 min. on a water-shed of 1 000 acres, depending on the slope, the shape of the area, and the degree of development. Lower velocities are responsible for longer time of flow through the drains themselves, so that, for drainage areas of 300 sq. miles, one is dealing with concentration periods, and, therefore, with rainfall duration periods, of 4 000 min., or more.

The slow movement of run-off over the ground, through furrows and unimproved drains, and the relatively greater effect of retention and retardation, tend to equalize the rate of run-off resulting from a series of downpours, which combined make up the typical storm period of from 1 day to 4 days' duration. There is little to dispute the fact that the net effect of a 3-day storm having three to five phases, is the same as that of a hypothetical storm of average intensity, determined by dividing the total rainfall depth, in inches, by the duration of the storm, in hours. Adolph F. Meyer, M. Am. Soc. C. E., has reached this conclusion[3] regarding rainfall-duration periods of from 30 to 120 min., and the conception may be carried into the longer periods with added certainty.

A knowledge of the frequency with which a rainfall intensity of a given duration will be reached or exceeded in a locality, is as essential to the intelligent design of the conduit which is to remove the run-off resulting from such rainfall, as a knowledge of the degree of the intensity itself. Formulas which classify rainfall intensities as "maximum", "ordinary", and reached or exceeded "occasionally" or "frequently", have no place in the design of a sewer or drain where all other factors are given determinate values.

---

[3] "The Probable Frequency of Given Rates of Rainfall", *Engineering News-Record*, Vol. 87, p. 1068.

Frequency is the measure of the efficiency; it explains and justifies the occasions when the structure is taxed beyond the capacity for which it was designed.

Frequency, when applied to meteorological phenomena, does not mean that at regular intervals the phenomenon will occur. Instead, for a record covering a selected period of years, it can be expected that a rainfall of given average intensity and duration will be reached or exceeded, about once in so many years. Dividing the period by the number of such happenings, the quotient obtained is the average length of time, in years, during which the phenomenon has happened once.

This conception of frequency should not detract from its importance. In establishing the limit beyond which the designed capacity will be exceeded, the capitalized loss or damage will be no different at the end of the period analyzed whether a particular event occurred at regular intervals or in two consecutive years.

INTENSITY FORMULA TYPES

The almost exclusive interest in rainfall-duration periods of from 5 to 120 min. has developed the following type of rainfall intensity formula,

$$i = \frac{a}{t + b} \qquad \text{(2)}$$

in which,

t = duration of rainfall, in minutes.

a and b = coefficients having definite values for any locality.

The general application of Equation (2) to rainfall intensities for this range of duration periods is demonstrated by Mr. Meyer,[4] who has made available for the Eastern United States a series of such formula groups.

The results of this and similar studies,[5] however, show that the relation between rainfall intensity and duration is more accurately expressed by the hyperbolic formula of the type:

$$i = \frac{C}{t^n} \qquad \text{(3)}$$

in which,

C = a coefficient, depending for value on locality.

n = an exponent, depending for value on locality.

A distinction between these types may be drawn by considering Equation (2) as applying to single storm phases lasting less than 2 hours, while Equation (3) is intended to express average intensities of storm periods of longer duration.

One cannot be extended into the field of the other without confusion and error. Fig. 1 compares the Meyer curves, extended, with those of the writer for Columbia, Mo. The curve groups are in agreement between the 60-min.

---

[4] "Elements of Hydrology," 1928.

[5] "Storm Rainfall of Eastern United States", Technical Repts., Miami Conservancy Dist.; "Rainfall and Run-Off Studies," by C. E. Grunsky, Past-President, Am. Soc. C. E., *Transactions*, Am. Soc. C. E., Vol. LXXXV (1922), p. 76; "American Sewerage Practice," Vol. 1, Second Edition, p. 260; and "A Method of Determining Storm-Water Run-Off", by Charles B. Buerger, M. Am. Soc. C. E., *Transactions*, Am. Soc. C. E., Vol. LXXVIII (1915), p. 1182.

and 200-min. intervals, but the former rapidly falls when produced beyond its intended limit of 120 min. It will be noted, from the plotted observational data, that all but three of the points lie above the Meyer 5-year frequency curve, which, considering the length of the record, shows the position of the curve to be in error.

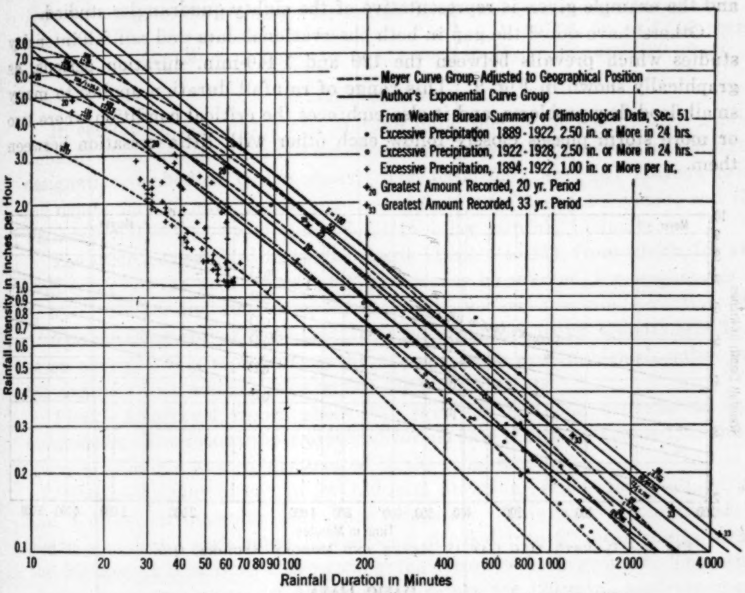


FIG. 1.—COMPARISON OF RAINFALL INTENSITIES FOR COLUMBIA, MO.

It is seen from Fig. 1 that, for duration periods shorter than 60 min., the curves separate. The exponential values increase rapidly until, at 10 min., they are about twice the Meyer values and obviously in error.

C. E. Grunsky, Past-President, Am. Soc. C. E., has made[6] a comparison of two intensity formulas: The New York formula,

$$I = \frac{150}{t + 20}$$

and the parabolic formula,

$$I = \frac{14.5}{\sqrt{t}}$$

which brings out the limitations of the former type when applied to periods longer than 2 or 3 hours.

The slight advantage involved in the easy determination of a value for t does not warrant the use of 0.5 as its exponent. The proper value ranges from 0.70 to 0.82, as will be shown.

---

[6] "Rainfall and Run-Off Studies", *Transactions*, Am. Soc. C. E., Vol. LXXXV (1922), pp. 75–76.

At several of the United States Weather Bureau stations in the area, records of rainfall intensity and duration for storms of 2.50 in., or more, in 24 hours have been kept and are tabulated to 1922. The average length of record is 32 years. Fig. 1 compares the writer's curve group with such observational data at Columbia, Mo. The time interval, 120 to 1 440 min., is well represented and the example given is representative of the eighty quadrangles studied.

Others[7] have noted the gap in both observational data and rainfall intensity studies which prevails between the 120 and 1 440-min. duration periods, as graphically shown in Fig. 2. This range of rainfall duration applies to many small flood-flow problems, and it also embraces the critical condition where two or more storm phases closely follow each other with little cessation between them.

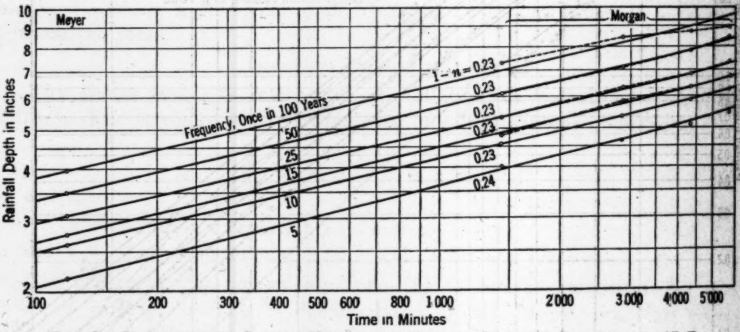


FIG. 2.—ILLUSTRATING GAP IN MEYER AND MORGAN STUDIES, QUADRANGLE 15-*E*.

BASIC DATA

Two studies of the rainfall records of the Eastern United States, outstanding in magnitude, have been made. One is by Mr. Meyer[4] who has analyzed approximately 2 000 storms and as a result has presented a series of formulas expressing the relationship between rainfall depth, intensity, and duration, and geographical location. The other is the work of Arthur E. Morgan, M. Am. Soc. C. E., who, in the interest of maximum flood flow, made an exhaustive study and analysis of the rainfall records of the Eastern United States for the Miami Conservancy District.[8]

These works are similar in the following respects:

(1) Both cover the eastern half of the United States.
(2) Both are based largely on Weather Bureau data.
(3) Both assume that, within a limited area, the sum of the records of the several stations involved is equivalent to a long-time record of one representative station.

---

[7] "Additional Meteorological Data Needed by Engineers", by Robert E. Horton, M. Am. Soc. C. E., *Engineering News-Record*, Vol. 82, p. 614; and "The Frequency of High Rates of Rainfall", by the late Allen Hazen, M. Am. Soc. C. E., *Engineering News-Record*, Vol. 87, p. 858.

[8] "Storm Rainfall of Eastern United States", Technical Repts., Miami Conservancy Dist.

The studies differ in that:

(a) Meyer's purpose was the development of intensity formulas for heavy storms of short duration, while Morgan was interested in the longer storm periods of 1 to 6 days.

(b) Meyer has presented his work in the form of intensity formulas. Morgan has presented his in a series of isopluvial charts which give the relation of rainfall depth to duration and frequency.

(c) The Meyer frequencies are once in 1, 2, 5, 10, 25, 50, and 100 years. The Morgan frequencies are once in 15, 25, 50, and 100 years.

METHOD OF ANALYSIS

In combining the Meyer and Morgan data, the writer has utilized a relationship which has proved to be consistent in eighty quadrangles of the Morgan designation, embodied in his study. The basic data are: (a) Meyer's rainfall depths for 120 min., for frequencies of 5, 10, 25, 50, and 100 years; and (b) the isopluvial charts, by Morgan,[9] for 1 to 4-day rainfall, inclusive.

The study has resulted in twelve charts (Figs. 4 to 15), from which, for any locality within the area, values of C and n may be selected, for frequencies of 5, 10, 15, 25, 50, and 100 years, for use in the general formula, Equation (3).

The development of the charts will be explained using Quadrangle 15-*E* as an example, this representing the average as to geographical location and the accuracy with which the curves fit the plotted points.

First.—A printed record sheet was prepared and one assigned to each quadrangle. The record sheet provided for all data shown in Tables 1, 2, and 3. Every quadrangle was rigidly treated in the same manner.

Second.—Isopluvial charts for 120-min. duration periods for 5, 10, 25, 50, and 100-year frequencies were then prepared. This was accomplished by plotting on large scale maps of the area, divided into 2° quadrangles as are the Morgan charts, lines representing the mean of each group.[10] These isopluvial lines were given the Meyer depth values for 120 min., separate charts being used for each frequency. The pluvial index for each quadrangle, referred to the center, was interpolated and recorded on the charts. This, then, made available isopluvial charts for 120, 1 440, 2 880, 4 320, and 5 760-min. duration periods.

Third.—The pluvial indices for each quadrangle were tabulated. Table 1 gives these data for Quadrangle 15-*E*. The numerical indices of the Morgan charts were recorded, but only the interpolated values were used in plotting the curves. In so doing the writer has taken advantage of the careful adjustment of the isopluvial lines to the weighted length of record of each quadrangle.

Fourth.—The data in Table 1 were plotted logarithmically as shown in Fig. 2. First, frequency was plotted against depth for time, 120 min. (Fig. 3), and the depth value for a frequency of 15 years was obtained by interpolation. Then, for frequencies of 15, 25, 50, and 100 years, time was plotted against depth (Fig. 2), resulting in the points controlling the upper four curves which were placed so as to fit the points best. The 120-min. depth values were allowed to control the lower ends of the curves in each case.

---

[9] Technical Repts., Pt. V., Miami Conservancy Dist.

[10] "Elements of Hydrology," 1928, p. 166.

The difference in the values given by any point and its equivalent curve value did not exceed 10% and the average difference in the eighty quadrangles was 3 or 4 per cent. As a general rule, the percentage of difference varied inversely with the record length of the quadrangle.

TABLE 1.—PLUVIAL INDEX, QUADRANGLE 15-*E*.

(A = interpolated values; B = numerical value from isopluvial charts.)

Frequency, in years	TIME, IN MINUTES.								
	120	1 440		2 880		4 320		5 760	
		A.	*B.*	*A.*	*B.*	*A.*	*B.*	*A.*	*B.*
5	2.13	....	....	....	....	....	....	....	....
10	2.60	....	....	....	....	....	....	....	....
15	2.75	4.8	5.1	5.8	5.7	6.3	6.0	6.7	6.5
25	3.05	5.3	5.6	6.3	6.2	6.8	6.8	7.3	7.5
50	3.50	6.1	6.3	7.1	7.0	7.8	7.5	8.4	8.7
100	3.97	7.3	7.8	8.4	8.8	8.7	8.9	8.9	9.2

Fifth.—As the next step (Fig. 3), the depth values given by the four upper curves in Fig. 2 have been plotted against frequency for 1 440, 2 880, 4 320, and 5 760-min. durations and the resulting curves extrapolated through the 10-year to the 5-year frequency.

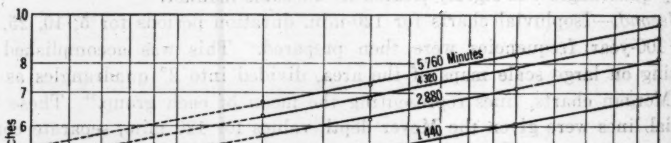


FIG. 3.—CONSTRUCTION AND EXTENSION OF RAINFALL CURVES, QUADRANGLE 15-*E*.

Sixth.—The resulting depth values for 5 and 10-year frequencies were plotted in Fig. 2, giving the two lower curves.

When it is realized that the upper and lower limits of these curves are established by separately prepared data, the writer believes the close agreement of points and curves in all of the eighty quadrangles studied to be unusual.

Seventh.—The slope of each curve in the group was measured carefully and the values for depth, corresponding to the time intervals, 500 min. and 2 000 min., were noted. From data given in Table 2 values of C and n were computed.

TABLE 2.—VALUES FOR C AND n, QUADRANGLE 15-E.

F.	$1 - n$.	500^{1-n}.	d.	c_1.	$2\,000^{1-n}$.	d.	c_2.	c.	C.	n.
5	0.24	4.444	3.03	0.682	6.198	4.28	0.692	0.687	41.22	0.76
10	0.23	4.176	3.60	0.862	5.744	4.93	0.858	0.860	51.60	0.77
15	0.23	4.176	3.83	0.917	5.744	5.25	0.914	0.915	54.90	0.77
25	0.23	4.176	4.21	1.008	5.744	5.73	0.998	1.003	60.18	0.77
50	0.23	4.176	4.83	1.157	5.744	6.58	1.146	1.151	69.06	0.77
100	0.23	4.176	5.49	1.313	5.744	7.52	1.311	1.312	78.72	0.77

Eighth.—The record sheet for each quadrangle was completed by noting the depth and intensity formulas for each frequency. These formulas, for Quadrangle 15-E, are given in Table 3.

TABLE 3.—RAINFALL DEPTH AND INTENSITY FORMULAS, QUADRANGLE 15-E.

Frequency.	Depth.	Intensity.	Frequency.	Depth.	Intensity.
5	$d = 0.687\ t^{0.24}$	$i = \frac{41.22}{t^{0.76}}$	25	$d = 1.003\ t^{0.23}$	$i = \frac{60.18}{t^{0.77}}$
10	$d = 0.860\ t^{0.23}$	$i = \frac{51.60}{t^{0.77}}$	50	$d = 1.151\ t^{0.23}$	$i = \frac{69.06}{t^{0.77}}$
15	$d = 0.915\ t^{0.23}$	$i = \frac{54.90}{t^{0.77}}$	100	$d = 1.312\ t^{0.23}$	$i = \frac{78.72}{t^{0.77}}$

PREPARATION OF CHARTS

The writer has selected, as a means of presenting the results of this study, a series of charts comparable with the isopluvial charts in the Miami Conservancy report. These charts will make the work of practical value, permitting a simple and rapid method of selecting intensity formulas for any locality within the area under consideration.

Lines of equal value were located by considering the variation to be uniform between the numerical values at the quadrangle centers. The resulting charts are shown in Figs. 4 to 15.

The "iso-C" and "iso-n" lines fell naturally into contours and nowhere was it necessary to disregard isolated or erratic values.

CHARACTERISTICS

The Morgan isopluvial charts, while adaptable to many uses, do not present the relation of rainfall intensity to duration and frequency in a form best suited for small stream-flow analysis or agricultural drain design. Frequency, in a particular problem, becomes a constant, being fixed by the conditions, leaving intensity to vary as, $\frac{C}{t^n}$. In the application of the rational method, t

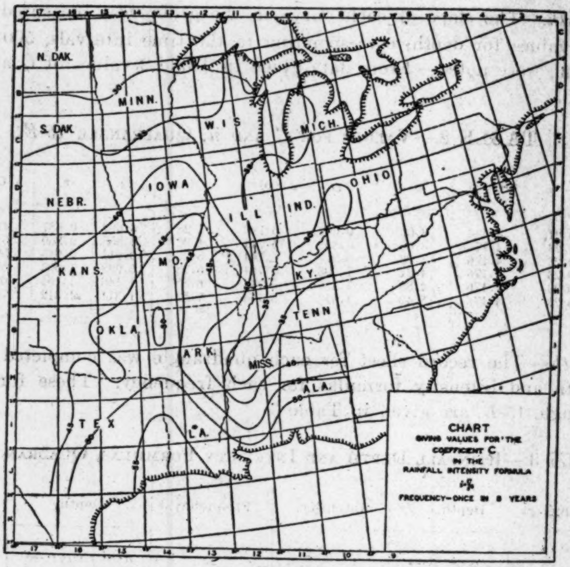

N. DAK.
MINN.
S. DAK.
WIS
MICH.
IOWA
NEBR.
OHIO
ILL
IND.
MO.
KANS.
KY.
TENN
OKLA.
ARK.
MISS.
ALA.
TEX
LA.
CHART
GIVING VALUES FOR THE
COEFFICIENT C
IN THE
RAINFALL INTENSITY FORMULA
i = C/t^n
FREQUENCY—ONCE IN 5 YEARS

Fig. 4.

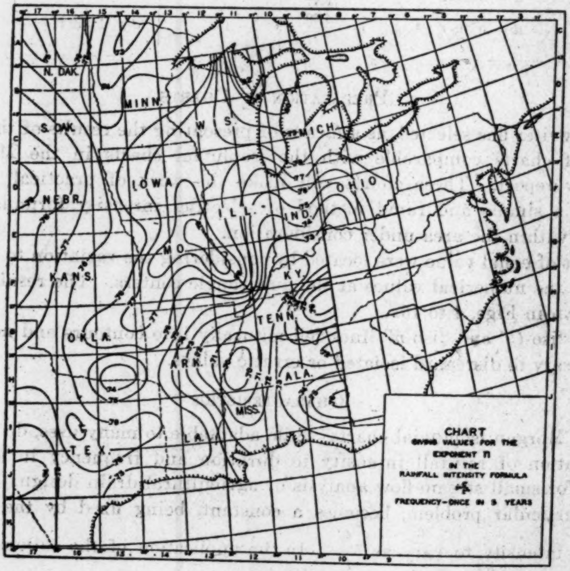

N. DAK.
MINN.
S. DAK.
WIS
MICH
IOWA
OHIO
NEBR.
ILL
IND.
MO.
KANS.
KY.
TENN.
OKLA.
ARK.
ALA.
MISS.
LA.
TEX.
CHART
GIVING VALUES FOR THE
EXPONENT n
IN THE
RAINFALL INTENSITY FORMULA
i = C/t^n
FREQUENCY—ONCE IN 5 YEARS

Fig. 5.

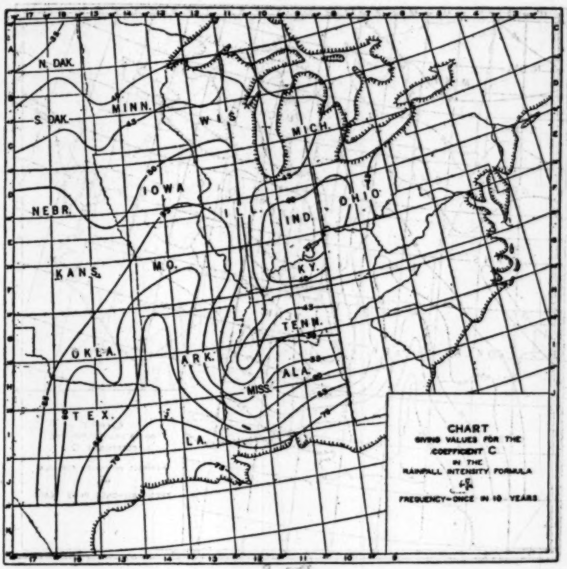

N. DAK.
S. DAK.
MINN.
WIS.
MICH.
IOWA
NEBR.
ILL.
IND.
OHIO
KANS.
MO.
KY.
TENN.
OKLA.
ARK.
ALA.
MISS.
TEX.
LA.
CHART
GIVING VALUES FOR THE
COEFFICIENT C
IN THE
RAINFALL INTENSITY FORMULA
$i=\frac{C}{t^{n}}$
FREQUENCY—ONCE IN 10 YEARS

Fig. 6.

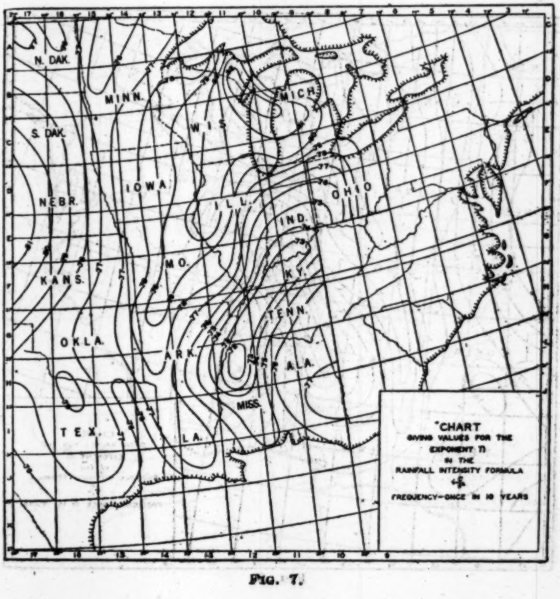

N. DAK.
MINN.
MICH.
S. DAK.
WIS.
IOWA
OHIO
NEBR.
ILL.
IND.
KANS.
MO.
KY.
TENN.
OKLA.
ARK.
ALA.
MISS.
TEX.
LA.
CHART
GIVING VALUES FOR THE
EXPONENT n
IN THE
RAINFALL INTENSITY FORMULA
$i=\frac{C}{t^{n}}$
FREQUENCY—ONCE IN 10 YEARS

Fig. 7.

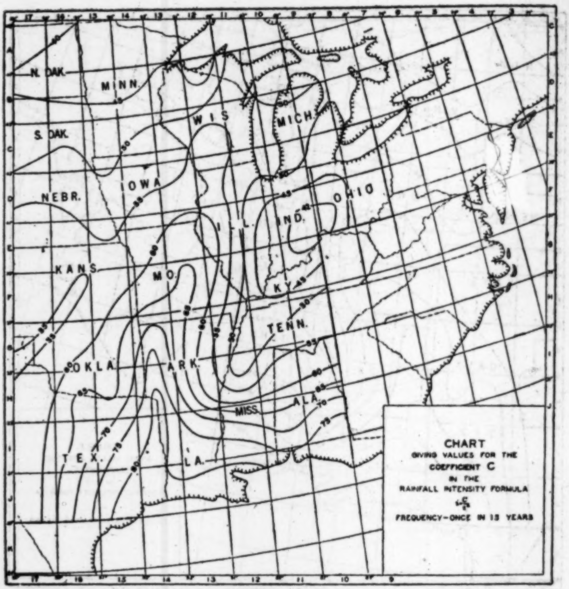

N. DAK.
MINN.
S. DAK.
WIS.
MICH.
IOWA
NEBR.
OHIO
ILL.
IND.
KANS.
MO.
KY.
TENN.
OKLA.
ARK.
MISS.
ALA.
TEX.
LA.
CHART
GIVING VALUES FOR THE
COEFFICIENT C
IN THE
RAINFALL INTENSITY FORMULA
$i = \frac{C}{t^n}$
FREQUENCY—ONCE IN 15 YEARS

FIG. 8.

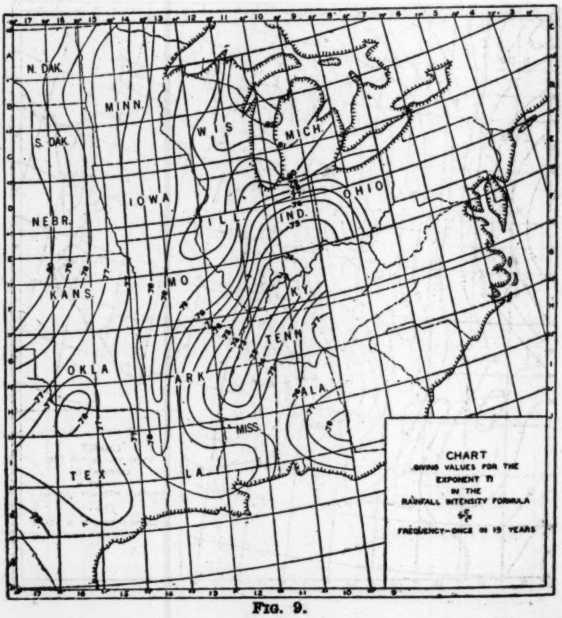

N. DAK.
MINN.
S. DAK.
WIS.
MICH.
IOWA
NEBR.
OHIO
ILL.
IND.
KANS.
MO.
KY.
TENN.
OKLA.
ARK.
MISS.
ALA.
TEX.
LA.
CHART
GIVING VALUES FOR THE
EXPONENT n
IN THE
RAINFALL INTENSITY FORMULA
$i = \frac{C}{t^n}$
FREQUENCY—ONCE IN 15 YEARS

FIG. 9.

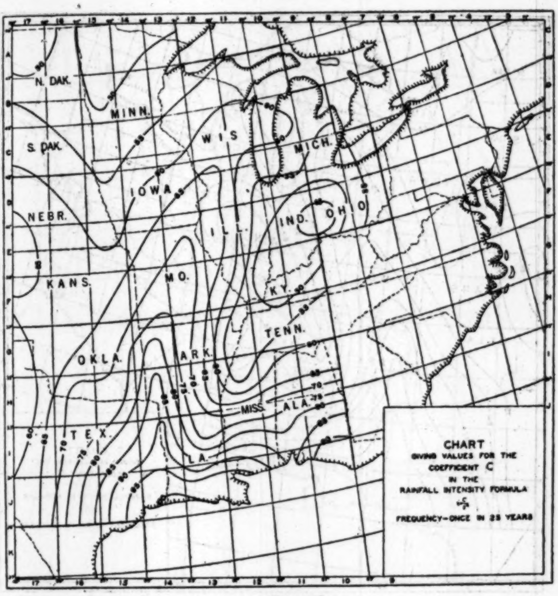

N. DAK.
MINN.
S. DAK.
WIS.
MICH.
IOWA
NEBR.
ILL.
IND.
OHIO
KANS.
MO.
KY.
TENN.
OKLA.
ARK.
MISS.
ALA.
TEX.
LA.
CHART
GIVING VALUES FOR THE
COEFFICIENT C
IN THE
RAINFALL INTENSITY FORMULA
$i=\frac{C}{t^n}$
FREQUENCY—ONCE IN 25 YEARS

Fig. 10.

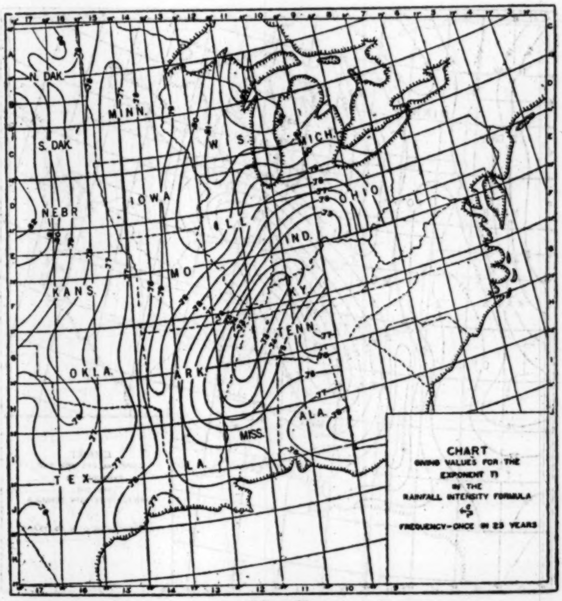

N. DAK.
MINN.
S. DAK.
WIS.
MICH.
IOWA
OHIO
NEBR.
ILL.
IND.
KANS.
MO.
KY.
TENN.
OKLA.
ARK.
ALA.
MISS.
TEX.
LA.
CHART
GIVING VALUES FOR THE
EXPONENT n
IN THE
RAINFALL INTENSITY FORMULA
$i=\frac{C}{t^n}$
FREQUENCY—ONCE IN 25 YEARS

Fig. 11.

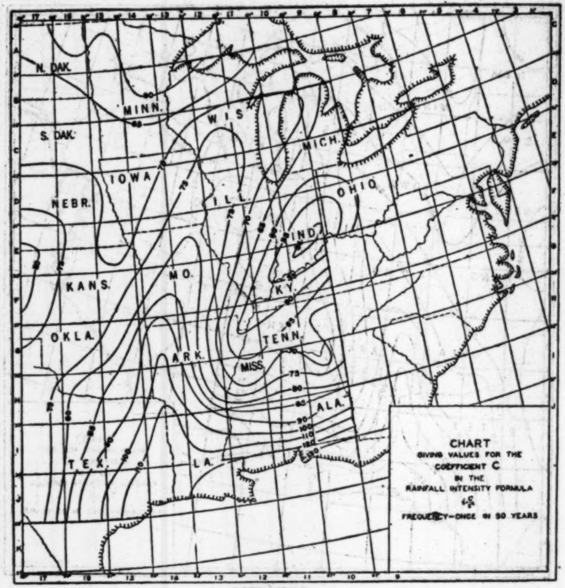

N. DAK.
MINN.
S. DAK.
WIS.
MICH.
IOWA
NEBR.
ILL.
OHIO
IND.
KANS.
MO.
KY.
OKLA.
TENN.
ARK.
MISS.
ALA.
TEX.
LA.
75
80
90
100
110
120
CHART
GIVING VALUES FOR THE
COEFFICIENT C
IN THE
RAINFALL INTENSITY FORMULA
$i=\frac{C}{t^n}$
FREQUENCY—ONCE IN 50 YEARS

FIG. 12.

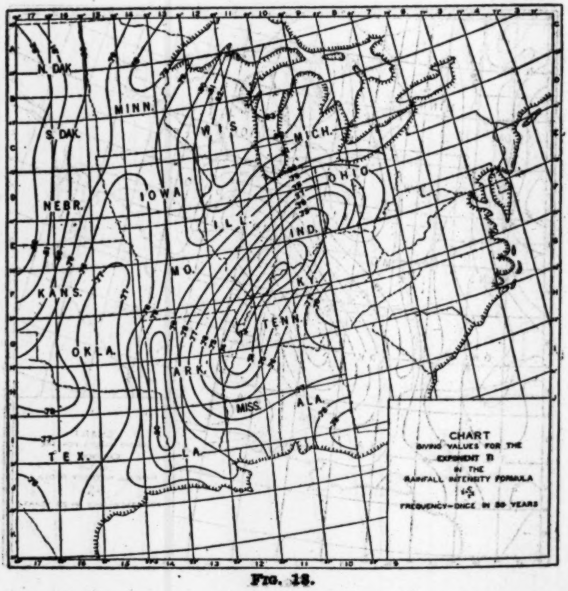

N. DAK.
MINN.
S. DAK.
WIS.
MICH.
IOWA
OHIO
NEBR.
ILL.
IND.
KANS.
MO.
KY.
TENN.
OKLA.
ARK.
MISS.
ALA.
TEX.
LA.
.75
.77
CHART
GIVING VALUES FOR THE
EXPONENT n
IN THE
RAINFALL INTENSITY FORMULA
$i=\frac{C}{t^n}$
FREQUENCY—ONCE IN 50 YEARS

FIG. 13.

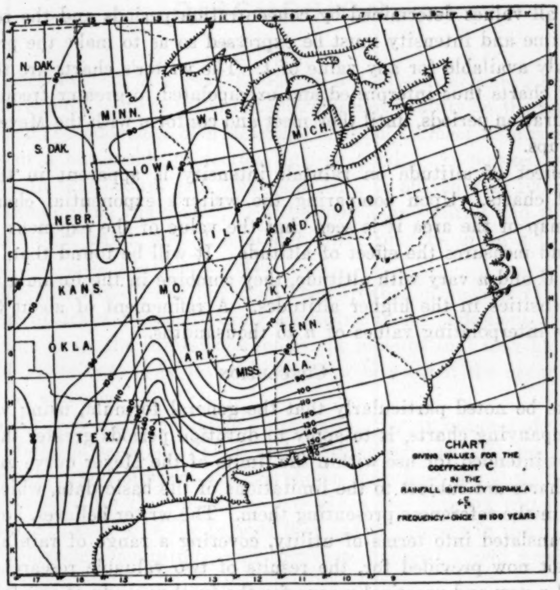

N. DAK.
MINN.
S. DAK.
WIS.
MICH.
IOWA
NEBR.
ILL.
IND.
OHIO
KANS.
MO.
KY.
TENN.
OKLA.
ARK.
MISS.
ALA.
TEX.
LA.
CHART
GIVING VALUES FOR THE
COEFFICIENT C
IN THE
RAINFALL INTENSITY FORMULA
$i=\frac{C}{t^n}$
FREQUENCY—ONCE IN 100 YEARS

FIG. 14.

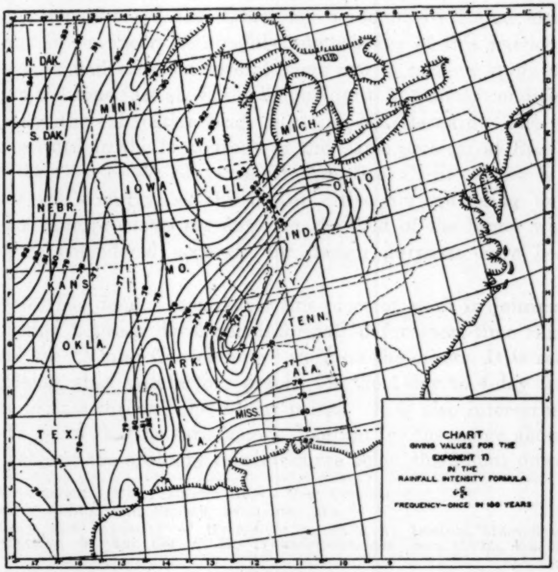

N. DAK.
MINN.
S. DAK.
WIS.
MICH.
IOWA
ILL.
OHIO
NEBR.
IND.
KANS.
MO.
KY.
TENN.
OKLA.
ARK.
ALA.
MISS.
TEX.
LA.
CHART
GIVING VALUES FOR THE
EXPONENT n
IN THE
RAINFALL INTENSITY FORMULA
$i=\frac{C}{t^n}$
FREQUENCY—ONCE IN 100 YEARS

FIG. 15.

will take all values determined by concentration periods and the relationship between time and intensity must be expressed so as to make the proper value of i quickly available for any value of t. The writer's charts are the Morgan isopluvial charts thus interpreted and extrapolated to greater frequencies and shorter duration periods, until they meet and conform with the Meyer intensity curve groups.

The effect of altitude on rainfall intensity is apparent in the Morgan isopluvial charts. Upon comparing the writer's exponential charts with a contour map of the area it is seen that the value of the exponent apparently reflects and measures the effect of altitude. It will be found that, while both values of C and n vary with altitude, they combine in the formula to give the lower intensities in the higher altitudes. A refinement of about 5% can be gained by interpolating values of n to thousandths.

CONCLUSION

It is to be noted particularly that the general formula, using values from the accompanying charts, is to apply to duration periods greater than 2 hours and is not intended for use within the limits of the Meyer curve groups.

The charts are subject to the limitations of the basic data, which are fully set forth in the references presenting them. The writer believes, however, that he has translated into terms of utility, covering a range of rainfall duration periods not now provided for, the results of two valuable researches and has suggested a new and practical means for the further study of rainfall intensity and its relation to the length of storm period, frequency of occurrence, and locality.

DISCUSSION

W. B. Gregory,[11] **M. Am. Soc. C. E.** (by letter).—Concerning problems of rainfall and run-off there is often considerable uncertainty in the mind of the designer as to the adequacy of a drainage channel to meet requirements that will represent a reasonable protection against floods. As pointed out by the author the rain that occurs as a maximum once in 50 years or 100 years may occur any time during the period. Furthermore, the rainfall and run-off data are not always at hand, and if available there may be quite a bit of work required to put them into usable form.

The subject of run-off has too often been treated in an empirical manner, with the size of drain designed purely according to the judgment of the engineer in charge or by the limitations set by the amount that may be expended. Even in the latter case it is desirable to know how often the capacity of the drains will be exceeded.

The author has made available information for intelligent design for a large part of the Mississippi Valley and portions outside. He has connected the work of Meyer and Morgan, and has shown that these two authorities agree in basic material. By this means he has given the data on which times of concentration are known in the periods that are of interest in the drainage of areas of considerable magnitude, and thus has rendered a real service to the profession.

W. W. Horner,[12] **M. Am. Soc. C. E.** (by letter).—In developing certain data for his personal need, the author has undoubtedly found himself in a large company of engineers thoroughly appreciative of this particular situation. The writer had occasion several years ago to make a study of rainfall frequencies in the zone just above the 2-hour limit, but under conditions which did not warrant the exhaustive study made by Mr. Bernard. The writer has also been interested in the local study of this same problem at Boston, Mass., as reported[13] by Charles W. Sherman, M. Am. Soc. C. E. There is no doubt that all over the country, as the bigger cities are spreading out and as municipal drainage problems are entering the field of the larger water-sheds, data heretofore considered accurate for design purposes have been found insufficient.

Mr. Bernard has done a remarkably fine piece of work in joining together and interpolating between developed information for short-time rains on the one hand and the "Miami Conservancy" work on the other. It is remarkable, almost surprising, that satisfactory graphs for the 1-day to 4-day frequencies can be made to pass through the 2-hour points. It is also interesting to note Mr. Bernard's belief that no satisfactory diagram for the range above 2 hours can be applicable to the ordinary rainfall curve below the 2-hour duration. In comparison with this, should be noted Mr. Sherman's belief[14] that a single formula can be developed which would cover the entire range up to several days. In a discussion[15] of Mr. Sherman's paper, the writer suggested that a formula devised for Boston did not seem to be satisfactorily in accord with the data for durations in excess of 24 hours.

---

[11] Prof. Experimental Eng., Tulane Univ., New Orleans, La.

[12] Chf. Engr., Sewers and Paving, St. Louis, Mo.

[13] "Frequency and Intensity of Excessive Rainfalls at Boston, Massachusetts," by Charles W. Sherman, M. Am. Soc. C. E., *Transactions*, Am. Soc. C. E., Vol. 95 (1931), p. 951.

Mr. Sherman's work in Boston is based upon complete local information covering the full range of duration. In Mr. Bernard's studies no actual data are available for the duration period between 2 and 24 hours, and his results are of value because of the remarkable concurrence at the two edges of this zone. It supplies a new tool for general use in this intermediate field, a working instrument apparently much more reliable than anything that has been published before. There is no doubt that to municipal engineers as well as land drainage engineers the results of Mr. Bernard's work will be exceedingly helpful. It can be used to good advantage until the time comes when some project of the general magnitude of the Miami Conservancy work may justify the expense of determining frequencies in this intermediate zone directly from recorded rainfall information. When this time comes, authoritative studies will be available throughout the entire range, and it will be possible to determine whether any single formula will be found to be completely satisfactory. In the meantime, Mr. Bernard's work should receive the apreciation of all practicing engineers interested in storm-water drainage.

ADOLPH F. MEYER,[16] M. AM. SOC. C. E. (by letter).—It is always a pleasure to read a paper like that of Mr. Bernard which sets forth the results of an extensive study in a clean-cut manner, ready for practical application.

Of the fundamental principles referred to by the author, the writer desires to emphasize the following, expressed in his own language:

> 1.—From the viewpoint of most structures involved, the effect of actual rainstorms, varying in intensity as they do, is practically the same as if the precipitation had been uniformly distributed over the given time interval.
>
> 2.—The sum of the records of several stations within a limited area, even if of considerable extent, gives a better indication of the probable rainfall experience of a given locality than the record of a single station in that locality. Such combined records are reasonably equivalent to a long-time record for that station.

The writer hopes that these two fundamental principles, supported as they are by ample proof in the literature of hydrology, will soon receive general acceptance by the profession. Such acceptance will be in the interest of both safe and economical structural design.

Another factor in design, which the author rightfully emphasizes, but which is often neglected or cloaked in generalities, is the element of probable average frequency of recurrence of given phenomena. Although at best a prediction can only represent a probable average experience, that is no reason why the best possible prediction should not be made in reasonably specific terms and these terms used as the basis for design.

---

[14] *Transactions*, Am. Soc. C. E., Vol. 95 (1931), p. 954.

[15] *Loc. cit.*, p. 961.

[16] Cons. Hydr. Engr., Minneapolis, Minn.

In his opening discussion, the author concludes that the relation between rainfall intensity and duration is more accurately expressed by the parabolic formula, $i = \frac{C}{t^n}$, than by the formula, $i = \frac{a}{t + b}$. This conclusion is apparently in conflict with what follows in the paper. The author specifically points out (under the heading "Intensity Formula Types") that the Meyer formulas of the type, $i = \frac{a}{t + b}$, give correct results over time intervals to about 200 min. They are intended for application only to time intervals up to 120 min. Fig. 1 shows that Mr. Bernard's parabolic type of formula can be used only for time intervals longer than 40 min. The writer has never suggested the application of his formulas to time intervals longer than 120 min.

At best, all formulas of rainfall intensity are empirical. Since they are derived from a given group of physical data, it follows as an elementary principle that they cannot be extended, with the slightest assurance of accuracy, more than a few per cent. beyond the limits of the data on which they are based. Discussions tending to show that certain formulas cannot be applied to data lying far outside the limits of the group of base data used in their derivation, are beside the mark. All too frequent attempts are being made to extend formulas and principles far beyond the scope of the base data. When such attempts lead to error, the formula or the principle often is blamed. The author, however, specifically points out the limitations of the several formulas.

Natural phenomena such as rainfall intensity and duration are virtually haphazard occurrences. The derived formulas are empirical, and since they do not embody a fundamental physical relationship, such formulas are necessarily limited to the base data from which they are derived.

The writer would like to see the author's conclusion respecting the limits of application of his charts added to the title of each chart lest these charts be reprinted and used without the author's qualifying statement that they are intended for application to duration periods greater than two hours.

R. W. POWELL,[17] ASSOC. M. AM. SOC. C. E. (by letter).—The author's charts give in condensed form a mass of valuable information regarding the probable intensity of rainfall for periods of from 2 hours to 3 days for an important section of the United States. It is to be hoped that some one will develop similar information for the remainder of the country. It is questionable, however, whether it is worth while to distinguish different values of n for different frequencies because, for most places, the values in Fig. 11 (for a frequency of once in twenty-five years) could be used for all frequencies without introducing errors larger than those which are probably already present in the data. For example, the values of n shown on the charts for Detroit, Mich., vary from 0.780 to 0.805, while Milton F. Wagnitz, M. Am. Soc. C. E., and Lewis C. Wilcoxen, Assoc. M. Am. Soc. C. E., from a study of Detroit data alone found a value of 0.855.[18] On the other hand, C is clearly a function of the frequency, but it seems probable that this relationship could be represented by an equation which would require the use of only one map. Charles W. Sherman,[19] M. Am. Soc. C. E., has suggested a formula for Boston, Mass., which can be expressed as:

---

[17] Asst. Prof. of Mechanics, Ohio State Univ., Columbus, Ohio.

$$C = k\,F^{0.27} \quad \text{......(4)}$$

Equation (10) for Detroit[18] is practically the same formula. The writer has read values for Columbus, Ohio, from the author's charts and finds a relationship that can be expressed either by,

$$C = 24\ F^{0.23} \quad \text{......(5)}$$

or

$$C = 13 + 27 \log_{10} F \quad \text{......(6)}$$

Equation (6) gives the best fit, but Equation (5) is the more convenient for combining with other equations. Putting it in the general form,

$$C = k\ F^{x} \quad \text{......(7)}$$

and substituting in the author's Equation (3):

$$i = \frac{k\,F^{x}}{t^{n}} \quad \text{......(8)}$$

For long periods the total precipitation is more useful than the rate of precipitation. If P = total inches of precipitation in t min.,

$$P = \frac{t\ i}{60} = \frac{k\ t^{(1-n)}\ F^{x}}{60} = K\ t^{(1-n)}\ F^{x} \quad \text{......(9)}$$

in which, $K = \frac{k}{60}$. For most points considered n seems to be approximately 0.75 and x, approximately 0.25, which leads to the interesting approximate formula,

$$P = K\ (t\ F)^{0.25} \quad \text{......(10)}$$

This would require only one map giving values of K. For more accurate values Equation (9) would be used. This would require another map giving values of $(1 - n)$. The coefficient, x, would probably vary so little with the location that an average value could be used.

The question presents itself as to whether these formulas could be used for periods longer than three days. This was tested for Columbus, Ohio, by computing the values given for a period of one month and comparing them with observed values. Using $n = 0.77$, $k = 24$, and $x = 0.23$, Equation (9) becomes,

$$P = 0.40\ t^{0.23}\ F^{0.23} \quad \text{......(11)}$$

The equality of the exponents in Equation (11) is probably only a coincidence. Taking the average month as 43 830 min., the probable maximum monthly precipitation occurring once in F years is computed in Table 4. Values in Column (3), Table 4, were taken from Fig. 16. These curves were plotted from records of the maximum recorded monthly rainfall at the Columbus Weather Bureau from 1878 to 1921, at Ohio State University from 1882 to 1921, and at Westerville (13 miles north of Columbus), from 1854 to 1902.[20] The data were combined to make one composite record of a total of 1 555 months which was considered as typical of the vicinity.

---

[18] *Transactions*, Am. Soc. C. E., Vol. 95 (1931), p. 964.

[19] *Loc. cit.*, p. 955.

TABLE 4.—PROBABLE MAXIMUM MONTHLY PRECIPITATION AT COLUMBUS, OHIO

Frequency of occurrence once in F years (1)	Total precipitation, P, in inches, by Equation (11) (2)	From the curve of observed values, Fig. 16 (3)
5	6.77	7.7
10	7.94	8.4
15	8.71	9.0
25	9.80	9.8
50	11.49	10.9
100	13.48	12.1

The fact that these formulas give such reasonable values when extrapolated so far beyond the range for which they were derived, speaks well for the soundness of the author's method and indicates that his formulas could be used safely for periods of more than three days. It is interesting to note, however, that the formulas do not hold for periods as long as one year. Equation (11) gives a 1-year maximum of 12.0 in. once in 5 years, and 23.9 in. once in 100 years. The actual values from Fig. 16 are 42.0 and 53.3 in., respectively. These conclusions are based on Columbus, Ohio, only and need confirmation from other points.

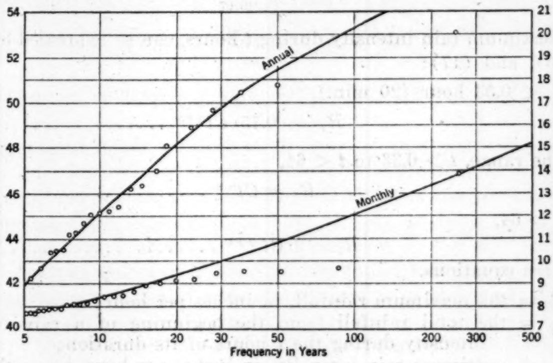


FIG. 16.—MAXIMUM RAINFALL AT COLUMBUS, OHIO.

---

[20] Data from "A Climatological History of Ohio," by W. H. Alexander, pub. by the Eng. Experiment Station, Ohio State Univ., 1923.

C. E. GRUNSKY,[21] PAST-PRESIDENT, AM. SOC. C. E. (by letter).—As the outcome of long experience in the use of formulas for the approximation of probable maximum rain intensity during time periods not sufficiently covered by actual observation, the writer suggests the following formulas which differ from those heretofore in common use mainly in that the factor, time, is expressed in hours instead of in minutes.

This departure appears desirable because the rain intensity is commonly expressed in inches per hour and not in inches per minute. Furthermore, the coefficient which appears in the formula most frequently used, thus becomes equal to the maximum number of inches of rain in 1 hour, a fact which facilitates a first approximation thereof. The formulas herein suggested should commend themselves by reason of their simplicity.

The maximum rain intensity during t hours, can be expressed by Equations (12), (13), and (14):

For $t < 0.33$ hour (20 min.),

$$i = \frac{0.75\ C}{t^{0.75}} \qquad (12)$$

For the range, $t > 0.33$ to $t < 64$,

$$i = \frac{C}{t^{0.50}} \qquad (13)$$

For $t > 64$,

$$i = \frac{2\ C}{t^{0.67}} \qquad (14)$$

The maximum rain intensity during t hours, can be expressed by Equations (15), (16), and (17):

For $t < 0.33$ hour (20 min.),

$$R_t = 0.75\ Ct^{0.25} \qquad (15)$$

For the range, $t > 0.33$ to $t < 64$,

$$R_t = Ct^{0.50} \qquad (16)$$

For $t > 64$,

$$R_t = 2\ Ct^{0.33} \qquad (17)$$

In these equations,

R = the maximum rainfall, in inches per hour;
R_t = the total rainfall from the beginning of a rain of maximum intensity during the t hours of its duration;
i_t = maximum rain intensity, in inches per hour, being the maximum average rate of rainfall during t hours;
t = the time, in hours, for which maximum rain intensity or the maximum total rainfall is to be determined; and
C = a coefficient which is to be ascertained for any locality from records of measured maximum rainfall.

These formulas for maximum rain intensity as well as for the maximum quantity of rain, if plotted, will be found to represent continuous lines with slight changes in direction at the 0.33-hour and 64-hour points. Fig. 17 shows these lines on logarithmic paper for values of t, ranging from 0.01 hour (0.60 min.) to periods far beyond any that ordinarily come under consideration.

---

[21] Cons. Engr. (C. E. Grunsky Co.), San Francisco, Calif.

The formulas are to be adapted to any place by the selection of a proper value of C. In each case this will be based, of course, on the best information relating to measured heavy rainfall at, or near, or in the general region of, the place in question. They should be found satisfactory in all cases in which the probable maximum rain is to be determined at any place for short or long time periods, from records which have already established the probable maxima for other comparable short or long periods, as the case may be. A computation sheet, as shown in Table 5, will be found convenient.

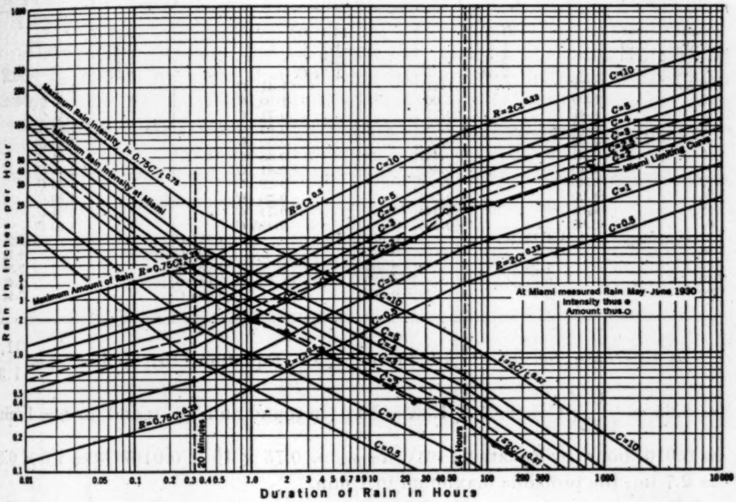


FIG. 17.—PROBABLE RAINFALL MAXIMA IN VARIOUS TIME PERIODS AND CURVES OF MAXIMUM RAIN INTENSITY.

These formulas are intended to replace formulas of the types (Equations (2) and (3)) referred to by the author. The coefficients a, b, and C, and the exponent, n, in Mr. Bernard's formulas, all depend for value on locality. The time, t, in these formulas is expressed in minutes while i is expressed in inches per hour.

The simplified Equations (12) to (17) are intended to obviate the inconvenience and limited application of the older types.

The following examples of exceptionally heavy rainfall are illustrative:

Example 1.—At Campo, Calif., on August 12, 1891, 11.5 in. of rain fell in 1.33 hours (80 min.). What was the probable maximum rainfall in 1 min. during this storm?

TABLE 5.—FORMULAS FOR MAXIMUM RAIN INTENSITY, COMPUTATION SHEET

Time, in hours, t	$t^{0.25}$	$t^{0.50}$	$t^{0.33}$	$\frac{1}{t^{0.75}}$	$\frac{1}{t^{0.50}}$	$\frac{1}{t^{0.67}}$	MAXIMUM RAIN INTENSITY, IN INCHES PER HOUR			MAXIMUM QUANTITY OF RAIN, IN INCHES		
							$\frac{0.75\ C}{t^{0.75}}$	$\frac{C}{t^{0.50}}$	$\frac{2\ C}{t^{0.67}}$	$0.75\ C\ t^{0.25}$	$C\ t^{0.50}$	$2\ C\ t^{0.33}$
0.0167	0.36			21.5			16.1 *C*			0.27 *C*		
0.083	0.54			6.5			4.7 *C*			0.40 *C*		
0.167	0.64			3.8			2.9 *C*			0.48 *C*		
0.333	0.76			2.28			1.71 *C*			0.57 *C*		
0.50		0.71			1.41			1.41 *C*			0.71 *C*	
1		1.00			1.00			1.00 *C*			1.00 *C*	
2		1.41			0.71			0.71 *C*			1.41 *C*	
3		1.73			0.58			0.58 *C*			1.78 *C*	
4		2.00			0.50			0.50 *C*			2.00 *C*	
5		2.24			0.447			0.45 *C*			2.24 *C*	
6		2.45			0.408			0.41 *C*			2.45 *C*	
12		3.46			0.289			0.29 *C*			3.46 *C*	
24		4.90			0.204			0.20 *C*			4.90 *C*	
48		6.94			0.144			0.14 *C*			6.94 *C*	
64		8.00			0.125			0.125 *C*			8.00 *C*	
72			4.13			0.0585			0.117 *C*			8.26 *C*
96			4.58			0.0475			0.095 *C*			9.16 *C*
120			4.94			0.0410			0.082 *C*			9.88 *C*
240			6.21			0.0259			0.052 *C*			12.42 *C*
720			8.97			0.0124			0.025 *C*			17.94 *C*

In this case, because $t > 0.33$ and < 64, Equation (13) applies: $i_t = \frac{11.5}{1.33} = 8.65 = \frac{C}{1.33}$; $C = 8.65 \times 1.33 = 10.0$ (in round numbers); and for $t = 1$ min. $= 0.0167$ hour (by Equation (16)), $R_{0.0167} = 0.75 \times 10 \times 0.0167^{0.25} = 7.5 \times 0.36 = 2.7$ in., the probable maximum in 1 min.

Example 2.—At Orpid's Camp, about 20 miles northeasterly from Los Angeles, Calif., observations with two recording rain gauges, about 4 ft. apart, showed rainfall in 1 min. (from 4:43 to 4:44 A. M.) of 1.03 in. in one gauge and 0.92 in. in the other. What coefficient does this indicate for the formulas of maximum rainfall?

Taking the maximum rainfall of this storm at 1.00 in. per min. (0.0167 hour), $i_{0.0167} = 1.00$ in. per min. $= 60$ in. per hour. By Equation (15), $R_{0.0167} = 1 = 0.75\ C\ (0.0167^{0.25})$; from which, $C = 4.2$ and $R = 4.2$ in., the probable maximum for any storm at this place during 1 hour.

The storm produced 1.17 in. of rain in 10 min. and 2.20 in. in 1 hour. For the 10-min. period (0.167 hour) Equation (15) would give, $R_{0.167} = 4.2 \times 0.167^{25} = 2.7$ in., the probable maximum, to be compared with the 1.17 in. as measured.

Example 3.—In a letter to the writer, Mr. J. F. Voorhees, Associate Meteorologist of the U. S. Weather Bureau, has stated that:

"On November 18, 1930, an automatic recorder belonging to the U. S. Geological Survey and situated in Moanalua Valley, Honolulu, measured 15.2 in. in 3 hours and 5.6 in. in 1 hour."

In this case, $C = 5.6$, and the probable maximum fall of rain during any single minute was $5.6\ \sqrt[3]{0.0167} = 1.4$ in.

Example 4.—At Miami, Fla., there was an unusually heavy rain from May 29 to June 2, 1930.[22] This storm established a near record for the periods of 1 hour, 2 hours, 4 hours, and a full day, as well as for the longer period of 44 hours. A comparison of the measured rain, during this storm, with the limiting curve deduced therefrom, is presented in Table 6. In this table, R_{44} (43.93 hours), equals 16.49 in. Then, by Equation (16), $R_{44} = C \sqrt{43.93} = 16.49$; from which, $C = 2.48$. Similarly, $R_4 = C \sqrt{4} = 4.39$; from which, $C = 2.20$.

These results indicate that at Miami the value of C should be taken at 2.50. It is with this value that the comparisons shown in Table 6 have been made.

TABLE 6.—RAIN AT MIAMI, FLORIDA, MAY 27 TO JUNE 19, 1930, COMPARED WITH COMPUTED MAXIMA

Period, hours	MEASURED RAINFALL		COMPUTED BY FORMULAS		Remarks, $C = 2.5$
	Intensity, in inches per hour	Quantity, in inches	Maximum intensity, in inches per hour	Maximum quantity, in inches	
0.0167	.....	......	40.	0.67	1 min.
0.167	.....	......	7.2	1.20	10 min.
0.50	.....	......	3.5	1.77	30 min.
1.	1.87	1.87	2.50	2.50	....
2	1.54	3.07	1.77	3.54	....
3	.....	......	1.44	4.33	....
4	1.10	4.39	1.25	5.00	....
5	0.91	4.53	1.12	5.59	....
6	.....	......	1.02	6.13	....
12	.....	......	0.94	8.65	....
24	0.38	9.36	0.51	12.25	....
44	0.375	16.49	0.378	16.60	43.93 hours.
48	.....	......	0.360	17.3	....
72	.....	......	0.286	20.6	....
96	.....	......	0.242	23.2	....
120	0.161	19.28	0.205	24.6	....
240	.....	......	0.108	26.	....
545*	0.061	33.16*	0.075	41.	....
720	.....	......	0.062	45.	30 days.
1 440	.....	......	0.0395	57.	....
8 640	.....	......	0.0119	103.	1 year.

*This period from May 27 to June 19, includes 49 consecutive hours on May 27, 28, and 29, after the beginning of the storm in which no rain fell.

It appears from the foregoing that while the greatest measured rainfall in 1 min. may well be credited to Orpid's Camp, there have, no doubt, been many other rainstorms with short-time rainfall of still greater intensity, as indicated by Examples 1 and 3.

More is known of the maximum rate of precipitation to be expected at single stations than of the maximum average rate in any selected period of time over extensive water-sheds. Thus, for example, during the entire storm at Campo in which 11.5 in. of rain fell in 80 min., the rainfall at another point only 1.25 miles distant, was only 3 in. What correction factors should be used for area and for location and orographic features? There is still ample field for study and such contributions as that of Mr. Bernard are valuable additions to the literature on this subject.

---

[22] See *Monthly Weather Review*, June, 1930.

CHARLES W. SHERMAN,[23] M. AM. SOC. C. E. (by letter).—In utilizing the Miami Conservancy District records of 1-day, 2-day, 3-day storms, etc., allowance ought to be made for the fact that few if any of these storms lasted exactly 24, 48, or 72 hours. Nearly all the figures are based on records of ordinary (not recording) rain-gauges, observed once daily. All the rain accumulated between two such readings is assumed to have fallen in 24 hours. There is nothing in the records themselves to show that a 1-day precipitation may not have fallen in a single hour or even less, a 2-day collection in, say, 2 hours, one preceding and one following the time of observation, and a 3-day collection within about 25 or 26 hours. It is unlikely that maximum rainfalls can occur in such short periods, but it is not improbable that some of the high 1-day rainfalls actually occurred in periods not exceeding 20 hours; the 2-day rainfalls in 40 to 42 hours, etc.

The writer believes, therefore, that Mr. Bernard was not justified in using 1 440 min., 2 880 min., etc., as representing the durations of such storms. In no case could the durations exceed those assumed, and in an appreciable number of cases they must have been considerably shorter. Would it not have been more correct to assume that the actual durations were at least 10% shorter than the number of minutes corresponding to the length of the storm in days? Actually, the storms must have varied in duration, so that curves based upon the assumption that all 1-day storms lasted exactly 1 440 min., or any other definite number, can be at best only approximate.

Another inaccuracy is in the treatment of all rainfall records within an arbitrary quadrangle, as though they represented observations at a single station, with a length of record equal to the sum of the periods of observation at the several stations. In the first place, the quadrangles are so large that wide variations in storm-producing conditions are likely to exist at various points in many of them; and, second, the combination of a number of short records obtained during a period when either an excess or a deficiency in rainfall is generally prevalent, cannot possibly be equivalent even to one long-term record covering a wide range of climatological conditions. In the major part of the United States, such a wide range is experienced when the term of years is great enough. The writer does not mean to imply that no advantage results from the combination of records, but rather to call attention to the fact that implicit confidence should not be placed in the formulas derived.

Examination of the charts, for values of the exponent, n (Figs. 5, 7, 9, 11, 13, and 15) shows a comparatively small range in these values—roughly from 0.72 to 0.83. In view of the inaccuracies inherent in the records, and the method of combining them, it appears to the writer that a single value, and that a convenient fraction, such as $\frac{3}{4}$ or $\frac{4}{5}$, might have been assumed for n, which would leave only one factor varying with location, namely, the coefficient, C.

---

[23] Cons. Engr. (Metcalf & Eddy), Boston, Mass.

The writer has shown elsewhere[13] that at least for some localities an intensity-duration equation of the form,

$$i = \frac{a}{(t + b)^n} \text{..........................} (18)$$

is much better for storms of short duration when the exponent approximates 0.7 than the Meyer form in which the exponent is 1.0. If b is less than 10, as seems probable from such data as have been studied, this does not differ materially in the case of long storms from,

$$i = \frac{a}{t^n} \text{..........................} (19)$$

As a matter of fact, the writer found a single formula applicable to Boston storms of both long and short durations, at least up to about 30 hours. With longer storms, especially those of rare occurrence, the formula was not as good a representation of the observed intensities as might be desired.

These criticisms are intended primarily as warnings against putting too great faith in the formulas derived by Mr. Bernard. They are not intended to throw discredit upon the results for practical use in drainage problems. The immense amount of detailed work which the author has devoted to the study is obvious, and the writer believes that, for the section of the country to which the figures are applicable, the available data have been arranged in such shape that the designer of drainage works can make use of them with much greater confidence than has been possible heretofore.

MERRILL M. BERNARD,[24] M. AM. SOC. C. E. (by letter).—The closing discussion will be confined to the several important points raised and to a number of interesting facts which developed in consolidating, extending, and checking the rainfall intensity charts.

Agricultural drainage and stream-flow analysis have opened a comparatively new field of use for the "rational" method, and have created the need for formulas expressing the relationship between rainfall intensity, duration, and frequency for time intervals longer than 120 min. The time of concentration for a system of storm sewers is seldom more than 180 min.; concentration time at the upper end of an agricultural drain is never less than 60 min. The two fields will seldom overlap in the same problem, and there would be little advantage in covering all duration periods in one expression. The Meyer formulas, which may be safely extended to durations of 200 min., adequately meet the needs for design in the field of storm sewers. The writer's formulas, with values of t ranging from 60 min. to several days, may be used in the design of extensive drainage systems and in the analysis of stream flow of considerable magnitude.

Factor b was not introduced into the intensity formula for the following reasons:

(1) Its influence on intensity rapidly diminishes for duration periods beyond 100 min.

(2) The refinement gained in introducing this factor is not warranted when many greater discrepancies are accepted in the course of design.

---

[24] Cons. Civ. Engr., Crowley, La.

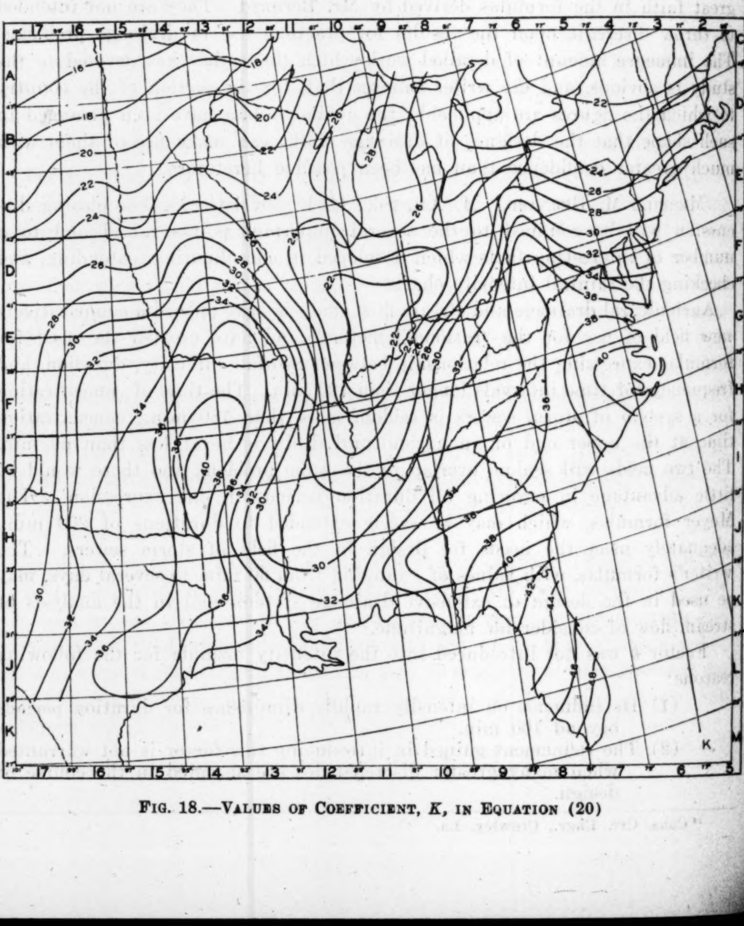
(3) The study, so extensive as to embrace one hundred and seven 2°-quadrangles, could only be undertaken by utilizing the facility of logarithmic plotting, which permitted the rapid and accurate adjustment of the curves, as straight lines, to the plotted points.

In extending the limits of the charted area, the writer has followed the suggestion that n need not vary with frequency, but may be considered to be constant and need only vary with geographical location, as may the exponent, x, in the formula,

$$i = \frac{K F^x}{t^n} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (20)$$

in which,

i = average rainfall intensity, in inches per hour.
F = frequency, in years.
t = duration of rainfall, exceeding 1 hour, in minutes.
K = a coefficient, depending for value on locality.
x and n = exponents, depending for value on locality.



17
16
15
14
13
12
11
10
9
8
7
6
5
4
3
2
A
B
C
D
E
F
G
H
I
J
K
C
D
E
F
G
H
I
J
K
L
M
16
18
18
20
20
22
22
24
24
26
26
28
28
30
32
34
36
38
40
36
34
32
30
32
34
36
30
32
34
36
38
40
42
44
46
48
22
24
26
28
30
32
36
40
K
6
7
8
9
10
11
12
13
14
15
16
17
5

FIG. 18.—VALUES OF COEFFICIENT, K, IN EQUATION (20)

The range in value of both the exponents is too great to permit further generalization by the use of average values. The formula may easily be modified if it becomes necessary to express time in hours. Rainfall depth is obtained by multiplying the computed average rainfall intensity by duration in hours.

Figs. 18, 19, and 20 have been constructed similar to Figs. 4 to 15, inclusive. Their limits have been extended to embrace all that part of the United States east of the 101st Meridian, completing the area covered by the Meyer and Morgan studies. Fig. 18 gives values for the coefficient, K; Fig. 19, values for the exponent, n; and Fig. 20, values for the exponent, x, in Equation (20).

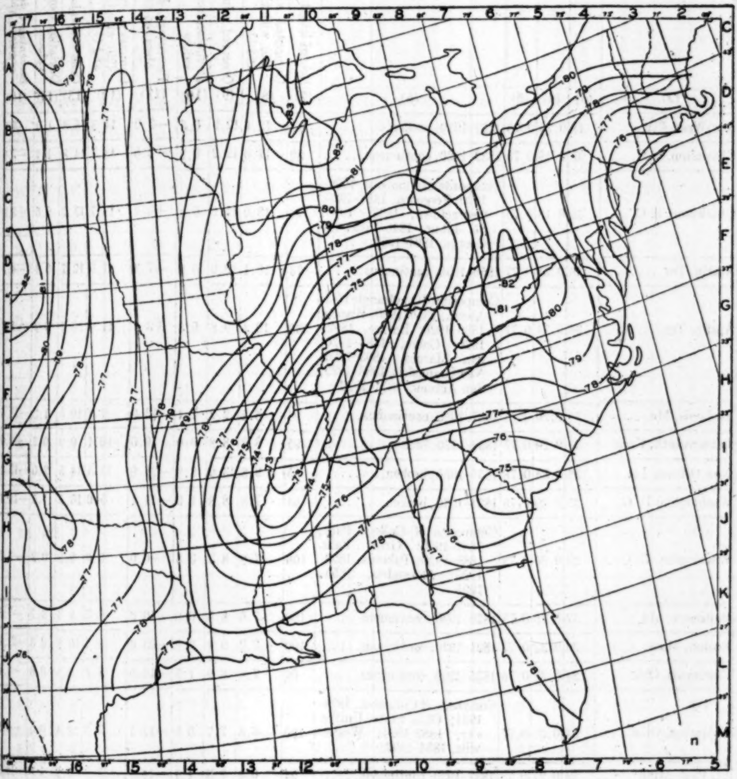


FIG. 19.—VALUES OF EXPONENT, n IN EQUATION (20)

Mr. Powell has shown that, at Columbus, the 30-day rainfall depths, as computed from the charts, agree quite accurately with the observed values taken from a composite record for the locality. The writer has followed the suggestion that this be verified at other points. The results are shown in Table 7, in which thirteen well-distributed points are listed according to

TABLE 7.—COMPARISON OF RAINFALL

Item No.	Station	VALUES OF COEFFICIENT AND EXPONENTS, EQUATION (20)			Rainfall record, U. S. Weather Bureau	Record length, in years	RAINFALL INTENSITIES FOR							
							F=5				F=10			
		K	x	n			Computed values, Equation (20) with Columns (3), (4), and (5)	Observed values	Deviation from observed values	Percentage, deviation from observed values	Computed values, Equation (20) with Columns (3), (4), and (5)	Observed values	Deviation from observed values	Percentage, deviation from observed values
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)
1	Key West, Fla.......	48	0.25	0.78	1832–1930, broken..........	81	12.4	12.9	0.5	—3.9	14.8	15.8	1.0	—6.3
2	Charleston, S. C.....	38	0.18	0.75	1832–1920, continuous......	89	12.0	12.2	0.2	—1.6	13.7	14.8	1.1	—7.4
3	Charleston, S. C.....	38	0.18	0.75	Composite of Pinopolis, 1900–1920; Ferguson, 1886–1920; Georgetown, 1893 – 1920; St. Johns, 1846–1861; St. George, 1894–1920	126	12.0	11.6	0.4	+3.4	13.7	13.2	0.5	+3.8
4	Austin, Tex.........	36	0.24	0.77	1856–1930, continuous......	73	10.1	10.9	0.8	—7.3	11.9	12.7	0.6	—4.7
5	Austin, Tex.........	36	0.24	0.77	Composite of Quadrangle 16-J, Austin, 1856–1920; Blanco, 1896–1920; Boerne, 1892–1920; Cuero, 1883–1920; San Marcos, 1893–1920; New Braunsfels, 1888–1920; San Antonio, 1871–1920	330	10.1	9.8	0.3	+3.1	11.9	12.1	0.2	—1.7
6	St. Louis, Mo.......	34	0.20	0.78	1837–1930, continuous......	94	8.1	9.2	1.1	—12.0	9.3	10.5	1.2	—11.4
7	Leavenworth, Kans..	33	0.20	0.77	1836–1930, broken..........	95	8.8	8.8	0.0	0.0	10.1	9.9	0.2	+2.0
8	New Orleans, La.....	33	0.34	0.78	1836–1930, broken..........	85	9.8	13.6	3.8	—28.0	12.5	14.5	2.0	—13.8
9	Washington, D. C...	33	0.20	0.78	1824–1930, broken..........	91	7.8	8.8	1.0	—11.4	9.0	10.5	1.5	—14.3
10	Washington, D. C...	33	0.20	0.78	Composite of College Park, 1861–1920; Great Falls, 1888–1920; Takoma, 1899–1920; Alexandria, 1853–1905	106	7.8	8.3	0.5	—6.0	9.0	9.2	0.2	—2.2
11	Baltimore, Md......	32	0.20	0.78	1820–1920, continuous......	100	7.6	8.5	0.9	—10.6	8.7	9.4	0.7	—7.5
12	Boston, Mass.......	26	0.23	0.77	1821–1920, continuous......	100	7.2	9.4	2.2	—23.4	8.5	10.8	2.3	—21.3
13	Cincinnati, Ohio.....	24	0.23	0.76	1835–1930, continuous......	96	7.4	8.9	1.5	—16.9	8.7	9.5	0.8	—8.4
14	Columbus, Ohio.....	24	0.23	0.77	Composite of Columbus, 1878–1921; Ohio State University, 1882–1921; Westerville, 1854–1902	130	6.8	7.7	0.9	—11.7	7.9	8.4	0.5	—6.0
15	St. Paul, Minn......	24	0.27	0.78	1837–1930, continuous......	94	6.4	7.5	1.1	—14.7	7.7	9.2	1.5	—16.3
16	Jamestown, N. Dak..	17	0.33	0.79	Composite of Ashley, 1892–1920; Edgerly, 1901–1920; Fullerton, 1898–1920; Fort Totten, 1869–1890; Jamestown, 1892–1920; Manfred, 1903–1920; Melville, 1898–1920; Napolian, 1892–1920: New Rockford, 1893–1915	200	4.5	5.1	0.6	—11.8	5.6	6.7	1.1	—16.4
17	Average deviation as a percentage of the observed values.	......	......	......	....	......	......	......	......	10.4	......	......	......	—9.0
18	Algebraic average of deviations as a percentage of the observed values.	......	......	......	....	......	......	......	......	—9.5	......	......	......	—8.2

INTENSITIES FOR VARIOUS FORMULAS

VARIOUS FREQUENCIES, *F*.																		
F=15				*F*=25				*F*=50					*F*=100					
Computed values, Equation (20), with Columns (3), (4), and (5) (16)	Observed values (17)	Deviation from observed values (18)	Percentage, deviation from observed values (19)	Computed values, Equation (20), with Columns (3), (4), and (5) (20)	Observed values (21)	Deviation from observed values (22)	Percentage, deviation from observed values (23)	Computed values, Equation (20), with Columns (3), (4), and (5) (24)	Observed values (25)	Deviation from observed values (26)	Percentage, deviation from observed values (27)	Record length, in years (28)	Computed values, Equation (20), with Columns (3), (4), and (5) (29)	Observed values (30)	Deviation from observed values (31)	Percentage, deviation from observed values (32)	Average percentage deviation from observed values (33)	Algebraic average deviation percentage of observed values (34)
16.3	16.6	0.3	−1.8	18.5	17.1	1.4	+8.2	22.0	18.2	3.8	+20.8	81	24.7	19.7	5.0	+25.4	11.1	+7.1
14.7	15.1	0.4	−2.6	16.1	16.0	0.1	+0.6	18.3	17.2	1.1	+6.4	89	20.2	19.1	1.1	+5.8	4.1	+0.2
14.7	15.1	0.4	−2.6	16.1	18.2	2.1	−11.5	18.3	21.5	3.2	−14.9	100	21.5	27.2	5.7	−21.0	9.5	−7.1
13.1	12.9	0.2	+1.7	14.8	16.2	1.4	− 8.6	17.4	20.0	2.6	−13.0	73	19.2	20.8	1.6	−7.7	7.2	−6.6
13.1	12.6	0.5	+4.0	14.8	14.8	0.0	0.0	17.4	16.2	1.2	+7.4	100	20.6	19.4	1.2	+6.2	3.7	+3.2
10.1	10.7	0.6	−5.6	11.2	11.1	0.1	+0.9	12.8	11.8	1.0	+8.5	94	14.5	17.0	2.5	−14.7	0.8	−5.7
10.9	10.6	0.3	+2.8	12.1	11.1	1.0	+9.0	13.8	12.6	1.2	+9.5	95	15.9	15.8	0.1	+0.6	4.0	+4.0
14.3	14.7	0.4	−2.7	17.0	17.0	0.0	0.0	21.5	19.5	2.0	+10.3	95	25.7	22.5	3.2	+14.2	11.5	−3.3
9.8	10.7	0.9	−8.4	10.8	11.5	0.7	−6.1	12.5	14.2	1.7	−12.0	91	14.3	14.4	0.1	−0.7	8.8	−8.8
9.8	9.6	0.2	+2.1	10.8	9.9	0.9	+9.1	12.5	10.3	2.2	+21.3	100	14.3	11.6	2.7	+23.3	10.7	+7.9
9.5	9.5	0.0	0.0	10.5	10.5	0.0	0.0	12.0	11.0	1.0	+9.1	100	13.9	12.3	1.6	+13.0	6.7	+0.7
9.3	11.1	1.8	−16.2	10.4	11.8	1.4	−11.9	12.3	12.1	0.2	+1.7	100	14.0	12.4	1.6	+12.9	14.6	−9.7
9.6	9.9	0.3	−3.0	10.7	10.5	0.2	+1.9	12.6	11.6	1.0	+8.6	96	14.8	12.0	2.8	+23.3	10.3	+0.9
8.7	9.0	0.3	−3.3	9.8	9.8	0.0	0.0	11.5	10.9	0.6	+5.5	100	13.5	12.1	1.4	+11.7	6.4	−0.6
8.6	9.5	0.9	−9.5	9.9	10.5	0.6	−5.7	11.9	11.2	0.7	+6.3	94	14.8	11.8	2.2	+18.6	11.8	−3.6
6.4	7.3	0.9	−12.3	7.6	8.0	0.4	−5.0	9.6	8.9	0.7	+7.9	100	12.0	9.3	2.7	+29.1	15.5	−1.4
......	......	....	4.9	......	......	....	4.9	......	......	....	10.2	......	......	....	....	14.3	9.0	.....
......	......	....	−3.6	......	......	....	−1.2	......	......	....	+5.2	......	......	....	....	+8.8	....	−1.4

the magnitude of the coefficient, K, in their intensity formulas. These are followed by the computed and observed 30-day rainfall depths, the deviation of the computed from the observed being expressed as a percentage of the observed. The study has a threefold interest:

First.—It has submitted the charts to a rather extensive check.

Second.—It indicates that the intensity and depth curves taken from the charts may be extrapolated to durations considerably beyond four days.

Third.—Considering monthly rainfall depth as a rate-duration relationship, it presents the opportunity to compare single station records of nearly 100 years, with composite records of equal or greater length.

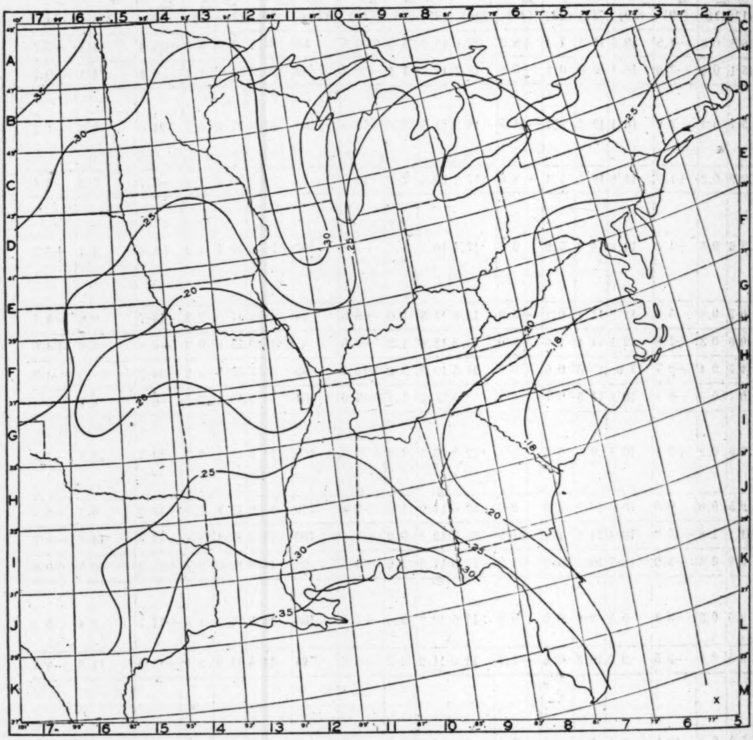


FIG. 20.—VALUES OF EXPONENT, x, IN EQUATION (20)

Because of meager intensity-duration data the theory of the composite record has not been fully established or explained, although it is rapidly being accepted as the basis for design. Composite records, varying in length, number of stations, distribution of recorded years, and arrangement of stations within the area represented, are compared with single station records

TABLE 8.—COMPARISON OF RESULTS TAKEN FROM FIGS. 18, 19, AND 20, WITH OTHER VALUES FOR BOSTON, MASS., AND DETROIT, MICH.

Station	Frequencies, F, in years	5				10				15				25				50				100			
	Duration of rainfall, t, in minutes	60	200	500	1 000	60	200	500	1 000	60	200	500	1 000	60	200	500	1 000	60	200	500	1 000	60	200	500	1 000
Boston, Mass..	Sherman equation: $i=\frac{16\,F^{0.27}}{(t+7)^{0.70}}$	1.30	0.59	0.32	0.20	1.57	0.71	0.38	0.24	1.75	0.80	0.42	0.26	2.01	0.91	0.49	0.30	2.42	1.10	0.59	0.36	2.92	1.36	0.71	0.44
Boston, Mass..	Writer's Figs. 18, 19, and 20, equation: $i=\frac{26\,F^{0.23}}{t^{0.77}}$	1.61	0.63	0.31	0.18	1.89	0.75	0.37	0.22	2.07	0.82	0.40	0.24	2.33	0.92	0.46	0.27	2.73	1.08	0.53	0.31	3.20	1.27	0.63	0.37
Detroit, Mich.	Wagnits and Wilcoxen equation: $i=\frac{37.6\,F^{0.\text{[illegible]}}}{(t+8)^{0.\text{[illegible]}}}$	1.56	0.60	0.28	0.16	1.86	0.72	0.34	0.19	2.08	0.80	0.37	0.21	2.33	0.92	0.43	0.24	2.85	1.10	0.51	0.28	3.43	1.32	0.61	0.34
Detroit, Mich.	Writer's Figs. 18, 19, and 20, equation: $i=\frac{25\,F^{0.23}}{t^{0.80}}$	1.37	0.52	0.25	0.14	1.61	0.61	0.29	0.17	1.76	0.67	0.32	0.19	1.98	0.76	0.36	0.21	2.32	0.89	0.43	0.24	2.72	1.04	0.50	0.29

at Charleston, S. C., Austin, Tex., Washington, D. C., and Columbus, Ohio, the latter being compared with the single station record at Cincinnati, Ohio. It should be noted that Table 7 compares computed values, by formulas based on composite records and duration periods to four days, with observed values from single station records for a duration period of thirty days. For this reason it is thought fair to show the algebraic averages of the deviations.

U. S. Weather Bureau data giving accumulated daily and monthly rainfall depths must be accepted as representing the time interval observations and the calendar month, respectively. Maximum 24-hour and 30-day values would be slightly higher.

The writer's formulas are shown in Table 8, with those published by Charles W. Sherman, M. Am. Soc. C. E., for Boston, Mass.,[19] and by M. F. Wagnitz, M. Am. Soc. C. E., and Lewis C. Wilcoxen, Assoc. M. Am. Soc. C. E., for Detroit, Mich.[18] respectively. They are in practical agreement for the lower frequencies and shorter durations, but the writer believes the comparatively short records, upon which they are based, have given them exponential values which are too high and which will result in intensity rates for the longer durations that will prove to be excessive when their individual records have extended to 100 years.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

TRANSACTIONS

Paper No. 1802

ECONOMICAL DESIGN OF HYDRAULIC PIPE-LINE DREDGE[1]

By John Francis Cushing,[2] M. Am. Soc. C. E.

With Discussion by Messrs. R. L. Vaughn, Arthur L. Collins, and John Francis Cushing.

Synopsis

The hydraulic pipe-line dredge described in this paper[3] is possibly the largest and most powerful built to date (1930) and embodies some new and novel equipment and construction features. It was designed for the Great Lakes and ocean service as well as for river work. The steel hull is made exceptionally heavy and rigid not only to withstand the machinery vibrations, but to support the dredge without failure if it sinks on an uneven bottom. The curved ends of the hull are the result of experiments to determine that shape which would require the minimum power for towing.

The cutter drive is of unusual flexibility and power, with a specially designed cutter, digging to a depth of 50 ft. The method of control is entirely new.

In order to excavate in a definite area at all times, even in a heavy fog when fixed sights could not be used, a gyroscopic compass is installed which insures accurate and positive operation, as well as an assured return to the site, if the dredge is compelled to seek harbor because of a storm.

The dredge is designed for the maximum economy of capital, labor, and power, and the extensive and thorough investigations by the writer and Mr. R. V. Hoopes to determine the economy of these three elements developed the following conclusions:

(1) That a dredge with a discharge pipe 30 in. in diameter is more economical than any smaller size;
(2) That for this size a Diesel-electric installation results in lower production costs than a direct Diesel engine drive; and,
(3) That a single-stage dredge pump driven by the maximum power applicable is most desirable.

---

[1] Published in November, 1930, *Proceedings*.

[2] Pres., Great Lakes Dredge & Dock Co., Chicago, Ill.

[3] The Diesel-electric dredge, *New Jersey*, constructed for, and under the specifications of, the Great Lakes Dredge and Dock Company.

INTRODUCTION

There has been considerable development in the design and manufacture of dredges of the hydraulic pipe-line type in the last few years. Until about twenty years ago (1910) the use of this type of dredge was limited to the transportation of sand or other loose material through comparatively short distances, after mixing the material in a stream of water through a centrifugal pump. By the adoption of cutting machinery, the field was gradually widened to include more refractory materials, such as clay.

At the same time, the effective range of transportation was extended by improvements in pump design and by an increase in power applied to the pump. This increase was limited for a time to the capacity of reciprocating steam engines with speed characteristics suitable for this class of work.

The application of steam turbines followed the reciprocating engine with very beneficial results.

The more recent improvements of the Diesel type of oil engine has opened a new field of development in the design of dredging equipment, and some installations of Diesel-driven pumps have been made.

The first installation of any considerable size, with Diesel-electric power applied to this service, was made in 1925 by the Port of Portland Commission, Portland, Ore., when the dredge, *Clackamas,* was constructed. It has a pump with a 30-in. discharge, driven by a 2 700-h.p. direct-current motor, the power being developed by four Diesel engines, connected to direct-current generators.

This design and installation had some very special features. The conditions under which the dredge was to operate were not as widely variable as to the nature of material, elevations, distances, exposure, etc., as would be encountered in the field of contracting. The dredge was designed primarily for river operation and it was not necessary to design for lake or ocean conditions.

In general, the design of a dredge of this type for contractors' use involves numerous other considerations which are of vital importance and are not included in a special design for rather fixed conditions. The installation described hereafter, covers a dredge for contractors' use and represents the result of many years of experience, combined with the best engineering and manufacturing skill.

NOTATION

The following notation has been adopted consistently throughout this paper:

A = cross-sectional area of the discharge pipe, in square feet.

$b = \dfrac{700\,e}{n\,w\,f\,d^{(2-p)}}$, a variable coefficient of the ratio of the cost of labor to the cost of power.

B = a variable coefficient $= \dfrac{700\,e}{n\,w\,f}$.

c = unit cost of labor and power, in dollars per effective pumping hour; c_l = unit cost of labor; and c_p = unit cost of power used by the main pump.

C = unit cost of labor and power, in dollars per cubic yard.

d = diameter of discharge pipe, in feet.
D = a variable coefficient $= \dfrac{c_l}{c_p}$.
e = efficiency of pump.
f = coefficient of friction in the pipe.
H = hydraulic head, in feet; H_s = suction head; H_e = elevation head; and H_f = friction head.
L = length of discharge line, in feet.
n = exponent of V in the formula for friction head.
O = output, in cubic yards, per effective pumping hour.
p = exponent of d used in the formula for friction head.
P = horse-power; P_m = water horse-power of the main pump; and P_s = shaft horse-power of the main pump driver $= \dfrac{P_m}{e}$.
R = rate of discharge, in cubic feet per second.
S = fraction by volume of solid material.
V = velocity of discharge, in feet per second; V_e = economic velocity.
w = weight of the mixture of water and solids, in pounds per cubic foot $= (S\,W + U - S)\,62.5$.
W = weight of dredged material, in pounds per cubic foot.

Economies of Capital, Labor, and Power

Reduced to its fundamentals, the basis for designing a dredge for commercial use must be one of economy—economy of capital, economy of labor, and economy of power. The results of investigation in these three fields will determine the type, size, and capacity of the tool.

Each of these elements is made up of inversely varying factors. The greater the size and capacity, the more refined the type, the greater is the initial investment and, of consequence, the attendant interest and depreciation charges. Against these must be set the decrease in cost of dredging. These items which may be called "fixed charges", are converted into the unit cost of output by dividing the annual fixed charges by the annual capacity. This item of cost varies, therefore, in direct proportion to the initial investment.

The cost of labor required is likewise proportional to the size and refinement of type, and the unit cost of labor is inversely proportional to the capacity. In the same manner, the cost of power varies with the size and type and the unit cost of power is inversely proportional to the capacity.

The hydraulic dredging method is unique as compared to that involving the use of other excavating machines. The dredge excavates its material and immediately transports it to comparatively great distances and there distributes it. The operation is continuous and direct, without the need of intermediate storage of the excavated material common in most other methods of dry or wet excavation. Because of this feature of directness, a fine co-ordination of all the parts or units is essential. Continuous functioning of all the elements or parts is also essential because when one part is out of commission or impaired, the entire dredge is in the same measure out of commission and its ability to do work is impaired. If the cutter is idle, the transportation system, namely, the discharge line, is likewise out of service. If the main pump is out of service, the cutting unit and the transportation unit are likewise out of service. If either the suction line or the discharge line is plugged, broken, or otherwise idle, the remainder of the dredge is out of commission, etc., each one of the units or parts being highly dependent on all the others.

The total power of the dredge is used largely in cutting or dislodging the material, sucking it up to the pump with a high percentage of water, and there energizing the burdened stream with velocity and pressure to sustain the transportation of the dredged material at a velocity of 10 to 25 ft. per sec. through pipe lines of from 1 000 to 12 000 ft. in length. More than three-fourths of the power is used in overcoming the friction in the pipe line, and this friction varies inversely as about the square of the diameter. Consequently, it is economical to use as large a pipe line as practicable, because the greater the diameter, the less is the friction, and also the range of the dredge. The distance it will transport the dredged material economically increases with the diameter of the pipe line. For the same reason, the use of straight, smooth pipes as far as practicable is desirable.

A diameter of 30 in. for the discharge is not necessarily the limit in economy from the standpoint of power consumption per unit of output or from the standpoint of the labor involved in operating the dredge alone. Practically, however, the shore discharge pipe can be handled satisfactorily in diameters up to this size.

The waste water from the enclosed spoil-bank areas can well be taken care of, and the annual capacity of such a dredge is more in keeping with the average demand of a hydraulic dredge than one of a larger size.

THE POWER PLANT

As the power plant of the hydraulic dredge, *New Jersey*, represented upward of 25% of the total cost of the dredge, it was a matter given most thorough consideration.

Four two-cycle, air-injection, cross-head type of Diesel engines, each direct-connected to a generator, was the final selection of power plant to fill the many requirements for this dredge in the best way. Among the principal considerations in this design are: (1) The fuel economy of the Diesel engine selected; (2) its convenient and balanced installation in the dredge hull; and (3) the comparatively light weight of the two-cycle engine per unit of power developed in this large size of engine. A four-cycle engine weighing 70 to 80% more would require much higher mean effective cylinder pressures. The cross-head type and comparatively low-speed engines were favorably considered because of their longer life and more reliable service as compared to the trunk type or other high-speed engine.

The range and flexibility of the power were developed by using any combination of the four units, by the convenient pooling of the power, and by its wide-ranged distribution to the different motors on the dredge to meet the almost momentarily changing demands. Particularly is this true in clay work, now quite common, where a wide range of quality of material is encountered frequently in a single swing of the dredge. These alluvial clay deposits, bordering and on the bottoms of rivers and lakes, are mixtures of solid material and water. The solid material is similar to that of shale and slate, which have developed from the clay and mud particles in geological times.

PHYSICAL CHARACTERISTICS OF CLAY AND SHALES

Shales probably are produced from the soft clays and mud. They weigh from 160 to 175 lb. per cu. ft., and slates, which have a certain small percentage of porosity, weigh 175 lb. per cu. ft. Considering the influence of heat, and also that the hardest clay particles contain some water, 165 lb. per cu. ft. seems the most probable weight of the solid particles in very soft clay and mud, and this is equal to the weight of most stones. The specific gravity of these solid clay particles is 2.64, assuming the specific gravity of water as 1, and its weight as $62\frac{1}{2}$ lb. per cu. ft. If v is the volume of water in a cubic foot of clay or mud then $(1 - v)$ is the volume of solids, and the weight of the clay per cubic foot will be, $w = 62.5\,v + (1 - v)\,165$. If, for example, the clay is 10% water by volume then the weight would be 156 lb. per cu. ft. If the clay contains 80% of water, the weight, w, equals 83 lb. This shows the wide range of 73 lb., or about 90% variation in the weights of clays commonly encountered in reclamation work.

These values illustrate why clays and muds are more difficult to predetermine and classify and also why they are relatively so costly to handle. Sands or other similar materials do not retain permanently this high content of water and hence are more uniform in weight and other characteristics. In the hydraulic dredging operation, the water content of clays may be changed or it may not be changed substantially, depending on the character of the clay. If it is changed, it will probably return to its original condition after it has been placed in the fill area. In pumping clay the liquid mixture of the suspended particles encountered does not weigh 160 to 175 lb. per cu. ft. as in the case of sands, gravels, and other similar materials; but while these particles rather weigh less, depending on the normal, natural content of water in the clay, nevertheless the weight per cubic foot of clay is greater and the work required to move it from the cut to the fill is greater. This accounts for some of the difficulties of retaining clay fills within the confines of the filled area, because it is manifest that, for higher water contents, the clay is lighter and more fluid. Furthermore, if the fill is a deep one the problem of the compression of clay is involved. When clay is compressed so that the water oozes out, the solid particles remain the same; but the height or depth of the fill is decreased, and it is important to note that the subsidence in the clay fill is not exactly in a direct proportion to the decrease in water content. The subsidence is at an increasing rate. For example, if a clay with 50% of water is compressed to 40% water, the subsidence or decrease in h, the height, would be $\frac{(0.50 - 0.40)}{1 - 0.40}\,h = \frac{1}{6}\,h$. Therefore, it will be seen that if the water content in the fill is to be reduced 10% from that found in the cut, 17% more cubic yards of clay must be pumped from the borrow area than will be paid for in the filled area as commonly measured.

In 1924 and 1925, the writer had an experience with a clay fill on the lake front at Chicago, Ill., in which the loss was in excess of 10% in the pay units of output for the dredge. This resulted largely from such compression of clay. The reclaimed or filled area was approximately 3 000 000 sq. ft., quite regular in shape, and would correspond to a circular area about 1 960 ft. in diameter, requiring about 110 000 cu. yd. for each vertical foot of fill over the area. The average depth of water before the fill was deposited was about 16 ft. The fill was of the same material as that of the ground below, and when hydraulically deposited, it could be considered as of the same quality after its natural compression. The clay was soft, with a natural slope of between 4 on 1 and 5 on 1 near the surface of the bed of the lake in the cut and in the fill, the lower strata in the cut naturally becoming a little firmer. The dredging was done during two years, settlement in each case continuing over the winter periods when the dredging operation could not be continued because of the exposure to lake storms and ice conditions. Successive surveys were taken over the cut and fill areas.

TABLE 1.—COMPRESSION IN CLAY FILL.

Year.	Average height of fill, in feet.	Quantities, in Cubic Yards.			Percentage of compression.
		Cut.	Fill.	Discrepancy.	
1924......	18.2	2 030 750	2 002 590	28 160	1.39
1925......	9.9	1 459 250	1 086 403	372 847	25.5
Total....	28.1	3 490 000	3 088 993	401 007	11.5

Table 1 shows the progress by years. In the placing of the first two-thirds or approximately 18 vertical ft. of the fill, a compression of only 1.39% was produced; however, during the second year in the placing of the remaining one-third (corresponding to a layer about 13 ft. over the previously placed fill) an average of less than 10 vertical ft. was actually found by survey, which constitutes an apparent compression of about 25 per cent. The height of this fill is only about 1½% of the width, which is a comparatively thin layer. It is conclusive, of course, that much of the compression took place in the lake bed and the first 18 ft. of fill above it.

The average apparent settlement on the fill was 3.65 ft. The water content of the clay was approximately 45%, and its weight was 119. lb. per cu. ft., producing an average pressure on the lake bottom of about 1.7 tons per sq. ft. By quite an extensive and careful analysis of this lake-bed clay and by computing the compression, it was possible to account for 266 000 cu. yd., or about 66% of the total discrepancy between the cut and fill measurements given in Table 1.

The subsidence that would result in a compression of 66% is equal to 2.32 ft. in vertical height. This corresponds to a 4% reduction in the water content, so that if clay with 49% water is compressed to 45%, the subsidence would be, $31.75 \left(\frac{0.49 - 0.45}{1 - 0.45}\right) = 2.31$.

The additional power required to handle medium or hard clays is cumulative and places a comparatively great demand on the power plant of the dredge.

More power is required for cutting, much more power for transporting the larger pieces of solids through the pipe, and considerably more power is required for handling throughout this denser and heavier material. More man-power is required on the fill for the distribution of the material. The high velocity required in the pipe line to transport these larger pieces of clay is quickly lost at the end of the discharge pipe. The clay lumps are immediately dropped and pile up near the end of the pipe, and this requires much additional pipe laying for distributing the fill, but, more important, it causes delays and lost working time of the dredge while the pipe is extended or the discharge is switched to another line.

To reduce delays to the dredge, due to these shore operations, a Y-valve that was developed by the writer is being used in the shore line for shifting the discharge from one line to another, without interrupting the operation of the dredge. This valve is a simple device, consisting of a spoon-shaped blade revolving on the axis of the pipe that enters the Y from the dredge so that material is deflected from one branch of the Y to the other. The sealing edges are built so that they will clear themselves, and will not jam, due to the heterogeneous burden in the stream. While the valve is in operation the cross-sectional area (and, therefore, the velocity of the discharge through it) is constant while the discharge is being shifted.

If the fill area is large, the natural slope may vary with the water ratio, and it is very expensive, if not impractical, to place the fill so that it differs much from its natural slope, which varies with the softness or water-ratio in the clay or mud. Clay with 40% of water (weight, 124 lb. per cu. ft.) is considered soft. This water-ratio, or porosity, is like that of average sand (35 to 45%) and the material is nearly of the same weight per cubic foot as sand and water. The natural slope of sand in water is about 25°, but the more fatty clay particles, lacking the sharpness of sand, will only have a natural slope of about 20 degrees. Only the best clays, when thoroughly wet, will reach a slope of 25 degrees. Mud or silt containing 60% of water (104 lb. per cu. ft.), of old formation in lakes, has a natural slope of about 4°, and in newly formed river mud the slope is probably even less. Clays of different solid matter may vary a little with the same water-ratio, but in clays of the same composition, the slope could be considered as decreasing approximately in a ratio that varies directly as the increase of water percentage.

Such alluvial clays range from very soft to very hard, and frequently contain boulders. Some clays are easy to pump, and others are difficult, developing a high friction coefficient. Some of these clays are broken up into small pieces by the pump, and others are tough and pass through the entire discharge line unbroken or in approximately the same sizes as sliced from the bank by the cutter. It is highly desirable to feed excavated material or burden into the pump and the line stream uniformly so as to reduce the shock to the machinery; and it is more important to reduce the delay due to "slugging" or plugging the suction, pump, or discharge line; but under the conditions described it is very difficult to approach uniformity in cutting or feeding the material into the dredge. In operation a practical limit is reached by working to a margin so that a momentary overload of burden can be passed through without slugging. The ability to shift power and speed quickly between the swinging motor, cutter motor, ladder hoist motor, and main pump motor is of great advantage. It permits giving to the temporarily overloaded unit a "shot" of power and to the overloading unit a reduction at the propitious moment.

In order to study the problem of designing a dredge it is necessary to have an understanding of the physical properties of materials encountered and of the field operating conditions on the dredge and on the fill. Furthermore, it is necessary to know something of the effect of storms, winds, tides, low temperatures, and slides in the cuts and fills.

PRELIMINARY CONSIDERATIONS

In any undertaking, the development of a plan or program for a short period of a few days is simple; such a program for a year is more involved; and for a 5-year period it is much more difficult, as many new problems are thereby brought forward for consideration, to be weighed and fitted into the program in a fashion that will make the entire project feasible and most desirable. These longer programs are necessarily subject to modification from time to time and this must be provided for, but it will be found that the practice of looking many years ahead will result in much more stable and economic progress than otherwise could have been possible. The plans and specifications for a dredge of the character described in this paper must cover a program for a period of 30 to 35 years, or the anticipated life of the dredge.

The application and distribution of power in the Diesel-electric installation are so flexible that the increased fixed charges on the additional electrical equipment and the loss of power in the double conversion are more than offset by the reduction in cost of output.

In a direct Diesel-engine installation it is necessary to provide maximum power capacity for both the main pump and the cutter and other auxiliaries independently. When the full cutter power, however, is not required, the engine supplying this power is at part load and any surplus is not available for other purposes. Similarly, when the full power is not required on the main pump, its engine is operating at part load and the surplus from this source is also not available for other purposes.

Practically, this means that when handling refractory materials, such as hard clay, the power requirement for the cutter is at a maximum while that of the main pump may be reduced, due to a decrease in the percentage of solids in the outflow and possibly to the fact that there may be a lower friction coefficient than with more pliable material. In this case, a proper and efficient distribution of power can be made by motors to all units while drawing current from a common pool of power.

Both the main pump and cutter requirements call for maximum horse-power over a considerable variation of speed. This cannot be accomplished except to a limited extent, with either a reciprocating engine or a turbine; but it can be taken care of almost automatically with motor drive. For instance, in this design, the main pump motor will deliver full-rated horse-power from 250 to 360 rev. per min., without over-heating; the cutter motor will deliver full-rated horse-power from 250 to 500 rev. per min. at full load with armatures in parallel, and from 125 to 250 rev. per min., at full load with armatures in series. The slow-speed range of the cutter is required only in very soft material so that power supply is ample for that condition.

The operating economy as between labor and power depends, in general, upon the velocity of discharge; that is, between the limits of a minimum velocity required to transport the dredged material and the maximum which can be provided by the pump.

Assuming any fixed set of conditions as to length of line, elevation, total head, percentage of working time, and percentage of solids in the stream, the daily output will be directly proportional to the velocity of the discharge. The power required for pumping will be proportional to the velocity and the total head. The total head consists of the suction head, the static or elevation head, the friction head, and the velocity head—the last being of such a low order as to be negligible in the computation.

For comparison, the suction head and the static head may be assumed as constant for any comparable condition. The friction head is an exponential function of the velocity and the diameter of the pipe. It may be demonstrated mathematically that there is a point of maximum economy between labor and power, a velocity at which the combined cost of labor and power is a minimum. By definition, the total cost for labor and power will be,

$$C = \frac{c_1 + c_p P_s}{O} \qquad (1)$$

The horse-power required to move a given quantity of liquid against a given head is equal to $\frac{w R H}{550}$, in which, w is the weight of the liquid per cubic foot; R, the rate of discharge, in cubic feet per second; and H, the head, in feet. The shaft horse-power of the driver is this value divided by the efficiency of the pump, or $\frac{w R H}{550\ e}$. Furthermore, R is a function of V, such that $R = A\ V$, or,

$$P_s = \frac{w\ A\ V\ H}{550e} \qquad (2)$$

The total head, H, is made up of the static head, the suction head, and the friction head. Equation (2) may then be written,

$$P_s = \frac{w\ A\ V\ (H_s + H_e + H_f)}{550\ e} \qquad (3)$$

The friction head is an exponential function of the velocity and is also a function of the length of line through which the mixture is pumped. Therefore,

$$H_f = \frac{L\ f\ V^n}{d^p} \qquad (4)$$

Substituting this value in Equation (3),

$$P_s = \frac{w\ A\ V\left(H_s + H_e + \frac{L\ f\ V^n}{d^p}\right)}{550e} \qquad (5)$$

The output is also a function of the velocity, since it is proportionate to the rate, R, which is equal to $A\ V$. Therefore,

$$O = 133.3\ S\ A\ V \ldots\ldots (6)$$

Substituting Equations (5) and (6) in Equation (1):

$$C = \frac{c_l}{133.3\ S\ A\ V} + \frac{c_p\ w\ (H_s + H_e)}{133.3\ S \times 550\ e} + \frac{c_p\ w\ (L\ f\ V^n)}{133.3\ S \times 550\ e\ d^p} \ldots\ldots (7)$$

Differentiating Equation (7),

$$\frac{d\,C}{d\ V} = \frac{-\ c_l\ V^{-2}}{133.3\ S\ A} + \frac{n\ c_p\ w\ f\ L\ V^{n-1}}{550\ e \times 133.3\ S\ d^p} \ldots\ldots (8)$$

in which, $A = 0.7854\,d^2$. This value is zero for a minimum, and,

$$V_e = \sqrt[n+1]{\frac{c_l}{c_p}\left(\frac{700\ e}{n\ w\ f\ L\ d^{2-p}}\right)} = \sqrt[n+1]{\frac{b}{L}\frac{c_l}{c_p}} \ldots\ldots (9)$$

Equation (9) can be written in a more convenient form, as follows:

$$\log V = \frac{\log (b\,c_l) - \log c_p}{n+1} - \frac{\log L}{n+1} \ldots\ldots (10)$$

Therefore, V_e varies directly as the $(n+1)$ root of c_l and inversely as the $(n+1)$ root of c_p.

The practical use of Equation (10) is to determine the horse-power of the main pump driver that will produce at least the economic velocity for the existing costs of labor and power on any length of line which may be met in ordinary operation.

ECONOMIC VELOCITY UNDER TYPICAL SERVICE CONDITIONS

In the Great Lakes region labor is highly organized and, consequently, is costly, and the power developed by Diesel engines is relatively cheap. Consequently, the economic velocity for dredging under those conditions is high, and it is important to supply the dredge with sufficient power to attain it.

In Table 2, Items 1, 2, and 3 give the economic velocity when the dredge is handling medium clay and the horse-power required for a 30-in. Diesel-electric dredge based on the labor and power costs which obtained in 1925 when the design was first considered.

TABLE 2.—ECONOMIC VELOCITIES UNDER VARIOUS CONDITIONS OF DREDGING.

Item	L, in feet.	V_e.	P_e.	P.
1	2 000	22.5	2 599	3 000
2	6 000	15.3	2 427	3 000
3	11 000	12.4	2 397	3 000
4	2 000	24.0	3 057	3 000
5	6 000	16.3	2 858	3 000
6	11 000	13.3	2 808	3 000
7	2 000	24.3	2 683	3 000
8	6 000	16.5	2 463	3 000
9	11 000	12.7	2 145	3 000

These values were determined by means of Equation (9) and (5), in which, $\frac{c_l}{c_p} = 3.940$; $b = 373$; $e = 0.70$; $n = 1.86$; $w = 68$; $f = 0.523$; $d = 2.5$; $p = 1.25$; $S = 0.067$; $H_s = 16$; and $H_c = 10$.

Anticipating an increase in labor cost, a 3 000-h.p. motor was installed. Under the conditions obtaining in 1930 the corresponding values of V and P_s are as given in Table 2, Items 4, 5, and 6. The actual velocity was checked at times in the field by injecting a salt solution at the dredge pump and observing the time required for it to travel through the pipe line, its passage at the discharge end of the shore pipe being detected by a galvanometer.

From the general Equation (10) developed for finding the economic velocity, it is possible to study the effects on the cost and shaft horse-power requirements, etc., for different values of the other factors involved. No effort has been here made to catalog partial unit costs of hydraulic dredging work as frequently printed, but rather to make a valuable contribution to the extension of the field of knowledge itself. A very careful and comprehensive estimate of the entire situation surrounding each piece of hydraulic dredging work is necessary in order to arrive at a worth-while unit cost. In the absence of such estimate, unit costs are generally misunderstood and misleading, because frequently, knowing the unit cost on one piece of work, a futile attempt is made to correct for the difference in conditions and the situation on another job by mere interpolation without fully recognizing the principles and fundamental conditions partly set out in this paper.

In developing Equation (10) only the costs of labor and power are included. To these must be added the maintenance and depreciation on the plant and the interest on the investment; the cost of shore labor for distributing the fill material; the cost of the attendant plant; wear of the shore discharge line; and the overhead. This cost of labor and power may vary from 30 to 70% of the total, depending on the nature of the material handled, local conditions of disposal, and weather hazards. It depends also on the cost of moving the equipment on and off the work, the cost of marine insurance, the cost for work done outside the section, and effects of shrinkage and of settlement. The miscellaneous items of cost are practically independent of the velocity and the power and may be considered either as current or fixed annual charges, depending on the annual output only.

Table 2 shows that the velocity and the horse-power required for economic operation is greater on short lines than on long lines, although the head is less. It is a characteristic of a centrifugal pump that the peripheral velocity of the impeller, and hence the revolutions per minute, is a function of the square root of the total head. For values of L from 2 000 to 11 000 ft. at the economic velocity of discharge, the head will increase in each case from about 130 ft. to about 220 ft. or, the square root of the head changes from about 11.4 to 14.8, a variation of about 23% using the upper value as a base. Table 2 shows that, in order to maintain the economic velocity the pump driver must deliver about 7% more power at this reduced speed on short lines than at rated full-load speed; on long lines, the pump must deliver 23% more power under the same conditions. In the case of a steam engine this can only be done by putting in an over-sized engine and using it at less than full load on long lines, or by using a smaller sized impeller, or by doing both. The disadvantage of this is that there is a lack of flexibility. This is also true in case of direct Diesel-engine drive. In either case the horse-power is proportional directly to the revolutions per minute and the revolutions per minute cannot be varied independently of the horse-power without considerable loss. This condition can be met with an electric motor of proper design. Computations indicated that the value of this flexibility more than offset the loss of energy in the Diesel-electric conversion.

EFFECT OF DECREASING THE UNIT WEIGHT OF EXCAVATED MATERIAL

The weight, W, of dredged material may be determined from the formula, $w = S\,W + (1 - S)\,62.5$, or,

$$S' = \frac{w - 62.5}{W - 62.5} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (11)$$

For example, if $w = 68$ lb. per cu. ft. and $S = 0.067$, by Equation (11) $W = 144.5$ lb. per cu. ft. This corresponds to a hard heavy clay. If $W = 124$ lb. per cu. ft., corresponding to a medium sand, then, by Equation (11), $S = 0.0894$.

In Equation (6), the output, $O = S\,A\,V = \left(\frac{w - 62.5}{W - 62.5}\right) A\,V$, varies directly as S, but inversely as W. Therefore, a decrease of 14% in the specific weight of the material corresponds to an increase of about 34% in the carrying capacity of the discharge, assuming that f remains constant. That is, for $W = 144.5$, $V = 22.5$, and $S = 0.067$, $O = 14\,700\,S = 14\,700 \times 0.067 = 985$ cu. yd. per hour, and for $W = 124$, $V = 22.5$, and $S = 0.0894$, $O = 14\,700 \times 0.0894 = 1\,315$ cu. yd. per hour.

EFFECT OF VARIATION IN f UPON V_e AND P_s

Under typical service conditions for which the *New Jersey* was constructed, the value of f has been previously given as 0.523. If this value is changed to 0.420, the value of $b = \frac{700\,e}{n\,w\,f\,d^{0.75}}$, will change proportionately, and the economic velocities for Items 7, 8, and 9 of Table 2 are found by Equation (9).

These data show that for a decrease in f of 20%, when $L = 2\,000$ ft., V is increased 8% and P_s is increased 2%; and for a decrease in f of 20%, when $L = 11\,000$ ft., V is increased $2\frac{1}{2}$% and P_s is decreased $12\frac{1}{2}$ per cent.

APPROXIMATE EFFECT ON V_e AND O OF DECREASE OR INCREASE IN DIAMETER OF PIPE LINE

From Equation (9),

$$V_e^{n+1} = \frac{c_l}{c_p}\left(\frac{700\,e}{n\,w\,f\,L\,d^{(2-p)}}\right) = \frac{700\,D\,e}{n\,w\,f\,L} \div d^{(2-p)} \ldots\ldots (12)$$

and,

$$V_e = \left(\frac{700\,D\,e}{n\,w\,f\,L}\right)^{\frac{1}{n+1}} \div d^{\left(\frac{2-p}{n+1}\right)} \ldots\ldots\ldots\ldots (13)$$

For typical service conditions, previously described, the exponent, $\left(\frac{2-p}{n+1}\right) = \left(\frac{2-1.25}{1+1.86}\right) = 0.262$; therefore, V_e varies approximately as $\frac{1}{d^{0.262}}$, and, for

$_u = 2.5$, $L = 2$, and $V = 22.5$. If d is reduced from 2.5 to 2.0, then $V_e = \dfrac{22.5}{\left(\dfrac{2}{2.5}\right)^{0.262}} = 23.8$ for $d = 2.0$; that is, a 20% decrease in d corresponds to a 5.8% increase in V_e.

From Equation (6), $O = (133.3 \times 0.067)AV = 8.9AV$. For $d = 2.5$, $A = 4.90$, and $V = 22.5$. Consequently, $O = 8.9 \times 4.9 \times 22.5 = 985$ cu. yd. per hour; and for $d = 2.0$, $A = 3.14$, and $V = 23.8$, and, therefore, $O = 8.9 \times 3.14 \times 23.8 = 665$ cu. yd. per hour. This illustrates that, for a decrease of 20% in d, O is reduced 320 cu. yd. per hour, or 32½ per cent.

EFFECT ON C OF DECREASING THE DIAMETER OF DISCHARGE LINE

In Equation (9), $V = \left[\dfrac{b\ c_l}{L\ c_p}\right]^{\frac{1}{n+1}}$; and $b = \dfrac{700\ e}{n\ w\ f\ d^{(2-p)}} = \dfrac{B}{d^{0.75}}$. Therefore,

$$V_e = \left[\frac{c_l B}{c_p\ L\ d^{0.75}}\right]^{\frac{1}{2.86}} = c_l^{\frac{1}{2.86}}\ B^{\frac{1}{2.86}}\ c_p^{-\frac{1}{2.86}}\ L^{-\frac{1}{2.86}}\ d^{-\frac{0.75}{2.86}} \text{.......} (14)$$

From Equation (7),

$$C = \frac{c_l}{133.3 \times 0.7854 \times 0.067\ d^2\ v} + \frac{c_p \times 68 \times 0.523\ L\ V^{1.86}}{133.3 \times 550 \times 0.7 \times 0.067\ d^{1.25}} + \frac{c_p \times 68 \times 26}{133.3 \times 550 \times 0.7 \times 0.067}$$

or,

$$C = \frac{c_l}{7.014\ d^2\ v} + \frac{0.01034\ c_p\ L\ V^{1.86}}{d^{1.25}} + 0.5142\ c_p \text{.........} (15)$$

Substituting the value of V_e in Equation (14) for V in Equation (15), and simplifying:

$$C = \frac{c_l^{0.65}\ [c_p L]^{\frac{1}{2.86}}}{d^{1.737}}\left[\frac{1}{7.014\ B^{\frac{1}{2.86}}} + 0.01034\ B^{0.65}\right] + 0.5142\ c_p \text{.....} (16)$$

The value of C for any economic velocity varies inversely as the 1.737 power of the diameter. The unit cost of labor, c_l, is also some function of the diameter; it is larger for large diameters. However, c_l may be considered constant for values of d between 2.0 and 2.5 ft. and a constant smaller value for diameters less than 2.0 ft. The unit cost, c_l, does not decrease proportionately with d, and even if it did, it appears in the numerator to a fractional power, while d appears in the denominator as a power greater than 1. Hence, the unit cost may be decreased by increasing the diameter of the discharge of the dredging pump.

EFFECT OF OPERATING AT VELOCITIES HIGHER AND LOWER THAN V_e

The value of V for minimum cost is given by Equation (9) assuming $n = 1.86$; thus,

$$V_e^{2.86} = \frac{c_l\ b}{c_p L} \text{.........................} (17)$$

Under the typical service conditions previously described, the shaft horse-power, P_s, is determined from the relation:

$$P_s = 22.54\ V + 0.1442\ V^{2.86}\ L \ldots\ldots\ldots\ldots\ldots\ldots (18)$$

The value of P_s corresponding to a minimum value of C occurs when $V = \sqrt[2.86]{\frac{c_l\, b}{c_p\, L}}$. Substituting this latter quantity in Equation (18) and writing D for $\frac{c_l}{c_p}$,

$$P_s = 22.54 \sqrt[2.86]{\frac{D\ b}{L}} + 0.1442\ D\ b \ldots\ldots\ldots\ldots\ldots (19)$$

The economical shaft horse-power varies directly with D and b and inversely with L. For all values of V greater than, or less than, $\sqrt[2.86]{\frac{D\ b}{L}}$, C will increase. Since P_s varies directly with V, the application of any shaft horse-power greater than that corresponding to $V = \sqrt[2.86]{\frac{D\ b}{L}}$ will increase the unit cost instead of reducing it. Fig. 1 shows the effect on C of a change in V and P_s for a particular instance, with L equal to 6 000 ft. It illustrates most clearly how definite Points V_e and P_{se} are, and how sharply the value of C rises with the application of power either less or greater than P_{se}, the economic power.

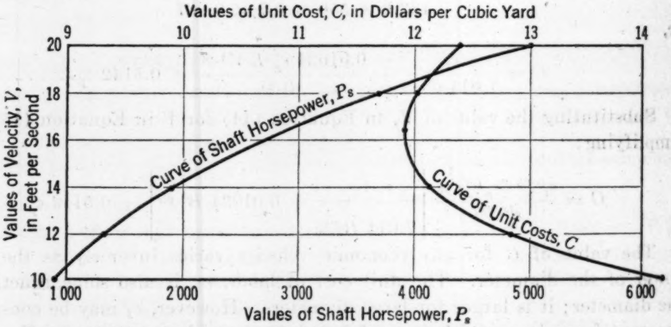


FIG. 1.—CURVES SHOWING THE RELATIVE EFFECTS OF VARYING THE VELOCITY OF DISCHARGE AND THE SHAFT HORSE-POWER.

EFFECT OF BOULDERS IN HYDRAULIC DREDGING

The unpleasant consideration of this element of the dredgeman's work was left to the last so that the foregoing engineering principles and finer analysis might be given calm and deliberate thought. Boulders are frequently mixed with clay and sand in natural deposits. Many times they are promiscuously distributed; sometimes they are more or less uniformly scattered; and at other times they are found in nests as if to make a concentrated attack on the dredge for disturbing their peaceful seclusion of millions of years. A relatively small percentage of boulders by volume in the proposed cut eliminates it for the hydraulic dredging method of excavation. In fact, the only sufficient reason

for considering the hydraulic dredging of ground containing boulders is to secure that part of the work not involving the handling of them.

All boulders are highly objectionable in this method of excavation, but the size most detrimental is that critical one which will pass through the opening in the cutter and will not pass through the opening in the pump flyer. It becomes lodged and sometimes tightly jammed, thus badly unbalancing the flyer and making it necessary to shut down the dredge for the removal of the boulder. These are expensive delays and, in terms of unit of excavation, may cost as much as $200 to $500 per cu. yd. for the boulders encountered. Boulders that are too large to enter the cutter, or become jammed in it, or smaller boulders that will pass through the pump and discharge line, are not so objectionable; but they are highly undesirable, and their presence seriously upsets all the calculated economies set forth in this paper.

There is, of course, a relation between the size of opening in the cutter and that in the pump flyer. The cutter designed for maximum efficiency in clay will have a maximum opening or clearance between blades of approximately six-tenths the diameter of the pipe line. For low-speed dredge pumps the minimum opening or clearance through the flyer vanes would be approximately five-tenths the diameter of the discharge pipe, and for high-speed pumps somewhat less, because of the flatter or less radial shape of the vanes. As the efficiency of the main dredge pump is so important, an efficient design of the impeller cannot well be compromised for this boulder condition. Similarly, the efficient design of the cutter is very important and, therefore, these clearance dimensions for both cutter and pump are more or less fixed. The particular size of boulder that will become lodged in the cutter or in the pump is the critical size and, of course, this varies with the size of the dredge.

The frequency of delays to the dredge caused by the removal of boulders is, of course, in proportion to the percentage of the critical size of boulders in the area to be excavated, and also to the output of the dredge. While the presence of boulders has a great effect on the cost of the work, there is no formula, either theoretical or empirical, that is applicable for determining the effect. Only by careful analysis and long experience can this element of the cost be even roughly estimated for any job.

The critical size of boulder for the *New Jersey* is 3 to 5 cu. ft., or 400 to 700 lb.; but, at times, and on short-line work, much larger ones are taken from the pump. For example, a granite boulder weighing 1 050 lb. was taken from this pump at Toledo, Ohio, on July 5, 1929, while dredging heavy hard clay mixed with sharp gravel. This combination of material is somewhat unusual and difficult to handle. Mysteriously, the minimum dimension of this large boulder (19 in.) was more than an inch greater than the maximum distance between the cutter blades, and if this is accounted for by springing these heavy short blades, it furnishes a new measure of the forces to be resisted and the sturdy design required.

THE DESIGN ELEMENTS

Having determined, from the preceding analysis, the relation between the different factors involved and their relative importance for economic operation, the conclusions, tempered with experience, have been applied in the design and construction of the *New Jersey*. Another factor considered of great importance, of course, is the general time efficiency of the entire dredge. Such dredges are usually operated 24 hours per day, and the time efficiency is the ratio of the number of working hours to the 24 hours, and for obvious reasons too numerous to mention, this has been kept in mind in the design of every unit of the dredge.

Having determined that a dredge with a 30-in. discharge, with Diesel-electric drive, is the most economical within practical limits and the best commercial type of the available sizes or types of power installations, the problem remains to design the component parts necessary for the successful operation of the entire installation.

Hull.—The first consideration in design, of course, is the hull, which must be constructed so as to provide buoyancy for the weight of machinery, fuel, supplies, etc. It was necessary that it be sufficiently rigid to withstand the vibrations set up by the Diesel engines, as well as the stresses incident to dredging operations, and that it be able to support the dredge without failure if stranded on an uneven bottom.

The limitations were that the hull should not draw too much water which would eliminate it from working on shallow projects, and that its length and breadth should not exceed the capacity of locks in canals between the Great Lakes and the Atlantic Coast. It was found impossible to keep within the required beam, so the hull was constructed with a detachable side section to bring it within the required width. The dimensions finally decided on were: Length, 245 ft.; beam, 50 ft.; depth (moulded), 14 ft. 4 in.; and draft (working condition), 9 ft. 3 in.

The hull proper is divided into compartments by a system of water-tight bulkheads. This affords maximum protection against sinking from collision or other mishap as well as protection against damage to the electrical equipment should any part of the hull be flooded. The side compartments are used for the storage of oil, water, and supplies; the larger center compartments are used for the machinery. The main pump and main pump motor are in separate compartments to avoid damage to the motor in case the pump should be broken by a boulder, or from some other cause. These bulkheads add considerable strength and rigidity to the hull structure.

In addition, there are two longitudinal trusses, at 23-ft. centers, the full length of the hull. They are extended at the bow to form the gantry for supporting the dredging ladder, and at the rear they support the stern spud anchors. The depth of these trusses is less than that ordinarily found in bridge practice. It was desired to keep down the height of structures above water to reduce wind resistance and maintain low bridge clearance. To have given the truss greater depth would have necessitated running it through the roof of the lower house which would have made it unsightly and left opportunity for leaks. In addition, it would have projected into the living quarters which would have made that part of the layout quite complicated. The additional weight due to the reduced depth is not great and adds to the rigidity of the entire structure.

Another novel feature is the shaping of the ends of the hull. This is done to reduce towing resistance. As early as 1914 the writer's Company had tests made at Rensselaer Polytechnic Institute, Troy, N. Y., to determine the relative resistance of square hulls with ends varying in shape. The models had square ends with differently shaped rakes. It was found at that time that a section of compound curvature increasing at the bottom had the least resistance. Later tests made by W. C. Sadler, M. Am. Soc. C. E., at the University of Michigan, developed the fact that a rounding of the deck section still further reduced the resistance. The results of both these series of tests have been included in this design.

Another feature of the hull design is the recess placed at the after starboard corner to accommodate the stern ball connection in the discharge line. Two methods are ordinarily used for this connection, either (1) to bring the discharge pipe through the hull near the water-line and support the connection by an A-frame, or gantry; or (2) to bring the pipe above deck and use a swinging elbow in a gooseneck. Both methods have serious faults. The first has the disadvantage of a connection that is more or less inaccessible due to its being in or near the water; the second involves short-radius elbows, and brings the discharge too high above the water. The new method meets the objections to both, eliminating the elbows and putting the discharge pipe at the proper level for joining to the pontoon line.

Deck House and Crew Quarters.—The main deck house is of steel and provides a complete enclosure for the machinery, leaving a deck space of approximately 9 ft. on each side. This space has guarded bulwarks 30 in. high around the ends and extended about 6 ft. along each side, with chains supported by removable steel stanchions between the bulwarks. The roof is supported by the longitudinal trusses. The doors in the lower house are of steel, and the windows are fitted with wire glass in steel sash.

The upper house rests on the steel roof of the lower house. It is of wood construction throughout and is divided into 39 rooms, comprising 18 staterooms, 3 toilets, 3 baths, 2 washrooms, 2 dining-rooms, 2 storerooms, wireless room, office, galley, pantry, room for ice machine, refrigerator room, officers' lounge, crews' "social hall", and laundry.

It has been found that comfortable quarters for the crew on a machine of this kind where the men spend most of their time on board, is a great asset; it keeps them satisfied and more inclined to stay with their job. The provision of an officers' lounge and a social hall for the crew may appear extravagant at first, but it must be realized that it is only necessary to take a very little from each of the staterooms to make this space available, and that the common social halls afford opportunities for recreation which could not be made possible in individual quarters.

All the rooms are heated, lighted, and well ventilated, and open into a common corridor in the middle of the house. Over this corridor for its full length is a monitor with ventilators. The galley is fitted with sink, tables, lockers, shelves, and an oil-burning range. Complete culinary equipment is provided for a crew of fifty men.

Cutter and Ladder.—A heavy plate-girder ladder of sufficient length for a digging depth of 50 ft. is hinged to brackets in the bow well, with its outer end suspended through wire-rope tackle blocks from the bow gantry extension of the main trusses. The lower end of the ladder is formed by the steel casting suction head made in two pieces with two bearings for the cutter shaft about 10 ft. apart. The outer bearing extends well inside the cutter head and consists of a sealed and water-lubricated cutless bearing. The suction head has a pressure-water chamber with jets directed into the suction inlet to assist in freeing a clogged pipe. Lugs are provided for the purpose of anchoring the ladder suspension tackle and the swinging sheaves. Heavy flanges serve to tie the suction head thoroughly to the structural part of the ladder. The upper ladder end has three heavy annealed steel castings which form the ladder hinges and foundations for the cutter motor and gear box.

The cutter head (Fig. 2) was designed and built by the writer's Company. It is an integral alloy steel casting with eight spiral arms connecting the hub and back ring. Replaceable chrome vanadium blades are riveted to the arms. The hub is taper-bored and keyed to a 13½-in. cutter shaft which is supported by two bearings in the suction head, two in the gear box, and four intermediate bearings mounted on transverse ladder girders. The lower bearing is designed to transmit the axial thrust of the shaft to the ladder.

The 600-h.p. cutter-drive motor is mounted on the ladder and drives the cutter shaft through a flexible coupling in combination with a slipping friction. The first set of reduction gears is of the herring-bone type; the second set, consisting of the cutter shaft gear and its pinion, is of the single helical type. Both sets are encased and run in oil, with all gears and pinions mounted between two bearings built into the gear casing. The cutter shaft is made in six sections held together with bolted and keyed couplings.

Suction and Discharge Pipe.—The suction pipe has an inside diameter of 34 in., and enters the hull through a packed and water-sealed ball joint which is bracketed to the end of the well plating and the deck concentric with the ladder trunnions. A readily removable manhole section next to the pump has a hydraulically-operated hinged cover, the mount of which permits it to function as a suction line relief valve.

The discharge pipe has an inside diameter of 30 in., and is located immediately outside the house on the starboard deck. It is connected to the discharge line with a rubber-packed, quick-opening ball joint mounted in a gimbal ring, the deck being tapered off to the stern to receive the bracket and ring. The joint is provided with compression spring bumpers which contact with a structural frame to limit its transverse angular movement. A gate-valve section is placed in the line immediately forward of the ball joint, and there are two manhole sections adjacent to the pump discharge. Replaceable plate liners are spot-welded in the lower half of each straight pipe section. All cast-pipe fittings on the dredge have cast alloy steel liners. The discharge pipe sections are held together by special rubber-sealed pressure joints.

Pump.—The pump (Fig. 3) is somewhat unusual; a number of novel features have been introduced in its design. It is of the square-casing lined type recommended by J. H. Polhemus, M. Am. Soc. C. E., who has had a

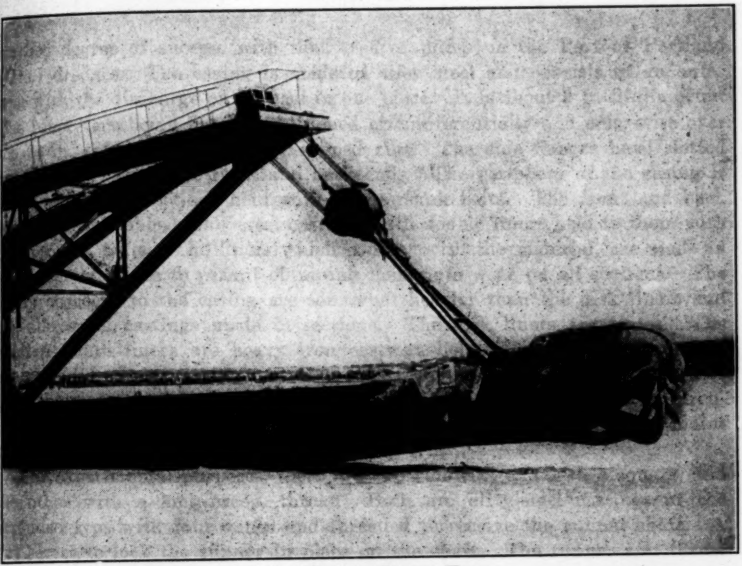
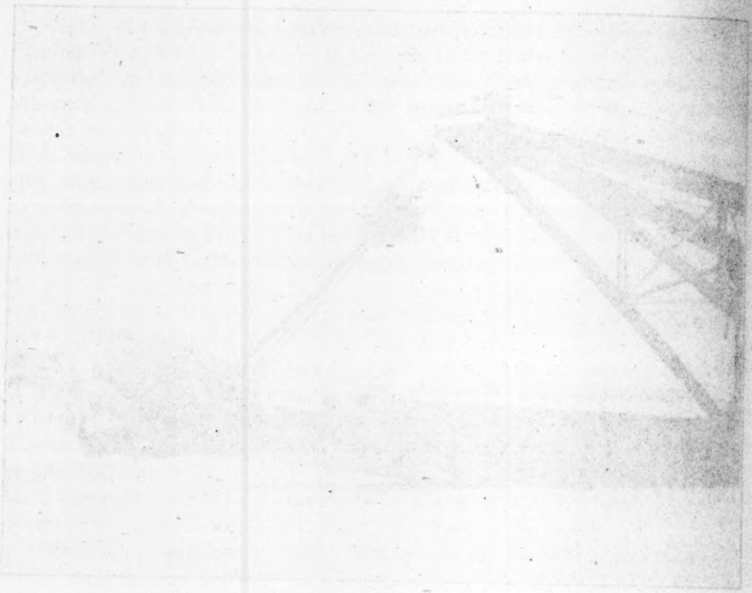
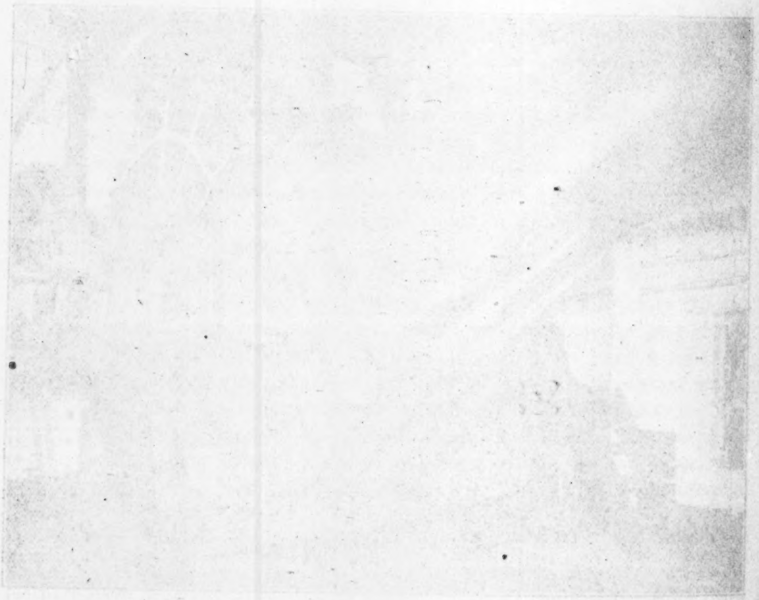


FIG. 2.—VIEW OF CUTTER HEAD.

FIG. 3.—THIRTY-INCH DREDGE PUMP.



[illegible]



[illegible]

marked degree of success with that type of pump on the Port of Portland (Ore.) dredges. The casing is made of alloy steel and consists of an outer ring with the discharge outlet cast in one piece. It is about 9 ft. in diameter. The inner face being flat, it is stiffened circumferentially and cross-wise over the 3½-in. thick metal with rail-shaped ribs. The side flanges have slotted holes for bolting to the front and back ends. The periphery of the casing is lined with plate sections held with countersunk bolts. The front and back heads are well-ribbed alloy-steel castings with double liners held to them with countersunk bolts. The liners which contact with the material, are made in a number of pieces to permit obtaining maximum wear on all sections. The liners adjacent to the casing are somewhat heavier than the disk liners and are alloy-steel castings made in sections. The disk liners are plates. The suction-mouth liners are heavy iron castings held in place with studs. A rubber-faced adjusting ring may be set up from outside the suction mouth to limit the clearance between it and the runner. This tends to prevent a circulation from the discharge pressure chamber to the suction inlet, thereby maintaining the pump efficiency and eliminating wear.

For experimental purposes, there are two runners; one with a square, and the other with a long-sweep, throat. Both are alloy-steel castings of the shrouded type with four vanes and threaded to receive the runner shaft. A nut serves to lock the runner in place on the shaft. The runner and thrust shafts are 14⅜ in. in diameter and are made in three sections bolted together through flanges forged with the shafts. The intermediate or short section is removable to permit of readily withdrawing the runner shaft through the back head. The shaft has a replaceable sleeve where it passes through the water-sealed stuffing-box in the back head. A motor-operated turning device consisting of a worm and wheel which may be readily thrown in or out of engagement, serves to assist in backing the runner off the shaft or to lock the shaft against turning. The thrust shaft has one collar which transmits the thrust to the base through an over-sized two-way, oil-lubricated, water-cooled thrust bearing in combination with a radial bearing. The radial bearing next to the pump back head is of the same type.

Three heavy cast-iron bases are securely bolted together to form a rigid foundation for the motor, pump casing, and bearings.

Bow Winch.—The bow winch consists of five drums loosely mounted on a single shaft. The middle drum hoists the ladder and is driven from a 110-h. p. motor with control for dynamic lowering. The two drums next to the ladder hoist drum are used for swinging the dredge in normal operation, and the two outer drums may be used for side-breasting or for crane service. A 125-h. p. motor drives the swinging and the auxiliary drums. The motor pinions are mounted on stub shafts. Each set of first-reduction gears are of the herring-bone type encased and running in oil. The second set of intermediate gears and the pinions which mesh with the drum gears, together with an independent friction and brake, for each drum, are mounted on a single intermediate shaft which has a winch head on each side of the house. The intermediate shaft is made in coupled sections for alignment purposes. The hoisting drum is grooved for its rope; all other drums have plain barrels. All spur gears have cut teeth.

The frictions, which are of the internal expanding, reversible driven type, are air-operated. The machinery with the driving motors is mounted as a unit on a substantial, structural steel base.

Stern Winch.—The stern winch is mounted on the deck aft of the main engine hatch and forward of the spuds. It consists of two drum shafts upon each of which is loosely mounted a grooved spud drum and a plain barrel auxiliary drum. Each drum has an independent friction and brake which, together with the drum gear pinions and intermediate gears, are mounted on an intermediate shaft, coupled in three sections, with a winch head on the port side. The first reduction gears are of the herring-bone type encased and running in oil. The auxiliary drums may be used with the forward auxiliary drums for breasting the dredge, for warping the pontoon line, or for auxiliary crane service. The gears, frictions, and other winch details are similar to those of the forward winch. The machinery and motor are mounted as a self-contained unit on a substantial, structural base.

All clutches and brakes for the bow and stern winches are controlled from the pilot-house which is located above the crews' quarters at the forward end. Each lever line is distinctly marked. If desired, the stern winch (including the motor) may be controlled from the stern as well as from the pilot-house. The pilot-house is the full width of the deck-house and has windows on all sides. It houses the controls for the cutter, ladder hoist, forward and stern winch motors, a push-button cut-out for the main pump motor, as well as all meters and gauges for the motor and main pump. The pilot-house is in communication with the engine-room and the other parts of the dredge through signal, telephone, and speaking-tube systems.

Spuds.—There are two spuds mounted in lined steel casting keepers in wells cut into the stern with a space of approximately 20 ft. between the upper and lower keepers. The upper keeper reactions are taken directly into the hull trusses. The stern of each keeper has a pin-connected gate which may be opened to unship the spuds. The spuds are 48 in. square, made with corner angles and side-plates, and two internal latticed trusses in the fore and aft direction. The lower spud end is of novel design; it consists of a heavy cylindrical steel casting, the outer point of which is clamped to, and rotates about, an inner point to which the square structural spud is securely riveted. Bushings and washers which form the contacts between the inner and outer points, are made of special brass. The spud-hoisting sheave is mounted in the upper end of the inner casting and is thoroughly rope-guarded. The rotating-point feature relieves the spud of all torsional stresses which otherwise would have to be transmitted through the spud to the keepers when the dredge swings.

Another very unusual feature is the provision for pinning up on the spud as is done on the forward spuds of a dipper dredge. This assures a firm grip for the spud in any reasonable bottom. The point remains fixed while the spud rotates, thus tending to prevent loosening the grip by enlarging the hole which has been made by the spud point.

The design of the spud machinery is such as to utilize the weight of the idle spud for "pin-up" reactions on the digging spud. The "pin-up" sheave is mounted in a steel casting bracket riveted to the upper spud end.

Power Plant.—All main dredging units are driven from a 600-volt direct-current power line energized by four 800-kw. parallel-operated, direct-current generators. Each of the latter is driven by an 1 150 b. h. p. vertical, 6-cylinder, 180 rev. per min., Diesel engine of the 2-cycle, air-injection, cross-head type. The engines are mounted on heavy longitudinal girders which are tied into the deep transverse floor system with plate intercostals. The generators and outboard bearings are mounted on steel-casting bed-plates.

The main Diesel engines are right and left hand with the operating gear for each on the inside. The air compressor, scavenging pumps, lubricating oil pumps, and fuel-oil injection pumps are built integral with the engines. A fresh-water cooling system consisting of re-coolers, filters, sea and fresh-water pumps, and a tank into which the water is discharged from the engines and redrawn by the circulating pumps, maintains the engines at a proper temperature by circulating water through the cylinder jackets and pistons. If desired, sea water may be used for cooling purposes in which case a small pump preserves a constant level in the discharge tank by discharging the cooling water overboard.

The dredge is equipped with 53 motors having a total normal rating of about 4 600 h. p. The main motors are operated from the 600-volt main generator busses, and this includes all motors of 5 h. p., or more, except the 7½-h. p. bilge-pump motors. The other motors are for auxiliary service and are of units less than 5 h. p. They are operated from the 125-volt bus which is energized from a 75-h. p. motor generator set, or the 75-h. p. Diesel engine set.

The main pump motor is of the adjustable speed type and will normally develop 3 000 h. p. continuously. Resistance is used for starting duty only, which means a sufficient period of time to fill the pipe line. Push-buttons in the engine-room serve to regulate the speed of the main motor through a motor-operated field rheostat between the limits of 250 and 360 rev. per min. This range is sufficient to allow for all reasonable service variations due to the pumping of different grades of material or variable line lengths with the same runner. This type of motor and control permits of efficient operation at full-load rating at any speed within the range, thus using the entire output of the power plant for variable duty, which assures maximum output at all times. The motor may be stopped by a push-button in the operating room.

The cutter motor has a full-load continuous rating of 600 h. p., and will develop overloads for time intervals up to a maximum of 1 100 h. p., for 5 min. This is considerably more power than has ever been applied to a cutter and the method of control is entirely new. The motor is of the adjustable-speed, double-armature type with resistance for only heavy-duty starting. The normal speed range with armatures in parallel is from 250 to 500 rev. per min. for full loads or overloads. With the armatures in series, the speed range is from 125 to 250 rev. per min. with about 50% of the normal load or overload ratings. This arrangement with the 4:1 speed range gives efficient and flexible control. The power developed by the motor at slow speeds is ample to satisfy the service condition, as the slow cutter speeds are required for easy digging only. The cutter and swinging motor controls are interlocked, which automatically slows down the swinging speed when the cutter is overloaded and restores it after the cutter motor has pulled through the overload condition.

The swinging motor has a continuous rating of 125 h. p., and is operated from the variable voltage motor-generator set with a Ward-Leonard system of control for starting, stopping, and regulating the speed from zero to 700 rev. per min. This system is used on the swing motor because of the necessity for a very wide speed variation and more reliable protection for the cutter motor through the interlocking control than could be obtained by a resistance control. A solenoid brake is provided on the swinging motor to prevent the dredge from drifting through wind or current action.

The ladder and spud-hoist motors are of the series-wound type with magnetic control and dynamic lowering. Solenoid brakes are provided, which set on the neutral position of the control lever. Both motors are controlled through the pilot-house, but the spud-hoist motor may be controlled from the stern if unlocked through the pilot-house controller. Both motors have a 75°, half-hour, 110-h. p. rating.

Miscellaneous Equipment.—There are two 10-ton, hand-operated traveling cranes, of 21 ft. span, mounted on overhead runways and supported by the truss posts. The runways extend over the upper end of the ladder to the bow gantry. For making heavy lifts, both cranes may be used together at any point in the length of the runway. There are six 5-ton, deck-mounted, cargo cranes, three forward and three aft. Blocks, lead sheaves, and anchorages are provided for operating these cranes by hand, from the winch head, the auxiliary drums of the bow, or the stern winches. A jib crane is pin-connected to the suction head at the lower end of the digging ladder, and is designed to fold back so as to be held by the ladder suspension bars when not in use.

A 3-ton, stockless anchor with 150 fathoms of stud-link chain, operated from a 30-h. p., worm-driven anchor hoist, with a locking gear and hand-operated brake, is located between the spuds on the deck, with a chain locker below the deck. A 30-h. p. capstan, with hand-operating mechanism, is located on the deck at the stern.

APPLICATION OF THE GYRO-COMPASS TO DREDGING WORK

Under numerous circumstances, especially when working in the lake at some distance from shore and at times when weather conditions prevent the use of ordinary ranges placed on shore, it is desirable and often essential that a means be provided on the dredge both for determining its location with reference to fixed points on shore and the width and direction of the cut being made by the dredge.

A partial solution of this problem was suggested in the use of the compass. To be of value in this respect the compass would have to be under the constant view of the operator and, because of the many cables carrying heavy electric currents which pass through and about the operating room, the magnetic compass proved to be of little value as it could not be adjusted to maintain true readings.

The next suggestion was the use of a gyro-compass which is not affected by the presence of electric currents and which has an additional value in that its readings are always given in true azimuth. In this manner the compass readings can be adapted to a survey based on co-ordinates the axes of which are parallel to and perpendicular to the true azimuth without correction.

The installation consists of a master compass which contains the gyroscope and all its mechanism, together with one or more repeaters, which has the advantage of providing a relatively small instrument in the repeater which can be placed in a convenient location. These repeaters are provided either with fixed horizontal compass cards or are mounted in gimbal rings for tilting in the plane most convenient for reading. An azimuth circle attachment is also provided by use of which the position of the dredge may be determined by triangulation, with fixed points on shore, the co-ordinates of which are known. More exactly, this process gives the location of the compass from which, by reading the heading of the dredge, the exact location can be determined by means of a small template of the same scale as the survey map.

The installation of this instrument proved a complete solution of this problem, and it was found that the dredge could be located within reasonably accurate limits by this means. Likewise, having established the location of the dredge, the cut can be maintained in any predetermined direction and of any predetermined width by controlling the swing of the dredge an equal distance either side from a predetermined average heading or direction of the cut.

After this installation was made and found to be satisfactory, a further addition was made to the installation which consisted of a recording repeater. The record of this repeating instrument was made on a paper strip, 7 in. wide, which gave the azimuth readings across the sheet, passed under the recording pen at a rate of about ¼ in. for each 10 min. of time. The clock mechanism was operated by an 8-day movement, and the record roll was of sufficient length to last a month.

In ordinary use on board ship this record is almost a straight line, showing only variations from the established course of the vessel due to windage, wave action, or other disturbing forces which alternate with the corrections applied to these deviations either by the wheelsman or by the automatic steering mechanism which is often applied to it. The curve produced by this recording and dredging operation, however, is quite different. It consists of a series of diagonal lines which record each swing of the dredge, indicating the azimuth reading at the end of each swing. Delays are recorded by a straight line in the direction of the chart so that a complete record of the operation of the dredge is maintained, indicating the exact number of minutes occupied in actual dredging, during which time the dredge is swung either in one direction or the other, and the actual delay time.

In addition, the operation of moving the dredge ahead in the cut by walking it on the two stern spuds leaves a characteristic record. The swing of the dredge is interrupted on the chart by a delay of about 1 min. or 2 min., during which time one spud is lowered and the other raised. This is followed by a continuation of the swing line through a certain number of degrees, and followed again by a delay of from 1 min. to 2 min., during which time the spuds are reversed. From the distance between the centers of the spuds and the angle moved through in the walking operation, the number of feet which the dredge will be advanced, can be determined. Checking the computed advance of the dredge over a considerable period of time by location from fixed points on shore, it was found that this method was especially accurate. This compass record gives an accurate determination of the width of the cut and the advance of the dredge so that, knowing the elevation of the bottom before and after dredging from soundings, the quantity of material moved can be checked with fair accuracy from day to day and from hour to hour.

As an essential part of the dredge operation, especially designed pontoons lashed together support the discharge pipe, and a special derrick boat at the discharge end permits of a wide range of free flow from the end of the pipe when discharging over a breakwater, or otherwise.

ACKNOWLEDGMENTS

This entire problem has been worked out by the organization of the Great Lakes Dredge and Dock Company, of which the writer is President, by R. V. Hoopes, Vice-President, and A. Neveling, Chief Mechanical Engineer, with the assistance of James H. Polhemus, M. Am. Soc. C. E., Chief Engineer of the Port of Portland Commission, as Consulting Engineer, together with the Engineering Staffs of the Bucyrus-Erie Company, of South Milwaukee, Wis., the Manitowoc Shipbuilding Corporation, at Manitowoc, Wis., the Busch-Sulzer Brothers Diesel Engine Company, of St. Louis, Mo., and the General Electric Company, of Schenectady, N. Y.

DISCUSSION

R. L. Vaughn,[4] M. Am. Soc. C. E. (by letter).—This paper, the most valuable which the writer has seen on the subject, is, perhaps, unique. It points the way to what has seemed a hopeless task, the application of scientific reasoning to that exceedingly complicated affair—a pipe-line dredger.

The writer has made several attempts to apply hydraulic theory to the dredger pump, and invariably has come to an abrupt stopping place at Equation (2). This is the customary formula given by many authorities on pump design, but the writer always has doubted whether it was even approximately true when applied to the pump on a pipe-line dredger. If the material passing through the pump were a uniform one, such as water or mercury, the formula would be unquestionable, but the mechanical mixture generally handled may present an entirely different set of problems.

The reason for introducing the factor, *w*, is to convert the fundamental relation, "power equals force times distance divided by time", into a form which can be applied readily to the conditions of a centrifugal pump. If the pump is handling a mechanical mixture the weight of which may (and often does) vary widely from moment to moment, the question arises as to what value should be assigned to *w*. In the ordinary texts, *w* is used as if it were the weight of material actually passing through the pump, although several observations indicate that this may not be correct.

Indicating watt-meters have frequently been observed while connected to pump motors of constant speed working on long pipe lines. When the digging was stopped and clear water pumped, there was no observed change in power, except such as could be accounted for by change in weight lifted to the pump, until such time as the line began to clear. If the ordinary equations are correct, there should have been a marked change in power as soon as clear water reached the pump.

Not infrequently some inspired enthusiast, with a background of theory on the design of centrifugal pumps, comes forth with a proposal to revolutionize the dredging business by arranging to introduce the material into the water after it has passed the pump. Mechanical details vary with the originality of the inventor, but the basic idea is that the pump will be handling clear water and can be designed to give much higher efficiency than an ordinary dredger pump will show. Then too, of course, the added advantage of greatly decreased wear on the pump is stressed.

If any advantage were to be gained by keeping the value of *w* low, that is, if the weight of the material actually passing through the pump were related to the question, there would be a great possible saving in power by keeping it low in the equation for power, and the idea might not seem so far-fetched.

A number of hopper dredgers have been built with arrangements for pumping the load ashore. The general arrangement has been a long suction line passing under the hoppers, with valves in each hopper discharging into the pipe. The material was dumped into the suction because this was the obvious and simple way to do it. If any real advantage existed in keeping the value of w low while the mixture passed the pump, it would be comparatively easy, and not too expensive, to insert the material into the discharge line instead of into the suction. No one seems to have discovered any such advantage.

---

[4] Cons. Engr., San Francisco, Calif.

Designers have assured the writer that, if a pump were handling clean water and a boulder were introduced into the line, there would be a momentary fluctuation in the watt-meter, corresponding to the change in weight of the mixture, while the boulder passed through the pump. The absurdity of this contention will become evident if two extremes are considered. Suppose that a pump is pumping water against a considerable lift and friction head. Imagine that the liquid (in the pump only) is replaced momentarily by mercury. According to the theory the power input should increase at once about eightfold. Again, suppose the water is replaced by some incompressible but weightless liquid. Then the power should drop to zero. Evidently, in either hypothetical case, the total work being done on the system would remain unchanged, except for internal pump losses.

For these reasons, when considering a dredger pump, the writer has always felt reluctant to accord to Equation (2) that veneration and respect which the authoritative elements of a subject are supposed to merit. Furthermore, the texts then proceed to develop the equations for the pump by inserting a large number of constants, for which a value is never given, consequently the writer's opinion of the texts with which he is familiar, is not high. Evidently, the power consumed by a dredger pump is not wholly independent of the weight actually passing through it; but it seems probable that the relation is a complicated one, involving, among other things, perhaps, the characteristics of the actual pump in each case. It will be appreciated if the author will offer any explanation which will make this question clearer.

If it is understood that w, as used in the paper, is the average weight of mixture handled, and that this average remains sensibly constant, then the foregoing objections do not apply to the mathematical treatment given. The assumption is not true in practice because as a rule a dredger does not dig on the return swing, and never digs when setting ahead; in fact, it often happens that the line is entirely cleared of material while "fleeting ahead".

Equation (4) expresses the condition that the friction head varies directly as the first power of the length of pipe line. This, for some kinds of clay, is far from true. If clay is pumped 3 000 or 4 000 ft., the greater portion of it will be discharged on to the fill in the form of lumps and balls, while if it is pumped 10 000 or 12 000 ft. (even without a booster), in many cases a large portion will come out as mud. Therefore, to state that the friction head is a linear function of the length of line is to state that, for a mixture of a given weight, the friction is the same whether the mix is a thin mud or a combination of water and balls. Manifestly, this is not true, and yet Equation (4) is part of the basis for Equation (10).

There appears to be some confusion in the meaning of C. From the notation, and in the text under the heading "Economic Velocity under Typical

Service Conditions", it would appear that C is the unit cost for labor plus the unit cost for all power consumed on the dredger. In Equations (1) to (10), it appears that only the unit cost of power consumed by the main pump is included. Experience indicates that, generally, when digging clay, at least one-quarter as much power will be required by the cutter as is used by the pump, and the ratio is often higher. This would seem to be too large a factor to be neglected, even in an approximate solution.

Nothing is more certain than that w will vary with the velocity in the pipe line, and yet it seems to be treated as a constant when the equation is differentiated. Thus, the writer looks upon the mathematical treatment resulting in Equation (10) with some misgivings and would like much more explanation of it. However, it is interesting, and worthy of careful study, as it may indicate correctly the effect of varying certain factors in the general problem.

Moreover, it must be confessed that the results which the author gets by using Equation (9) are in line with the writer's experience. A decrease in weight of material gives a relatively great increase in output. A considerable variation in pipe-line diameter does not produce much change in desirable pipe-line velocity. Again, on the average, large pipe lines, with suitable increases in power, make for reduced unit costs, and many contractors are obtaining larger machines.

In general, the paper describes a fine dredger which affords little to criticize or question in regard to its mechanical features. The need for strength and rigidity in the hull is illustrated by the experience of the dredger, *Houston*, which sank in a depth of 30 ft. of water in Galveston Bay during the hurricane in 1915. At the time she sank both spuds and the ladder were down and the hull came to rest nearly level longitudinally, upside down, with the bottom making an angle of 55° with the horizontal. The hull was of wood and was exceptionally well designed and well built. It was 165 ft. long, 44 ft. beam, and 14 ft. deep, with no trusses extending above the deck. The stresses which this hull underwent in rolling over as it sank, then being rolled back and raised, can be imagined. (In explanation of how a hull 44 ft. wide could roll over in 30 ft. of water, it may be well to state that the hurricane had raised the water to a depth of about 44 ft.) Rolling the hull back again after the storm, when there was only 30 ft. of water, was difficult. Extensive re-caulking and some repairs around the ladder and spud-wells put the dredger back in service, and, with some additional repairs, the hull is in good condition to-day (1931).

The installation of a gyroscope compass is a refinement not hitherto considered justified on a dredger, although as early as 1913 a device was proposed, consisting of a recording mechanism actuated by a gyroscope and also connected to the ladder, so that the device would draw a cross-section of the cut at each set-up. This proposal was rejected on the ground of expense, and for the further reason that it required the best energies of the organization to keep the dredgers running, without adding further complications.

Some mechanically successful Diesel-electric dredgers have been built. They are not without advantages. Practical Diesel speeds do not correspond to suitable dredger-pump speeds, and if direct-connected, it is necessary to use gears. The advantages of flexibility to be had with direct current are recognized, while the shocks which a Diesel engine gets when directly geared to a cutter are not conducive to trouble-free operations. However, when the added capital cost, the additional skilled labor required, and the additional complications are all considered, the writer has never been fully convinced that the Diesel-electric is the best design. On a large machine it probably is the best, due to the possibility of using several moderate sized duplicate main engines. Up to a size of at least 1 000 h.p. on the pump, the direct Diesel design seems to be satisfactory.

No information is at hand concerning the power distribution conditions throughout the territory in which this dredger is to be used. On the Pacific Coast power is abundant, cheap, and available nearly everywhere at 11 000 volts, alternating current. As a result, most successful dredgers there are electric ones. These are simple, relatively cheap to build, and inexpensive to operate. Power is brought to the dredger on pole lines (often built over the water) and by 1 000 ft. or so of armored cable; it is then reduced to 2 200 volts, the transformers being located on the dredger or on a scow moored alongside. All the principal motors are usually 2 200-volt alternating-current machines. Any other voltage is readily available through small auxiliary transformers.

Rotary converters could be installed and the power converted to direct current on the dredger, but this has never been done. Direct current has advantages as to flexibility, but these have not been considered sufficient to warrant the additional investment, complications, and power loss. The main motors are of the phase-wound induction type. Some variation in speed is obtained by means of variable resistances in the rotor circuits (external). However, variable pumping conditions are usually met by changing the diameter of the pump runner.

A recently developed device will give these motors a range of speed from 25% below to 25% above the rated speed, maintaining constant torque, with little loss of energy. The device has not yet been applied to a dredger as far as the writer knows, but it is being considered for that purpose.

The first turbine dredger to be built had a turbo-generating unit producing alternating current at 2 200 volts, and was otherwise arranged as an ordinary electric machine. It was a simple matter to install transformers on a scow, bring power from shore when available, and dispense with the steam plant. It was found that operating costs could be much reduced in this way, and the dredger is still so operated. At present (1931), larger motors have been installed and the steam plant could not begin to give the output which can be had by taking power from shore.

On paper, it is easily possible to show that power can be generated on board more cheaply than it can be bought ashore. These calculations, when applied to a dredger, usually prove to be "a delusion and a snare". Dredging contractors find it advantageous to abstain from the power business whenever possible.

It is understood that the anticipated conditions were such that the *New Jersey* must have her own power plant, and that, when considering the design, the advantages of direct current were attractive; but it would be interesting to know whether the possibility of obtaining power from shore, occasionally, was fully considered when selecting the type of electrical equipment which was installed.

The author's statement that the largest practical diameter of pipe is the most economical, may be misleading. It is true as regards power expenditure per unit of material transported. It is far from being true as regards a dredger of stated power. For a given machine, when pumping mud, the largest line within reason will be the best, but in pumping any mixture that will segregate, there will be a minimum velocity which must be maintained. This velocity will vary with the size of the particles in the mixture, and with the percentage of dilution. If the line is too large, a much diluted mixture must be pumped, and there is no economy in that. In fact, most dredgers do not have power enough for the size of the line; very few of them can pump all the clay they can dig for a distance of 6 000 ft. It is often a question of whether or not more power should be added. More power means more pressure, which causes trouble. The ball joints freeze; there is more trouble in holding the line together; and heavier pipes are required. Often, it is better to put in a booster.

Selecting an economical size of dredger for a contractor involves so many practical considerations, that a mathematical analysis can only be used as a guide to the judgment on a few factors. The tendency is toward larger sizes, undoubtedly. The author has selected a 30-in. machine. For jobs of some magnitude this is probably not too large, although for a series of scattered jobs, each of, say, from 100 000 to 500 000 cu. yd., coming up from time to time, some contractor with a well-designed 22-in. machine, having a 500-h.p. cutter and a 1 500-h.p. pump, could furnish plenty of competition.

Be that as it may, the 30-in. diameter having been decided upon, the next step is the question of how much pump power is suitable for this size of line. This may be approximated by remembering how much power the designer wishes he had had on some previous dredger which worked fairly well, and estimating that the powers should vary, roughly, as the squares of the respective pipe-line diameters. The next step is to estimate how much digging power is needed to keep this pumping outfit busy on a reasonably short line. Experience indicates that at least one-quarter the pump power should be available for the cutter, and more may be needed. The *New Jersey* has a nominal ratio of one-fifth which may be found ample, particularly as this is an intermittent service and the drive will stand a generous overload.

The writer fully shares Mr. Cushing's opinion concerning clay with boulders in it. Any dredger which will perform, even passably well, in such material is a good machine. During the early days of the dredging business it was considered necessary to install a "stone box" in the suction line just ahead of the pump. This device was provided with grate bars intended to prevent the entrance into the pump of anything that could not pass through. Eventually, it was found that the trouble with the stone box was that it worked too well;

the delays caused by cleaning it were intolerable. Runners were obtained which would pass anything that could come up the suction line; the gain in running time far more than made up for any loss in pumping efficiency. Present practice is to have a suction larger than the discharge, in which case it is not necessary to have a runner which will pass larger objects than can pass down the line. With a cutter having a maximum opening of only six-tenths the diameter of the pipe, no rocks should stick in the pump, although trash may. Other cutters are used, having larger openings. In such cases, where necessary, the condition is met by inserting intermediate rings in the cutter, or by placing a bar across the mouth of the suction pipe.

An extended experience with trash, buried forests, saw-mill slabs, pile stubs, etc., might modify the author's opinion concerning the relative value of high efficiency in a dredger pump. Large rocks can be excluded from the suction, but these other things can not, and the manner in which they can plug up a runner is truly remarkable. Ordinarily, if the runner can pass anything which can go through the line, there will not be much trouble from this source. Runners with smaller openings can often be used, and are better. A few years ago a two-vane runner was developed at New Orleans, La., for pumping sewage. Built somewhat after the fashion of a weedless propeller, it would pass just about anything, and when adapted to a dredger it did yeoman service in the swamps. The output of the dredger increased from 1 600 to 10 000 cu. yd. per day, and the operators spent little time in vain regrets over the sacrifice in mechanical efficiency.

The author uses an efficiency of 70% for his pump, which is to perform against heads ranging from 125 to 220 ft., with no alteration other than a variation in speed. Few centrifugal pump designers would undertake to design a water-pump to maintain such an efficiency over this range. When it is remembered that in a dredger pump the disk friction is much higher, that the turbulence is necessarily far greater, that the passages through the pump must be much larger than is desirable, that the surfaces of these passages are anything but smooth and highly polished, and that wear is constantly changing the theoretical lines, it is very doubtful whether any dredger pump even approaches this efficiency. If the author has any data to sustain such an efficiency, such information would be interesting.

Table 2 has been computed using one set of values for w, f, b, and p, and variable values of L. It is certain that w will vary with the length of line and with the velocity, while, for some materials, f will vary with the length of line independently of any variation in w. When this last factor varies, then f, b, and p are all variables. It would be a contribution of the greatest value if Mr. Cushing could give any experimentally determined values for e, f, n, and p.

The author states that the volume of a clay will be a linear function of the water content, and, of course, if this is true, then the statement follows, as given, that in a fill where the change in dimension is all in one direction the settlement will be in greater ratio than the change in water content. If there are any observations to substantiate this relation between water content and volume they would be of value. Experiments on the bulking of sand give different results. Anything approaching accuracy in estimating the volume of a cut, in clay or mud, required to make a given fill, is very difficult, and contractors as a rule are reluctant to bid on fill measurements.

A combination of causes probably contribute to the great difference in shrinkage between the first and second fill shown in Table 1. For one thing, the description indicates that the first fill did not come much above the water level. Under such circumstances a comparatively light load is placed on the original bottom, and there is little chance for the newly deposited fill to dry out. Furthermore, with less than two-thirds of the fill placed, a large settling basin must have remained and, consequently, there was only a small loss through the spillway. When the second fill was placed, it was mostly above water where it could dry out and shrink; also, the superimposed load expelled water from the first fill and from the underlying bottom. Again, during the second season's work the settling basin was becoming filled, and there were greater losses through the spillway. Under some circumstances these losses become quite appreciable, and no computations of volume ratio are of much value unless they are taken into account.

The description of liners in the hull pipe and fittings brings to mind an experiment made by the writer, which may be of interest. During the World War the problem of supplies and repairs was especially difficult on the Pacific Coast and any desperate expedient was justified. One job on hand involved the moving of about 2 000 000 cu. yd. of mud and 750 000 cu. yd. of clay which was in a fairly level strata under a portion of the mud. The mud was a fine slippery bay silt, containing less than 1% of heavier particles, grains of sand, and small pieces of broken shell. The clay contained considerable small sharp gravel. This last combination works on the principle of the diamond drill bit and is guaranteed to wear out anything rapidly, but there was no reason to anticipate more than nominal abrasion from the mud. It was proposed to strip as much of the mud as possible and deposit it in one fill and then dredge the clay and deposit it in another, and closer, fill.

Pipe was scarce, exorbitant in price, and was only available in low carbon steel which wore out rapidly. Records were on file showing cases in which at least 2 000 000 cu. yd. of sand had been handled through a single installation of wooden stave pipe. Several manufacturers of this pipe were located in the the vicinity, and were in a position to fill orders promptly. If wooden pipe would handle 2 000 000 cu. yd. of sand, there was no reason to think it would not give equally good service with soft slippery mud. Of this pipe, 1 000 ft. was purchased and installed in the most permanent part of the line. It lasted just five days during which it carried about 50 000 cu. yd. of mud and then blew up. More steel pipe was coming through the shops but there was none to spare on hand, and the experience was aggravating to say the least. No clay at all had been pumped.

What had happened was that the coarser particles settled to the bottom of the pipe where they were dragged along by the stream of mud and did a maximum amount of cutting. A width of about 6 in. on the bottom of the pipe was practically all cut away while the remainder of the pipe showed no wear at all. This circumstance suggested another experiment. Another 500 ft. of wooden pipe was purchased and sent to the pipe shop where the bottom third was lined with soft steel plates ⅛ in. thick. All the liners were punched to template and held in place with small bolts, the purpose being that when the plates wore out they could be easily replaced. This experimental pipe was installed in the line and it never did wear out. It carried about 1 000 000 cu. yd. of mud and 750 000 cu. yd. of clay, and outlasted two sets of steel pipe.

The method used by the author to determine velocity in the pipe line is interesting, but can be used only in fresh water. Permanganate and several different dyes have been used with satisfaction in salt water. The coloring matter is easily introduced into the suction. A rough rule which is nearly correct is as follows: The velocity in the line, in feet per second, is equal to four times the horizontal distance, in feet, traveled by the stream while it falls the first foot after leaving the end of the pipe. That the rule may be correct, it is necessary that the pipe be full at the end, that it be discharging horizontally, and that the stream be falling freely.

A swivel joint near the lower end of a steel spud has often been discussed, but this is the first time the writer has heard of its actual use. While it is true that there is comparatively small stress on a spud near its point, still this sounds like an additional complication which may give trouble. Unless they have a round section of some length at the lower end, square spuds are subject to considerable torsion. Round spuds are free from this trouble. On the other hand, it is much easier to build a square steel spud than a round one. One steel spud was built 24 in. square from 1-in. plates and 8 by 8 by 1-in. angles. This spud was lagged to a round shape with hard wood; it was used in a 36-in. round spud well, and gave complete satisfaction.

The use of two steel spuds is open to one objection. Sooner or later the leverman will get careless and swing with both spuds down. When this happens it is preferable to have one wooden spud to replace rather than to have to deal with two bent steel ones. Even if the leverman never makes a mistake, a swinging wire has been known to carry away when the dredger was in a cross-current with both spuds down. One spud is used only to "fleet ahead", and there is no necessity for having this one as strong as the digging spud.

Arthur L. Collins,[5] Assoc. M. Am. Soc. C. E. (by letter).—In changing from the steam-operated pipe-line dredge to the motor-driven units, it is well to bear in mind the peculiar characteristics of the centrifugal pump.

In present-day steam-dredge operation the efficiency of the pump is given little consideration. It is noted that in determining the power requirements for the dredge, *New Jersey*, the pump efficiency was assumed to be 70 per cent. While pumps with an efficiency as high as 90% can now be obtained for clear water, it would appear to the uninitiated that 70% could be depended upon for the dredge. However, when the dredge pump is properly tested, it is generally found that the efficiency is somewhere between 55 and 65 per cent.

---

[5] Cons. Engr., Berkeley, Calif.

This is about the minimum at which a centrifugal pump can be expected to operate.

In passing from the steam-driven to the motor-driven pump, whether the latter receives its electrical energy from a power station or from Diesel engines, it is of great importance that the temperamental operating characteristics of the pump be thoroughly understood.

Fig. 4 represents the characteristics of a typical dredge pump driven by electric motor. Only the fundamental information regarding the test is shown. In the motor-driven pump it is assumed that the speed is practically constant. This is the point that accounts for the condition of undesired instability, as indicated by the test. For example, the curve of total lift and capacity is very flat. At 24 000 gal. per min., the head is 131 ft. At 18 000 gal. per min., the head is 138 ft. In other words, the pump capacity falls 25% in a change of 7 ft. The power input to the motor falls from 1 060 to 880, or 180 kw. In normal operation the suction lift on the pump will fluctuate more than the specified 7 ft. from minute to minute, depending upon the material which is picked up by the pump. The fluctuation is 7 ft., more or less, and, consequently, the pump is in a continuous unstable condition, regarding capacity and power.

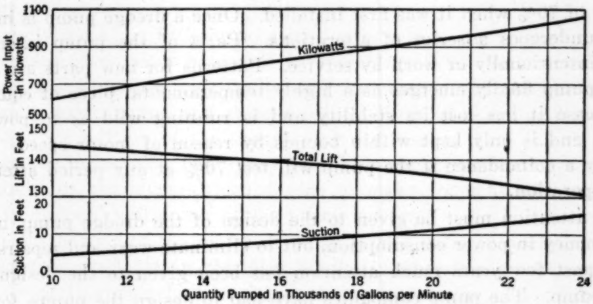


FIG. 4.—CHARACTERISTICS OF A DREDGE PUMP.

Ordinarily, there is a recording suction gauge installed on the pump. This gauge should show considerable variation, but this effect should not be instantly transmitted to the pressure gauge in a well-designed pump. A widely fluctuating pressure gauge is an indication of the sensitive pump.

A steam-operated pump, with its more or less flexible control, has a tendency to ease the characteristics of the pump. There is a balance between power and speed, as well as between head and capacity. The electrically-driven pump cannot slow down, but must maintain a nearly constant speed. The motor-driven pump is likely to be noisy, due, in part, to the constantly changing internal flow velocities.

The variable-speed motor does not eliminate the objectionable features of the constant-speed motor. The motor is operating at reduced efficiency when on any contact less than full speed. While on this contact, it is not far from being a constant-speed motor, the same as when operating at full speed.

In passing from the steam-engine drive to the electric drive, the engineer must take into consideration the difference between a constant-speed and the variable-speed, steam engine. Of course, the steam engine will be controlled within certain limits by the governor, but the governor can be set at any desired speed.

There are centrifugal pumps now in various parts of the United States which were installed in the late Seventies, and afterward, and which were operated by steam. Strange to say, some of these pumps have a very good efficiency. They are efficient because they can be driven at a selected speed. Try to drive them electrically with a constant speed, and it will be found that they cannot be satisfactorily operated, due to the conditions shown in Fig. 4.

It can be said that probably the pump given as an example did have an efficiency of 70% when it was first installed. Once a dredge pump is in operation, it undergoes a series of alterations. Parts of the pump interior are chipped intentionally or worn by service. Patterns for new parts are altered and the pump finally emerges as a highly temperamental piece of equipment. In this case it has lost its stability and is running wild as to power and capacity, and is only kept within bounds by reason of motor speed. It will be merely a coincidence if the pump will test 70% at any period after it has started operation.

More attention must be given to the design of the dredge pump, not only to save money in power consumption, but to eliminate wear and repairs. During the past few years much attention has been given to the design of the sewage pump. The pump companies have had to design the pumps for motor drives and they have developed pumps with characteristics that should be copied by the dredge designer. By making use of the similarity of the sewage pump, it is possible to build the larger sizes as used on dredges, and anticipate the desired characteristics. The electric drive will require a properly designed pump.

The writer does not consider the injection of salt or color into a line as the best method for obtaining data which will enable one properly to analyze the operations of the pump. The most satisfactory method is first to test the pump with clean water. The curve similar to that in Fig. 4 is obtained by pumping against a partly closed valve, and noting the action of suction and pressure gauge. The Pitot tube is used in the suction pipe for the capacity measurements. This method provides the true operating characteristics of the pump which are comparable from time to time.

From the viewpoint of the engineer endeavoring to check the characteristics, the color or salt-water injection method is not to be preferred. Its only justification would be to check or calibrate the gauges under load conditions after the fundamental information is obtained. The fundamental information is important, because one can check the condition of the pump within, say, 2%, from time to time, and can do this in a few minutes and with no loss of dredge time.

No doubt there are many dredge owners with steam-driven pumps who have their eye on the motor drive. It would be disastrous to assume the pump efficiency as 70 per cent. In view of the uncertainty of the operating characteristics of the pump unit, it is advisable to base the power requirements on a pump of low efficiency.

Too much stress cannot be placed upon the importance of knowing the exact performance of the pump. In operating a pump direct by a Diesel engine, the owner is very likely to be gravely disappointed, if the pump efficiency is rated too high. To begin with, the gas engine or Diesel engine is not generally given an overload rating. An electric motor with plenty of reserve power from the line can readily provide an overload of 25 per cent. The Diesel engine must be liberally rated, otherwise the centrifugal pump may be found to require more power than was anticipated by the assumed efficiency.

Generally speaking, little or no attention is paid to the matter of efficiency in the dredge pump after it is once in operation. The characteristics of the pump can change within wide limits, and the pump under the new conditions will require more power and more repairs. Without a systematic plan for checking the condition of the pump every few weeks, there is no clue as to how it can be improved. There may have been some justification for the lack of incentive on the steam-driven dredge to make frequent tests, due to the flexibility in speed and power of the prime mover.

With the advent of the motor drive, generally little or no information of importance is available in the files of the engineering department of the dredge owner to enable him to anticipate accurately the performance of the same pump under the motor drive. To operate a motor-driven dredge which will show good efficiency over a long period of time, will require that a constant check be kept on the characteristics of the pump.

It will require the concentrated effort of a technical engineer who has a full working knowledge of the meaning of the characteristic curve of the pump. The dredge captain should not step from the steam dredge to the motor-driven dredge without recognizing the value of the technical engineer in analyzing the pump problem. The cost of handling the material is not only affected by an excess cost of power, but also by repairs, shut-downs, and improper pipe-line velocities, all contingent upon the design of the pump; and which design can be altered quite unintentionally.

JOHN FRANCIS CUSHING,[*] M. AM. SOC. C. E. (by letter).—Referring to Mr. Vaughn's discussion, it is probably true that there is and always will be difficulty in applying hydraulic theory to a dredge pump, but Equation (2) does represent the principal factors (and their relation one to the other) necessary to obtain results of practical value in dredging. Some authorities, however, question the exactness of this equation when the pump is used for handling uniform material, such as water and mercury. The power consumed by a dredge pump is dependent on the weight actually passing through it. It is likewise dependent on many other factors, and their relation one to the other may be complicated, but in hydraulic dredging operations there is an average condition, the limits of which are within a range that makes this equation of practical value. Many data are available on the performance of centrifugal water pumps, and they are available, probably, because of the general use of such pumps and because it was a simpler and easier problem to make such observation than to determine similar data for a pump handling a liquid burdened with a miscellaneous quantity and character of solid material. Nevertheless, this burdened stream of water passing through the dredge pump is made to act as a liquid and, therefore, the theory and data available on centrifugal water pumps are, in a large measure, made applicable to the dredge pump by developing new and different values for some of the constants in the formula.

---

[*] Pres., Great Lakes Dredge & Dock Co., Chicago, Ill.

It is true, as stated by Mr. Vaughn, that Equation (4) assumes that the friction head is a linear function of the length of line for a mixture of given weight. Even in dredging a material, such as a uniform clay or a uniform sand, this weight of burden is not constant in the pipe line, but varies materially, principally because of the lack of control of the operation of the cutter at the intake under water. Consequently, such loading of burden into the line cannot be done uniformly, but only within such limits as may be determined by observing the vacuum and pressure gauges. However, in the usual dredging operations, these limits are promptly established by the experienced operator and hence an approximate average condition is maintained under any given set of conditions.

By means of constants, Equation (4) also takes into account the character of the material; soft or light material requires one value for *f*, and hard clay balls or gravel another. These must be determined by experience with the particular dredge being used; the constants for one dredge are not essentially applicable to another, but it is feasible and practicable to develop such constants for one dredge and re-use them for predetermining economic performance on future contracts and under similar conditions.

The constant, *C*, in Equation (7) is the unit cost for labor plus the unit cost of all power consumed on the dredge, and in this equation the unit cost of power used by the pump is considered as this total cost divided by the power consumed by the pump only.

It is understood that in clay work, a large part of the total power used on the dredge is consumed by the cutter. Furthermore, a large part of the total power is used to operate the other auxiliaries on the dredge. Nevertheless, the pump consumes the main part of the power, and it is considered that power for these auxiliaries, including the cutter, is properly included in the cost of that for the dredge and that the complication resulting from developing a formula segregating the cost for these different units that absorb so much less power, would not be warranted over and above the practical results obtained by treating it as part of a whole or in keeping with the character of the operation.

In addition, it is usually found that where refractory material is being dredged, such as hard clay, and where much cutter power is necessary, that the pump power required also increases in proportion in handling the larger pieces through the pump and for transportation through the line. Likewise, in soft clays, or in fine sand that is not sharp, little cutter power is necessary and less pump power is required due to the lesser friction in the pump and line. These differences are adequately covered in Equations (1) to (10), inclusive, by variation in the values determined for the constants therein. It is probable that future practice will develop more accurate formulas for dredge pumps, but Equation (7) has been found reliable for determining economic performance.

The effect of w varying with the velocity in the pipe line would be important for low velocities, or for velocities just high enough to transport the solid material or particles in the pipe line; but in commercial dredging operations much higher velocities must be provided to reach some of the necessary economies, and for the velocities usually maintained—particularly in the more modern and high-powered dredges—this effect of the variation of w is not of high order. It has been determined in practice repeatedly that w increases up to a certain velocity. From this point there is a certain practical range of velocity in which w is more or less constant; that is, w is not increased materially by an increase of velocity. Other conditions remaining constant, the dredge operator frequently finds this practical range of velocity by actual experimentation. The results of similar experimentations may be used to determine the constants in Equations (1) to (10).

When a new contract is being started, estimates and calculations are made from the best data available by Equations (1) to (10), in order to find the practical and economic method of operation, including the economic velocity to be used. From the output over a few hours it is easy to determine whether the economic velocity has actually been reached and when the dredge begins to exceed it. Much of the work that has been done by the *New Jersey* has been done with only three of its main engines, and these three not developing full power. On other work that it has performed the full power as developed by all four engines has been utilized. From this experimentation in a great volume of work, however, mostly clay to date (1931), the writer is convinced that the theory and practice experienced in the development of Equation (10) are thoroughly applicable to such hydraulic dredging.

The writer believes with Mr. Vaughn that when electric power is available it is more simple and economic to use it on dredging. However, this power supply varies widely in different communities, and for general utility it is better to have the power plant aboard the dredge. In special localities where electric power is readily available, the building of a straight electric dredge would be advisable.

In the pump of the *New Jersey*, efficiency was determined as follows: At a time when working conditions were uniform, arrangements were made for conducting a velocity test, as described in the paper under "Economic Velocity Under Typical Service Conditions," by injecting a salt solution at the dredge pump and noting the discharge of this solution at the end of the line by the use of a small galvanometer at that point. During the period between the injection of the solution and its discharge from the line, readings were taken simultaneously at short intervals of the vacuum in the suction line, pressure in the discharge line, motor input amperes, line voltage, and motor revolutions per minute. If conditions fluctuated very widely during this period, the test was disregarded. However, if these various readings remained fairly uniform over the period, sufficient data were at hand to compute the efficiency of the pump by reducing them to values of shaft horse-power input to the pump and water horse-power output for the pump.

In two typical tests the following conditions were observed: The observed velocity was 20.1 ft. per sec.; the motor speed, 300 rev. per min., the shaft horse-power input to the pump, 2 760; the total head, 165.4 ft.; and the percentage of solids in the stream, 9.2. The computed efficiency of the pump in this instance was 68.8 per cent. This test was made on the pump after it had been in service approximately five months.

In the second case, the observed velocity was 20.5 ft. per sec.; the motor speed, 327 rev. per min.; the shaft horse-power input to the pump, 3 091; the total head, 197.4 ft.; and the percentage of solids in the stream, 7.0. The computed efficiency in this test was 74.5 per cent. This test was made shortly after the pump had been relined and with an impeller of a slightly different design from that in use when the previous test was made. The material being handled in each instance was a hard and abrasive blue clay which required an average of practically the full 600 rated h. p. of the cutter motor to maintain the percentages of solid materials indicated in these tests. Likewise, the velocities indicated in the tests were required to move the material uniformly in the pipe line without undue precipitation of the solid material to the bottom of the pipe.

In view of these and other similar tests made under varying conditions, it is believed that an average pump efficiency for this installation of 70% was reasonably justified.

In Table 2, "Economic Velocities Under Various Conditions of Dredging," it is true as stated by Mr. Vaughn that one set of values was used for w, f, b, and p, and variable values of L. It is true that w will vary with the length of the line and the velocity, and, for some materials, f will vary with the length of the line independent of any variation in w, but the variation in all these factors is not great if the velocity is maintained approximately at the economic velocity. Further, Table 2 shows the results of handling medium clay only; for a variation of material of course other values of some of these constants in the formula are used. The value of e was determined experimentally as described previously; the value of f by the reading of pressure gauges at intervals along the discharge line; the value of n was taken at 1.86 and that of p at 1.25 from the Schroder and Dawson formula for friction head in pipes.

Referring to the discussion by Mr. Collins, the matter of justification for assuming a pump efficiency of 70% for the purposes of computation and comparison has been covered to a certain extent in the preceding remarks. However, there are several other details which may be discussed. It is quite true that the ordinary run of dredge pump carries with it an efficiency of 55 to 65 per cent. One reason for this appears to be that some of the possibilities for higher efficiencies have been sacrificed in the interest of cheaper manufacturing costs and simpler operating conditions. As explained in the paper under "The Design Elements," the pump for the *New Jersey* was a specially designed unit, aiming at higher efficiency than that of common practice. The fundamental assumptions were based on the accomplishments of another pump designed along the same lines in which the efficiency averaged about 70% over normal operating ranges. This increase in efficiency was obtained by several refinements of manufacture and operation, some of which are frequent inspection and renewal of liners and the adjustment of clearances to a practical operating minimum by rubber-faced rings in the pump.

By the use of renewable liners throughout the pump its characteristics are maintained in accordance with the original design within close limits and, differing very much from the ordinary plain cast shell and impeller, such characteristics are not altered materially by wearing or intentional chipping.

It is quite true and likewise quite justifiable that the design, not only of the pump, but of the driving motor, should be given exceptional attention and care in the size of a unit, such as is used in the dredge, *New Jersey*. In smaller installations such refinement in design is not ordinarily obtained. It is not proposed that a pump efficiency of 70% can be assumed for any size or type of installation, but in this case the assumption was justified, as proved by subsequent test conditions.

Preliminary tests on this installation were conducted by pumping clean water and establishing various values of head by attaching a series of conical nozzles to the discharge. In this way the characteristics of the pump and motor as a function of head or as a function of quantity of discharge were determined. However, it is possible that the conditions obtained in such preliminary test would not maintain under the varying conditions of actual dredging service. For this reason, velocity tests by means of salt solution were used under actual working conditions where there were momentary variations of suction and pressure as well as fluctuations in the quantity of solid material passing through the pump and through various points in the line. In this regard, it is considered that the use of the salt solution in the stream would have an integrating influence, so to speak, in covering the momentary variations.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

TRANSACTIONS

Paper No. 1803

THE BUTTRESSED DAM OF UNIFORM STRENGTH[1]

BY HERMAN SCHORER,[2] ASSOC. M. AM. SOC. C. E.

WITH DISCUSSION BY MESSRS. JAMES GIRAND, FRED A. NOETZLI, B. F. JAKOBSEN, CALVIN V. DAVIS, A. A. EREMIN, EUGENE KALMAN, A. FLORIS, FREDRIK VOGT, A. C. SCHWAGER, PAUL BAUMAN, I. M. NELIDOV, AND HERMAN SCHORER.

SYNOPSIS

Any investigation of the principal stresses in buttressed dams, as designed at present, reveals the fact that the maximum compressive stress occurs only at one single point in each horizontal section. At the same time, the up-stream portion of the buttress may disclose considerable tension in the direction of the second principal stress.

The object of this paper is to present the general theory of a buttressed dam of definitely known and uniform strength—to discuss its dimensional and quantity laws and to make suggestions for incorporating the theoretical findings in practical designs.

The theory is based on the conception of elementary arched column units, which transmit the water pressure from their respective deck areas directly and in uniform compression to the ground, together with the accumulated dead load. The elementary column units can be combined to a monolithic buttress system, in which the first principal stress is uniform compression throughout and the second principal stress is zero.

Secondary stresses due to shrinkage, temperature changes, etc., are largely eliminated by dividing the buttress system into several arched column units of practical dimensions, which units, together with their monolithically connected decks, are completely separated from each other by continuous joints. Climatic influences are further reduced by designing the down-stream column units so as to enclose the structure completely.

The independent column units will also permit the construction of the dam to the ultimate height in independent steps at convenient time intervals. The dangerous and expensive necessity of anchoring new concrete to the older structure is thus successfully avoided.

A buttress system thus designed may be used in connection with any of the common deck types. The method is illustrated in a tentative example for

---

[1] Published in November, 1930, *Proceedings*.

[2] Chf. Engr., Thebo, Starr & Anderton, Inc., San Francisco, Calif.

a dam 300 ft. in height, using a simple cantilever deck. Diagrams have been prepared, which will enable the designer to determine quickly the most important elements for any desired combination of the main design variables.

Considerable saving in cost can be realized for any height to which a gravity dam with the same apparent factor of safety may be built. With a design stress of 500 lb. per sq. in., the maximum possible height of the buttressed dam of uniform strength is about 800 ft.

INTRODUCTION

Hollow dams of the reinforced concrete type generally consist of three distinct members: First, the deck, forming a water-tight up-stream face; second, the buttress, or supporting system, transmitting the combined water and dead load to the foundation; and, third, the foundation, which transmits the total load to the ground.

The design of the foundation structure is governed entirely by local geological conditions which determine the permissible bearing pressures. The following analysis assumes that the dam is founded on solid rock, so that no special foundation structure will be required. The most common deck types are the simple beam slab of the Ambursen dam and the multiple-arch deck. The supporting system of both types generally consists of triangular buttresses with rectangular, horizontal sections.

Any analysis of the relative yardage in deck and buttress reveals the fact that with increasing height the volume of the buttress becomes by far the largest item. The cost of high hollow dams, therefore, is governed largely by an economical design of the supporting system.

The arbitrary, triangular buttress, which is commonly used, has been borrowed directly from the gravity dam. The same may be said of the ordinary buttress analysis, based on the stress determination in horizontal sections, when stability is assumed to be assured if the vertical stress components and the horizontal shearing stresses do not exceed safe limits.

Mr. B. F. Jakobsen first called attention[3] to the necessity of determining the principal stresses in the buttresses of multiple-arch dams, because the ordinary analysis may not reveal the existence of large tensile stresses. Later, Messrs. N. Kelen[4] and Carl Boegh[5] also gave solutions for the principal stresses in wedge-shaped buttresses. The older method of analysis, which considers horizontal sections only, must now be regarded as obsolete and unreliable.

Unfortunately, the principal stress determination does not simplify the design, because the analysis must begin with dimensions already known. For this reason all the present design methods are based on repeated trial analysis, a procedure which is well described[6] by Fred A. Noetzli, M. Am. Soc. C. E.

---

[3] *Transactions*, Am. Soc. C. E., Vol. LXXXVII (1924), p. 276.

[4] "Die Staumauern," by N. Kelen, p. 81, Julius Springer, 1926.

[5] "Beitrag zur Berechnung der Spannungsverteilung in Stuetzpfeilern von Stauwehren aufgeloester Bauweise," by Carl Boegh, *Beton und Eisen*, 1927, p. 331.

[6] "The Design and Construction of Dams," by Edward Wegmann, M. Am. Soc. C. E., Eighth Edition, 1927, p. 439.

For economic reasons, and for reasons of safety, the ideal design must always aim at a structure of uniform strength. It is evident that the arbitrary, triangular buttress shape, with rectangular horizontal sections, does not necessarily approach the structural ideal. The trial method also involves a large amount of labor, and the derivation of general dimensional and quantity laws, based on the arbitrary shapes, would lead to a waste of material in higher dams.

In order to eliminate the trial computations the following direct design method for the supporting system is proposed. The water pressure, acting on an elementary deck area, is transmitted to the foundation in the most direct way by elementary arched columns, dimensioned so that the compressive stress is constant throughout. These elementary columns can then be combined to a monolithic buttress of uniform strength. They may also be built as adjacent members, separated from each other by continuous joints, so that secondary stresses due to shrinkage, temperature changes, etc., can practically be eliminated.

The available space does not permit the full presentation of the analysis, and the mathematical expressions, therefore, must be limited to the fundamental results. For the detailed derivations the reader is referred to a more complete manuscript of the paper entitled, "The Hollow Dam of Uniform Strength," filed by the writer in the Engineering Societies Library, New York, N. Y.

NOTATION

The notation adopted for the discussion of this paper is as follows:

a = a ratio expressing the relation, $\frac{v}{c}$.

A = cross-section area of an elementary column; a subscript, 1, denotes the section at the point, P_1.

b = length of elementary deck area.

c = a ratio expressing the relation, $\frac{f_c}{\gamma_c}$; this unit is a length.

d = horizontal distance, top and bottom, of an arched column at the center line.

D = total height of buttress, identical with maximum depth of water.

e = base of Naperian system of logarithms.

f = unit stress; a subscript, c, denotes compression.

F = a sliding factor; F' is the sliding factor for the buttress of absolute minimum weight.

g = the gravity constant.

h = vertical distance from top of deck to the center of an elementary area of deck slab.

H = horizontal component of force; a subscript, 0, identifies this component with P_0 and a subscript, w identifies this component with R_w.

i = a ratio expressing the relation, $\frac{D}{c}$, called buttress ratio.

k = a factor signifying the relative deck weights.

m = a variable coefficient that expresses a relation, $\frac{v}{D}$.

P = axial thrust on an arched column; a subscript, 0, denotes water pressure; a subscript, k, denotes top thrust due to water and slab combined; and P'_k denotes top thrust due to water and normal slab component combined.

p = pressure at the top of an elementary arched column; a subscript, v, denotes the vertical component.

r = a coefficient expressing the ratio, $\dfrac{V'_w}{H_w}$.

R = resultant of all forces above the foundation; R_w is the resultant thrust on deck due to water.

S = a force that represents the weight of the elementary deck area; a subscript, B, indicates the component perpendicular to the deck; and a subscript, D, indicates the component transmitted through the slab to the base.

t = thickness of a catenary column; the subscript, 0, indicates a section at any point, N_0, and a subscript, 1, indicates thickness at Section N_1, etc.

v = vertical distance from the toe of the deck slab to the center of an elementary area in that slab; Δv = horizontal projection of w_0, the width of the elementary area, and $d\,v$ is the horizontal projection of a differential width of deck area.

V = vertical component of force; a subscript, 0, identifies this component with P_0; a subscript, w, identifies this component with R_w; and V'_w = total vertical component of water pressure of buttress of absolute minimum weight.

w = width of an arched column; a subscript, 0, identifies the width with that at N_0; and subscripts, 1, 2, etc., identify corresponding sections at N_1, N_2, etc.

W = weight of structure; a subscript, c, denotes the arched column; a subscript, D, denotes the total deck; and a subscript, B, denotes the total buttress.

x = horizontal distance from the toe of the slab to the center line of the arched column at its base; a subscript, 0, denotes horizontal distance from toe of slab to the center line of the arched column at its top; $d\,x$ is the vertical projection of a differential width of deck area; and Δx = small horizontal interval.

β = angle of R and V.

γ = unit weight; a subscript, c, refers to concrete; a subscript, w, denotes water; and γ'_w is an imaginary unit weight.

λ = weight factor: subscripts, D and B, are for deck and buttress, respectively; λ_0 = factor for buttress with zero deck weight; λ'_0 = factor for buttress of absolute minimum weight; and λ_g = factor for gravity section.

ϕ_0 = angle that deck slab makes with the vertical.

ϕ_1 = angle of column thrust with horizontal at N_1; ϕ_k = angle of combined top thrust, P_k, with horizontal, $\Delta\ \phi_k = \phi_k - \phi_0$; and $\Delta\ \phi$ = angular change of column axis corresponding to $\Delta\ x$.

ARCHED COLUMN UNIT

Referring to Fig. 1 consider an elementary deck area, $A\ B\ C\ D$, and the water pressure, P_0, acting on it. Assume for the present that the weight of the deck is zero. Similar to the thrust of an ordinary bridge arch the thrust, P_0, can be transmitted to the ground by direct compression by the arched column, $N_0\ N_1$. The crown of the arch may be imagined at O, and the right abutment at N_1. The water pressure, P_0, simply replaces the arch thrust at N_0.

If the cross-sectional area of the arch is varied so that the compressive stress is constant at any point, the arch axis will conform to a curve known as the catenary of uniform strength.[7] The elementary arch, $N_0 N_1$, then is defined as an arched column.

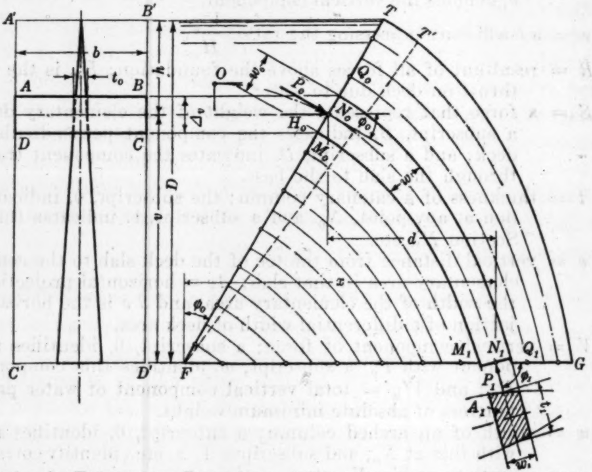


FIG. 1.—AN ELEMENTARY DECK AREA, *A B C D*, SUPPORTING A WATER LOAD.

Designating by f_c the unit compressive stress and by γ_c the unit weight of the arched column, the expression,

$$c = \frac{f_c}{\gamma_c} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

represents a parameter, the unit of which is a length. Furthermore, let the ratio, $\frac{v}{c}$, equal a. Designating by P_1 the resultant thrust of the arched column at N_1, and with reference to the dimensions of Fig. 1, the following relations can be derived,

$$\sec \phi_1 = \sec \phi_0 e^a \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$$

or,

$$P_1 = P_0 e^a \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3)$$

The horizontal distance, d, is given by the expression,

$$d = c\,(\phi_1 - \phi_0) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (4)$$

in which, the angles, ϕ_1 and ϕ_0, are measured in radians.

Designating by W_c the weight of the arched column between N_0 and N_1, and by H_0 the horizontal component of P_0; then, also,

$$W_c = H_0\,(\tan \phi_1 - \tan \phi_0) \ldots\ldots\ldots\ldots\ldots\ldots (5)$$

---

[7] "Higher Mathematics," by Mansfield Merriman and Robert S. Woodward, Third Edition, p. 147, John Wiley & Sons, 1902.

The top thickness of the elementary column becomes,

$$t_0 = \frac{b\,\gamma_w\,(D - v)}{f_c} \quad\text{.........................(6)}$$

Equation (5) can now be written in the form,

$$W_c = b\,\gamma_w\,(D - v)\left[\sqrt{e^{2a}\sec^2\phi_0 - 1} - \tan\phi_0\right]\Delta v \quad\text{.........(7)}$$

Multiple-Column Buttress of Uniform Strength

Assume that the whole deck strip, $A'\,B'\,C'\,D'$, is divided into a large number of elementary deck areas, and that the water pressure acting on each elementary area is transmitted by corresponding elementary arched columns, $Q_0\,Q_1$, $N_0\,N_1$, etc. If the cross-sectional area of the independent supporting units is shaped so that adjacent column faces just touch each other, the whole buttress can be conceived as built up of a large number of arched column units. Such a buttress is defined as a multiple-column buttress of uniform strength.

The cross-sectional area, A_1, of the elementary column at N_1 is found from Equation (3),

$$A_1 = \frac{P_1}{f_c} = \frac{P_0\,e^a}{f_c} \quad\text{.........................(8)}$$

If the width, w, of the column at any point is defined as the mean distance between the axis of adjacent column units, the column thickness at any point is definitely determined. Substituting,

$$v = m\,D \quad\text{............................(9)}$$

and designating the column thickness at N_1 by t_1, and assuming that $\frac{D}{c} = i$, the following expression results,

$$\frac{t_1}{b} = \frac{D\,\gamma_w\,(1 - m)\,e^{2\,mi}\sec^2\phi_0}{c\,\gamma_c\,[\tan\phi_0\sqrt{e^{2\,mi}\sec^2\phi_0 - 1} + 1]} \quad\text{............(10)}$$

From Equation (4), the horizontal distance, $x = F\,N_1$, and,

$$\frac{x}{D} = m\tan\phi_0 + \frac{1}{i}\,(\phi_1 - \phi_0) \quad\text{..................(11)}$$

The angle, ϕ_1, is found from Equation (2); ϕ_1 and ϕ_0 are expressed in radians.

By varying the factor, m, in Equations (10) and (11), the thickness of the buttress along the horizontal plane, $F\,G$ (Fig. 1), can be determined and by repeating the same process for other values of D it is possible to evaluate the shape of the entire buttress.

Equations (10) and (11) indicate that the shape of the buttress is mainly a function of the ratio, $\frac{D}{c}$ (or i), and the angle, ϕ_0. Considering ϕ_0 as a constant, buttresses with the same ratio, $\frac{D}{c}$, have a similar geometrical shape. From Equation (1),

$$\frac{D}{c} = \frac{D}{f_c}\,\gamma_c = i \quad\text{.........................(12)}$$

This important ratio is called the buttress ratio. Since γ_c is constant for a given case, the ratio, $\frac{D}{c}$, is simply a function of height and stress.

Fig. 2 gives an axonometric view of a monolithic buttress of uniform strength, with horizontal sections at different elevations computed by Equations (10) and (11). In this case, let c = 200 ft.; f_c = 200 lb. per sq. in.; γ_c = 144 lb. per. cu. ft.; and γ_w = 62.4 lb. per cu. ft. It will be noted that for D = 80 ft. the horizontal section approaches an isosceles triangle. As the height of the buttress increases the horizontal sections bulge out more and more, so that for D = 320 ft. and D = 400 ft., the maximum thickness, designated by $t_{\text{max.}}$, occurs at a considerable distance from the up-stream face. For D = 400 ft., $t_{\text{max.}}$ even exceeds the width of the deck strip.

Fig. 2 also shows the buttress ratio for the different values, D, computed by Equation (12) with the given values of γ_c and f_c. Since this ratio determines the shape of the buttress, Fig. 2 also represents the buttress shapes obtained with other unit stresses. Assuming, for instance, f_c = 400 lb. per sq. in., D = 320 ft., and γ_c = 144 lb. per sq. in., then the buttress ratio becomes 0.80. (See Equation (12).)

The corresponding buttress is, therefore, identical in shape with that of a buttress with D = 160 ft. and f_c = 200 lb. per sq. in., as shown.

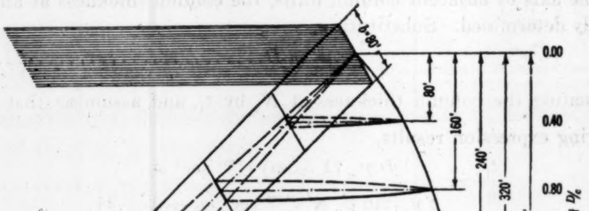


FIG. 2.—DIAGRAM SHOWING A MONOLITHIC BUTTRESS OF UNIFORM STRENGTH.

STRESSES IN THE MONOLITHIC BUTTRESS

The multiple-column buttress (Fig. 1) is built up of independent column units, which can deform freely under the applied load. Since the unit compressive stress is constant, each column will shorten in proportion to its length and the up-stream face moves into the deformed position, FT', which is almost a straight line. Opposite points along adjacent column faces, therefore, will closely retain their relative position after the deformation has taken place.

Neglecting the influence of Poisson's ratio, the same deformation of the buttress can evidently take place if adjacent column units are connected monolithically with each other, and no shearing forces along the imaginary column faces will be induced. Furthermore, in order to maintain equilibrium, no forces normal to the axis of the catenary columns are required.

Since the buttress is subject mainly to direct stresses in one direction the internal work must approach a possible minimum. From Castigliano's principle of least work it can then be concluded that the stress distribution in the monolithic buttress is very nearly the same as that in the multiple-column buttress composed of independent arched column units.

For practical purposes extreme accuracy is not required, because shrinkage and temperature stresses may easily destroy a theoretically refined stress analysis.

From these considerations the following conclusions are derived:

> (1) The catenaries of uniform strength represent principal stress trajectories;
> (2) The first principal stress, at any point of the buttress, is a constant compressive stress and equals the design stress, f_c; and,
> (3) The second principal stress is zero throughout.

It should be kept in mind that such a stress condition applies only to a monolithic buttress with dimensions determined as for Fig. 2. Any other distribution of the buttress material must necessarily produce a different system of principal stresses. The larger the deviation from the ideal shape, the more distorted the resulting stress system must be.

A buttress, approaching more or less uniform strength, could also be designed by the trial analysis. After the principal stresses have been determined by any of the current methods, the shape of the buttress must be corrected until the final analysis gives a practically uniform first principal stress and a zero second principal stress.

Fig. 3 shows a reproduction of the Vöhrenbach buttress,[8] the heavy lines indicating the principal stress trajectories. The magnitude of the principal stresses along the trajectories is shown by light lines, the plus sign indicating compression.

The striking similarity between the first principal stress trajectories in Fig. 3 and the catenaries of uniform strength in Fig. 1 will be noted. The magnitude of the first principal stress in Fig. 3 also indicates that a more uniform stress distribution could be obtained by decreasing the thickness toward the down-stream face and increasing the thickness toward the lower elevations near the up-stream face. After several trial computations a shape similar to a multiple-column buttress of uniform strength would result. The trial computations would involve considerable labor; the advantage of the direct method is evident.

---

[8] "Die Staumauer des Kraftwerkes Vöhrenbach," by Fritz Maier, *Beton und Eisen*, Vol. 23, 1924, p. 13.

INFLUENCE OF DECK WEIGHT

In analyzing the influence of the deck weight on the shape and weight of the buttress, two extreme cases of deck support are considered.

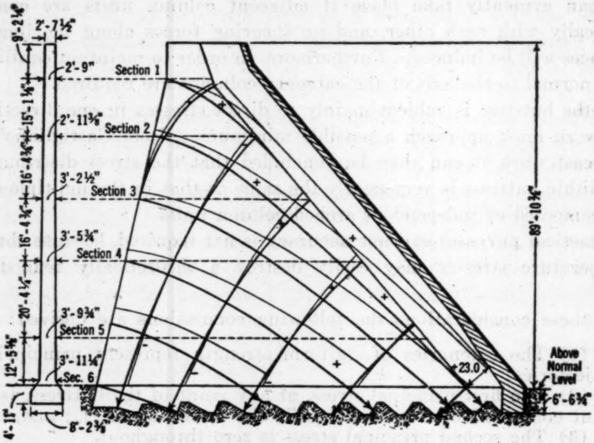

9'-2¼"
2'-7½"
2'-9"
Section 1
15'-1⅛"
2'-11⅜"
Section 2
16'-4⅜"
3'-2½"
Section 3
16'-4⅜"
3'-5¾"
Section 4
20'-4½"
3'-9¾"
Section 5
12'-5⅝"
3'-11¼"
Sec. 6
4'-11"
8'-2⅜"
89'-10⅜"
23.0
Above Normal Level
6'-6¾"

FIG. 3.—THE VOHRENBACH BUTTRESS.

Case 1.—Assume that the deck is completely separated from the buttress by a frictionless joint along the up-stream face of the buttress, as indicated in Fig. 4. In this case the initial top thrust of the catenary column, designated by P'_k, is composed of the water pressure, P_0, and the normal component, S_B, of the corresponding deck weight, S. The weight component, S_D,

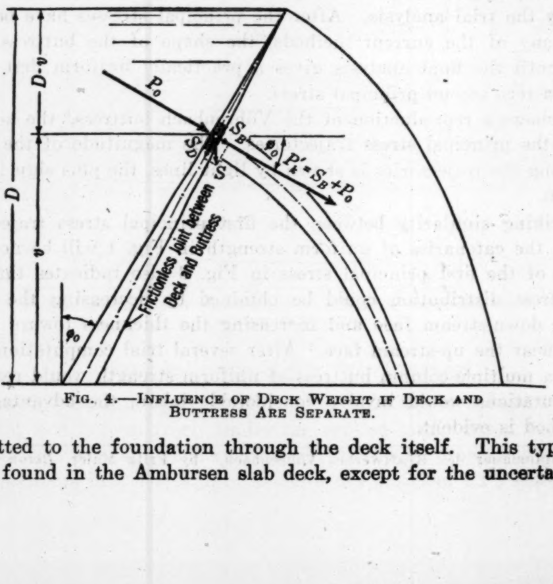

D - y
D
y
P_0
S_B
S_D
S
φ_0
$P'_k = S_B + P_0$
Frictionless Joint between Deck and Buttress
φ_0

FIG. 4.—INFLUENCE OF DECK WEIGHT IF DECK AND BUTTRESS ARE SEPARATE.

is transmitted to the foundation through the deck itself. This type of deck support is found in the Ambursen slab deck, except for the uncertainties due to friction forces, but the assumption of a frictionless joint will be on the safe side.

The weight of the deck is dependent upon the type, working stresses in the deck, span, etc. The relative deck weight is expressed by a factor, k, defined as follows,

$$k = \frac{S}{P_0} \dots\dots\dots\dots\dots\dots\dots\dots (13)$$

For a triangular deck weight distribution, here considered, the factor, k, is constant for any elevation, v. Since its value is also practically independent of the inclination, ϕ_0, of the deck, the factor, k, is called the deck constant. The initial top thrust, P'_k, of the catenary column then becomes

$$P'_k = P_0 (1 + k \sin \phi_0) \dots\dots\dots\dots\dots\dots\dots\dots (14)$$

Equations (1) to (12) also apply to Case 1 if P'_k is substituted for R_0, or by replacing the unit weight, γ_w, by the unit weight γ'_w; thus,

$$\gamma'_w = \gamma_w (1 + k \sin \phi_0) \dots\dots\dots\dots\dots\dots\dots\dots (15)$$

Case 2.—Assume that the elementary deck strip is to be monolithically connected with the corresponding catenary column, as shown in Fig. 5. Adjacent column units are assumed to be completely separated from each other by continuous joints through deck and buttress.

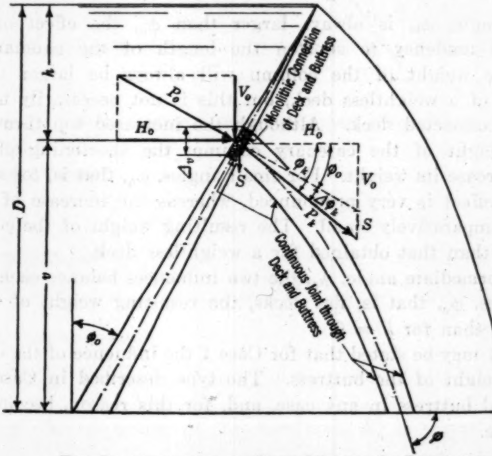


FIG. 5.—INFLUENCE OF DECK WEIGHT IF DECK AND BUTTRESS ARE MONOLITHIC.

In this case the full deck weight, S, is combined with the water pressure, P_0, and the resultant of these two forces, designated by P_k, is inclined at the angle, ϕ_k, to the horizontal. The angle, ϕ_k, is determined from the relation,

$$\tan \phi_k = \tan \phi_0 + k \sec \phi_0 \dots\dots\dots\dots\dots\dots\dots\dots (16)$$

For small values of ϕ_k the following approximate expression, measured in radians, can be derived,

$$\phi_k = \phi_0 + \frac{k \cos \phi_0}{1 + k \sin \phi_0} \ldots\ldots\ldots\ldots\ldots\ldots (17)$$

The shape of the buttress, as modified by the deck weight, is given by the following equations, the derivations of which are similar to Equations (10) and (11);

$$\frac{t}{b} = \frac{D\,\gamma_w\,(1 - m)\,e^{2mi}\sec^2\phi_k}{c\,\gamma_c\left(\tan\phi_0\sqrt{e^{2mi}\sec^2\phi_k - 1} + 1\right)} \ldots\ldots\ldots (18)$$

and,

$$\frac{x}{D} = m \tan \phi_0 + \frac{1}{i}\,(\phi - \phi_k) \ldots\ldots\ldots\ldots (19)$$

The angle, ϕ, is given by the relation,

$$\cos \phi = \frac{\cos \phi_k}{e^{mi}} \ldots\ldots\ldots\ldots\ldots\ldots (20)$$

It may be noted that for $k = 0$, or $\phi_k = \phi_0$, Equations (18) and (19) become identical with Equations (10) and (11) derived under the assumption of a weightless deck.

The weight of the arched column unit is given by the following expression, which is similar to Equation (7),

$$W_c = b\,\gamma_w\,(D - v)\left[\sqrt{e^{2m}\sec^2\phi_k - 1} - \tan\phi_k\right]\Delta v \ldots\ldots (21)$$

Since the angle, ϕ_k, is always larger than ϕ_0, the effect of the deck weight has the tendency to shorten the length of the catenary column. For Case 1 the weight of the column will always be larger than under the assumption of a weightless deck, but this is not necessarily true for the monolithically connected deck. Although the increased top thrust tends to increase the weight of the catenary column, the shortening effect has a tendency to decrease its weight. For small angles, ϕ_0, that is, for steep decks, the shortening effect is very pronounced, whereas the increase of the initial top thrust is comparatively small. The resulting weight of the column then is actually less than that obtained for a weightless deck.

At some intermediate angle, ϕ_0, the two influences balance each other, and for larger angles, ϕ_0, that is, flat decks, the resulting weight of the column becomes greater than for $k = 0$.

In general, it may be stated that for Case 1 the influence of the deck always increases the weight of the buttress. The type described in Case 2 gives a more economical buttress in any case, and, for this reason, becomes of practical importance.

WEIGHT FACTOR AND SLIDING FACTOR DIAGRAMS

The total weight of the buttress is the sum of the weights of all the elementary, arched columns, and is found by integrating Equation (21), which contains the differential equation of the total buttress weight. Since it is not possible to present the integral in explicit form, Equation (21) has been integrated by Simpson's rule.

The main purpose of any dam consists in providing a structure which will resist the horizontal component of the total water pressure. The weight of the structure, therefore, may be expressed conveniently as a factor of the horizontal water pressure. With reference to Fig. 6, the deck constant, k, is then given by the ratio,

$$k = \frac{W_D}{R_w} \text{.........................} (22)$$

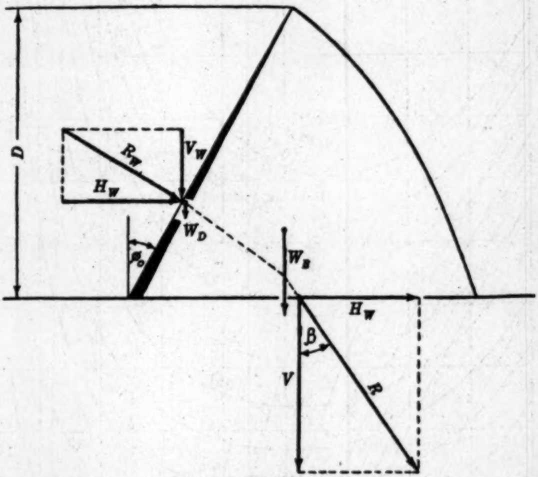


FIG. 6.

The weights of deck, buttress, and combined weights are defined by the following weight factors:

Deck weight factor:

$$\lambda_D = \frac{W_D}{H_w} = k \sec \phi_0 \text{.....................} (23)$$

Buttress weight factor:

$$\lambda_B = \frac{W_B}{H_w} \text{..........................} (24)$$

Combined weight factor:

$$\lambda_{BD} = \frac{W_D + W_B}{H_w} = \lambda_D + \lambda_B = k \sec \phi_0 + \lambda_B \text{.........} (25)$$

In addition, the direction of the resultant, R, is given by the sliding factor, F,

$$F = \tan \beta = \frac{H_w}{V} = \frac{1}{\tan \phi_0 + \lambda_{BD}} \text{...............} (26)$$

The deck weight factor, Equation (23), is a function of the deck constant, k, and the inclination, ϕ_0, of the deck. The buttress weight factor, Equation (24), is a function of k, ϕ_0, and the buttress ratio, i, defined by Equation (12).

The combined weight factor, Equation (25), is also a function of k, ϕ_0, and i, and the same is true for the sliding factor, F, Equation (26).

The weight and sliding factors can be presented in graphical form as functions of the variables, k, ϕ_0, and i, as shown in Figs. 7, 8, and 9.

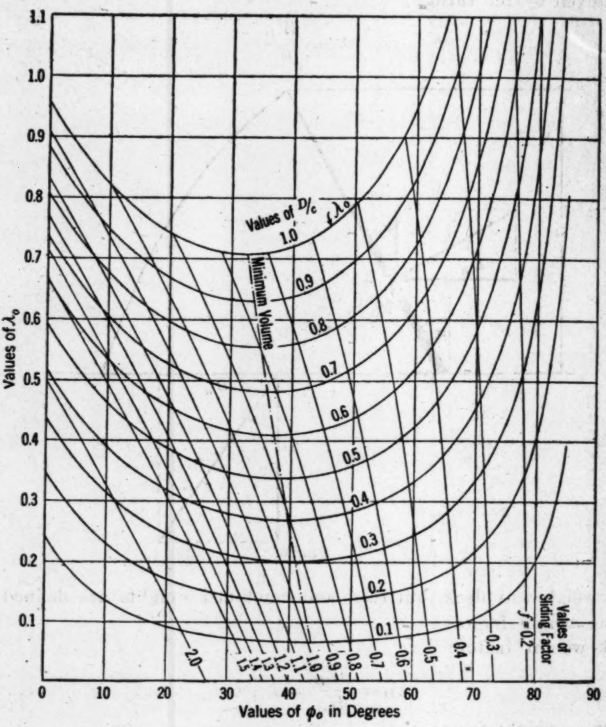


FIG. 7.—GRAPH FOR DETERMINING WEIGHT AND SLIDING FACTORS WHEN $k = 0$.

These diagrams illustrate the quantity laws of the buttressed dam of uniform strength for Case 2. It may be noted that the total volume becomes a minimum for deck inclinations ranging approximately from $\phi_0 = 20$ to $\phi_0 = 40$ degrees. However, the minimum quantities can only be realized in combination with excessively large sliding factors. The deck inclination, therefore, will be governed largely by the limiting sliding-factor values.

The buttress quantities for Case 1 can be obtained from Fig. 7, by computing the horizontal component of the water pressure from Equation (24), with an imaginary unit weight of water as obtained from Equation (15).

The diagrams can be used as guides in the design and for the purpose of preliminary estimates. They are equally applicable to any system of measurements, because the different factors represent pure ratios.

As an example, find the most economical deck inclination as well as the quantities in the deck and buttress of a hollow dam, with the following design data: $D = 200$ ft.; $b = 50$ ft.; $k = 0.10$; $f_c = 400$ lb. per sq. in.; $\gamma_c = 150$ lb. per cu. ft.; $\gamma_w = 62.5$ lb. per cu. ft.; and $F = 0.70$ (maximum permissible value).

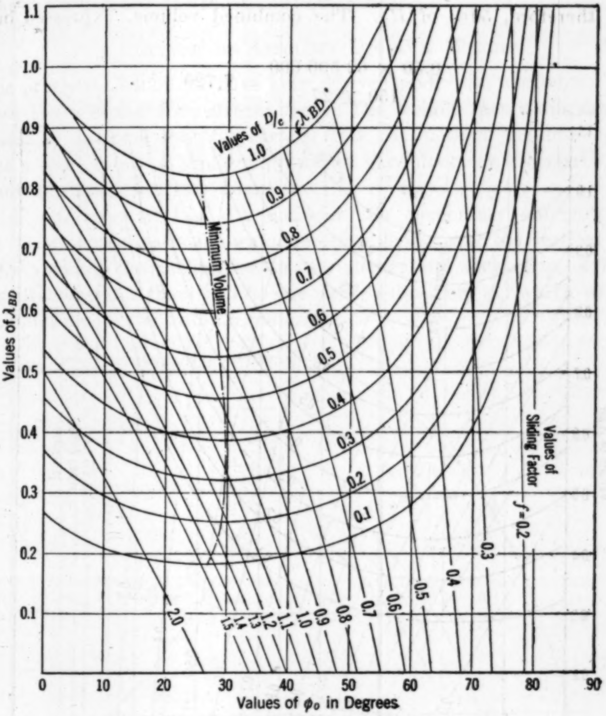


FIG. 8.—GRAPH FOR DETERMINING WEIGHT AND SLIDING FACTORS WHEN $k = 0.10$

By Equation (12) the buttress ratio is found to be 0.521. Entering Fig. 8, corresponding to the deck constant, $k = 0.10$, it will be noted that, for $\frac{D}{c} = 0.521$, the minimum quantities in the dam require a deck inclination, $\phi_0 = 29$ degrees. The combined weight factor, read at the intersection of $\frac{D}{c} = 0.521$ and $\phi_0 = 29°$, is $\lambda_{BD} = 0.47$. The same intersection gives a sliding factor, $F = 0.97$, which is too large.

In order not to exceed a value, $F = 0.70$, the deck inclination must be limited to $\phi_0 = 43°$; the corresponding combined weight factor is $\lambda_{BD} = 0.50$ which is only 6% larger than the possible minimum.

The total horizontal component of the water pressure, acting on the deck strip under consideration (in pounds), is,

$$H_w = \frac{50 \times 200^2 \times 62.5}{2} = 62\ 500\ 000$$

With $\lambda_{BD} = 0.50$, the combined weight of deck and buttress, from Equation (25), is, therefore, 50% of H_w. The combined volume, expressed in cubic yards, is,

$$\frac{0.50 \times 62\ 500\ 000}{150 \times 27} = 7\ 720$$

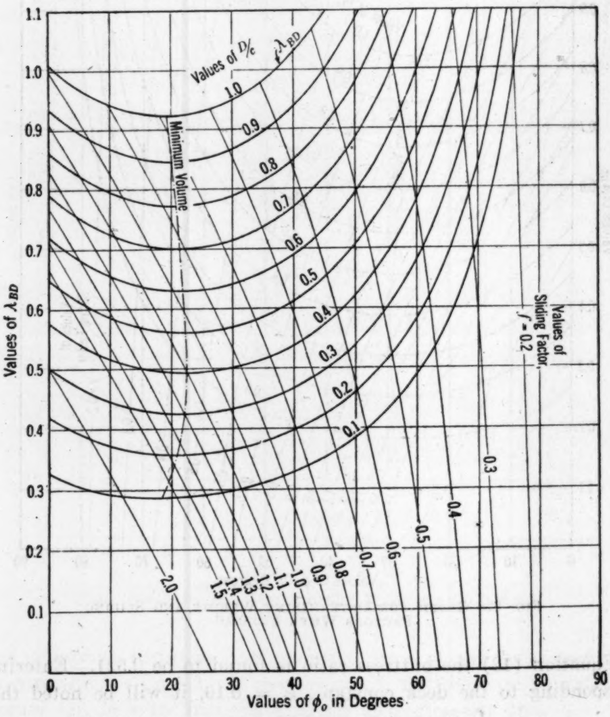


FIG. 9.—GRAPH FOR DETERMINING WEIGHT AND SLIDING FACTORS WHEN $k = 0.20$.

The quantities in the buttress alone are found from Equation (25), which gives the following percentage for the buttress volume,

$$\frac{\lambda_B}{\lambda_{BD}} = \left(1 - \frac{k \sec \phi_0}{\lambda_{BD}}\right) 100 = 72.6$$

These quantities represent the minimum possible volume that can be obtained with the given data. Because of the minimum practical wall thickness the actual quantities will be somewhat larger. If the buttress is provided with auxiliary stiffening structures, not designed as integral parts of the catenary columns, then the additional volume due to this cause must be added.

Generally, the larger the buttress ratio, $\frac{D}{c}$, the closer the agreement between the actual and the theoretical quantities.

BUTTRESS OF ABSOLUTE MINIMUM WEIGHT

The previous deductions have been derived under the assumption that the buttress has a straight up-stream face. The weight factor diagrams (Figs. 7, 8, and 9) indicate that for certain deck inclinations the total quantities become a minimum. A straight up-stream face does not necessarily give the absolute minimum buttress quantities.

With reference to Fig. 10, consider the elementary catenary column, $A\,B\,C\,D$, which supports the water pressure acting over the vertical distance, dv. By varying the inclination of the elementary deck strip, AB, and by keeping dv constant, there will be one deck inclination, ϕ_0, which will require a minimum amount of column material.

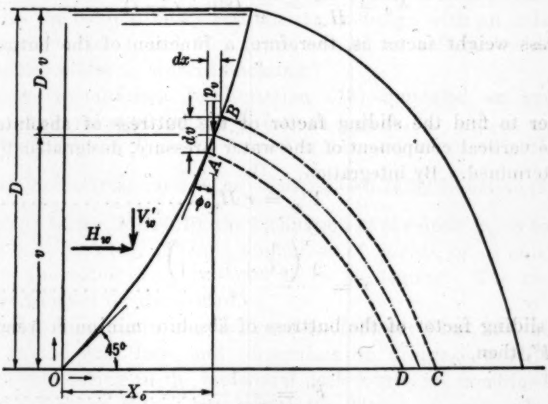


FIG. 10.

In case of a weightless deck this inclination can be determined by differentiating the column weight, W_c, Equation (7), with respect to ϕ_0, which leads to the following relation,

$$\tan\phi = \frac{1}{e^{a}} \qquad (27)$$

Equation (27) shows that the most economic deck slope is a function of the elevation, v, and the parameter, c $\left(\text{since } a = \frac{v}{c}\right)$. At the bottom, $v = 0$, the deck inclination is $\phi_0 = 45$ degrees. As v increases, the deck becomes steeper and steeper.

Equation (27) also represents the differential equation for the up-stream curve of the buttress, because tan ϕ_0 is identical with the first derivative of x_0 with respect to v. By integration the following relation is obtained:

$$x_0 = c\left(1 - \frac{1}{e^a}\right) \dots\dots\dots\dots\dots\dots\dots (28)$$

By substituting $v = mD$, from Equation (9), Equation (28) can be written in the form,

$$\frac{x_0}{D} = \frac{1}{i}\left(1 - \frac{1}{e^{mi}}\right) \dots\dots\dots\dots\dots\dots\dots (29)$$

Equation (29) shows that the buttress ratio, $\frac{D}{c} = i$, entirely determines the curvature of the up-stream face.

By substituting ϕ_0, from Equation (27), in Equation (7), the minimum weight of the elementary column is obtained. The total weight of the buttress, designated by W'_0, is found by integrating over the total height, D. Designating by λ'_0 the weight factor of the buttress of absolute minimum weight, then,

$$\lambda'_0 = \frac{W'_0}{H_w} = \frac{4}{i^2}\,(\sin h\, i - i) \dots\dots\dots\dots\dots\dots (30)$$

The buttress weight factor is, therefore, a function of the buttress ratio, i, or $\frac{D}{c}$, only.

In order to find the sliding factor of the buttress of absolute minimum weight, the vertical component of the water pressure, designated by V'_w, must be first determined. By integration,

$$V'_w = r\, H_w \dots\dots\dots\dots\dots\dots\dots\dots (31)$$

in which,

$$r = \frac{2\left(e^{\frac{1}{i}} + i - 1\right)}{i^2} \dots\dots\dots\dots\dots\dots\dots (32)$$

If the sliding factor of the buttress of absolute minimum weight is designated by F', then,

$$F' = \frac{1}{r + \lambda'_0} \dots\dots\dots\dots\dots\dots\dots\dots (33)$$

A comparison with the straight-face buttress shows that the difference between the minimum quantities for $i < 1.0$ is less than 1 per cent. However, the buttress of absolute minimum weight has a sliding factor which is appreciably less than that of the straight-face buttress. The comparison becomes more favorable as the buttress ratio, $\frac{D}{c}$, increases. If the buttress is built with independent column units completely separated from each other by continuous joints along adjacent faces, then the individual bottom columns of Fig. 10 will have a maximum sliding factor value, $F = 1.0$.

With the same quantity of material, the stability of a buttress of absolute minimum weight is superior to that of a straight-face buttress under all conditions.

The influence of the deck, monolithically connected with the corresponding column head, is similar in effect to that derived for a straight up-stream face. The minimum weight of deck and catenary column unit combined is best determined by trial by varying the angle, ϕ_0. If the resulting sliding factor is too large, then the slope of the deck must be decreased until safe sliding-factor values are obtained.

In narrow V-shaped canyons, where the heights of adjacent buttresses change rapidly, the application of the foregoing principles may lead to practical difficulties. Under these conditions the deck of the dam would assume a warped surface, with lateral thrusts acting on the buttress heads.

In case of comparatively wide cross-sections the warping effect is small, so that the foregoing features may be advantageously incorporated in the design of the dam.

DESIGN EXAMPLE

Fig. 11 represents a tentative design for a buttressed dam of uniform strength, consisting of five catenary column units, completely separated from each other by continuous joints extending through deck and buttress system.

The deck consists of symmetrical cantilever beams or disks, monolithically connected with the column heads and similar in design with an ordinary wall footing. Columns No. 1 and No. 2 are treated as cellular sections, built up of rectangular plates to prevent buckling.

Preliminary computations by Equation (13) indicated an approximate deck weight factor value of from $k = 0.10$ to $k = 0.15$.

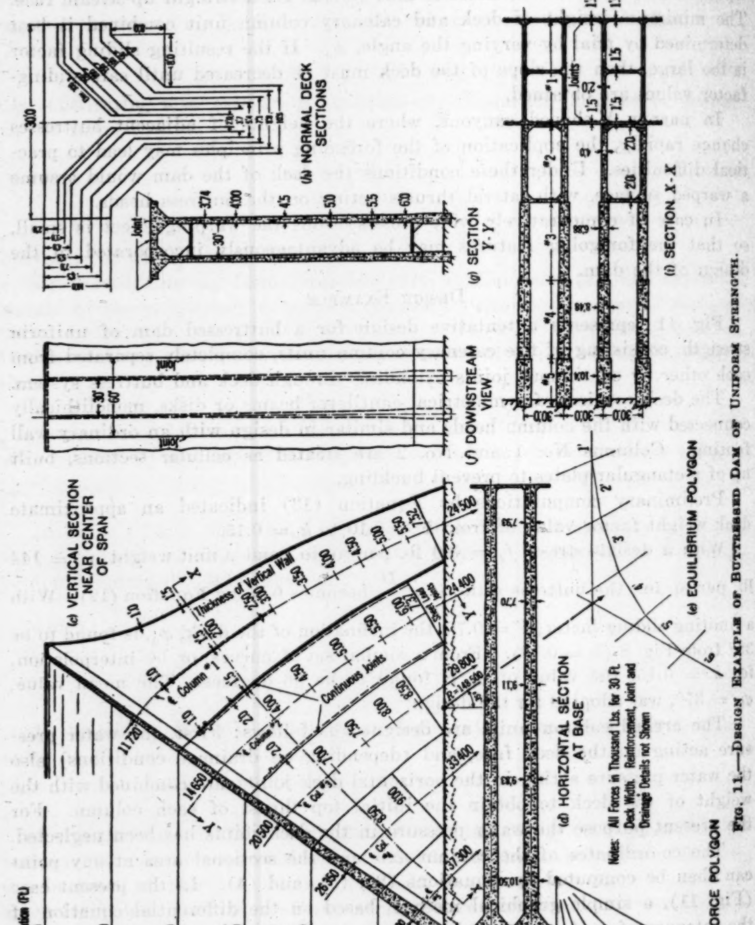
With a design stress, $f_c = 400$ lb. per sq. in., and a unit weight, $\gamma_c = 144$ lb. per sq. in., the buttress ratio, i, or $\frac{D}{c}$, becomes 0.75 by Equation (12). With a limiting sliding factor, $F = 0.70$, the inclination of the deck, ϕ_0, is found to be 38° from Fig. 8 ($k = 0.10$). From a similar set of curves, or by interpolation, for $k = 0.15$, the value of ϕ_0 is found to be 36 degrees. The mean value, $\phi_0 = 37°$, was adopted for the design.

The arched column units are designed as follows: First, the water pressure acting on the deck face, and (depending on drainage conditions) also the water pressure acting in the horizontal deck joints, are combined with the weight of the deck to obtain the initial top thrust of each column. For the present purpose the water pressure in the deck joints has been neglected.

The co-ordinates of the column axis and the sectional area at any point can then be computed by Equations (2), (3), and (4). In the present case (Fig. 11), a simple graphical method, based on the differential equation of the catenary of uniform strength, has been used. This method is illustrated in Fig. 12 in which P_0 represents the given top thrust, applied at N_0, and acting at the given angle, ϕ_0, to the horizontal. The horizontal through N_0 is divided into equal intervals, and the verticals, AB, CD, EF, etc., are drawn as shown.

The small angle, $\Delta\phi$, expressed in radians, is given by the equation,

$$\Delta\phi = \frac{\Delta x}{f_c}\gamma_c \qquad (34)$$


(a) UPSTREAM VIEW
Joint
30'
(b) FORCE POLYGON
Column Reactions at Base
R
1
2
3
4
5
6
(c) VERTICAL SECTION NEAR CENTER OF SPAN
Elevation (Ft.)
300
250
200
150
100
50
Thickness of Vertical Wall
Column #1
Continuous Joints
11,720
14,650
20,500
26,350
32,200
37°
31,300
33,400
29,800
24,400
24,500
R = 149,500
(d) HORIZONTAL SECTION AT BASE
(e) EQUILIBRIUM POLYGON
(f) DOWNSTREAM VIEW
(g) SECTION Y-Y
(h) NORMAL DECK SECTIONS
(i) SECTION X-X
Joints
30.0'

Notes: All Forces in Thousand Lbs. for 30 Ft. Deck Width. Reinforcement, Joint and Drainage Details not Shown.

FIG. 11.—DESIGN EXAMPLE OF BUTTRESSED DAM OF UNIFORM STRENGTH.

Assuming $\Delta x = 20$ ft., $f_c = 400$ lb. per sq. in., and $\gamma_c = 144$ lb. per sq. in., then $\Delta\phi = 0.05$.

Since P_0 is tangent at the point, N_0, the construction must begin with $\frac{\Delta\phi}{2}$, as indicated in Fig. 12, and N_1 then is the first point on the catenary of uniform strength. Additional points, N_2, N_3, N_4, etc., are found by equal deflections, $\Delta\phi$, as shown. A continuous curve drawn through N_0, N_1, N_2, N_3, etc., then represents the column axis. The thrust at any point is best determined from Equation (3) by means of a log log slide-rule. Its direction is found graphically by considering that the horizontal component is constant and equal to the horizontal component of P_0.

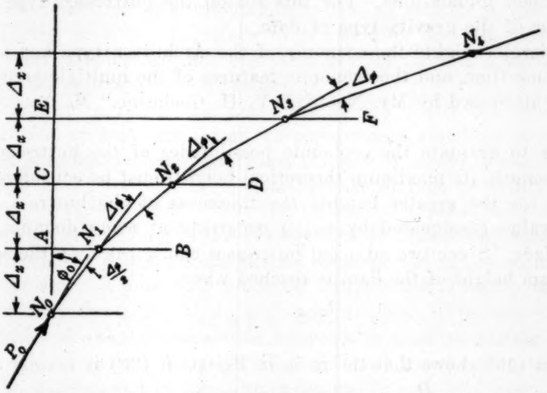


FIG. 12.—GRAPHICAL METHOD OF DETERMINING THE CO-ORDINATES OF THE COLUMN AXIS AND THE SECTIONAL AREA AT ANY POINT.

The resultant of all the column thrusts at the base and its direction are determined by a force and equilibrium polygon, as shown in Fig. 11(b) and Fig. 11(c).

For high dams the influence of secondary stresses due to shrinkage, temperature changes, yielding of the foundation, etc., deserves careful consideration in design and construction. In order to reduce the danger from this source, Mr. Jakobsen has advised[8] contraction joints in buttresses and arches of high multiple-arch dams.

The arched-column principle lends itself without difficulty to the elimination of secondary stresses because the logical location of such joints is given by the column outlines. In addition, the bottom joints, shown in Fig. 11, allow slight rotation of the individual columns, whereby the secondary stresses are further reduced. In certain cases temporary hinges, as used in arch bridges, may be provided near the foundation. Each column unit thus represents a statically determinate member, the action of which is definitely known under all conditions of loading.

Temperature changes in the interior of the dam are reduced to a minimum by the designing column, No. 1 (Fig. 11 (*d*)) so that complete enclosure of the structure is obtained. In severe climates this feature also effectively prevents freezing of the deck from the down-stream side, as reported[9] by F. W. Scheidenhelm, M. Am. Soc. C. E.

The down-stream face can also be designed as an overflow section, with most of the material acting as a column member. In locations subject to earthquake shocks, all the columns may be designed as adjacent cellular units, whereby it is possible to increase the lateral stability to any desired extent.

ECONOMIC CONSIDERATIONS

Buttressed dams are especially adapted for comparatively wide dam sites with solid rock foundations. For this reason the buttressed type is mainly a competitor of the gravity type of dam.

For moderate heights the economy of the Ambursen type has been recognized for some time, and the economic features of the multiple-arch type have been amply discussed by Mr. Noetzli,[10] V. H. Cochrane,[11] M. Am. Soc. C. E., and others.

In order to evaluate the economic possibilities of the buttressed dam of uniform strength, its maximum theoretical height must be considered. Fig. 2 shows that for the greater heights the thickness of the buttress assumes a maximum value (designated by $t_{\text{max.}}$) occurring at some distance from the up-stream face. Since two adjacent buttresses can not occupy the same space, the maximum height of the dam is reached when,

$$\frac{t_{\text{max.}}}{b} = 1.0 \ldots\ldots\ldots\ldots\ldots\ldots (35)$$

Equation (10) shows that the ratio in Equation (35) is mainly a function of the buttress ratio, $\frac{D}{c}$. Designating the maximum height of the dam by $D_{\text{max.}}$, then by interpolation,

$$\frac{D_{\text{max.}}}{c} = 1.75 \ldots\ldots\ldots\ldots\ldots\ldots (36)$$

Assuming, for instance, a design stress of $f_c = 500$ lb. per sq. in., and a unit weight of $\gamma_c = 144$ lb. per cu. ft., the maximum theoretical height, in feet, becomes,

$$D_{\text{max.}} = 1.75 \times \frac{500 \times 144}{144} = 875$$

The maximum practical height is somewhat lower, due to the deck weight, but it may be concluded that the buttressed dam of uniform strength can be built to considerable heights with ordinary working stresses.

The relative volume of a buttressed dam and that of a gravity dam are best compared by means of the weight factors. The weight factor of a gravity section, designed for full uplift at the up-stream face, with uniform decrease to zero at the down-stream face, is designated by λ_g. Using the previous designations for the unit weight of water and concrete,

---

[9] *Transactions*, Am. Soc. C. E., Vol. 89 (1926), p. 772.

[10] *Loc. cit.*, Vol. LXXXVII (1924), p. 342.

[11] *Loc. cit.*, p. 370.

$$\lambda_g = \frac{\gamma_c}{\gamma_w} \sqrt{\frac{\gamma_w}{\gamma_c - \gamma_w}} \ldots\ldots\ldots\ldots\ldots\ldots (37)$$

With $\gamma_w = 62.5$ lb. per cu. ft. and $\gamma_c = 144$ lb. per cu. ft., $\lambda_g = 2.1$. The weight factor of the gravity section is practically constant for usual heights.

Figs. 7, 8, and 9 indicate that the combined weight factor of the buttressed dam increases about in direct proportion with the buttress ratio, $\frac{D}{c}$, but for usual sliding factors its value is only a fraction of λ_g. For the tentative design of Fig. 11 the combined weight factor value is, $\lambda_{BD} = 0.68$, or about one-third that of a gravity section. Therefore, even assuming that the unit cost of the buttressed dam is twice the unit cost of a gravity section, a saving of about one-third in the concrete cost is realized.

As the height increases the unit cost of the buttressed type decreases, because the cost of deck and formwork represents a smaller and smaller percentage, and the elimination of secondary stresses by means of expansion joints and complete enclosure results in considerable savings of reinforcing steel. Preliminary estimates indicate that the buttressed type of uniform strength will be superior in economy to the gravity dam in almost any case.

The independent column units permit construction to the ultimate height in convenient steps at long-time intervals, without the necessity of anchoring green concrete to the older concrete already in place. The uncertain action, due to different shrinkage and elastic behavior, is thus completely eliminated. At the same time spillway provisions for different construction steps may be conveniently incorporated in the column design.

Since the column units are statically determinate, their stress conditions and factors of safety are definitely known in advance for any contemplated height. To estimate the actual stress condition in the large masses of high gravity dams, even with the provision of vertical contraction planes, is not a simple problem, because successive layers are poured at comparatively slow rates, so that the lower portions may shrink and cool, while the upper portions expand due to chemical heat. It is also evident that the buttressed dam presents more favorable curing conditions because of the possibility of installing effective sprinkling systems, which will not only reduce the shrinkage, but also the setting temperatures.

Considering economy and safety combined it may be stated that the buttressed dam of uniform strength can be built to greater heights than the gravity dam.

CONCLUSIONS

From evidence presented in this paper, the writer has come to the following conclusions:

(1).—The arched column principle presents a simple and exact means for designing buttressed dams of uniform strength.

(2).—In case of monolithic buttresses designed by this method the first principal stress is constant compression and the second principal stress is zero.

(3).—Secondary stresses are practically eliminated by building the arched columns as independent, statically determinate units, separated from each other by continuous joints extending through deck and buttress system and complete enclosure of the dam.

(4).—The buttressed dam of uniform strength contains the minimum possible amount of material that can be obtained with a given design stress.

(5).—The exact quantities and design elements can be obtained from diagrams, in which the main variables are expressed as pure ratios.

(6).—The independent arched column units permit construction of the dam in economical steps at long-time intervals—no anchorage between the old and new concrete is required, and spillway facilities may be incorporated in each step with no waste of material for this purpose.

(7).—The buttressed dam of uniform strength is superior in safety and economy to the gravity type for any height to which a concrete dam can be built.

ACKNOWLEDGMENTS

The writer deeply appreciates the encouraging assistance and valuable suggestions received during the preparation of the original manuscript from Mikael Juul Hvorslev, Assoc. M. Am. Soc. C. E., Walter Ruppel, Assoc. M. Am. Soc. C. E., and A. C. Schwager, Test Engineer, Pacific Electric Manufacturing Company, San Francisco, Calif.

DISCUSSION

JAMES GIRAND,[12] ASSOC. M. AM. SOC. C. E. (by letter).—The design of large massive structures has fallen short of the precision developed by the steel designers. There has been much groping for the fundamentals underlying the design of these monolithic masses, in which the weight of the structure is the sole resistance to the load. The author has presented a method, complete in every detail, which appears to be the solution so long sought. Difficulties of design and construction have pointed to a buttress of this shape. Other attempts have been made to approach this method of design, including a dam on the Choapa Project in Chile, in whch a graphical method was used instead of the analytic.

In discussion of this paper, two points may be emphasized. The first deals with the factor of safety of the structure, and the second, with the inclined open joints.

Factor of Safety.—It is apparent that in the author's development of the unit column theory and the use of γ_c = 150 lb. per cu. ft., that each unit column is in equilibrium with the water load balanced against its own weight. A decrease of load will allow part of the weight to rest on the underlying columns. An increase in load will overturn the entire column. In other words, the factor of safety of the structure against overturning is 1.0. This is due to the design, considering the full weight of the concrete as effective. Any desired safety factor can be secured by assuming the proper proportion of the concrete to be resisting the load. Various investigations by the writer on the trajectories of stress and the principle of least work have led to the conclusion that at the point of least work the stresses will distribute themselves so that about two-thirds of the weight of the structure will be useful in resisting the load. This gives a logical distribution of stress and a factor of safety of 1.5. This use of two-thirds of the unit weight of the concrete in the design will not materially increase the total quantity of concrete in the buttress; it will change the shape of the catenaries and slightly lengthen the buttress, but it will not increase the thickness. A high factor of safety against overturning is not essential, as the maximum load on a dam is well fixed. If an ample allowance is made for maximum high water, then the only additional load which could occur, would be an earthquake load which is generally accepted at 0.1 g.

Inclined Open Joints.—In his paper, the author indicates continuous joints between adjacent column units. This method of putting joints on the lines of first principal stress is being widely accepted, and the Coolidge Dam is an example of buttresses so constructed. This technique offers several inducements to the designer. It simplifies the units of design and permits more accurate use of the older methods. From the construction standpoint, it localizes shrinkage cracks and directs them so as to do the least harm to the structure. The latter is probably the principal excuse for introducing planes of weakness into a structure.

---

[12] Engr., Arizona Hydro-Elec. Syndicate, Phœnix, Ariz.

Why should designers intentionally remove some of the inherent strength of the structure? Many dams show considerable tension along the lines of second principal stress, and joints on the lines of first principal stress would seriously weaken the structure. This ability of the concrete to take tension is an additional factor of safety and should not be sacrificed. There are other reasons why the division into columns does not accomplish the best results:

(*a*) If the design is absolutely correct, the first principal stresses uniform, and the second principal stresses zero, it is immaterial whether or not, a joint is present, therefore, it may as well be omitted.

(*b*) The lines of stress will not conform to the shape of the column except at exactly the designed head. If the columns are totally free, this will lead to bending stresses and other undesirable conditions. If the columns are in contact with the adjacent ones, the stresses will be transmitted across the joint, in which case, it would be preferable to have no joint.

(*c*) In any case of a laminated body supporting a load, it is better to put the joints parallel to the lines of second principal stress rather than to the first. This might be exemplified by considering a foot cube of steel necessary to support a load. Such a cube would undoubtedly be made of plates lying flat and not standing on edge.

All these premises lead to the conclusion that no joints should be used; but how should one deal with the concrete as it is being placed to-day? Shrinkage undoubtedly occurs and even the finest structures with exceptionally high test concrete may have high initial tension due to it. Even if no cracks appear, the tension will radically affect the stresses and make futile the labors of the designer.

As the concrete must be placed in units, these units should be bounded by the lines of first and second principal stress.

The ideal way to place concrete would be in diamond-shaped units, as shown in Fig. 13. As the concrete sets and shrinkage takes place, the entire unit will settle down into the notch. This should relieve all tension in the unit and might actually cause a small initial compression. This method would not cause as many construction difficulties as would appear at first; the forms could be left open on the top edges until they were half full; strips nailed inside the forms would support short lengths of sheeting put in place as the pour rises. To bound the pour by the lines of first and second principal stress approaches this ideal method and allows the unit poured to free itself of some of the shrinkage stresses. If it is still considered desirable to introduce continuous joints to take up any movements and control cracks, the joints should be introduced along the lines of second principal stress instead of the first. In this location, they will cause the least weakening of the structure.

The idea of the horizontal construction joint with large form units, which may be sluiced full of concrete, is difficult to dispel. It is a fact, however, that at many dams the bed-rock has been stepped, cleaned, and treated in every manner to assure a good bond and to prevent sliding, while a few feet above it, there is a horizontal construction joint which does not have one-tenth the strength of the bed-rock joint.

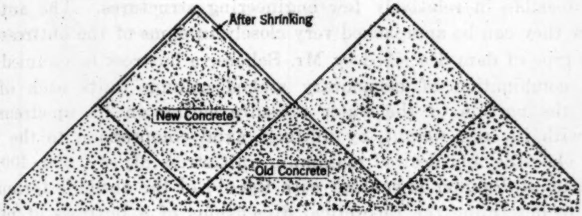

After Shrinking
New Concrete
Old Concrete

FIG. 13.—AN IDEAL WAY TO PLACE CONCRETE.

The buttress of absolute minimum weight, as designed by the author, has many advantages. As pointed out, the sliding factor is less than that for a straight-faced buttress. The curved up-stream face is also advantageous if a multiple-arch deck is used, as the curve will cause the arches to be of constant intrados. The writer suggests modifications, as shown in Fig. 14. These

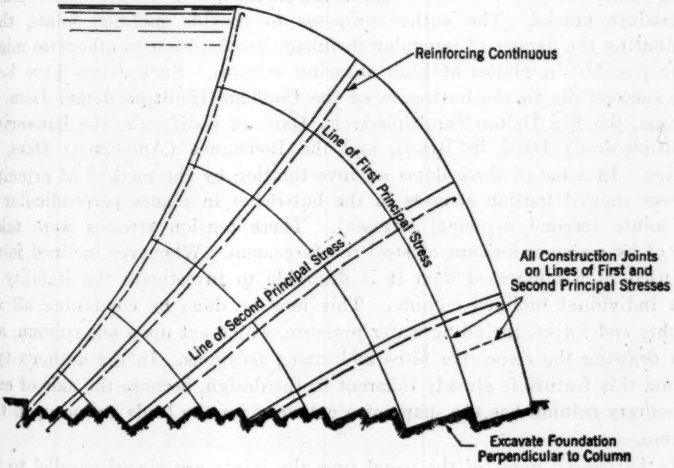

Reinforcing Continuous
Line of First Principal Stress
Line of Second Principal Stress
All Construction Joints
on Lines of First and
Second Principal Stresses
Excavate Foundation
Perpendicular to Column

FIG. 14.

principally involve: (1) Using a unit weight of about two-thirds that of the concrete for γ_c; (2) the specification that the concrete units of pour shall be bounded by the lines of principal stress; and (3) the use of continuous reinforcement from the bed-rock through to the down-stream face of the buttress.

Fred A. Noetzli,[13] M. Am. Soc. C. E. (by letter).—It is a well established fact that the highest degree of economy of material in a structure is obtained when this can be designed so that for full load the material is subjected at all points to the same maximum permissible unit stress. Such ideal conditions are feasible in relatively few engineering structures. The author has shown how they can be approached very closely in dams of the buttressed type.

In the type of dam described by Mr. Schorer a buttress is assumed to consist of a combination of elementary arched column units each of which transmits the portion of the water pressure acting on its up-stream deck, together with its own weight, by uniform axial compression to the foundation. By choosing any desired maximum unit stress of, say, 300, 400, or 500 lb. per sq. in., a designer is enabled with the aid of the author's formulas and diagrams to determine the theoretical dimensions of a buttress of minimum volume for any given height and spacing. Only the tallest buttress at a given dam site must be determined in this manner. The buttresses of lesser height are then merely portions of the tallest buttress cut off at the corresponding elevations.

The theoretical shape of a buttress of uniform strength, such as that shown in Fig. 2, requires some modification for practical applications, for instance, such as that incorporated in the author's design example shown in Fig. 11.

In buttresses of considerable length the shrinkage of the concrete is likely to produce cracks. The author proposes to provide inclined joints, thus eliminating the danger of irregular shrinkage cracks, such as otherwise might occur possibly in planes of high shearing stresses. Such joints have been used successfully in the buttresses of the Coolidge (multiple-dome) Dam, in Arizona, the Big Dalton (multiple-arch) Dam, in California, the Bananeiras (multiple-arch) Dam, in Brazil, and the Rodriguez (Ambursen) Dam, in Mexico. In some of these dams an investigation by the method of principal stresses showed tension stresses in the buttresses in planes perpendicular to the joints (second principal stresses). These tension stresses were taken care of by a proper amount of steel reinforcement. Whenever inclined joints are used in a buttressed dam it is desirable to investigate the stability of each individual inclined column. This may be done by combining all the weights and forces, including water pressure, which act upon one column, and then drawing the respective force and string polygons. In the author's type of dam this feature is already inherent in the design, because the axis of each elementary column has the shape of a catenary for the loads acting upon this column.

In buttressed dams of the usual type the joints are placed parallel to the direction of the first principal stresses because in these planes the shear stresses are zero. Each individual inclined column must be stable by itself. The safety against overturning can be increased materially at relatively small expense by placing reinforcing bars parallel to and immediately beneath the up-stream deck.

---

[13] Cons. Hydr. Engr., Los Angeles, Calif.

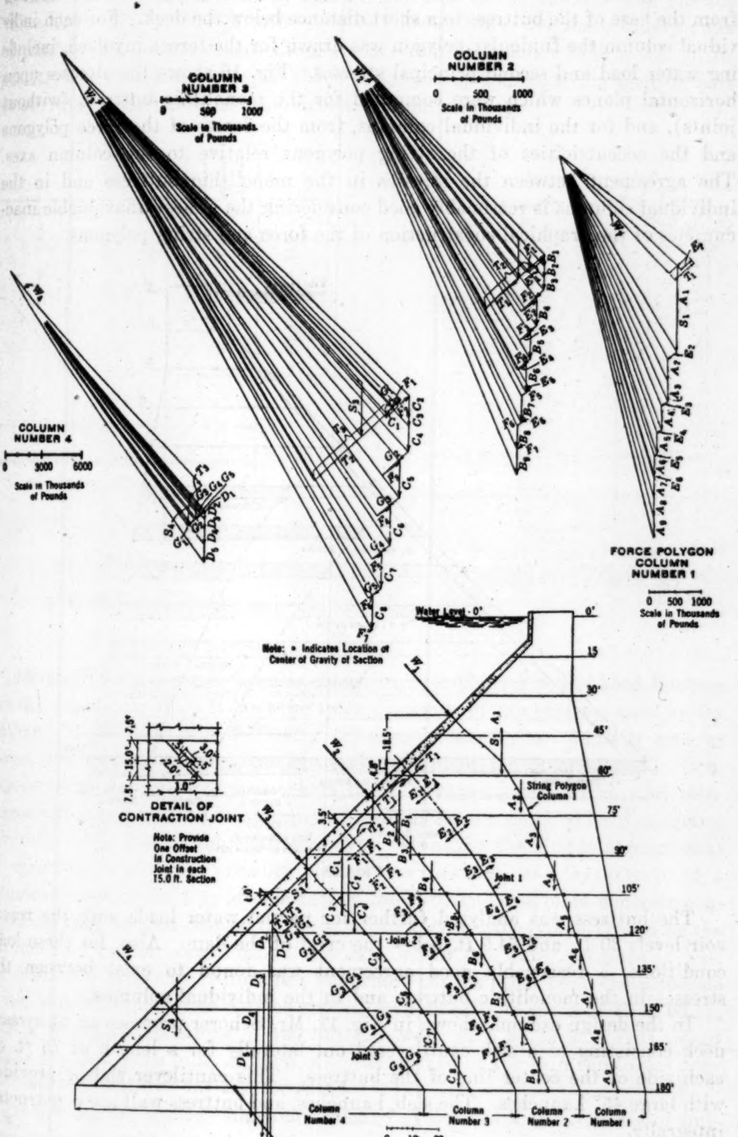

COLUMN NUMBER 3
0 500 1000
Scale in Thousands of Pounds
COLUMN NUMBER 2
0 500 1000
Scale in Thousands of Pounds
COLUMN NUMBER 4
0 3000 6000
Scale in Thousands of Pounds
FORCE POLYGON COLUMN NUMBER 1
0 500 1000
Scale in Thousands of Pounds
Note: • Indicates Location of Center of Gravity of Section
Water Level - 0'
0'
15
30'
45'
60'
75'
90'
105'
120'
135'
150'
165'
180'
String Polygon Column 1
Joint 1
Joint 2
Joint 3
Column Number 4
Column Number 3
Column Number 2
Column Number 1
0 10 20
Scale in Feet
DETAIL OF CONTRACTION JOINT
Note: Provide One Offset in Construction Joint in each 15.0 ft. Section
7.5'
15.0'
7.5'
3.0'
1.0'
18.5'
4.8'
3.5'
1.6'

FIG. 15.

In Fig. 15 there are three inclined joints extending upward and forward from the base of the buttress to a short distance below the deck. For each individual column the funicular polygon was drawn for the forces involved, including water load and second principal stresses. Fig. 16 shows the stresses upon horizontal planes which were computed for the monolithic buttress (without joints), and for the individual columns, from the forces of the force polygons and the eccentricities of the string polygons relative to the column axes. The agreement between the stresses in the monolithic buttress and in the individual columns is reasonably good considering the almost unavoidable inaccuracies of the graphical construction of the force and string polygons.

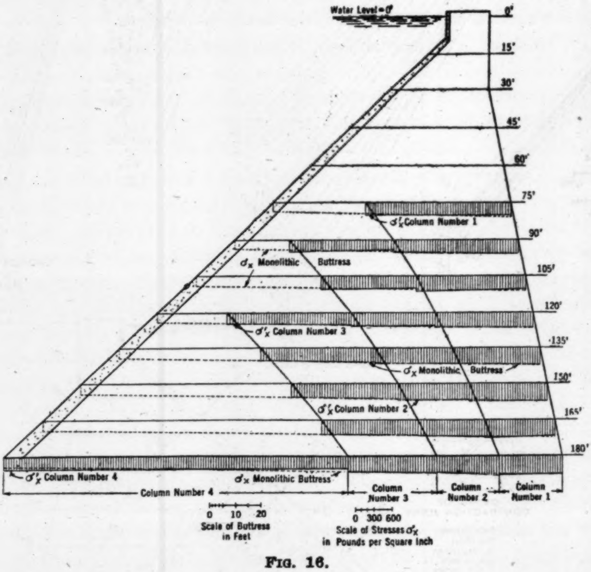
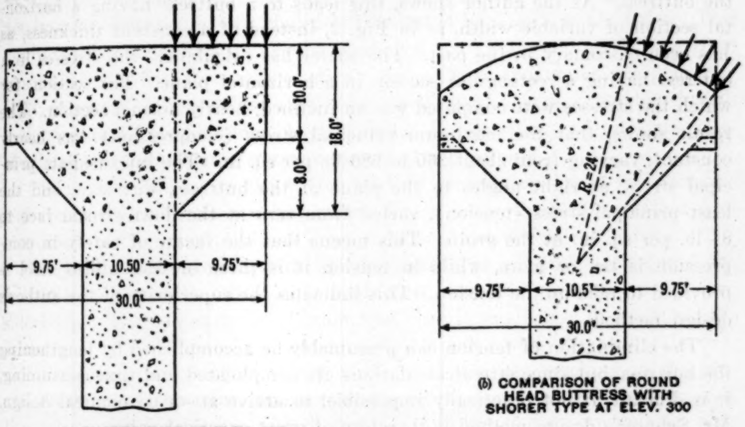

Water Level = 0'
0'
15'
30'
45'
60'
75'
90'
105'
120'
135'
150'
165'
180'
σ'_x Column Number 1
σ_x Monolithic Buttress
σ'_x Column Number 3
σ_x Monolithic Buttress
σ'_x Column Number 2
σ'_x Column Number 4
σ_x Monolithic Buttress
Column Number 4
Column Number 3
Column Number 2
Column Number 1
0 10 20
Scale of Buttress in Feet
0 300 600
Scale of Stresses σ_x in Pounds per Square Inch

FIG. 16.

The buttress was analyzed further for partial water loads with the reservoir levels 30 ft. and 54.5 ft. below the crest of the dam. Also, for these load conditions, a reasonably good agreement was found to exist between the stresses in the monolithic buttress and in the individual columns.

In the design example shown in Fig. 11, Mr. Schorer proposes an up-stream deck consisting of a slab cantilevered out laterally for a length of 15 ft. on each side of the center line of the buttress. This cantilever slab is provided with large 45° haunches. The slab, haunches, and buttress wall are constructed integrally.

The normal sections of the deck slab at various elevations are shown by the author in Fig. 11(*h*). For instance, at the 300-ft. depth below the crest the normal section has the dimensions shown in Fig. 17(*a*). The bending moment per foot in slab and haunch on one side of the buttress wall has the value, $M = 62.5 \times 300 \times 9.75^2 \times \frac{1}{2} = 890\,000$ ft-lb. By the ordinary rules of reinforced concrete design the necessary reinforcement would be 3.5 sq. in. per ft. The maximum shear is about 80 lb. per sq. in., requiring bent-up bars or stirrups to take care of the diagonal tension stresses.



(a) SHORER TYPE AT ELEV. 300

(b) COMPARISON OF ROUND HEAD BUTTRESS WITH SHORER TYPE AT ELEV. 300

FIG. 17.

By way of comparison there is shown in Fig. 17(*b*) a round-head buttress for the same span of 30 ft. and the same thickness of the buttress used by the author. In such a case the water pressure is transmitted radially and by direct compression through the buttress head upon the buttress wall. Consequently, no bending moment occurs in the buttress head and no steel reinforcement is required. The quantity of concrete in the rounded head compares favorably with that of the cantilever type. Thus, by the simple arrangement of directing the water pressure toward the buttress wall by means of a rounded (or polygonal) up-stream face of the buttress head, plain concrete construction may be used instead of the reinforced cantilever type proposed by the author. At higher elevations of the dam where the water pressure is less, the cantilever type may require less material. On the other hand the principle of the round-head buttress may be applied economically to spans of 50 to 60 ft., with a minimum thickness of the buttress walls of 8 to 10 ft. This would enable the dam to be built of mass concrete instead of the thin reinforced concrete usually required for shorter spans.

Of course, the author's ingenious method of determining buttresses of uniform strength containing the minimum possible amount of material for a given design stress, may be used to advantage also for dams of the round-head buttress type.

B. F. JAKOBSEN,[14] Esq. (by letter).—The author presents a design method for a multiple-arch dam buttress, which should result in a structure of uniform strength. In general, at least, it should be possible to eliminate the possibility of buckling by providing counterforts, and since in Mr. Schorer's design two principal stresses are zero, the requirement of uniform strength is met when the third principal stress is constant throughout the buttress. As the author shows, this leads to a buttress having a horizontal section of variable width, as in Fig. 2, instead of a constant thickness, as has been customary in the past. The writer has calculated[15] the stresses in a buttress having a rectangular section in a horizontal plane. The section, for which the stresses were computed was an inclined, nearly normal section. The result shows, that the maximum principal stress (compression) was nearly constant, varying from about 260 to 286 lb. per sq. in. The intermediate principal stress, at right angles to the plane of the buttress, was zero, and the least principal stress (tension), varied from zero at the down-stream face to 84 lb. per sq. in. at the groin. This means that the factor of safety in compression is ten or more, while in tension it is three or less, unless steel is provided to take up the tension. This indicates the superiority of the author's design method.

The elimination of tension can presumably be accomplished by lengthening the buttress, but since stress calculations are complicated and time-consuming, it is difficult, if not practically impossible, to arrive at an economical design. Mr. Schorer's design method is therefore of great practical value.

In connection with the design of a fairly high multiple-arch dam the writer has had an opportunity to compare Mr. Schorer's method of design with the customary one. The consulting engineer, who prepared the design, first submitted one of the usual type having a constant thickness in a horizontal plane. This design had a small amount of tension, and it was decided to apply Mr. Schorer's method. The result was highly satisfactory to both parties, and a design with a tapered buttress in a horizontal section was adopted.

In a paper entitled "Stresses in Multiple-Arch Dams"[3] the writer treated the arch as monolithic with the buttress, as the author has done in Fig. 5. This treatment gives a shorter buttress than the assumption that the deck transmits load to the buttress only in a direction normal to the groin, as in Fig. 4. This latter involves the assumption of a frictionless joint between the deck and the buttress, and the writer questions whether such a joint is a practical possibility, considering that it must function for an almost unlimited time and under any conditions. These considerations would lead to the adoption of the arrangement shown in Fig. 4, as the safer design assumption, if it were not for the possibility of an upward thrust of the deck, due to water-soaking, combined with drying out of the buttress, and the consequent shrinkage. This tendency will be more pronounced when the temperature of the air is less than that of the water.

---

[14] Cons. Engr., Los Angeles, Calif.

[15] *Transactions*, Am. Soc. C. E., Vol. LXXXVII (1924), p. 306.

Most multiple-arch dams, of which the writer has knowledge, show cracks in the buttresses which, near the groin, are normal to the direction of the groin.[16] These cracks may be due to shrinkage and swelling, as just outlined, or they may be due to tensile load stresses. In some of these dams the writer inclines to the opinion, that the cracks are due primarily to shrinkage and swelling. If the deck could be constructed of segments, say, 10 ft. high, which were provided with open joints above and below, so that these segments could not interact lengthwise, the condition illustrated by Fig. 5 would exist. This might involve difficulties for the multiple-arch type, but could easily be accomplished for the flat deck type of dam. As multiple-arch dams are generally built, it seems to the writer, that some swelling of the deck combined with shrinkage of the buttress must be considered as a possible, and probable, state of affairs and the designer must be governed accordingly.

The author states that the writer advised contraction joints in buttresses and arches in 1924. This is not quite correct, since the writer advised consideration of "temporary" joints[17] and suitable construction methods.

Recently, in connection with the design of two multiple-arch dams, the writer, in collaboration with other engineers, gave this question careful consideration. The consensus of opinion was, that it was better to reinforce the buttresses in an effort to prevent unsightly cracks, than to attempt to provide shrinkage joints, which would be likely to promote cracking in the general direction of the joint, unless they started at the deck and went entirely through to the foundation (see Fig. 11). Additional steel was placed near the groin line and parallel to it, in order to counteract the swelling-shrinkage effect already mentioned. In addition, it was agreed that the buttresses should be constructed as far ahead of the arches as possible, in order to give them a chance to shrink before the arches were poured.

CALVIN V. DAVIS,[18] ASSOC. M. AM. SOC. C. E. (by letter).—The author has made a valuable contribution to the art of dam design. This paper emphasizes the important principle that a dam is a series of elementary curved, inclined columns, each of which transmits part of the water pressure from the water-bearing surface through the structure to the foundation. This discussion will treat several phases of this principle, as developed by Mr. Schorer, and will also demonstrate that the same principle may be developed by the trial method of design.

In his "Synopsis", the author makes the following statement:

> "The theory is based on the conception of elementary arched column units which transmit the water pressure from their respective deck areas directly and in uniform compression to the ground, together with the accumulated dead load. The elementary column units can be combined to a monolithic buttress system in which the first principal stress is uniform compression throughout and the second principal stress is zero."

---

[16] *Transactions*, Am. Soc. C. E., Vol. LXXXVII (1924), p. 311; see Fig. 42, showing cracks in Lake Hodges Dam.

[17] *Loc. cit.*, p. 311, last paragraph.

[18] Chf. Designer, Ambursen Dam Co., New York, N. Y.

As the writer understands this, Mr. Schorer considers that if each of a series of inclined elementary columns were in equilibrium under their respective loading this same condition would apply when the columns were combined to form a monolithic buttress. In the writer's opinion the stress conditions in a buttress considered as a monolith will not be similar to the stress conditions in the elementary columns when they are considered independently. The elastic properties of each elementary column will have some relation to the elastic properties of each adjacent column and, consequently, in arriving at the stresses in the structure, the deformations in the buttress as a whole must be considered.

Buttresses are not ordinarily constructed as rectangular sections and their structural properties are materially influenced by combining haunch or monolithic deck areas with the buttress section. For instance, the monolithic deck shown in Mr. Schorer's design example (Fig. 11) will have the effect of moving the center of gravity of a horizontal buttress section in an up-stream direction. When the buttress is considered as a monolith the effect of this added up-stream area will be to increase the eccentricity of the resultant loading. If the inclined columns were designed so as to show zero second principal stress along the joints when considered independently and neglecting the monolithic action of the deck, the second principal stresses would be in tension down stream from the deck if the buttress were analyzed monolithically and the effect of the horizontal deck area were included in the computations. In the monolithic analysis the increased eccentricity would lower the vertical normal stresses in the up-stream portion of the buttress and this, in turn, would change the second principal stresses from a value of zero down stream of the deck to some value in tension. On the other hand, the second principal stress throughout the deck will be compression. The writer believes that the results obtained by considering the buttress as a monolith will be less favorable than those obtained by Mr. Schorer's method and, therefore, would be more conservative to use in buttress dam design.

The writer is in thorough agreement with Mr. Schorer in the statement that the greatest economy may be obtained through the use of a buttress dam in which the first principal stresses are of uniform intensity and approach the allowable value throughout the structure. The results of several years of research, however, have convinced the writer that the same structural properties, that is, uniform strength, zero second principal stress, etc., may be obtained in a buttress of triangular shape with approximately rectangular horizontal sections. These properties may be readily controlled through the proper selection of both the up-stream and down-stream buttress slopes and also through the use of pilasters on the down-stream face of the dam. The second principal stresses in the designs developed from the research already mentioned were relatively high values in compression and this property, in the writer's opinion, adds a substantial margin of safety to the structure. Failure to include the effect of possible second principal stress in compression or tension is perhaps the greatest objection to the use of Mr. Schorer's method, which gives results only with reservoir full. Under all other loadings the second principal stresses will be compression. The inclined joints will only be on trajectories of first principal stress with reservoir full, and with the reservoir partly filled light shearing stresses will be developed across these joints. The writer believes it is important to determine these stresses and to provide keys in the joints or other means to take care of them in order that the monolithic character of the structure as a whole may be preserved.

For the reasons outlined the writer prefers the trial method of design, even if its use is somewhat tedious and involves considerably more labor than Mr. Schorer's method. The trial method is general and may be applied to the analysis of any type of dam. It is understood, of course, that it must be carried far enough to determine completely the principal stresses and their directions and also the shearing stresses in all parts of the structure. It is no longer adequate, as Mr. Schorer points out, to base a design on the determination of vertical normal stresses alone, as the second principal stresses may have

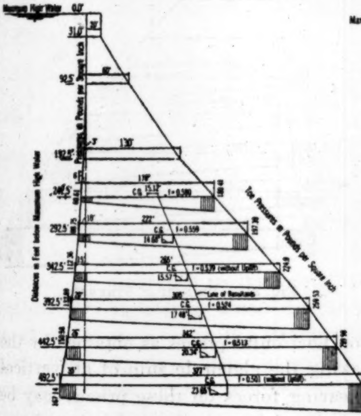

Maximum High Water
0.0'
31.0'
30'
92.5'
67'
192.5'
130'
242.5'
178'
C.G.
15.12'
f = 0.580
292.5'
18'
222'
C.G.
f = 0.558
14.68'
342.5'
266'
C.G.
f = 0.539 (without Uplift)
15.57'
392.5'
309'
Line of Resultants
C.G.
f = 0.524
17.48'
442.5'
342'
C.G.
f = 0.513
20.34'
492.5'
30'
377'
C.G.
f = 0.501 (without Uplift)
23.65'
347.96
Distances in Feet below Maximum High Water
Pressures in Pounds per Square Inch
Toe Pressures in Pounds per Square Inch

FIG. 18.

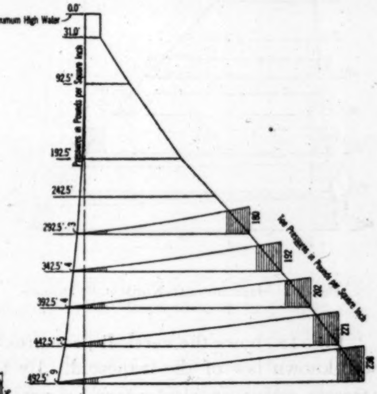

0.0'
Maximum High Water
31.0'
92.5'
192.5'
242.5'
292.5'
342.5'
392.5'
442.5'
492.5'
Pressures in Pounds per Square Inch
180
192
202
221
228
Toe Pressures in Pounds per Square Inch

FIG. 19.

high values in tension with the trapezoid of vertical normal stress entirely in compression.

In applying the trial method the principal stresses and shearing stresses may be determined directly from the vertical normal stresses by means of successive integrations. The writer believes that this procedure was first introduced in the United States by the late William Cain, M. Am. Soc. C. E., in 1909.[19] Mr. Jakobsen first showed the necessity for taking into consideration the principal stresses in a multiple-arch dam.[3]

Figs. 18 to 23 illustrate the application of the trial method of analysis to the design of a typical high dam of the gravity type; and at the same time they illustrate the development of the inclined column principle. The computations were not carried out for the upper half of the structure, but were carried far enough to illustrate the points raised in this discussion. No uplift pressure was assumed in making these computations. It is essential, of course, to consider uplift pressure in gravity dam design. However, these pressures are more important in considering sliding and overturning than stress intensity and distribution. It is possible that uplift pressures may vary from zero to 100% of the full static head at the heel and, of course, any assumption of this kind may give fictitious values. Inasmuch as the principal stresses may have maximum values with no uplift pressures, this assumption was used.

---

[19] *Transactions*, Am. Soc. C. E., Vol. LXIV (1909), p. 208.

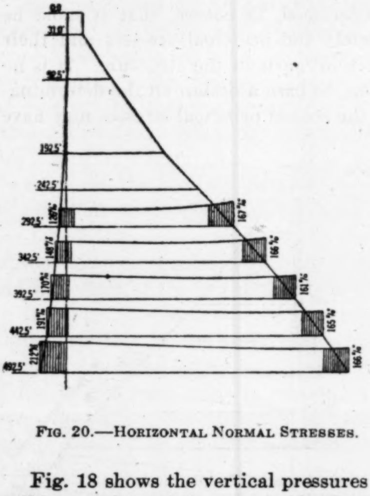


FIG. 20.—HORIZONTAL NORMAL STRESSES.

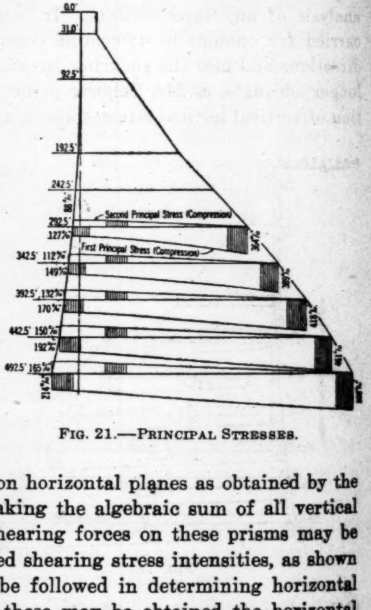


FIG. 21.—PRINCIPAL STRESSES.

Fig. 18 shows the vertical pressures on horizontal planes as obtained by the well-known law of the trapezoid. By taking the algebraic sum of all vertical stresses acting on elementary prisms, shearing forces on these prisms may be obtained, and from these may be obtained shearing stress intensities, as shown on Fig. 19. The same procedure may be followed in determining horizontal forces on elementary prisms and from these may be obtained the horizontal normal stresses shown on Fig. 20. The necessary elements are now at hand for determining the principal stresses and their directions. These may be obtained from either the principal stress equations, or by the more convenient method of using Mohr's circle. Fig. 21 shows the intensity of the first and second principal stresses; Fig. 22 shows curves of equal principal stress intensity; and Fig. 23 shows the trajectories of the first and second principal stresses.

It will be observed that the directions of the principal stresses follow definite mathematical laws and form regular curves which are normal to the up-stream face of the dam and asymptotic to the down-stream face. This calls attention again to the fact that the dam is a series of curved, inclined columns, each transmitting part of the water pressure from the up-stream face to the foundation. The curvature, of course, represents the weight influence of the masonry. These illustrations also serve to show the uneconomical use of material in a gravity dam design. Maximum principal stresses and maximum shearing stresses are attained only at the down-stream face. In a dam of the buttress type it is possible to keep all these values uniform throughout any horizontal plane and thus make the most efficient use of the material. The difference in cost of the buttress type and the gravity type of dams represents, after all, the difference in the value of the material required for stabilizing purposes for the gravity dam. In the gravity dam dependence is placed on the weight of material in the structure alone to attain stability. This means an unsatisfactory stress distribution. In the buttress type, stability is obtained through the use of an inclined up-stream face which introduces heavy stabilizing water loads on the structure. This allows an economical use of material in the structure and makes possible the attainment of satisfactory stress distribution.

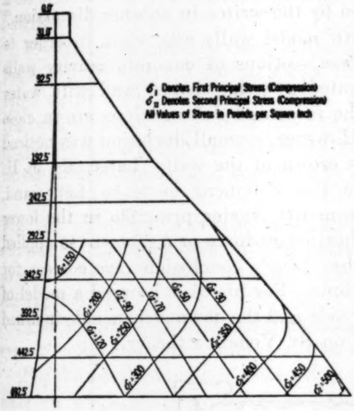

0.0'
31.0'
92.5'
192.5'
242.5'
292.5'
342.5'
392.5'
442.5'
492.5'
σ_I Denotes First Principal Stress (Compression)
σ_II Denotes Second Principal Stress (Compression)
All Values of Stress in Pounds per Square Inch
σ_I=150
σ_I=200
σ_II=90
σ_II=70
σ_II=50
σ_II=30
σ_II=120
σ_I=250
σ_I=350
σ_I=300
σ_I=400
σ_I=450
σ_I=500

FIG. 22.

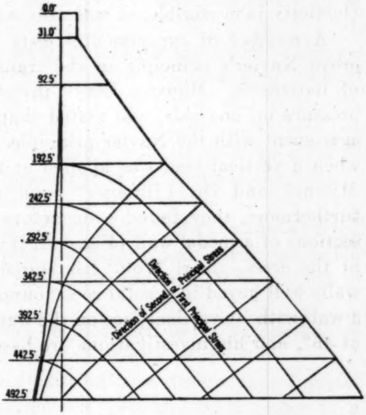

0.0'
31.0'
92.5'
192.5'
242.5'
292.5'
342.5'
392.5'
442.5'
492.5'
Direction of Second Principal Stress
Direction of First Principal Stress

FIG. 23.

A. A. EREMIN,[20] ASSOC. M. AM. SOC. C. E. (by letter).—The author has developed a new method of designing buttresses in which first principal stress compression is uniform and second principal stress is zero.

The present method of computing principal stresses is based on assumptions which have numerous limitations. The lines of uniform principal stresses in a buttress vary, due to changes in temperature, shrinkage of concrete, and lateral stiffness of the buttress. Professor Otto Mohr[21] has stated that in ten dams the stresses will be different regardless of the fact that they may be constructed under similar conditions.

---

[20] Asst. Designing Engr., Bridge Dept., State Highway Comm., Sacramento, Calif.

[21] "Die Staumauern," by Dr. N. Kelen, p. 85.

Principal stresses in buttresses are commonly computed with the assumption that the normal stresses in the transverse section vary linearly. Therefore, the deformations in a buttress must obey the principle expressed by Navier, that transverse plane sections will remain plane after loading and in Hooke's law, that strain is proportioned to stress or that the modulus of elasticity of material is constant at all stresses.

The modulus of elasticity of concrete is variable. However, in sections with all compressive stresses, the error from using a constant modulus of elasticity is negligible, as was also stated by the writer in another discussion.[22]

A number of experimental tests with model walls was made in order to prove Navier's principle in the transverse sections of concrete gravity walls or buttresses. Maurice Lévy[23] investigated a wedge-shaped wall with water pressure on one side, and stated that the measured deformations are in close agreement with the Navier principle. However, a small deviation was noticed when a vertical load was applied at the crown of the wall. Later, Mr. J. H. Mitchell and Dr. Fillunger[23] confirmed the statement made by Lévy, and, furthermore, they found a departure from the Navier principle in the lower sections of a model wall (Fig. 24), if a horizontal force or a moment is applied at the crest. Karl Wolf[23] has stated that Lévy's conclusions are correct for walls well keyed into solid rock foundations. For his tests he used a model of a wall with its vertical face on the water side and the down-stream side inclined at 45°, and his investigations are based on St. Venant's theory.

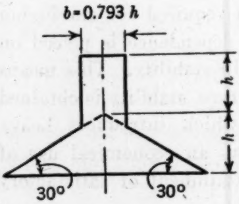


FIG. 24.

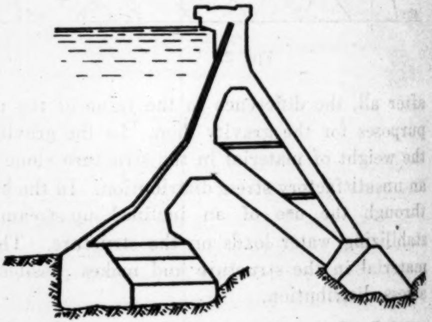


FIG. 25.—SECTION OF BUTTRESS USED BY DR. ROSSIN.

Experimental tests have proved also that shear stress in a transverse section of a buttress does not follow a linear distribution.[23] Thus far, however, all experimental tests to determine the distribution of stresses in gravity dams or buttresses have been performed with small-sized models and their results may be in error when applied to the massive sections used at the present time.

Involved mathematical analyses of buttress sections based on an uncertain distribution of stresses may be worthless unless a more precise method of stress computation is discovered.

---

[22] *Transactions*, Am. Soc. C. E., Vol. 95 (1931), p. 679.

[23] "Die Staumauern," by Dr. N. Kelen.

The author has properly indicated the advantages of the buttress composed of the elementary arched columns; however, the following factors should be considered in the design of the arched columns:

1.—If the deck is monolithic and separated from the arched columns by a frictionless joint, as shown in Figs. 4 and 5, secondary stresses in the deck will occur due to unequal settlement of the columns or to a variable change in the temperature of the buttress.

2.—A rock foundation is seldom homogeneous or isotropic and, therefore, a deformation of the foundation should be considered in any precise design of buttresses.

3.—If the elementary deck strips are monolithic with the arched columns there will be reaction forces along the center line of the deck strips where these strips touch each other (Figs. 5 and 6). These forces will vary with the swelling effect of the deck due to the moisture in the concrete, and with the temperature variation in the deck, and the result will be bending stresses in the arch columns, and eccentricity of pressures in the foundations.

The idea of a unit system of buttresses was used by Dr. Rossin[24] (see Fig. 25). This buttress is more economical than a monolithic buttress and the stresses in the members of the frame may be determined with reasonable precision by the method of least work or any other design method applied to the rigid frame.

The height of Dr. Rossin's buttress may be increased by using the hollow type of columns or by columns stiffened with counterforts or struts.

For the advancement of knowledge concerning buttress design, experimental tests on stress distribution on a large scale are needed. This paper is a meritorious contribution to the advancement of the theory, and the application of the author's new design methods to actual practice is highly desirable.

EUGENE KALMAN,[25] M. AM. SOC. C. E. (by letter).—Although the writer cannot agree with all the conclusions drawn by Mr. Schorer, he considers this paper a valuable contribution to the study of dam design. The ordinary buttress shape defined by two more or less parallel planes has been discarded and replaced by a more complicated form. The choice of that form constitutes a remarkably ingenious effort, and it proves that sometimes by making structures more complicated it is possible to simplify their analysis, due to the special shape.

Arch Column Unit.—The author's analysis is rather difficult to understand, and it will be useful, therefore, to review some of the underlying principles briefly. In Fig. 1 the two loads, P_0 and P_1, acting on the upper and lower section of the arch column, are necessarily the closing sides of the string polygon formed by the axis of the column, the intermediate forces being represented by the weights of the single elements in the column. Since all those weights are acting vertically downward, the force polygon will have the shape illustrated in Fig. 26(*b*). The vectors, $O\,B$ and $O\,D$, represent the magnitude and direction of the end loads, P_0 and P_1. The vector, $O\,C$, represents the resultant acting at a point along the axis of the column at which the tangent to the axis is parallel to $O\,C$, as shown in Fig. 26(*a*).

---

[24] "Die Staumauern," by Dr. N. Kelen, p. 264.

[25] Visiting Prof. of Civ. Eng., California Inst. of Technology, Pasadena, Calif.

First, the shape of the column axis must be known and, therefore, its equation must be determined. This may be accomplished by equating the inclinations of an arbitrary intermediate force in the funicular polygon and in the force polygon. Let ε and η denote the co-ordinates of the column axis. Then, the slope of the force, P, in Fig. 26(*a*), may be expressed by,

$$\tan\phi = -\frac{d\eta}{d\varepsilon} \quad\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (38)$$

On the other hand, the inclination of $O\,C$ is expressed as follows:

$$\tan\phi = \frac{A\,C}{O\,A} = \frac{A\,B + B\,C}{O\,A} \quad\ldots\ldots\ldots\ldots\ldots\ldots (39)$$

Since $B\,C$ is the total weight of the column between the points of application of P_0 and P, its value will be given by the sum,

$$w = \gamma_c \int_{\varepsilon_0}^{\varepsilon} w\ t\ ds \quad\ldots\ldots\ldots\ldots\ldots\ldots\ldots (40)$$

in which, *w*, *t*, and *ds* are the width, the thickness, and the elementary arch length of the column, respectively, at an arbitrary point on the axis.

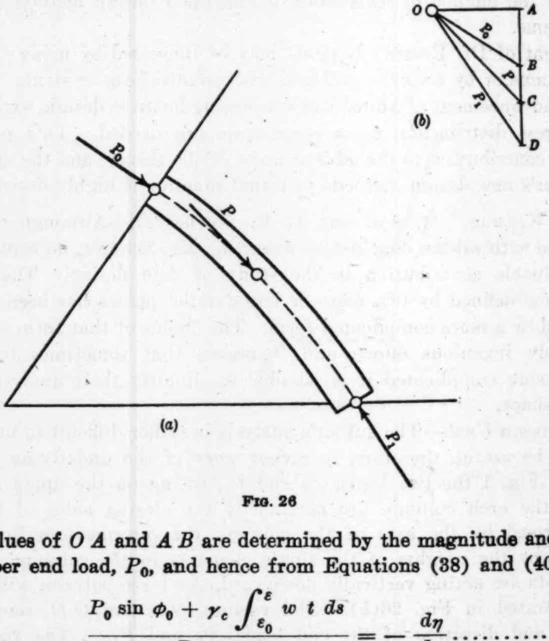


FIG. 26

The values of $O\,A$ and $A\,B$ are determined by the magnitude and direction of the upper end load, P_0, and hence from Equations (38) and (40),

$$\frac{P_0 \sin\phi_0 + \gamma_c \int_{\varepsilon_0}^{\varepsilon} w\ t\ ds}{P_0 \cos\phi_0} = -\frac{d\eta}{d\varepsilon} \quad\ldots\ldots\ldots\ldots (41)$$

Equation (41) expresses one of the two fundamental relations of the problem.

The other follows from the specification of a constant unit compressive stress, f_c, at all sections of the column. That stress may be obtained by dividing the force, P, by the area, $w\,t$, on which it is distributed. The magnitude of P can be evaluated from Fig. 26(*b*) as follows:

$$P = O\,A \sec\phi = P_0 \cos\phi_0 \sec\phi \ldots\ldots\ldots\ldots\ldots(42)$$

from which the second fundamental relation is,

$$\frac{P_0 \cos\phi_0 \sec\phi}{w\,t} = f_c \ldots\ldots\ldots\ldots\ldots\ldots\ldots(43)$$

In order to obtain the equation of the axis, both sides of Equation (41) must be differentiated. This operation may be performed after the substitution of the arch element, ds, by $\sec\phi\, d\varepsilon$, thus deriving the equation,

$$-\frac{d^2\,\eta}{d\,\varepsilon^2}\,\frac{P_0\cos\phi_0}{\gamma_c} = w\,t\sec\phi \ldots\ldots\ldots\ldots\ldots(44)$$

The product, $w\,t$, may be evaluated from Equation (43) for use in Equation (44), with the following result:

$$-\frac{d^2\,\eta}{d\,\varepsilon^2}\,\frac{P_0\cos\phi_0}{\gamma_c} = \frac{P_0\cos\phi_0\sec^2\phi}{f_c} \ldots\ldots\ldots\ldots\ldots(45)$$

and applying the well-known formula,

$$\sec^2\phi = 1 + \tan^2\phi = 1 + \left(\frac{d\eta}{d\varepsilon}\right)^2 \ldots\ldots\ldots\ldots\ldots(46)$$

the ultimate form of Equation (45) will be:

$$c\,\frac{d^2\,\eta}{d\varepsilon^2} + \left(\frac{d\eta}{d\varepsilon}\right)^2 + 1 = 0 \ldots\ldots\ldots\ldots\ldots\ldots(47)$$

This is a differential equation of the second order, for which the following general solution applies:

$$\eta = c\log\cos\frac{\varepsilon + C_1}{c} + C_2 \ldots\ldots\ldots\ldots\ldots\ldots(48)$$

A simple substitution will show that Equation (48) is really an integral of Equation (47), in which, C_1 and C_2 are two arbitrary constants. Equation (48), expressing the curve of the column axis, indicates that two geometrical conditions may be imposed upon it, and, since the position of the upper end is always given, as well as the slope of the up-stream face which yields the first derivative of the catenary, the curves of Equation (48) are entirely determined. With the form of the up-stream face given, there is one, and only one, catenary that passes through each point of that curve. The buttress is, then, subdivided into a certain number of independent catenaries, for which the width, w, is known at every point. Consequently, Equation (43) enables the designer to select a thickness at every section of each independent column unit, such that the unit stress at any section will remain constant.

Multiple Column Buttress of Uniform Strength.—A structure is obtained by the foregoing method that represents an ideal buttress. At every section the stress is constant, provided the columns are not in contact with each other and the proper loads, P_0 and P_1, are applied at their upper and lower ends.

Obviously, this ideal structure cannot be duplicated in actual practice. In the first place, the independent column units cannot be built; and, in the second place, the practical buttress will have to rest on the foundation and not on a system of needles, such as P_1.

In constructing the buttress one column unit must be placed upon the other; and after the construction, when the reservoir is filled, the column units will undergo certain deformations. Will they act independently of each other? In a buttress built as a monolithic structure, will the trajectories follow the ideal catenary curves? The influence of the minor principle stress has a tendency to disturb the equilibrium of the individual column units. Will the effect of this tendency be negligible?

These are the questions that are raised by a study of the author's method and there is no satisfactory answer to them in his paper. Unfortunately, Mr. Schorer makes extensive use of a refined mathematical tool to analyze the case of a buttress subdivided into column units and to explain their statical behavior (which does not interest the designer very much). On the other hand he drops the theoretical considerations entirely in treating the reconstruction of the buttress from the column units and in explaining the statical behavior of the buttress (which is the only thing that really does interest the designer).

The author merely limits himself to a few generic remarks which are absolutely not convincing. He writes, for instance:

> "Since the buttress is subject mainly to direct stresses in one direction, the internal work must approach a possible minimum. From Castigliano's principle of least work it can then be concluded that the stress distribution in the monolithic buttress is very nearly the same as that in the multiple column buttress composed of independent arched column units."

The foregoing statement is nebulous and the writer has been entirely unable to interpret it. The paper does not state whether the foundation will re-act exactly or approximately in conformity with the load system, P_1, which was selected, without any consideration of the foundation. It is assumed that the catenaries are trajectories and that the minor principle stresses are negligible everywhere; but these assumption have not been proved by the author.

Comparison with the Principle of Linear Stress Distribution.—Since the paper does not include a quantitative analysis of the stresses in the proposed buttress, the most simple check at hand might be attempted, and that is to compare it with the principle of linear stress distribution at a horizontal section of the buttress.

In the case of a plane up-stream face, all catenaries are identical curves, since $\phi_0 = 0$. The lower catenaries are obtained from the uppermost one by a simple translation. This fact follows since that portion of the upper catenary unit lying to the right of any arbitrary section may be removed and the corresponding thrust, P, at that section may be substituted for it.

The only difference between the remaining portions of the highest catenary unit and the lower column units is that the upper end-points of the first are acted upon by the constant, P_0, while the load, P_0, acting on the upper ends of the lower column units will depend on the depth. This will involve a corresponding increase in the thickness in the lower columns in proportion to the depth of their upper end points. The derivation of the curve for the lower catenaries from the higher one by means of the foregoing system of translation is illustrated in Fig. 27. The slopes of the two catenaries, R and R', are equal. Consequently, the slope of the reaction at R' is obtained by drawing the tangent to the catenary, $S\,Q$, at the point, R. In this way the direction of the reaction at any point of the horizontal section under consideration may be determined.

Information concerning the distribution of vertical normal stresses on the horizontal section is lacking. The foregoing graphic procedure permits a designer to avoid the more cumbersome algebraic operations. The directions of the reactions, P_1, are now known and since the major principal stresses are assumed to be equal to a given f_c, the magnitude of the reactions, P_1, also, is known.

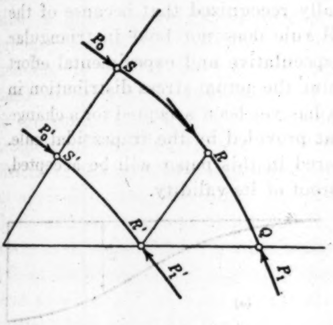


FIG. 27

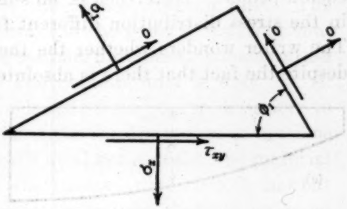


FIG. 28

The magnitudes of the vertical normal stresses at different points of the horizontal section may be evaluated with the assistance of Fig. 28, which represents the equilibrium of the stresses acting on an elementary prism. The catenaries are assumed to represent special trajectories without minor principal stresses. This means that the major principal stress is the only one acting on the two smaller sides of the prism, provided the smaller sides are parts of trajectories. In Fig. 28, note the vertical normal stress, σ_x, and the horizontal shearing stress, τ_{xy}. From the condition of equilibrium on the prism, the following equations may be written:

$$\sigma_x = f_c \sin^2 \phi_1 \qquad (49)$$

and,

$$\tau_{xy} = f_c \sin \phi_1 \cos \phi_1 = \frac{1}{2} f_c \sin 2\,\phi_1 \qquad (50)$$

These values may be determined, since the angles, ϕ_1, at any point are known. They are plotted for illustration in Fig. 29(*a*) and Fig. 29(*b*).

The corresponding curves referring to the buttress with a vertical upstream face are particularly interesting. The tangent to all catenaries at the left end point of the horizontal sections will be horizontal. The value of ϕ_1 in Equations (49) and (50) is equal to 0. Consequently, both the vertical normal and the horizontal shearing stress will be equal to zero at the left end points of all horizontal sections, as indicated in the curve, Fig. 30(*a*) and Fig. 30(*b*).

It will be noticed that the stress distribution does not follow the trapezoidal rule. The diagrams are peculiar especially in the case of the vertical up-stream face. Engineers are inclined to accept the trapezoidal rule as a matter of fact, and they do not like to discard it unless its fallacy has been proved. Mr. Schorer, however, does not supply this deficiency, since he fails to prove that there is no shear and no minor principal stress along the catenaries. In fact, there are both normal and tangential loads acting on the sides of the five columns in Fig. 11. Furthermore, it is quite probable that the effect of the foundation is to create a still greater discrepancy between the actual stress distribution and that shown in Figs. 29 and 30, in which the buttress alone is considered. It is universally recognized that because of the influence of the foundation the trapezoidal rule does not hold in triangular dams. It is also known that considerable speculative and experimental effort has been directed toward an effort to determine the actual stress distribution in such a profile.[26] Nevertheless, no suggestion has yet been accepted for a change in the stress distribution different from that provided by the trapezoidal rule. The writer wonders whether the theory offered in this paper will be accepted, despite the fact that there is absolutely no proof of its validity.

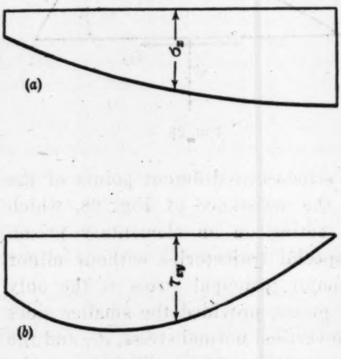


FIG. 29

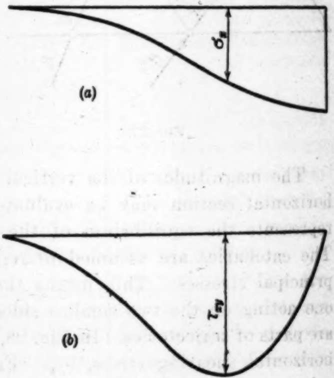


FIG. 30

The author states that his direct procedure makes it possible for a designer to proportion the entire buttress so that it will have a uniform strength; but engineers like to check the structures after they have been proportioned. How should one proceed to check the structure in this case? Should the buttress be treated as a column when it is monolithic? Decidedly, it is not a column. Should it be treated as a beam? If so, the trapezoidal rule should be applied to the individual sections, and the author is opposed to the application of this rule.

---

[26] See p. 489.

The buttress units might be treated as laminæ, approximately, but their complicated shapes introduce insurmountable difficulties to such treatment. It would be valuable if Mr. Schorer would suggest some way of checking the dimensions selected.

Design Example.—The design example illustrated in Fig. 11 contains nothing revolutionary. Its characteristic features are the subdivision of the buttress into five parts, a somewhat strange "neck", and a decrease in thickness from the up-stream face toward the down-stream face. It contains no sections of the shape indicated in Fig. 27. Of all the innovations only the back portion is new, which, by the way, is not a convincing innovation. A perfectly triangular profile of the same volume could supplant it without the least appreciable change in the statical conditions.

There are two construction features that should be discussed. The continuous plate, or slab, that covers the buttresses on the down-stream side is quite useful as a means of preventing buckling. However, its presence makes it impossible to inspect the buttresses and, in the writer's opinion, it should be removed entirely, or, at least, it should be confined to certain isolated parts of the dam. The same reasoning applies to the continuous slab at the inside face of the highest column unit. The prospect of removing these slabs directs attention to the general resisting conditions of the structure and especially to the column unit under consideration as applied to the possibility of buckling. The column unit is only 1.5 ft. thick, and its length is about 300 ft. Such a length would cause designers to worry even with stiffening cross-walls and even if the thickness were as much as 8 ft. at the bottom. By removing the cross-walls at alternate cells, the material thus saved could be used in proportioning the column unit with somewhat greater thickness. In the construction of dams, certainly, there is no room for quibbling over the addition or subtraction of every inch of concrete in place.

Another construction feature may be mentioned in connection with Section *X-X* of Fig. 11. Fig. 11(*i*) shows that the column units change abruptly in thickness from one to the other, while Fig. 11(*d*) shows that the horizontal section at the base has a uniform width. No section is given to show where the difference between Fig. 11(*d*) and Fig. 11(*i*) becomes compensated. The writer does not approve the arrangement in Fig. 11(*i*), Section *X-X*. From a purely theoretical point of view, abrupt changes are not justifiable, because any two hypothetical elementary column units in any of the five columns are subjected to different loads and, therefore, require different cross-sections. From a practical point of view, abrupt changes in thickness do more harm than good.

Conclusions.—A continuous rear slab is not a desirable feature in this kind of design. Furthermore, the shape of the "neck" at the level of 250 ft. is not essential and the abrupt change in thickness at the joints is a poor feature. The writer does not favor the introduction of joints in the design of recent dams for two reasons: First, they weaken the buttress from a structural point of view without making the statical conditions any clearer. Indeed, they make the conditions of static so complicated that they cannot be comprehended at all. The second reason for distrusting the use of joints is that they do not improve the conditions in the buttress in view of the hazards caused by shrinkage and temperature changes. A further remark may be added concerning the buttresses. They should not be made too slender near the down-stream face where the relatively great height of the column unit is most conducive to buckling.

As for the theoretical considerations in the paper, the author does not furnish any proof as to the validity of his theory for the monolithic buttress, or even for a definite number of columns resting on each other. The writer has established the fact that in monolithic buttresses the author's theory involves proving that the rule of the trapezoid is not valid. For this reason, the proposed method cannot be used for the final selection of the shape of the buttress. It is an excellent way to estimate, without much preliminary effort, an approximate shape for the buttress. It will be necessary, however, to analyze a certain number of horizontal sections by means of the trapezoidal rule. Although, admittedly, this rule is not correct it is to be preferred to any other stress distribution until the newer plan has been proved valid. Obviously, the process of analyzing horizontal sections by means of the trapezoidal rule will be extremely laborious by analytical methods, owing to the irregular change in thickness throughout the entire buttress. That is the disadvantage of this procedure in contrast to the alluring possibility of choosing, directly, the shape of the buttress by Mr. Schorer's method. A graphical method, however, can be easily found for the successive investigation of the single horizontal section by adopting and extending the well-known graphical application of the trapezoidal rule to the analysis of the single horizontal section in a gravity profile of general shape.

The author deserves considerable credit for having introduced the idea of elementary columns, even if it cannot be applied to monolithic buttresses. His method undoubtedly permits a direct selection of a primary, approximate buttress shape so that when the method is completed by means of a supplementary graphical investigation, as suggested in the foregoing remarks, it represents a positive progressive step in the design of dams.

A. FLORIS,[27] ESQ. (by letter).—It is well known that tensile stresses are produced in dams during the cooling period of the concrete. Furthermore, it is generally recognized that in triangular dam profiles near the foundation the stress distribution, due to water pressure and the weight of the dam, does not remain linear. To reduce the shrinkage in concrete dams to any considerable extent is scarcely possible at present, and the problem of determining stresses at the base of the dam produced by water pressure and the weight of the structure presents many mathematical difficulties that have not yet been overcome. For this reason attempts have been made to avoid these difficulties by introducing joints and thus allowing the concrete to contract, without cracking, along predetermined lines. The dam is then analyzed either approximately by considering these joints or by assuming that the structure is monolithic in spite of them.

---

[27] Civ. Engr., Los Angeles, Calif.

In the literature on dams the statement has been made repeatedly in recent years that inclined contraction joints should be located along the trajectories of the first principal normal stress caused by the water pressure and the load of the structure. These trajectories, on the other hand, are determined by means of the rule of the trapezoid and by assuming the dam to be monolithic. However, the introduction of such joints complicates the problem of stress distribution very much, because the boundary loads transmitted to the given column from the adjacent columns formed by the joints are not known.

One can be reasonably confident that in the near future the problem of determining the stress distribution in gravity dam profiles without joints, and which form a single unit with the foundation, will be solved. Then, the stress distribution in various types of monolithic buttresses will also be known within satisfactory limits. However, this cannot be expected in the case of dam profiles with joints.

Mr. Schorer states that from Castigliano's theorem of least work it can be concluded that the stress distribution in monolithic buttresses will be very nearly the same as that produced in the independent columns. Stresses can be found in prismatic bars (one-dimensional) by the use of Castigliano's theorem, but it cannot be applied to the stress distribution in gravity dam profiles (two-dimensional) and certainly not in the type of buttress suggested by Mr. Schorer, which varies in form even in the third dimension (three-dimensional).

The author has not proved that the catenaries of the independent arched columns will coincide with the trajectories of the first principal normal stress in a monolithic buttress. Furthermore, it is not correct to state that the first principal normal stress in a monolithic buttress will be equal to that produced in the structure with independent arched columns; nor is it correct to state that the second principal stress will vanish in a monolithic structure the same as it does in independent arched columns. It is not at all correct to maintain that the influence of shrinkage and temperature changes will be so great that the effect of the principal normal and shearing stresses, Poisson's ratio, etc., can be neglected.

The effort to discard the rule of the trapezoid in designing dams by introducing the arched column theory is not quite new. In addition to the fact that dams with inclined joints have been built in recent years, other writers before Mr. Schorer have treated the same subject. In studying failures of dams, Paul Ziegler came to the conclusion that a kind of arch action takes place in gravity and buttressed dams.[28] He points out that cracks produced in the remaining buttresses of the Gleno Dam which failed and also in other gravity dams, indicate that arches are probably formed in gravity and buttressed dams under stress. Such arches apparently have a horizontal abutment at the up-stream face while the other is resting either on rock in the lower

---

[28] "Volle Strebenmauer und Pfeiler-Strebenmauer," von Paul Ziegler, *Schweizerische Bauzeitung*, June 29, 1929, p. 315.

part of the dam, or is carried in the upper parts of the profile by the lower arches as shown in Fig. 31. The author attempts to locate these arches and thus to determine the stresses in the dam graphically by using funicular polygons.

The method proposed by Fredrik Vogt, Assoc. M. Am. Soc. C. E., is exactly the same as that presented in this paper.[29] By dividing the buttress of a monolithic dam into a number of independent columns subjected to the load, p_0 (see Fig. 32), Dr. Vogt derives the following differential equation for the column axis:

$$\frac{d^2 y}{dx^2} = q\left[1 + \left(\frac{dy}{dx}\right)^2\right] \qquad (51)$$

in which, $q = \frac{\gamma_w}{\sigma}$, γ_w = the water pressure, and σ = the allowable compressive stress. Let $y = 0$, and let $\frac{dy}{dx} = \tan\alpha$; when $x = 0$, Equation (51) becomes:

$$y = \frac{1}{q}\log\frac{\cos\alpha_0}{\cos\alpha} \qquad (52)$$

in which, $\alpha = \alpha_0 + q\,x$.

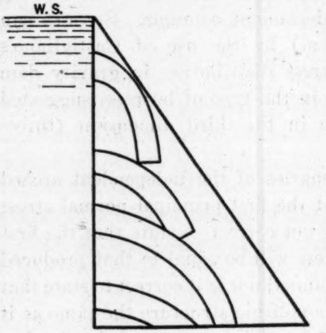


Fig. 31

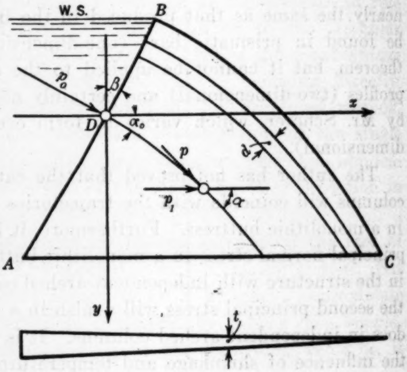


Fig. 32

The curve represented by Equation (52) is, as in the author's case, a catenary. Let the thickness equal,

$$t = \frac{A}{\alpha} \qquad (53)$$

in which, $A = \frac{p}{\sigma} = \frac{p_1}{\sigma\cos\alpha}$, and p_1 = the constant horizontal component of p. By selecting the thickness of the columns properly, they may be designed so as to be subjected to the same compressive stress at every point. Because the width, d, of the columns decreases toward the base of the dam directly in proportion to the increasing pressure, p, to maintain uniform strength through-

---

[29] "Economical Design of Buttresses for High Dams and of Cellular Gravity Dams," by Fredrik Vogt, Det Kgl. Norske Videnskabers Selskab, Trondhjem, Norway, Forhandlinger, Bd. II, No. 40, p. 141.

out, it is clear that the thickness of the column must be increased accordingly. Furthermore, by determining the minimum value for the expression of the sum of deck and buttress weights between two adjacent elevations, the following expression for the variable slope, β, of the up-stream curve, *ADB*, in Fig. 32, can be derived:

$$c^2 F^2 z^4 + 2\,c\,(1 + c^2)\,F^2 z^3 + [(1 + 4\,c^2 + c^4)\,F^2 - 1]\,z^2$$
$$+ 2\,c\,F\,[(1 + c^2) - 1]\,z = (F - 1)\,(1 - c^2 F) \dots\dots\dots\dots (54)$$

in which, $z = \sin \beta$, $F = e^{2qh}$, and c varies from 0 to 0.2. The constant, c, expresses the ratio of deck weight to water pressure, and in high multiple-arch dams it is approximately equal to 0.1. By solving Equation (54), with respect to z, the up-stream curve of the dam can be traced step by step.

The same procedure is followed by Mr. Schorer, and his proposed dam profile (see Fig. 10) is, therefore, necessarily the same as that presented by Dr. Vogt.

FREDRIK VOGT,[30] ASSOC. M. AM. SOC. C. E. (by letter).—The writer has studied this paper with the greatest interest, particularly because he has worked along the same line himself, and published a short paper on this subject in 1929.[29] It is gratifying to note that the general point of view is the same in both papers, and that even the formulas check each other to a great extent, except for differences in nomenclature.

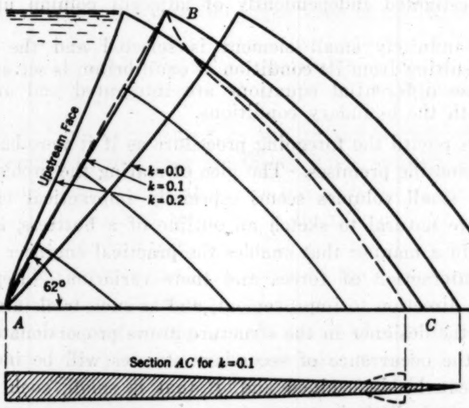


FIG. 33.—MOST ECONOMICAL SHAPE OF BUTTRESSES WITH DECK FACTORS 0, 0.1, AND 0.2, AND BUTTRESS RATIO, $i = 0.5$.

The writer has not much to add, therefore, except to state that theoretically the most economical shape of buttresses approaches very close to that of a straight-faced dam when the cost of the arches is included in the analysis. Fig. 10 gives this shape for a deck factor, 0, defined by Equation (22), that is, neglecting the cost of the arches. For actual dams the deck factor averages 0.1. Fig. 33, taken from the writer's paper, illustrates the shapes corresponding to deck factors 0, 0.1, and 0.2. It may be noted that a deck factor, 0.1, makes the up-stream face, determined in this way, steeper than if this factor were zero. Furthermore, the up-stream face is almost entirely straight if the deck factor is 0.1, or more. For practical design the writer, therefore, suggested a straight-faced dam as shown by the dotted lines. This shape has also been adopted by the author.

---

[30] Prof. of Mechanics, Norges Tekniske Höiskole, Trondhjem, Norway.

Dams with steep up-stream faces require a minimum amount of concrete, but the sliding factor computed for horizontal joints does not satisfy the usual standards. Therefore, the saving in concrete work can only be effected if sliding is prevented by cutting large steps in the rock foundation approximately normal to the lines of thrust for each of the columns into which the buttress is divided. Furthermore, if sliding is to be prevented in this manner, the rock foundation should be perfectly sound.

A. C. SCHWAGER,[31] Esq. (by letter).—The strict solution of a problem, that has been sought since the beginning of modern dam design, is presented by the author. The writer proposes to analyze the mathematical parts of this paper without regard to the construction aspect described by Mr. Schorer. The method may be outlined concisely, as follows:

> (1) The design specifies that only compression stresses, constant throughout any buttress, shall exist. Therefore, each hypothetical column in a buttress may be assumed to be a unit, and its equilibrium can be investigated independently of adjacent column units in the buttress.
>
> (2) An infinitely small element is selected and the differential equation resulting from its condition of equilibrium is set up.
>
> (3) These differential equations are integrated and are made to coincide with the boundary conditions.

Some engineers regard the foregoing procedure as if it were based entirely on abstract and scientific premises. The idea of stating the problem in terms of infinitesimally small volumes seems especially impractical to them. It appears much more natural to sketch an outline of a buttress, analyzing it and dissecting it in a manner that enables the practical engineer to visualize the action and interaction of forces and their variations. Repeated trial analyses show the direction to improvement, and as such trials are duplicated the confidence of the designer in the structure grows proportionately. At the various sections the occurrence of secondary stresses will be indicated, but their magnitude can be determined and, if that is small, such stresses are considered of relatively little importance.

Which of the foregoing two methods should be given preference? From a scientific standpoint there is no doubt of the extreme superiority of the method of exact integration. In the history of technical advancement the same situation has often been repeated. Electrical engineering was the first technical science to apply the exact principles brought over from the field of pure physics in the form of the Maxwell equations. It is scarcely conceivable that a power transmission line could be designed without them. A problem encountered in mechanical engineering is especially comparable to that of the author's buttress of uniform strength. When the development of steam turbines began to increase most rapidly, speeds advanced into a range previously unknown. As a consequence, the stresses in the disk wheels produced by their own centrifugal forces represented the major design problem.

---

[31] Oil Circuit-Breaker Design Engr., Pacific Elec. Mfg. Corporation, San Francisco, Calif.

Trial methods based on successive graphical solutions were inaccurate and time-consuming. A small proportion of the total weight, placed incorrectly, caused inefficiency and resulted in designs that were dangerous to life. It was De Laval and Stodola who introduced the requirement of constant radial and tangential stresses. They integrated the differential equations and obtained the disk of uniform strength. The resulting formulas have made it possible to eliminate all guesswork. They are considered definitely correct and have been accepted for several decades as the tools of every turbine designer.

In the case of the buttressed dam of uniform strength the same arguments apply as in that of the transmission line or the turbine wheel. The scientific analysis offers the only correct solution eliminating waste of materials and time. The success of Mr. Schorer's method of solution from a technical standpoint cannot be doubted, but it is just as important from a scientific standpoint, since once more it has proved the tremendous power that is available in this use of differential and integral mathematics. It is not possible to over-emphasize the fundamental importance of stating a problem in differential equations wherever possible as set up by the exact science, and of finding the integral effects applicable to the particular condition.

PAUL BAUMAN,[32] M. AM. SOC. C. E. (by letter).—The method of designing buttressed dams as proposed by the author is well adapted to the flat-slab type, because in that case his theory holds without modification. With a multiple-arch dam, a joint between the arch barrel and the face of the buttress would be required to make it directly applicable, which is not in accordance with standard practice. The greater the span of multiple-arch dams, the greater becomes the prominence of the arch over the buttress, both as to area and moment of inertia.

In a hoof dam, for example, with spans of 200 ft., or more, between buttresses, the principle of uniform strength would scarcely prove advantageous because the greater part of the dam is in the arches, which can only be designed, safely and economically, as such. The relative load taken by the buttresses of such a dam is materially smaller than with a flat-slab dam. Even with a flat-slab dam the safe height would probably be considerably less than the one based on the author's formulas, because of stresses due to elastic deformations of the arched column which, as far as the writer is aware, are not covered by the paper.

If a dam of considerable height is designed for full load, a uniform compression of the arched columns of maximum length does not necessarily take place even though the slab during the deformation may remain practically straight, as suggested by Mr. Schorer.

---

[32] Chf. Designer, Quinton, Code & Hill-Leeds & Barnard, Engrs., Consolidated, Los Angeles, Calif.

Less work is involved in shortening these arched columns by bending than by concentric pressure. To enable this the columns must shorten their average radius of curvature by bending toward the air side of the dam and by opening up the joints. The latter will take place due to eccentricity of the line of support, primarily caused by a restraining moment at the base due to deflection of the buttress. The maximum stresses thereby produced would be materially greater than those due to uniform pressure, but would not necessarily be critical since the basic, uniform pressure could be rather low even with a strictly economical design.

A further tendency toward eccentricity in the arched columns exists during partial load, provided they are designed for full load. The resulting stresses may again be greater than those for full load, and the factor of safety may be smaller for partial load than for full load. This may sound somewhat paradoxical, but similar conclusions may be drawn from the study of structures of entirely different types, such as a continuous beam over numerous supports, etc. If designed for uniform load only, the latter may promptly fail under partial load.

To establish the principal trajectories by the author's method is more satisfactory than to determine them from the stresses (horizontal plane), assumed to be linearly distributed because, even if their distribution is nearly linear, they are more or less in error, except for a flat-slab dam with triangular buttresses. From a practical standpoint, the elimination of friction from the joints of a buttressed dam of uniform strength would probably be the outstanding task.

I. M. NELIDOV,[33] ASSOC. M. AM. SOC. C. E. (by letter).—The author's method of proportioning the buttress with the aid of the catenary curves is a process opposite to that ordinarily used. Instead of assuming loading and dimensions of a buttress and expressing stresses as functions of co-ordinates (ultimately principal stresses), the principal stresses and loadings are assumed and dimensions of the buttress are expressed as their functions.

This process is applicable only to the cases in which the continuity of loading is assured, that is, when the water level is at the crest of the dam or higher. It is applicable only for the particular loading assumed, because only with this loading is the direction of the first principal stresses at the base coincident with that of tangents to the catenary, and only for this case is there no difference as to whether or not the elementary columns are fixed or hinged at the base. If the base is fixed then for any other loading the afore-mentioned tangents must remain unchanged.

The writer wished to investigate the difference between the stress distribution computed by the author's method and by the ordinary method, by taking moments of weights and loads about the center of gravity of the base, computing moments of inertia of the base, etc. For this purpose a buttress was selected, with the following data given: $D = 100$ ft., $b = 20$ ft., $k = 0.1$ (constant, although in actual practice k increases with v, causing a "hump" at the upper part of the buttress), $\gamma_w = 62.5$ lb. per cu. ft., $\gamma_c = 144$ lb. per cu. ft., $f_c = 250$ lb. per sq. in., and $\phi_0 = 30$ degrees.

---

[33] Senior Engr. of Hydr. Structure Design, State Dept. of Public Works, Sacramento, Calif.

With these data and notation in Fig. 34; $\phi_k = 34.71512°$, $\alpha = 2.15286°$, and $\frac{h}{h_v} = 0.97564$.

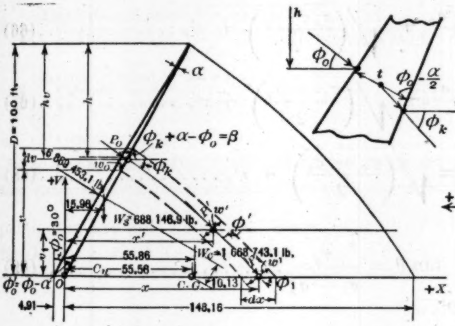
Furthermore, let $\frac{v - v_1}{2} = q$; then,

$$t = \frac{\gamma_w b h e^{2q} \sec^2 \phi_k}{f_c [\tan \phi'_0 \tan \phi' + 1]} \quad \text{.....................(55)}$$

$$x' = v \tan \phi'_0 + c (\phi' - \phi_k) \quad \text{.....................(56)}$$

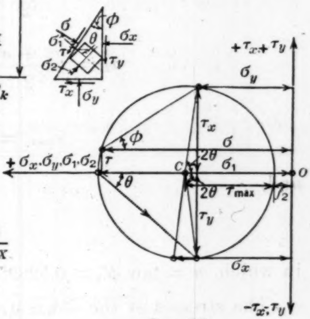
and,

$$W_c = \gamma_w \; b \; h \; dv \; [\tan \phi' - \tan \phi_k] \quad \text{.................(57)}$$



D=100 ft.
hv
h
α
h
ϕ_o
t
$\phi_o - \frac{\alpha}{2}$
h_v
ϕ_k
P_o
$\phi_k + \alpha - \phi_o = \beta$
dv
16 869 453.1 lb.
w_o
ϕ_k
+V
15.96
W_s= 688 146.9 lb.
w'
ϕ'
$\phi_o = 30°$
x'
v
55.86
W_c=1 668 743.1 lb.
C_v
55.56
w'
$\phi_o' = \phi_o - \alpha$
O
C.G.
10.13
ϕ_1
x
dx
+X
4.91
148.16

FIG. 34.



σ
φ
σx
σ1
θ
τ
τy
σ2
τx
σy
+τx,+τy
σy
τx
φ
σ
+ σx,σy,σ1,σ2
τ
2θ
σ1
C
O
θ
2θ
τmax
σ2
τy
σx
−τx,−τy

FIG. 35.

Taking moments about Point O (Fig. 34):

$$M_{co} = \gamma_w \; b \; h \; d \; v \; [v \tan \phi'_0 (\tan \phi' - \tan \phi_k) + c \tan \phi' (\phi' - \phi_k) - (v - v_1)] \quad \text{.............(58)}$$

and,

$$\tan \phi' = \sqrt{e^{2q} \sec^2 \phi_k - 1}$$

In case $v_1 = 0$, then $\phi' = \phi_1$.

The stresses as computed by the catenary method are:

Vertical normal stress:

$$\sigma_y = f_c \sin^2 \phi' \quad \text{...................................(59)}$$

Horizontal normal stress:

$$\sigma_x = f_c \cos^2 \phi' \quad \text{...................................(60)}$$

and,

Shearing stress:

$$\sigma_x = f_c \sin \phi' \cos \phi' \quad \text{..........................(61)}$$

Due to the fact that the face of the buttress makes an angle, $\beta = \phi_x - \phi_0 + \alpha = 6.86845°$, with a plane normal to the catenary at that point there is very slight shearing stress at the face, $\tau = f_c \sin \beta \cos \beta = 29.7$ lb. per sq. in., the compressive normal stress being $\sigma = f_c \cos^2 \beta = 246.4$ lb. per sq. in.

Vertical normal stresses by the ordinary method were computed by the formula:

$$\sigma_y = \frac{\Sigma V}{A} \pm \frac{\Sigma M}{S} \quad \ldots\ldots (62)$$

The stresses at the up-stream face are:

$$\sigma_y = \frac{\Sigma V}{A} - \frac{\Sigma M}{S} \quad \ldots\ldots (63)$$

$$\tau_x = \tau - (\sigma_y - \sigma)\, n \quad \ldots\ldots (64)$$

$$\sigma_x = (\sigma_y - \sigma)\, n^2 - 2\, n\, \tau + \sigma \quad \ldots\ldots (65)$$

$$\sigma_1 = \frac{\sigma_x + \sigma_y}{2} + \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau^2_x} \quad \ldots\ldots (66)$$

$$\sigma_2 = \frac{\sigma_x + \sigma_y}{2} - \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau^2_x} \quad \ldots\ldots (67)$$

$$\tau_{\text{max.}} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau^2_x} \quad \ldots\ldots (68)$$

and,

$$\tan \theta = \frac{\tau_y}{\sigma_1 - \sigma_y} \quad \ldots\ldots (69)$$

in which, $n = \tan \phi'_0 = 0.52828$.

The stresses at the down-stream face are:

$$\sigma_y = \frac{\Sigma V}{A} + \frac{\Sigma M}{S} \quad \ldots\ldots (70)$$

$$\tau_x = m\, \sigma_y \quad \ldots\ldots (71)$$

$$\sigma_x = m\, \tau_x \quad \ldots\ldots (72)$$

$$\sigma_1 = \sigma_y\,(1 + m^2) \quad \ldots\ldots (73)$$

$$\sigma_2 = 0 \quad \ldots\ldots (74)$$

and,

$$\tau_{\text{max.}} = \frac{1}{2}\,\sigma_1$$

in which, $m = \cot \phi'$.

Analytical stress computations were checked by Mohr's circle as adapted by the writer to the general condition of stress as shown on Fig. 35. Table 1 shows that the two methods give essentially the same results, at least for the

TABLE 1.—Comparison of Stresses Computed By Ordinary Method and by Catenary Method

(Pounds per square inch)

Item No.	Kind of stress	Reference Points on the Dam (See Figs. 36 and 37)							
		A	*a*	*B*	*b*	*C*	*c*	*D*	*d*
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
	Vertical Normal Stress, σ_y:								
1	Catenary method..........	81.1	174.1	81.1	153.5	81.1	136.8	81.1	117.1
2	Ordinary method..........	79.3	178.7	78.9	155.8	78.3	139.1	77.7	123.8
	Horizontal Shearing Stress, τ_x:								
3	Catenary method..........	117.0	115.0	117.0	121.7	117.0	124.5	117.0	124.8
4	Ordinary method..........	118.0	118.0	118.3	123.5	118.5	126.5	118.8	131.9
	Horizontal Normal Stress, σ_x:								
5	Catenary method..........	168.9	75.9	168.9	96.5	168.9	113.2	168.9	132.9
6	Ordinary method..........	168.4	78.0	168.2	97.9	168.1	115.1	168.0	140.5
	First Principal Stress, σ_1:								
7	Catenary method..........	250.0	250.0	250.0	250.0	250.0	250.0	250.0	250.0
8	Ordinary method..........	250.0	256.6	250.0	253.7	250.0	254.2	250.0	268.3
	Second Principal Stress, σ_2:								
9	Catenary method..........	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
10	Ordinary method..........	—2.2	0.0	—2.9	0.0	—3.5	0.0	—4.3	0.0
	Maximum Shear Stress, $\tau_{\max}$:								
11	Catenary method..........	125.0	125.0	125.0	125.0	125.0	125.0	125.0	125.0
12	Ordinary method..........	126.1	128.3	126.5	126.8	126.7	127.1	127.1	132.1

particular example selected. Figs. 36 and 37 will serve to identify Points *A* to *D* and *a* to *d* in this table.

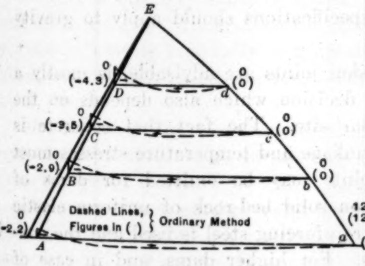

E
0
(-4.3)
D
0
(0)
d
0
(-9.5)
C
0
(0)
c
0
(-2.9)
B
0
(0)
b
Dashed Lines,
Figures in ( )
Ordinary Method
0
(-2.2)
A
a
0
(0)

Fig. 36.—Diagram of Second Principal Stresses, σ_2.

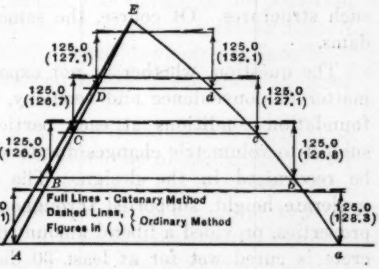

E
125.0
(127.1)
125.0
(132.1)
125.0
(126.7)
D
d
125.0
(127.1)
125.0
(126.5)
C
c
125.0
(126.8)
B
b
Full Lines - Catenary Method
Dashed Lines,
Figures in ( )
Ordinary Method
125.0
(126.1)
125.0
(128.3)
A
a

Fig. 37.—Diagram of Maximum Shearing Stresses, τ_{max}.

In conclusion, the writer wishes to mention that joints should not enter the problem until shrinkage and other structural items have been discussed because if the buttress is treated as an inelastic body the separate catenary columns will be in equilibrium regardless of whether or not they are joined together.

HERMAN SCHORER,[34] ASSOC. M. AM. SOC. C. E. (by letter).—Mr. Girand properly calls attention to the factor of safety against overturning and suggests that two-thirds of the actual weight of the concrete be used for design purposes. This assumption evidently is identical in effect with that of using a normal unit weight for concrete, but a unit weight of water and a design stress, which are both 50% larger than the usual values.

Since the unit weight of concrete is not subject to changes, the latter assumptions would perhaps be somewhat more logical, because the unit weight of the impounded fluid may actually increase, due to gradual or sudden silting of the reservoir, depending on the character of the drainage area. According to Charles Terzaghi,[35] M. Am. Soc. C. E., the consolidation of mud deposits is an extremely slow process, so that the full fluid pressure may be exerted over a considerable period of time.

The French Commission, which reported on the third failure of the La Habra Dam,[36] which was of the gravity type, has estimated a fluid weight of 81 lb. per cu. ft. for the impounded mud and water caused by a sudden torrential flood.

Under such conditions, a higher compressive stress would be entirely justified on account of the increase in the compressive strength of concrete with age. It is much more important to avoid tensile stresses in the concrete and sliding of the buttresses under such extreme, but not impossible, load conditions. As pointed out by Mr. Girand, this large factor of safety can be obtained with comparatively small additional quantities, probably not exceeding 10% in most cases.

A conservative building code for dams, specifying a fluid weight of 90 lb. per cu. ft., a weight of concrete of 140 lb. per cu. ft., a principal compressive stress of 600 lb. per sq. in., no tension, and a maximum sliding factor of 0.70, would lead to an adequate design, consistent with the importance of such structures. Of course, the same specifications should apply to gravity dams.

The question, whether or not expansion joints are advisable, is mostly a matter of convenience and economy, a decision which also depends on the foundation conditions at each particular site. The fact that concrete is subject to volumetric changes due to shrinkage and temperature stresses must be recognized in the design. The joints may be omitted for dams of moderate height, supported throughout on solid bed-rock of uniform elastic properties, provided a liberal amount of reinforcing steel is used and the concrete is cured wet for at least 30 days. For higher dams, and in case of non-uniform foundation conditions, the provision of permanent joints has decided advantages, because large masses of concrete are thereby subdivided into convenient construction units, and the amount of reinforcing steel may be considerably reduced.

---

[34] Chf. Engr., Thebo, Starr & Anderton, Inc., San Francisco, Calif.

[35] "The Science of Foundations—Its Present and Future," by Charles Terzaghi, *Transactions*, Am. Soc. C. E., Vol. 93 (1929), p. 278.

[36] "La Rupture du Barrage de L'Habra," *Le Génie Civil*, March 24, 1928, p. 283.

Mr. Girand suggests locating expansion joints, if any, in the direction of the second principal stress. Evidently, a joint so placed could not act as intended, because the first principal stress would prevent movement on account of friction. His proposed method of placing concrete would certainly be the ideal way. In case of a monolithic buttress most of the reinforcing steel should be placed in the direction of the second principal stress, as pointed out by Mr. Girand in Fig. 14.

Mr. Noetzli has evidently worked along similar lines in the design and, therefore, has come to fairly identical conclusions. The writer is aware of the fact that inclined joints have been used in several dams before this paper was published; but not, however, before the time of his original conception and formal presentation. This feature is inherent with the principle of self-supporting arched column units, where the columns may be built integral or adjacent to each other, and therefore, the writer does not hesitate to claim priority in this matter.

The joints shown by Mr. Noetzli in Fig. 15 and Fig. 16 stop a short distance below the deck, causing a local stress concentration, which could be eliminated by extending the joint through the deck. Mr. Noetzli mentions that in some cases the stress analysis indicated tension in the direction of the second principal stress which undoubtedly made it necessary to stop the joint below the deck and to provide extra steel reinforcement in the deck zone.

It is not sufficient that only the shearing stresses be zero along the plane of the joint; it is still more important that the normal stresses (second principal stress) should not be tension. Steel reinforcement cannot generally be relied upon to transmit tensile stresses to the foundation and a slight amount of compression in the direction of the second principal stress is, therefore, desirable.

With the ordinary buttress shape of rectangular horizontal cross-section and other arbitrary dimensions the elimination of tension is a rather difficult problem. This is especially true in case of a steep down-stream face combined with a steep up-stream face, as in the case of the Gleno Dam, analyzed by A. Stucky.[37]

Mr. Noetzli's Fig. 16 is very instructive because it indicates a fairly close agreement between the σ_x stresses, as derived from the arched columns and those computed for the monolithic buttress by means of the trapezoidal rule.

His comparison of the round-head buttress with the cantilever type of deck is interesting, but not convincing. The stresses in a disk-shaped member, such as Fig. 17(*a*), cannot be computed by the ordinary beam theory. The tests by E. Moersch[38] have shown conclusively the beneficial influence of the haunches on the reduction of the shearing stresses. For a more exact analysis, supported by photo-elastic stress measurements, the reader is referred to the work of Akira Miura.[39]

---

[37] "Der Talsperrenbruch im Val Gleno," by A. Stucky, *Schweizerische Bauzeitung*, Vol. 83 (1924), p. 74.

[38] "Der Eisenbetonbau," by E. Moersch, Vol. I, Second Half, Fifth Edition, pub. by Konrad Wittwer, Stuttgart, 1922, p. 197.

[39] "Spannungskurven in rechteckigen und keilförmigen Trägern," by Akira Miura, pub. by Julius Springer, Berlin, 1928.

The rounding of the up-stream face may have a favorable influence on the stress distribution, but the total saving in concrete, if any, will be small, because the yardage in the deck is a small percentage of the total yardage in the dam. Since the deck is a very vital part of the structure the writer would not advocate the omission of reinforcing steel in a deck of this type. It should be pointed out that the dead load stresses are not radial and the possibility of shrinkage and temperature cracks, as well as earthquake stresses, may lead to serious consequences in the case of a plain concrete deck. Since reinforcing steel should be provided in either case, the advantages of a straight cantilever deck in the ease of placing steel and form work are evident.

On account of his pioneer work in the analysis of principal stresses in multiple-arch dams,[40] Mr. Jakobsen is well qualified to appreciate the possible advantages of a direct design method. The result of his comparison in a practical case is gratifying and demonstrates the usefulness of the proposed principle.

As pointed out by Mr. Jakobsen, the arrangement shown in Fig. 5 introduces the possibility of an upward thrust on the deck. This uplift force should be combined with the other forces acting on the monolithic column head, as mentioned by the writer under the heading, "Design Example."

The writer thoroughly agrees with Mr. Jakobsen that inclined joints, if used at all, should be continuous through deck and buttress down to the foundation.

Mr. Davis prefers the use of a more laborious method, because in his opinion the results obtained by considering the buttress as a monolith will be less favorable than those obtained from the independent arch columns. If the adjacent arched columns were not designed for the same compressive stress throughout, then, the monolithic and independent stress system could not be identical. By computing an actual design example, Mr. Davis will convince himself that the results are almost identical with those obtained by the trapezoidal rule. In order to be consistent, the same assumption regarding the action between deck and buttress, either according to Fig. 4 or Fig. 5, must be made in both stress computations.

Mr. Davis correctly states that the method can only be applied for "reservoir full." For lower reservoir levels the principal stress system must necessarily be different. Perhaps he overlooks the fact that the proposed principle is not a stress analysis method, but only a means to dimension the structure directly for the most dangerous case of loading under given design assumptions. If several possible load conditions could produce maximum or minimum stresses of the same order, then, such a direct method could not give the desired result.

In the design of dams there are generally only two important cases of loading, namely "reservoir full" and "reservoir empty." In gravity dams, both these load conditions produce maximum and minimum stresses of the same order. In arch dams and buttressed dams, however, the most dangerous load condition is found with reservoir full, so that a structure dimensioned for this case will generally be safe under all other load conditions. In no case, however, should the designer neglect to investigate other conditions of loading, even if they are apparently safe. The analysis by means of the trapezoidal rule will be sufficient for such cases.

---

[40] "Stresses in Multiple-Arch Dams," by B. F. Jakobsen, M. Am. Soc. C. E., *Transactions*, Am. Soc. C. E., Vol. LXXXVII (1924), p. 276.

Mr. Eremin points out the possibility of secondary deck stresses due to unequal column deformations. If the deck is separated from the monolithic buttress by means of a continuous joint, as shown in Fig. 4, then the buttress can only deform as a whole. Since the deck is comparatively much more flexible than the stiff buttress, it is evident that the secondary deck stresses are small, but their magnitude may be determined from the buttress deformation.

The writer agrees with Mr. Eremin that the arrangement of Fig. 4 (independent deck) should not be combined with independent column units because the relative deformation of the individual column units may introduce high local deck stresses near the column corners. With the arrangement of Fig. 5 the possible interaction between individual deck strips can practically be eliminated by providing a plastic filler in the deck joint area.

Mr. Eremin points out that the foundation deformations should be considered in an exact design of the buttress. These deformations may be estimated according to the methods proposed[41] by Fredrik Vogt, Assoc. M. Am. Soc. C. E.

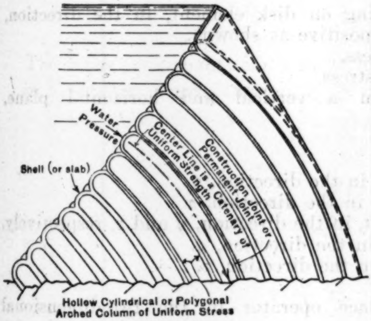

Water Pressure
Shell (or slab)
Center Line is a Uniform Strength Catenary of
Construction Joint or Permanent Joint
Hollow Cylindrical or Polygonal Arched Column of Uniform Stress

FIG. 38.

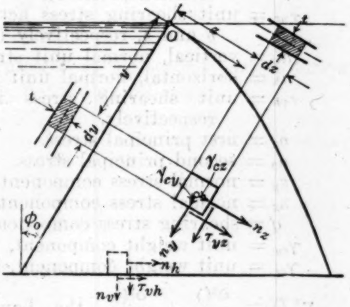


FIG. 39.

Mr. Eremin's suggestion to increase the possible height of Dr. Rossin's buttress type by the use of hollow columns, is interesting. Fig. 38 shows the adaptation of the arched column principle to a tentative design with reinforced hollow column sections. The construction of Dr. Rossin's type, or a type similar to that of Fig. 38, is economical in locations where cheap skilled labor is available in connection with comparatively expensive material.

Professor Kalman's able discussion is based mostly on theoretical considerations and, therefore, can be answered only on that basis. For this purpose it will be necessary to introduce the fundamental relations of the theory of elasticity, as applied to structures of this type, with the usual assumptions made for the elastic properties of the material.

---

[41] *Transactions*, Am. Soc. C. E., Vol. 90 (1927), p. 554.

Under the heading "Comparison with the Principle of Linear Stress Distribution," Professor Kalman states:

> "Should the buttress be treated as a column when it is monolithic? Decidedly, it is not a column. Should it be treated as a beam? If so, the trapezoidal rule should be applied to the individual sections, and the author is opposed to the application of this rule."

The buttress is neither a column nor a beam, but it is a three-dimensional elastic body. Since one dimension is small compared to the other dimensions it may be defined as a disk of variable thickness, symmetrical about its middle plane. This disk, in contrast to a plate or slab, is assumed to be loaded by forces acting in the direction of the middle plane only. If the thickness is comparatively small, then the stresses in the direction normal to the middle plane can be neglected, resulting in a two-dimensional, or plane (not necessarily straight-line) stress system, as defined by Föppl.[42]

With reference to Fig. 39, the following definitions are introduced:

O = origin of co-ordinate system.
y, z = co-ordinates of a small element, dy, dz.
t = thickness of buttress, given as a function of y and z.
n_y = normal unit stress on small element, acting in the direction, y (tension positive).
n_z = normal unit stress on small element, acting in the direction, z (tension positive).
τ_{yz} = unit shearing stress acting on disk element, in the direction, y and z, respectively (positive as shown).
n_v = vertical, normal unit stress.
n_h = horizontal, normal unit stress.
τ_{vh} = unit shearing stress in a vertical and horizontal plane, respectively.
n_1 = first principal stress.
n_2 = second principal stress.
s_y = normal stress component in the direction, y.
s_z = normal stress component in the direction, z.
q = shearing stress component, in the direction, y and z, respectively.
γ_{cy} = unit weight component, in the direction, y.
γ_{cz} = unit weight component, in the direction, z.
$\nabla^2 () = \frac{\partial^2()}{\partial y^2} + \frac{\partial^2()}{\partial z^2}$, the Laplace operator in the two-dimensional system, y, z.

For the purpose of the following analysis it will be most convenient to consider the stress components, instead of the unit stresses, acting on a small element. The stress components are:

$$s_y = t\, n_y \qquad (75)$$

$$s_z = t\, n_z \qquad (76)$$

$$q = t\, \tau_{yz} \qquad (77)$$

---

[42] "Drang und Zwang," by August Föppl and Ludwig Föppl, Vol. I, Second Edition, pub. by R. Oldenbourg, Munich and Berlin, 1924, p. 244.

In order to maintain equilibrium on a small element the following well-known fundamental equations apply,

$$\frac{\partial s_y}{\partial y} + \frac{\partial q}{\partial z} + t\,\gamma_{cy} = 0 \quad \text{.....................(78)}$$

and,

$$\frac{\partial s_z}{\partial z} + \frac{\partial q}{\partial y} + t\,\gamma_{cz} = 0 \quad \text{.....................(79)}$$

As shown by Föppl,[42] the compatibility between stresses and deformations also requires compliance with the following equation,

$$\nabla^2 (n_y + n_z) = 0 \quad \text{.........................(80)}$$

or, expressed by the stress components,

$$\nabla^2 \left(\frac{s_y}{t} + \frac{s_z}{t} \right) = 0 \quad \text{...................(81)}$$

For a given load system and given buttress dimensions, the three unknowns, s_y, s_z, and q, must be determined by integration of the three simultaneous, differential Equations (78), (79), and (81).

Since t is expressed as a given function of y and z, a function, $T\,(y, z)$, may be determined by partial integration of t with respect to y and z, so that,

$$t = \frac{\partial^2 T}{\partial y\, \partial z} \quad \text{..........................(82)}$$

The stress components can now be expressed as the second partial derivatives of a common stress function, $F\,(y, z)$, defined as follows:

$$s_y = \frac{\partial^2 F}{\partial z^2} - \frac{\partial T}{\partial z}\gamma_{cy} \quad \text{.........................(83)}$$

$$s_z = \frac{\partial^2 F}{\partial y^2} - \frac{\partial T}{\partial y}\gamma_{cz} \quad \text{.........................(84)}$$

and,

$$q = -\frac{\partial^2 F}{\partial y\, \partial z} \quad \text{..............................(85)}$$

By substituting Equations (82), (83), (84), and (85) for s_y, s_z, q, and t, in Equations (78) and (79), it may be seen that the equilibrium equations are satisfied.

The compatibility Equation (81) then becomes, by substituting s_y and s_z from Equations (83) and (84),

$$\nabla^2 \left(\frac{1}{t}\frac{\partial^2 F}{\partial y^2} + \frac{1}{t}\frac{\partial^2 F}{\partial z^2} - \frac{1}{t}\frac{\partial T}{\partial y}\gamma_{cz} - \frac{1}{t}\frac{\partial T}{\partial z}\gamma_{cy} \right) = 0 \quad \text{.....(86)}$$

Equation (86) represents the general differential equation of the desired stress-function in the most general case of a buttress of variable thickness, subject to any external loads acting in the middle plane, and gravity action.

---

[42] "Drang und Zwang," by August and Ludwig Föppl, Vol. 1, Second Edition, 1924, pp. 247 and 326.

If the gravity action is neglected, then Equation (86) becomes,

$$\nabla^2\left(\frac{1}{t}\frac{\partial^2 F}{\partial y^2}+\frac{1}{t}\frac{\partial^2 F}{\partial z^2}\right)=0 \quad \ldots\ldots\ldots\ldots\ldots (87)$$

Equation (87) is given by Föppl,[44] but the general integral is not known and, therefore, neither is that of Equation (86). For a disk of constant thickness (t constant), Equation (87) is reduced to the well-known differential equation for Airy's stress function,

$$\nabla^2\left(\frac{\partial^2 F}{\partial y^2}+\frac{\partial^2 F}{\partial z^2}\right)=0 \quad \ldots\ldots\ldots\ldots\ldots (88)$$

or, written in the more common form,

$$\frac{\partial^4 F}{\partial y^4}+2\,\frac{\partial^4 F}{\partial y^2\,\partial z^2}+\frac{\partial^4 F}{\partial z^4}=0 \quad \ldots\ldots\ldots\ldots\ldots (89)$$

The general integral of Equation (89) is a complex function, as given by Föppl,[45] but such a solution only applies to a buttress of constant thickness, or a slice of a gravity dam section, contained between two parallel vertical planes, normal to the up-stream face.

Any exact solution of the most general case must be based on Equation (86). For a given load system and given buttress dimensions there can exist only one stress function, which satisfies this differential equation and such a stress function then defines the exact stress system.

As pointed out by Professor Kalman, the proposed method of dimensioning a buttress of uniform strength results in a deviation from the trapezoidal rule, especially in the case of a vertical up-stream face. The following analysis, therefore, will be derived for a vertical up-stream face, in which case the maximum discrepancy from the exact solution can be expected. With $\phi_0 = 0$, and the designations of Fig. 39, from Equations (2), (4), and (10), in the case of a weightless deck, the thickness of the buttress at any point is defined by the equation,

$$t=\frac{b\,\gamma_w}{c\,\gamma_c}\left(y+c\log\cos\frac{z}{c}\right)\sec^2\frac{z}{c} \quad \ldots\ldots\ldots\ldots\ldots (90)$$

The function, T, defined by Equation (82), is found by partial integration of Equation (90) with respect to y and z, which gives,

$$T=\frac{b\,c\,\gamma_w}{\gamma_c}\left[\tan\frac{z}{c}\left(1+\frac{y}{2\,c}+\log\cos\frac{z}{c}\right)-\frac{z}{c}\right]y \quad \ldots\ldots (91)$$

For $\phi_0 = 0$, $\gamma_{cy} = \gamma_c$ and $\gamma_{cz} = 0$, therefore,

$$\frac{\partial T}{\partial y}\,\gamma_{cz}=0 \quad \ldots\ldots\ldots\ldots\ldots (92)$$

and,

$$\frac{\partial T}{\partial z}\,\gamma_{cy}=b\,\gamma_w\left(\frac{y}{2\,c}+\log\cos\frac{z}{c}\right)y\sec^2\frac{z}{c} \quad \ldots\ldots\ldots\ldots (93)$$

---

[44] "Drang und Zwang," Vol. I, Second Edition, 1924, p. 326.

[45] *Loc. cit.*, p. 252.

The desired stress function, F, as given by the following equation has been found by trial:

$$F = -b\,\gamma_w \left[\frac{y^3}{6} + \frac{c\,y}{2}\left(y + c \log \cos \frac{z}{c} \right) \log \cos \frac{z}{c} \right.$$
$$\left. + c \int \int \tan^2 \frac{z}{c} \log \cos \frac{z}{c}\, dz\, dz \right] \ldots\ldots\ldots\ldots\ldots (94)$$

With reference to Equations (83), (84), (85), (92), and (93), the following values of the stress components are obtained from Equation (94),

$$s_y = -\,b\,\gamma_w \left(y + c \log \cos \frac{z}{c} \right) \tan^2 \frac{z}{c} \ldots\ldots\ldots\ldots\ldots (95)$$

$$s_z = -\,b\,\gamma_w \left(y + c \log \cos \frac{z}{c} \right) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (96)$$

and,

$$q = -\,b\,\gamma_w \left(y + c \log \cos \frac{z}{c} \right) \tan \frac{z}{c} \ldots\ldots\ldots\ldots\ldots (97)$$

At the up-stream face, with $z = 0$, the stress components give the following values,

$$s_y = 0 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (98)$$

$$s_z = -\,b\,\gamma_w y \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (99)$$

$$q = 0 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (100)$$

These boundary conditions evidently correspond to a buttress of unlimited height, with the water level at the origin, O, of the co-ordinate system, as shown in Fig. 39, and a weightless deck of the width, b.

With reference to Equations (75), (76), (77), (90), and c from Equation (1), the unit stresses at any point of the buttress become,

$$n_y = \frac{s_y}{t} = -\,f_c \sin^2 \frac{z}{c} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (101)$$

$$n_z = \frac{s_z}{t} = -\,f_c \cos^2 \frac{z}{c} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (102)$$

and,

$$\tau_{yz} = \frac{q}{t} = -\,f_c \sin \frac{z}{c} \cos \frac{z}{c} \ldots\ldots\ldots\ldots\ldots (103)$$

The sum of the normal stresses at any point becomes, therefore,

$$n_y + n_z = -\,f_c \left(\sin^2 \frac{z}{c} + \cos^2 \frac{z}{c} \right) = -\,f_c \ldots\ldots\ldots\ldots (104)$$

Substituting Equation (104) in the compatibility Equation (80), and considering that f_c is a constant,

$$\nabla^2 (n_y + n_z) = \frac{\partial^2 (-f_c)}{\partial y^2} + \frac{\partial^2 (-f_c)}{\partial z^2} = 0 \ldots\ldots\ldots\ldots (105)$$

Since the stress function, Equation (94), complies with the compatibility Equation (80), it also satisfies the differential Equation (86), as well as the equilibrium Equations (78) and (79) and, therefore, defines the actual stress system for the given buttress shape and the aforementioned boundary conditions.

From Equations (101), (102), and (103), the principal stresses, n_1 and n_2, at any point, are found from the well-known equations;

$$n_1 = \frac{n_y + n_z}{2} - \frac{1}{2}\sqrt{4\,\tau^2_{yz} + (n_y - n_z)^2} = -f_c \;\ldots\ldots\ldots(106)$$

and,

$$n_2 = \frac{n_y + n_z}{2} + \frac{1}{2}\sqrt{4\,\tau^2_{yz} + (n_y - n_z)^2} = 0 \;\ldots\ldots\ldots(107)$$

Designating the direction of the first principal stress by the angle, β, with the direction, z, then,

$$\beta = \frac{1}{2}\tan^{-1}\frac{2\,\tau_{yz}}{n_z - n_y} = \frac{z}{c} \;\ldots\ldots\ldots(108)$$

For a vertical up-stream face, from Equation (4),

$$\frac{z}{c} = \phi_1 \;\ldots\ldots\ldots(109)$$

Equation (109) indicates that the direction of the first principal stress at any point coincides with the direction of the tangent of the catenary of uniform strength through that point. Equation (106) shows that the first principal stress at any point is constant, compression, and equal to the design stress, f_c. Equation (107) shows that the second principal stress is zero at any point.

From the foregoing derivations it must be concluded, therefore, that the buttress dimensioned according to Equation (90) actually gives the exact solution for the buttress of uniform strength in the case of unlimited height, vertical up-stream face, weightless deck, and full reservoir.

A similar solution also can be derived in case of a buttress of unlimited height, the up-stream face of which is inclined at the angle, ϕ_0, to the vertical, and the thickness at any point defined by the equation,

$$t = \frac{b\,\gamma_w \cos\phi_0\, y}{f_c} \;\ldots\ldots\ldots(110)$$

The shape of such a buttress is similar to that of the upper part of Fig. 2, consisting of a triangular up-stream face, triangular horizontal cross-sections, and a straight down-stream face, normal to the plane of the up-stream face. The proposed method gives such a shape in case the unit weight of the concrete is assumed to be zero and the corresponding stress function is,

$$F = -\,b\,\gamma_w \cos\phi_0 \frac{y^3}{6} \;\ldots\ldots\ldots(111)$$

The unit stresses, by similar derivations as the preceding, are:

$$n_y = 0 \;\ldots\ldots\ldots(112)$$

$$n_z = -\,f_c \;\ldots\ldots\ldots(113)$$

$$\tau_{yz} = 0 \;\ldots\ldots\ldots(114)$$

Since the shearing stress is zero, the co-ordinate system, in this case, also defines the principal stress system, the first principal stress being constant at any point, and again equal to the design stress, f_c, the second principal stress being zero. Such a buttress, therefore, can be conceived as built up of straight, weightless columns, the axes of which are normal to the up-stream face. The vertical stresses in a horizontal section are also uniform and, in this case only, are identical with those obtained from the trapezoidal rule.

Since the proposed design principle represents the exact solution for the two most extreme cases it is logical to conclude that the same theory also applies to intermediate cases and, therefore, Professor Kalman's sweeping statement, to the effect that there is absolutely no proof of its validity, must be completely rejected.

With the designations of Fig. 39, the stresses in a horizontal section, from Equations (98), (99), (103), and (109), become,

$$n_v = -f_c \sin^2 \phi_1 \qquad (115)$$

$$n_h = -f_c \cos^2 \phi_1 \qquad (116)$$

and,

$$\tau_{vh} = -f_c \sin \phi_1 \cos \phi_1 \qquad (117)$$

Equations (115) and (117) are identical with Professor Kalman's Equations (49) and (50). However, the writer is unable to check Professor Kalman's stress diagrams, Fig. 29 (*b*) and Fig. 30 (*b*), because the angle, ϕ_1, (see Fig. 1), can never assume a value of 90° and for this reason the shearing stresses, as computed from Equation (50), or Equation (117), can never become zero at the down-stream face.

Professor Kalman's general conclusions, based on these diagrams, are therefore misleading, especially also due to the fact that the assumptions of Figs. 29 and 30 are not clearly stated.

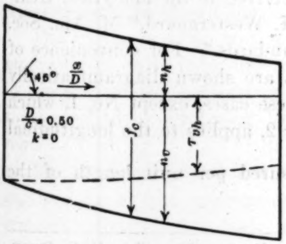


FIG. 40.

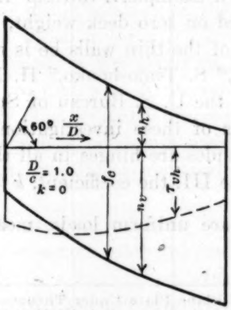


FIG. 41.

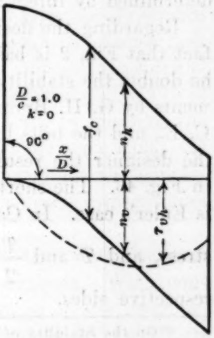


FIG. 42.

Fig. 40 shows the normal and shearing stresses in a horizontal section, based on a buttress with $\frac{D}{c} = 0.50$, $\phi_0 = 45°$, and $k = 0$. These conditions, except for the weightless deck, are typical within the ordinary practical range. The stress diagram indicates that, in this case, the assumption of the trapezoidal rule for the n_v stresses would give a satisfactory agreement with the actual stress conditions.

Fig. 41 is based on a buttress with $\frac{D}{c} = 1.0$, $\phi_0 = 30°$, and $k = 0$, and represents approximately the conditions at the upper practical range, because an even steeper up-stream slope would generally give sliding factors that are too high. It will be noted that the deviation from the trapezoidal has increased. Although the discrepancy is not large, the shape of the n_v-stress diagram indicates that the trapezoidal rule would not necessarily give stresses on the safe side.

Fig. 42 illustrates the conditions in the case of a vertical up-stream face which is only of theoretical interest due to excessively high sliding factors combined with a waste of material. The trapezoidal rule would indicate a triangular n_v-stress diagram, which, in this case, is a rather poor approximation to the actual stress conditions.

Of course, the foregoing comparisons with the trapezoidal rule only apply to a buttress of uniform strength and do not permit any general conclusions for other buttress shapes, especially if such shapes deviate greatly from those obtained with the proposed design method.

However, these typical examples sufficiently demonstrate that in a reasonably economical design a fairly close agreement with the actual conditions can be expected from a principal stress system derived by means of the trapezoidal rule, provided the up-stream slope is not too steep. The satisfactory results obtained by Mr. Noetzli and Mr. Jakobsen seem to support this point. There is absolutely no objection to a comparison of the findings in this manner as long as the designer keeps in mind the fact that the trapezoidal rule is an assumption and that the actual stress system can only be determined by integration of the differential Equation (86).

Regarding the design example, Professor Kalman evidently overlooks the fact that Fig. 2 is based on zero deck weight, whereas Fig. 11 is not. Since he doubts the stability of the thin walls he is referred to the analytical treatments by G. H. Bryan,[46] S. Timoshenko,[47] H. M. Westergaard,[48] M. Am. Soc. C. E., and the tests by the U. S. Bureau of Standards.[49] For convenience of the designer the results of these investigations are shown diagrammatically in Fig. 43. The short sides are hinges in all these cases, except No. I, which is Euler's case. In Case III, the coefficient, $k = 2$, applies to the longitudinal stress, and T and $\frac{T}{2}$ are uniform loads, measured per unit length of the respective sides.

---

[46] "On the Stability of a Plane Plate Under Thrusts in Its Own Plane," by G. H. Bryan, *Proceedings*, London Mathematical Soc., Vol. XXII, 1890, p. 54.

[47] "Einige Stabilitätsprobleme der Elastizitätstheorie," by S. Timoshenko, *Zeitschrift für Mathematik und Physik*, Vol. 58, 1909-10, p. 337; and "Applied Elasticity," by S. Timoshenko and T. M. Lessels, Westinghouse Technical Night School Press, Pittsburgh, Pa., 1925, p. 291.

[48] "Buckling of Elastic Structures," by H. M. Westergaard, *Transactions*, Am. Soc. C. E., Vol. LXXXV (1922), p. 576.

[49] "Compressive Strength of Column Web Plates and Wide Web Columns," *Technologic Papers No. 327*, U. S. Bureau of Standards, Washington, D. C., Govt. Printing Office, 1926.

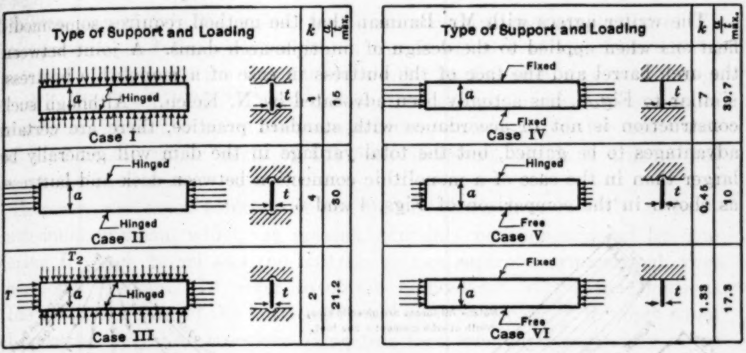


FIG. 43.—BUCKLING STRESS OF THIN PLATES: $F_{cr} = k \frac{\pi^2 E}{12} \left(\frac{t}{d}\right)^2$

Mr. Floris doubts that Castigliano's theorem can be applied to the stress distribution in gravity dam profiles and still less to a buttress type, as suggested in this paper, which is three-dimensional. Föppl[50] has shown conclusively also that in a three-dimensional elastic body the actual stress system will produce the least amount of work. In fact, every elastic body has, of course, three dimensions and the definition of one-dimensional, or two-dimensional stress systems only serves to introduce permissible simplifications in the analysis when certain dimensions are comparatively small.

Since the general Equation (86) defines the actual stress system, it also conforms to the principle of least work and so do Equations (94) and (111) in the special case of a buttress of uniform strength.

The writer is aware of the fact that other authors have treated the same subject before this paper was published. In connection with Paul Ziegler's work, however, it should be noted that the writer's paper was first submitted March 12, 1929. The arched columns proposed by Mr. Ziegler, as illustrated by Fig. 31, appear to be a rather crude solution of the problem, because they intersect each other, a condition which gives an entirely impossible principal stress system, as shown for instance by Mr. Davis in Fig. 23.

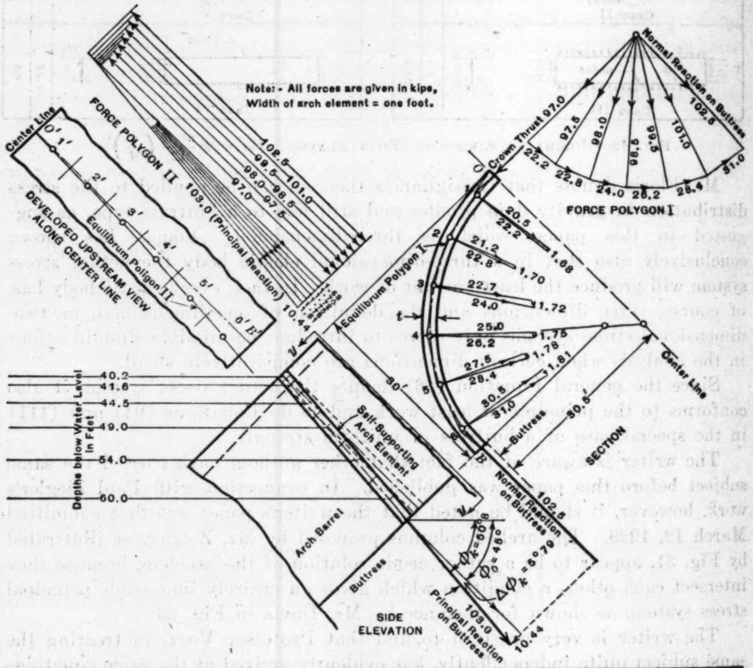
The writer is very gratified to find that Professor Vogt, in treating the same subject quite independently, has evidently arrived at the same equations in the case of the buttress of absolute minimum weight. Since the conclusions reached by Professor Vogt are almost identical in every respect there remains nothing to be added.

Mr. Schwager points out the progress made in mechanical and electrical engineering by the application of exact mathematical analysis. With concrete as a construction material and a foundation with more or less irregular elastic properties even a theoretically correct solution becomes necessarily an approximation. However, the stress system obtained from exact analytical methods under ideal elastic conditions must always serve as the most reliable guide in a practical design.

---

[50] "Drang und Zwang," by August Föppl and Ludwig Föppl, Vol. I, Second Edition, 1924, p. 99.

The writer agrees with Mr. Bauman that the method requires some modifications when applied to the design of multiple-arch dams. A joint between the arch barrel and the face of the buttress in case of a monolithic buttress, similar to Fig. 4, has actually been advocated by N. Kelen.[51] Although such construction is not in accordance with standard practice, there are certain advantages to be gained, but the total yardage in the dam will generally be larger than in the case of a monolithic connection between deck and buttress, as shown in the comparison of Figs. 4 and 5.


Note: - All forces are given in kips, Width of arch element = one foot.
FORCE POLYGON II
102.6
101.0
99.5
98.5
98.0
97.5
97.0
103.0 (Principal Reaction)
10.44
Center Line
0'
1'
2'
3'
4'
5'
6' B'
Equilibrum Polygon II
DEVELOPED UPSTREAM VIEW ALONG CENTER LINE
Normal Reaction on Buttress
102.6
Crown Thrust 97.0
97.5
98.0
98.5
99.5
101.0
22.2
22.8
24.0
26.2
28.4
31.0
FORCE POLYGON I
Equilibrum Polygon I
O
1
2
3
4
5
6
B
t
20.5
22.2
21.2
22.8
22.7
24.0
25.0
26.2
27.5
28.4
30.5
31.0
1.68
1.70
1.72
1.75
1.78
1.81
30.5
Buttress
SECTION
Center Line
40.2
41.6
44.5
49.0
54.0
60.0
Depth below Water Level in Feet
Self-Supporting Arch Element
Arch Barrel
Buttress
Normal Reaction on Buttress
102.6
φ = 45°
φ_k = 50°8'
Δφ_k = 5°79'
Principal Reaction on Buttress
103.0
10.44
SIDE ELEVATION

Arch Voussoir	Upstream Length ft.	Water Depth ft.	Unit Weight lb. per cu. ft.	Water Pressure lb.	Mean Length ft.	Mean Thickness ft.	Unit Weight lb. per cu. ft.	Weight of Voussoir lb.	Component normal and parallel to Arch Axis
1	8.15	40.2	62.5	20 500	7.9	2.01	150	2 380	1 680
2		41.6		21 200		2.03		2 410	1 700
3		44.5		22 700		2.05		2 430	1 720
4		49.0		25 000		2.08		2 470	1 750
5		54.0		27 500		2.12		2 520	1 780
6		60.0		30 500		2.18		2 580	1 810

FIG. 44.

In order to apply the proposed method to a monolithically connected arch barrel it will be consistent to consider self-supporting arch elements of uniform strength, which are curved in two directions, as shown in Fig. 44.

---

[51] "Die Staumauern," by N. Kelen pub. by Julius Springer, Berlin, 1926, p. 171.

It follows then that in the case of long span arches a large proportion of the water pressure and the arch weight in the lower part of the deck will be transmitted directly to the foundation, similar to the arch ring action suggested by Professor Vogt.[52] Nevertheless, the yardage in the buttress proper will still exceed that of short span construction, on account of the additional dead load in the upper parts of the deck structure, which is transmitted through the buttress.

However, the exact solution of the multiple-arch dam is truly a three-dimensional problem, which, at present, can only be approximated by considering the arch barrel and the buttress as two separate, symmetrical, two-dimensional systems. In order to obtain coincidence of the deformations at the up-stream face of the buttress, it will then be necessary to apply equal and opposite, corrective reactions along this face, whereby a secondary stress system can be superimposed to the main stress systems in the arch-end buttress, respectively. The proposed method enables the designer to determine the main buttress deformation due to the shortening effect of the arched columns, as indicated in Fig. 1, Line *FT′*. In a more exact analysis the foundation deformations should also be taken into consideration in similar manner.

Mr. Bauman's statement that less work is required in shortening the arched columns by bending rather than concentric pressure, is not confirmed by the exact analysis, as derived in answer to Professor Kalman's discussion in case of a monolithic buttress. However, the writer agrees with Mr. Bauman regarding the possibility of eccentric column pressures due to partial loads in case of inclined joint construction. For this reason, a normal joint has been provided near the foundation in Fig. 11, containing a sheet-lead filler to allow slight plastic deformations and a redistribution of secondary stresses.

The elimination of friction in the inclined joints does not present any serious problem. With reservoir full there are no normal forces acting across the joint, and the columns can deform freely. With partial loads and reservoir empty there will be a normal component acting across the joint. However, with the usual up-stream slopes, the friction coefficient will seldom be less than 1.0, so that movement could still take place, even with reservoir empty.

Mr. Nelidov contributes a very desirable and instructive comparison of the buttress stresses as given by the proposed catenary method and those obtained by analyzing the same buttress by the ordinary method. According to Mr. Nelidov's Equation (62), the latter method is identical with the trapezoidal rule, commonly assumed to be valid for the vertical normal stress distribution in horizontal sections.

For the buttress investigated, with $\frac{D}{c} = 0.40$, $\phi_0 = 30°$, and $k = 0.10$, the agreement between the two methods is quite satisfactory, whereby the conclusions regarding the possible accuracy of the trapezoidal rule, in answer to

---

[52] *Transactions*, Am. Soc. C. E., Vol. 93 (1929), p. 1288.

Professor Kalman's discussion, are further confirmed by Mr. Nelidov's careful analysis for the given case.

In conclusion, the writer wishes to express his sincere thanks to all those who have so freely discussed this paper. It is gratifying to note that the discussions have been constructive throughout, thereby contributing many important points not treated in the original paper.

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
TRANSACTIONS

Paper No. 1804

RELOCATING LAND SURVEYS IN TEXAS[1]

By B. F. Williams,[2] M. Am. Soc. C. E.

With Discussion by Messrs. J. M. Howe, Arthur D. Kidder, and W. J. Powell.

Synopsis

This paper describes the problems encountered by engineers in Texas who are engaged in relocating old survey lines. The conflicting influence of Spain and England on the land laws are discussed, and the modern equivalents of old and obsolete units of measurement are given. The paper includes also an interesting anecdote concerning the monumenting of the 100th meridian.

The principal emphasis, however, is directed toward outlining some practical rules to govern the engineer on relocation surveys. Of what use to the surveyor is the evidence pertaining to annual rings of old trees? How is it possible to obtain direct evidence of the location of a timber line? How was the land cleared? The answers to these and other important questions are given in this paper.

Original Marks

It is a well fixed principle of law that in relocating surveys it is the duty of the surveyor to follow the boundary lines as laid on the ground in the original survey. "Move not the ancient land marks" is an old legal maxim given by Moses in Deuteronomy. Land boundary litigation has burdened the Civil Court dockets since the beginning of private ownership of the soil, and many wars between kings, princes, barons, and lords have been fought over disputed boundaries.

Indefiniteness in description and inaccuracy in laying down the original boundaries cause most of such litigation. As an example the following is cited from the archives of the Texas General Land Office:

"In the same Town on the same day, month, and year that precedes, Don Juan de la Garza and Don Pedro de Salvana, the Surveyors, the former on the part of His Majesty and the latter on that of the Town, came into our presence and said: That in conformity with the Act made known to them on the third, they proceeded to mark out the boundaries assigned to the Town on the south, and using the cord composed of fifty Mexican varas recognised by the Commissioners present, they stretched the same one hundred times (beginning at the center of the Town) which make five thousand Mexican varas or one complete league, with which they arrived at the point known as La Cantera, which they marked out; with as many more they reached a vale with woods which are transitable and which is named the stopping place called Senor San Jose; with as many more to the gully at the foot of the hills of Los Quatitos and they dedicated it to our Senor of San Juan; with as many more to the stopping place called Agua-pas and dedicated it to our Lady of La Sotemano; with as many more to a hollow in front of Abra del Quemado which they dedicated to our Lady of Dolores; with as many more to the margin of the River Blanco which they dedicated to our Lady of Loreto, and they are the six leagues which in accordance with the order communicated to them they have surveyed and marked out, each league for itself on said course. And from said plain they returned to the Town and directing themselves to the North from its center they stretched the cord one hundred times and arrived at the Palitos Blancos and dedicated it to our Lady of Santa Ana; and with another hundred cords commencing on the other bank of the Rio Grande del Norte (for the reason that but a few varas remained between the said league and the river) they arrived at a hill which they dedicated to San Joachin and marked out the second league; with as many more to an opening in the hills (Abra) called El Yachupin which they dedicated to San Francisco de Borja; with as many more to the end of the said opening in the hills to the place called San Lorenzo; with as many more to the opening in the hills of San Yldepfonso which they dedicated to Santa Theresa de Jesus; and with as many more to a hill which is on one side of the stopping place called San Yldepfonso which they dedicated to La Santissima Trinidad, which six leagues they surveyed and marked out in the same manner as those above specified, always following the said course North. And this is what they did in compliance with their duties, and without having any thing to add, they signed with us and those of our assistance.=Palacio.=Lizos Ossino.=Juan Jph de la Garza Falcon.=Pedro Joseph Ygnacio de Salvana.=of Assistance, Duran.=of Assistance Federico Losada."

---

[1] Presented at the meeting of the Surveying and Mapping Division, Dallas, Tex., April 25, 1929.

[2] State Reclamation Engr., Austin, Tex.

Such deficiencies as these are the causes of the greatest difficulties to the surveyor in making a relocation. The boundaries of a survey are usually a series of connected straight lines having given lengths and bearings, and designated natural or artificial objects, marks, and monuments; or they follow the meanderings of streams or divides of water-sheds. The natural marks are streams, mounds, hills, rocks, trees, etc. The artificial marks are crosses, blazes, hacks, drill holes, stakes, letters, figures, mounds, pits, a pile of buffalo bones, a cow's head, or other conventional signs and characters on standing timber, rocks, ledges, or on the ground. In order to retrace the lines of a survey it is necessary to have the description of the original boundaries made by the surveyor who laid them down. This description, called the field notes, is usually found in the archives of the General Land Office or in the records of deeds in the county where the property is situated. It is also necessary to know the laws, rules, regulations, and conditions under which the survey was made.

TEXAS LAWS

All early surveys in Texas were made under Spanish and Mexican laws and regulations. The territory within the present boundaries of the State—

under the discoveries of Columbus, the conquests of Cortez, and the explorations of Coronado, De Soto, and others—was claimed by Spain until the successful Mexican Revolution of 1821. This claim, however, was disputed by the French on the basis of La Salle's explorations in 1684. Therefore, the land laws of this State have been influenced by the Spanish and French laws, which have a common origin in the Roman Civil Law. After the Texas Revolution of 1836 the principles of the English Common Law took precedence over the Spanish and French influences, but the land titles granted by Spain and Mexico were recognized and ratified by the Republic and the State of Texas. Prior to the Texas Revolution, title to about 35 000 000 acres had been granted.[3] Texas was attached to and administered as part of the Mexican States of Coahuila and Tamaulipas from 1821 to 1836, when it became an independent republic. In 1845, it was admitted into the American Union of States. By a provision of the act of admission, Texas retained title to all her public vacant land, and the State has administered these lands chiefly for the benefit of railroad development and the endowment of a public educational system. The United States has never owned or administered the public lands of the State and, therefore, the Federal system of land surveys has never been adopted.

The Arabs and Moors who helped to develop the Spanish laws and customs, having established governments in the arid and semi-arid territories of Arabia, Northern Africa, and the Iberian Peninsula, impressed upon the Spanish law rules for laying out surveys of land, giving each tract or grant a front on a stream of permanent water wherever possible. The Spaniards followed this policy in North America.

The streams of the State of Texas all have a general southeasterly course; therefore, all Spanish and Mexican surveys make an angle with the meridian and seldom does the width northwest and southeast exceed one-half the length northeast and southwest; and the statutes now provide that in laying surveys along a stream they shall have a stream front of not more than one-half the square. Stephen F. Austin, impresario of one of the earliest colonies, in a letter of instruction to Elias Wightman, a surveyor, directs that surveys shall be laid along the streams, giving each a water-front, and if any land could not be covered, but should be left vacant, it would be on the prairies where the land had no value.

MEASURING UNITS

The Spanish and Mexican laws prescribed the units of land measurement and the methods of laying out surveys and of marking the lines and corners. The linear unit is the "vara" (in the Austin collection of documents frequently called "bar"), which is equal to 33⅓ in. and is defined as 3 "geometrical" ft.[4] or "pie". The term, "pie", is a Spanish unit of length, corresponding roughly to 1 ft. In Castile it compares with the Old Roman foot, as 923 is to 1 000.[5] The vara had different values at different dates and in different countries. For example, in Peru, it equals 33 in.; in Spain, 33.384 in.; in

---

[3] Wooten's, "History of Texas."

[4] "Lamar Papers," edited by Harriet Smither, Vol. V, p. 129.

[5] See, Velazquez Spanish Dictionary.

Argentina, 34.092 in.; and in Portugal, 43.7007 in.[6] When the Americans began to make surveys in Texas, each surveyor had his own idea of the length of the vara. Stephen F. Austin[7] prescribed the length as 33⅓ English in. and this value has become the standard.[8] The geometric foot was generally used in the Sixteenth Century. DeMorgan[9] gives its value as 9.8 in. and Locke[9] as 10 in. The units of area are the square "pie", square vara, "labor", and "league" or "sitio". An English acre is equivalent to 5 645.376 sq. varas. A labor contains 1 000 000 sq. varas, or 177.13 acres. It is 1 000 varas square. The league is 5 000 varas square and contains 25 labors, or 4 428 acres. Many of the Spanish surveys contain a league and labor, or 26 labors, which is equal to 4 605 acres. Five leagues or sitios is a hacienda. In the theory of the Spanish laws and customs the labor is a farm, or an area upon which to labor and to produce crops. In contradistinction to the labor, a league is land upon which to conduct ranching and stock-raising operations.

It is also interesting to note a peculiar custom of ancient origin practiced in Latin countries in connection with granting and delivering land. The patent contains a certificate over the signature of the administrative officer that upon a certain date he took the grantee by the hand and led him around the land and pointed it out, and the grantee went upon the land and pulled herbs and threw stones and proclaimed in a loud voice that he was the owner of the said land. However, a similar custom was practiced by many ancient peoples.

In the Spanish laws, natural objects received preferential importance over artificial marks as monuments and witnesses to the boundaries in the specifications for laying out surveys, and, in numerous opinions, the State Courts have uniformly held that natural objects, other elements of the proof of the locations of the survey being equally certain, shall be given superior importance. In relocating surveys the Courts consider four elements in the following descending order of importance: Natural objects, such as rivers, creeks, mountains, etc.; artificial objects, such as marks on trees, marked lines, stakes, etc.; course and distance; and quantity. These rules are founded on reason, experience, and observation, and the surveyor must approach the problem of relocation with them well in mind, because he may be, and frequently is, required to defend the accuracy of his work before the Courts. While the surveyor must know the law, he has done his duty when he has found all the facts relating to the original boundary, relocated it upon the ground, and reported his findings of fact to his principal, or to the Court. In many cases he will be unable to harmonize the description of the original boundaries with the facts he finds on the ground.

Two cases will illustrate the point. A surveyor returned field notes to the General Land Office for 116 sections of land along the right bank of the Canadian River. His certificates state that the work was done on the ground, that the boundaries were marked by natural and artificial objects, and that he began at a definite point, and progressed toward the west giving each survey a river front of ½ mile. Each succeeding survey begins at the northwest corner of the preceding survey on the right bank of the Canadian River. In making a relocation on the ground it was found that one of these sections would cross the river twice, and a part of it, at its northwest corner, would lie on the left bank of the stream and all succeeding surveys in the series would lie on the left bank if joined to the northwest corner of the preceding survey.

---

[6] See, Fourteenth Edition, Encyclopedia Britannica.

[7] Baker's "Life of Austin."

[8] "Lamar Papers," edited by Harriet Smither, Vol. V, p. 129.

[9] Century Dictionary, under "Geometrical Foot."

During a lawsuit involving the location of some of the surveys in this series the surveyor testified that the only field work done was the taking of the few bearings on the course of the river, while an assistant kept watch at the top of the cliff to give warning of Indians. After taking these bearings on the course of the river and assuming that it continued indefinitely on the same course, he went into the safety of his office and wrote out the field notes. When asked why he did not make the surveys on the ground as certified, he said, "It would have taken a company of United States soldiers to have kept the Indians off of me while making these surveys, and the land was not worth it." In Fig. 1 the solid lines show the location as called for in the field notes; the dotted lines show the actual river with the surveyed tracts as they would lie in relation to it.[10]

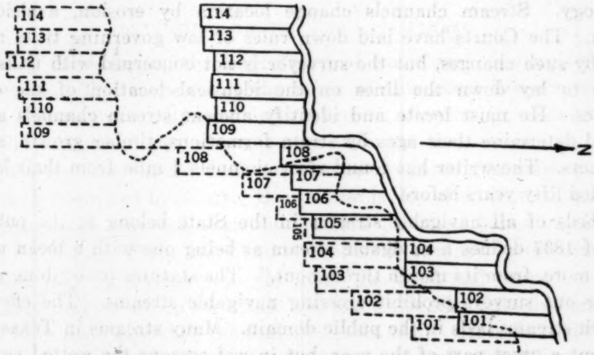


FIG. 1.—SECTION OF THE CANADIAN RIVER IN THE TEXAS PANHANDLE, SHOWING EFFECT OF INACCURACY IN OLD SURVEYS.

In 1886 or 1887, a district or county surveyor, in order to locate tracts of school land in the Panhandle, sold by the State, ran a connecting line from a monument on the 100th meridian west (at right angles to that meridian at the initial point) for 50 or 60 miles to the school surveys, and located the tracts in that manner. It is plain that he did not cross the next meridian at right angles, and his corners were located several hundred feet too far south. These corners conflicted with the adjoining alternate sections previously located and patented to a railroad company for constructing its lines, and the owners had the surveyor enjoined by Court proceedings. The result was that a settlement of various conflicting claims became necessary and by co-operation with the State a re-survey of all the railroad and alternate school sections in that part of the State was made.

---

[10] Sanborn *et al.* *v.* Gunter *et al.*, Southwestern Reporter, Vol. 20, p. 72.

It is interesting to note that the position of the initial monument on the 100th meridian, located by Jones and Brown in 1859, by observations of lunar culminations, when checked in 1892 by Messrs. H. S. and H. C. Pritchett and T. U. Taylor, M. Am. Soc. C. E., for C. A. Culberson, Attorney General of Texas, to secure evidence in the trial of a suit brought by the United States against Texas to recover the territory in Greer County, was found to be 3 797.3 ft. west of the 100th meridian. In 1929 a new monument was set in correct geodetic position by Mr. S. S. Gannett under the directions of a United States Supreme Court decision, and the true 100th meridian was run and monumented.[11] This adds a strip of territory about 134 miles long, about 880 ft. wide at its north end and 4 040 ft. wide at its south end, to the eastern edge of the Texas Panhandle, over which Oklahoma had theretofore exercised sovereignty. The Legislatures of the two States are now (1929) adjusting the civil affairs of this strip.

QUALIFICATIONS OF SURVEYOR

In making investigations of natural objects called for in a description of a boundary, the surveyor is confronted with problems in physiography, geology, and ecology. Stream channels change location by erosion, avulsion, and accretion. The Courts have laid down rules of law governing titles to lands affected by such changes, but the surveyor is not concerned with titles. It is his duty to lay down the lines on the identical location of the original boundaries. He must locate and identify ancient stream channels and terraces and determine their ages by strata formations, timber growth, and collateral facts. The writer has found river channels ¾ mile from their locations as recorded fifty years before.

The beds of all navigable streams in the State belong to the public. A statute of 1837 defines a navigable stream as being one with a mean width of 30 ft. or more, from its mouth throughout.[12] The statutes prescribing methods of laying out surveys prohibit crossing navigable streams. The effect is to leave such streams beds in the public domain. Many streams in Texas are dry throughout a great part of the year, but in wet seasons the wetted perimeters are several hundred feet long and the width between the cut banks of the freshet flows makes them navigable under this statutory definition. It appears that surveyors understood the definition of a navigable stream to mean a 30-ft. width of water and gave no importance to the width between the cut banks of the flush flow. These flashy streams are located in the arid or semi-arid regions, which at the time the surveys were made, were sparsely settled. The work was done under great difficulty of transportation and accessibility of supplies, and with great dangers from savage marauding Indians. It was usually done in the warm, dry season, and the surveyors in many instances found these streams dry. They located the surveys across the streams in violation of the statute, if it is assumed that the Congress of the Republic meant to prescribe a width between cut banks and without regard to the width of water. The Supreme Court of the United States, however, in the Texas-Oklahoma Boundary Case laid the common boundary between these States on the south cut bank of the Red River as located under the treaty of 1819 and as subsequently changed by erosion and accretion.[13] The surveyor is frequently called upon to locate the public lands in the stream beds of the State, and the work requires highly technical knowledge and skill. It is necessary to locate the original boundaries of the adjoining surveys and determine the changes made by the forces of the stream.

---

[11] See, Rept. of Samuel S. Gannett, Boundary Commr., Washington, D. C.

[12] Article 5302, Texas Rev. Civ. Statutes, 1925.

An investigation of artificial objects requires no less skill and experience. A fair knowledge of botany, plant anatomy and physiology, and ecology is required. Many efficient, capable, and skillful surveyors have no definite or organized knowledge of these sciences and probably have never heard or seen the names of some of them; they are students in them nevertheless. A little native wit and common sense, that is, sense about common things, supplies a great deficiency in knowledge of organized science. There is probably no technical branch of work that requires more common sense and self-reliance than surveying. Species, genera, habits, and methods of growth of trees must be known and ages of marks must be determined in many cases by inspection.

IDENTIFYING SURVEY LINES ON TIMBER

Consider, for example, the importance of knowledge concerning annual rings on trees. The exogenæ, as the name of this great division of plants indicates, enlarge outwardly, each year's growth showing in cross-section as a ring. The cambium layer, which is next under the bark, is the only growing part. Since the inner wood does not change by growth, a mark put on it remains until destroyed by decay. Scar tissue is deposited in the matrix; the wound heals, but a scar will show on the bark for many years. Neither the character nor the age of the mark can always be determined by the scar on the bark, and it may be necessary to cut away the wood and count the annual rings back to that on which the mark was made. Marks sixty-five or seventy years old have been positively identified in this way. Often it is difficult to determine definitely the presence of a mark on a tree; the lateral enlargement of the trunk distorts it, and on an exposed side it may be more or less obliterated by disintegration of the bark; therefore, it may happen, and frequently does, that two competent and credible witnesses will disagree as to the presence of a mark.

The field notes describing a boundary usually give the sizes and genera of the trees called for, and sometimes they will include descriptions of timber lines and character of flora. While magnitude of size varies directly with magnitude of age, the surveyor must not assume that all large trees are very old or that all small trees are very young and, therefore, refuse to examine small trees for old marks. In 1920, in making a relocation, the writer found a cedar elm which was marked in 1842 as a witness to a corner. The locating surveyor gave the size as 6 in. in diameter. In 1920, the diameter was 8 in. On another occasion, a post oak, 3½ in. in diameter, was compared with a sweet gum, 18 in. in diameter. The oak had 52 annual rings, while the larger sweet gum had only 60 annual rings.

---

[13] State of Oklahoma *v.* State of Texas, U. S. Intervenor, U. S. Repts., Vol. 256, p. 70.

Often it becomes highly important to determine the locations of old timber lines on some of the early surveys. If the notes call for a timber line at a given distance from a corner, or for timber at the corners, and these locations are found in fields or on prairies, it is necessary to determine whether the conditions called for existed at these locations when the survey was made. To make the investigation pits are dug 2 or 3 ft. in the ground. By careful examination of the vertical banks of these pits the forms of tree roots can be traced by the presence of root bark or coloration of the earth long after all trace of the wood fiber of the roots has perished. Most hardwood trees have large tap roots, penetrating deep into the substratum of clay. These roots persist long after all surface traces of the trees are gone, and even after the roots have decayed completely, their forms can be easily traced by the residual humus and the surface soil washed and settled into their clay moulds.

The writer's experience indicates that the pioneer surveyors ran most of their lines long, and that the bearings vary on the ground considerably from those given in the field notes; but in making a relocation of an old survey, if the surveyor will use a light, open-sight compass with an ordinary Jacob staff he will have little difficulty in following the marked line.

Duties to Clients

A proper approach to a difficult relocation is very important. The surveyor is usually called in after a controversy has arisen to which his client is a party. The client expects the surveyor to find the facts in his favor, but the only interest the surveyor should have is in finding the facts. He should make his position clear to the contending parties and with the record before him approach the problem very much as a detective investigates the mysteries of a great crime. He should run down clues and test out theories until some one or more facts have been established. Then by applying the descriptive matter of the record, he should find corroborative facts and make out the complete case. He should observe the rules of legal evidence, because they are the results of experience from time so remote that the memory of Man "runneth not to the contrary." Hearsay testimony without corroborative facts cannot be accepted, except as clues.

When the investigation is finished, the surveyor must be certain of the results beyond a reasonable doubt. If he has found sufficient facts to justify it, he should relay the boundary on the location of the old one regardless of interests or conflicts. On the other hand, if the facts are not sufficient, he should not make a relocation, but should report to his client the conflicting evidence, giving the client the benefit of all clues, theories, and facts found during the investigation. Doubtful relocations and conflicting interests can be settled by agreements or adjudicated by the Courts.

CONCLUSION

In making relocations the surveyor must have a definite conception of his duty. If the field notes state that the original surveyor began at a definite point and ran the lines on given bearings for given distances; that he marked certain objects and monuments at the initial, terminal, and intermediate points; and if those objects can be found on the ground and can be identified by the marks, the lines joining them fix the original boundary regardless of course, distance, or quantity of land. Therefore, it is the duty of the surveyor to find and identify the objects called for and lay the relocated boundary on the ground accordingly. The surveyor is employed to find and report the facts. His duties do not require him to become an advocate of his principal, but it is his privilege and duty to defend the facts set out in his report.

DISCUSSION

J. M. Howe,[14] **M. Am. Soc. C. E.**—Texas land-line location is so interwoven with Texas land law that the surveyor or engineer who depends upon the rules of the U. S. General Land Office will soon be completely lost. Some one has said that "land surveying is an art and not a science", and if this applies anywhere, it certainly does in Texas.

The earliest surveys were so placed as to give riparian rights and formed a border along the principal streams with their back lines serrated so as to be roughly parallel to the course of the stream. The later surveys, to fill the unappropriated areas, tie into the riparian grants and make the land map look like an old-fashioned crazy quilt. As the early work was poorly done, with corners and lines inadequately marked, or frequently not marked at all, the fill-in work often laps over or fails to reach the earlier lines. When the first surveys were made the land had values of less than $1 per acre, and scarcely more than that for the later fill-in tracts. In consequence, chaining was carelessly done and courses were only approximate.

The Texas law requires that precedence be given to senior surveys, and many of the conflicts in title are due to failure or inability of the later surveyors to identify the earlier lines. Theoretically, the surveyor should be able to re-locate them by giving the senior surveys their distances and thus retrace the entire map structure on the ground; but original monuments, when identified, hold precedence over both course and distance. Therefore, regardless of the recorded length of lines, when junior surveys are in conflict, the junior must give way. When one appreciates that the early surveyors were working for the grantee and not for the sovereignty of the soil, he can readily understand that many of the early grants have lines on the ground considerably longer than the distances called for in the patent, and it is these lines that hold and not the ones called for in the field notes.

Unfortunately, one has to contend not only with uncertainty of distance, but also with uncertainty of course. North in Texas is seldom true north. The early surveyors, or at least most of them, knew that the magnetic needle had a declination, but their ideas were at considerable variance as to what allowance to make. Not infrequently one will find well identified original lines of surveys in the same county, each calling for the same course, but put in by different surveyors and varying by as much as 5° from each other. When the work of different surveyors comes together, the result is what a facetious judge once termed the "Texas rectangle", which he defined as a "rectangular parallelogram, no two sides of which are the same length and no angle of which is a right angle". As far as the speaker knows, no attempt was ever made to allow for earth curvature, and the change in length between meridians due to curvature has to be absorbed in errors in course or distance. This, however, is a minor matter, as the errors in execution of the work are sufficient to make curvature corrections negligible.

---

[14] Cons. Engr. (Howe & Wise), Houston, Tex.

In his closing paragraph the author states that the duty of the surveyor is to find and report the facts, and that his duties do not require him to become an advocate. This is true if he is working for the State or qualifying as a witness, but when he is employed by a land owner, his employer expects to be shown the outline of his property and the surveyor is supposed to have sufficient skill to determine these lines. The land owner wants to know the number of acres to which he has title, and what lines he can sustain on the ground.

The surveyor, therefore, must have a working knowledge of the essentials of Texas land law. He must decide in the field, without legal advice, whether he should follow a well marked, and apparently well recognized, crooked line or connect the two corners with a straight line. When calls are inconsistent with measurements on the ground, he must know which calls he should change. Where fencing is found, he must know what weight to give to it as fixing the line, and this involves an acquaintanceship with the 3, 5, and 10-year limitation laws and the decisions interpreting them. His success as a surveyor depends upon his ability to place the lines so that they will be sustained in Court.

Arthur D. Kidder,[15] M. Am. Soc. C. E. (by letter).—The important observations to be noted in this paper are the great need for durable survey monuments, for a complete field-note record of the surveys, and for experience in the work. The need is present both with reference to the establishment of the original boundaries and sub-divisions, and in making retracements of lines for restoration of lost corners and other purposes.

It is made plain that the official survey and plat are the means constituted by law for the identification and description of the public lands of the State of Texas as well as within the Federal domain. The locations are controlled by the established original monuments, and the difficulties of the surveying procedure greatly increase when the monuments disappear. The difficulties become complicated in the event of unexpected discrepancies between the field-note record and the evidence of the survey to be found upon the ground.

The author has not overstressed the need for extended experience in meeting the practical questions which are to be dealt with, the scope of which could only be outlined, and it is presumed to be obvious that a good technical training is requisite, as well as broad field experience. The specialized character of the surveyed work described, may well be emphasized; similar conditions, with their attendant problems in making restorations, are encountered in other localities.

W. J. Powell,[16] M. Am. Soc. C. E. (by letter).—Little has been written, outside of transcripts of Court proceedings, concerning the surveying of land in Texas. Therefore, this paper is of great value to engineers in this State. When it was admitted to the Union the State of Texas retained all its public domain, and the Federal system of surveys has never been established. Probably this is due to the difficulty of co-ordinating that system, or, for that matter, any other system of surveys, with the old Spanish, Mexican, and Republic of Texas land grants which are discussed by Mr. Williams. Nor is it easy to determine to what extent, if any, the Federal Land Laws affect surveys in Texas. In fact, it is difficult to learn the laws governing such surveys because published information on the subject is difficult to obtain.

---

[15] Assoc. Superv. of Surveys, Gen. Land Office, Interior Dept., Washington, D. C.

[16] Cons. and City Planning Engr. (Wood & Powell), Dallas, Tex.

The principles stated by Mr. Williams, of course, have general application in all parts of the State, but some of the methods discussed in the paper have no such general application. For example, it would be quite difficult to retrace an old survey line by the marking of the timber, or to determine the age of a witness tree by counting its rings, in vast areas in the western part of the State where the trees are scarcely large enough to afford shade to a jack-rabbit. Furthermore, in that region the usual problem is not solved when the original survey lines have been retraced because there still remains the important and sometimes very difficult work of reconstructing the surveys based on the original survey lines, which are often only random lines run across a large group of surveys.

The author has shown, in a most interesting manner, the vagueness in the descriptions of many of the old land grants of former Governments. In such cases it is frequently impossible to retrace or determine the original boundaries of the grants, and the only way of settling disputes in regard to them is by agreement and compromise or by lawsuit. By far the greatest portion of the public domain of Texas, however, was disposed of and surveyed during a later period, when education and transportation were the crying needs of the State. During the three decades following the Civil War large grants of land were made to the public schools and also to railroad companies to encourage the building of railroads in the State.

The school lands were usually granted in solid blocks varying in size and shape. Many of them were not surveyed at all, but were delineated by "paper field notes," filed in the General Land Office of the State. In the case of the railroads the State issued certificates or scrip to the railroad company on the basis of 16 sections, containing 640 acres each, for each mile of track built in the State, and to provide for such grants "reservations" were set aside within which the railroad company could locate its lands and apply the certificates. Under the laws in force at that time the railroad companies were required to survey the lands. For fear the companies would take up all the good land, all the water, and all the mineral prospects, the State required that they take alternate surveys and, in surveying the land, to return field notes for two sections; the odd numbered ones for the railroad company and the even numbered ones for the State. This resulted in a checker-board arrangement.

In one of these large grants it is interesting to note the manner in which the regulations were met. When the work was started a number of surveying parties were put in the field and for a time all section lines were actually run out on the ground. Somewhat later a system of surveys consisting of blocks, 6 miles wide and 8 miles long, was developed, and the surveyors ran out the block lines. Later, and farther West, they ran out every other block line, and, finally, contented themselves with running a line across a large group of blocks; in some cases, no lines were run at all.

The methods used by these early surveyors were crude and the difficulties under which they worked, were severe. The land was beyond the frontier and the survey parties were usually accompanied by small military escorts to protect them from Indians. There were no roads and transportation was by wagon and team. Water and forage were scarce, and the country was sometimes a sandy, barren plain and at others rugged mountain topography. A light compass (without vernier), a Jacob staff, and a 10-vara chain sometimes constituted the surveying equipment. Under such circumstances it is surprising that they got as good results as they did; but, at best, the work was inaccurate, and many conflicts and vacancies have developed. Excess is almost universally found in these old surveys. The lines were not run straight, and the brush was not cut; and it is said that it was the practice of many of the old surveyors to add a chain at every "out" to allow for the shortcomings of the chaining.

The law requires that a re-survey shall follow as nearly as possible in the footsteps of the original surveyor. If the original surveyor left marks on the ground that can be found and identified by bearings or witnesses, this requirement can be complied with; and if he actually ran all the boundaries of a survey and set corners that can be re-established beyond reasonable doubt, the problem of the re-survey is solved at once. If, however, he ran only one boundary line, or only a random line across a group of surveys, and protracted the other lines, where are his footsteps on the lines that he did not run on the ground?

Many surveyors hold to the theory that the variation actually used by the original surveyor should be determined and, with that variation, the other lines of the survey should be run on the bearings called for in the original field notes. This theory is all right for small tracts, but in surveys covering large areas where the curvature of the earth becomes a considerable factor it is likely to produce incorrect results.

In a specific case, the original surveyor started at the northeast corner of a group of surveys which is two blocks (16 miles) wide north and south, and ten blocks (60 miles) long east and west. The north lines of these blocks are supposed to lie in the Thirty-Second Parallel of Latitude. The surveyor ran a zigzag line, following the valley of a stream from east to west through this group of surveys. His field notes call for the lines to be north and south and east and west. His corners, as found on the ground, are sometimes north, sometimes south, of their correct positions, so that no two corners could be accepted as determining his actual east-west lines, and since he apparently neglected to correct his variation from time to time, a general average of his corners does not give a true east-west line. The

approved construction in this case is as follows: Where two or more corners are found on a line they determine that line between them; all other lines are meridians and parallels passing through the controlling corners. This sounds simple, but working it out on the ground is no small job.

Such problems as this are frequently encountered in the western part of the State. Many surveyors still apparently believe that the earth is square, and some weird results have been obtained.

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
TRANSACTIONS

Paper No. 1805

FLOW OF WATER IN TIDAL CANALS[1]

BY EARL I. BROWN,[2] M. AM. SOC. C. E.

WITH DISCUSSION BY MESSRS. R. L. FARIS, G. T. RUDE, EUGENE E. HALMOS, W. M. BLACK, H. F. FLYNN, RALPH G. DAVIS, AND EARL I. BROWN.

SYNOPSIS

The passage of the ocean tidal wave before the mouth of an estuary, river, or inlet sets up therein a derived wave, which is propagated into the mouth in accordance with certain laws not yet clearly understood by engineers. The simplest example of a channel in which a derived wave may be propagated is a canal of uniform width and depth. This paper is devoted to a consideration and further study of the phenomena resulting from the propagation of derived waves in inland channels.

The derived wave is considered to be a wave of translation with successive alternating positive and negative phases, propagated in accordance with known laws. Proper allowances are made for the effects of friction on the bottom and sides of the channel. The problem is treated from the standpoint of the expenditure of the energy imparted to the derived wave by the generating wave in overcoming this friction.

The length, width, and depth of the canal have an effect in determining the rate at which this energy is expended. The conditions under which the derived wave is propagated are also modified by the character of the body of water with which the canal communicates at its farther end. Three cases are considered, namely, (*a*) a body of water at constant level; (*b*) a body of water with varying level, but with no proper tide of its own; that is, one that depends for its variation on the wave propagated through the canal; and (*c*) a body with varying level, but having a tide of its own derived from some source independent of the tide propagated through the canal.

Theoretical considerations are discussed, first, in order to enable fundamental conceptions to be established as to the mechanism of propagation, and as to the influence of width, depth, and canal outlet on propagation. These fundamental conceptions lead to the proposal of what may be termed the "Reflected Wave Theory" to account for the effect on the derived wave of the characteristics of the body of water at the outlet of the canal. This theory is believed

---

[1] Published in December, 1930, *Proceedings*.

[2] Col., Corps of Engrs., U. S. A., U. S. Engr. Office, Philadelphia, Pa.

to be new; the writer has not heretofore encountered it in his researches in literature on tidal subjects.

As they usually exist, no canals are so long that friction will absorb all the energy of the derived wave. Therefore, the presence of the unexpended energy of the wave, when it arrives at the end, or the outlet, must be explained. According to the reflected wave theory this remaining energy is propagated back into the canal in the form of a reflected wave the energy of which is also subject to frictional losses; if the remaining energy is not entirely consumed in the course of the transit of the reflected wave, a second reflected wave is set up, and so on indefinitely until practically all the energy of the derived wave shall have been absorbed by friction within the limits of the canal.

If the canal outlet is restricted or closed, the reflected wave will be of the same sign with respect to height as the derived wave. If the outlet is wider than the width of the canal, it will be of the opposite sign.

These various waves, derived and reflected, exist simultaneously in the canal, and accordingly they will interfere at all points in accordance with the laws of the interference of simultaneous waves. The derived wave as actually observed in the channel is the resultant formed from this interference.

Wave energy is manifested both in the height of the wave above or below mean sea level and in the velocity of the water set up by the wave propagation. In the simple wave of translation, the height of the wave and the accompanying velocity of the water are roughly proportional. When two simple waves of translation interfere while being propagated in opposite directions, the heights will be added to form the resultant height, and the velocities will be subtracted to form the resultant velocity, since the two component velocities are traveling in opposite directions. Hence, in the resultant, the height and velocity are no longer proportional to each other.

While the longitudinal profile of the generating wave is not usually truly sinusoidal, it may be so considered without appreciable error; hence, the derived wave, including the resultant and all reflected waves, may be considered as sinusoidal. This approximation makes it unnecessary to compute the effect of friction in reducing each element of the derived wave, but only of the summit of the wave. All other elements follow from that by an application of the cosine law.

To apply this theory, an equation is first deduced by which the height of the wave summit, as reduced by friction, may be found for any point in the canal. An extension of this computation by the same formula gives the height at the selected point of as many reflected waves as have sufficient magnitude to warrant their consideration. The cosine law is applied to each of these with proper allowances for phase differences resulting from the various times required for each wave to be propagated from the entrance to the selected point, and the summation of these results gives the resulting wave height. A similar process gives the resulting velocity at the same point.

The method of applying the theory to canals of various characteristics is shown, and methods of determining the coefficient of friction for each case are indicated.

Lastly, the theory is tested by making the computations in accordance with it for certain known canals and then comparing the computed results with the results of observation.

DEFINITIONS

The term, "tide," relates to the rise and fall of the water due to the propagation of a derived wave.

The movement of a "simple wave of translation" causes a particle of water to remain permanently displaced in one direction or the other. This wave is usually referred to under the same "wave of translation", in which the height of the surface and the velocity of the water at a given point are approximately proportional.

A "compound" or "resultant wave of translation" is formed by the interference of two or more simultaneous simple waves of translation.

The "speed" of a wave is the rate at which its form is propagated. It is not to be confused with the velocity of the water.

"Mean sea level" is that at which the body of water in which the generating tide is propagated would stand if there were no tidal action in it. It is the plane of reference used herein.

"Mean canal", or "mean river level", is the mean of the high and low-water levels at any point.

The "depth" of the channel referred to herein is the reduced depth. It is the ratio of the area of cross-section referred to mean sea level to the mean width at the same level.

"High-water stand," or "low-water stand" refers to the instant at which the vertical tidal variation changes from rising to falling, or *vice versa.*

The "summit" of a wave is that element at its greatest rise or fall.

The "velocity" is the rate of motion of a particle of water acted on by a wave of translation. Velocity due to other causes is neglected in this study.

The "strength of the tide" is its maximum mean velocity.

"Slack water" refers to the instant at which the mean velocity in any cross-section is zero.

The "turn of the current" is the instant at which the current at any point changes direction. It usually occurs at or near the instant of slack water.

The "local tidal curve" expresses the relation between height of tide and time at any point in the canal.

The "instantaneous tidal curve" indicates the height of the tide at all points in the canal at a given instant.

The "geometric loci of high and low water" are the curves enveloping the local tidal curves.

The "local velocity curve" expresses the relation between the velocity and time at any point in the canal.

The "instantaneous velocity curve" indicates the velocity at all points in the canal at a given time.

The "geometric loci of slack waters" are the curves indicating the height of tide at which high or low-water slack occurs.

NOTATION

C = speed at which the wave is propagated; C' denotes the rate at which a wave is propagated between two given points.

c = time before the instant, $t = 0$, when the tide was at mean sea level; that is, when $t + c = 0$.

D = reduced depth of the canal.

d = difference in times between the arrival of high water at the ends of the canal.

E = energy of any unit length of a wave.

e = elevation of mean sea level corresponding to the staff reading, R.

F' = work done by frictional resistance.

F = friction per square foot of the water on the wetted surface of the bed of the canal.

f_0 = coefficient of friction depending on the nature of the bottom, the water depth, and the velocity of flow.

g = acceleration due to gravity.

h = height of the tide referred to mean sea level at any point, x, corresponding to any time, t.

H = height of the tide at the summit of the wave (Point x), at the time, t_m.

H_0 = maximum height of the generating wave (and, consequently, of the derived wave) at the canal entrance; H_1, H_2, H_3, etc., denote the value to which H_0 becomes reduced by friction in the first, second, third, etc., passages of the simple derived wave of translation. They each have different values for each value of x.

h_v = velocity height, or the height of a simple wave of translation which, by itself, would create a velocity, V.

K = a constant.

l = width of the canal at sea level.

L = length of a wave.

M = surface area of a reservoir.

p = wetted perimeter of any cross-section at mean sea level.

Q = the discharge.

R = a staff reading.

t = time, in seconds, counted from the instant of high water at the canal entrance.

t_m = time at which maximum high water occurs at the point, x. It is, therefore, the time required for the resultant wave to be propagated from the entrance of the canal to the point, x. t_1, t_2, t_3, etc., denote the time required for the wave of translation and the second, third, etc., reflected waves to reach the point, x, counted from the instant, $t = 0$.

t_{mv} = time of occurrence of the maximum velocity at a point, x. It is, therefore, the time required for the maximum resultant velocity to be propagated from the entrance of the canal to the point, x.

T = total tidal period from one high water to the next (44 715 sec.).

t_x = time required for the crest of the wave to travel from the entrance of the canal to the point, x, in question.

t_y = time required for the propagation of a reflected negative wave from the reservoir toward the sea to a point, x.

t_L = time required for the positive wave to be transmitted from the sea to the reservoir.

u = velocity of any current of water not of tidal origin.

U = horizontal displacement of the water molecules.

v = actual velocity of the water at any time, t, and at any point, x.

V_m = maximum velocity at any point corresponding to the time, t_{mv}.

V_0 = velocity at the entrance to the canal corresponding to the height, H_0. V_1, V_2, V_3, etc., equal the velocities at any point, x, due to the passage of a wave of translation, or the second, third, etc., reflected simple waves of translation, with heights equal to H_1, H_2, H_3, etc.

V_h = velocity due to height, h, in an infinitely long canal.

x = abscissa on the sine curve, or the distance required for wave propagation in the canal.

y = ordinate on the sine curve.

Y = height of water above the mean level of the canal, due to the propagation of a wave of translation at any instant, t, counted from the time of high water in the tidal sea.

ρ = density of the water.

WAVES

To treat of this subject, it is first necessary to determine the character of the flow through canals subject to tidal influences. Most textbooks treat only of the flow of water under the influence of a surface slope, without much reference to tidal influences and the manner in which their effects are manifested. Tidal currents are sometimes attributed to the effects of the surface slope of the tidal wave, but both theory and observation disprove that explanation. Omitting from consideration for the present all questions of fresh-water flow or discharge, and taking into consideration only the flow directly attributable to tidal influences, consider the manner in which such influences are manifested.

All tidal action in coastal waters is clearly generated by the great ocean tidal wave formed directly under the combined attraction of the moon and the sun. This great wave gives rise to other waves which are propagated in greater or less degree to all waters connected with the sea at a suitable level. This paper is not concerned with any of the theories of the action of the ocean tide, except as it is manifested along the coast line, usually in a semi-diurnal rise and fall, during which the graph of the surface variations is approximately a cosine curve.

The only waves of importance affecting tidal canals besides the tidal waves are wind waves, or secondary waves generated by wind waves, and these are too irregular and variable to be considered herein.

Waves of Oscillation.—Every force which acts upon a liquid mass develops waves in it, among which waves of oscillation and waves of translation may be distinguished.

Waves of oscillation occur whenever the water is subjected to a vertical force which causes a momentary depression or elevation upon a relatively extensive area of the surface. They may be ordinary or periodical. Periodical waves follow each other like ordinary waves, but each has its own particular cause; their periodicity is inherent in the force which acts upon the water, and which produces a wave each time it acts. The undulations of the water oscillate above and below the level of equilibrium, and are of the same height or semi-amplitude in both directions.

The agitation produced by waves of oscillation makes itself manifest below the surface of the water. Undulations of the same length as those upon the surface occur in the body of the water, but their amplitude diminishes from the surface. Waves of oscillation, therefore, cause motion in the water only for a limited depth.

Two different undulatory motions, which occur together in a liquid mass, are propagated independently of each other, without coalescing. The resultant wave has a different form from that of the two components; the variation of the level of the water at any given point is the algebraic sum of the superelevations and depressions due to the two waves. The heights are added together or subtracted according as they are above or below the mean level.

Waves of Translation.—A wave of translation will originate in the waters of a canal, at the extremity of which there has been introduced suddenly a given mass of liquid, or when there has been impressed upon a transverse section an instantaneous effect of compression. Thus, a laden boat in motion, on entering into a narrow section of the canal, of sufficient length, carries before it a certain quantity of water, and should the boat stop abruptly, the intumescence thus formed above the surface of the water detaches itself, becomes an undulation, and continues its progress in the direction of the motion of the boat.

The wave of translation is single for each effort imposed upon the liquid mass; for this reason it is also called the solitary wave. It has the characteristic of being contained entirely above the surface of the water.

The most notable experiments which have been made upon the solitary wave date from 1837 and are due to Sir Scott Russell;[3] the results which he obtained have been confirmed by Bazin.[4] The works of these two scholars have disclosed the different properties of waves of translation.

Although the horizontal force which creates such a wave may act upon only a portion of the water, the wave spreads over the entire width of the surface, and puts the entire mass of liquid in motion from top to bottom.

When the wave is propagated in a canal of variable cross-section, it takes a greater height at the narrow points, while it becomes lower in the places where the canal is wider; but it always occupies the entire section of the canal.

In still water of sufficient depth each wave produced by a force exercising pressure on the liquid mass in the horizontal direction gradually takes a certain form which is distinguished by its great stability; this is influenced only by the attrition resulting from friction, which is quite slow.

The speed or velocity of propagation of the wave of translation, observed under these conditions, has for its expression,

$$C = \sqrt{g\,(D + h)} \qquad (1)$$

If the depth is variable in the same cross-section, the wave takes a single velocity of propagation corresponding to the mean depth of the section. In a wide rectangular canal, the waters of which are endowed with uniform motion, any wave of translation, when propagated in the direction of the current, takes a regular form, of remarkable longevity, which tends to approach that of the solitary wave traversing still water of constant depth. Its speed is given by the formula,

---

[3] Rept., Fourteenth Meeting, British Assoc. for the Advancement of Science, September, 1844, Lond., 1845.

[4] "Recherches Expérimentales sur la Propagation des Ondes," *Mémoires* des Savants Etrangers, Tome XIX.

$$C = \sqrt{g\,(D + h)} + u \quad\text{.........................}(2)$$

For a wave which is propagated in a direction opposed to that of the current the formula becomes,

$$C = \sqrt{g\,(D + h)} - u \quad\text{.........................}(3)$$

In this case the wave soon departs from the original regular form, differing more as the current is swifter, and when the speed has diminished to the point of approaching the velocity of the current, it loses its vitality rapidly, the mass of water composing it being gradually dissipated in the current.

Boussinesq has determined by analysis a general theoretical formula which permits the computation of the speed of each layer, dx, of a wave of translation, when the wave has not acquired the permanent form which is assumed in the case of the experimental formulas, Equations (1), (2), and (3), which assume the same speed for all layers. Boussinesq's formula[5] is,

$$C^2 = g\left(D + \frac{3\,h}{2} + \frac{1}{h}\,\frac{D^3}{3}\,\frac{d^2\,h}{d\,x^2}\right) \quad\text{.................}(4)$$

Among all the intumescences of various forms the propagation of which may be imagined, there is one for which the heights, h, and the curvatures, $\frac{d^2\,h}{d\,x^2}$, are so distributed that the speed of propagation is the same for all its elements. That is the solitary wave—the wave of translation—studied by Sir Scott Russell.

Tidal Waves in the Sea.—Tidal waves in the open sea take the form of waves of oscillation. The propagation of the wave sets up currents due to the displacements of the liquid involved in the motion. Tidal currents have no relation to the surface slope of the water; they are exclusively undulatory phenomena.

Tidal currents in shallow waters near the shore line are mass currents, acting for the entire depth of the sea; they are directed successively in the direction of travel of the undulation, and then in the opposite direction. They are called flood currents in the former case and ebb currents in the latter. They are alternating currents; barely perceptible in the deep sea, they increase in intensity as the depth of water diminishes. The currents sometimes become gyratory because of the combination of longitudinal currents with lateral currents which come and go, corresponding to the alternate rise and fall of the water on shore, or of the crossing of two co-existent waves the directions of which intersect at an angle.

The stage of the tide that corresponds to the moment when flood and ebb currents cease to run (and when they are about to be replaced by currents in the opposite direction) has received the name "flood slack", or "ebb slack", as the case may be. Under normal conditions the tidal wave is periodical and its curve takes a regular sinusoidal form; the currents which it generates turn at mean tide level, reaching their maximum velocity or strength at high or low water. These phenomena are completely modified when the depth decreases rapidly, and the propagation of the wave is hindered or checked in its travel, as in approaching the coast line, in estuaries, and especially in tidal rivers.

---

[5] "Essai sur la Théorie des Eaux Courantes," par J. Boussinesq; see, also, "Hydraulique," par A. Flamant, Paris, 1923, pp. 433–435 and pp. 438–440.

In general, near the coast line, the situation is intermediate between the state of free propagation of the tidal wave, and that of filling a basin of small dimensions; the regimen of the tide becomes changed. The reversal or turn of the tide is then no longer effected at mid-tide, but approaches closer to the times of high and low water, and the maximum velocity approaches mid-tide. The velocities then differ from their theoretical value and, in order to interpret the phenomena observed, one is tempted at times to have recourse to considerations based upon the hydraulics of running water and the influence of surface slope. Under these conditions the tidal wave loses its usual characteristics, entirely or in part, and in order to account for observed facts, it must be considered as formed by a series of small waves of translation each of which is propagated according to its own proper law.

Waves in Tidal Channels.—In the propagation of a wave there is only a simple transmission of motion, during which the molecules of water are successively raised and lowered on the passage of a wave, without participating in any way in its velocity of propagation. These transmissions of motion cannot take place without being accompanied by transportation of liquid molecules occurring step by step in the horizontal direction, which gives rise to currents in the direction of propagation.

For waves in tidal channels, the horizontal force which produces them resides in the rise of the tide at the mouth. The derived wave in tidal channels is not a tidal wave in the sense in which that term is used in connection with the ocean tidal wave; it is not a periodic wave of oscillation, a chop, or a seiche. It exhibits the traits of a simple wave of translation, resembling it notably in the means of its formation, but differing from it as soon as the conditions of propagation are considered. The velocity of propagation of the derived wave is generally less than is indicated by the usual theoretical formulas. Furthermore, in the true simple wave of translation, all the velocities are in the same direction, whatever point of the wave is considered, while in the derived wave the currents flow alternately in one direction and then in the other, and the velocities may be greater or less than for a simple wave of translation.

The various discrepancies, however, are more apparent than real, and arise from the fact that formulas for the speed and velocity of the wave of translation, such as Equations (1) to (4), inclusive, are applicable to small waves which are propagated in relatively wide and deep channels. In cases of this kind the influences of resistances and of reflections are inappreciable. The equations are not applicable to waves of which the height is comparable to the depth of the channel and which are propagated in short, crooked channels with irregular banks. These conditions offer very great resistance to the propagation of the wave. It is to these resistances and to reflections within the channel that the discrepancies observed between the behavior of the derived wave and that of small simple waves of translation, must be attributed.

Effect of the Canal on Waves in It.—Accepting these conclusions as to the identity of the canal wave with the wave of translation, such a wave entering into a canal from another body of water will be confronted by a set of conditions differing from those prevailing when it enters a tidal river or estuary from the sea. Its character is changed by the expenditure of energy necessary to overcome each resistance to its progress, and this expenditure must then cause either a diminution of the volume of the wave, a reduction of its height, or both simultaneously.

A tidal river or estuary will always be found closed at its upper end. This limits the distance in which the wave may be propagated and it limits the volume of the wave that may be propagated past any given point. In such cases the wave may also be opposed by a large fresh-water discharge. In addition to friction losses, the usual funnel shape of the river or estuary causes a continual decrease in the volume of the wave. Such lost volume, together with friction losses and any fresh-water discharge, is ejected down stream.

These conditions, however, are not usually encountered in a canal. The canal is not ordinarily of a funnel shape; it is usually of constant depth and cross-section. There is seldom any fresh-water discharge to be considered.

The physical characteristics of a canal have a preponderating influence upon the propagation of the tide within it. Assuming a horizontal canal of constant width and depth, communicating at one end with a body of tidal water, the character of the tide within the canal will be further controlled by the length of the canal and the character of the body of water with which it communicates. In this study it will be assumed that the canal communicates at its far end with some body of water, and is not merely a closed channel.

Regardless of the length of the canal, three cases will arise with respect to the body of water with which it communicates:

(1) The body of water may be at a constant level;
(2) It may vary in level due to tidal flow through the canal; and
(3) It may vary in level due to a proper tide of its own.

Case (1), as stated, implies a non-tidal reservoir sufficiently large in area so as not to be affected appreciably by any tidal flow it may receive through the canal. Case (2) may be a rather small reservoir which will be partly filled and emptied through the canal at each tide. In Case (3) the canal will be the meeting place of two opposed independent waves, the phases and amplitudes of which may differ widely.

These three comprise practically all cases of tidal flow in canals. The flow in tidal rivers and estuaries will not be included in this study. However, before beginning these practical studies, it is necessary to consider the theory of tidal action in a canal of infinite length, in order that the theory may be of assistance.

THE THEORETICAL CANAL, INFINITELY LONG

An infinitely long canal is a fiction that has been invented to meet the necessities of theory. Except in rare instances, such as the Amazon River, the St. Lawrence River, or Chesapeake Bay, the length of channel influenced by the tide does not exceed that of an entire wave. It is nevertheless of interest to follow scientists into the hypothetical domain and to examine the theoretical regimen of the tide in such a canal assuming that its bottom is horizontal and that its cross-section remains constant for its entire length. Under these restrictions the question has been treated by several investigators, and it is of interest to review the results at which they have arrived.

Theory of Airy.—Airy has devoted an important work[6] to the general study of the tides and of waves of different kinds, in which he has discussed in the greatest detail the circumstances of the propagation of the tidal wave in tidal rivers and estuaries. The results which he obtained may be summed up in the following equations:

$$h = H_0 \sin \frac{2\,\pi}{T}\left(1 - \frac{x}{\sqrt{g\,H_0}}\right) + \frac{3}{4}\,\frac{2\,\pi\,H_0^2}{T\,D} \sin \frac{4\,\pi}{T}\left(t - \frac{x}{\sqrt{g\,H_0}}\right) \ldots (5)$$

The speed or velocity of propagation of the wave equals, at high-water stand,

$$C = \sqrt{g\,(D + 3\,H_0)} \ldots\ldots (6)$$

and at low-water stand,

$$C = \sqrt{g\,(D - 3\,H_0)} \ldots\ldots (7)$$

The horizontal displacements of the molecules, which are the same from the bottom to the surface, are:

At high-water stand,

$$v = \left(1 - \frac{5\,H_0}{8\,D}\right) \frac{H_0}{D} \sqrt{g\,D} \ldots\ldots (8)$$

and at low-water stand,

$$v = \left(1 + \frac{5\,H_0}{8\,D}\right) \frac{H_0}{D} \sqrt{g\,D} \ldots\ldots (9)$$

Airy gave the changes to be made in these formulas when the water has a seaward current and when its depth is variable. He recognized that a bore should occur in such a case unless an additional horizontal force should intervene to combine its action with that of gravity, and he suggested that friction might supply such a force. He attempted to evaluate the influence of friction of the liquid on the bottom of the canal, assuming it to be proportional to the velocity, but he found that an evaluation thus made, did not possess the desired reliability for use in practice; nor was it capable of explaining the phenomena noted by observation.

Theory of Saint-Venant.—Saint-Venant has treated the same question[7] based upon a hypothesis previously proposed by Bremontier and confirmed by Bazin and Partiot. It consists in considering the tidal wave (which is of the class termed "periodical") as the result of the superposition of a series of very small waves of translation being propagated according to the laws established by Scott Russell and developed by Bazin. During the flood wave, these elementary waves would be formed by intumescences or fluxes, and during the ebb by depressions or refluxes.

---

[6] "Tides and Waves," Encyclopedia Metropolitana, Vol. V, 1845, p. 241.

[7] *Comptes Rendus* de l'Academie des Sciences, Vol. LXXIII, 1871, pp. 147 and 189.

Under this hypothesis, and neglecting the influence of friction upon the walls of the canal (assumed to be rectangular), he obtained the following results:

$$v = 2\sqrt{g\,(D + h)} - 2\sqrt{g\,(D - h)} \quad \text{.............} (10)$$

$$C = 3\sqrt{g\,(D + h)} - 2\sqrt{g\,(D - h)} = v + \sqrt{g\,(D + h)} \quad \text{....} (11)$$

and,

$$x = C\left(t - \frac{T}{\pi}\,\text{arc sin}\,\sqrt{\frac{2\,h}{D}}\right) \quad \text{...............} (12)$$

Theory of Maurice Lévy.—The theory of Saint-Venant was verified and confirmed by Maurice Lévy[8] who also treated the general question of the regimen of the tide in canals both of constant and of variable width. He made an application of the theory to river tides in the case in which the width is constant or slowly variable, due regard being given to the friction exerted upon the tidal wave.

He established the principles that:

1.—The wave is propagated with a decreasing height according to a definite coefficient of extinction.

2.—The velocity of propagation is given by a simple expression as a function of the height of the tide and of the depth of the water in the canal at half-tide.

3.—The strength of the current occurs before the high and the low waters, and its value diminishes as the point considered is farther from the mouth.

His results are more in accord with the results of observation than those of the others. The superiority of his theory arises principally from the fact that Lévy has succeeded in taking into account the effects of friction which Saint-Venant had neglected and which Airy thought inadequate to explain the discrepancies between facts and the results of his analysis. This influence of friction upon the phenomena of the tide, formerly regarded rather vaguely, is now an established fact. It can be computed in a fairly precise manner, under simple circumstances, it is true, but nevertheless in such a way as to permit rational inductions to be made for the most complicated cases.

In attempting to apply his equations to the Suez Canal, Lévy found that the computed values of the velocity were far less than the actual velocities, particularly at the end next to the Bitter Lakes. He attempted to remedy this discrepancy by the use of equations to a second approximation, but still without a complete agreement with observation.

The theory of Lévy is quite fully set forth and explained in an article[9] on the Cape Cod Canal by the late William Barclay Parsons, Hon. M. Am. Soc. C. E. This paper gives the deduction of Lévy's equations and makes a practical application of them to the Cape Cod Canal. Recognizing that the frictional coefficient is not a constant quantity, but varies slightly in an inverse ratio with the velocity, Mr. Parsons proposed a frictional coefficient which, as a linear function of the mean velocity, will absorb the same total energy in the interval during which the velocity changes from zero to maximum.

---

[8] "Leçons sur la Théorie des Marées," par Maurice Lévy, Gauthier-Villars, 1898.

[9] *Transactions*, Am. Soc. C. E., Vol. LXXXII (1918), pp. 1–157.

Writings of Bourdelles.—In a remarkable series of memoirs,[10] M. Bourdelles, Inspector-Général des Ponts et Chaussées, has fully and clearly set forth the theory of the propagation of waves of translation in canals, and has pointed out how closely observed facts confirm that theory. Bourdelles' exposé of the theory is so clear and vigorous that much of what follows has been adapted from his writings. However, Bourdelles, as well as Lévy, was unable to explain satisfactorily certain anomalies of velocities observed at points where canals empty into tideless seas. In such cases, where the observed velocities exceeded the computed velocities, those authors explained the anomaly by attributing it to the influence of flow due to surface slope in the undulatory motion of the tide. The writer advances the theory of a reflected negative wave to explain it.

Tidal Rivers.—M. Bonnet, Chief Engineer and Director of the Ponts et Chaussées of Belgium, has made a most valuable contribution[11] to the theoretical study of tidal rivers. He treats the propagation of the tidal wave as a wave of translation continually modified by friction (which he evaluates) and by the shape of the bed. These causes which act to reduce the energy of the wave give rise to a current directed down stream the volume of discharge of which represents at each instant the volume of water lost in a unit of time by the portion of the wave which has passed into the river. This he terms counter-current. He also shows that the propagation of the wave of translation and the action of the counter-current permit an explanation of all the characteristics of the river tide. Bonnet's method of showing how the energy of the wave is reduced by friction has been adopted in this paper, with necessary modifications.

Propagation of Elementary Waves of Translation in an Infinitely Long Canal.—Assuming that the tidal wave entering a canal is a wave of translation, it is necessary to consider some of the properties of the latter. To do so intelligently, the first step is to study the propagation of such a wave in a canal of constant depth and cross-section, but of infinite length.

Suppose the water is without motion at the moment of the introduction of the first flux emitted by the tidal wave on the flood. In Fig. 1, which represents a longitudinal profile of the canal, $A\,B$ is the mouth with a mean depth, D; $A\,M$, the horizontal level of the water in the canal at low water; and $h = A\,a$, the height of the first flux the propagation of which is being studied.

By virtue of the properties of the wave of translation, after a time which will be assumed as unity, this flux, propagating itself with a speed of $C = A\,M$, will have arrived at $M\,N$, introducing into the canal a volume, $A\,a\,m\,N = C \times h$ per unit of width, and in impressing upon all the vertical filaments of the canal a horizontal displacement of translation the velocity of which will be designated by v. This displacement will have the effect of moving the molecules of water of the section of the mouth, $B\,A$, to $B'\,A'$, and of introducing into the canal a volume, $B\,a\,a'\,B' = v\,(D + h)$. Since this volume is necessarily equal to that of the flux, $C\,h = v\,(D + h)$, or,

---

[10] Bourdelles contributed the following articles published in *Annales des Ponts et Chaussees*: (1) "Etude du Regime de la Maree dans le Canal de Suez," 1898, 3^{me} trimestre; (2) "Distribution des Vitesses Suivants la Verticale dans les Courants de Maree," 1898, 4^{me} trimestre; (3) "Etude du Regime de la Maree dans la Manche," 1899, 3^{me} trimestre; and (4) "Etude sur le Regime de la Maree dans les Estuaires et dans les Fleuves," 1900, 2^{me} trimestre.

[11] "Contribution à l'Etude Théorique des Fleuves à Marée," par L. Bonnet, *Annales* des Travaux Publiques de Belgique, 1922, Fascicules 3, 4, 5, and 6, and 1923, Fascicules 1, 2, 3, and 5.

$$v = C\,\frac{h}{D + h} \qquad \text{(13)}$$

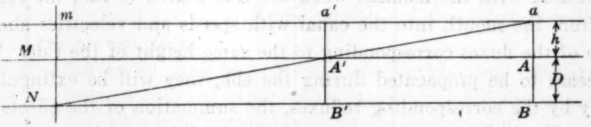


FIG. 1.—LONGITUDINAL PROFILE OF A CANAL.

Thus, a known relation is found between the velocity of displacement common to all the molecules and the speed of the undulatory motion which causes this displacement. Furthermore (see Fig. 1), the velocity is represented by the right line, $A\,A'$, since $\frac{h}{D + h} = \frac{A\,A'}{B\,N} = \frac{A'\,A}{C}$. The effect of the undulatory motion thus becomes quite clear. The impetus is propagated successively to all the vertical filaments of the canal with a rapidity equal to the speed, giving to each of these filaments the same velocity, so that the compression of the water comprised between two neighboring filaments produces the height of the flux corresponding to it. Hence, the total compression, equal to the volume of the flux, provokes an equivalent supply of water through the mouth.

Fig. 1 also demonstrates that, for the same depth, D, the supply of water to the canal and the horizontal displacements increase with the height of the flux. The same is true when (the height of the flux and the mean depth remaining constant) the speed is nevertheless increased by extraneous circumstances.

It is to be noted finally that, in the infinitely long canal, the first flux will continue to be propagated during the entire period of the flood until the moment when the ebb, beginning to be formed at the mouth, will be opposed to the introduction of the necessary water for the supply of the flux. In spite of this obstacle, it will be able, however, to continue its propagation to some degree; but then the action of friction will result in the gradual diminution of its height and its final extinguishment.

It is needless to add that all that precedes is also applicable to the refluxes on the ebb. There is no difference except in the lesser speed of the propagation and also in the mode of supplying the wave volume. This is effected by expelling water from the canal, creating a current which is opposite in direction to that of propagation.

Friction has the effect of diminishing the height, the speed, and the velocity of a flux or of a reflux, by an amount proportional to the velocity and to the duration of the undulatory motion.

The first flux will be propagated during the entire flood tide, under the conditions just indicated. The second flux will be propagated in the same manner as the first, with the difference, however, that it will flow in the current created by the preceding flux, which will increase its normal speed by a quantity equal to the velocity of that current. It is the same for the other fluxes which follow, up to the moment of high water, so that the velocity of the flood current will continue to increase in all parts of the canal during the rising tide.

Beginning with the moment when the tide begins to fall, the refluxes will travel from the mouth into the canal with speeds and velocities almost equal to those of the fluxes corresponding to the same height of the tide. When the fluxes cease to be propagated during the ebb, they will be extinguished successively by the corresponding refluxes, the summation of the effects of which will accordingly be to evacuate to the sea, almost completely, a quantity equivalent to the water introduced by the flood. The fluxes continue their propagation, however, when the length of the canal allows it, and they will escape entirely from the action of the refluxes if the length of the canal is infinite. Under the hypothesis as defined, therefore, it is apparent how a portion, or even all, of the volume, introduced by the fluxes, may remain in the canal after the end of the ebb, without being evacuated by the succeeding refluxes. Then the surface of the canal becomes a succession of positive and negative waves of translation, both being propagated in the same direction. The positive waves leave each particle of water moved forward a distance dependent upon the volume of the wave, and each succeeding negative wave, if it is of equal volume, returns the particle to its original location. If for any reason, such as increased frictional resistances, the negative wave should not equal the positive wave in volume, the particle will suffer a permanent displacement in the direction of propagation after the passage of each positive wave and its succeeding negative wave.

Friction.—The sum of all friction losses in the water flowing in an open canal—whether flowing under the influence of a surface slope or under the impulse of the propagation of a wave of translation—may be reduced to the friction on the sides and the bottom of the canal, which is to say, on the wetted perimeter.

Observation has resulted in the formulation of general laws as to friction losses. For example, they are proportional to the length of the channel; they increase with the roughness of the walls; they decrease as the mean depth increases; they increase nearly as the square of the velocity; and they are independent of the pressure of the water.

It is usually assumed that the friction per unit of wetted surface may be represented by,

$$F = f_0 \, v^2 \quad \dots\dots\dots\dots\dots\dots\dots\dots (14)$$

In this study, F will be considered as a constant for any canal, and will be given such value as shall have been found experimentally sufficient to reduce the summit of each wave to its observed value.

The reason for this is that the coefficient of friction is applied only to the value of the maximum height or summit of the wave to determine its value at a given point in the canal, and the remaining heights are assumed to be found from this maximum height by the cosine curve. As this maximum height varies quite slowly from point to point under the influence of friction, the corresponding maximum velocities do not vary rapidly. Accordingly, the value of the frictional coefficient varies slowly enough to be considered as constant without any great error.

The influence of friction is manifestly of more importance as the depth of the channel becomes smaller. In a narrow strait, several hundred feet deep, it might be negligible, but in artificial canals and the more prevalent tidal channels, it has a great effect.

Effect of Friction in Reducing Height of Wave in an Infinitely Long Canal.—It has been established experimentally that, in the case of a long flat wave of translation, with little curvature (as in the case of a tidal wave of translation), the speed of propagation, C, may be found from the so-called Scott-Russell formula (Equation (1)). The current velocity, v, may be represented by,

$$v = C\frac{H}{D+H} \dots\dots\dots\dots\dots\dots\dots\dots \text{(15)}$$

When the velocity of propagation is not very great, and when only the interior frictional resistances of liquids are considered as a main cause, the resistance due to friction per unit length of wave is equal to $F\,p\,v$.

The theoretical value of v is expressed by,

$$v = \sqrt{g\,(D+H)} \times \frac{H}{D+H} = \frac{\sqrt{g}\,H}{\sqrt{D+H}} \dots\dots\dots\dots \text{(16)}$$

Hence,

$$F\,p\,v = \frac{F\,p\,\sqrt{g}\,H}{\sqrt{D+H}} \dots\dots\dots\dots\dots\dots\dots\dots \text{(17)}$$

The elementary work of frictional resistances during the time, dt, represented by dF', is,

$$dF' = F\,p\,v \times v \times dt = F\,p\,v^2\,dt = \frac{F\,p\,g\,H^2\,dt}{D+H} \dots\dots\dots \text{(18)}$$

The work, dF', represents the loss of energy of the tidal wave per unit of length in the time, dt. Hence,

$$dE = -\,dF' = -\,\frac{F\,p\,g\,H^2\,dt}{D+H} \dots\dots\dots\dots\dots\dots\dots \text{(19)}$$

It can be shown from theoretical considerations that the energy of any section of wave of unit length may be expressed as,

$$E = \rho\,g\,l\,H^2 \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \text{(20)}$$

If x is the distance of propagation in the channel, $\frac{dx}{dt} = C = \sqrt{g\,(D+H)}$; whence,

$$dt = \frac{dx}{\sqrt{g\,(D+H)}} \dots\dots\dots\dots\dots\dots\dots\dots\dots \text{(21)}$$

Differentiating Equation (20), substituting for dt in Equation (19) its value from Equation (21), and simplifying,

$$dE = \frac{(D+H)^{\frac{3}{2}}\,dH}{H} = -\,\frac{F\,p\,dx}{2\,\rho\,l\,\sqrt{g}} \dots\dots\dots\dots \text{(22)}$$

By integrating Equation (22), an expression is obtained which shows how the height of a wave of translation in a canal of infinite length and constant section is reduced from the effects of frictional resistance as the wave is propagated along the axis of x.

Equation (22) may be expanded and reduced as follows:

$$\frac{D^{\frac{3}{2}}\, dH}{H} + \frac{3}{2} D^{\frac{1}{2}}\, dH + \frac{3}{8} \frac{H\, dH}{D^{\frac{1}{2}}} - \frac{1}{16} \frac{H^2\, dH}{D^{\frac{3}{2}}} + \ldots = - \frac{F\, p\, dx}{2\, \rho\, l \sqrt{g}} \quad \text{(23)}$$

As the expanded equation is rapidly convergent, all terms after the third may be omitted in practice. Integrating Equation (23),

$$D^{\frac{3}{2}} \log_e H + \frac{3}{2} D^{\frac{1}{2}} H + \frac{3}{16} \frac{H^2}{D^{\frac{1}{2}}} - \frac{1}{48} \frac{H^3}{D^{\frac{3}{2}}} + \ldots = - \frac{F\, p\, x}{2\, \rho\, l \sqrt{g}} + \text{a constant} \ldots\ldots\ldots\ldots \text{(24)}$$

When $x = 0$, let $H = H_0$; then,

$$D^{\frac{3}{2}} \log_e H_0 + \frac{3}{2} D^{\frac{1}{2}} H_0 + \frac{3}{16} \frac{H_0^{\,2}}{D^{\frac{1}{2}}} - \frac{1}{48} \frac{H_0^{\,3}}{D^{\frac{3}{2}}} + \ldots = \text{a constant} \ldots \text{(25)}$$

Subtracting Equation (24) from Equation (25),

$$D^{\frac{3}{2}} \log_e \left(\frac{H_0}{H}\right) + \frac{3}{2} D^{\frac{1}{2}} (H_0 - H) + \frac{3}{16} \frac{(H_0^{\,2} - H^2)}{D^{\frac{1}{2}}} - \frac{1}{48} \frac{(H_0^{\,3} - H^3)}{D^{\frac{3}{2}}} + \ldots = \frac{F\, p\, x}{2\, \rho\, l \sqrt{g}} \ldots\ldots\ldots\ldots \text{(26)}$$

which is the desired formula.

Equation (26) shows that the wave would be propagated in an infinitely long canal for an infinite distance; that is, theoretically, the continually reducing effect of friction would cause the wave to disappear only at an infinite distance; but, practically, it would disappear within appreciably finite distances. Except for friction the wave would be propagated indefinitely at full volume. However, the effect of friction is continually to reduce the volume of the wave by a varying quantity per foot. Assuming for the moment that this lost water could be held at the point at which it was left behind by a single positive wave, it would appear upon the original canal surface as a thin wedge, thickest at the canal entrance, and diminishing to zero only at an infinite distance. Actually, of course, this water left behind by the wave in its travel will tend to flow back to the source and add its volume to that of the succeeding negative wave.

Since waves of translation may be either positive or negative, a depression in the water surface may be propagated as a wave, as well as an intumescence, in which case the height of the wave, h, will have a negative value. Since Equation (26) is of general application, by giving proper positive and negative values to H, curves will be found which will be the theoretical loci of high and low tide for the infinitely long canal. (See Fig. 2.) It is apparent at once that these curves are not symmetrical. The effect of friction in destroying the wave is greater for the negative than for the positive wave, since the depth of water is less in the canal during the propagation of such a wave.

Time Required for Crest of Wave to Reach a Given Point.—If Equation (21) is written, $\frac{x}{t} = C = \sqrt{g\,(D + H)}$, the rate of propagation to a given point is found to depend on the height of the wave which, due to the action of friction, is variable. If, however, the values of C are computed for given values of H selected quite close together, the rate of propagation, C, may be assumed constant between these points, and a mean value for the points in question may be assumed as correct without appreciable error. This gives a new value, C', for the rate of propagation between two such points,

$$C' = \frac{1}{2}\sqrt{g}\left[\sqrt{D + H_1} + \sqrt{D + H_2}\right] = \frac{(x_1 - x_2)}{t} \ldots\ldots (27)$$

whence,

$$t = \frac{2\,(x_1 - x_2)}{\left(\sqrt{D + H_1} + \sqrt{D + H_2}\right)\sqrt{g}} \ldots\ldots\ldots\ldots\ldots (28)$$

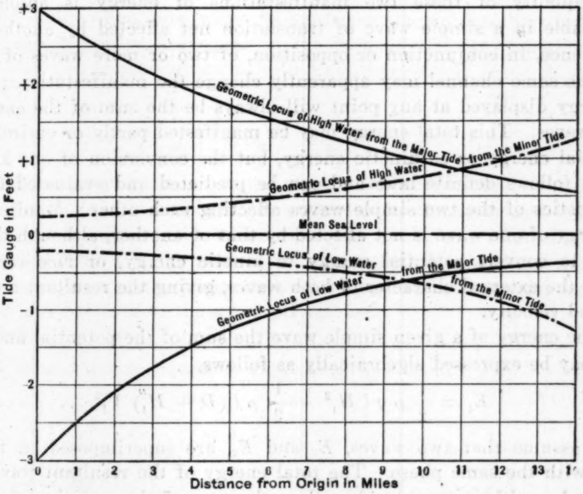


FIG. 2.—GEOMETRIC LOCI OF HIGH AND LOW WATER FOR TWO SIMULTANEOUS WAVES PROPAGATED IN A CANAL OF INFINITE LENGTH.

For the negative wave, of course the values of H in Equation (28) would be negative.

The equations show that, considering friction, the rate of propagation, or the speed, falls off as the wave progresses, and the wave constantly tends to check its speed. Furthermore, this effect is greater for the negative than for the positive wave.

If, therefore, a series of successive waves of translation, positive and negative alternating, are being propagated in the canal one would expect to find the positive waves tending to outrun the negative waves slightly, the effect being the more apparent the greater the distance traveled.

Energy.—The total energy of a wave of translation,[13] or the work which it could do in being brought to a state of rest, would be constant were it not for friction; but while continually reduced by friction, the remaining energy at any point of an indefinite canal consists of two equal parts, potential energy as represented by the height of the intumescence above the mean level, and kinetic energy as represented by the velocity of the mass of water. The expression for the potential energy of one unit of length of wave is $\frac{1}{2}\,\rho\,g\,l\,h^2$, and that for its kinetic energy is $\frac{1}{2}\,\rho\,l\,(D + h)\,v^2$. The equality of these two kinds of energy may be made apparent by substituting in the latter expression for v its value from Equations (13) and (15). This equality of energy may be upset, however, by any cause which impresses a change upon either height or velocity of the wave.

The equality of these two manifestations of energy is absolute and unchangeable in a simple wave of translation not affected by another wave. The existence, in conjunction or opposition, of two or more waves of translation in the same channel may apparently change the manifestations; but the total energy displayed at any point will always be the sum of the energies of the two waves. This total energy may be manifested partly or entirely either as potential energy or as kinetic energy, but the conversion of one kind into the other follows definite laws and can be predicted and evaluated from the characteristics of the two simple waves affecting each other. Manifestly, the total energy of one wave is not affected by that of another, although the latter may act to convert potential energy to kinetic energy, or *vice versa,* thus changing the external character of both waves, giving the resultant a different height and velocity.

For the energy of a given simple wave the sum of the potential and kinetic energy may be expressed algebraically as follows,

$$E_1 = \frac{1}{2}\,\rho\,g\,l\,H_1^2 + \frac{1}{2}\,\rho\,l\,(D + H_1)\,V_1^2 \qquad (29)$$

Now assume that two waves, E_1 and E_2, are superimposed in the same channel with the same phase. The total energy of the resultant wave at this point can be neither more nor less than the sum of the energies of the component waves, neglecting friction; hence,

$$E_3 = E_1 + E_2 = \frac{1}{2}\,\rho\,g\,l\,(H_1^2 + H_2^2) + \frac{1}{2}\,\rho\,l\,(D + H_1 + H_2)\,(V_1^2 + V_2^2) \qquad (30)$$

It will be noted that in Equation (30) the total resulting depth of water $(D + H_1 + H_2)$, is used instead of $(D + H_1)$ or $(D + H_2)$. In compound waves, the total depth must always be used in computing the respective velocities of component waves from their respective heights. Failure to do so would involve an error in the expression for the total amount of kinetic energy.

It will be interesting to see whether any change as between kinetic and potential energy has occurred in this superposition. Assume, in accordance with the usual understanding as to the formation of a resultant wave from two components, that $E_3 = E_1 + E_2$; $H_1 + H_2 = H_3$; and $V_1 + V_2 = V_3$. If the resultant wave is assumed to be a simple wave of translation, its equation will be similar to Equation (29), thus:

---

[13] "Hydraulique," par Flamant, Paris, 1923, p. 434.

$$E_3 = \frac{1}{2}\rho\, l\, g\, H_3^2 + \frac{1}{2}\rho\, l\, (D + H_3)\, V_3^2 \ldots\ldots\ldots\ldots\ldots (31)$$

or,

$$E_3 = \frac{1}{2}\rho\, l\, g\, (H_1^2 + 2\, H_1\, H_2 + H_3^2)$$

$$+ \frac{1}{2}\rho\, l\, (D + H_1 + H_2)\, (V_1^2 + 2\, V_1\, V_2 + V_2^2) \ldots\ldots\ldots\ldots (32)$$

Subtracting Equation (31) from Equation (30) to solve for V_3 and H_3, $V_3^2 = \sqrt{V_1^2 + V_2^2}$ and $H_3^2 = \sqrt{H_1^2 + H_2^2}$.

This result seems to be at variance with the understanding previously expressed, that when one wave is superposed upon another the resulting heights and velocities are equal to the sum of the respective heights and velocities of the component waves. In Equation (32) if $H_1 = H_2$, and hence $V_1 = V_2$,

$$E_3 = 4\left[\left(\frac{1}{2}\rho\, l\, g\, H_1^2\right) + \frac{1}{2}\rho\, l\, (D + 2\, H_1)\, V_1\right] \ldots\ldots\ldots (33)$$

which is twice as great as the actual energy available in the two waves. Such a result violates the principle of the conservation of energy and is manifestly incorrect. It invalidates the assumption that in the case of superposed waves the resultant height is the sum of the component heights, and the resultant velocity is the sum of the component velocities. In other words, the resultant wave can not conform to the assumptions made.

Next, assume the two equations expressing the relations, $V_1 = \dfrac{\sqrt{g}\, H_1}{\sqrt{D + H_1 + H_2}}$ and $V_2 = \dfrac{\sqrt{g}\, H_2}{\sqrt{D + H_1 + H_2}}$. Multiplying these together, $V_1\, V_2 = \dfrac{g\, H_1\, H_2}{D + H_1 + H_2}$, and, consequently,

$$\frac{1}{2}\rho\, l \times 2\, V_1\, V_2\, (D + H_1 + H_2) = \frac{1}{2}\rho\, l\, g \times 2\, H_1\, H_2 \ldots\ldots (34)$$

Subtracting Equation (34) from Equation (30),

$$E = \frac{1}{2}\rho\, l\, g\, (H_1 + H_2)^2 + \frac{1}{2}\rho\, l\, (D + H_1 + H_2)\, (V_1 - V_2)^2 \ldots (35)$$

It is apparent that if $H_1 + H_2 = H_3$, then must $V_1 - V_2 = V_3$, or if $V_1 + V_2 = V_3$, then must $H_1 - H_2 = H_3$, in order that the principle of conservation of energy may not be violated. From these results one may infer that any energy that may appear to have been added as a reinforcement to the actual height of the resultant wave must disappear as a loss from the total energy of the velocities of the component waves, or *vice versa*.

The general conclusions to be drawn from these facts are that:

1.—For two or more waves traveling together in conjunction, one wave is formed, the height and velocity of which are equal to the square root of the sum of the squares of the respective component heights and velocities.

2.—For waves traveling in opposite directions at the points of conjunction, if both waves are either positive or negative, the heights will be the sum of the heights of the component waves, and the velocity will be the difference between the component velocities.

3.—If one wave is positive and one negative—that is, if the waves are in opposition—the resultant height will be the difference between the component heights, and the resultant velocity will be the sum of the component velocities.

***Influence of Change of Width.*—Consider next the effect upon the tidal wave of an abrupt change in the width of a canal.**

Let the canal, *A B*, in Fig. 3, with a width, l, suddenly become narrower at *B*, so that its width is l' in the section, *B C*. If h_0 is the height of the generating wave at *A* and h_1 is its height as reduced by friction at *B*, then the energy of the wave at *B* will be $E = \rho\, g\, l\, h_1^2$. Due to the reduced width of *B C*, only a portion of this energy can be propagated into *B C*. Let the actual height of the tide at *B* equal h; then, this energy, E_1, is equal to $\rho\ g\ l_1\ h^2$.

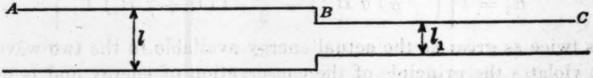


FIG. 3.—PLAN OF CANAL WITH SUDDENLY CONTRACTED SECTION.

The energy which is not transmitted past *B* will manifest itself in a reflected wave traveling back from *B* toward *A*, and will have the effect at *B* of raising the water level at that point higher than it would have been had the width been constant, and it accounts for the difference between h_1 and h.

Therefore, this reflected energy may be expressed by the formula,

$$E_2 = E - E_1 = \rho\, g\, (l\, h_1^2 - l_1\, h^2) = \rho\, g\, l\, (h - h_1)^2 \quad \text{.......(36)}$$

Reducing,

$$l\, h_1^2 - l_1\, h^2 = l\, h^2 - 2\, l\, h\, h_1 + l\, h_1^2$$

and,

$$h = \frac{2\, l\, h_1}{l + l_1} \quad \text{..........................(37)}$$

If l_1 is less than l, h will be greater than h_1; for example, if $l_1 = 0, h = 2\, h_1$. If l_1 is greater than l, h will be less than h_1; for example, if $l_1 = \infty$, $h = 0$. In the former case the reflected energy is positive and causes an additional rise in level between *A* and *B*. In the latter case, energy is abstracted from the canal, or the reflected energy is negative, and causes a lowering of level between *A* and *B*.

THEORY OF THE INTERFERENCE OF TWO OPPOSED WAVES

It is desirable to consider theoretically the results of superposing two tidal waves that are propagated in opposite directions in a horizontal canal of

uniform section and infinite length, neglecting the effects of friction.[13] It will be assumed that the waves are of the class called periodic, exactly sinusoidal, and of the same length, and that, conformably with theory, the velocities of the currents which they generate are approximately proportional in any vertical to the height of the wave referred to its mean level.

Under these conditions, it is easy to find the instantaneous profiles as well as the distribution of the velocities of the resulting wave during the entire period of its propagation. Considering as positive the heights of the two component waves above the mean level, it will only be necessary to add them algebraically to obtain the corresponding heights of the resulting wave. Likewise, the distribution of the velocities in the wave will be obtained by adding algebraically the velocities of the component waves in the same vertical. Those in the direction of the propagation of the wave which has the largest vertical amplitude, are considered positive and those which have an opposite direction are negative. There are then three relative positions of the two waves to be considered principally.

First consider what happens in such a case when the waves are in coincidence, or in conjunction.

In Fig. 4, let *A* 2 *C* 2 *E* be a wave being propagated toward *E,* and let *E* 1 *C* 1 *A* be a wave being propagated toward *A.* Since both waves are tidal, they will have equal lengths, and there will be points such as *A, C,* and *E,* at which their points of zero elevation will arrive simultaneously. It is evident that these points will be spaced at distances apart equal to one-half a wave length. Half way between these points, and, consequently, distant from them by one-quarter of a wave length, there will be points, *B* and *D,* at which the maximum height of each wave will arrive simultaneously.

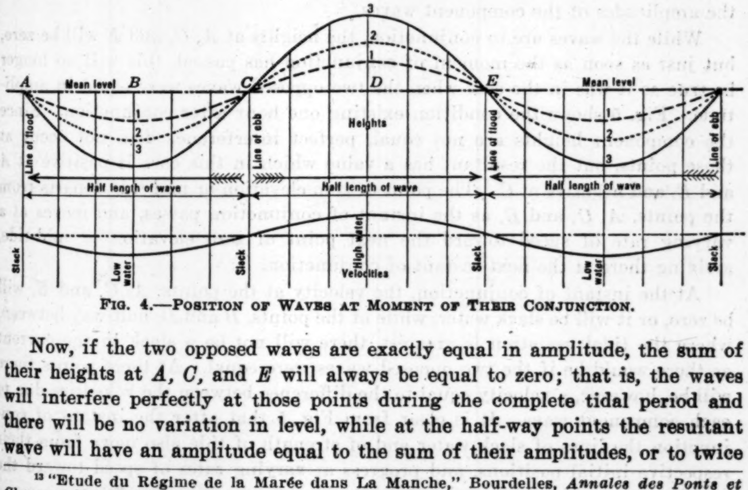


FIG. 4.—POSITION OF WAVES AT MOMENT OF THEIR CONJUNCTION.

Now, if the two opposed waves are exactly equal in amplitude, the sum of their heights at *A, C,* and *E* will always be equal to zero; that is, the waves will interfere perfectly at those points during the complete tidal period and there will be no variation in level, while at the half-way points the resultant wave will have an amplitude equal to the sum of their amplitudes, or to twice their common amplitude. On the other hand, at *A, C,* and *E,* the respective velocities will add their values to give a new velocity equal to twice that of a single wave, while at *B* and *D* the velocities will annul each other, being equal in value and opposite in direction.

---

[13] "Etude du Régime de la Marée dans La Manche," Bourdelles, *Annales des Ponts et Chaussees,* 1899, 3^me trimestre.

Then, the surface of the hypothetical canal, when subjected to the action of two equal and opposed waves, will present the appearance of being subdivided into sections the length of one half wave in which the surface oscillates up and down in amplitudes increasing from zero value at *A* to a maximum at *B* and then decreasing to zero again at *C,* and so on indefinitely.

At the same time the mass of water in the canal will flow to and from the points, *B* and *D,* increasing in velocity in both directions from these points as a starting place to a maximum at *A, C,* and *E,* but always decreasing to zero again at *B* and *D* and reversing this action in the next half of the wave.

Points such as *A, C,* and *E* are nodes, and *B* and *D* are summits. Therefore, as a result of the interference of two opposed equal waves, there is a series of alternating nodes and summits, the surface variations being zero and the velocities greatest at the nodes, and the surface variations greatest and the velocities zero at the summits.

This state of affairs gives rise to a resultant wave termed the stationary wave or "chop." It is called stationary because the wave form does not progress in the canal, but remains fixed in position, and it is always characterized by these peculiarities. Its special laws will be summarized later.

For waves of unequal height in conjunction, the crests and the troughs of the two component waves will be found in the same vertical, the swells of the waves being in the same phase. (See Fig. 4.) In this position, the vertical amplitude of the resultant wave is a maximum and is equal to the sum of the amplitudes of the component waves.

While the waves are in conjunction, the heights at *A, C,* and *E* will be zero, but just as soon as the moment of conjunction has passed, this will no longer be true as it was in the case when the two opposed waves were equal in amplitude. Fig. 5 shows the condition existing one hour after conjunction. Since the component heights are not equal, perfect interference does not occur at those points, but the resultant has a value which in this case is positive at *A* and *E,* and negative at *C.* The point of zero elevation or mid-tide departs from the points, *A, C,* and *E,* as the instant of conjunction passes, and moves at a varying rate of speed toward the next point of zero elevation or mid-tide, arriving there at the next instant of conjunction.

At the instant of conjunction, the velocity at the points, *A, C,* and *E,* will be zero, or it will be slack water, while at the points, *B* and *D,* half-way between, where the tidal variation is greatest, there will not be a slack in the current as there would be if the two opposed waves were equal. At these points there will be, however, a velocity equal to the difference between the velocities due to each component wave. It is clear from Fig. 5 that after the instant of conjunction the lines of slack water and of strength of tide also move from their respective initial positions, and progress at varying rates of speed toward the positions proper for the next instant of conjunction. (See Fig. 6.)

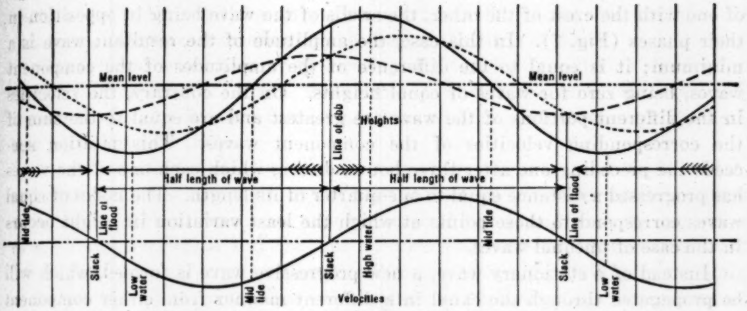

Mean level
Mean level
Heights
Line of ebb
Half length of wave
Half length of wave
Mid tide
Line of flood
Slack
Low water
Mid tide
Slack
High water
Mid tide
Slack
Low water
Velocities

FIG. 5.—POSITION OF WAVES ONE HOUR AFTER THEIR CONJUNCTION.

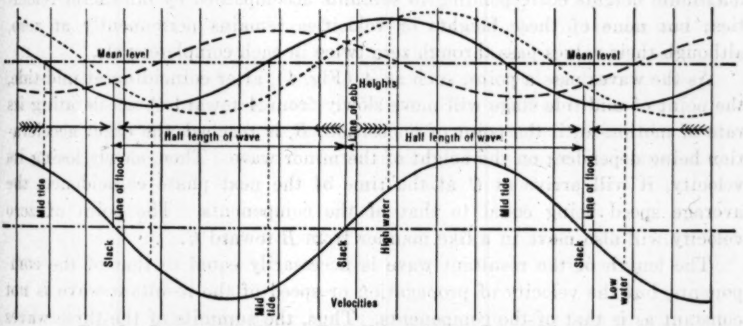

Mean level
Mean level
Heights
Line of ebb
Half length of wave
Half length of wave
Mid tide
Line of flood
Slack
Mid tide
Slack
High water
Mid tide
Slack
Low water
Velocities

FIG. 6.—POSITION OF WAVES TWO HOURS AFTER THEIR CONJUNCTION.

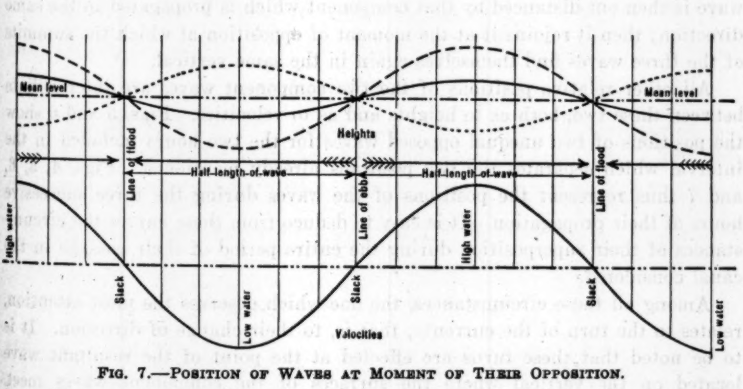

Mean level
Mean level
Heights
Half length of wave
Half length of wave
Line of flood
Line of ebb
Line of flood
High water
Slack
Slack
Low water
High water
Slack
Low water
Velocities

FIG. 7.—POSITION OF WAVES AT MOMENT OF THEIR OPPOSITION.

The second relative position of the waves which it is important to notice, is that which corresponds to the coincidence, in the same vertical of the trough of one with the crest of the other, the swells of the wave being in opposition in their phases (Fig. 7). In this case, the amplitude of the resultant wave is a minimum; it is equal to the difference of the amplitudes of the component waves, being zero for waves of equal heights. On the contrary, the velocities in the different portions of the wave are greatest and are equal to the sum of the corresponding velocities of the component waves. This position succeeds the preceding one after three hours, during which each one of the waves has progressed a distance equal to one-quarter of its length. The nodes of equal waves correspond to those points at which the least variation in height occurs in the case of unequal waves.

Instead of a stationary wave, a new progressive wave is formed, which will be propagated through the canal in a different manner from either component wave. This progressive wave will have points of minimum heights corresponding to nodal points accompanied by maximum velocities, and points of maximum heights corresponding to summits accompanied by minimum velocities; but none of these heights or velocities remains permanently at zero, although their values pass through zero twice in each complete cycle.

As the waves pass a point, such as *A* (Fig. 4), after coinciding at mid-tide, the point of mid-tide stage will move slowly from *A* toward *C*, accelerating its rate of motion until the point of opposition, *B*, is reached, the exact acceleration being dependent on the height of the minor wave. Then, slowly losing its velocity, it will arrive at *C* at the time of the next phase coincidence, the average speed being equal to that of the components. The point of zero velocity will also move in a like manner from *B* toward *C*.

The length of the resultant wave is necessarily equal to that of the components, but the velocity of propagation or speed of the resultant wave is not constant as is that of the components. Thus, the summits of the three waves are in the same vertical at the moment of their conjunction. The resultant wave is then out-distanced by that component which is propagated in the same direction; then it rejoins it at the moment of opposition at which the summits of the three waves find themselves again in the same vertical.

All other relative positions of the two component waves are intermediate between these two, both as to heights and as to velocities. Figs. 5 and 6 show the positions of two unequal opposed waves for the two hours included in the interval which separates the two positions already mentioned. Figs. 4, 5, 6, and 7 thus represent the positions of the waves during the three successive hours of their propagation. It is easy to deduce from these curves the circumstances of their superposition during the entire period of their passage in the canal considered.

Among all these circumstances, the one which deserves the most attention, relates to the turn of the currents; that is, to their change of direction. It is to be noted that these turns are effected at the point of the resultant wave located on the vertical where the surfaces of the component waves meet. Furthermore, when this point is found in that portion of the resultant wave corresponding to the falling tide, the currents diverge or separate from that point which marks what is termed the "line of separation" of the currents, or "line of ebb". They approach or meet when the point of intersection occurs on that portion of the resultant wave corresponding to the rising tide, marking the line termed the line of meeting of the tide, or the "line of flood".

The lines of separation or of meeting are equally spaced at a distance of one-half wave length. Upon these lines the currents turn; their velocity increases with the distance from these lines, and it is a maximum at the half-way point between them.

In the resultant wave, the currents turn at half-tide at *A, B, C, D,* and *E,* and they reach their strength at the high and at the low water at the moment when the component waves are superposed at *B* and *D* and when in opposition at *A, C,* and *E* (Figs. 4 and 7). For all other points in the canal, the turns of the currents occur a greater or less time after half-tide, depending on the hour, or the position of the component waves, and the strength of the current comes one-quarter of the tidal period later. When the waves are equal, the currents turn simultaneously throughout the entire canal.

Therefore, if the water of the infinitely long canal is subjected to the action of two unequal and opposed waves, its surface will present the appearance of transmitting a wave having a direction and average speed equal to that of the major wave, but which will continually vary in speed and amplitude as it progresses. Furthermore, its velocity in any given vertical will not be in direct proportion to the amplitude at that point. The phenomenon of the interference of two opposed waves, having the same horizontal amplitude, the same speed of propagation, and the same period, but of different vertical amplitude, lends itself easily to mathematical demonstration.[14]

Assuming that, at the instant $t = 0$, the two profiles of the waves pass through the origin of co-ordinates taken upon the mean level, with the axis of x in that level, and that they correspond to the conjunction of the two waves which would accordingly be in the same phase—the equations of the profiles of the two waves would be:

$$y_1 = H_a \sin 2\pi \left(\frac{x}{L} - \frac{t}{T}\right)$$

and,

$$y_2 = H_b \sin 2\pi \left(\frac{x}{L} + \frac{t}{T}\right)$$

In these equations, L is the length of each of the waves; T, their period; and H_a and H_b, their semi-amplitudes above and below the mean level. The ordinate, y, of the resultant wave is:

$$y = y_1 + y_2 = H_a \sin 2\pi \left(\frac{x}{L} - \frac{t}{T}\right) + H_b \sin 2\pi \left(\frac{x}{L} + \frac{t}{T}\right) \quad \text{..(38)}$$

Let,

$$K^2 = H_a^2 + H_b^2 + 2 H_a H_b \cos \frac{4\pi t}{T}$$

and,

$$\tan \frac{2\pi\phi}{L} = \frac{H_a - H_b}{H_a + H_b} \tan \frac{2\pi t}{T} \quad \text{.................(39)}$$

---

[14] "Hydraulique," Flamant, Paris, 1923, p. 486.

then Equation (38) can be written in the form:

$$y = K \sin \frac{2\pi}{L}(x - \phi) \qquad (40)$$

It follows from Equation (40) that at every point the maximum amplitude of oscillation has a special value, dependent upon the position of the point, and that it is comprised between $H_a + H_b$ and $H_a - H_b$. The maximum amplitude, $H_a + H_b$, is found at intervals equal to L, and the minimum amplitude, $H_a - H_b$, at the middle point of these intervals. At a given point, the duration of a complete oscillation is always equal to T. Likewise, at a given instant, the total length of the undulation is always equal to L; its mean speed of propagation is, therefore, equal to that of each of the component waves, but the speed varies with time. Differentiating Equation (39):

$$\frac{d\phi}{dt} = \frac{\frac{L}{T}}{\frac{H_a - H_b}{H_a + H_b} \sin^2 \frac{2\pi t}{T} + \frac{H_a + H_b}{H_a - H_b} \cos^2 \frac{2\pi t}{T}} \qquad (41)$$

which is a minimum for $t = 0$, $t = T$, etc., and a maximum for $t = \frac{T}{2}$, $t = \frac{3T}{2}$, etc.

The meeting points of the component waves and the positions of the lines of flux are found from the equality, $y_1 = y_2$; whence, $H_a \sin 2\pi \left(\frac{x}{L} - \frac{t}{T}\right) = H_b \sin 2\pi \left(\frac{x}{L} + \frac{t}{T}\right)$, which gives,

$$\tan 2\pi \frac{x}{L} = \frac{H_a + H_b}{H_a - H_b} \tan 2\pi \frac{t}{T} \qquad (42)$$

Equation (42) shows that the lines are spaced at one-half wave length apart. The speed of their displacement is given by the expression,

$$\frac{dx}{dt} = \frac{\frac{L}{T}}{\frac{H_a - H_b}{H_a + H_b} \cos^2 \frac{2\pi t}{T} + \frac{H_a + H_b}{H_a - H_b} \sin^2 \frac{2\pi t}{T}} \qquad (43)$$

which is minimum for $t = \frac{T}{2}$, $t = \frac{3T}{2}$, etc., and a maximum for $t = 0$, $t = T$, $t = 2T$, etc.

When the two waves have the same vertical amplitude, $H_a = H_b$, the resultant motion is the "chop". Its special laws which result from the preceding equations are summarized as follows:

1.—At the instant of conjunction, the velocities are annulled, or slack-water prevails for the entire length of the canal, and the amplitude of the resultant wave is double that of the component waves.

2.—At the instant of opposition, the velocities reach their maximum in all that part of the canal where the mean level prevails. There is no change of level produced, during the entire tide, at those points corresponding to the summits of the waves in opposition, and the velocity there is greater than elsewhere.

3.—The positions of the lines of flux and of reflux remain invariable for the entire tide; they coincide with that of the summits of the waves in conjunction.

General Remarks.—It must be recognized that theory, even if perfect, would still remain inadequate to solve completely the complex problems which the regimen of the tide raises in practice. The ideal canal is not found in Nature; waterways have a more or less irregular bed, and almost always they have a length less than that of the tidal wave which traverses it. This latter fact alone has hitherto sufficed to upset all theories, because the tidal wave cannot be propagated in them as has been assumed for an infinitely long canal. In this hypothetical canal the waves always travel forward, without mutually disturbing each other; each one before and after its passage finds and leaves any point in its original condition; they then disappear without leaving any trace of their existence. It is not so when the length of the canal is limited; in introducing itself the wave encounters the effects of preceding waves; the circumstances of its propagation, consequently, are modified by reason of these effects, which will be found impossible to follow in all their consequences. A complex and variable regimen results from all this, according to circumstances, controlled by the length and characteristics of the bed, which it seems must forever escape satisfactory analysis.

On the other hand, by a consideration of the theoretical canal, it is possible to treat some of the influences which affect the tidal wave, and to forecast what general effect may be expected from variations of length, width, depth, and the material of the bed of the canal. By means of this study it is possible to determine relative if not absolute values of such effects. It is in this spirit that the problem is approached in the following pages.

Propagation of the Tide in a Canal Communicating with an Infinitely Large Reservoir

Insufficiency of Theory That Flow Caused by Surface Slope Accounts for Tidal Flow.—Assume a horizontal canal of constant cross-section communicating at one of its ends with the sea, and at the other with a reservoir sufficiently large to be considered as infinite; that is, so large that its level remains constant. Let $A B$ in Fig. 8 be the mean level of the sea assumed to be the same as that of $B C$ in the hypothetical reservoir, and A_2, A_4, the extreme levels attained by the tide. Assume that, at a given instant, a state of equilibrium of the entire liquid mass is established, the level of the sea being at A, and the surface of the water horizontal for the entire length of the canal. Suppose the level then rises to A_1. If it should remain immobile for a sufficiently long time at this point, a permanent motion would be set up in the canal between A and B, defined by the equations of constant uniform flow. The free surface between these points would take the form of a curve tangent at A_1 to the horizontal, and convex upward.

However, things do not happen in this manner. The level is constantly variable and the permanent regimen, corresponding to any such height as A_1, never has time to become established. Even the state of equilibrium which was assumed when the level was at A, at the height, $B C$, would never be realized unless the level should remain motionless at A for a sufficiently long time.

During the period when the level rises from A toward A_2, it reaches some height A_1, before the water entering through the mouth can have filled completely the triangle, AA_1B. It rises relatively faster at A than at any other intermediate point between A and B, and the curve of level takes a concave form upward, such as that shown by the broken line in Fig. 8.

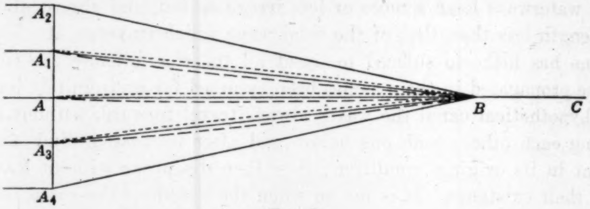


FIG. 8.

On the other hand, when the level falls from A_2 toward A, as it is continually variable, it leaves any position before the corresponding permanent state can become established in the canal, and the curve of level is then convex upward, as is shown by dotted lines. The convexity of this curve will be even greater than that which would correspond to a permanent regimen.

The same effects will be produced in the portion of the oscillation that takes place below the level of equilibrium, between A and A_4. When the level comes to A_3 in falling, the surface of the water in the canal is convex upward and, conversely, it is concave when the level passes through A_3 in rising. The curves in broken lines then represent the levels during the flood, and those in dotted lines, the levels during the ebb.

Under these conditions, the velocities at each point of the canal will be nil because of reversal at the moment of half-tide, when the level passes through the mean level; they will be a maximum, whether on the flood or the ebb, at the moment the difference of level is greatest.

The phenomenon is in reality much less simple than the preceding description might indicate. The tidal wave is not transmitted instantaneously in the entire length of the canal, so that the high waters, the slack waters, and the low waters are not simultaneous in the entire stretch. They are successive because of the retardation of the propagation of the wave. The speed of this propagation itself is different from what it would be in a canal of constant depth. If the level of the sea is considered to be held motionless during a sufficient time at a height, A_1, and the corresponding permanent regimen is established with a certain mean velocity of flow, U_1, an undulation created at A_1 will be propagated toward B with a speed greater by U_1, approximately, than that which would correspond to the depth. On the other hand, the tidal wave entering into the canal possesses a certain energy which, except for the losses due to friction, remains constant. Because the depth diminishes from A_2 toward B an increase of the velocities results from the constancy of this energy, which again has for a consequence an increase of the speed of propagation. For this reason, the speed of propagation of the wave and the velocities of the water will be greater in the following sections than they would be in a horizontal canal with free surface and of the same depth as this section; and the difference will go on increasing as the point, B, is approached. (From the origin of the canal toward the point, A, this speed of propagation and the velocities will have adjusted themselves with regard to local circumstances and to the depth corresponding to the level, A_2.)

Influence of an Infinitely Large Reservoir.—What influence is exerted upon the wave of translation by the intervention at any point of an infinitely large reservoir? This hypothetical body of water has been defined as one sufficiently large to be unaffected by the emptying of the volume of the tidal wave into it. Consequently, its surface remains at a constant level, which is also assumed to be the mean level of the tidal sea and of the canal.

Equation (20) shows that no matter what the energy per unit length of the wave may be, the square of its height will vary inversely with the width of the canal, l. It follows from this and from the demonstration given under the heading, "Influence of Change of Width" (see Fig. 3), that if the width of the canal is made very large, h will become very small. Since the reservoir constitutes an infinite expansion in the width of the canal, l becomes very large, approaching infinity, and h becomes correspondingly small, approaching zero. Therefore, the wave of translation, whether positive or negative, must disappear at the outlet of the canal into the reservoir. Observation reveals that not only does the wave disappear at the outlet, but that the influence of the reservoir in decreasing the height of the wave is reflected back for the entire length of the canal, causing both heights and velocities at all points in the canal to differ from their theoretical values.

Explanation.—To find an explanation of this phenomenon, it is desirable to revert to the theory of the propagation of the tide in a canal as being formed by a series of elementary waves of translation, or successive fluxes and refluxes. Neglecting friction, the first flux being created on the rising tide would propagate itself indefinitely without change, until it reached the reservoir, where it must disappear. The flux is propagated at a definite speed through the canal, and a certain period is required to elapse from the time it appears at the entrance of the canal until it disappears at the outlet. Why does not the flux manifest itself simultaneously throughout the canal immediately upon its appearance? The answer, of course, is that the front or anterior portion of the flux must successively overcome the friction of the banks and the resistance or inertia opposed to its propagation by each particle of water in the canal. The height and the form of the flux are moulded by these resistances. The decrease of these resistances of inertia and the sudden cessation of all resistance to the flux upon entering into the reservoir permit a more rapid speed of propagation between the entrance and the outlet; but even this more rapid speed is never instantaneous, and a certain time would be required for the surface of the flux to acquire the slope it would assume if it were allowed time to establish the conditions of permanent flow. The flux is accompanied by a given velocity of flow of the water in the canal to supply the intumescence. The cessation of the forces of inertia opposing the flux then results in an increase of this velocity of flow.

Now, if a piston that just fits the cross-section of the canal were inserted at any point and given a push in a direction to increase the volume already flowing under the influence of the flux, the effect of this applied energy would be to generate a negative wave propagating itself in the direction opposite to that of the flux. This would be in addition to the positive wave propagated in the same direction, which for the time being is neglected.

The cessation of the resistances of inertia at the outlet will have the same effect as the piston or the addition of energy as assumed, thus creating a negative flux at the outlet which is propagated back through the canal toward the entrance.

The height of the negative flux (or depression) at the outlet will be exactly equal to that of the flux, so that their sum shall be zero, and the resultant velocities of the two fluxes, being in the same direction, will add their effects. Hence, the final velocity will be twice the theoretical, which is the velocity that would correspond to the computed height of the wave of translation for the point in question under the assumption of an infinitely long canal.

Source of the Energy Generating the Negative Flux.—The energy of the wave of translation is reduced only by friction, and in the infinitely long canal it will consist of equal parts of potential energy represented by the height of the wave and of kinetic energy as represented by the velocity of the water. Under proper conditions one kind of energy is convertible into the other; yet the total sum of available energy at any given point is constant.

The hypothetical reservoir is of sufficient size to absorb all the energy of the wave, with no effect upon it, and the energy represented by velocity of the water is expended at the outlet in the whirls, boils, and eddies that usually occur when flowing water is brought to rest in still water. If no change had occurred in the flux until it had reached the reservoir, one-half its energy, (kinetic), would have been so expended. The other half, the potential, must be converted into kinetic energy before it can be dissipated in this manner, and it is this energy being converted into kinetic energy—manifested by increased velocities of the water—which generates the opposed negative flux.

Resultant Effect.—Each flux following the first must be subject to the same requirement of disappearance in the reservoir, and must generate an opposite negative flux. Hence, it follows that, as the complete wave of translation is being propagated toward the reservoir, it is being continually modified by an opposed negative wave propagated from the reservoir toward the sea, and generated by the influence of the reservoir upon the original wave.

A similar reasoning would show that the negative wave of translation propagated from the sea toward the reservoir, and which may be said to absorb energy from the water into which it is propagated, will, when reaching the reservoir, cause an opposite positive wave to be generated there which will be propagated toward the sea.

A rigorous application of this reasoning would lead to the conclusion that the propagation of a single positive wave through the canal would result in an infinite series of alternating positive and negative waves being set up. Each positive wave from the sea would generate a corresponding negative wave from the reservoir, and *vice versa.* Under the influence of friction, however, each succeeding wave would become smaller and smaller, so that the series would finally die out, for all practical purposes. The resultant profile of the canal at any instant would be the algebraic sum of all the wave heights at that instant, and the resultant velocity would be the sum of all the velocities.

Formula for the Variation of the Tide at a Given Point.—At a given point the cycle of variation of any wave surface, being a result of the great ocean tidal wave, will be expressed approximately by a sinusoidal law, and its trace may be expressed by the equation,

$$Y = H \cos 2\pi \left(\frac{t - t_x}{T}\right) \dots\dots\dots\dots\dots\dots\dots (44)$$

Each point of the canal will have a different set of values for H and t_x. It is likewise manifest that values of H and t_x for the same point will be different for the positive and negative waves.

Changing the point of origin for counting the time, t_x, and computing the time, t_y, for the propagation of a wave from the reservoir toward the sea, the same formulas can be used to compute the variation of the reflected wave from the reservoir to the sea, the proper constants being inserted in the general equations.

Let t_L be the time required for the positive wave to be transmitted from the sea to the reservoir, and t_y the time required for the reflected negative wave to be transmitted to the point represented by a given set of values of H and t_x (the factor, $-H'$, being the corresponding value of the height of the reflected wave at that point), the following is the equation of the variation of the reflected wave at the same point:

$$-Y' = -H' \cos 2\pi \left(\frac{t - (t_L + t_y)}{T}\right) \dots\dots\dots\dots (45)$$

Adding Equations (44) and (45), the actual height, h, of the tide at the selected point at the time, t, is,

$$h = Y - Y' = H \cos 2\pi \left(\frac{t - t_x}{T}\right) - H' \cos 2\pi \left(\frac{t - (t_L + t_y)}{T}\right) \dots (46)$$

This equation may be written in the shorter form,

$$h = \sqrt{H^2 + H'^2 - 2\, H H' \cos 2\pi \left(\frac{t_x - (t_L + t_y)}{T}\right)} \cos 2\pi \left(\frac{t - t_m}{T}\right) . (47)$$

in which, t_m may be found from the equation,

$$\tan 2\pi \frac{t_m}{T} = \frac{H \sin 2\pi \frac{t_x}{T} - H' \sin 2\pi \left(\frac{t_L + t_y}{T}\right)}{H \cos 2\pi \frac{t_x}{T} - H' \cos 2\pi \left(\frac{t_L + t_y}{T}\right)} \dots\dots (48)$$

It will be observed that t_m is that value of t for which h will be a maximum. It, therefore, represents the time required for the resultant wave to be propagated from the sea to the point considered.

It is evident from the preceding that for any number of reflected waves that may be considered, a general expression for the resultant wave may be written as follows:

$$h = H_1 \cos 2\pi \left(\frac{t - t_1}{T}\right) + H_2 \cos 2\pi \left(\frac{t - t_2}{T}\right)$$
$$+ H_3 \cos 2\pi \left(\frac{t - t_3}{T}\right) + \ldots + H_n \cos 2\pi \left(\frac{t - t_n}{T}\right) \ldots\ldots (49)$$

When any two waves simultaneously affect the same point and their heights are additive because they have the same sign, the resultant velocity must be the difference between the component velocities. A general formula, similar to Equation (49), may be written for the resultant velocity at the same point, as follows:

$$V = V_1 \cos 2\pi \left(\frac{t - t_1}{T}\right) - V_2 \cos 2\pi \left(\frac{t - t_2}{T}\right)$$
$$- V_3 \cos 2\pi \left(\frac{t - t_3}{T}\right) - \ldots - V_n \cos 2\pi \left(\frac{t - t_n}{T}\right) \ldots\ldots (50)$$

in which, V_1, V_2, V_3, etc., are found directly from the corresponding values of H_1, H_2, H_3, etc., by means of the equation,

$$V_n = H_n \sqrt{\frac{g}{D + h}} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (51)$$

the value of h being that given by Equation (49).

Equation (49) may be made to represent conditions at the outlet of the canal into the reservoir by making substitutions, as follows: $H_2 = -H_1$; $H_4 = -H_3$; $H_n = -H_{n-1}$; $t_1 = t_2$; $t_3 = t_4$; and $t_{n-1} = t_n$. Likewise, Equation (50) may be written so as to represent velocities at the same point by substituting $V_2 = -V_1$; $V_4 = -V_3$; and $V_n = -V_{n-1}$. At the sea entrance to the canal, $H_1 = H_0$; $H_3 = -H_2$; $H_5 = -H_4$; $H_n = -H_{n-1}$; $t_1 = 0$; $t_2 = t_3$; $t_1 = t$; $t_n + 1 = t_n$; $V_1 = V_0$; $V_8 = -V_2$; $V_5 = -V_4$; and $V_n = -V_{n-1}$.

In a similar manner Equations (49) and (50) may be applied to all other points in a tidal canal opening into a tideless reservoir by changing signs in alternate (second, fourth, sixth, etc.) quantities. Thus, in Equation (49), change the sign from plus to minus in alternate quantities, in which n is even. Conversely, under the same circumstances change the sign from minus to plus for alternate quantities in Equation (50).

To determine the values of t for which these values of h and v are maxima, differentiate the equations with reference to t, place the first differential equal to zero, and solve for t, the particular value of which will be designated t_m:

$$\frac{dh}{dt} = -H_1 \sin 2\pi \left(\frac{t - t_1}{T}\right) + H_2 \sin 2\pi \left(\frac{t - t_2}{T}\right)$$
$$- H_3 \sin 2\pi \left(\frac{t - t_3}{T}\right) \ldots \mp H_n \sin 2\pi \left(\frac{t - t_n}{T}\right) = 0 \ldots\ldots (52)$$

whence, $\tan 2\,\pi \dfrac{t_m}{T}$ equals,

$$\frac{H_1 \sin 2\,\pi \dfrac{t_1}{T} - H_2 \sin 2\,\pi \dfrac{t_2}{T} + H_3 \sin 2\,\pi \dfrac{t_3}{T} \ldots \pm H_n \sin 2\,\pi \dfrac{t_n}{T}}{H_1 \cos 2\,\pi \dfrac{t_1}{T} - H_2 \cos 2\,\pi \dfrac{t_2}{T} + H_3 \cos 2\,\pi \dfrac{t_3}{T} \ldots \pm H_n \cos 2\,\pi \dfrac{t_n}{T}} \ldots (53)$$

Similarly, for t_{mv}, the time of maximum velocity, $\tan 2\,\pi \dfrac{t_{mv}}{T}$ equals,

$$\frac{V_1 \sin 2\,\pi \dfrac{t_1}{T} + V_2 \sin 2\,\pi \dfrac{t_2}{T} - V_3 \sin 2\,\pi \dfrac{t_3}{T} \ldots \mp V_n \sin 2\,\pi \dfrac{t_n}{T}}{V_1 \cos 2\,\pi \dfrac{t_1}{T} + V_2 \cos 2\,\pi \dfrac{t_2}{T} - V_3 \cos 2\,\pi \dfrac{t_3}{T} \ldots \mp V_n \cos 2\,\pi \dfrac{t_n}{T}} \ldots (54)$$

It is quite evident that the value of t_m for heights is not the same as that of t_{mv} for velocities, and, therefore, the time of maximum velocities will not coincide with the time of maximum or of zero heights.

After $-V_{n-1}$ has been substituted for V_n in Equation (50) it is possible to evaluate the frictional coefficient, *F*, in an easy and rapid manner. Combining it with Equation (51):

$$V = 2\,V_1 \cos 2\,\pi \left(\frac{t - t_1}{T}\right) = 2\,H_1 \sqrt{\frac{g}{D + h}} \cos 2\,\pi \left(\frac{t - t_1}{T}\right) \ldots (55)$$

Whence,

$$H_1 = \frac{1}{2}\,V_0 \sqrt{\frac{D}{g}} \ldots\ldots\ldots\ldots\ldots\ldots\ldots (56)$$

in which, V_0 equals the maximum value of V occurring when $t = t_1$. The value of V_0 can be measured readily at the outlet by any of the usual methods of determining the maximum mean velocity, and this value when determined enables H_1 to be found from Equation (56). Then when H_1 is substituted in Equation (26), in which all factors except F will be known, the frictional coefficient for the particular case will be given. Bonnet found this value to be 2.48 for the Scheldt River (0.51 English measure). The writer has found 3.05 (or 0.60 English measure) to be the usual value for canals of 6 to 12 ft. in depth, while Bonnet's value appears to apply to larger canals, such as the Cape Cod and the Suez Canals.

Since the direct and reflected waves at the outlet have all their ordinates exactly equal and of opposite sign, thus completely annulling each other, it is apparent that they are in opposition at that point, and that the time of maximum velocity corresponds to the time when the maximum ordinates are in opposition. The instant of this occurrence, therefore, measured from the time when the wave entered into the canal at its mouth, gives the time, t_1, consumed by the wave in propagating itself from the mouth to the outlet, and this time can also be determined by observation.

In Equation (50) change the signs of the alternate second, fourth, sixth, etc., quantities from negative to positive and divide by $\sqrt{\dfrac{g}{D + h}}$. To the result add Equation (49) with the signs of alternate second, fourth, sixth, etc., quantities changed from positive to negative. The resulting equation, simplified, takes the form,

$$h + v\sqrt{\frac{D+h}{g}} = 2\,H_1 \cos 2\,\pi\left(\frac{t-t_1}{T}\right)\ldots\ldots\ldots\ldots(57)$$

Now, $v\sqrt{\frac{D+h}{g}}$ may be considered as the height of a simple wave of translation which alone would give rise to a velocity, v; it will be designated the velocity height and represented by h_v.

Equation (57) then takes the form,

$$h + h_v = 2\,H_1 \cos 2\,\pi\left(\frac{t-t_1}{T}\right)\ldots\ldots\ldots\ldots\ldots\ldots(58)$$

Equation (58) is general, and will be true for any moment, at any point on the canal. The conclusion is drawn from it that in a uniform canal communicating with a tidal sea at one end, and with a tideless basin at the other, the actual height of the water in the canal at any instant plus a height equivalent to the velocity of the water at the same instant will be equal to double the height of the original wave of translation as it would have been transmitted at the same instant in a canal of infinite length; that is, in the hypothetical canal as defined in this paper.

Similarly, if Equation (50) is multiplied by $\sqrt{\frac{g}{D+h}}$ and added to Equation (49) as before (with alternate signs changed), the resulting equation takes the form:

$$v + \frac{\sqrt{g}\,h}{\sqrt{D+h}} = 2\,V_1 \cos 2\,\pi\left(\frac{t-t_1}{T}\right)\ldots\ldots\ldots\ldots(59)$$

In Equation (59), $\frac{\sqrt{g}\,h}{\sqrt{D+h}}$ equals a velocity which would be due to the height, h, in an infinitely long canal of the same dimensions. Denoting this quantity by v_h,

$$v + v_h = 2\,V_1 \cos 2\,\pi\left(\frac{t-t_1}{T}\right)\ldots\ldots\ldots\ldots\ldots\ldots(60)$$

From Equation (60) it is conclusive that the velocity at any point in the canal at any instant, added to the velocity which would accompany—in an infinitely long canal—a wave of the same height as is found at the same instant, the sum will be equal to twice the velocity which the wave would have generated at that instant in the hypothetical canal.

Since the second members of Equations (58) and (60) are constant for any point at a particular instant, it shows that any decrease in the height of water above or below the theoretical height for the long canal must be accompanied by an increase in velocity, or *vice versa,* and that the total amount of energy manifested is the same in any case.

For the compound or resultant wave there is no approximately constant value for the ratio between h and v, as in the case of the simple wave of translation. Failure to comprehend this departure of the compound wave from the laws pertaining to the simple wave has heretofore made the computation of the tidal characteristics of any given point in a tidal canal quite unsatisfactory. It is now proposed to show that a satisfactory agreement between observed tidal phenomena and the computed values of such phenomena can be reached, and that the computation in any case can be made sufficiently close to enable all tidal phenomena in canals of fairly uniform size to be predicted.

Application of the Theory to the Suez Canal

That section of the Suez Canal between the Red Sea and the Bitter Lakes is an example of the type of canal under consideration. It is 26 km. (16 miles) long, and has an average semi-amplitude of tide of 2.24 ft. at the Suez entrance. The amplitude gradually diminishes until at the Bitter Lakes all tidal variation disappears. The Bitter Lakes have an area of about 48 500 acres, which is large enough to receive the tidal flow without appreciable change in level.

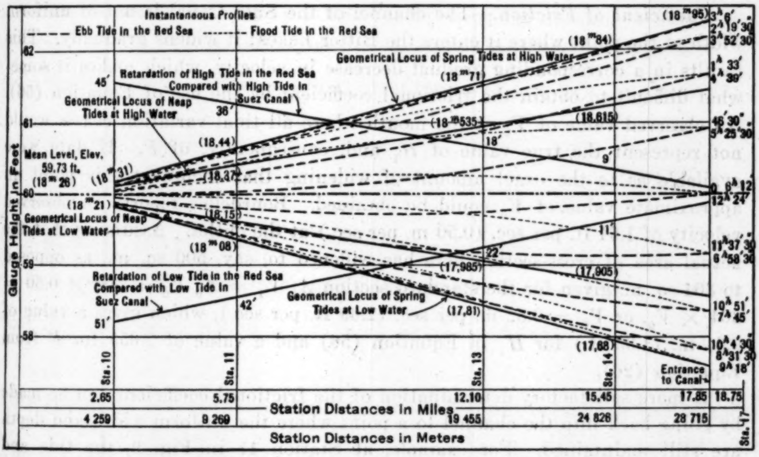


Fig. 9.—Graphical Representation of the Mean Instantaneous Profiles of the Suez Canal Between the Red Sea and the Bitter Lakes: Third Quarter of 1898.

Lévy tested his theory[8] by comparison with observations on the Suez Canal. He found that while it could account for the observed heights of the tide, it could not account for the observed velocities, particularly those at the outlet of the canal into the Bitter Lakes. This canal was also the subject of rather complete tidal observations, some of the results of which are given by Bourdelles[15] in an idealized graphical representation of the mean instantaneous profiles of the canal between the Red Sea and the Bitter Lakes (Fig. 9).

---

[15] "Etude sur le Régime de la Marée dans le Canal de Suez," *Annales des Ponts et Chaussees*, 1898, 3me trimestre.

In his memoir, Bourdelles directed attention to the failure of any previous theory to account for the high velocities at the outlet where all existing theories called for the disappearance of the velocity when the height of the wave vanished. He found a satisfactory agreement with theory for velocity near the Suez end, but the discrepancies increased toward the lake end. To explain this, he was forced to resort to the theory that the excess velocity over that caused by the height of wave must be due to a flow under the influence of the surface slope of the wave, but no proof of such flow could be adduced.

Under the reflected wave theory, the computed velocities agree quite well with the observed velocities, and the computed heights are almost identical with those given by Bourdelles. Unfortunately, Bourdelles gives very little information as to velocities, simply naming the maximum velocities observed at certain points without giving the amplitude of the tide at the Suez Station during the tidal period which gave rise to those velocities. It is assumed that the velocities which he does give were generated during an average tide such as he indicates upon his profiles, and the comparisons are based upon that assumption.

Coefficient of Friction.—The channel of the Suez Canal is not of uniform width at the point where it enters the Bitter Lakes; it widens gradually. This results in a corresponding gradual decrease in velocity, which makes it somewhat difficult to obtain the frictional coefficient by the use of Equation (56). An observed value of V_0 at the mouth where all tidal variation ceases would not represent the true value of H_1 and, consequently, of F. If data were available as to the exact amount of widening that occurs at this point an approximate value of V_0 could be obtained. Bourdelles reports an observed velocity of 1.64 ft. per sec. (0.50 m. per sec.) at the outlet. Assuming that the actual area of cross-section here has widened to, say, 500 sq. m., as opposed to 304 sq. m. given for the standard section $A_1 V_1 = A_0 V_0 = 500 \times 0.50 = 304 \times V_0$, or $V_0 = 0.82$ m. per sec. (2.69 ft. per sec.), which gives a value of 0.31 m. (1.33 ft.) for H_1 in Equation (56) and a value of 2.655 for F from Equation (26).

A more satisfactory determination of the frictional coefficient can be made by going back into the channel to a point where the uniform width and depth are still maintained. For example, at Station 11 in Fig. 9, the tide and velocity of the current are both given. In this case it is necessary to proceed in a somewhat different manner, since the heights of the direct and reflected waves will not be equal at this point like they are at the outlet. It is now necessary to make use of Equation (57). In that equation, substitute t_m for t; v is not the maximum velocity in this case, but the velocity at high water, observed simultaneously with h_m.

At the time of the observations, the canal was 22 m. (72 ft.) wide on the bottom, with side slopes of 1 on 2. The depth was 8 m. (26 ft.) below the datum plane of mean tide, which was at an elevation of +18.26 m. (59.73 ft.). The following, therefore, are the basic data on which the computations are based:

	Metric units.	English units.
Area of cross-section at mean sea level, in square meters and square feet............	304	3 272
Width of canal at mean sea level, in meters and feet..............................	54	177
Wetted perimeter, in meters and feet........	57.77	189.54
Semi-amplitude of the tide at Suez, in meters and feet..............................	0.69	2.26
Length of canal, in kilometers and miles.....	26	1.6
Reduced depth, $\frac{304}{54}$, in meters and feet.......	5.63	18.47

For Chalouf, Bourdelles gives the following data: Height, 0.18 m. (0.59 ft.), and velocity, 0.80 m. per sec. (2.62 ft. per sec.).

As t_1 is the time required for the wave of translation to travel to the point of observation under the assumption of an infinitely long canal,

$$t_1 = \frac{x}{c} = \frac{x}{\sqrt{g\,(D + h)}} = \frac{19\,446}{\sqrt{(5.63 + 0.18)\,(98.06)}} = 2\,576 \text{ sec.} = 43 \text{ min.}$$

The other term, t_m, is the time at which high water occurred at the point of observation, which is given by Bourdelles as occurring 36 min. later than at Suez, or about 30 min. later than at the canal entrance, making $t_m - t_1 = (-43 + 30) = -13$ min., which gives the value, $2\pi\left(\frac{t_m - t_1}{T}\right) = 6° \, 17'$.

Substituting these values in Equation (57), $0.18 + 0.80\sqrt{\frac{5.63 + 0.18}{9.806 +}}$

$$= 2\,H_1 \cos 6° \, 17'; \text{ whence, } H_1 = \frac{0.18 + 0.80\sqrt{\frac{5.63 + 0.18}{9.806 +}}}{2 \cos 6° \, 17'} = 0.4023.$$

Substituting this value in Equation (26), in which, $H_0 = 0.69$, $x = 19\,446$, and $D = 5.63$; then, $8.2439 = \frac{F\,p\,x}{2\,\rho\,l\,\sqrt{g}}$; and $F = 2.4816$.

The value of F as given by Bonnet for the Scheldt River is 2.4802. The close agreement between these values is noteworthy, but is probably accidental.

Computation of Tidal Curves.—Using this value of F, the computations in Tables 1 and 2 may be made. These tables give the values of H and of t for various selected stations which are the points of observation for the data upon which Bourdelles' curves are based. In Table 1 the value of $\frac{F\,p}{2\,\rho\,l\,\sqrt{g}}$ equals 0.000423935. The data contained make it possible to write the following general equations of the tide for those stations; for example, at Station 3889 (Table 1), the equation for the positive wave is:

$$h = +\,0.622 \cos 2\pi\left(\frac{t - 496}{T}\right) - 0.159 \cos 2\pi\left(\frac{t - 6\,298}{T}\right)$$
$$+\,0.123 \cos 2\pi\left(\frac{t - 7\,290}{T}\right) - 0.01 \cos 2\pi\left(\frac{t - 13\,092}{T}\right) \ldots\ldots \quad (61)$$

TABLE 1.—COMPUTATIONS FOR DETERMINING THE GENERAL EQUATIONS FOR POSITIVE WAVES, THE SUEZ CANAL.

H, in meters.	$D^{3/2} \log_e H$.	$\frac{3}{2} D^{1/2} H$	$\frac{3 H^2}{16 D^{1/2}}$	$-\frac{F p \Delta x}{2 \rho l \sqrt{g}} + K$.	$\frac{F p \Delta x}{2 \rho l \sqrt{g}}$.	Δx*, in meters.	Cumulative distance, in meters.	$D + h$, in meters.	RATE OF PROPAGATION, IN METERS PER SECOND. At station.	RATE OF PROPAGATION, IN METERS PER SECOND. Mean.†	Time of propagation, in seconds.	Cumulative time, in seconds.	Equivalent phase, in degrees.
FIRST COMPONENT WAVE, POSITIVE.													
+0.690	− 4.9569	+2.4557	+0.0376	− 2.4636	0.	0	0	6.52	7.8727	......	0	0	0
+0.650	− 5.7543	+2.3134	+0.0333	− 3.4076	0.9440	2 226	2 226	....	......	......	....	......	......
+0.622	....	....	....	....	....	......	3 889	6.21	7.8038	7.8382	496	496	3.99
+0.600	− 6.8230	+2.1354	+0.0284	− 4.6602	1.2526	2 955	5 181	....	......	......	....	......	......
+0.550	− 7.9864	+1.9575	+0.0239	− 6.0050	1.3448	3 172	8 353	....	......	......	....	......	......
+0.537	....	....	....	....	....	......	9 260	6.07	7.7153	7.7595	692	1 188	9.56
+0.500	− 9.2594	+1.7796	+0.0198	− 7.4600	1.4550	3 432	11 785	....	......	......	....	......	......
+0.467	....	....	....	....	....	......	14 260	5.94	7.6323	7.6738	652	1 840	14.81
+0.450	−10.6659	+1.6015	+0.0160	− 9.0484	1.5884	3 747	15 532	....	......	......	....	......	......
+0.403	....	....	....	....	....	......	19 446	5.81	7.5483	7.5903	683	2 523	20.31
+0.400	−12.2394	+1.4236	+0.0126	−10.8032	1.7548	4 139	19 671	....	......	......	....	......	......
+0.350	−14.0225	+1.2456	+0.0097	−12.7672	1.9640	4 633	24 304	....	......	......	....	......	......
+0.348	....	....	....	....	....	......	24 456	5.68	7.4633	7.5058	667	3 190	25.68
+0.333	−14.6880	+1.1852	+0.0088	−13.4940	0.7268	1 714	26 018	5.63	7.4303	7.4468	207	3 397	27.35
SECOND COMPONENT WAVE, NEGATIVE.													
−0.333	−14.6880	−1.1852	+0.0088	−15.8644	....	......	26 018	....	......	......	....	3 397	27.35
−0.316	....	....	....	....	....	......	27 544	....	......	......	207	3 604	29.02
−0.300	−16.0822	−1.0677	+0.0071	−17.1428	1.2784	3 015	29 033	....	......	......	....	......	......
−0.267	....	....	....	....	....	......	32 554	....	......	......	667	4 271	34.39
−0.250	−18.5175	−0.8897	+0.0049	−19.4023	2.2595	5 330	34 363	....	......	......	....	......	......
−0.225	....	....	....	....	....	......	37 740	....	......	......	683	4 954	39.89
−0.200	−21.4970	−0.7118	+0.0032	−22.2056	2.8033	6 612	40 975	....	......	......	....	......	......
−0.190	....	....	....	....	....	......	42 740	....	......	......	652	5 606	45.14
−0.159	....	....	....	....	....	......	48 111	....	......	......	692	6 298	50.71
−0.150	−25.3403	−0.5339	+0.0018	−25.8724	3.6668	8 649	49 624	....	......	......	....	......	......
−0.139	−26.3579	−0.4835	+0.0015	−26.8399	0.9675	2 283	51 907	....	......	......	....	......	......
−0.139	....	....	....	....	....	......	52 000	....	......	......	496	6 794	54.70
THIRD COMPONENT WAVE, POSITIVE.													
+0.139	−26.3579	+0.4835	+0.0015	−25.8729	....	......	52 000	....	......	......	....	6 794	54.70
+0 123	....	....	....	....	....	......	55 889	....	......	......	496	7 290	58.69
+0.120	−28.3217	+0.4271	+0.0011	−27.8935	2.0206	4 766	56 673	....	......	......	....	......	......
+0.105	....	....	....	....	....	......	61 260	....	......	......	692	7 982	64.26
+0.100	−30.7571	+0.3559	+0.0008	−30.4004	2.5069	5 913	62 586	....	......	......	....	......	......
+0.089	....	....	....	....	....	......	66 260	....	......	......	652	8 634	69.51
+0.085	−32.9270	+0.3025	+0.0006	−32.6239	2.2235	5 245	67 831	....	......	......	....	......	......
+0.076	....	....	....	....	....	......	71 446	....	......	......	683	9 317	75.01
+0.070	−35.5213	+0.2491	+0.0004	−35.2718	2.6479	6 246	74 077	....	......	......	....	......	......
+0.065	....	....	....	....	....	......	76 456	....	......	......	667	9 984	80.38
+0.062	−37.1425	+0.2207	+0.0003	−36.9215	1.6497	3 891	77 968	....	......	......	....	......	......
+0.062	....	....	....	....	....	......	78 000	....	......	......	207	10 191	82.05
FOURTH COMPONENT WAVE, NEGATIVE.‡													
−0.062	....	....	....	....	....	......	78 000	....	......	......	....	10 191	82.05
−0.050	....	....	....	....	....	......	79 544	....	......	......	207	10 398	88.72
−0.040	....	....	....	....	....	......	84 554	....	......	......	667	11 065	89.09
−0.030	....	....	....	....	....	......	89 740	....	......	......	683	11 748	94.59
−0.020	....	....	....	....	....	......	94 740	....	......	......	652	12 400	99.83
−0.010	....	....	....	....	....	......	100 111	....	......	......	692	13 092	105.41
−0.000	....	....	....	....	....	......	104 000	....	......	......	496	13 588	109.40

---

* Δx is the distance between stations corresponding to selected values of H.

† These figures are the means of those in the preceding column.

‡ As this wave is very small, it is assumed to become extinguished progressively in this reflection.

TABLE 2.—COMPUTATIONS FOR DETERMINING THE GENERAL EQUATIONS FOR NEGATIVE WAVES, THE SUEZ CANAL.

H, in meters.	$D^{2/3}\log_e H$.	$\frac{3}{2}D^{1/3}H$.	$\frac{3H^2}{16\,D^{1/3}}$.	$-\frac{Fp\,\Delta x}{2\rho l\sqrt{g}}+K$.	$\frac{Fp\,\Delta x}{2\rho l\sqrt{g}}$.	Δx*, in meters.	Cumulative distance, in meters.	$D+h$, in meters.	RATE OF PROPAGATION, IN METERS PER SECOND.		Time of propagation, in seconds.	Cumulative time, in seconds.	Equivalent phase, in degrees.
									At station.	Mean.†			
FIRST COMPONENT WAVE, NEGATIVE.													
−0.690	− 4.9569	−2.4557	+0.0376	− 7.3750	0.	0	0	4.94	6.9601	......	0	0	......
−0.650	− 5.7543	−2.3184	+0.0333	− 8.0344	0.6594	1 555	1 555	....	......	......	....	......	......
−0.600	− 6.8230	−2.1354	+0.0284	− 8.9300	0.8956	2 113	3 668	....	......	......	....	......	......
−0.595	....	....	....	....	....	......	3 889	5.07	7.0511	7.0056	554	554	4.46
−0.550	− 7.9864	−1.9575	+0.0239	− 9.9200	0.9900	2 335	6 603	....	......	......	....	......	......
−0.500	− 9.2594	−1.7796	+0.0198	−11.0182	1.0982	2 591	8 594	....	......	......	....	......	......
−0.487	....	....	....	....	....	......	9 260	5.23	7.1616	7.1063	756	1 310	10.55
−0.450	−10.6659	−1.6015	+0.0160	−12.2514	1.2332	2 909	11 503	....	......	......	....	......	......
−0.408	....	....	....	....	....	......	14 260	5.38	7.2636	7.2126	693	2 003	16.13
−0.400	−12.2394	−1.4236	+0.0126	−13.6504	1.3990	3 300	14 803	....	......	......	....	......	......
−0.350	−14.0225	−1.2456	+0.0097	−15.2584	1.6080	3 793	18 596	....	......	......	....	......	......
−0.340	....	....	....	....	....	......	19 446	5.49	7.3373	7.3004	710	2 713	22.26
−0.300	−16.0822	−1.0677	+0.0071	−17.1428	1.8844	4 445	23 041	....	......	......	....	......	......
−0.286	....	....	....	....	....	......	24 456	5.59	7.4040	7.3706	680	3 393	27.73
−0.270	−17.4755	−0.9609	+0.0058	−18.4406	1.2978	3 061	26 102	5.63	7.4303	7.4171	208	3 601	29.40
SECOND COMPONENT WAVE, POSITIVE.													
+0.270	−17.4755	+0.9609	+0.0058	−16.5088	....	......	26 000	....	......	......	....	3 601	29.40
+0.258	....	....	....	....	....	......	27 544	....	......	......	208	3 809	31.08
+0.250	−18.5175	+0.8897	+0.0049	−17.6129	1.1041	2 604	28 706	....	......	......	....	......	......
+0.225	....	....	....	....	....	......	32 554	....	......	......	680	4 489	36.55
+0.200	−21.4970	+0.7118	+0.0032	−20.7820	3.1691	7 475	36 181	....	......	......	....	......	......
+0.192	....	....	....	....	....	......	37 740	....	......	......	710	5 199	42.68
+0.166	....	....	....	....	....	......	42 740	....	......	......	693	5 892	48.26
+0.150	−25.3403	+0.5339	+0.0018	−24.8046	4.0226	9 488	45 669	....	......	......	....	......	......
+0.140	....	....	....	....	....	......	48 111	....	......	......	756	6 648	54.34
+0.123	−27.9914	+0.4378	+0.0012	−27.5524	2.7478	6 481	52 150	....	......	......	554	7 202	58.80
THIRD COMPONENT WAVE, NEGATIVE.													
−0.123	−27.9914	−0.4378	+0.0012	−28.4280	....	......	52 150	....	......	......	....	7 202	58.80
−0.120	−28.3217	−0.4271	+0.0011	−28.7477	0.3197	754	52 904	....	......	......	....	......	......
−0.109	....	....	....	....	....	......	55 889	....	......	......	554	7 756	63.26
−0.100	−30.7571	−0.3559	+0.0008	−31.1122	2.3645	5 577	58 481	....	......	......	....	......	......
−0.092	....	....	....	....	....	......	61 260	....	......	......	756	8 512	69.34
−0.085	−32.9270	−0.3025	+0.0006	−33.2289	2.1167	4 993	63 474	....	......	......	....	......	......
−0.078	....	....	....	....	....	......	66 260	....	......	......	693	9 205	74.92
−0.070	−35.5213	−0.2491	+0.0004	−35.7700	2.5411	5 994	69 468	....	......	......	....	......	......
−0.066	....	....	....	....	....	......	71 446	....	......	......	710	9 915	81.05
−0.060	−37.5805	−0.2135	+0.0003	−37.7937	2.0237	4 763	74 231	....	......	......	....	......	......
−0.056	....	....	....	....	....	......	76 456	....	......	......	680	10 595	86.52
−0.053	−39.2370	−0.1815	+0.0002	−39.4183	1.6246	3 815	78 046	....	......	......	208	10 803	88.20
FOURTH COMPONENT WAVE, POSITIVE.‡													
+0.053	....	....	....	....	....	......	78 000	....	......	......	....	10 803	88.20
+0.050	....	....	....	....	....	......	79 544	....	......	......	208	11 011	89.88
+0.040	....	....	....	....	....	......	84 554	....	......	......	680	11 691	95.35
+0.030	....	....	....	....	....	......	89 740	....	......	......	710	12 401	101.48
+0.020	....	....	....	....	....	......	94 740	....	......	......	693	13 094	107.06
+0.010	....	....	....	....	....	......	100 111	....	......	......	756	13 850	113.14
+0.000	....	....	....	....	....	......	104 000	....	......	......	554	14 404	117.60

* Δx is the distance between stations corresponding to selected values of H.

† These figures are the means of those in the preceding column.

‡ As this wave is very small, it is assumed to become extinguished progressively in this reflection.

and for the negative wave (Table 2):

$$h = -\,0.595 \cos 2\,\pi \left(\frac{t - 554}{T}\right) + 0.14 \cos 2\,\pi \left(\frac{t - 6\,648}{T}\right)$$

$$-\,0.109 \cos 2\,\pi \left(\frac{t - 7\,756}{T}\right) + 0.02 \cos 2\,\pi \left(\frac{t - 13\,850}{T}\right) \ldots\ldots (62)$$

At Station 9260, the equation for the positive wave is:

$$h = +\,0.537 \cos 2\,\pi \left(\frac{t - 1\,188}{T}\right) - 0.190 \cos 2\,\pi \left(\frac{t - 5\,606}{T}\right)$$

$$+\,0.105 \cos 2\,\pi \left(\frac{t - 7\,982}{T}\right) - 0.02 \cos 2\,\pi \left(\frac{t - 12\,400}{T}\right) \ldots\ldots (63)$$

and for the negative wave:

$$h = -\,0.487 \cos 2\,\pi \left(\frac{t - 1\,310}{T}\right) + 0.166 \cos 2\,\pi \left(\frac{t - 5\,892}{T}\right)$$

$$-\,0.092 \cos 2\,\pi \left(\frac{t - 8\,512}{T}\right) + 0.02 \cos 2\,\pi \left(\frac{t - 13\,094}{T}\right) \ldots\ldots (64)$$

These general equations may then be transformed—by the methods described for Equation (47)—into the simpler forms given below. Each one gives, directly, the maximum value of h, and the time required for the resultant wave to be propagated from the entrance to the point being considered, that is, the time of occurrence of high water.

The transformed equations are as follows: For Station 3889, the positive wave is,

$$h = +\,0.5864 \cos 2\,\pi \left(\frac{t - 190}{T}\right) \ldots\ldots\ldots\ldots\ldots\ldots (65)$$

and the negative wave is,

$$h = -\,0.5648 \cos 2\,\pi \left(\frac{t - 263}{T}\right) \ldots\ldots\ldots\ldots\ldots\ldots (66)$$

At Station 9260, the positive wave is:

$$h = +\,0.4456 \cos 2\,\pi \left(\frac{t - 483}{T}\right) \ldots\ldots\ldots\ldots\ldots\ldots (67)$$

the negative wave:

$$h = -\,0.4073 \cos 2\,\pi \left(\frac{t - 556}{T}\right) \ldots\ldots\ldots\ldots\ldots\ldots (68)$$

By giving proper values to t, instantaneous profiles may be constructed as shown in Fig. 10. Similar equations for velocity at the same stations may be found from the Equations (61) to (64) as explained for Equation (49), and from those equations instantaneous velocity curves may be plotted as shown in Fig. 11.

Comparing Figs. 9 and 10, it will be noted that the instantaneous profiles are very similar and that all the discrepancies are of a minor character.

Heights.—Bourdelles' curves are based on a complete symmetry between the positive and negative wave curves.[15] Under any known theory of wave propagation complete symmetry can exist only when the height of the wave is negligible with respect to the depth, which is not the case in the present instance. It is

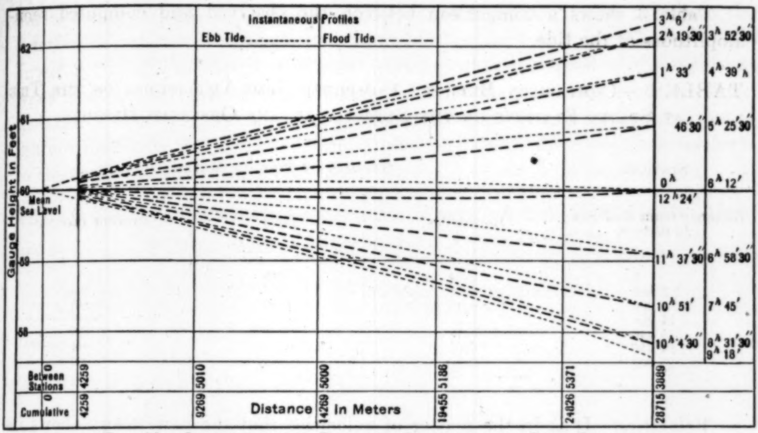


FIG. 10.—INSTANTANEOUS PROFILES COMPUTED FOR SUEZ CANAL.

the practice of the United States Coast and Geodetic Survey to allow for lack of symmetry between positive and negative waves in making tidal predictions by introducing a "shallow-water" component into the computations.[16] The new theory automatically allows for the shallow-water component when a separate computation is made for the negative wave.

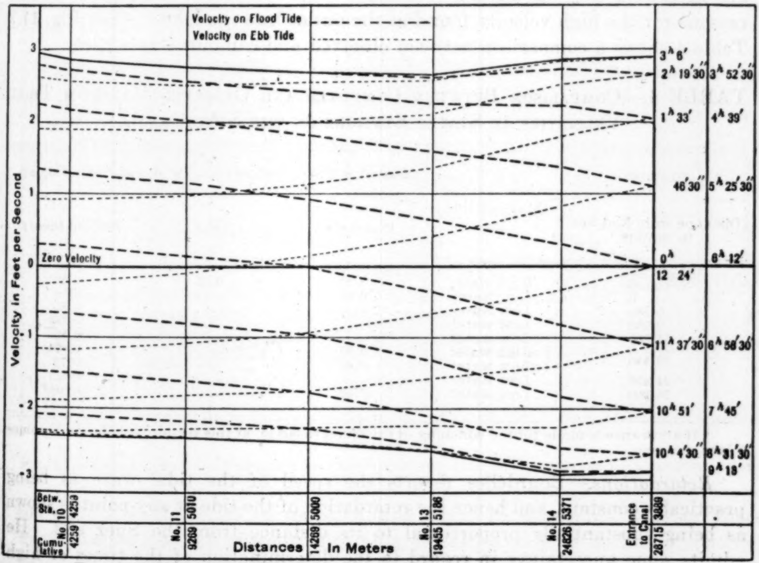


FIG. 11.—INSTANTANEOUS VELOCITY CURVES CORRESPONDING TO FIG. 10.

---

[16] *Special Publications, No. 111,* U. S. Coast and Geodetic Survey, p. 14.

Table 3 shows a comparison between the observed and computed semi-amplitudes of the tide.

TABLE 3.—Comparison Between Computed Semi-Amplitudes of the Tide at Stated Stations on the Suez Canal, and Observed Heights.

Station.	Heights of Tide, in Meters.		
Distance from Red Sea, in meters.	Bourdelles.	Levy.	Present theory.
0	0.69	0.69	0.69
3 889	0.58	....	0.586
9 260	0.45	0.47	0.445
14 260	....	....	0.313
19 446	0.18	0.23	0.178
24 456	0.05	....	0.044
26 000	0.00	....	0.000

Velocities.—It is in the matter of velocities that the new theory shows its superiority over the previous theories. Both Lévy and Bourdelles were compelled to confess the failure of any theory of tidal propagation to explain the discrepancies noted between observed and computed velocities. These discrepancies were small at the Suez end of the canal, but they increased in magnitude until at the end at the Bitter Lakes there was complete disagreement. The velocities at all points in the canal, as computed by the new theory, are in satisfactory agreement with those observed, and it fully explains the reasons for the high velocity found at the outlet into the lakes. (See Fig. 11.) Table 4 shows a comparison between observed and computed velocities.

TABLE 4.—Comparison Between Computed and Observed Maximum Tidal Velocities at Stated Stations on the Suez Canal.

Station.	Stage of tide.	Maximum Tidal Velocities, in Meters per Second.		
Distance from Red Sea, in meters.		Bourdelles.	Levy.	Present theory.
0	High water	0.90	0.28	0.860
0	Low water	0.80	....	0.71
3 889	Low water	....	....	0.855
9 260	Low water	....	....	0.785
14 260	Low water	....	....	0.805
19 446	High water	0.85	....	0.824
	Low water	0.80	....	0.72
24 456	Low water	....	....	0.858
26 000	Low water	0.50	....	0.880*

* If allowance is made for the widening of the canal to 500 m. at this station, this figure becomes 0.54 m. per sec.

Retardations.—Bourdelles reports the speed of the tidal wave as being practically constant, and hence the retardation of the tide at any point is shown as being substantially proportional to its distance from the Suez end. He admits some uncertainty in regard to the determination of the times of high and low water "by reason of the imperfections in the mode of observation used, which consisted only in readings upon graduated scales".[15] The theory of the interference of waves as explained herein under the heading, "Theory of the Interference of Two Opposed Waves", requires that the speed of the resultant wave shall be variable, and not constant, although it will be constant for a distance of one wave length. The retardations as computed by the new theory are not markedly different from those given by Bourdelles; yet they conform to theoretical requirements (see Table 5).

TABLE 5.—THE RETARDATIONS OF HIGH WATER ON THE SUEZ CANAL; COMPUTED VALUES COMPARED WITH THOSE OBSERVED.

STATION.	RETARDATIONS, IN MINUTES.		
Distance from Red Sea, in meters.	Bourdelles.	Levy.	Present theory.
0	0	0	0
3 889	8	10	3
9 260	17	22	8
14 260	....	....	14
19 446	35	38	15
24 456	44	42	32
26 000	....	....	55

Inland Waterway from Pamlico Sound to Beaufort Inlet, N. C.—This is an artificial canal connecting the waters named, which was constructed by the United States about 1910. In 1911, some observations[17] were made of the action of the tidal currents in this canal to determine whether there were prospects of the currents attaining velocities such as to hinder navigation, and to require the installation of guard locks.

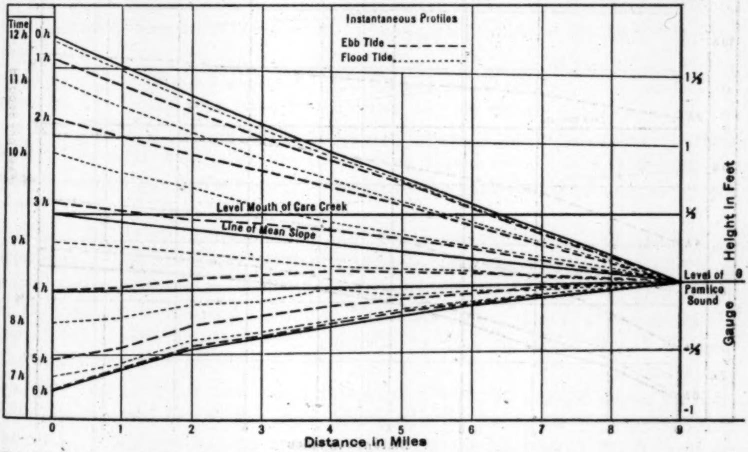


FIG. 12.—INSTANTANEOUS PROFILES AS COMPUTED FOR THE WATERWAY FROM BEAUFORT INLET TO PAMLICO SOUND, NORTH CAROLINA.

---

[17] *Professional Memoirs*, Corps of Engrs., U. S. A., 1912, vol. 4, p. 216.

Briefly, this waterway comprises three sections: The tidal stream known as Core Creek at the southern end; the connecting artificial canal; and the non-tidal Adams Creek at the northern end. The dimensions of the canal were 90 ft. wide on the bottom, with side slopes of 1 on 2, and 10 ft. deep referred to mean low water in Beaufort Harbor, near-by. The mean level of Adams Creek was 1.4 ft. referred to the same datum. The portion of the canal herein considered is about 8½ miles long.

The mean amplitude of the tide was found to be about 2.6 ft. at the mouth of Core Creek; thence it gradually decreased until all tidal variation of level practically disappeared at Hooker's Landing at the head of Adams Creek. Conversely, tidal velocities were mildest at the mouth of Core Creek, and greatest at Hooker's Landing. Some change of level was found at Hooker's Landing, but it was due entirely to wind effects on the level of Pamlico Sound and River into which Adams Creek empties. Curves of instantaneous heights and velocities for this canal have been computed under the proposed theory, as shown in Figs. 12 and 13, respectively. These may be compared with similar curves of observed data prepared by the Corps of Engineers, U. S. Army.

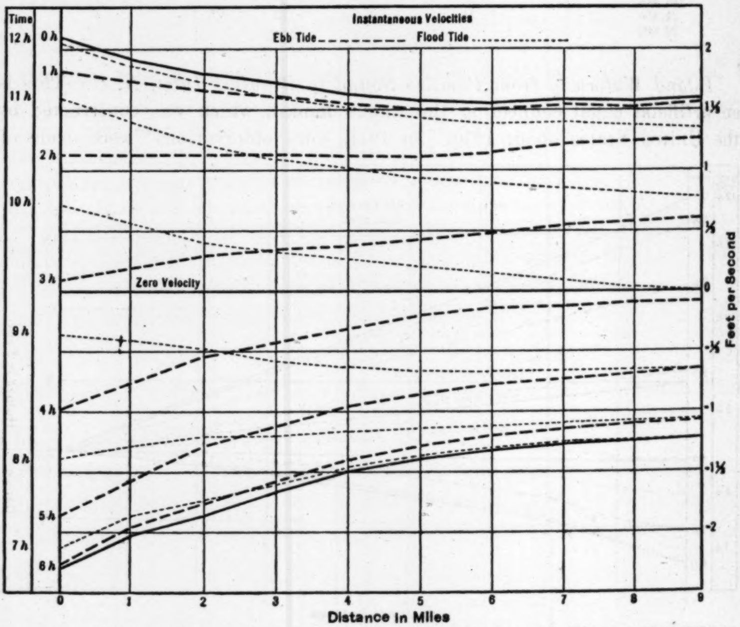


FIG. 13.—INSTANTANEOUS VELOCITIES AS COMPUTED FOR THE WATERWAY FROM BEAUFORT INLET TO PAMLICO SOUND, NORTH CAROLINA.

It will be noted that the mean level at the mouth of Core Creek is higher than that at Hooker's Landing, thus increasing the tidal velocities by a permanent set of the current northward due to this slope. This difference in level has been taken into account in computing the curves in Figs. 12 and 13.

The Rehoboth Canal.—It is now proposed to test the foregoing theory in a more complex example. The Inland Waterway between Rehoboth Bay and Delaware Bay, Delaware, is a tidal canal situated in the southeasterly portion of Sussex County, Delaware. It extends, for a distance of 12 miles, northward from Rehoboth Bay, through the highlands west of the Town of Rehoboth; thence through the marshes back of Cape Henlopen to Lewes River, a small tidal creek; and thence it follows this creek to Broadkill River at a point near the mouth of that stream, through which it empties into Delaware Bay about 5 miles from Cape Henlopen.

This canal was constructed by the United States about 1913, under a project which provides for a waterway 6 ft. deep at mean low water, 50 ft. wide through Lewes River and the marshes, and 40 ft. wide where it passes through deeper cutting.

Rehoboth Bay and Indian River Bay are two "drowned" valleys which formerly communicated with the Atlantic Ocean through an inlet. This inlet has closed and the two bays are completely separated from the ocean except by the Rehoboth Canal on the north and the Assowoman Canal on the south. The two bays, which are of about equal size, and which have a combined area of about 20 sq. miles, are partly separated from each other by a series of islands known as Burton's Islands, the connecting channels between the islands being locally known as the "Ditches". Those portions of the bays adjacent to the littoral cordon are quite shoal, but the average depth of the two bays elsewhere is about 6 ft.

The Assowoman Canal is a small artificial cut, originally intended to be 6 ft. deep and from 40 to 50 ft. wide. It connects Chincoteague Bay, Virginia, with Indian River Bay by way of Sinepuxent Bay, Maryland.

The surface level of Rehoboth and Indian River Bays is now quite constant; it is affected very little by anything but the wind. Tidal flow reaches the bays through the Rehoboth Canal, but has no apparent effect on their level. The current through the Assowoman Canal is not tidal, although the normal outflowing current may occasionally be reversed in direction by the wind.

The canal is naturally divided into two parts differing in physical characteristics. The northerly portion, 23 663 ft. long, has an average width of water surface at mean sea level of 100 ft., and an average sectional area of 900 sq. ft., making the reduced depth 9 ft. This portion of the canal comprises the bed of the former Lewes Creek, a stream heading up near Cape Henlopen. The canal bed narrows to a width of 80 ft. at Lewes, but maintains about the same depth. From the end of this section 3 000 ft. east of Lewes an artificial cut extends, for 33 404 ft., to the jetties in Rehoboth Bay terminating the canal. This cut has an average width on the surface of 78 ft. and an average cross-section at mean tide of 548 sq. ft., or a reduced depth of 7 ft.

In the artificial cut, the propagation is regular, and in accordance with theory. The height of the wave gradually decreases until it disappears in Rehoboth Bay, while the tidal velocities retain higher values than a direct proportion to the height of tide. The surface heights and velocities conform to the requirements of the theory of a resultant wave caused by the superposition on a primary wave of a reflected negative wave from the bay.

In view of the difference in depth and area of the two sections of the canal as just mentioned, the tidal wave cannot be expected to be propagated through the entire length of the canal as simply as it would if both depth and cross-sections were constant for the entire distance. At the point of junction of the artificial cut with the old creek bed, a portion of the wave will continue on to the head of the old creek where a reflection of the wave back upon itself must be expected. This reflected wave, combining with the original wave, may be expected to cause higher levels and smaller velocities in that portion of the canal near Lewes than would have occurred normally. Furthermore, the constriction of the channel at Lewes must generate a reflected positive wave, and the division of the wave at the junction point must generate a reflected negative wave. These reflected waves may be expected to have a more pronounced effect near Lewes than near Broadkill, and to exert that effect at an earlier instant. The writer's observations amply confirm this expectation. Fig. 14 shows curves of instantaneous heights as given by hourly gauge readings made in August, 1925. These curves reveal clearly the large gauge heights and the advanced phases of the tide at Lewes resulting from this change of section.

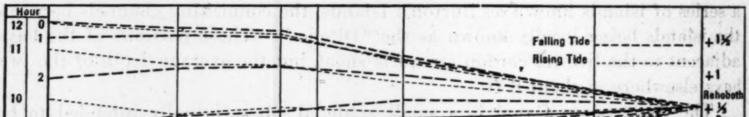


FIG. 14.—INSTANTANEOUS PROFILES IN THE LEWES-REHOBOTH CANAL, DELAWARE.

To find the resultant wave in this portion of the canal, first compute the direct wave, the positive reflected wave coming from the constriction at Lewes, the positive reflected wave from the head of the old creek, and the negative reflected wave from the junction with the artificial cut. A summation of these waves will give the tidal characteristics of the Lewes River section. The remaining portion of the canal can be treated as being affected solely by the direct positive and the reflected negative wave.

Computation of Tidal Curves.—The tidal characteristics of the remainder of the canal may be determined by using the tidal curve at the junction as a generatrix of the tide between this point and the jetties.

Observations made in August, 1925, disclosed a maximum mean velocity of 1.41 ft. per sec. at the jetties, and a tidal amplitude of 2.8 ft. at the junction. Therefore, $H_1 = \frac{1}{2} \times 1.41 \sqrt{\frac{7}{32.17}} = 0.33$ ft., and the following values may then be determined for substitution in Equation (26): $H_0 = 1.4$ ft.; $x = 33\,404$ ft.; $D = 7$ ft.; $H_1 = 0.33$ ft.; $l = 78$ ft.; and $p = 83.4$ ft.

Making these substitutions, the constant, $\frac{F\,p}{2\,\rho\,l\,\sqrt{g}}$, is found to have the value, 0.00087877, from which, F is found to have the value, 0.5883. With this value of F in Equation (26), the height, H, at any point, x, along the canal may be found for both direct and reflected waves. Since the values of l and p are, respectively, 100 and 105 ft. for the Lewes River Section, the corresponding value of $\frac{F\,p}{2\,\rho\,l\,\sqrt{g}}$ will be 0.0008629.

The heights for each wave at selected points, as well as the times of propagation, may be computed as illustrated in Tables 1 and 2, and from these the phase angles are found. The general equations are then written for each point in a manner similar to the use made of Tables 1 and 2.

Fig. 13 is a graphical comparison between observed and computed instantaneous tidal curves.

Mean Level.—Attention has been called to the lack of symmetry between the geometric loci of high and low waters. If "mean canal level" is defined as a point midway between mean high water and mean low water at any point, it is quite evident that the locus of mean canal level so defined will not coincide with mean sea level which is the plane of reference. Tangent to mean sea level at the mouth, the locus of mean canal level will gradually ascend, and will be higher than mean sea level at the outlet in the reservoir, provided the reservoir can adapt itself to this condition. If not of unlimited capacity, it will so adapt itself in course of time.

If the reservoir surface should be below mean canal level at the outlet, the direct negative wave cannot return to the sea all the water brought in by the positive wave and an excess volume will remain in the reservoir; this excess must eventually bring it to the critical level. If the reservoir surface is above the mean canal level, the negative wave will return more water to the sea than was brought in by the positive wave, and eventually this must reduce the level to the proper elevation.

If the reservoir capacity is too great for such an adjustment to become effected, then the locus of mean canal level will incline downward from a point of maximum height and will become tangent to the reservoir level at the outlet.

It takes this form in any case during the periods of adjustment previously mentioned. Such a condition has been noted in connection with Adams Creek.

A fresh-water discharge into a comparatively small bay, such as those under consideration, will tend to raise the mean level above mean sea level.

The mean level of Rehoboth Bay is 0.55 ft. higher than mean sea level and the mean level of the canal rises gradually from one end to the other. This difference of level is to be attributed to the two causes just mentioned. Sufficient data do not exist upon which to determine the portions of this difference attributable to each cause. Measurements made at the jetties show a flood discharge of 11 070 090 cu. ft. per tide, and an ebb discharge of 12 230 330 cu. ft. per tide, a difference of 1 160 240 cu. ft. per tide, which represents a part of the fresh-water drainage into the basin. Proper corrections must be made to all computed values of tidal variation and tidal velocities in order to allow for the canal slope and fresh-water discharge. These corrections are made on the curves of instantaneous heights and velocities.

Comparison of Computed and Observed Data.—A series of tidal observations was taken upon the Rehoboth Canal at six selected stations during August, 1925, and gaugings were made at two stations near the respective ends of the canal. The height of the tide was read every 15 min. on a staff gauge at each station, the zero of which was established at the level of mean low water for Delaware Breakwater. These readings were used to determine mean sea level, mean range of tide, and mean high and mean low waters at each station.

Reduction of Observations.—The following method was used to check these determinations, and to give in addition the time of propagation of the tide from station to station.

If the time of making any reading upon a staff gauge is observed, and the height of the reading above mean sea level is known, the results of this observation may be used to write the equation,

$$h = H \sin 2\pi \left(\frac{t + c}{T}\right) \qquad \text{(69)}$$

in which, h and t are the observed height and time, respectively; H is the local half amplitude of the tide; and c is the time before the instant, $t = 0$, when the tide was at mean sea level; that is, when $t + c$ was equal to 0. It is clear that an independent equation of this form can be written for each observation, and, consequently, as many observation equations may be formed as are desirable for each tide, and they may be reduced by the methods of least squares to find the mean values of H and c.

If a series of observations are taken at several different stations, all reckoned from the same instant when $t = 0$, a value of c may be found for each point of observation, and the difference between the values of c for any two given points will be the time required for the tidal wave to be propagated between those stations.

In case the elevation of mean sea level is not known, the staff reading may be used directly, without a reduction, to determine the value of h.

Let R be the staff reading, the zero of the gauge being set at any desired elevation. Then, if e represents the elevation of mean sea level on the staff, $h = R - e$, and the observation equation may be written,

$$R = e + H \sin 2\pi \left(\frac{t + c}{T}\right) \ldots\ldots\ldots\ldots\ldots\ldots (70)$$

in which, e, H, and c are unknown.

Expanding,

$$R = e + \sin 2\pi \frac{t}{T} H \cos 2\pi \frac{c}{T} + \cos 2\pi \frac{t}{T} H \sin 2\pi \frac{c}{T} \ldots (71)$$

If n observations are made during any tide, n independent observation equations may be written, from which normal equations for each unknown quantity may be formed by the usual methods of least squares. These equations will be in the form,

$$\Sigma R = n e + H \cos 2\pi \frac{c}{T} \Sigma \sin 2\pi \frac{t}{T} + H \sin 2\pi \frac{c}{T} \Sigma \cos 2\pi \frac{t}{T} . (72)$$

$$\Sigma R \sin 2\pi \frac{t}{T} = e \Sigma \sin 2\pi \frac{t}{T} + H \cos 2\pi \frac{c}{T} \Sigma \sin^2 2\pi \frac{t}{T}$$
$$+ H \sin 2\pi \frac{c}{T} \Sigma \sin 2\pi \frac{t}{T} \cos 2\pi \frac{t}{T} \ldots\ldots\ldots\ldots (73)$$

$$\Sigma R \cos 2\pi \frac{t}{T} = e \Sigma \cos 2\pi \frac{t}{T} + H \cos 2\pi \frac{t}{T} \Sigma \sin 2\pi \frac{t}{T} \cos 2\pi \frac{t}{T}$$
$$+ H \sin 2\pi \frac{c}{T} \Sigma \cos^2 2\pi \frac{t}{T} \ldots\ldots\ldots\ldots\ldots\ldots (74)$$

In the observations at the Rehoboth Canal, hourly readings were used for this computation, and n was made 14 so as to embrace one complete tide in one set of equations. In this case, Equations (72), (73), and (74) become reduced to the following forms:

$$37.4 = 14\, e + 0.3267\, H \cos 2\pi \frac{c}{T} + 1.5376\, H \sin 2\pi \frac{c}{T} \ldots\ldots (75)$$

$$11.1686 = 0.3267\, e + 6.2924\, H \cos 2\pi \frac{c}{T} + 0.2135\, H \sin 2\pi \frac{c}{T} .. (76)$$

$$10.3442 = 1.5376\, e + 0.2135\, H \cos 2\pi \frac{c}{T} + 7.7076\, H \sin 2\pi \frac{c}{T} .. (77)$$

The left-hand members of Equations (75), (76), and (77) are the only ones that change for different stations, as t is counted for each station in all cases from the same instant. This makes the solution of a series of equations for observations at different points an easy process.

Table 6 gives the results of the solution of five sets of simultaneous observations at five points on the canal.

The values of c are given in Table 7, from which the time of propagation of the tide from the Broadkill Station to each of the others is obtained.

The differences are as listed in Table 8.

TABLE 6.—Computations for Simultaneous Observations at Five Points on the Canal.

(Observations are in feet.)

Item No.	Broadkill.		Greenhill.		Lewes.		Halfway.		Railway.	
	H.	*e.*	*H.*	*e.*	*H.*	*e.*	*H.*	*e.*	*H.*	*e.*
1	1.92	2.53	1.65	2.57	1.41	2.74	0.69	2.99	0.17	3.03
2	1.90	2.24	1.76	2.25	1.44	2.49	0.77	2.79	0.27	3.01
3	1.63	2.13	1.76	2.27	1.58	2.43	0.93	2.78	0.27	2.96
4	1.71	2.78	1.82	3.05	1.34	2.79	0.79	2.84	0.46	2.83
5	1.86	2.67	1.84	2.70	1.55	2.81	0.89	2.88	0.28	2.97
Mean	1.80	2.47	1.76	2.57	1.46	2.65	0.81	2.85	0.29	2.96
Computed	1.80	....	1.60	....	1.47	....	0.88	....	0.29	....

TABLE 7.—Values of *c* for Use in Determining Times of Propagation of the Tide.

Station.	Observation Set.				
	No. 1.	No. 2.	No. 3.	No. 4.	No. 5.
Broadkill	29° 55′	60° 17′	255° 06′	40° 14′	53° 39′
Greenhill	16° 36′	52° 09′	252° 08′	31° 10′	50° 53′
Lewes	17° 57′	54° 00′	245° 09′	36° 44′	50° 46′
Halfway	7° 48′	45° 04′	244° 10′	29° 26′	43° 41′
Railway	0° 10′	51° 42′	203° 51′	10° 52′	46° 23′

TABLE 8.—Differences in Values of *c* as Referred to the Corresponding Value at Broadkill.

Set No.	From Broadkill to:			
	Greenhill.	Lewes.	Halfway.	Railway.
1	13° 19′	11° 58′	22° 07′	30° 05′
2	8° 08′	6° 17′	15° 13′	8° 35′
3	2° 58′	9° 54′	10° 56′	51° 15′
4	9° 04′	3° 30′	10° 48′	29° 12′
5	2° 46′	2° 53′	9° 58′	7° 16′
Mean	7° 25′	6° 54′	13° 48′	25° 17′
In seconds of time	920	857	1 714	3 142
Computed values, in seconds	574	1 248	1 450	1 900

It was noted during the period of observations that the canal surface was quite sensitive to the wind, and this fact is reflected in Table 8, but it is quite probable that that effect has been partly eliminated in the averages, and that they may be accepted as representative values. Since the computed times of propagation of the wave do not differ from the mean observed time by amounts greater than the mean error of the observed values, they may be accepted as confirmatory to the theory.

CANAL COMMUNICATING WITH A TIDAL RESERVOIR OF LIMITED CAPACITY

It is now proposed to consider the case of the flow in a tidal canal having no proper tide of its own, but which connects a tidal sea with a basin of limited capacity such that, under the influence of the tide transmitted to it through the canal, its own surface undergoes periodic oscillations of level.

To explain the phenomena observed during the propagation of the tide in the type of canal now considered, assume as before that the wave propagated in the canal is assimilated to a series of small waves of translation. Consequently, it is formed during the flood by a series of successive intumescences or influxes of small height and by a similar series of depressions or refluxes during the ebb. These influxes and refluxes are propagated with the speeds and velocities determined by Equations (1) and (13), already given, conformably with the theory of waves of translation.

Assuming the sea, the canal, and the basin to be in equilibrium and at the same mean level, and that the first influx is then introduced, its action will be analogous to that of a canal communicating with a limitless reservoir except that the volume of the intumescence now has a finite relation to the volume of the basin, and will suffice, when introduced therein, to raise its level a certain amount. The rise of level will oppose the continued introduction of the intumescence into the basin, but this opposition will not be sufficient to counteract it completely until the volume of the intumescence poured into the basin shall equal in quantity the area of the basin multiplied by the height of the intumescence. When that occurs, the basin will be emitting into the canal in the direction of the sea, an opposed intumescence of equal height, the velocity due to which will completely annul that due to the first intumescence so that all flow will cease. Between the time of first arrival of the original intumescence at the outlet into the basin and the time when the basin intumescence exactly equals the original, the magnitude of the opposing basin intumescence will be increasing from zero to its final magnitude. Consequently, the effective magnitude of the original intumescence as an instrument for filling the basin to its own height will gradually diminish and will disappear when the basin is filled to that height.

In the case of an infinitely large reservoir as previously described, it was shown that as the intumescence was unable to raise the basin level, it was compelled to disappear in the basin through a lowering of its height and an increase of its velocity, resulting in a reflected depression of equal amplitude. Similarly, in the present instance, the intumescence must disappear in the basin, but the surface level of the basin will be rising, and, consequently, the reflected depression will not be equal to the original intumescence, but will be equal only to the difference between the height of the intumescence and that of the basin surface at the same instant.

If the height of the intumescence as transmitted in the infinitely long canal and reduced by friction is h_l on its arrival at the basin, and if the height of the basin level at the same moment is h_b, the disappearance of the intumescence in the basin requires that any difference between h_l and h_b give rise to a reflected depression of height equal to $-(h_l - h_b)$, which is equivalent to considering a depression and an intumescence with respective heights equal to $-h_l$ and $+h_b$. The depression with a height equal to $-h_l$ is clearly the reflected depression corresponding to h_l already mentioned in the case of an infinitely large reservoir, and h_b is a new intumescence emitted by the basin, due to the rising of its level.

Similarly, a second intumescence propagating itself on the first one will raise the basin surface again to a point of equilibrium, thus giving rise to a similar series of phenomena, etc., for each influx.

As long as the successive influxes propagated into the canal and thence into the basin are greater in volume than the opposed fluxes emitted by the basin, the tide will be in flood, and the surface of the basin will continue to rise.

When the influxes shall have ceased to be further propagated, at high water, and the refluxes shall have begun to be propagated, these refluxes will add their velocity to that of the opposed fluxes from the basin. When the time arrives at which this sum is exactly equal to the combined velocity of the influxes, or when the total height of all intumescences is equal to the sum of the heights of all intumescences emitted from the basin, the basin level will cease to rise, and the velocity of inflow will be zero. Thenceforward, as the refluxes gain over the influxes, the surface of the basin will fall.

Manifestly, a similar reasoning on the refluxes of the negative wave would indicate an emptying of the basin until the influxes of a rising tide should cause a cessation of the emptying, and the resumption of a filling.

Previously, in discussing the properties of the wave of translation propagated in an infinitely long canal, the result of limiting the length by the introduction of an infinitely large reservoir was considered, and it was found that the result was to generate an opposed negative wave of magnitude equal to that of the original positive wave. The effect of limiting the length of such a canal by the introduction of a like reservoir, of limited capacity, will be similar, but with the added difference of causing the generation of a third wave—that emitted from the basin—which will be positive for a positive generatrix.

Therefore, the canal connecting the tidal sea with a basin of limited capacity will always have within it at least three simple waves of translation, and the actual heights and velocities at any point in it will be the resultant of those due to these three waves. The first will be the simple wave of translation generated by the tide in the sea, which will be propagated with proper friction losses as already explained for an infinitely long canal. The second wave will be the reflected negative wave corresponding to the first one exactly as previously explained. The third wave will be that emitted by the basin, positive for a positive tide and negative for a negative tide, and of a magnitude at the canal outlet in the basin that will be exactly equal to the height of the basin level, but subject to the usual friction losses while being propagated in the canal itself.

Let the following additional notations be assumed:

H_b = the semi-amplitude of the basin tide.
t_b = the time of high water of the basin tide.
V_b = the velocity due to the height of wave, H_b.

The following equations may now be written for the tidal variation at the outlet into the basin,

$$h = H_1 \cos 2\pi \left(\frac{t - t_1}{T}\right) - H_2 \cos 2\pi \left(\frac{t - t_2}{T}\right) + H_b \cos 2\pi \left(\frac{t - t_b}{T}\right) . \quad (78)$$

When $t = t_b$, then $h = H_b$; hence,

$$H_b = H_1 \cos 2\pi \left(\frac{t_b - t_1}{T}\right) - H_2 \cos 2\pi \left(\frac{t_b - t_2}{T}\right) + H_b \ldots\ldots \quad (79)$$

Therefore,

$$H_1 \cos 2\pi \left(\frac{t_b - t_1}{T}\right) - H_2 \cos 2\pi \left(\frac{t_b - t_2}{T}\right) = 0 \ldots\ldots\ldots\ldots \quad (80)$$

Since the direct wave and the reflected wave must have the same phase at the outlet into the basin, $t_1 = t_2$; hence, $H_1 = H_2$, and Equation (78) becomes

$$h = H_b \cos 2\pi \left(\frac{t - t_b}{T}\right) \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (81)$$

The values of H_b and t_b will depend upon two factors, the first of which is the amplitude of the original wave of translation, and the second, the area of the basin. The first of these factors determines for a specific case the volume of the wave capable of being transmitted into the basin and available for filling it. The second determines the depth to which the filling may proceed, and, therefore, the proportion of the total volume of the wave of translation which is used for that purpose.

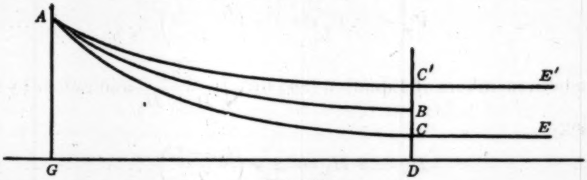


FIG. 15.

In Fig. 15, let $A\,B$ be the geometric locus of H as computed for the distance, $G\,D$, in an infinitely long canal having a semi-amplitude, $G\,A$, at G. The height, $D\,B$, will be the computed semi-amplitude of the wave, $A\,G$, as reduced by friction. Let $D\,B$ be the point at which the canal enters a basin the surface of which is represented by $C\,E$, and let $D\,C$ be the semi-amplitude of the surface variation of the level in the basin. Let it be assumed, also, for simplicity, that the area M, of the basin, is constant for all heights of the tide, so that its cubical contents above low-water level at any moment may be given by the expression, $M \times H_b \left(1 + \cos 2\pi \left(\frac{t - t_b}{T}\right)\right)$.

The heights, *D B* and *D C*, will seldom be equal. Since the actual height of water in the canal will automatically adjust itself to that of the basin, the theoretical geometric locus of *H* will seldom coincide with the actual locus. If the basin rises only to *C E*, the geometric locus of high water in the canal will be *A C* instead of *A B*. This means that the basin is of such large capacity that it receives not only the volume of the wave, *D B*, but also the volume equivalent to a wave the height of which is *C B*. If the basin is smaller, *D C* may become larger than *B D* as, for example, *D C′*. This condition would denote a basin with an area too small to afford a capacity to receive the entire volume of the wave, *D B*, and a surplus would be stored in the canal equal in volume to that of a wave with a height, *B C′*.

As shown in developing Equation (50), the velocity equation corresponding to Equation (78) may be written,

$$v = V_1 \cos 2\pi \left(\frac{t - t_1}{T}\right) + V_2 \cos 2\pi \left(\frac{t - t_2}{T}\right) - V_b \cos 2\pi \left(\frac{t - t_b}{T}\right) . \quad (82)$$

and,

$$v = 2\,V_1 \cos 2\pi \left(\frac{t - t_1}{T}\right) - V_b \cos 2\pi \left(\frac{t - t_b}{T}\right) \ldots\ldots\ldots (83)$$

It has already been seen that when $t = t_b$, $v = 0$, hence,

$$0 = 2\,V_1 \cos 2\pi \left(\frac{t_b - t_1}{T}\right) - V_b$$

or,

$$V_b = 2\,V_1 \cos 2\pi \left(\frac{t_b - t_1}{T}\right) \ldots\ldots\ldots\ldots\ldots\ldots (84)$$

Denoting the maximum velocity at the outlet by V_m, v will be equal to V_m when $t = t_b + \frac{T}{2}$. Hence,

$$V_m = 2\,V_1 \sin 2\pi \left(\frac{t_b - t_1}{T}\right) \ldots\ldots\ldots\ldots\ldots\ldots (85)$$

Divide both members of Equation (84) by $\sqrt{\frac{g}{D + H_b}}$ and reduce by Equation (16), giving,

$$H_b = 2\,H_1 \cos 2\pi \left(\frac{t_b - t_1}{T}\right) \ldots\ldots\ldots\ldots\ldots\ldots (86)$$

Attention has already been called to the fact that the values of H_b and t_b depend on H_1 and *M*. Equation (86) shows the relation between these quantities and H_1.

An expression for H_b in terms of *M* is now required. By definition, the cross-sectional area through which the canal discharges into the reservoir at any instant, *t*, may be written, $a = A' + H_b\, l \cos 2\pi \left(\frac{t - t_b}{T}\right)$, *A′* being the area of the canal outlet at mean sea level, and equal to *D l*. The quantity of water discharged through this area in the time, *dt*, will be given by the equation, $dq = a\, v\, dt$.

Substituting in this equation the proper values of a and v expressed in terms of the variable, t, the equation for the differential discharge becomes,

$$dq = \left[D\,l + H_b\,l \cos 2\,\pi \left(\frac{t - t_b}{T}\right)\right] V_m \sin 2\,\pi \left(\frac{t - t_b}{T}\right) dt$$

$$= D\,l\,V_m \sin 2\,\pi \left(\frac{t - t_b}{T}\right) dt + H_b\,l\,V_m \cos 2\,\pi \left(\frac{t - t_b}{T}\right) \sin 2\,\pi \left(\frac{t - t_b}{T}\right) dt. \quad (87)$$

Whence, by integration,

$$Q = \int_{t_b}^{t_b + \frac{T}{2}} dq = \frac{D\,l\,V_m\,T}{\pi} \ldots\ldots\ldots\ldots\ldots\ldots (88)$$

If Equations (84) and (85) are squared and added, $4\,V_1^2 = V_b^2 + V_m^2$. Whence,

$$V_m = \sqrt{4\,V_1^2 - V_b^2} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (89)$$

From Equation (16), $V_1 = \dfrac{\sqrt{g}\,H_1}{\sqrt{D + H_b}}$ and $V_b = \dfrac{\sqrt{g}\,H_b}{\sqrt{D + H_b}}$.

Substituting in Equation (89),

$$V_m = \sqrt{\frac{4\,g\,H_1^2}{D + H_b} - \frac{g\,H_b^2}{D + H_b}} = \sqrt{\frac{4\,g\,H_1^2 - g\,H_b^2}{D + H_b}} \ldots\ldots (90)$$

giving for Q,

$$Q = \frac{D\,l\,T}{\pi} \sqrt{\frac{4\,g\,H_1^2 - g\,H_b^2}{D + H_b}} = 2\,M\,H_b \ldots\ldots\ldots\ldots (91)$$

in which, H_b is the only unknown quantity.

Let it be assumed that $\dfrac{g\,D^2\,l^2\,T^2}{4\,M^2\,\pi^2} = K$, and reduce Equation (91) to the form, $H_b^2\,(D + H_b) = K\,(4\,H_1^2 - H_b^2)$, or,

$$H_b^3 + (D + K)\,H_b^2 = 4\,K\,H_1^2 \ldots\ldots\ldots\ldots\ldots\ldots (92)$$

which is the required solution, K being the factor the value of which depends on M.

Should the semi-amplitude of any one or of all three of these principal component waves be of such magnitude, or should the canal be of such a short length that their subsequent reflections would be of sufficient importance to come within the usual limits of observation, they would require to be accounted for in the computation. For this purpose the corresponding semi-amplitudes of these various reflections will be designated H_3, H_4, etc., as before, for the various reflections of the sea wave, and H_{b_3}, H_{b_4} etc., for the semi-amplitudes of the reflections of the basin wave. The general equations for heights of tide and velocity of flow at any point in the canal will be, therefore,

$$h = H_1 \cos 2\,\pi \left(\frac{t - t_1}{T}\right) - H_2 \cos 2\,\pi \left(\frac{t - t_2}{T}\right) + H_3 \cos 2\,\pi \left(\frac{t - t_3}{T}\right)$$

$$- H_4 \cos 2\,\pi \left(\frac{t - t_4}{T}\right) + \text{etc.}, + \left[H_{b1} \cos 2\,\pi \left(\frac{t - t_{b1}}{T}\right)\right.$$

$$\left. - H_{b2} \cos 2\,\pi \left(\frac{t - t_{b2}}{T}\right) + H_{b3} \cos 2\,\pi \left(\frac{t - t_{b3}}{T}\right) - \text{etc.}\right] \ldots\ldots (93)$$

$$v = V_1 \cos 2\pi\left(\frac{t - t_1}{T}\right) + V_2 \cos 2\pi\left(\frac{t - t_2}{T}\right) - V_3 \cos 2\pi\left(\frac{t - t_3}{T}\right)$$
$$+ V_4 \cos 2\pi\left(\frac{t - t_4}{T}\right) + \text{etc.}, - \left[V_{b1} \cos 2\pi\left(\frac{t - t_{b1}}{T}\right)\right.$$
$$\left. + V_{b2} \cos 2\pi\left(\frac{t - t_{b2}}{T}\right) - V_{b3} \cos 2\pi\left(\frac{t - t_{b3}}{T}\right) + \text{etc.}\right] \ldots\ldots(94)$$

The times of occurrence of the maximum heights and velocities at any point may be found by equations similar to Equations (53) and (54).

Inspection of Equations (84) and (86) shows that the maximum possible value of V_b is $2\,V_1$, and of H_b is $2\,H_1$, occurring when $t_b = t_1$. These substitutions in Equation (83) show the value of v to be zero for all values of t. This is manifestly the limiting case when the reservoir area has become so small as to disappear, and the canal has a "dead end".

The value of v is maximum at a time, $\frac{1}{4}\,T$, later than t_b. Its maximum possible value is $2\,V_1$, which can occur only when t_b and t_1 differ by $\frac{1}{4}\,T$; that is, when the basin is completely filled, or after the wave of translation has poured its entire volume into the basin. This can occur only when the basin is of such large extent that its level cannot be raised by this volume. It is apparent that if the level should be raised, an opposed wave would be generated, which would prevent the entire volume from entering the basin. This limiting condition then becomes the same as the case of an infinitely large reservoir.

It will probably be possible in any practical example of canal and basin of this type to observe both H_b and t_b directly, from which, H_1 can be found at once by means of Equation (86), and, consequently, the frictional coefficient by Equation (26). No example of this case in Nature is known to the writer with the exception of some small coastal bays connected with the sea by channels of various lengths and characteristics. Usually these channels are very short, since the bays are generally close to the ocean. The shorter the length of such channels, the greater will be the number of reflected waves that will be generated by the rise of the bay tide as well as by that of the sea tide. Equations (93) and (94) will then contain many terms in the last members, and their solution will be long and tedious. For such short canals the current may be assimilated, with possibly equal accuracy, to flow under the influence of a surface slope, always assuming the canal to be of short length. This method of solution has been described in a previous article.[18] It is the solution that will generally be found to be more suitable for this case, and of sufficient accuracy for all practical purposes.

CANAL CONNECTING TWO BODIES OF WATER
EACH HAVING AN INDEPENDENT TIDE OF ITS OWN

When a canal connects two bodies of water each having an independent tide of its own, no matter what the source of either tide may be, there occurs in the canal an interference of two opposed systems of waves, each system completely independent of the other. In the two cases heretofore discussed, all the various waves simultaneously affecting the surface of the canal could be considered as pertaining to a single system, since each and every wave was dependent in some manner on the primary wave of the sea as a generator. If the generating wave disappeared, all the various reflected waves also disappeared. The present case is that in which two such systems simultaneously affect the surface of the canal and are propagated in opposite directions.

---

[18] "Inlets on Sandy Coasts," by Earl I. Brown, *Proceedings*, Am. Soc. C. E., February, 1928, Papers and Discussions, pp. 505-553.

Since the two systems are independent, it is apparent that the two primary generating waves—that is, the respective primary waves pertaining to the two bodies of water connected by the canal—may have any possible phase relation at a given point in it from complete coincidence to complete opposition, and their amplitudes may have any ratio between their respective magnitudes. If such amplitudes are equal and opposite in phase at certain points, there will be complete disappearance of variation in tidal height at these points. In general, the amplitudes at the two ends of the canal will not be equal, and the phase differences of the two generating waves will be greater than zero.

The method of solving the problem of determining the actual or resultant amplitude of the tides and velocity of flow at any given point in this case must be apparent from what has already been demonstrated in the two preceding cases.

If the subscripts, *a* and *b*, are used to characterize each wave system, respectively, the equation for height of the major and minor wave systems each will be,

$$h_a = H_{a1} \cos 2\pi\left(\frac{t - t_{a1}}{T}\right) - H_{a2} \cos 2\pi\left(\frac{t - t_{a2}}{T}\right) + H_{a3} \cos 2\pi\left(\frac{t - t_{a3}}{T}\right) - H_{a4} \cos 2\pi\left(\frac{t - t_{a4}}{T}\right) + \text{etc} \ldots\ldots\ldots\ldots\ldots\ldots \quad (95)$$

and,

$$h_b = H_{b1} \cos 2\pi\left(\frac{t - t_{b1} + d}{T}\right) - H_{b2} \cos 2\pi\left(\frac{t - t_{b2} + d}{T}\right) + H_{b3} \cos 2\pi\left(\frac{t - t_{b3} + d}{T}\right) - H_{b4} \cos 2\pi\left(\frac{t - t_{b4} + d}{T}\right) + \text{etc} \ldots (96)$$

in which, *d* equals the difference in time between the arrival of high water at the respective ends of the canal. It is positive when the minor wave arrives the earlier.

The corresponding equations for velocity will be,

$$V_a = V_{a1} \cos 2\pi\left(\frac{t - t_{a1}}{T}\right) + V_{a2} \cos 2\pi\left(\frac{t - t_{a2}}{T}\right) - V_{a3} \cos 2\pi\left(\frac{t - t_{a3}}{T}\right) + \text{etc} \ldots\ldots\ldots\ldots\ldots\ldots \quad (97)$$

$$V_b = V_{b1} \cos 2\pi\left(\frac{t - t_{b1} + d}{T}\right) + V_{b2} \cos 2\pi\left(\frac{t - t_{b2} + d}{T}\right) - V_{b3} \cos 2\pi\left(\frac{t - t_{b3} + d}{T}\right) + \text{etc} \ldots\ldots\ldots\ldots\ldots\ldots \quad (98)$$

According to the principles discussed in connection with Equation (82), $h = h_a + h_b$ and $v = v_a - v_b$, or,

$$h = H_{a1} \cos 2\,\pi \left(\frac{t - t_{a1}}{T}\right) - H_{a2} \cos 2\,\pi \left(\frac{t - t_{a2}}{T}\right) + H_{a3} \cos 2\,\pi \left(\frac{t - t_{a3}}{T}\right)$$
$$- H_{a4} \cos 2\,\pi \left(\frac{t - t_{a4}}{T}\right) + H_{b1} \cos 2\,\pi \left(\frac{t - t_{b1} + d}{T}\right) - H_{b2} \cos 2\,\pi \left(\frac{t - t_{b2} + d}{T}\right)$$
$$+ H_{b3} \cos 2\,\pi \left(\frac{t - t_{b3} + d}{T}\right) - H_{b4} \cos 2\,\pi \left(\frac{t - t_{b4} + d}{T}\right) + \text{etc.} \ldots (99)$$

and,

$$v = V_{a1} \cos 2\,\pi \left(\frac{t - t_{a1}}{T}\right) + V_{a2} \cos 2\,\pi \left(\frac{t - t_{a2}}{T}\right) - V_{a3} \cos 2\,\pi \left(\frac{t - t_{a3}}{T}\right)$$
$$- V_{b1} \cos 2\,\pi \left(\frac{t - t_{b1} + d}{T}\right) - V_{b2} \cos 2\,\pi \left(\frac{t - t_{b2} + d}{T}\right)$$
$$+ V_{b3} \cos 2\,\pi \left(\frac{t - t_{b3} + d}{T}\right) \ldots\ldots\ldots\ldots\ldots\ldots\ldots (100)$$

To find the coefficient of friction: If Equation (100) is divided by $\sqrt{\dfrac{g}{D + h_a + h_b}}$ and then added to the actual head or height of tide (Equation (99)), thus,

$$h + v\sqrt{\frac{D + h_a + h_b}{g}} = 2\,H_{a1} \cos 2\,\pi \left(\frac{t - t_{a1}}{T}\right) - 2\,H_{b2} \cos 2\,\pi \left(\frac{t - t_{b2} + d}{T}\right)$$
$$+ 2\,H_{b3} \cos 2\,\pi \left(\frac{t - t_{b3} + d}{T}\right) - 2\,H_{b4} \cos 2\,\pi \left(\frac{t - t_{b4} + d}{T}\right) + \text{etc.} \; (101)$$

It will be noted from Equation (96) that this expression may be written:

$$h + v\sqrt{\frac{D + h_a + h_b}{g}} = h + h_v = 2\,H_{a1} \cos 2\,\pi \left(\frac{t - t_{a1}}{T}\right)$$
$$+ 2\left[h_b - H_{b1} \cos 2\,\pi \left(\frac{t - t_{b1} + d}{T}\right)\right] \ldots\ldots\ldots\ldots (102)$$

whence,

$$h_a - h_b + h_v = 2\,H_{a1} \cos 2\,\pi \left(\frac{t - t_{a1}}{T}\right) - 2\,H_{b1} \cos 2\,\pi \left(\frac{t - t_{b1} + d}{T}\right) . (103)$$

At the end of the canal from which the major wave comes, $H_{a1} = H_{a0}$; $t_{a1} = 0$; $H_{b1} = H_{bL}$; $t_{b1} = t_{bL}$; $h_b = 0$; hence, $h_a + h_v = 2\,H_{a0} \cos 2\,\pi \dfrac{t}{T} - 2\,H_{bL} \cos \pi \left(\dfrac{t - t_{bL} + d}{T}\right)$. Substituting for h_a its equivalent, $H_{a0} \cos 2\,\pi \dfrac{t}{T}$, the value of h_v is found to be equivalent to,

$$h_v = H_{a0} \cos 2\,\pi \frac{t}{T} - 2\,H_{bL} \cos 2\,\pi \left(\frac{t - t_{bL} + d}{T}\right) \ldots\ldots (104)$$

For $t = 0$,

$$h_v = H_{a0} - 2\,H_{bL} \cos 2\,\pi \left(\frac{t_{bL} + d}{T}\right) \ldots\ldots\ldots\ldots (105)$$

At the end of the canal from which the minor wave comes, $H_{a1} = H_{aL}$; $t_{a1} = t_{aL}$; $H_{b1} = H_{b0}$; $t_{b1} = 0$; $h_a = 0$; whence,

$$- h_b + h_v = 2\,H_{aL} \cos 2\,\pi \left(\frac{t - t_{aL}}{T}\right) - 2\,H_{b0} \cos 2\,\pi \left(\frac{t + d}{T}\right) .. (106)$$

Substituting, $h_b = H_{b0} \cos 2\pi \left(\frac{t+d}{T}\right)$, and $t = -d$, in Equation (106):

$$h_v = 2\, H_{aL} \cos 2\pi \left(\frac{d + t_{aL}}{T}\right) - H_{b0} \ldots\ldots\ldots\ldots\ldots (107)$$

It is apparent from Equations (105) and (107) that the frictional coefficient can be computed from either of these combined with Equation (26), after measurement of the velocity at high water at either or both ends of the canal, and substituting the equivalent velocity head in the proper one of these equations, in which all terms will be known but H_{bL} or H_{aL}, and t_{bL} or t_{aL}. The latter may be computed with sufficiently close approximation from the known depths and heights of tide, thus fixing H_{bL} or H_{aL} for substitution in Equation (26). The measurement at the end from which the minor wave comes will usually be found the more satisfactory, thus giving H_{aL} which is larger than H_{bL}.

Application to Existing Canals.—There are a great many examples of canals of this type, such as the English Channel, the St. Georges Channel, Seymours Narrows, Deception Pass, the Cape Cod Canal, and the Chesapeake and Delaware Canal.

The English Channel is an example of the longer canals of this type. Its tides are the resultant of the interference of two unequal opposed waves. The major or Atlantic wave enters from the south end, with an amplitude of 16 to 18 ft. The minor wave enters from the North Sea with an amplitude of about 10 ft.

The length of the Channel is great compared with that of the wave, hence there is sufficient room in it for the development of all the phenomena of wave interference anticipated from theoretical considerations. A complete description of these phenomena is given by Bourdelles[18] and will not be repeated here.

The case of the interference of two equal opposed waves is realized in the St. Georges Channel of the Irish Sea. Two flood currents having opposite directions, exist simultaneously and proceed from the ends of the channel toward the line of flood which forms a line joining the Isle of Man with Morecambe Bay, so that the sea rises on that line to a maximum height, but is accompanied by no current. Likewise, on the ebb, two opposed ebb currents start from this line directed toward the ends of the channel. At the place where the trough of one of the waves encounters the summit of the other, the vertical variation of the tide is zero, while the currents there reach their greatest velocity. This occurs at Coustown and at a point 50 miles south of Dublin. These ebb and flood currents turn throughout the channel at the time of high and low water at the line of flood; likewise, the time of maximum velocity throughout the channel corresponds to that of half tide at this line.

The Cape Cod Canal is an example of the shorter canals subject to unequal opposed waves.[19] The canal is about 8 miles long. The amplitude of the tide in Cape Cod Bay is about 9 ft. and of that in Buzzards Bay, 4 ft. This canal is so short with respect to the magnitude of its major wave that the number of reflections of the wave through the canal must be large, and the equations corresponding to Equations (99) and (100) would be long and tedious to solve. The writer has not attempted to compute by this theory the tidal characteristics of this canal because data obtained by observation known to him are not sufficiently complete for purposes of comparison.

---

[19] "The Cape Cod Canal," by William Barclay Parsons, M. Am. Soc. C. E., *Transactions*, Am. Soc. C. E., Vol. LXXXII (1918), p. 1.

The Chesapeake and Delaware Canal occupies an intermediate status as to length and depth between the Suez Canal and the Rehoboth Canal. It is the one by which the theory will be further tested. This canal connects the Delaware River with Chesapeake Bay.[30] It begins about 40 miles below Philadelphia, Pa., near Delaware City, Del., and extends for about 14 miles in a westerly direction to Chesapeake City, Md., on Back Creek, an arm of Chesapeake Bay. The canal was designed to afford a sea-level channel between these bodies of water, 12 ft. deep at mean low water, with a bottom width of 90 ft. The sea-level connection was made in February, 1927, and some observations were made on May 13, 1927, to ascertain the tidal conditions.

From previous records there was found to be an average higher high water of 6 ft. above mean low water at the Delaware entrance of the canal, and of 2.4 ft. at Chesapeake City. High water occurs two hours earlier at Chesapeake City than at the Delaware entrance.

The side slopes of the excavated prism are constructed at 2 ft. horizontal to 1 ft. vertical, and the bottom of the canal is uniformly 15 ft. below mean sea level, hence the following data result (see Fig. 16):

$$p = 33.5 + 33.5 + 90 = 157 \text{ ft.}$$

$$l = 30 + 30 + 90 = 150 \text{ ft.}$$

$$A = \frac{1}{2}(150 + 90) \times 15 = 1\,800 \text{ sq. ft.}$$

$$D = \frac{1\,800}{150} = 12 \text{ ft.}$$

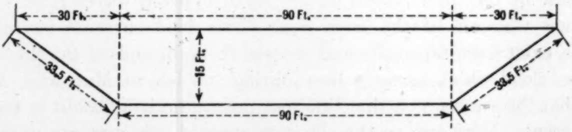


FIG. 16.—DIAGRAMMATIC CROSS-SECTION OF DELAWARE-CHESAPEAKE CANAL.

No opportunity was had to determine the exact value of the coefficient of friction for this canal. It has been shown, however, that a value of 0.51 was proper for channels of the cross-section and depth of the Suez Canal and of 0.58 for the Rehoboth Canal. The value of 0.55 appears to give results in accordance with observations for the Chesapeake and Delaware Canal, and that is the value adopted.

The first three miles of the canal at the Delaware end were constructed wider than the remainder, primarily to obtain more width for vessels to lie at the banks while awaiting passage, and, secondarily, to obtain material to build a dike in the vicinity. This change in width results in the heaping up of the tide at Mile 3, which must be noted in considering the effect of width. It is

---

[30] *Transactions*, Am. Soc. C. E., Vol. 95 (1931), p. 758.

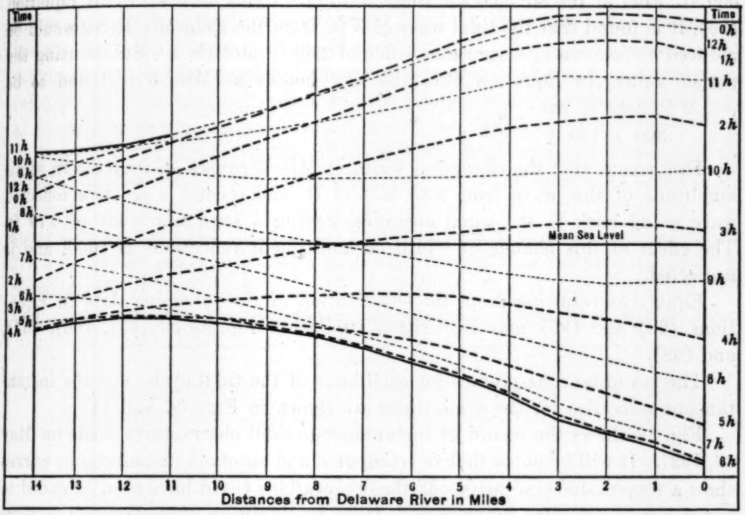


FIG. 17.—COMPUTED INSTANTANEOUS PROFILES IN THE CHESAPEAKE AND DELAWARE CANAL.

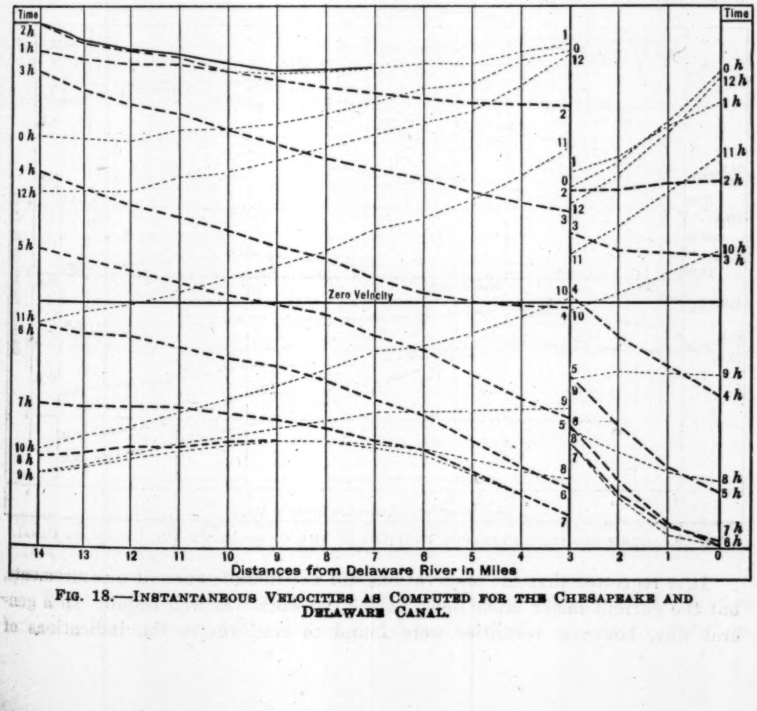


FIG. 18.—INSTANTANEOUS VELOCITIES AS COMPUTED FOR THE CHESAPEAKE AND DELAWARE CANAL.

300 ft. wide at the surface for these 3 miles. Using this width in Equation (26), it is found that the tidal wave of 3 ft. from the Delaware River would be reduced by friction to a semi-amplitude of 2.39 ft. at Mile 3. Substituting the proper values in Equation (37) the new height at Mile 3 is found to be

$$h = \frac{2 \times 2.39 \times 300}{300 + 150} = 3.18.$$

This means that the change of width at Mile 3 causes an increase in semi-amplitude of this wave from 2.39 to 3.18 ft. and creates a positive reflected wave going back to the canal entrance, having a semi-amplitude of 0.79 ft. The effect of this change of width on subsequent reflections is small and is neglected.

Equations resulting from the substitution of the foregoing data in Equations (26) and (99) may be written similar to Equations (65), (66), (67), and (68).

The instantaneous profiles for each hour of the tidal cycle, and the instantaneous velocities for the same times are shown in Figs. 17 and 18.

Fig. 19 shows the record of instantaneous tidal observations made on May 13, 1927. It will be noted that the computed and observed instantaneous curves show a remarkable similarity. If the observations could have been extended to cover, say, a complete lunar period, so as to obtain an average result, a more complete comparison could have been made; yet the results are quite in agreement with the theory.

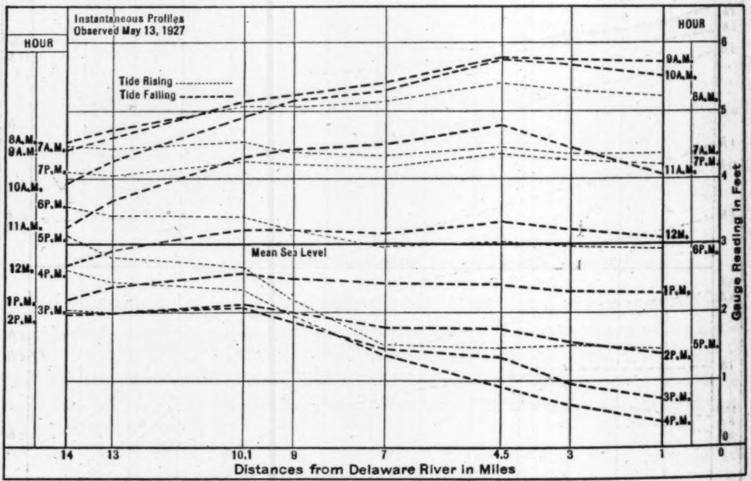
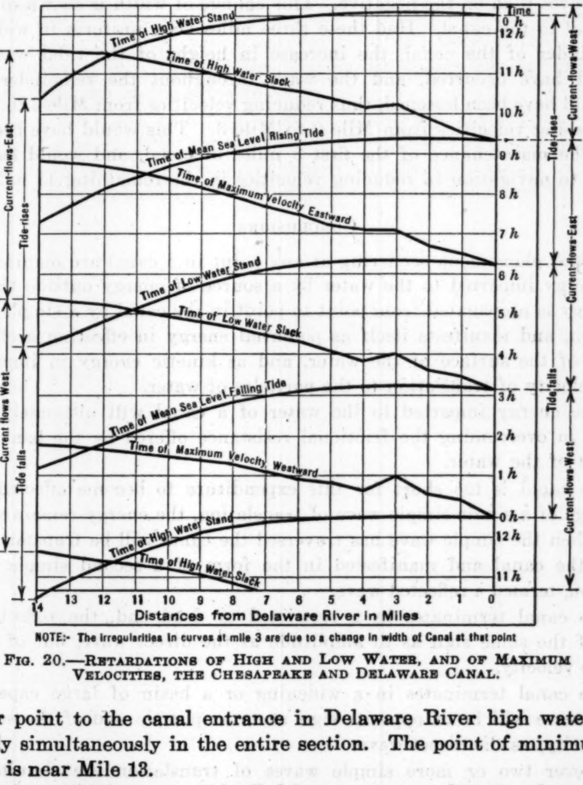


FIG. 19.—OBSERVED INSTANTANEOUS PROFILES IN THE CHESAPEAKE AND DELAWARE CANAL.

It is regretted that the observations did not include current measurements, but the current meter broke down before the work was well begun. In a general way, however, velocities were found to conform to the indications of theory. For instance, in the first three miles of the canal, the velocities are so low that silt brought in and deposited from the Delaware River which here has a very muddy bottom, has caused such rapid shoaling of the channel in this portion as to require much maintenance dredging. The sudden increase in velocity at Mile 3 is quite marked, and the greater velocity at Mile 14 was such as to cause shipping much concern.

Fig. 20 shows the computed retardations of high and low water, and the times of slack water and maximum velocities. These features were noted in the observations of May 13, 1927, and are also shown in Fig. 20. Here, again, the general appearance and trend of the curves is such as to show that the results given by theory agree quite well with observations. In conformity with the requirements of theory, the rate of propagation of high water is quite slow beginning at Mile 14, from which the rate gradually increases to Mile 6. From the latter point to the canal entrance in Delaware River high water occurs practically simultaneously in the entire section. The point of minimum tidal variation is near Mile 13.



NOTE:- The irregularities in curves at mile 3 are due to a change in width of Canal at that point

FIG. 20.—RETARDATIONS OF HIGH AND LOW WATER, AND OF MAXIMUM VELOCITIES, THE CHESAPEAKE AND DELAWARE CANAL.

Beginning at high water at the Delaware entrance, the rate of propagation of the maximum velocity westward is fairly uniform in a westerly direction.

The only change of importance in this rate is at Mile 3, where irregularities are due to the change of width of the canal at that point. It will be noted that the time of maximum velocity westward occurs at Mile 14, somewhat after the level has fallen below mean sea level. Between Miles 11 and 12, slack water occurs on high and low-water stands, so that, in general, the period of falling tide corresponds to that of a westerly current and the period of rising tide to that of an easterly current. At the Delaware entrance, on the contrary, the current is westerly as long as the tide is above, and easterly as long as it is below, mean sea level. Mean canal level will be found above mean sea level at all points between its two ends.

Theory and observation both reveal a lack of similarity between the geometric loci of high and low water, due to the greater effect of friction on the negative waves. The effect of change of width at Mile 3 is more marked on the positive waves than on the negative. This change of width is shown to be very detrimental to the canal. Had these three miles been uniform in width with the remainder of the canal, the increase in height of the tidal wave there would not have occurred, and the wave throughout the remainder of the canal would have been lessened, thus reducing velocities from Mile 3 to Mile 14, and increasing velocities from Mile 0 to Mile 3. This would have been beneficial to the maintenance of the first 3 miles of canal, and would have been beneficial to navigation in reducing velocities in the remaining 11 miles.

CONCLUSIONS

The tidal phenomena occurring at any point in a canal are manifestations of the energy imparted to the water by a source of energy outside the canal. This energy is propagated from point to point in the canal by a simple wave of translation, and manifests itself as potential energy in effecting a change of elevation of the surface of the water, and as kinetic energy in imparting a certain velocity of translation to the particles of water.

All the energy imparted to the water of a canal will ultimately become expended in overcoming the frictional resistance offered by the banks to the movement of the water.

If the canal is too short for this expenditure to become effected during the passage of a single simple wave of translation, the energy remaining unexpended when the simple wave has traversed the canal will be transmitted back through the canal and manifested in the form of a second simple wave of translation, termed a reflected wave.

If the canal terminates in a restricted or dead end, the reflected wave will be of the same sign as to amplitude as the direct wave, but of opposite sign as to velocity.

If the canal terminates in a widening or a basin of large capacity the reflected wave will be of opposite sign as to amplitude and of the same sign as to velocity, as the direct wave.

Whenever two or more simple waves of translation are simultaneously propagated in a canal, whether in the same or in opposite directions, a compound wave is formed, which is the resultant of all the simultaneous simple

waves. The resultant amplitude at any point is the algebraic sum of all the component amplitudes at that point, and the resultant velocity, the algebraic sum of all the component velocities, provided all velocities in the direction of propagation of the major wave are considered as positive, and all in the opposite direction, negative. In general, the algebraic sum of the component amplitudes will not bear the same simple relation to the algebraic sum of the component velocities as expressed in Equation (17) for the relation of amplitude to velocity in the simple wave of translation.

The simple wave of translation seldom exists in a state of Nature, and, consequently, can be but rarely observed. Practically all tidal propagation as observed in Nature is in the form of a resultant wave, in which the varying effect of the different components as modified by friction accounts for the varying relation so frequently observed between amplitude of tide and velocity of current at any given point.

Shallow channels are conducive to high velocities of tidal currents, with rapid absorption of energy, which reduces velocity. Deep channels are conducive to moderate current velocities, but absorb energy slowly, which tends to maintain currents at the same velocity.

As long as the width of a tidal channel is uniform, the magnitude of the width has little or no effect on velocity or other phases of tidal propagation. Any variation in the width of a tidal channel will affect the rate of expenditure of energy, and, consequently, it will affect both amplitudes and velocities in any canal.

DISCUSSION

R. L. Faris,[21] M. Am. Soc. C. E. (by letter).—This paper is an important contribution to a subject that is beset with many complexities. Throughout the world, and more especially in the United States, the subject of tides has been developed very largely by the mathematician whose interest, primarily, was in other than the engineering phases of the subject. It is gratifying, therefore, to find the fundamental problems of the subject successfully attacked from the engineering point of view.

In introducing Table 3, under the heading, "Heights", the author states that,

"It is the practice of the United States Coast and Geodetic Survey to allow for lack of symmetry between positive and negative waves in making tidal predictions by introducing a 'shallow-water' component into the computations."

This may leave an erroneous impression with the reader not familiar with the problem of the prediction of tides, and it appears desirable to amplify the subject somewhat.

In connection with the prediction of tides it is unnecessary to assume the existence either of positive or negative waves in the sense used by the author. The problem centers in the resolution of the tides, as actually observed at any given place, into a number of simple cosine curves each having a definite period, amplitudes, and phase. As soon as these become known, the tide for any future time can be predicted by combining these simple cosine curves for the time in question. Technically, each of these simple curves is spoken of as a component of the tide.

The periods of the primary components of the tide are derived from astronomic considerations, and, on the open coast, the tide as actually observed can be closely represented by the combination of a number of such components. However, in traversing the shallow waters of tidal rivers, or other like bodies of water, the open ocean tide becomes distorted or modified.

The modification that the tide undergoes in its progress up a tidal river is illustrated in Fig. 21 which represents the rise and fall of the tide at Fort Hamilton, New York, and Albany, N. Y., for the 2-day period, May 20 and 21, 1914. The lower curve is the tide curve for Fort Hamilton, which may be regarded as situated near the mouth of the Hudson River, while the upper curve is the tide curve for Albany, near the head of tide-water on the Hudson, about 130 nautical miles above Fort Hamilton.

The tide curve for Fort Hamilton is representative of the tide on the open coast in that the durations of rise and fall are approximately equal. This is evidenced by the symmetry of the curve. At Albany, however, the tide has become modified considerably. In overcoming the friction against its progress up stream, the duration of rise has become shortened, giving a corresponding increase to the duration of fall. In Fig. 21 this is represented by the steepness of the rise of the tide which contrasts markedly with the much gentler slope of the fall.

---

[21] Asst. Director, U. S. Coast and Geodetic Survey, Washington, D. C.

In resolving river tides into their constituent simple tides for the purposes of prediction, the so-called shallow-water components come to light, quite

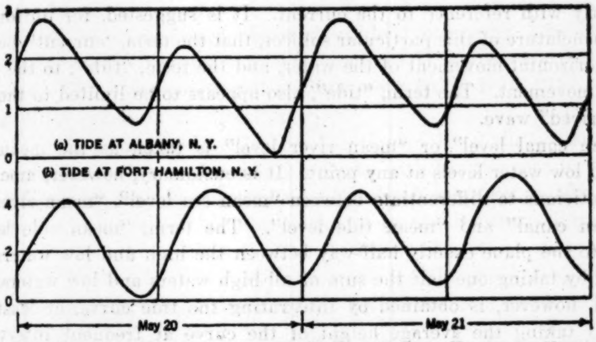


FIG. 21.—TIDE CURVES AT ALBANY, N. Y., AND FORT HAMILTON, NEW YORK, FOR MAY 20 AND 21, 1914.

apart from the causes which bring them about. In this connection, therefore, it is better not to speak of "introducing" shallow-water components, because these exist in, and must be taken into account in the prediction of, river tides.

G. T. RUDE,[22] M. AM. SOC. C. E. (by letter).—In the closing discussion of the paper, "Tides and Their Engineering Aspects",[23] the writer made the statement that Colonel Brown had had charge of the lowering of the Chesapeake and Delaware Canal to sea level, and that any studies he had made of wave propagation in the canal would be awaited with interest. The Engineering Profession is indeed indebted to Colonel Brown for this contribution to the subject of restricted and complex tidal flow.

The writer desires to preface this discussion with the explanation that, while exception is taken to certain statements regarding ocean currents and the dominant tides of the oceans, which are considered not in entire accord with these phenomena as accepted by tidal students, it is obvious that they are used by Colonel Brown merely to round out his paper. In no way do they detract from the value of the study as to the flow of water in tidal canals, assuming for the moment that the writer's statements as to these phenomena are the correct ones.

Undoubtedly, from his exhaustive study, Colonel Brown can easily and successfully clarify one phase of his "reflected wave" theory. A visualization of a reflected wave, in the case of a canal with a dead end, is not difficult, nor is its agency in consuming the unexpended energy of the wave upon arrival at the end. In the case of the canal with an outlet into a body of water, however, it would appear that the propagation of the wave would continue out into the outlet body in its original form and be dissipated in that body instead of being propagated back into the canal as a "reflected wave".

---

[22] Commander, U. S. Coast and Geodetic Survey, Washington, D. C.

[23] *Transactions*, Am. Soc. C. E., Vol. 92 (1928), p. 711.

In the definitions, Colonel Brown uses the term, "strength of the tide", apparently with reference to the current. It is suggested, for uniformity in the nomenclature of this particular subject, that the term, "current", be applied to the horizontal movement of the water, and the term, "tide", to the periodic vertical movement. The term, "tide", also appears to be limited to the motion of a "derived" wave.

"Mean canal level" or "mean river level" is taken as the mean of the high and low-water levels at any point. It is customary, however, among tidal mathematicians to differentiate between "mean sea level", "mean river level", or "mean canal" and "mean tide level". The term, "mean tide level", is applied to the plane exactly half-way between the high and low waters and is obtained by taking one-half the sum of all high waters and low waters. Mean sea level, however, is obtained by integrating the tide curve, at least to the extent of taking the average height of the curve at frequent intervals. It is customary to use the hourly heights for this purpose. Mean river level or mean canal level is the mean height of the water surface as influenced by the tide, freshets, and bottom slope, and is obtained in the same manner as mean sea level.

The difference between mean tide level and mean sea level, however, is small, and if the series of observations is short, the two planes may be considered for practical purposes as coincident. On the Atlantic Coast the plane of mean sea level lies above the plane of mean tide level—at Portland, Me., 0.03 ft.; at New York, N. Y., 0.05 ft.; at Atlantic City, N. J., 0.03 ft.; and at Fernandina, Fla., 0.10 ft. On the Pacific Coast mean sea level is the lower of the two—at Seattle, Wash., 0.01 ft.; at San Francisco, Calif., 0.06 ft.; and at San Diego, Calif., 0.05 ft. This difference in elevation of the two planes is due to the fact that the tide curve is not a simple sine curve; actually, the larger part of the curve may be above or below the mean level of the water as determined from the average of the high and low waters.

Apparently, the hydraulic characteristics in the canal are not taken into consideration. The tidal data for the Chesapeake and Delaware Canal, furnished the U. S. Coast and Geodetic Survey by the Army Engineer Corps, appear to indicate that the current is primarily hydraulic in the western portion of the canal, although it loses this characteristic at the Delaware Bay end. The present current predictions made by the Coast and Geodetic Survey in the Atlantic Coast Current Tables are for the location of the railroad bridge. These predictions are based on the assumption that the current is hydraulic at that point.

Under the paragraph, "Tidal Waves in the Sea", Colonel Brown states that tidal currents in shallow waters near the shore line are mass currents acting for the entire depth of the sea. Observations indicate that, in rivers and canals, the maximum velocity is just below the surface. Mr. Parsons calls attention to this fact in his paper on the Cape Cod Canal.[23] Current observations made by the U. S. Coast and Geodetic Survey have shown varying results in short series. The observations, however, over a longer series substantiate Mr. Parsons' statement. At the principal station of a section of the Boston Harbor Current Survey by the Coast and Geodetic Survey, near Deer Island Light, where fourteen days of observations were obtained, the following velocities were observed at flood strength: 7-ft. depth, 3.78 knots; 13-ft. depth, 3.81 knots; 32-ft. depth, 3.77 knots; and 51-ft. depth, 3.75 knots. The corresponding directions were 267°, 270°, 271°, and 268°, showing little variation in direction.

In the open ocean, however, where the terrain has little or no influence, the currents act with ever-decreasing velocity with the increase in depth and, too, the direction changes constantly as the depth increases, clockwise in the Northern Hemisphere and counter-clockwise in the Southern Hemisphere.

In the paragraph mentioned, Colonel Brown also states that:

> "The currents [in the open sea] sometimes become gyratory because of the combination of longitudinal currents with lateral currents which come and go, corresponding to the alternate rise and fall of the water on shore, or of the crossing of two co-existent waves, the directions of which intersect at an angle."

The work of an internationally known mathematician, corroborated by actual observations of ocean currents, demonstrates that this gyratory or rotary movement is a distinct and ever-present characteristic of these currents, not only along the coasts, but over the entire ocean. These phenomena were first suspected by Fridtjof Nansen in his historic drift in the Arctic ice pack during 1893-1896.

Nansen's deductions were reported to Helland-Hansen, who demonstrated mathematically that this rotary movement is due to the deflective force of the earth's rotation, the direction of rotation of the currents being clockwise in the Northern Hemisphere and counter-clockwise in the Southern Hemisphere. Nansen's theory of this rotary movement has been proved as an ever-present characteristic of ocean currents, not only by the mathematics of Helland-Hansen, but also by actual current observations made since 1915 by the U. S. Coast and Geodetic Survey on every light-vessel along both coasts of the United States. It has also been checked in the open ocean by Helland-Hansen, utilizing innumerable reports of current effect upon ocean-going vessels.

These ocean currents are never slack,[24] their direction changing continually in a rotary movement at an average rate of about 30° per hour for semi-diurnal currents. At the station of observation the velocities vary throughout a tidal cycle and the directions and velocities at a station, plotted to scale, roughly approximate an ellipse.

It appears that Colonel Brown has not accepted the "stationary wave theory" of Harris as an explanation of the changed regimen of the tide in certain bodies of water in which the tide occurs almost simultaneously throughout the entire length of the body and in which the turn of the current occurs near the time of high and low water instead of at the time of half-tide. Under the heading, "Tidal Waves in the Sea", Colonel Brown states:

---

[24] "Tides and Their Engineering Aspects," *Transactions*, Am. Soc. C. E., Vol. 92 (1928), p. 656.

> "In general, near the coast line, the situation is intermediate between the state of free propagation of the tidal wave, and that of filling a basin of small dimensions; the regimen of the tide becomes changed. The reversal or turn of the tide [current] is then no longer effected at mid-tide, but approaches closer to the times of high and low water, and the maximum velocity approaches mid-tide. The velocities then differ from their theoretical value and, in order to interpret the phenomena observed, one is tempted at times to have recourse to considerations based upon the hydraulics of running water and the influence of surface slope. Under these conditions the tidal wave loses its usual characteristics, entirely or in part, and in order to account for observed facts, it must be considered as formed by a series of small waves of translation each of which is propagated according to its own proper law."

Harris' stationary wave theory accounts for these and other apparent anomalies in the tides in a more rational manner than the older theories and it is being accepted both in the United States and abroad. This theory would explain the dominant tides of the world as the results of oscillations or stationary waves, set up and maintained by the periodic tidal forces of moon and sun. In most inland bodies of water connected with the ocean the tide is due to a progressive wave thrown out by a near-by stationary wave in the ocean. In a body of water, however, of a size and depth to maintain an independent stationary wave, the tides may be caused by its own stationary wave.

It could be shown that Harris' stationary wave theory[25] furnishes a logical explanation of the tidal characteristics of some regions, which differ from the usual tidal regimen, such as the reversal of the current at the time of high and low water instead of at the time of mid-tide, mentioned in Colonel Brown's paper; the almost simultaneous occurrence of high or low water throughout the length of the body; the increase of the range toward the head of the body, represented in the large range at the head of the Bay of Fundy; and the strictly solar tide of the Island of Tahiti, with its high and low waters occurring at about noon and midnight, and about six o'clock in the morning and evening.

Under the heading, "Canal Connecting Two Bodies of Water Each Having an Independent Tide of Its Own", Colonel Brown accepts the theory advanced in the past by some engineers, that the characteristics of the tidal movement in such canals, for example, Seymour Narrows, Deception Pass, East River, etc., are due to the interference of two opposed systems of waves, each independent of the other. The evidence, however, from direct tidal and current observations, supported by the accuracy of predictions of tides and currents based on hydraulic considerations, indicates that the movement of the water in such straits is primarily hydraulic, brought about by the difference of the tidal characteristics in the connected bodies of water and the differences in the time and height of the tide in the two bodies.[26]

---

[25] Manual of Tides, Pt. IV *A*, Appendix 7, Rept. of U. S. Coast and Geodetic Survey, 1900, and Pt. IV *B*, Appendix 5, Rept. of U. S. Coast and Geodetic Survey, 1904.

The writer would like to inquire of Colonel Brown whether the values in the columns headed "Bourdelles" and "Lévy" in Tables 3, 4, and 5, are observations made by Bourdelles and Lévy with which the computed values in the column, "Present Theory" are compared, or whether they are values computed by Bourdelles and Lévy compared with computed values by the "Present Theory"?

EUGENE E. HALMOS,[27] M. AM. SOC. C. E. (by letter).—The writer has given careful attention to this paper, not only because of its general appeal to engineers interested in the science of hydraulics, but particularly because of the references made therein to the paper describing the Cape Cod Canal,[28] in the development of the mathematical portion of which the writer had the privilege of collaborating with Mr. Parsons.

The analysis of the tidal flow in canals presented in Mr. Parsons' paper followed the theory of Professor Maurice Lévy, which, in turn, is based on d'Alembert's theorem defining the interdependence of the free surface of a fluid in motion and the forces acting on its mass. The derivation of the formulas is strictly rational, their application being restricted only by the requirement of comparatively small tidal variations in relation to the depth of the canal.

In his study Colonel Brown starts with an equation (Equation (15)) which the writer considers empirical and in which the velocity of the current is assumed to be a very simple function of the speed of propagation, the depth of the canal, and the height of the tide at the summit of the wave. He then uses this value of the velocity in expressing the energy contained in the wave and the work done by friction in destroying this energy. From this empirical beginning, the author, by the application of well-established principles of hydrodynamics, and especially by the introduction of his very ingenious theory of the reflected wave from the reservoir, arrives at formulas and methods of treatment of the problem which are applicable to the varying conditions that may be encountered in tidal canals.

The author's treatment of the subject does not limit the use of his method and formulas to comparatively small tidal variations. It permits the consideration of friction in such a manner as to prove that its effect in modifying the height and the speed of the wave is greater for the negative than for the positive wave. This the writer believes to be an important advance in the study of tidal canals. The demonstration of the effect of interference of opposed waves is clear, instructive, and useful.

The author has shown that by his theory a very close check on both tidal heights and current velocity is obtained when applied to the Suez Canal. However, as stated by Mr. Parsons,[28] a similar agreement of computed and observed results is found with the Lévy formulas if the frictional coefficient is so modified that friction, as a linear function of the mean velocity, will absorb the same total energy as if its variation depended on the square of the velocity. On the other hand, the two methods seem to give appreciably different results in the case of more shallow canals with comparatively large tidal variations. An inspection of Fig. 10 in the author's paper on "The Chesapeake and Delaware Canal"[29] and Fig. 17 and Fig. 19 of his present paper, at once reveals the fact that the tidal elevations computed by his method more closely approximate the observed profiles.

---

[26] "Tides and Their Engineering Aspects", *Transactions*, Am. Soc. C. E., Vol. 92 (1928), pp. 713-715.

[27] Chf. Engr., Parklap Constr. Corp., New York, N. Y.

[28] *Transactions*, Am. Soc. C. E., Vol. LXXXII (1918), p. 121.

It appears that, while the mathematical basis of the analysis given in Mr. Parsons' paper is more rational, the results obtained from the formulas deduced by the author may conform better to actual observation for a wider range of conditions.

In the writer's opinion, the empiricism of Equation (15) does not seriously impair the validity of the author's formulas. He believes, however, that the method shown by Colonel Brown for the evaluation of the frictional coefficient, F, is of doubtful practical value. This method calls for the actual measurement of the velocity, V_0, at the outlet of the canal, which, of course, cannot be done before the canal is completed. For the design, therefore, this basic quantity is not available and must be evaluated by comparison with existing canals. As it is not likely that such canals, with closely similar cross-section, length, tidal conditions, and consistency of canal bed, could be easily found, the writer feels that it would be better to follow Mr. Parsons' suggestion and determine V_0 by one of the approximate methods given in his paper on the Cape Cod Canal.

Under "General Remarks", the author states that no theory can adequately and completely solve the complex problems which the regimen of the tide raises in practice and that such complete solution "must forever escape satisfactory analysis". In this, there is a complete agreement between the author and Mr. Parsons, who concluded that numerous necessary assumptions, which are never quite true in practical cases, "*a priori* reduce all analytical results to a grade of a more or less fair approximation". However, Mr. Parsons went further than the author when he definitely stated that, in his opinion, approximate methods of calculation should be used for the practical evaluation of the velocities in canals subjected to tidal influences.[30] On the basis of the theoretical studies, Mr. Parsons suggested the use of particular approximate formulas for different types of canals characterized by the product of their length and the factor, q, the value of which was given in his Equation (8).[31]

The writer found that this product for the Chesapeake and Delaware Canal amounts to 0.972 for the 11 miles between Mile 3 and Mile 14. It, therefore, would come under the Classification (c), as defined[32] in the paper on the Cape Cod Canal. For this type, Mr. Parsons recommended the use of the formulas of the French Academy of Sciences[33] as likely to give close results for practical purposes, both as to tidal elevations and maximum current velocities. The writer was unable to make a numerical check, by the use of these formulas, against the values computed by the author, because the published diagrams (Figs. 17 and 18) do not show the scale of either heights or velocities, nor could he check against observed current velocities as actual data are not yet available. The writer would suggest that the author make such a check when current velocity measurements will furnish sufficient information on the Chesapeake and Delaware Canal and in this manner establish whether or not the recommendations made by Mr. Parsons as to the limits for the use of the proper approximate formulas need to be revised.

---

[29] *Transactions*, Am. Soc. C. E., Vol. 95 (1931), p. 760.

[30] *Loc. cit.*, Vol. LXXXII (1918), p. 142.

[31] *Loc. cit.*, p. 112.

[32] *Loc. cit.*, p. 143.

[33] *Loc. cit.*, p. 138.

Nothing said by the writer in discussing this paper should detract from its value as a treatise of the highest order on the motion of water in tidal canals. The theory of the propagation of waves of translation in canals, conceived by Bourdelles and given mathematical expression by the author expands and completes the work of previous writers on the subject. In this paper, Colonel Brown has made a real and useful contribution to the science of hydrodynamics.

W. M. Black,[34] M. Am. Soc. C. E. (by letter).—The writer deems this paper of great value to the Engineering Profession. It is a distinct contribution to the very limited information heretofore available for determining in advance the movement of water in projected canals designed to connect bodies of water in which the tidal regimens differ. It will also assist in the determination of the effects of changes projected in the cross-sections of canals in which there is tidal action. From personal experience the writer knows how prone many engineers have been to treat tidal currents simply as water flowing under the laws which govern the uniform flow of water in pipes and channels under a fixed head. Too little weight has been given to the wave action propagated in advance of the current when the head in a stream is increased by an influx. McAlpine demonstrated this fact in the latter part of the Fifties and in the Nineties the late Brig.-Gen. Amos Stickney, U. S. A., reported the same phenomenon in his discussion of flood movements in the Mississippi River where, in the deep channel below the mouth of the Red River, the velocity of advance of the flood wave is five times greater than the maximum current velocity.

The velocity of wave propagation is greater than the velocity of the current and an increase of channel capacity increases the velocity of wave propagation and decreases the velocity of the induced current (Equations (2) and (16)). Therefore, the necessity of considering the changes of head due to the wave movement is apparent when the probable effect of changes in channel depths in waterways subject to tidal action is under study. The problem of the control of current velocities in the East River, New York, is an example.

Colonel Brown's theory seems to be supported in its application to the conditions in the several canals for which sufficient data are available. The writer believes that observations on other tidal canals will give further confirmation. He hopes that investigations along this line will be continued. He also hopes that similar analytical studies will be made of the tidal action in estuaries, such as the Hudson and Delaware Rivers, in which there is a gradual change in the form of the tidal wave from one of oscillation near the entrance from the sea to one of translation farther up the river.

---

[34] Maj.-Gen., U. S. A. (*Retired*), Washington, D. C.

It is only by a more accurate knowledge of the laws which govern the tidal wave action that the problems of the improvement of such estuaries can be solved with any degree of accuracy. Under the pressure of what seemed to be a local necessity, channel widths have been changed recently in the Lower Hudson. The effects of this change on the tidal regimen of the upper river cannot be foretold with any certainty at this time. The entire question of tidal action in inland waters is of the greatest practical importance.

H. F. FLYNN,[35] M. AM. SOC. C. E. (by letter).—In that portion of the paper dealing with the Chesapeake and Delaware Canal, a comparison is made between observation and prediction, but only one day's heights and times and no currents were then available. The purpose of this discussion is to make such a comparison based on reasonably long series of observations. The material that will be used and the derived tidal constants are shown in Table 9.

The elevations in Table 9 refer to the United States Engineer Department datum in the Delaware River. The locations of the tidal stations are as follows: (1) Delaware City, on the west bank of the Delaware River, 1.8 miles above Reedy Point; (2) Reedy Island, on Reedy Island on the west bank of the main channel of the Delaware River, 3.3 miles below Reedy Point; (3) Reedy Point, just inside the outer end of the north jetty at the Delaware River end of the canal; (4) Biddle Point, on the north bank of the canal, 3.1 miles from Reedy Point; (5) Summit, on the bank of the canal at Summit Bridge, 9.2 miles from Reedy Point; (6) and (7) Chesapeake City, on the bank of Back Creek, at the entrance to the canal, 14.0 miles from Reedy Point; (8) Court House Point, on the southeast bank of Elk River, about 1 mile below the mouth of Back Creek and 5 miles from Chesapeake City; (9) Town Point, on the southeast bank of Elk River, about 1.5 miles below Court House Point; (10) Reybolds Wharf, on the southeast bank of Elk River, about 4.5 miles below Town Point; and (11) Frenchtown, on the southeast bank of Elk River, about 5 miles above Court House Point and about 4 miles above the mouth of Back Creek.

Before proceeding with comparisons between the observed and predicted tides in the canal proper, it will be well to look into conditions at the entrances before and after the canal was opened to tidal flow. No preliminary observations were made right at Reedy Point, the nearest being those at Delaware City, in 1923, and those at Reedy Island. Interpolating between these values, the resulting lunitidal intervals at Reedy Point are 11.10 hours for high water and 5.76 hours for low water. Comparing these values with those in Table 9, it appears possible that the duration of rising tide may have become somewhat greater than before, which is to be expected. The elevations of high and low water observed at Reedy Point are normal for this part of the Delaware River, but those at Delaware City in 1923 appear to be somewhat low.

---

[35] Engr., U. S. Engr. Office, Philadelphia, Pa.

TABLE 9.—TIDAL DATA IN CHESAPEAKE AND DELAWARE CANAL, AND VICINITY

No.	Station	Year	Series	Lunitidal Intervals, in Hours		Elevations, in Feet		Mean tide level, in feet	Range, in feet
				High water	Low water	High water	Low water		
1	Delaware City..............	1923	4 months	11.27	5.93	5.97	0.47	3.22	5.50
2	Reedy Island..............	1929	12 "	10.78	5.45	6.20	0.63	3.42	5.57
3	Reedy Point..............	1928-29	9 "	11.19	5.59	6.16	0.79	3.48	5.37
4	Biddle Point..............	1928-29	8 "	11.46	5.90	6.10	1.01	3.56	5.09
5	Summit,..............	1928-29	9 "	11.18	4.56	5.30	1.93	3.62	3.37
6	Chesapeake City..............	1923	5 "	9.28	3.32	4.47	2.15	3.31	2.32
7	Chesapeake City..............	1928-29	9 "	9.81	3.64	4.53	2.23	3.38	2.30
8	Court House Point..............	1929	7 "	9.04	3.09	4.32	2.10	3.21	2.22
9	Town Point..............	1898-1927	26 days	8.97	3.01	....	....	....	2.42
10	Reybolds Wharf..............	1898-1927	58 "	8.79	2.55	....	....	....	1.93
11	Frenchtown..............	1899	14 "	8.87	2.93	....	....	....	2.43

At Chesapeake City conditions are very different. High water occurs 0.53 hours, and low water 0.32 hours, later than formerly. There are three stations in Elk River for which tidal constants are published by the U. S. Coast and Geodetic Survey; namely, Town Point, Reybolds Wharf, and Frenchtown. At each of these places one or more short series of observations were made between 1898 and 1927. The means of all results for each station are given in Table 9. Comparing the lunitidal intervals from seven months' observations at Court House Point in 1929 with the intervals at the neighboring stations, it is apparent that there has been no material change in the times of tide in Elk River since the canal was opened. There appears to have been no material change in tidal heights in Back Creek since the opening of the canal.

In Table 10 are given in parallel columns the observed and predicted times and heights of tide at the four canal stations. Times are referred to high water at Reedy Point, and heights, to mean tide level at the same point, 3.48 ft. above the datum.

The predicted values are in fair agreement with the observed values, following them in a general way, and are satisfactory for a preliminary computation. It is believed, however, that the method developed by the author is capable of bringing out closer results. In computing the predicted values, it was necessary to make certain general assumptions as to the conditions that would prevail in the canal. An examination of the discrepancies brought out by a comparison of the observed and predicted results leads to the belief that they can be reduced materially by a revision of the preliminary assumptions.

TABLE 10.—COMPARISON OF OBSERVED AND PREDICTED TIDES IN CHESAPEAKE AND DELAWARE CANAL

Station	HIGH TIDE				LOW TIDE				DURATION, IN HOURS				Range, in feet		Mean tide level, in feet	
	Time, in hours		Height, in feet		Time, in hours		Height, in feet		Rise		Fall					
	Observed	Predicted	Observed	Predicted	Observed	Predicted	Observed	Predicted	Observed	Predicted	Observed	Predicted	Observed	Predicted	Observed	Predicted
Reedy Point	0.00	0.0	+2.68	+2.90	6.82	6.2	−2.69	−2.89	5.60	6.2	6.82	6.2	5.37	5.79	0.00	0.00
Biddle Point	0.27	0.0	+2.62	+2.97	7.13	6.2	−2.47	−2.26	5.56	6.2	6.86	6.2	5.09	5.23	+0.08	+0.35
Summit	−0.01	−0.5	+1.82	+1.75	5.79	5.5	−1.55	−1.12	6.62	6.4	5.80	6.0	3.37	2.87	+0.14	+0.31
Chesapeake City	−1.38	−1.9	+1.05	+1.18	4.87	4.2	−1.25	−1.15	6.17	6.3	6.25	6.1	2.30	2.33	−0.10	−0.02

The computations are based on a canal 14 miles long, the length of the excavated portion. The eastern end of the canal connects with the broad estuary of the Delaware River, and such changes in the tidal regimen as may have occurred there are naturally small. At the western end, the connection is with Back Creek, a small tributary of the Elk River. The distance from the Chesapeake City end of the canal through Back Creek to the waters of Elk River is about 4 miles. Back Creek has a width of about 350 ft. at Chesapeake City, gradually increasing to about 1 700 ft. at its mouth; it has a mean depth of about 7 ft. at mean tide level. These dimensions show that this waterway, especially in its upper portion, is of the same order of magnitude as the canal itself. The Elk River, however, is far wider and deeper, forming a body of water so large that it can be expected to maintain its tidal regimen without material change from the comparatively small affect of the canal waters. That this expectation is reasonable is confirmed by the discussion of the changes in tidal times in the vicinity, given previously.

It seems clear that better results can be secured by computing the predictions for a canal 18 miles long than for one of 14 miles, even though this introduces some complications, due to the modifications necessary to take care of the continually changing dimensions in the 4 miles of channel lying in Back Creek.

It is believed preferable to use the observed differences in time of high tide and low tide at the two ends of the canal separately rather than a round value for both. Judging from the lunitidal intervals at Chesapeake City in 1923, and those at Court House Point in 1929, the values at the mouth of Back Creek probably would have been approximately 9.1 hours for high water and 3.1 hours for low water. The probable values at Reedy Point, as shown previously, were 11.1 hours and 5.8 hours, respectively. The differences for the ends of the 18-mile canal are, therefore, 2.0 hours for high water and 2.7 hours for low water.

In the Delaware River, in the vicinity of Reedy Point, the duration of rise of tide is about 1.7 hours less than the duration of fall; but in Elk River, at Court House Point, the difference is only 0.5 hours. In the computed predictions the durations of rise and fall are assumed to be equal at Reedy Point.

For a comparison between observed and predicted velocities in the canal, there are five sets of simultaneous observations at each of the four canal stations. Each set extends over an entire tidal cycle. In Table 11 the mean values of the westerly and easterly velocities, observed and predicted, are shown in parallel columns for each of the canal stations.

The double predictions at Biddle Point (Items Nos. 2 and 3, Column (4)) refer to the sudden change in velocity due to the change in canal cross-section. The values in Item No. 2 pertain to currents between the station and Reedy Point and those in Item No. 3 to currents between the station and Summit.

The relationship of tides and currents in the canal is shown in Fig. 22. Fig. 22(*a*) exhibits the observed tides and currents; and Fig. 22(*b*), the predicted tides and currents.

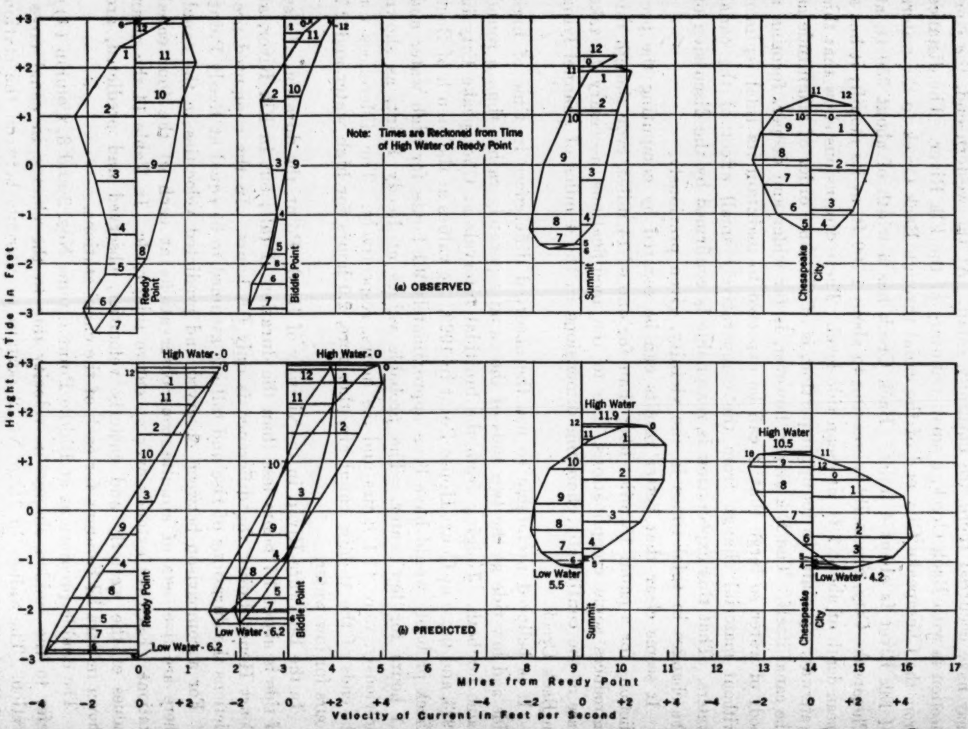


FIG. 22.—HEIGHTS OF TIDE AND VELOCITIES OF CURRENT FOR EACH HOUR (0 to 12), CHESAPEAKE AND DELAWARE CANAL.

The ordinates represent height of tide above and below mean tide level at Reedy Point, and abscissas, the corresponding velocities of current. The numbers on the abscissas indicate the hour after high water at Reedy Point. Plotted in this way, a characteristic curve is formed for each station, bringing out the relations between tidal heights and currents.

TABLE 11.—MEAN VALUES OF WESTERLY AND EASTERLY VELOCITIES

Item No. (1)	Station (2)	MEAN VELOCITY, IN FEET PER SECOND: Westerly, Observed (3)	Westerly, Predicted (4)	Easterly, Observed (5)	Easterly, Predicted (6)
1	Reedy Point.........	1.5	2.4	1.5	2.4
2	Biddle Point........	1.1	1.2	0.8	1.4
3	Biddle Point........	....	2.4	....	2.1
4	Summit..............	1.2	2.1	1.3	1.4
5	Chesapeake City....	1.9	2.4	1.6	1.7

Comparison of the corresponding diagrams, Fig. 22(*a*) and Fig. 22(*b*), shows how closely the type of tide at each station is brought out by the predictions. The most striking divergence is exhibited at Reedy Point, and is due to local conditions in the canal. Beginning about Mile 1 and extending somewhat beyond Biddle Point, the canal passes through wide marshes, forming basins, with bottom elevations ranging upward from about 1 ft. below half-tide level. These are filled and emptied at each tide through the Delaware River entrance, and the resulting hydraulic currents superimposed on the normal tidal currents are the principal cause of the unusual form of the upper part of the resultant curve at Reedy Point and the somewhat similar appearance of the curve at Biddle Point.

RALPH G. DAVIS,[38] M. AM. SOC. C. E. (by letter).—In this paper, the author has expressed regret that sufficient observations were not available for a close comparison between the theory and the observed tides and currents in the Chesapeake and Delaware Canal. Some observations have been made since those to which the author refers and it is the purpose of this discussion to compare the observations with the results computed by the theory.

Colonel Brown's results are not used for the comparison, as it was found that the canal cross-sections at the time (1928) the observations were made were not uniform and, in some places, gave appreciably different working data. Therefore, in order that a close comparison might be made, the theory has been used and extended to fit the conditions at the time of the observations, and new values have been derived. Additional work has been done on the canal since the preliminary observations (May 13, 1927), which affects the tidal regimen materially; so it seems desirable to include a brief description of the canal as it was at the time the later observations were made.

---

[38] Engr., Beach Erosion Board, Office of Chf. of Engrs., U. S. Army, Washington, D. C.

The channel was dredged with a horizontal bottom to a depth of 12 ft. below mean low water, Delaware River datum. It is 150 ft. wide, from the 12-ft. curve in the Delaware River to a point about 500 ft. west of Reedy Point Bridge (see Fig. 1); 90 ft. wide to the lift bridge at Chesapeake City; and 150 ft. wide in Back Creek, to the 12-ft. curve in Elk River. All side slopes were dredged to 1 ft. vertical on 2 ft. horizontal. As the cut was made through low-lying marsh areas on the eastern end, a high divide near the middle, and in the bed of Back Creek, which varies from 350 to 1 400 ft. in width, the different classes of materials excavated resulted in variations in the depth and width of the completed channel. In some places, considerable overcutting was done to provide for the slides and bulging of the bottom.

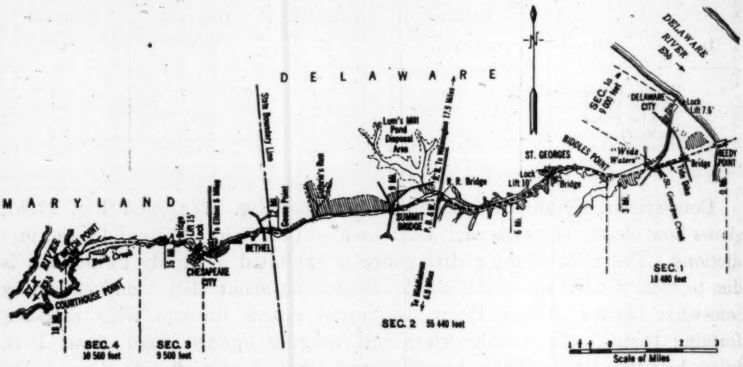

DELAWARE RIVER
Ebb
DELAWARE
MARYLAND
SEC. 1a 9 000 feet
DELAWARE CITY
Lock Lift 7.6'
State Boundary Line
Lum's Mill Pond Disposal Area
To Wilmington 17.9 Miles
ST. GEORGES
BIDDLES POINT
"Wide Waters"
REEDY POINT
Bridge
Tide Dale
St. Georges Creek
Lock Lift 10'
Bridge
R. R. Bridge
P. R. & W. R.
SUMMIT BRIDGE
9 Mi.
6 Mi.
3 Mi.
0 Mi.
Goose Point
BETHEL
To Elkton 6 Miles
Lock Lift 15'
Bridge
CHESAPEAKE CITY
15 Mi.
Back Creek
ELK RIVER
COURTHOUSE POINT
18 Mi.
To Middletown 6.5 Miles
SEC. 1 18 480 feet
SEC. 2 55 440 feet
SEC. 3 9 500 feet
SEC. 4 10 560 feet
0 1 2 3
Scale of Miles

FIG. 23.—CHESAPEAKE AND DELAWARE CANAL, SHOWING COMPUTING SECTIONS

Subsequent operations included the dredging of the Delaware City Branch Channel, 6 ft. deep at mean low water and 50 ft. wide on the bottom, from the Delaware River, at Delaware City, to the 12-ft. canal, a distance of about 1¾ miles. The point of junction of the two channels is about 2 miles west of the outer ends of Reedy Point jetties. The Branch Channel was constructed before the current observations were made and preliminary computations indicated that this "by-pass" has a marked influence on the tides and currents throughout the greater part of the canal. The 150-ft. channel at the western end is a component part of the sea-level canal and is affected, to some extent, by the Delaware River tides propagated through the canal. Therefore, this section has been included in the computation for the canal proper.

For the writer's computations, the canal was divided into four sections, exclusive of the Branch Channel, as shown by Fig. 23. Section 1, about 3½ miles in length, includes the 150-ft. channel at the eastern end and the wide waters through the low marshy area. In this section, it was found that the average width at high water is about twice the low-water width, and in the computations, the two widths were handled separately for the positive and negative waves. Section 2 extends from the 3½-mile point through the deep

TABLE 12.—DIMENSIONS OF CANAL FOR TIDE AND CURRENT COMPUTATIONS.

Section (see Fig. 23).	1(*a*)	(1)		(2)	(3)	(4)
Canal Stations (thousand feet):						
From	....	3+600		22+080	77+520	87+020
To	....	22+080		77+520	87+020	97+530
Distance from Reedy Point tide gauge to western end of section, in feet	10 500	18 480		73 920	83 420	93 980
Length of sections, in feet	9 000	18 480		55 440	9 500	10 560
		Negative wave.	Positive wave.			
Average cross-sectional areas, in square feet	743	2 032	4 323	2 446	3 705	6 932
Average width, in feet	122	240	461	185	450	1 130
Reduced depth, in feet	6.09	11.22	6.63	13.22	8.23	6.13
Average wetted perimeter, in feet	128	245	468	192	455	1 134

cut section of the canal to Chesapeake City and is about 10½ miles long. Back Creek was divided into two sections, according to the average widths. Section 3 is about 1⅞ miles long and Section 4, 2 miles in length. The Delaware City Branch Channel, Section 1(*a*), has a reduced depth nearly equal to that for the positive wave of Section 1, and a little greater than one-half the reduced depth for the negative wave, which affects the tides in the main 12-ft. canal considerably. The values for the various sections, as used in the computations, are shown in Table 12.

The relation between the results of the computations and the observed tides and currents is shown graphically on Figs. 24 to 28, inclusive. The observed tide curves (solid lines) represent the average for a long series of tide readings at the five stations indicated. The current curves are averages

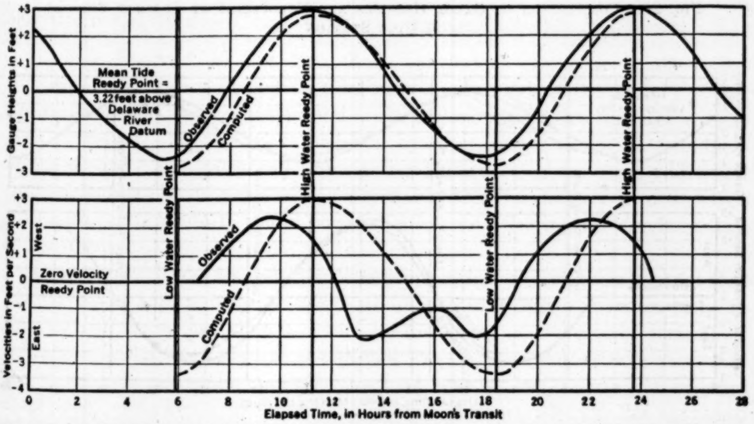


FIG. 24.—COMPARISON OF GAUGE HEIGHTS AND VELOCITIES, OUTER END OF JETTIES, REEDY'S POINT, STATION 0, MILES

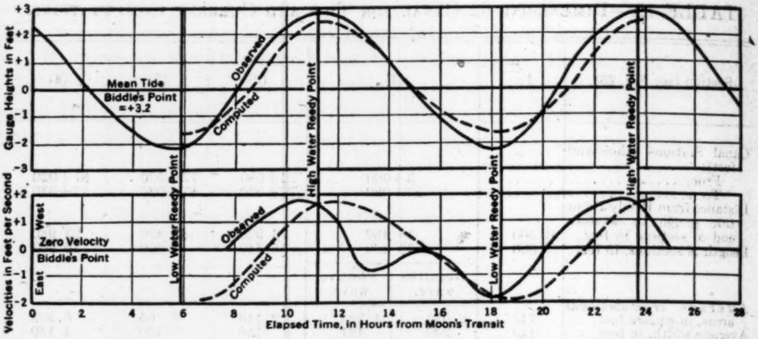


FIG. 25.—COMPARISON OF GAUGE HEIGHTS AND VELOCITIES AT BIDDLES POINT; 3-MILE STATION

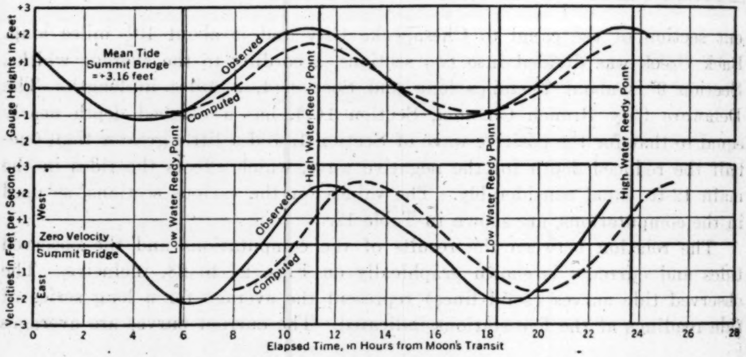


FIG. 26.—COMPARISON OF GAUGE HEIGHTS AND VELOCITIES AT SUMMIT BRIDGE; 9½-MILE STATION

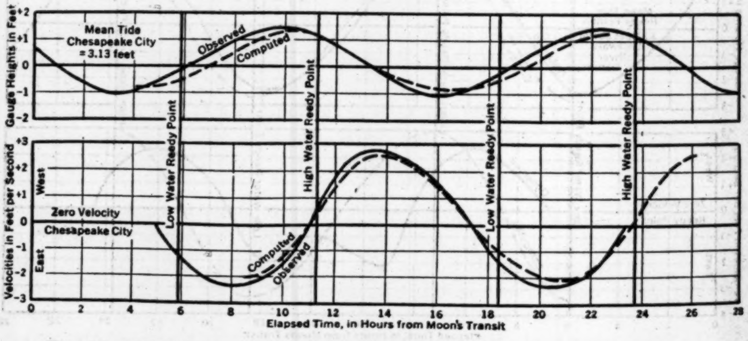


FIG. 27.—COMPARISON OF GAUGE HEIGHTS AND VELOCITIES AT CHESAPEAKE CITY; 14-MILE STATION

for five full tide cycle observations by current meters held in the center of the channel opposite the tide stations and at the six-tenths depth through the entire tide.

Theoretical values agree fairly well for the intermediate stations, the greatest difference occurring in the velocities at the ends. At Reedy Point and Biddle's Point Stations, at the eastern end of the canal, the observed negative velocity curves are somewhat irregular (see Figs. 24 and 25). These irregularities are apparently due to hydraulic phenomena producing alternate siphoning effects at the junction of the two channels. The divergence at the western end is probably due to the difference in the sectional area of Back Creek gauging station and the average area for Section 4, as used in the computations. The comparison demonstrates that the theory is in close agreement with the observations and that it is a practical method for predetermining the tidal heights and velocities in a tidal canal designed to connect two bodies of water, each having an independent tide of its own.

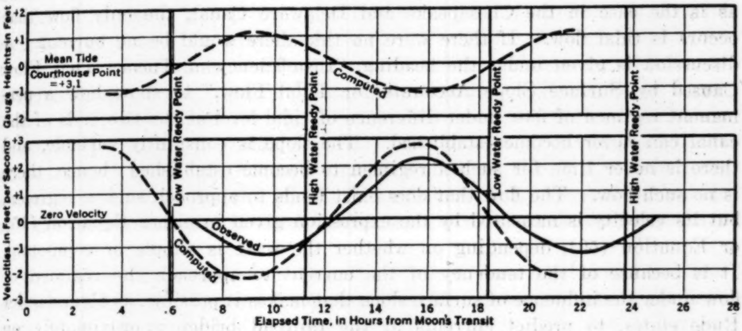


FIG. 28.—COMPARISON OF GAUGE HEIGHTS AND VELOCITIES AT COURTHOUSE POINT (ELK RIVER); 18-MILE STATION

EARL I. BROWN,[27] M. AM. SOC. C. E. (by letter).—In general, the reception which the theory of flow in tidal canals, advanced in the writer's paper, has received, has been quite cordial. No serious differences of opinion have been expressed, and no serious question has been raised as to the soundness of the theory. Mr. Faris and Commander Rude have given some additional information to clarify parts of the paper, but those parts were in connection with statements made to round out the paper, not affecting the fundamentals of the theory.

Commander Rude suggests that the propagation of a wave in a canal that has an outlet into a body of water would continue into the outlet body in its original form, and that it would be dissipated in that body instead of being propagated backward as a reflected wave. It is the writer's view that the wave does continue out into the outlet body, but not in its original amplitude. The amplitude must change in accordance with the law expressed by Equation (37), in which, if l_1 becomes very large or infinite, h_1, the amplitude at the outlet, must be very small or zero. This, of course, is because the canal wave does not possess enough energy to raise a wave in a wider expanse of water to an equal height.

---

[27] Col., Corps of Engrs., U. S. A., U. S. Engr. Office, Philadelphia, Pa.

Commander Rude also declares his understanding that "the hydraulic characteristics of the canal were not taken into consideration by the writer." Apparently, he means that no account is taken of flow under the influence of surface slope. When the two ends of the canal have the same mean level, as is the case in the Chesapeake and Delaware Canal, the only flow that occurs is tidal flow. If there were no tide there would be no current. A discussion is given under the heading, "Insufficiency of Theory That Flow Caused by Surface Slope Accounts for Tidal Flow," to show how a permanent regimen of flow under difference of tidal level at the two ends of the canal can never become established. The slope is constantly variable, and there is never time for such a regimen to become established; hence, there is no such flow. The flow that does exist tends to approach such a regimen, but its velocity is measured by the expression given in either Equation (15) or Equation (50), depending on whether the wave is simple or compound. It is because of the tendency of the current to approach the regimen of flow under the influence of surface slope that makes it possible, as Commander Rude states, to predict currents at the railroad bridge approximately on that assumption. The fact that this can be done at the railroad bridge is a fortuitous chance, however, as it does not follow by any means that it could be done elsewhere in the canal.

Commander Rude expresses a doubt as to whether the writer accepts the "'stationary wave theory' of Harris as an explanation of the changed regimen of the tide in certain bodies of water in which the tide occurs almost simultaneously throughout the entire length of the body * * *."

The "stationary wave" of Harris is nothing more than a resultant wave formed by the interference of two equal and opposed waves (see heading, "Theory of the Interference of Two Opposed Waves"). An illustration of it occurs when a simple wave of translation is propagated into a canal of fairly uniform cross-section, closed at one end, and of length just equal to one-half the wave length. (The positive and negative phases of the ordinary tidal wave are considered as separate waves in this theory.) The canal will then have a stationary wave with a regimen exactly as Harris defines it. Such a wave is known to exist in certain bays such as Long Island Sound, where the western end is almost closed, and in certain seas, where the depth and superficial dimensions bear the proper relationship to the period of the tide for reflections from its shores to interfere with the oncoming wave in the proper phase relationship to create the stationary wave. Harris' theory is, therefore, a special case of the more general theory of reflected waves in interference.

Replying to Commander Rude's inquiry as to Tables 3, 4, and 5, it may be stated that the data in the columns headed, "Bourdelles," are based on observation. Those in columns headed, "Levy," are values computed by Levy according to his formulas.

In commenting on the differences between the Levy formulas as presented by Mr. Parsons, and those of the writer, Mr. Halmos states:

> "The derivation of the [Parsons'] formulas is strictly rational, their application being restricted only by the requirement of comparatively small tidal variations in relation to the depth of the canal."

This restriction, by the requirement of comparatively small tidal variations in relation to the depth of the canal, is sufficient to exclude these formulas from use in most artificial tidal canals constructed for navigation, in which the depth will probably seldom exceed 35 ft., and in which tidal variations of 4 to 5 ft. must be expected. Variations of these magnitudes cannot be considered as small with respect to the depth. In addition to this defect, the writer urges another. The expressions, P and Q, in Equations (2) of the paper[33] by Mr. Parsons, are assumed to be logarithmic functions of x in the formulas, $P = AC^{ax}$, and $Q = BC^{ax}$. It is the writer's opinion that P and Q are not only functions of x, but also of h, which is the quantity sought to be determined. It is the failure of the formulas to include h in the values of P and Q which makes them give equal values for positive and negative waves.

Mr. Halmos suggests that the formulas of the French Academy of Sciences recommended by Mr. Parsons for canals of certain characteristics might have been useful in predicting approximate values of heights and velocities. Their use was rejected by the writer because they are based on the assumptions of a uniform slope of the water surface from end to end of the canal and of the velocity varying with that surface slope, both of which he holds to be erroneous.

The writer admits the difficulty apprehended by Mr. Halmos in finding a proper value of F by the proposed method. Its value for any proposed canal can be estimated with confidence only when it has actually been determined experimentally from existing similar canals.

General Black points out how the useful range of the theory may be extended to the study of the effects of changes projected in the cross-section of tidal canals, and cites the control of current velocities in the East River, at New York City, as an example. The writer may venture further and call attention to the way in which control of these current velocities might be obtained directly by the use of this theory.

The tide in Long Island Sound is of the stationary type due to the reflection of a wave back on itself from the nearly closed western end. The height of the tide increases at a fairly uniform rate from east to west. The depth of the Sound and its length have the required relationship with the tidal period to make the tidal phenomena almost a perfect example of the stationary type of tide. If this relationship were destroyed, the excessive amplitude at the point of reflection near Throgs Neck would be lessened, and this, in turn, would reduce the East River currents. The relationship could be destroyed by making one or more artificial contractions of Long Island Sound at proper intervals in the distance from Fishers Island to

---

[33] "The Cape Cod Canal," by William Barclay Parsons, M. Am. Soc. C. E., *Transactions*, Am. Soc. C. E., Vol. LXXXII (1918), p. 111.

Throgs Neck, breaking up the wave by reflections having a less favorable relationship to cause high amplitudes. A thorough study of this problem by the aid of this theory, it is believed, could lead to a solution giving any desired control of the East River currents.

General Black's hope that similar studies will be made of the tidal action in estuaries such as the Hudson and Delaware Rivers will be realized in the case of the Delaware River. This study is now (1931) in progress.

The readiness with which the effect of changed conditions in the canal are reflected in the results of computations is indicated in the curves given by Messrs. Flynn and Davis. Mr. Flynn showed that a material reduction of discrepancies between observed and computed results could probably be made by a revision of preliminary assumptions to conform more nearly with actual conditions. Mr. Davis has recomputed predicted results under revised assumptions corresponding to actual conditions. The close agreement of his computed and observed results is very gratifying.

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
TRANSACTIONS

Paper No. 1806

THE DON MARTIN PROJECT[1]

BY ANDREW WEISS,[2] M. AM. SOC. C. E.

WITH DISCUSSION BY MESSRS. C. H. HOWELL, ROBERT S. STOCKTON, AND ANDREW WEISS.

SYNOPSIS

On January 4, 1926, President Plutarco Elias Calles approved the existing Mexican Federal Irrigation Law. It provides in essence that the irrigation of private agricultural lands of whatever kind or extent of cultivation be declared a public interest, if subject to irrigation from Federal sources of water supply. Furthermore, it creates a National Commission of Irrigation composed of three members, to put into effect the studies of irrigation projects, the execution of all plans related thereto, and the administration of these within the provisions of this Act.

This paper gives a general description of the Don Martin Project, one of the largest so far undertaken by the Mexican Government under the provisions of this Act, including therein also a brief outline of the construction methods as well as of the settlement plans in mind and developed to date.

GENERAL FEATURES OF THE PROJECT

Location.—The Don Martin Project is on the Salado River in the States of Coahuila and Nuevo León. The Town of Rodriguez, about 45 miles southwest of Laredo, Tex. (see Fig. 1), is near the center of its irrigable area which embraces a total of 65 000 hectares (160 000 acres).

Irrigation Plan.—The storage feature of this system consists of a reservoir 100 km. (60 miles) directly west of Laredo; its normal capacity is 138 600 hectare-m. (1 123 600 acre-ft.) covering an area of 19 600 hectares (48 412 acres) when filled.

The dam affecting this storage is 66 km. (41 miles) up stream from Rodriguez and an equal distance west of Camaron at a place where a low range of hills known as "Lomerias de Caracol" crosses the Salado River in a general north-northeast and south-southwest direction. This dam will also

---

[1] Presented before the Texas Section of the Society at the Annual Meeting, Laredo, Tex., November 14, 1928.

[2] Res. Engr., J. G. White Eng. Corp., Estacion La Cruz, Chihuahua, Mexico.

serve to divert the water impounded by it into the main canal of the system. After leaving the head-works this canal diverges slightly northward from the river for a distance of 42 km. (26 miles) from which point one of its principal branches, the Camaron Lateral, extends in a general easterly direction to cover the first unit to be irrigated in the vicinity of Camaron and Rodriguez. From this point also the main canal turns almost directly south a distance of about 120 km. (75 miles), and about 5 km. (3 miles) up stream from the crossing of the Laredo-Monterrey Highway with the Salado River. (See Fig. 2.) The area to be irrigated covers 30 000 hectares north of the river and 35 000 hectares south of it, or a total of 65 000 hectares (160 000 acres).

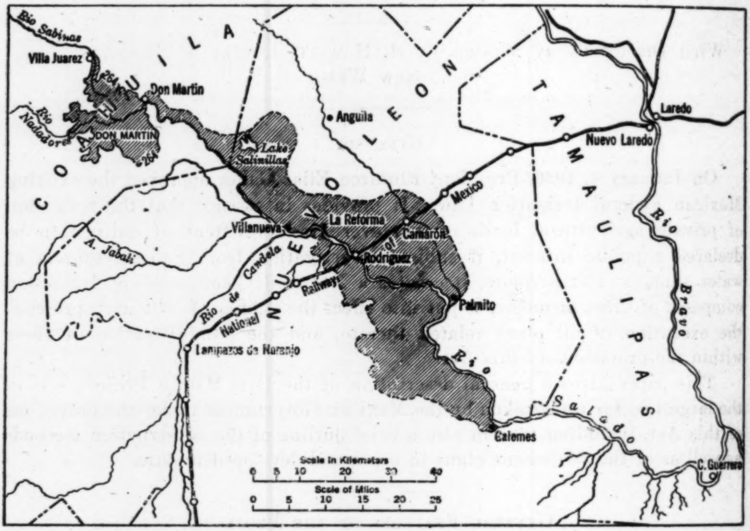

Rio Sabinas
Villa Juarez
264
Don Martin
Rio Nadadores
DON MARTIN
264
COAHUILA
NUEVO LEON
TAMAULIPAS
Anguila
Lake Salinillas
Laredo
Nuevo Laredo
Mexico
La Reforma
Villanueva
Camaron
Rodriguez
A. Jabali
Rio de Candela
National Railways of Mexico
Palmito
Lampazos de Naranjo
Rio Bravo
Rio Salado
Calemes
C. Guerrero
Scale in Kilometers
0 5 10 20 30 40
Scale of Miles
0 5 10 15 20 25

FIG. 1.—OUTLINE PLAN SHOWING THE DON MARTIN PROJECT.

In its original concept, the project embraced a much more compact area of which the Town of Camaron formed an approximate central point. Detailed soil investigations revealed the presence of extended tracts of land underlaid with a gypsum subsoil, or a soil which at a depth of about a meter disclosed undesirable strata of crystallized gypsum mixed with highly compacted clays forming a very impermeable structure. For this reason the soil experts selected the younger alluvial soils nearer the river and generally free from these gypsum bottoms. This choice of the major body of the project lands to the south of the river, although resulting in an appreciably higher estimated cost, seems to have been a wise move.

***Water Supply.*—The water supply for this project is almost wholly dependent upon the storage of flood waters of the Salado River and its upper tributaries, principal of which are the Sabinas and the Nadadores, which unite to form the Salado about 8 km. (5 miles) above the dam. The catchment basin**

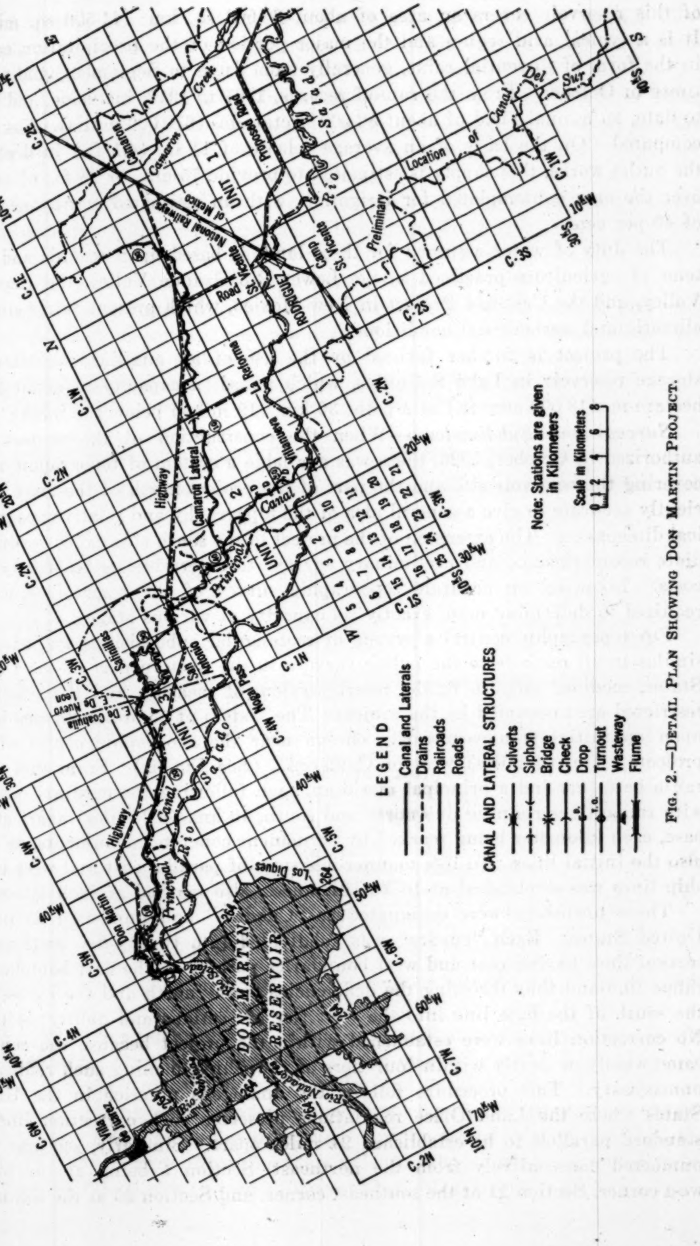

DON MARTIN RESERVOIR
Villa de Juarez
Rio Sabinas
Rio Salado
Rio Nadadores
Los Diques
264
Don Martin
Highway
Principal Canal
Rio Salado
UNIT 3
UNIT 2
UNIT 1
Salinillas
E. De Coahuila
E. De Nuevo Leon
San Antonio
Nogal Pass
Camaron Lateral
La Reforma
Villanueva
Road St. Vicente
St. Vicente
RODRIGUEZ
Camp Vicente
National Railways of Mexico
Proposed Road
Camaron Creek
Preliminary Location of Canal Del Sur
N
E
W
S
C-1N
C-2N
C-3N
C-4N
C-5N
C-6N
C-1E
C-2E
C-1S
C-2S
C-3S
C-1W
C-2W
C-3W
C-4W
C-5W
C-6W
C-7W
10KmW
20KmW
30KmW
40KmW
50KmW
60KmW
70KmW
20KmE
10KmS
20KmS
30KmS
40KmS
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25
LEGEND
Canal and Laterals
Drains
Railroads
Roads
CANAL AND LATERAL STRUCTURES
Culverts
Siphon
Bridge
Check
Drop
Turnout
Wasteway
Flume
Note: Stations are given in Kilometers
Scale in Kilometers
0 1 2 4 6 8

FIG. 2.—DETAIL PLAN SHOWING DON MARTIN PROJECT.

of this reservoir covers an area of about 30 000 sq. km. (11 500 sq. miles). It is a typical arid region and the major portion of the precipitation comes in the form of torrential rains, generally from June to September, and sometimes in October. From the run-off records, 1901 to 1912, inclusive, and 1923 to date, an annual yield of about 82 000 hectare-m. (670 000 acre-ft.) has been computed. On the basis of an average release of 1¼ m. (4.1 ft.) in depth at the outlet works, this supply is estimated to provide 75 cm., or 2½ ft., of water, over the area contemplated for irrigation, with an estimated conveyance loss of 40 per cent.

The duty of water assumed for these lands is based upon records and systems of agriculture practiced in the Lower Rio Grande Valley, the Imperial Valley, and the Carlsbad Project in New Mexico, which present fairly similar climatic and agricultural conditions.

The project is further favored by the use of an auxiliary equalization storage reservoir in Lake Salinillas, which affords a capacity of about 1 600 hectare-m. (13 000 acre-ft.) at a point 30 km. (19 miles) below the intake.

Surveys and Subdivisions.—When the construction of the project was authorized in October, 1926, there was available a system of topographic maps covering the reservoir site and the part of the project north of the river, sufficiently accurate to give a general idea of the topography and relative geographical dimensions. The extension of the project to the south side called for immediate reconnoissance and preliminary surveys to determine its feasibility and costs. Likewise, an accurate topographic survey of the reservoir site was required to determine more exactly its capacity at various stages.

For topographic control a system of rectangular subdivisions was adopted, similar in all respects to the public surveys in the western part of the United States, modified only to fit the metric system of measure and the relatively restricted area occupied by the project. The system of co-ordinates was based upon an initial point arbitrarily chosen near the geographic center of the project, 8 km. (5 miles) west of Camaron. (See Fig. 2.) From this point ran a base line and a principal meridian, each following the cardinal courses, with range lines running due north and south, 10 km. (6.2 miles) apart at the base, each kilometer being marked by permanent concrete monuments, as were also the initial lines. In like manner a system of parallel east and west township lines was established at 10-km. distances, also marked at each kilometer.

These townships were designated as "Cuadras" and numbered as in the United States. Each "cuadra" was subdivided into twenty-five sections, by section lines having east and west boundaries parallel to the east boundary or range line and thus throwing the deficiencies to the north and the excesses to the south of the base line into the west tier of sections and quarter-sections. No correction lines were established within the project because the confines came wholly or nearly within four tiers of townships, making such refinement unnecessary. This procedure follows the analogous practice in the United States where the Land Office regulations prescribe such correction lines or standard parallels to be established 24 miles apart. Similarly, sections were numbered consecutively from the northeast, Section 5 being at the northwest corner, Section 21 at the southeast corner, and Section 25 at the southwest corner of each cuadra. All interior section lines were marked at each kilometer both as to location and elevation, thereby forming an excellent control for the topography.

This system of subdivision forms also the skeleton upon which is being based the farm unit subdivision, the location of minor distributaries, and the network of secondary roads which will give an outlet to the principal, or market, roads where the topography permits.

In addition, the location of farm units will be greatly facilitated by alphabetical lettering within each section. Thus, Farm Unit A (Cuadra * * *) is in each case the northeast parcel or subdivision within the section, thus distinguishing it from any other parcel within the entire project.

Roads.—The Commission has planned a system of public roads to give easy access to each farm unit. These are to be of such ample width that ultimately a line of shade trees may border each side, to enhance the beauty of the landscape, as was done on the Salt River Project, in Arizona, for example.

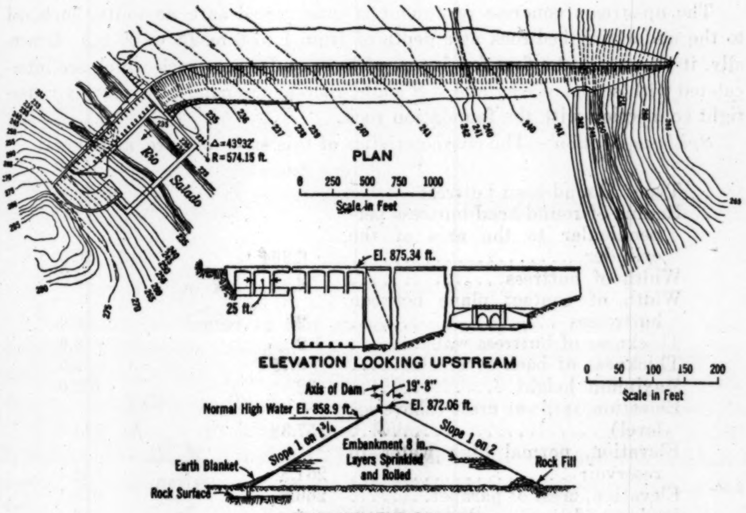

Rio Salado
Δ=43°32′
R=574.15 ft.
PLAN
0 250 500 750 1000
Scale in Feet
El. 875.34 ft.
25 ft.
ELEVATION LOOKING UPSTREAM
0 50 100 150 200
Scale in Feet
Axis of Dam
19′-8″
Normal High Water El. 858.9 ft.
El. 872.06 ft.
Slope 1 on 1¾
Slope 1 on 2
Embankment 8 in. Layers Sprinkled and Rolled
Earth Blanket
Rock Fill
Rock Surface
EMBANKMENT SECTION

FIG. 3.—PLAN OF DAM ACROSS RIO SALADO.

Right of Way.—The entire project is privately owned land. The reservoir site and a large part of the north side is held or controlled by the Bank of Milmo. The remainder is owned by perhaps less than forty persons or associations. Generally speaking, these lands were originally subdivided in lots of about 4 to 8 km. (2½ to 5 miles) in width and 30 km. (18¾ miles), or more, in length, marked by existing fence lines supplemented occasionally by corner monuments. Adjustment has been made for the lands comprised within the first unit. The manner of making these adjustments is further explained under the heading, "Settlement of and Repayment for Lands."

General Description of the Works

Rio Salado Dam—Earth and Gravel Fill.—This dam is a composite structure consisting of an earth and gravel embankment 985 m. (3 231 ft.) in length and a concrete overflow section 234 m. (768 ft.) in length. (See Fig. 3.) Following are the characteristics of the embankment:

	Meters.	Feet.
Length	985	3 231
Greatest height	34.8	114
Up-stream slope	1¾:1	1¾:1
Down-stream slope	2:1	2:1
Width of crown	6	19.7
Free-board to crown of embankment	4	13.1
Free-board to top of parapet wall	5	16.4
Revetment: Reinforced concrete slab varying in thickness from 11 in. at the base to 8 in. at the crown, with ¾-in. round bars, 12 in. center to center, horizontally and vertically.		

The up-stream concrete revetment is constructed as a monolith, anchored to the underlying bed-rock at a depth of from 1 to 6 m. (3 to 20 ft.) Generally, it penetrates the first few layers of loose and seamy rock which are intercalated with seams of clay and soft shale, thereby assuring a reasonably water-tight connection with the foundation rock.

Spillway Section.—The characteristics of this structure are, as follows:

	Meters.	Feet.
Type: Round-head buttress.		
Radius of round-head buttress perpendicular to the axis of the cylinder	6.364	20.9
Width of buttress	9	29.5
Width of contact plane between buttresses	2	6.6
Thickness of buttress walls	2	6.6
Thickness of back-cover slab	0.75	2.5
Maximum height	32	105.0
Elevation, spillway crest (above sea level)	257.38	844.2
Elevation, normal high water in reservoir	261.8	858.7
Elevation, crest of parapet	266.8	875.1
Bridge, width over spillway (clear)	5	16.4
Height of radial gates	4.42	14.5
Width of radial gates	7.62	25.0
Number of radial gates	26	26
Computed discharge capacity with surface elevation of water at 263.8 m. (865.3 ft.)	6 000 cu. m. per sec.	210 000 sec-ft.

The spillway is controlled by a system of twenty-two automatic radial gates, 25 by 14.5 ft., supplemented with four additional gates operated by direct mechanical lift. The water admitted through the 6-in. entrance at the bottom of the float-well exerts a buoyant force on the suspended float sufficient to bring about an overbalancing of the gate frame by action of the suspended counterweight attached to the lever arm which projects 16 ft. beyond the gate axis, as shown in Fig. 4. The admission of the water which effects the filling of the float-well is automatic when the level of the reservoir rises above an elevation of 261.8 m. (858.7 ft.), and, upon cessation of inflow, the float-wells are automatically drained by a 3-in. pipe or orifice at the bottom. The admission of the water may also be regulated at will by a small shutter at the entrance of the 6-in. pipe at spillway level, if for any reason it should be desired to lower any or all of the gates before the water has receded to the normal high-water level, 261.8 m. (858.9 ft.).

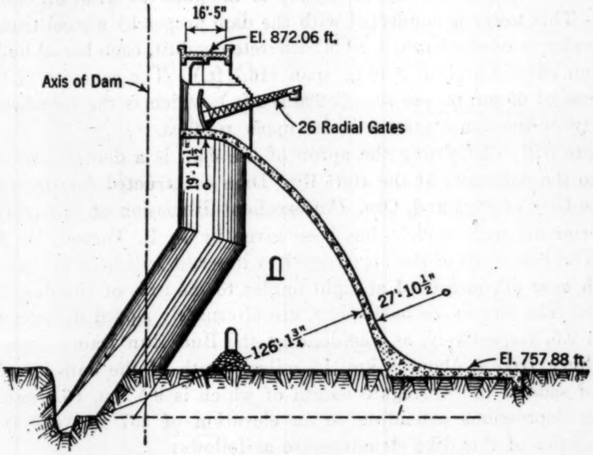


FIG. 4.—SECTION THROUGH SPILLWAY.

The principle adopted in this automatic control was independently conceived by the designing force of the J. G. White Engineering Corporation. A similar device has been used for some time in the Turlock Irrigation District, the Imperial Valley, in California, and elsewhere, with complete satisfaction.

The type of spillway here adopted follows in general a suggestion given[8] by Fred A. Noetzli, M. Am. Soc. C. E., with modifications to suit local requirements. Its distinctive features are the round-head buttresses conveying the water pressure to the counterforts by radial compression, thus avoiding bending moments and flexural stresses to be resisted by reinforcement. These were considered objectionable because of the highly corrosive action on steel of the waters in this stream. Furthermore, this type of independent units is peculiarly adapted to the local foundation conditions, each of the counterforts being deeply embedded into the limestone strata by a system of channeling, thus affording a maximum attainable sliding resistance within acceptable limits of cost.

Sluice-Gates.—Provision is also made for the complete drainage of the reservoir when desired through four 3.2 by 4-ft. rectangular sluice-gates, operated by geared, hand-lifting devices of suitable design in the interior of the dam at an elevation of 241.26 m. above the floor of the down-stream apron. The gate-sill is at an elevation of 232.1 m., which is somewhat below the ordinary level of the water surface in the river at this point. This gate is shown in Fig. 5.

---

[8] "The Design and Construction of Dams," by Edward Wegmann, M. Am. Soc. C. E., pp. 524-527.

Outlet Works.—Water may be released from the reservoir through two batteries of eight vertical, rectangular, 3.5 by 10-ft. sliding gates, each battery containing four regulating and four emergency gates. (See Fig. 6.) The gate stems extend vertically to mechanical lifting devices operated by electric motors located in a gate-house at the top of the gate tower, at an elevation of 265.8 m. This tower is connected with the dam proper by a steel truss bridge.

The water is released into a 2-bbl. concrete conduit, each barrel built in the form of an elliptic arch of 5.02-m. span (16.7 ft.). The capacity of the gates is in excess of 65 cu. m. per sec. (2 296 sec-ft.), which is the calculated maximum duty of the canal system in its upper reaches.

Dentate Sill.—Adjoining the apron of the dam is a dentate sill of a type similar to the deflectors at the Bull Run Dam constructed for the water supply of the City of Portland, Ore. An excellent discussion of this type, including experiments with models, has been given by B. E. Torpen, M. Am. Soc. C. E.[4] The height from the apron level to its crest will be 2 m. (6½ ft.) and its length over all, measured at right angles to the axis of the dam, 13.22 m. (43.4 ft.). The dentils, or baffle-piers, are alternately sloped up stream 1 on 5 and 1 on 2.5, respectively, as was done on the Bull Run Dam. (See Fig. 7.)

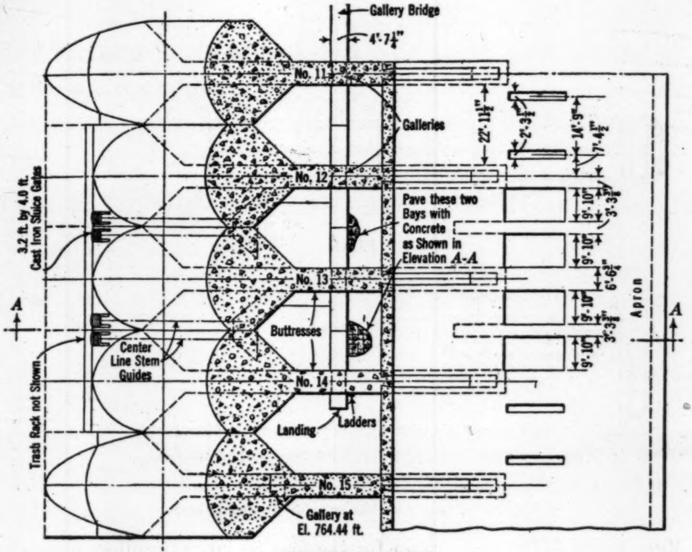
Auxiliary Dike.—About 8 km. (5 miles) south of the dam is a series of connected saddles the aggregate length of which is 9.5 km. (5.9 miles), with minimum depressions extending to an elevation of 257 m. (843 ft.). The characteristics of this dike structure are as follows:

	Meters.	Feet.
Maximum height	8.8	28.9
Up-stream slope	1¾:1	1¾:1
Down-stream slope	2:1	2:1
Width of crown....................	6.	19.7
Revetment: Mamposteria stone masonry of lime mortar 0.26 m. (10 in.) in thickness, connected at its footing with an up-stream cut-off wall extending to a variable depth, generally 0.8 H (H representing the height of the dike, in meters).		
Height, parapet wall above crown...	1.	3.28
Free-board to top of parapet wall....	5	16
Quantity of earth-fill..............	651 540 cu. m.	852 195 cu. yd.
Quantity of intermediate cut-off trench excavation	38 720 cu. m.	50 645 cu. yd.

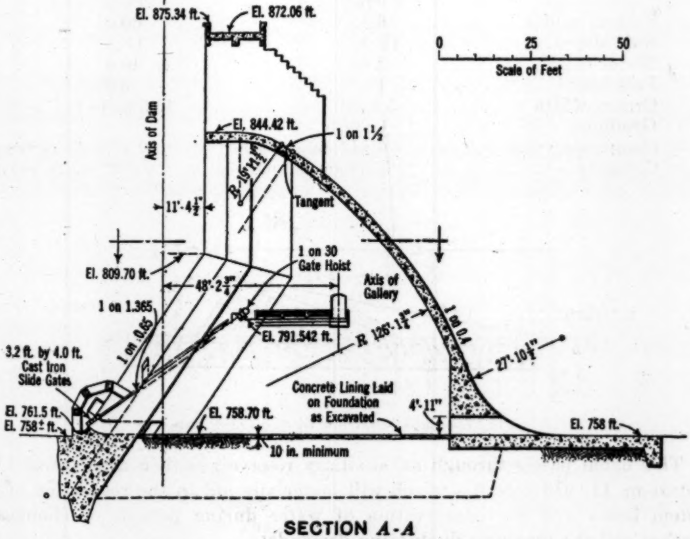
The body of this dike is composed of the earth material from shallow borrow-pits near-by and down stream. Because the ground here is underlaid at shallow depths with a gypsum stratum of undesirable composition for this purpose, it has been necessary to limit the depth of the borrow-pits.

---

[4] *Transactions*, Am. Soc. C. E., Vol. 93 (1929), pp. 487–496.


Gallery Bridge
4'-7¼"
No. 11
No. 12
No. 13
No. 14
No. 15
Galleries
22'-11½"
2'-3¾"
14'-9"
7'-4½"
9'-10"
3'-3⅞"
9'-10"
6'-6¾"
9'-10"
3'-3⅞"
9'-10"
Pave these two Bays with Concrete as Shown in Elevation A-A
Apron
A
A
3.2 ft. by 4.0 ft. Cast Iron Sluice Gates
Center Line Stem Guides
Buttresses
Trash Rack not Shown
Landing
Ladders
Gallery at El. 764.44 ft.

GENERAL PLAN


El. 875.34 ft.
El. 872.06 ft.
0
25
50
Scale of Feet
Axis of Dam
El. 844.42 ft.
1 on 1½
R. 15'-11"
Tangent
11'-4½"
El. 809.70 ft.
1 on 30
Gate Hoist
48'-2¾"
Axis of Gallery
1 on 1.365
1 on 0.65
El. 791.542 ft.
R 126'-1⅛"
1 on 0.4
27'-10¼"
3.2 ft. by 4.0 ft. Cast Iron Slide Gates
Concrete Lining Laid on Foundation as Excavated
El. 761.5 ft.
El. 758± ft.
El. 758.70 ft.
4'-11"
El. 758 ft.
10 in. minimum

SECTION *A-A*

FIG. 5.—PLAN AND SECTION OF SLUICE-GATES.

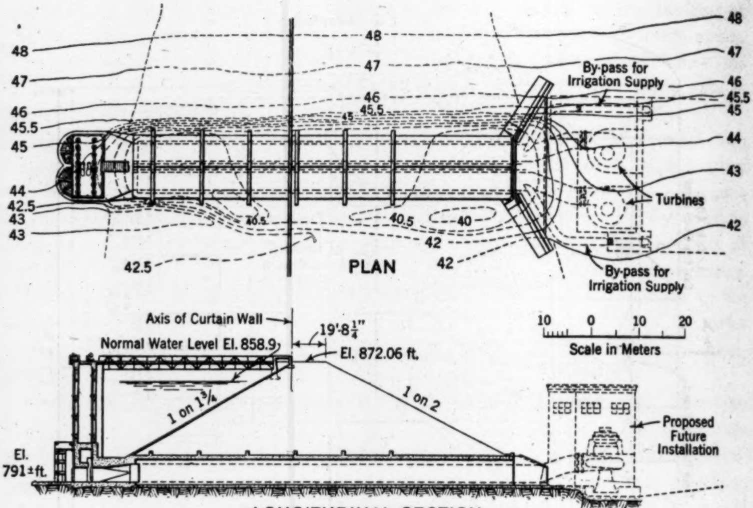

48
47
46
45.5
45
44
42.5
43
43
42.5
48
47
46
45.5
45
40.5
40.5
40
42
42
PLAN
By-pass for Irrigation Supply
Turbines
By-pass for Irrigation Supply
48
47
46
45.5
45
44
43
42
Axis of Curtain Wall
Normal Water Level El. 858.9
19'-8¼"
El. 872.06 ft.
1 on 1¾
1 on 2
El. 791± ft.
10 0 10 20
Scale in Meters
Proposed Future Installation
LONGITUDINAL SECTION

FIG. 6.—PLAN AND SECTION OF SLIDING GATES.

Main Canal.—The main canal for the first 30 km. (19 miles) maintains a uniform section, as follows (see Figs. 8 and 9):

	Meters.	Feet.
Bottom width	18.3	60.0
Side slopes	1½:1	1½:1
Water depth	3.23	10.6
Free-board	1.	3.28
Crown width	5 to 6	16.7 to 19.7
Gradient	0.0001	
Computed velocity	0.847 m. per sec.	2.78 ft. per sec.
Capacity	63.36 cu. m. per sec.	2 239 cu. ft. per sec.

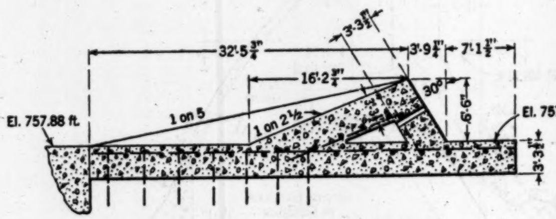

32'-5¾"
3'-3½"
3'-9¼"
7'-1½"
16'-2¾"
30°
6'-6"
El. 757.88 ft.
1 on 5
1 on 2½
El. 757.88 ft.
3'-3½"

FIG. 7.—CROSS-SECTION OF BAFFLE-PIERS.

This canal passes through an auxiliary reservoir with a capacity of 1 600 hectare-m. (12 970 acre-ft.) which will materially aid in the regulation of the system below and the conservation of water during periods of changeable weather and the resulting fluctuating demands.

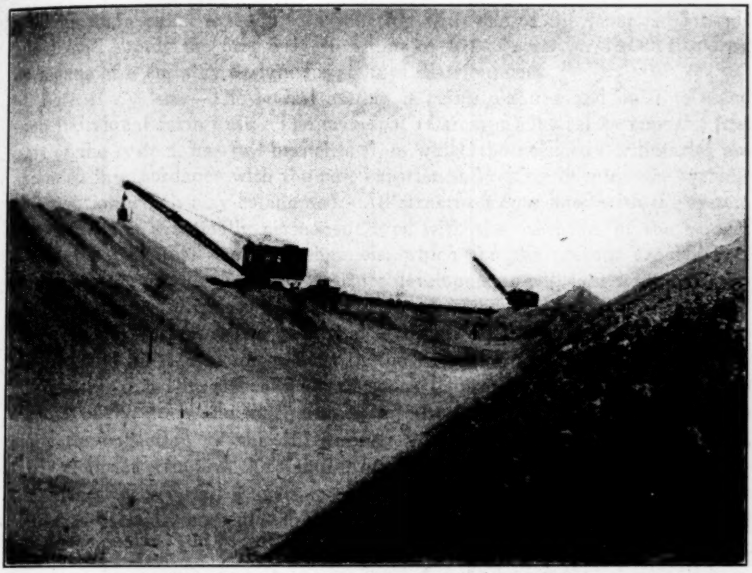


Fig. 8.—View of a Section of the Finished Canal.

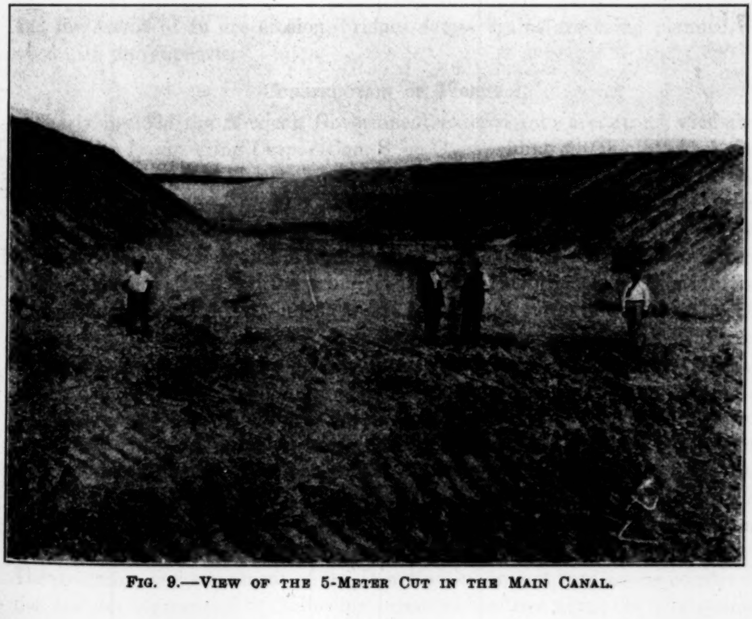


Fig. 9.—View of the 5-Meter Cut in the Main Canal.

This main canal extends to a junction with one of the principal laterals which will supply the first irrigation unit of 40 000 acres, or 16 000 hectares, by means of a suitably designed system of distributaries.

Lateral System.—The lateral system is being planned and built to reach each individual farm unit. The principal (Camaron) lateral serving the first unit of the system, has two branches from which the necessary tributaries are extended in accordance with the best experience, keeping in mind the systems of irrigation which may be adopted. All structures connected with the system are made of a practically permanent type with the exception of the bridges across the principal drainage channels, which for the present are built of timber resting on pile supports. Future developments will best serve to determine which may ultimately best be replaced with more permanent material.

Drainage System.—The principal arteries of the drainage system are planned and built in connection with the irrigation systems. They are constructed of sufficient size and depth so as to provide not only for the safe disposal of the storm and waste waters, but also for the carriage, and more especially the collection, of the soil drainage. As these soils are generally of a fairly compact structure the removal of surplus waters is believed to be essential. Experience on all irrigation projects thus far has demonstrated that this collection can only be made effective by means of relatively deep ditches (7 to 10 ft.) whether the system is of the open or the closed type. In the project area considered, only open drains are built, for the reason that these will have to take care of quantities of storm waters, the volumes of which are known to be large but indeterminate. On account of the uncertainties as to the floods and the extent of future erosion, bridges across drains are being planned of wood with pile supports.

CONSTRUCTION OF WORKS

Early in 1926 the Mexican Government entered into a contract with the J. G. White Engineering Corporation, S. en C., providing for the investigation and construction of a number of irrigation projects in different parts of the Republic. One of these projects, the Rio Mante, near Xicotencatl in the State of Tamaulipas, has been completed and is now in successful operation; another, the Calles Project in the State of Aguascalientes, on the Santiago River, is nearing completion; and the principal irrigation features of the Don Martin Project, including the distributing works for the first irrigation unit of 16 000 hectares (40 000 acres), will be available for the beginning of settlement and irrigation near the end of 1930. The Conchos Project in the State of Chihuahua, which has been given a most exhaustive line of investigation and study, is now (1930) under construction; it covers from 45 000 to 55 000 hectares (112 000 to 137 000 acres) of irrigable area, according to the scheme of development that may be adopted. Other projects in the course of construction and investigation are the Tula, in the State of Hidalgo, and the San Carlos, in the State of Coahuila. These two are old projects which require partial rehabilitation and the extension of the existing works constructed at an earlier date. The irrigation and settlement of the Don Martin Project require the construction and development of the following principal features now (1930) in course of construction or well advanced toward completion in so far as they relate to the first irrigation unit; Rio Salado Dam; Rio Salado Auxiliary Dike; Canal System; Lateral System; Drainage System; and Roads.

CONSTRUCTION OPERATIONS

Río Salado Dam.—The length of haul of the material used in the earth and gravel section of the dam was about 2 km. (1¼ miles), and this haulage was effected by a railway system with seven to eight trains, drawn by 20-ton oil-burning locomotives. Each train consisted of from twelve to fourteen 5-yd. dump cars. Cars were loaded by two 2½-yd. steam shovels. Following each successive unloading, 30-h. p. tractors shifted the track a sufficient distance to permit the spreading of the materials, using two 60-h. p. caterpillar tractors and graders. After suitable sprinkling the material was compacted by four 12-ton road rollers in 8-in. layers. It is of ideal composition for a dam embankment of this type, consisting of about 65% of gravel well graded to form a compact mass, mixed with about 35% of "fines". In its compacted state the material weighs approximately 135 lb. per cu. ft.

As is usual in construction of this character, the surface was first stripped of all vegetable matter. Where heavy brush and undergrowth prevailed, the stripping was extended to include the root zone of this undergrowth.

The aggregate for the main body of the concrete overflow section was limestone obtained from the right abutment. Here, an opportunity was afforded for lengthening the spillway and the installing of four additional radial gates of the same size as the twenty-two automatic gates, but operated by direct control. The materials, suitably selected, were passed through a No. 6 gyratory crusher driven by a 240-h.p. Diesel engine of the marine type, the engine also furnishing the motive power for the screens, sand mill, mixers, and auxiliaries. After passing the crusher, the material was elevated 70 ft. to a revolving screen from which it passed into the storage bins used both for supplying the mixer plant below and for loading trucks. A part of the crushed material was returned to a 36-in. cone crusher which furnished the sand for the concrete, no natural sand or gravels having been available. Immediately below the storage bins was the mixing plant, which consisted of two 1-cu. yd. mixers from which the concrete was conveyed to the forms in side dump-cars. (See Figs. 10 and 11.)

The concrete for the outlet works and retaining wall was mixed by a system of from three to four 14 S mixers, of ½-yd. capacity. Side dump-cars and chutes were used as for the overflow section.

Auxiliary Dike.—Starting in December, 1927, the construction of the dike progressed uninterruptedly. The major part of the work was performed with three standard 36-in. elevating graders and from eight to twelve 1½-yd. dump wagons to each machine, according to the haul. Because of the shallow soil mantle suitable for this embankment the haul ranged from 55 to 180 m. (180 to 600 ft.). In order to expedite progress and to employ available forces of drivers and light-weight teams, the smaller dike sections were built with slip scrapers and mules. As with the main embankment, the material was spread, leveled in 8-in. layers, wetted, and then rolled with 12-ton road rollers.

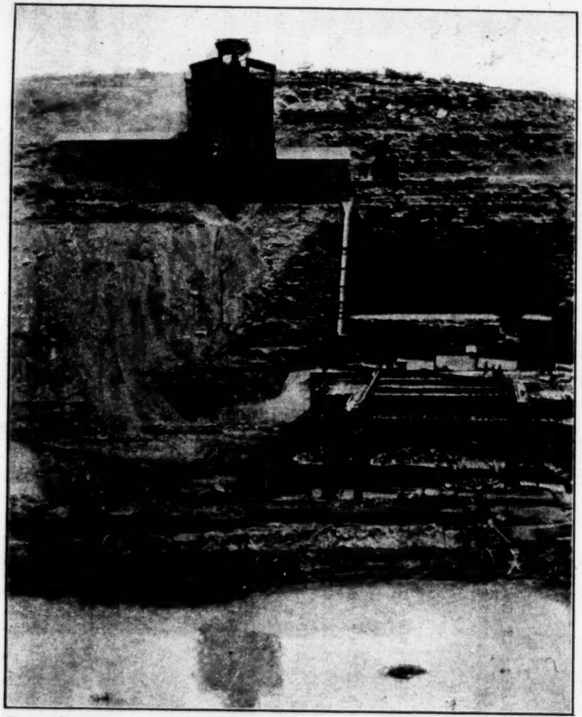


Fig. 10.—View of the Right Abutment of the Spillway Section.

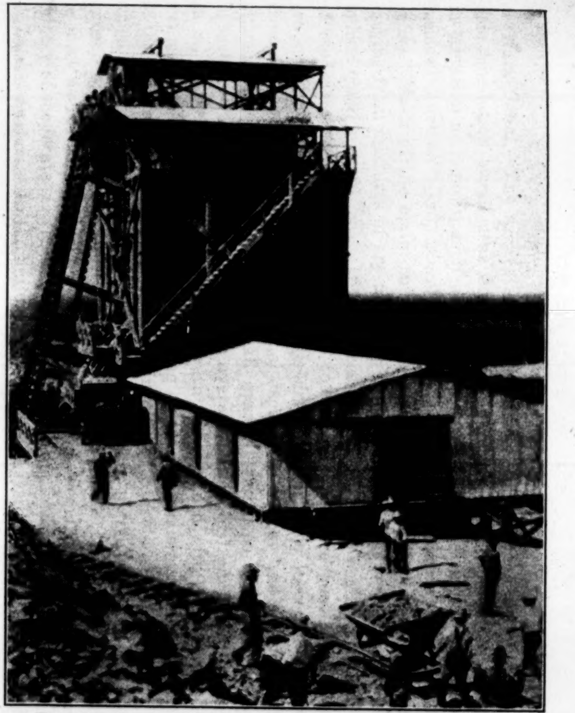


Fig. 11.—View of the Quarry at the Right Abutment.

Main Canal.—The excavation of the main canal and the stripping of the dam foundation began near the end of March, 1927. Equipment was placed in operation as rapidly as it could be procured from the manufacturers and included fourteen 1½-yd. Diesel and one 2½-yd. Diesel, electric drag-line excavators. Much of the main canal passes through uncommonly difficult ground ranging from mixtures of crystallized gypsum embedded in very compact clays to hard conglomerate rock. It was necessary, therefore, to use large quantities of explosives and a battery of from four to six portable gasoline air compressors supplying the necessary drills and auxiliaries to loosen and shatter these materials. As a result, the usual excavation output was not obtainable, although the operators were nearly all men of long experience and training in such work. Generally speaking, an average output of from 20 000 to 30 000 cu. m. (26 000 to 39 000 cu. yd.) per month was obtained with two 8-hour shifts and using 1½-cu. yd. machines, depending on the materials encountered. Much of the excavation had also to be made in cuts of considerable dimensions for which these machines were inadequate, but owing to the desire of the Commission of Irrigation to limit its equipment to such standard sizes as would be most useful in its expected operations, it was decided to choose only machinery of moderate size such as could be made applicable to future uses.

Lateral System.—In the building of the lateral system, advantage was taken of the excavating equipment on hand. The monthly excavation output was reduced 30% from that on the larger canals of 15 to 40-ft. bottom width and 4 to 8-ft. depth, in similar soils.

Drainage System.—The principal drains for disposing of soil drainage and storm waters are being built jointly with the distributing system and with the same excavating equipment, because of the relatively softer materials encountered. In these, the output and the unit costs have shown more nearly normal results than in the main canal system.

Road System.—Intimately connected with the laying out of the farm unit plan is the provision of an adequate system of public roads. This system as now planned and in process of construction provides for the building of a network of secondary or earth roads giving access to each individual farm unit. This network of secondary roads is united in a number of gravel surfaced roads leading to the principal highways and railroad stations, thereby facilitating marketing and shipments of produce as well as intercommunication with the principal population centers.

SETTLEMENT OF AND RE-PAYMENT FOR LANDS

Land Exchanges.—All the work on the Don Martin Project was under the supervision of the National Commission of Irrigation, created by the Enabling Act of January 4, 1926, to function under the Department of Agriculture and Development. This law provides that, upon the determination of the feasibility of any project composed of private lands, the owner or owners shall be given not to exceed three months within which they may voice their desire to build the necessary works or to co-operate with the Federal Government in such undertakings, under special conditions provided by this Act.

Omitting details, the law further provides that in case the Government builds the work covering private lands, it is to be reimbursed by a part of the owner's holdings such that its assessed unit value in the raw state multiplied by the area ceded to the Government shall be equal to the value of the area retained by the owner multiplied by the unit cost of reclamation or unit values arbitrarily applied. In making these adjustments, the Commission is authorized to add to the assessed value of the raw land in each project, a percentage so that the value of the irrigated land retained by the owner shall be equal to that of the property he possessed before the project was undertaken.

COLONIZATION

Mexican Citizens Favored.—The Commission desires above all things to make available the benefits of these irrigation developments to all Mexican citizens, many of whom live in adjoining States across the border where they have acquired considerable agricultural knowledge and experience. It is hoped that they may form the nucleus of a system of colonists, supplemented by others from within or without the confines of the Republic, the basic thought being to serve best a struggling population which is greatly in need of employment and home resources.

The National Commission of Irrigation has from the outset been conscious of the inherent difficulties attending the settlement of irrigation projects. For this reason every care has been taken to minimize the number of failures. To this end, the preliminary soil surveys upon which the outline of the project was based, were supplemented by a system of detailed soil surveys by competent experts. The precautions thus taken eliminate all doubt as to the adequacy of the soils or the system which is to furnish the water supply.

Following exhaustive studies of the problems of reclamation President Pascual Ortiz Rubio, under date of May 22, 1930, issued in effect the following resolution: That the first unit of the Rio Salado System shall be settled by colonists possessing the following primary requisites: They must be of Mexican nationality, have agricultural experience, be physically and morally capable, and have sufficient capital and resources to establish and support themselves on the land during the first year.

Further, that the Commission will select settlers according to the following preferences: (1) Original land owners within the limits to be colonized; (2) settlers or renters within the region; (3) Mexican farmers, including those outside of Mexican borders desiring to return; (4) Mexicans residing in cities who have the necessary primary qualifications; (5) alumni from the agricultural colleges who have completed their courses; and (6) persons not included within these items, but who have the requisite primary qualifications.

The prices to be paid for the irrigable lands cleared and ready for irrigation are 300 pesos per hectare ($54 50 per acre) for first class; 285 pesos per hectare ($52.00 per acre) for second class; and 270 pesos per hectare ($49.00 per acre) for lands of the third class.[5] To this may be added dry pasture land at the rates of 40, 15, and 7.50 pesos per hectare ($7.30, $2.74, and $1.37

---

[5] The values of 300, 285 and 270 pesos per hectare of irrigable land are based upon going values of similar lands within the Republic, subject to similar soil, climatic, and economic conditions.

per acre), according to classification fixed by the Commission. The colonist is to pay 5% of the cost price on signing the contract, the remainder in not to exceed twenty-five years; and, in addition, the annual operation and maintenance charges necessary for the upkeep of the system and 4% interest on the unpaid instalments.

With the listing of the applications by settlers it became increasingly apparent that a large number of otherwise desirable settlers found it difficult to make the initial payment of 5% of the construction payment, and also to meet the numerous additional items of expenses incident to the first season's operations. To meet this condition the President approved a resolution admitting colonists on a rental basis on a sale contract for a term of three years. The contract provides that the colonist pay the Government an annual rental of 20% of the gross crop returns for the use of the land and the irrigation system; 25% of the gross crop returns for the use of the irrigation system, including the furnishing of the necessary seeds; and 30% of the gross crop returns if the Commission also furnishes the necessary farm equipment, in addition to the seeds and the land as indicated.

It is to be understood that the value of these crops is to be credited to the colonist according to the market prices prevailing (the particular time is probably subject to future regulations by the Commission). Thus, if the value of the colonist's products should amount to 15 pesos per hectare ($2.74 per acre), and the cost of the water charges, including the rental, to 10 pesos per hectare ($1.83 per acre), the colonist is credited with the surplus 5 pesos ($0.91) on the construction charges. In other words, if the renter-colonist who furnishes his own seeds and implements gives his 20% of the gross produce, he is credited with the prevailing market price therefor from which is deducted the fixed charge of 5 pesos per hectare ($0.91 per acre) plus a charge of 50 centavos per 1 000 cu. m. ($0.64 per 100 000 cu. ft.) of water furnished to and measured at the land. Thus, an application of 60 cm. of water (2 ft. in depth) will cause a charge of 5 plus 3 pesos per hectare, making a total of 8 pesos per hectare ($1.46 per acre) to be deducted from his gross returns, the remainder to be credited to the construction charges applying to his lot.

The renter-colonist further obligates himself to cultivate not less than 50% of the irrigable land contained within his lot in a manner subject to the approval of and in accordance with the technical directions given him by the administrators of the local experiment farm. If he fails to accumulate a total of 10% of the value of the construction cost assessed against his land within the term of three years, his contract is then to be cancelled, and such payment benefits as may have accrued will be given to the succeeding settler.

Space does not permit of discussing the details many of which will have to be developed in the future as these and newer problems arise. The general statement gives one an idea of the extent the Mexican Federal Government is disposed to aid reclamation, which attitude appears to be generously supported by the Federal Congress.

The policies thus far pursued by the Presidents of Mexico since and including President Calles and so ably supported by the successive Commissions of

Irrigation are deserving of the highest praise. Without a doubt they will result in a material betterment of internal conditions. This should go far to halt emigration and the consequent neglect of the resources remaining undeveloped within the Republic, especially those relating to agriculture and the supplying of foodstuffs needed by its own people.

Authority and Direction of the Works

All the engineering and construction work described has been performed by the J. G. White Engineering Corporation, S. en C., under contract with the Government of Mexico and in conformity with the policies and general direction of the National Commission of Irrigation appointed by the President.

The original Commission appointed by President Calles was composed of Señores Francisco A. Salido, Director, and Javier Sanchez Mejorada and José Mares. During the administration of President Portes Gil, the Commission was composed of Roberto Gayol, M. Am. Soc. C. E., and Señores Alfonso González Gallardo and González Robles, with Sr. Ignacio López Bancalari as Director General of the Commission.

The present (1930) Commission is composed of Gen. Manuel Perez Treviño, President (who is also Secretary of Agriculture and Development), Sr. Bancalari, Executive Director, and Sr. Leopoldo Vasquez, Secretary-Member of the Commission.

Mr. G. W. Caldwell is General Manager for the J. G. White Engineering Corporation, S. en C., and H. V. R. Thorne, Construction Supervisor, F. E. Weymouth, M. Am. Soc. C. E., was Chief Engineer until March 1, 1929, when he was succeeded by C. H. Howell, M. Am. Soc. C. E., all with offices in the City of Mexico.

The Commission's representatives on the project were Sr. Jesús Oropesa, Technical Supervisor, who was succeeded in April, 1929, by Sr. Manuel Bancalari. R. M. Conner, M. Am. Soc. C. E., is Superintendent of Construction; Sr. A. Becerril Colin was Assistant Superintendent until March, 1930, and the writer, Resident Engineer.

DISCUSSION

C. H. Howell,[*] M. Am. Soc. C. E. (by letter).—The spillway of the Don Martin Dam is the first example of the round-head buttress type to be built. For this reason it may be interesting to give the reasons for its adoption.

Comparative designs were made for gravity, multiple-arch, and flat-slab or Ambursen sections. The foundation is stratified limestone. The strata are almost horizontal. The upper layers are separated by clay and shale seams. A worse condition for sliding can scarcely be imagined.

The gravity section proportioned to care for uplifting with these conditions was found to be exceedingly expensive. The possibility of unequal settlement of the buttresses, due to compression of the seam filling, with the attendant unknown stresses in the arches, eliminated serious consideration of the multiple-arch section. The flat-slab design, of course, was well adapted to meet the uplift and settlement conditions.

However, the round-head type was nearly as efficient as to uplift. Each unit being stable in itself it was equally efficient as to settlement. It required no reinforcing steel as all the material is in compression. There are no thin concrete sections exposed to water pressure. Due to the greater thickness and lower stresses of the up-stream face, a leaner concrete could be used in this portion than would be required for a flat-slab section. Also, due to the greater thickness, the operation of placing concrete is similar to the corresponding operation in a gravity dam. It is much cheaper than placing in the thin sections of the flat-slab design.

The comparative estimates, therefore, showed an appreciable saving in favor of the round-head buttress dam. Furthermore, it was and is considered to be structurally superior in that no tensile or bending stresses are involved.

In regard to uplift on the base of the heads and buttresses the following five conditions were assumed:

Case 1.—No uplift.

Case 2.—Full reservoir head on one-half the area of the buttress head; and full tail-water head on one-half the area of the buttress stem.

Case 3.—Average of full reservoir head and full tail-water head on one-half the area of the buttress head; and full tail-water head on one-half the area of the buttress stem.

Case 4.—Average of full reservoir head and full tail-water head on the entire area of the buttress head; and full tail-water head on the entire area of the buttress stem.

Case 5.—Full reservoir head on the entire area of the buttress head; and full tail-water head on the entire area of the buttress stem.

With the reservoir water surface at Elevation 858.70 (the top of the radial gates), computations indicated sliding coefficients as listed in Table 1 (Items 1 to 6, inclusive).

---

[*] Chf. Engr., J. G. White Eng. Corporation S. en C., City of Mexico, Mexico.

The weight of concrete was assumed at 140 lb. per cu. ft. It actually weighs an average of 142 lb. Tail-water was assumed at Elevation 764. With the water surface at Elevation 865.26, or 6.56 ft. above the top of the radials, the coefficients are as shown in Table 1 (Items 7 to 12, inclusive). Tail-water was assumed at Elevation 770. The weight of the over-pour on the back slab was neglected.

TABLE 1.—Sliding Coefficients, Don Martín Dam

Item	Elevation	Case 1	Case 2	Case 3	Case 4	Case 5
Water Surface at Elevation 858.70						
1	844.21	0.323	.....	.....	.....	.....
2	809.50	0.597	0.762	0.670	0.762	1.052
3	787.20	0.575	0.692	0.628	0.692	0.868
4	770.80	0.583	0.677	0.627	0.677	0.809
5	754.40	0.591	0.684	0.642	0.702	0.812
6	740.62	0.588	0.682	0.646	0.721	0.820
Water Surface at Elevation 865.26						
7	844.21	0.618	.....	.....	.....	.....
8	809.50	0.756	1.002	0.863	1.002	1.484
9	787.20	0.665	0.810	0.730	0.810	1.088
10	770.80	0.654	0.765	0.705	0.765	0.922
11	754.40	0.640	0.750	0.704	0.788	0.907
12	740.62	0.622	0.735	0.696	0.789	0.894

If this dam were to be designed over again, "diamond heads" would be used instead of round heads. The same results are secured with a slight saving in concrete and somewhat easier form work. (See Fig. 12.)

At first, there was some doubt as to the amount of leakage which might occur between the units. There was some fear that it might be large. The reservoir water surface is now (January, 1931) about 10 ft. below the top of the radials. To date, there has been no measurable leakage between the units. The copper seals between them have proved to be practically water-tight. Due to the excellent care used in construction there is no leakage through the horizontal joints between the various concrete lifts. A slight amount of moisture appeared at a few joints when the reservoir began filling, but these places have sealed themselves.

The concrete was approximately a 1:2.5:5 mix, and the coarse aggregate was crushed limestone passing a 3-in. screen. The sand was made of the same material, all of which passed a ¼-in. screen. The water-cement ratio was varied somewhat, but it averaged 1.20. The maximum length of a placement was about 100 ft., and the height of the lifts was 5 ft. The concrete was kept wet—not merely moist but wet—for a minimum of a week. The cement used on the project was manufactured near Monterrey, Mexico.

Practically no shrinkage trouble was experienced on this dam. In fact, practically no shrinkage has been noticed on any concrete placed on the entire project. This is believed to be due mostly to the extra care used in providing somewhat more than the usual quantity of curing water.

In a few of the buttresses hair cracks appeared in the crown of the arches in the lower gallery openings (Fig. 4). These cracks appeared during construction soon after the concrete was placed. They are vertical and are about 6 ft. long. After they were noticed, reinforcing steel was placed over the remaining openings. To date, no other cracks have been found. The existing cracks have not lengthened as the reservoir has filled.

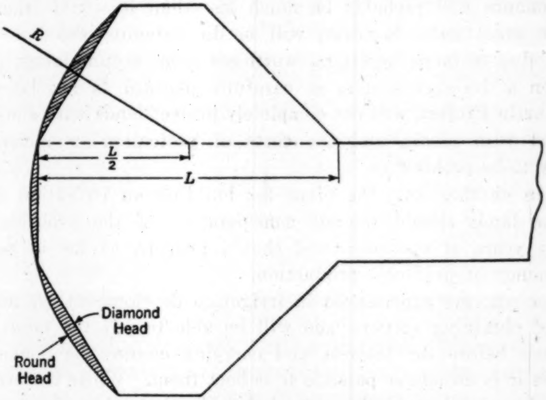


FIG. 12.—COMPARISON OF THE "DIAMOND-HEAD" AND ROUND-HEAD BUTTRESSES.

As stated previously, the water surface in January, 1931, was about 10 ft. below the top of the radial gates. Therefore, actual service conditions had not been reached. However, the automatic action was tested by building a bulkhead in front of one gate and pumping the space between the bulkhead and gate full of water. This reproduced initial starting conditions exactly. Water was then pumped into the float-wells. Movement of the gate took place almost exactly at the computed critical water depth in the float-well. The tests also indicated that an excess of power was available for operation, and that the gates would respond automatically to small changes in the reservoir water surface.

The writer believes that the round-head or diamond-head dam should be considered in all studies for sites where gravity, multiple-arch, or flat-slab sections would be considered. He also believes that the round or diamond-head buttress dam has many inherent merits, and that these will become generally recognized in the future.

ROBERT S. STOCKTON,[7] M. AM. SOC. C. E. (by letter).—The construction and settlement program for the irrigation system described by the author has evidently been planned with full knowledge of the latest ideas and technical information available. The description of the work, while not detailed, indicates the thoroughness and care with which the problems have been studied.

---

[7] Supt. of Operation and Maintenance, Western Section, C. P. Ry. Irrig.-Block, Strathmore, Alberta, Canada.

The exclusion of certain topographically attractive areas from irrigation development on account of alkaline subsoil conditions, is most wise, and while this increases the first cost of the system the future average cost of operation and maintenance will probably be much less than it would otherwise have been. The great gain, however, will be in reducing the loss in capital investment due to lands becoming worthless from accumulation of alkaline salts. Even a drainage system as carefully planned as has been done for the Don Martin Project, will not completely protect lands with subsoil heavily impregnated with alkali, and the costs of protection or reclamation are quite likely to be prohibitive.

Stated in another way, the plans for building an irrigation system and settling the lands should include consideration of the probable condition after many years of operation and should provide, as far as possible, for the permanency of profitable production.

All those who are experienced in irrigation development are aware of the difficulty of obtaining settlers who will be able to put the lands on a productive basis before the interest and carrying charges have mounted to a point where it is no longer possible to collect them. While Governments can afford to take some loss on the ground that, in the end, an increased population and production will accrue, the plans should provide, as far as can be foreseen, for the repayment of all costs, including a low rate of interest. In most cases the development should be deferred or abandoned if it can not "pay out" on a strictly business basis.

Another point, that is most essential, is that of supervision of the farm program of the first settlers. Mr. Weiss mentions that such supervision is to be provided for all settlers on the three-year rental basis. It may probably occur that for good settlers it will be found desirable to extend the rental period to, say, five years.

The matter of proper supervision and layout of the farm management and cropping system is so important that it would be well if all land and water contracts included a definite agreement, by which the settler would be obligated to follow the directions of the farm manager or agriculturist responsible for results, for a period of five years, or until a substantial equity in the holding had been established.

It is essential of course that the agricultural supervisor shall be experienced and thoroughly qualified in every way. He should have a personality sufficiently forceful and diplomatic to obtain results with a minimum amount of friction.

Some credit for the purchase of stock and equipment is most desirable, after the character of the settler has been demonstrated as worthy, and the management has been tested as to its soundness. If funds are made available, they should be handled through a bank and by strictly business methods, as this is the point at which mistakes are most easily made, but where good management will hasten and enhance prosperity.

ANDREW WEISS,[8] M. AM. SOC. C. E. (by letter).—The writer is indebted to Mr. Howell for the pertinent explanatory comments, stating more fully the reasons for the adoption of the buttress type of dam.

To date (1931) only two of the contact joints between the buttresses have shown leakage and this in negligible quantities. As the water was being raised within the reservoir, some leakages developed in varying amounts through the limestone seams around the south abutment, and these were effectually followed by drilling and grouting. At this writing, the reservoir is filled to within 3 ft. of the flowage line.

The automatic opening feature on all the radial gates was further tested, with satisfactory results, by pumping water into the float-wells, but incident to these operations a number of unforeseen details were discovered, which led to the adoption of certain further precautionary measures. One of these was the use of covers for the tubes admitting water to the float chambers, thus preventing careless or mischievous introduction of waste matter and obstacles. Some of the chains designed to connect the gates with the movable emergency lifting devices had become loosened from the upper corners of the gates and, with the automatic lift in operation, the suction of the escaping water had a tendency to drag the slack portion of these chains between the radial gate seats and the rubber seals, thus interfering with free movement. Hence, it was necessary to be certain of the locking of these latches which fasten the chains to the upper corners of the gates in order to avoid a recurrence of these happenings (see Fig. 13).

A view of the overflow section, looking up stream from the south end, is shown in Fig. 14. Fig. 15 shows the discharge of three gates under a 9-ft. head, each releasing about 2 200 cu. ft. per sec. It will be seen that immediately at the toe of the dam some spray and a turbulent wave action are formed; the latter, however, gradually reduces to a steady or normal flow condition about 20 ft. down stream from the dentated sill. With such quantities of water as have been released it is too soon to state how effective these dentated sills may be, but it appears that erosion below the toe will be kept entirely within harmless limits.

Mr. Stockton correctly points out the difficulties of obtaining settlers of the required financial capacity and training in agricultural pursuits, who will be able to put the lands on a productive basis before the interest and carrying charges have mounted to a point where it is no longer possible to collect them. This applies with particular emphasis to the new projects being built by the Mexican Federal Commission of Reclamation because of the greater scarcity of settlers with the requisite means. For this purpose the framers of the reclamation policy and the regulations expressing it, have wisely provided for three-year rental contracts and for farm supervision. It is too soon to predict the extent to which this supervision may be carried, but judging from the interest thus far displayed by the governmental authorities in charge, it is believed that such assistance will be freely provided and that the settlers will co-operate in a most responsive spirit.

---

[8] Res. Engr., J. G. White Eng. Corp., Estacion La Cruz, Chihuahua, Mexico.

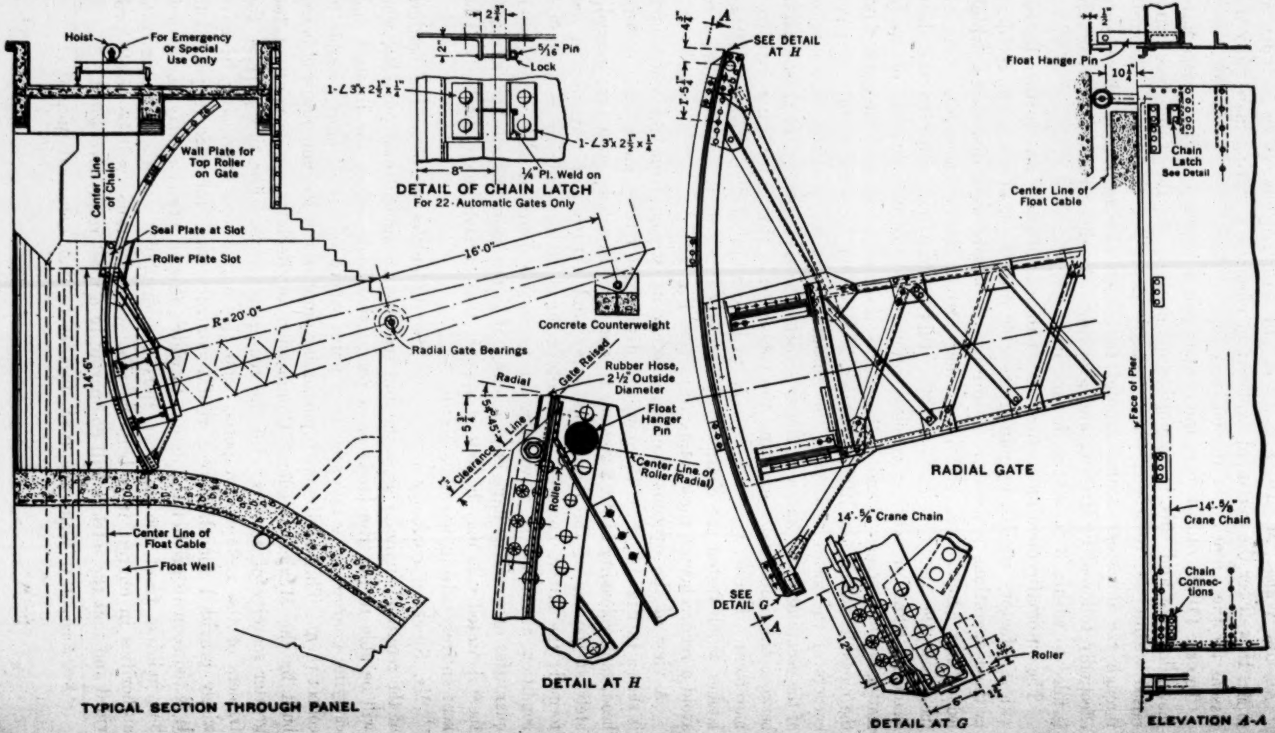

Hoist
For Emergency or Special Use Only
Wall Plate for Top Roller on Gate
Center Line of Chain
Seal Plate at Slot
Roller Plate Slot
16'-0"
R=20'-0"
14'-6"
Radial Gate Bearings
Concrete Counterweight
Center Line of Float Cable
Float Well
TYPICAL SECTION THROUGH PANEL
2¼"
½"
5/16" Pin
Lock
1-∠3"x 2½"x ¼"
1-∠3"x 2½"x ¼"
8"
¼" Pl. Weld on
DETAIL OF CHAIN LATCH
For 22-Automatic Gates Only
Radial
Gate Raised
Rubber Hose, 2½" Outside Diameter
Float Hanger Pin
5¾"
54°-45'
Clearance Line
1"
Center Line of Roller (Radial)
Roller
DETAIL AT H
4¾"
A
SEE DETAIL AT H
1'-5¼"
SEE DETAIL G
A
RADIAL GATE
14'-⅝" Crane Chain
12"
6"
Roller
DETAIL AT G
½"
Float Hanger Pin
10¼"
Center Line of Float Cable
Chain Latch See Detail
Face of Pier
14'-⅝" Crane Chain
Chain Connections
ELEVATION A-A

FIG. 13.—ASSEMBLY AND DETAILS OF CRANE CHAIN CONNECTIONS, RIO SALADO DAM.

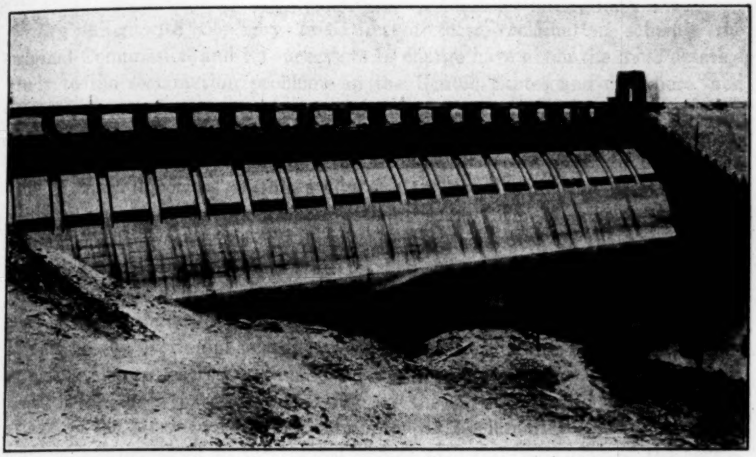


FIG. 14.—OVERFLOW SECTION OF DAM, LOOKING UP STREAM FROM THE SOUTH END.

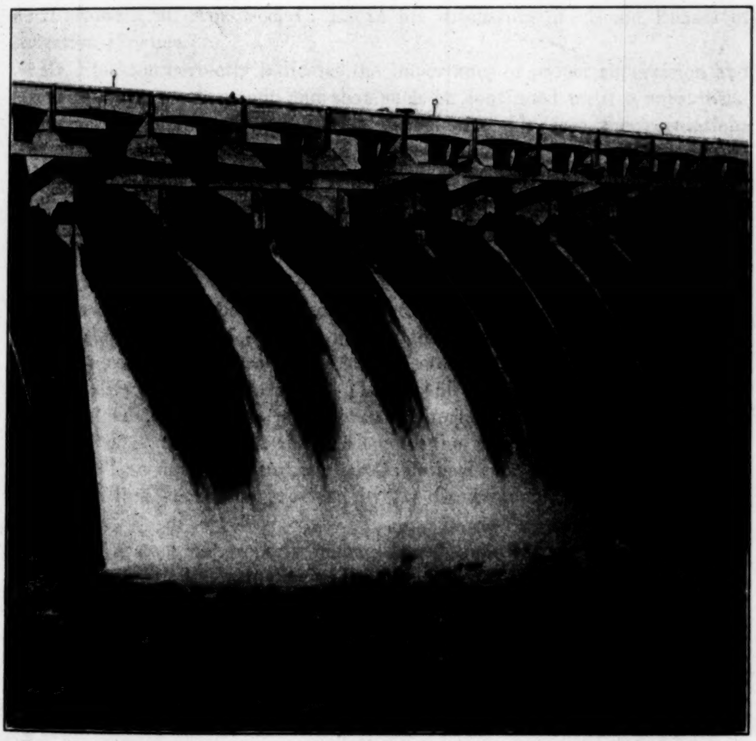


FIG. 15.—VIEW SHOWING DISCHARGE OF THREE GATES, EACH UNDER A 9-FOOT HEAD.

Beginning with the very initiation of these reclamation schemes the original Commission and its successors in charge have given the most detailed study to the reclamation problems in the United States and elsewhere, and have familiarized themselves with the principal results of the many investigations during the course of development—more especially with the report of the Fact-Finding Commission—to the end that errors might be minimized. One of the outgrowths of this study was the conclusion that most of these projects could not be undertaken without some form of subsidy from the Federal Treasury and, for this reason, it was decided to fix the construction charges commensurately with the market value of the land in question. Doubtless much will have to be added to the present regulations as experience and economic developments may dictate within the coming years. It can not be over-emphasized, however, that the definite fixing of the construction charges corresponding to the market values of the land rather than reclamation costs has obviated much of the criticisms and incentives for attack on the Mexican Federal Commission on Reclamation and its engineers such as was practiced in the United States with the beginning of the repayments. This train of events was pointed out exceedingly well by the late F. H. Newell, M. Am. Soc. C. E., in his discussion of "Some Phases of Irrigation Finance."[9]

Mr. Stockton correctly indicates the importance of proper supervision and layout of farm management and that such be continued until a substantial equity in the holding has been established. In this case the regulations provide that the settler follow the general guidance of the farm advisers and the experiment station, from which sources it is believed much good will result. Much misunderstanding has existed as to the proper functions, responsibilities, and results of agricultural advisers on these projects. It is pointed out with reason that no one man can combine the necessary experience and training to direct the farming operations of all the settlers. This is not and should not be expected. His functions and work are more nearly comparable with that of the field men as they are employed and directed by the sugar companies operating in the Western States. Few of these men would qualify as master farmers. They do fill an important need, however, as general advisers and as purveyors of information as well as keepers of statistical data relating to the needs and the prospects of the individual settlers. These data may furnish a reliable basis for the future credits and assistances that should be provided by the Government, or by other credit agencies.

Such a system, if properly managed and maintained, will in no sense impair the settler's individuality and self-reliance, as it is essentially co-operative and helpful. It is obviously unpopular with a certain minority of settlers and sympathizers who are chiefly interested in building and collecting pretexts and arguments in defense of non-payment and non-performance of just and practicable dues; but this fact does not argue against such a system of vigilance and help because the majority of the settlers on any project are essentially as honest and as anxious to meet their debts as they become due, as people are in other walks of life. Experience has shown that such a system of supervision soon becomes popular rather than otherwise if backed by the proper management and ideals.

---

[9] *Transactions*, Am. Soc. C. E., Vol. 92 (1928), pp. 563–568.

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
TRANSACTIONS

Paper No. 1807

A DISCHARGE DIAGRAM FOR UNIFORM FLOW IN OPEN CHANNELS[1]

BY MURRAY BLANCHARD,[2] M. AM. SOC. C. E.

WITH DISCUSSION BY MESSRS. LYNN PERRY, J. B. MACPHAIL, G. B. PILLSBURY, LYNN CRANDALL, BENJAMIN E. JONES, A. J. MATHESON, L. STANDISH HALL, RAY E. MACKENZIE, AND MURRAY BLANCHARD.

SYNOPSIS

This paper introduces a diagram of discharge stage and slope or fall relation for uniform flow in open channels. It is based on the Chezy formula wherein, for a constant stage, the discharge varies as the square root of the slope or fall and is designed to give the discharges at any stage for the falls that create the resulting discharges.

A discharge diagram consists of a series of fall curves referred to discharges as abscissas and stages as ordinates, from which the discharge for any observed stage and fall can be obtained.

The development of the diagram is shown by a practical application to a series of discharge measurements made in 1914-16 on the Chicago Sanitary District Canal with regulated flows in a uniform rock section.

Discharge diagrams are applicable to all uniform stream flows, whereas the ordinary discharge curve cannot be used where there is back-water effect or where the up-stream and down-stream waters do not rise or fall simultaneously.

THE DISCHARGE OR RATING CURVE

In many streams for uniform flow there is only one stage and a corresponding slope for any discharge in a reach between stations so spaced that the water-surface profile is a straight line. An increase in the discharge creates a higher stage with its corresponding slope. Increments of discharge have their accompanying increments of stage in such a manner that by plotting discharges as abscissas and corresponding stages as ordinates, the points will fall in a regular curve. This is the ordinary discharge curve shown in Fig. 1. The profiles that obtain at the various stages are of the type shown in Fig. 2.

---

[1] **Published in January, 1931, *Proceedings*.**

[2] **Cons. Engr., Chicago, Ill.**

They do not cross one another, are nearly parallel, and are approximately parallel to the bed of the stream.

Sometimes, it is necessary to make measurements of discharge when the flow is not uniform; that is, while the stage is rising or falling. Discharges thus obtained will fall, respectively, to the right or left of the curve for uniform flow, and methods have been devised for reducing these discharges to their positions on the discharge curve, in the directions indicated in Fig. 3.

WHEN THE DISCHARGE CURVE IS NOT APPLICABLE

There are streams in which the flow is varied by inflow and outflow such that, at any stage, a wide range of discharges may obtain. This may be due to various conditions, such as:

(*a*) The back-water effect of high, low, or intermediate stages of the receiving reservoir (river, lake, or pool).
(*b*) The effect of a large tributary inflow.
(*c*) The effect of high, low, or intermediate stages of an up-stream reservoir (lake or pool).

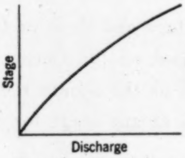


FIG. 1.—TYPICAL DISCHARGE CURVE, UNIFORM FLOW.

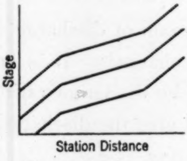


FIG. 2.—PROFILES, TYPE *A*.

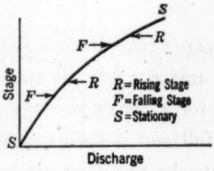


FIG. 3.—TYPICAL DISCHARGE CURVE FROM NON-UNIFORM FLOWS.

A few instances of stream gaugings made under these conditions are given in Table 1.

TABLE 1.—TYPICAL STREAMS THAT HAVE PROFILES OF THE TYPE *B*

Stream	Years	Authorities	Upper reservoir	Lower reservoir	Reaches	Gauging stations
Detroit River*....	1902-03.	U. S Lake Survey.	Lake St. Clair.	Lake Erie.	Fort Wayne, Mich. to Lake St. Clair.	Fort Wayne, Mich.
Illinois River.....	Various.	U.S. Geological Survey. U. S. Engineering Department. Sanitary District of Chicago.	Lake Peoria.	Mississippi River.	Peoria, Ill., to Pekin, Ill. Copperas Creek to Havana Ill., etc.	Peoria, Ill. Havana, Ill. Beardstown, Ill. Valley City, Ill. Hardin, Ill.
Sanitary District Canal†....	1914-16.	Sanitary District of Chicago	Lake Michigan.	Pool above dam at Lockport, Ill.	Willow Springs, Ill., to Lemont, Ill.	Lemont, Ill.

---

* Report of Chf. of Engrs., U. S. A., for 1902, Appendix EEE, pp. 2867–2872; and for 1903 Appendix EEE, pp. 2813–2817.

† "Hydraulics of Chicago Sanitary District's Main Channel," *Journal*, Western Soc. of Engrs., September, 1920, and discussion, August, 1921.

The instances listed in Table 1 are illustrated graphically in the profiles of Type *B*, Fig. 4. In the reach, *A-B*, the upper profiles have a common mean elevation of water surface and the lower profiles have another common mean stage.

In making a study and investigation of flood flows of the Illinois River in 1929, these conditions were encountered. It was found that at all the gauging stations in the lower 100 miles of the river, discharges that had been measured by competent observers under the direction of most reliable authorities, would appear to be grossly in error when compared with the values indicated by established discharge curves.

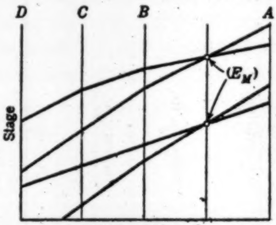


FIG. 4.—PROFILES, TYPE *B*.

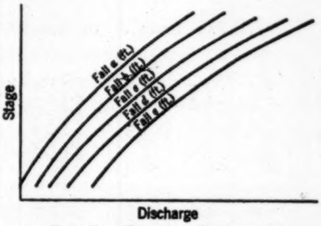


FIG. 5.—TYPICAL DISCHARGE DIAGRAM.

To obtain a substitute for the rating curve under these conditions the writer developed the discharge diagram described herein. It consists of a series of curves of slope which are referred to discharge and stage and from which the discharge can be obtained for a known stage and slope, or fall (see Fig. 5).

DEVELOPMENT OF THE DISCHARGE DIAGRAM

Consider, first, a reach in which the slope is unbroken, and a certain mean stage of reach. The Chezy formula for discharge, Q, in terms of mean area of section, A, mean hydraulic radius, R, and slope, S, is:

$$Q = A\,C\,R^{\frac{1}{2}}\,S^{\frac{1}{2}} = \frac{A\,C\,R^{\frac{1}{2}}\,F^{\frac{1}{2}}}{L^{\frac{1}{2}}} \ldots\ldots\ldots\ldots\ldots\ldots (1)$$

in which, C is a constant and the slope, S, is equal to the fall, F, divided by the length of reach, L. For a single stage under consideration the factors, (A), (C), (R), and (L), are constant. Therefore, K may be substituted in Equation (1) for the quantity, $\frac{A\,C\,R^{\frac{1}{2}}}{L^{\frac{1}{2}}}$, so that Equation (1) becomes,

$$Q = K\,F^{\frac{1}{2}} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$$

For any other discharge and corresponding fall (denoted by the subscripts, x),

$$Q_x = K\,F_x^{\frac{1}{2}} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3)$$

From Equations (2) and (3), $\frac{Q_x}{Q} = \frac{F_x^{\frac{1}{2}}}{F^{\frac{1}{2}}}$,

or,

$$Q_x = Q\,\frac{\left(F_x^{\frac{1}{2}}\right)}{\left(F^{\frac{1}{2}}\right)} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (4)$$

and,

$$F_x = F\left(\frac{Q_x}{Q}\right)^2 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (5)$$

The method of obtaining a discharge diagram from these formulas may be described in ten steps, as follows:

1.—Decide upon a base discharge (preferably about an average) to which all observations are to be reduced.

TABLE 2.—SAMPLE COMPUTATIONS REQUIRED FOR CONSTRUCTING DISCHARGE DIAGRAM FOR EQUATION (5)

Group No.	Observation No.	STAGE (ALL MINUS)			DISCHARGE	FALL		DISCHARGE		
		Observation, F_s, in feet	Observation, E_1, in feet	Mean, E_m, in feet	Observation, and mean, Q, in cubic feet per second	Observation, F, in feet	Base, F_x, in feet	Diagram, Q_x, in cubic feet per second	Residual, $Q_x - Q$, in cubic feet per second	Residual, $Q_x - Q$, percentage
	157	0.68	1.38	1.03	9 300	0.70	0.52	....	....	....
A				1.03	9 300	0.70	0.52	9 400	+100	1.1
	159	0.87	1.55	1.21	8 470	0.68	0.61	....	....	....
B				1.21	8 470	0.68	0.61	8 900	+430	4.8
	164	0.99	1.53	1.26	7 800	0.54	0.57	....	....	....
	156	0.97	1.58	1.27	8 232	0.61	0.58	....	....	....
	158	0.93	1.60	1.27	8 360	0.67	0.52	....	....	....
	155	0.92	1.61	1.25	8 550	0.69	0.60	....	....	....
	165	1.03	1.59	1.31	7 430	0.56	0.65	....	....	....
	60	1.11	1.54	1.33	7 130	0.43	0.54	....	....	....
C				1.23	7 920	0.58	0.58	8 200	+280	2.8
	147	1.16	1.65	1.40	7 370	0.48	0.57	....	....	....
	65	1.21	1.60	1.41	6 750	0.39	0.55	....	....	....
	49	1.23	1.63	1.43	6 900	0.40	0.54	....	....	....
D				1.41	7 010	0.42	0.55	6 600	−410	6.2
	153	1.24	1.81	1.52	7 740	0.57	0.61	....	....	....
	152	1.29	1.77	1.53	7 580	0.48	0.53	....	....	....
	160	1.26	1.86	1.56	8 250	0.60	0.57	....	....	....
E				1.54	7 860	0.55	0.57	7 700	−160	2.1

2.—From each discharge and corresponding fall, using Equation (5), compute the fall corresponding to the base discharge. This step is demonstrated briefly in Table 2 for a number of observations made on the Chicago Drainage Canal in 1914, 1915, and 1916.

3.—Compute the average elevation of the reach for each observation.

4.—Plot the falls as abscissas with the corresponding average stages as ordinates.

5.—Pass an average smooth curve through the points thus obtained and call it the base curve.

6.—From the base curve, take off the fall for the base discharge at desired stage intervals.

7.—By means of Equation (4) compute the discharge corresponding to a desired slope at each of the stage intervals.

8.—Compute the discharges corresponding to other desired falls, F_x, at each of the stage intervals.

9.—At the stage intervals plot the discharges as abscissas and the corresponding falls as ordinates.

10.—Join the points of the same slope value, and smooth slope curves will be obtained.

Fig. 6 in an example of a discharge diagram constructed from actual observations made on the Chicago Drainage Canal, some of which data are given in Table 2. The series of slope curves form the discharge diagram (see Fig. 7).

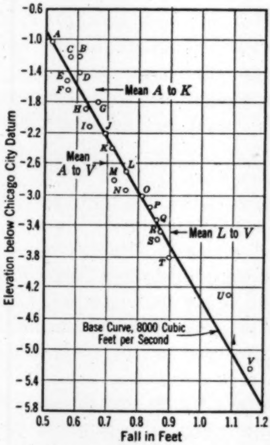


FIG. 6.—BASE DISCHARGE CURVE.

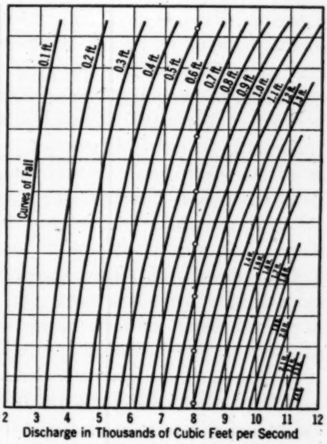


FIG. 7.—DISCHARGE DIAGRAM.

Since a stage at one end of a reach in connection with any certain slope or fall corresponds to a definite discharge, a diagram of discharge, an end-station stage, and a slope or fall relation can be constructed in a manner similar to that for the diagram of discharge, mean stage, and slope, or fall relation, herein described.

CONCLUSIONS

The construction of a discharge diagram involves the additional expense, over the discharge curve, of maintaining and reading two gauges in each reach instead of one. It would seem that in most cases the more reliable results obtained from the former would warrant the expenditure.

Even if the diagram is constructed for all cases of uniform flow there would also be the special uses for the discharge curve, which gives for any stage the discharge that normally or most frequently occurs. It is often desirable to know the increment of normal discharge for a rise or fall in stage. Thus, the increment of discharge in the outlet of a large lake, taking into account also the effect of evaporation, etc., is a measure of the effect of withdrawal from the lake of a volume equivalent to the increment.

DISCUSSION

LYNN PERRY,[3] M. AM. SOC. C. E. (by letter).—The author is to be commended and congratulated for the work he has done in connection with this paper. It is manifest that a great deal of careful thought has been given to a problem that has confronted many engineers who have been identified with the gauging of streams.

The Chezy formula (Equation (1)), seems to be in more common use than any other, and when the coefficient is carefully chosen it is probably as accurate as any that has been proposed. However, it is based on assumptions that do not usually obtain in an unregulated stream flowing naturally. Reference is made particularly to the mean hydraulic radius, R, and the slope, S. The writer has had some experiences that have tended to destroy the usual simple faith in Kutter's value for C.

The mean hydraulic radius varies with the stages of a stream at the particular point of observation. An identical value for R may be obtained for, say, an increased depth of the water at the observation point: (*a*) When the increased depth is caused by back-water; and (*b*) when it is caused by a flood flow in the stream. It should be perfectly obvious that such conditions are not logical. It may be argued that the slope, S, would be different in a case of this kind and that this difference will cause the Chezy formula to show the proper difference in discharge. This may be answered in two ways: (1) Actual observations, limited to the writer's experience, indicate that the difference in the slope, S, does not account for the difference in the actual measured discharge; and (2) R is not a constant and should not be so treated.

This formula (Equation (1)) was proposed by Chezy in 1775 following a series of observations of stream flow. Notwithstanding the fact that it is purely empirical, hydraulic engineers have used it, almost unchanged and unquestioned, during 150 years of the most extensive engineering activity since the dawn of history. Scores of engineers have lived during every generation subsequent to the time of Chezy, who are devoting more time, with more ably trained assistants, with more accurate instruments, and working on many more streams than Chezy. It certainly does seem to the writer that some engineer engaged in stream measurement and one who has at his disposal a large amount of data and some personnel, should have the initiative and courage to propose a more accurate and at the same time as economical and practical a method of measuring stream flow as Chezy.

When two observations are made in order to obtain a correct value for S at the time of reading the stage, a much more accurate value for the discharge can be computed than when the stage alone is read. The author refers to the additional expense of making the additional observation. Of course, where there is only one observation point on a stream, the expense would be doubled. Most of the large and important streams in the United States have a number of such points where the stage is read and reported every day. The writer believes that these rivers could be organized in such a way that the additional expense would be relatively small. Even if the cost is somewhat more per observation point, would it not be far better to have a smaller number of more accurate observations than a larger number of less trustworthy ones?

---

[3] Asst. Prof., Hydr. and San. Eng., Lafayette Coll., Easton, Pa.

The author does not state whether he is making use of the well known and frequently used Kutter formula for C. When Kutter first proposed his formula for the computation of C in the Chezy formula, he did not include S; that is, his own personal observations did not indicate to him that the slope would have any effect on the constant, C. Afterward, when reports of the flow of the Mississippi River were published by Humphreys and Abbot,[4] he was prevailed upon to alter his formula to absorb certain variations that could be taken care of by admitting that the slope had some influence on the numerical value of C in individual cases. If the Kutter formula is used, the author's expression for his constant, K, in Equation (2), will not be a constant. This must be obvious because the slope,

$$\frac{F^{\frac{1}{2}}}{L^{\frac{1}{2}}} = S \quad \text{......(6)}$$

is also to be included in C.

Assuming that the author has used some other more accurate and more reliable method for obtaining the coefficient, C, and that, therefore, his Equation (2) is correct; and if F is the total fall, in feet, in the run, L, in feet, so that Equation (6) is correct, then the value for K, as given by the author, is also correct, namely,

$$K = \frac{A\ C\ R^{\frac{1}{2}}}{L^{\frac{1}{2}}} \quad \text{......(7)}$$

in which, A is the area of the cross-section of the stream, in square feet; R, the mean hydraulic radius, in feet; and C complies with the requirements of the foregoing paragraph.

If Equation (7) is a true constant for all values of Q which the range of the observations is to cover, Equation (2) is a true parabola. Now, if this is plotted on logarithmic cross-section paper, it will be a right line; the position and direction will be fixed by the numerical values of K and the exponent of F, respectively. The numerical value of K is the point where the line intercepts the Y-axis and the exponent of F ($= +\frac{1}{2}$) is the slope of the line.

A considerable number of engineers who have had some experience in measuring the flow of water will refuse to grant that V (and, consequently, Q) varies as the square root of S. The results of much work along this line have been published.

Table 2 does not give the numerical values for A, C, R or L, but values for Q and F are given for a number of cases. Taking Group A as an example, $Q = 9\,300$, $F = 0.70$; therefore, $K = 11\,000$. Taking a sheet of logarithmic cross-section paper and plotting values of F along the horizontal scale and Q

---

[4] Report upon the Physics and Hydraulics of the Mississippi River, by A. A. Humphreys and H. S. Abbot, *Professional Papers*, Corps of Topographical Engrs., U. S. Army, No. 4, 1861.

vertically, the solid line on Fig. 8 can be obtained. Readings from this line will give values for Q corresponding to observed values for F as long as other factors are constant. With an increase in the stage reading, raising K, say, to 13 000, the first line above is obtained; with a decrease to 9 000, the line below, etc. A chart of this type appears to be much more direct and of much simpler construction than that proposed by the author.

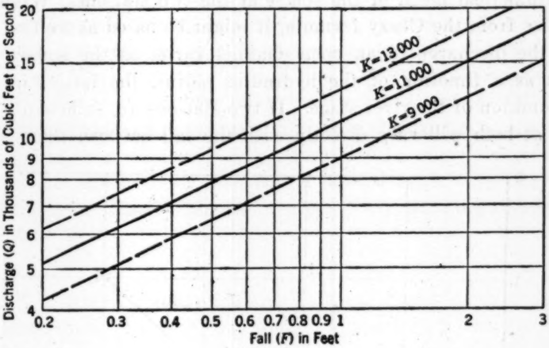


FIG. 8.

If, however, Mr. Blanchard prefers to use the ratio of the square roots of the fall as abscissas, it is necessary only to consider his Equation (5) which reduces to,

$$Q_x = Q \frac{F_x^{\frac{1}{2}}}{F^{\frac{1}{2}}} \qquad \text{.........................}(8)$$

Then, his base, Q, becomes the Y-intercept value, and the ratio of the square roots of the new fall to the base fall is the ordinates.

J. B. MACPHAIL,[5] ASSOC. M. AM. SOC. C. E. (by letter).—A slight simplification can be made in Mr. Blanchard's method. The coefficient, K, is a function of the physical dimensions of the reach at the stage under consideration. It varies with the mean elevation of the water surface of the reach, and a curve, therefore, can be plotted, giving the value of K for any given mean elevation.

Hence, this curve can be used with Equation (2) to plot the curves in Fig. 7. It has, however, a greater curvature than the base discharge curve proposed and, therefore, it may be less precisely definable in some cases in which experimental difficulties introduce noticeable residual errors.

G. B. PILLSBURY,[6] M. AM. SOC. C. E. (by letter).—In this paper the author points out that the discharge of a stream at a given gauging station is not necessarily a function of the river stage at that station alone, but may depend also on the river stage at a station down stream. Such a condition will occur in fact if the back-water effect of the lower station extends to the upper station, and the stage at the lower station does not result wholly from the discharge passing the upper one. It is a condition frequently arising, particularly in large rivers.

---

[5] With Power Eng. Co., Montreal, Que., Canada.

[6] Brig.-Gen., U. S. A.; Asst. Chf. of Engrs., U. S. A., Washington, D. C.

The author bases his deduction on the premise that the discharge varies with the square root of the fall between the two stations, and with a function of the arithmetical mean of the stages at the two stations. While he derives this premise from the Chezy formula, it might be based as well on the general law that the discharge in an open channel varies as the square root of the slope and as a function of the hydraulic radius, the latter, in turn, being a direct function of the river stage. If two stations are sufficiently contiguous, the slope at both will vary directly with the fall between them, and the discharge will vary as the square root of the fall. This condition will obtain, however, only when the fall between the stations is small in comparison with the river depth.

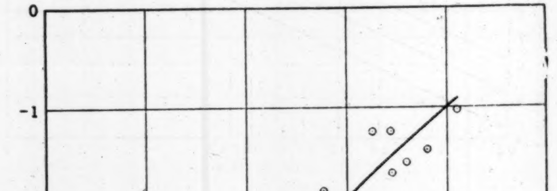


FIG. 9.

If sufficient data are available on the flow with a wide variation in fall with respect to the stage, it may be possible to derive a weighted mean of the upper and lower gauges which will afford a better measure of the equivalent hydraulic radius of the section than the arithmetical mean. In the usual case, however, the data are insufficient to justify anything but the arithmetical mean.

A diagram in the form presented by the author appears, therefore, to be based on correct hydraulic principles. The preparation of such a diagram is, however, quite a laborious procedure, and the taking off of discharge from the diagram involves estimation of proportionate intervals. A much simpler procedure is the construction of a discharge diagram for 1-ft. fall between the two stations. This diagram is used like any other discharge diagram, except that its readings are multiplied by the square root of the fall for which the discharge is desired—a very simple, slide-rule operation.

To construct such a diagram, each observed discharge is divided by the square root of the fall, and the quotient is plotted against the mean of the corresponding stages. Such a diagram, prepared from the author's data, is demonstrated in Fig. 9.

LYNN CRANDALL,[7] M. AM. SOC. C. E. (by letter).—The writer has been interested in this paper because of his experience in making discharge determinations under conditions somewhat similar to those described by the author. About 1916 he developed practically the same procedure in determining discharges on large irrigation canals in Idaho where the stage-discharge relation was affected by the operation of check-gates in the canal some distance below the measuring station. For convenience, however, the stage-fall-discharge diagrams were referred to one of the gauges only, instead of to the mean elevation between gauges as was done by the author.

The general method suggested by Mr. Blanchard, in various similar and modified forms, has been used to the writer's knowledge by many engineers in computing discharges under conditions of variable slope, and it is a valuable aid where such conditions obtain. Attention should be directed, however, to the fact that the method as outlined is applicable only to conditions where the channel cross-section and coefficient of roughness remain unchanged.

The usual stage-discharge rating curve may be affected at any time, not only by changes in slope but, also, by changes in the cross-section of the stream, or in the coefficient of roughness which, on many Western streams, results from the growth of moss and aquatic vegetation at certain seasons of the year.

The effect on the standard rating curve, at any station, of changes in cross-section or degree of roughness of the stream channel, can be determined effectively only by making current-meter measurements, and at stations of variable rating where accurate records are desired there is no general substitute for current-meter measurements carefully made at fairly frequent intervals.

If the standard rating curve is changed at any station, due to cutting or filling of the channel or to the growth of aquatic vegetation, the new rating curve will usually be approximately parallel in gauge height to the old curve over a considerable range of stage. Advantage may be taken of this fact to extend the use of the stage-fall-discharge diagram also to stations the ratings of which are affected by channel changes or growth of aquatic vegetation, in addition to the effect of variable slopes in the water surface at given stages resulting from back-water farther down the stream.

---

[7] Watermaster, Snake River Dist. 36, Idaho Falls, Idaho.

In such instances, first construct a diagram similar to the author's Fig. 7 for a standard cross-section and degree of channel roughness, referring it, for convenience, to one of the gauges only. From time to time, as current-meter measurements are made, plot them on this diagram, and the amount in gauge height by which they fail to plot on the proper fall curve is the correction factor to apply to the observed gauge heights before using the diagram to determine daily discharges. On alluvial streams this correction factor may vary considerably from one measurement to another, due to continuing changes in cross-section. The changes thus indicated may be applied, as to time, gradually from one measurement to the next, or the change may be concentrated at the time of sudden changes in stage between measurements in the manner customarily used in the so-called "shifting-channel" method of determining discharges at stations of variable rating.

BENJAMIN E. JONES,[8] ASSOC. M. AM. SOC. C. E. (by letter).—Back-water at river measurement stations has always been a source of complication to the hydraulic engineers of the U. S. Geological Survey, as it disturbs the relation of stage to discharge. An example of this effect was encountered at the station on the Tennessee River at Chattanooga, where the stage and slope are affected by back-water from Hales Bar Dam, by the operation of the power plant, by leakage under the dam, and by the operation of the flash-boards on the dam. To meet this situation M. R. Hall, W. E. Hall, and C. H. Pierce, Members, Am. Soc. C. E., at that time hydraulic engineers of the Geological Survey, developed a method of determining the discharge of rivers of variable slope.[9]

Their method is similar to that of Mr. Blanchard. Messrs. Hall and Pierce used two gauges to determine the slope or fall. They then selected a fall which they considered normal and for each current-meter measurement at the station they computed the flow for the so-called normal slope by the equation,

$$Q_n = \frac{Q_1}{\sqrt{\frac{F_1}{F_n}}} \dots\dots\dots\dots\dots\dots (9)$$

In this formula, Q_1 equals the actual discharge; Q_n, the "normal" discharge; F_1, the actual fall between gauges; and F_n, the "normal" fall. This is the same as Mr. Blanchard's Equation (4), but in slightly different form. A "normal" discharge curve is drawn through the points obtained in this way, and the daily discharge at any given stage is then found by the equation,

$$Q_1 = Q_n \sqrt{\frac{F_1}{F_n}} \dots\dots\dots\dots\dots\dots (10)$$

---

[8] Chf., Power Div., Conservation Branch, U. S. Geological Survey, Washington, D. C.

[9] *Water Supply Paper No. 345-E,* U. S. Geological Survey.

In applying the method the letter, z, was taken as representing $\frac{F_1}{F_n}$. A table was then prepared giving the values of z and $\sqrt{z}$ for each gauge height. The true discharge was obtained by multiplying the "normal" discharge by the $\sqrt{z}$ factor taken from the table. Mr. Blanchard obtains the discharge by a series of curves each of which shows the discharge for an assumed fall.

Both methods of procedure are based on the assumption that C in Chezy's formula is a constant for a given stage and section of channel regardless of the slope. The best information available on this point is that furnished by measurements by Mr. Blanchard on the Chicago Drainage Canal, which indicate that the slope has no effect on the value of C. These measurements, however, cover only one case, where the conditions were not as complicated as in a natural river channel, and they can not be taken as conclusive. Little fault can be found with the theory underlying either of the methods and they are being applied with good results at several stations maintained by the Geological Survey.

For several years Warren R. King, Assoc. M. Am. Soc. C. E., obtained the discharge at Chattanooga by the following method: A slope-velocity curve was determined by plotting mean velocities obtained from discharge measurements at Chattanooga against the total slope between Chattanooga and Hales Bar. A stage-area curve was determined by plotting areas obtained from discharge measurements at Chattanooga against corresponding gauge heights. To ascertan the daily discharge, the velocity was first obtained by applying the mean daily slope (Chattanooga to Hales Bar) to the slope-velocity curve. The area was then obtained by applying the mean daily gauge height for the gauge at Chattanooga to the stage-area curve. The product of the area and velocity thus obtained gave the daily discharge. This method gives satisfactory results except for low stages, when there is very little fall between the gauges. Theoretically, this method would seem to be applicable only where there is little change in stage.

A. W. Harrington, M. Am. Soc. C. E., supervises several gauging stations on canals where conditions similar to back-water are caused by growing vegetation. At these stations not only the slope, but the value of C changes, and he uses the Chezy formula directly in determining the daily discharge. The coefficient, C, is computed for each current-meter measurement and plotted against a time scale. A smooth curve, drawn as nearly as possible through the plotted points, shows the variations of C throughout the season and indicates the coefficients for intervening days. The other factors in the Chezy formula are obtained from gauge-height records and the cross-section of the canal. This method is said to give good results, except when the slope factor is too small to use satisfactorily.

For conditions of rapidly changing stage, C. E. McCashin, and E. D. Burchard, Assoc. Members, Am. Soc. C. E., have used a series of curves based on gauge height and discharge, with curves for various rates of rising and falling stage. These curves are based on actual measurements during periods of changing stage, without resort to any formula.

The writer devised a method of estimating the slope due to a rising or falling stage and then obtaining the discharge for normal conditions by the use of the Chezy formula. He assumed that a rise or fall in stage travels down stream at the same speed as the surface velocity of the water. This has been found to be true of streams that have an appreciable fall and velocity. In lakes or behind dams the change in stage probably travels as a wave at a high velocity, and the effect of any change of stage on the slope is negligible. The formula as developed is, as follows:

$$\frac{Q_1}{Q_2} = \frac{\sqrt{S_1}}{\sqrt{S_1 + \dfrac{\text{Rate of change of stage}}{\text{Surface velocity}}}} \quad \ldots\ldots\ldots\ldots (11)$$

in which, Q_1 and S_1 represent the discharge and slope at constant stage, and Q_2, the discharge at the given rate of change of stage. This method[10] has been used for many years with good results. Where large and rapid changes of stage are frequent it is better to install slope gauges and obtain the actual slope.

In using any method for computing stream flow in a section where back-water exists, it is necessary to study the local conditions that cause the back-water and to adjust the method to them. The method outlined by Mr. Blanchard is excellent and probably can be adapted to fit a variety of local conditions.

A. J. MATHESON,[11] ESQ. (by letter).—The writer has found this paper and the discharge diagrams presented therein, very interesting, particularly so because, since about 1916, he has been using a method of back-water computations which he developed, based on the same theory, but expressed by a different fundamental, or base-discharge curve, and extended upon somewhat different lines. Computations by either system, if based on the same precise data, should give exactly the same result.

In preparing his base-discharge curve for the example illustrated, Mr. Blanchard arranged the observations in groups (Table 2), in each of which the mean stage was approximately the same. He then plotted the mean elevation of each of these groups against the mean of the corresponding computed falls for a discharge of 8 000 cu. ft. per sec. The writer would suggest plotting the result of each individual observation, in obtaining the curve, because, by this process, any computed data which are obviously inconsistent can be eliminated and the general trend of the relations can be more closely analyzed. The conditions existing in the example he used for illustration are decidedly sensitive, since most of the recorded falls are well under 1 ft.

In the application of this method to any particular case, the field data required consist of a number of discharge measurements covering the range of flow, together with the corresponding records of the elevations of the water surface at the upper and lower ends of the reach. Since the accuracy of the final computations and curves is dependent upon the precision of these data, it would seem advisable to stress the following points:

---

[10] *Water Supply Paper No. 375*, U. S. Geological Survey.

[11] With Dominion Water Power and Hydrometric Bureau, Dept. of the Interior, Ottawa, Ont., Canada.

(1) There must not be any variation in the volume of water flowing through the channel from the upper to the lower gauge, due to streams flowing into it or channels leading out.

(2) The gauges must be tied in to a common datum so that the precise fall in the water surface between the two gauges may be obtained.

(3) The gauges, if possible, should be located so that during the time in which discharge measurements are being taken, there will be a fall of at least 2 ft. in the water surface between them. As the computed discharges are dependent on the square root of the fall, an error of, say, 0.10 ft. will cause a greater discrepancy with a total fall of 0.5 ft. than it would if the fall were 2 ft., or more.

(4) The gauges should be located so as to be as free as possible from the effect of local disturbances, such as eddies, wind, etc.

(5) The actual distance between the gauges is not important, as long as the flow is uniform throughout the channel.

(6) It is not essential that the cross-section of the channel should be the same throughout its length. It is essential, however, that there shall be no change in the channel cross-sections, such as erosion or silting of the sides or bottom, during or after the observations.

(7) Care should be taken to see that, during the time that measurements are being taken, the flow in the channel is not fluctuating and that the backwater conditions are not changing.

(8) This method of computation is not applicable to free falls, but its use is not confined to a straight-line, smooth water surface, between the recording gauges. The writer, using his own diagrams, computed the elevation at seven intermediate gauges along the rapids of the St. Lawrence River near Morrisburg, Ont., Canada, when levels had been taken after an ice jam had raised the water $10\frac{3}{4}$ ft. at the lowest gauge and 4 ft. at the upper gauge. His computed elevations at these seven gauges varied from an exact check, to a maximum difference of 0.14 ft. from the actual levels recorded by two independent parties of Government engineers. For the same flow under summer conditions the fall through these rapids is 10 ft. in a distance of about 3 miles.

L. STANDISH HALL,[12] ASSOC. M. AM. SOC. C. E. (by letter).—This paper offers a solution to a troublesome hydraulic problem frequently met in the measurement of stream discharge.

The influence of a water surface with variable slope on the discharge relation affects the results on streams more frequently than is ordinarily appreciated. When the change in slope is quite marked this condition is readily identified. However, in many instances, the effect of the variation in slope is so slight that it is confused with a shift of control. In cases where a variable surface slope is combined with a shifting control, the separation of the two effects is often quite difficult.

---

[12] Chf. Hydrographer, East Bay Municipal Utility Dist., Oakland, Calif.

A control is effected on a stream, when the water surface proceeding in the direction of flow changes from a flatter to a steeper slope. Some controls are so slight that they are "drowned out" when higher stages are reached, and the condition of variable surface slope is introduced.

Before classing a station as one affected by a variable surface slope, a careful check on the condition of the intake pipe to the recorder well should be made, since a partly plugged intake pipe can create the effect of larger discharges on rising stages than on falling stages.

In Fig. 10 is shown the discharge curve for a part of the measurements taken on the Mokelumne River, at Woodbridge, Calif., for the year 1927. The separation of the curves for rising and falling stages is easy to distinguish. The interpretation of measurements at this station is further complicated by shifts in the control. The points plotted on the graph have been adjusted to represent times when the condition of the control was particularly uniform.

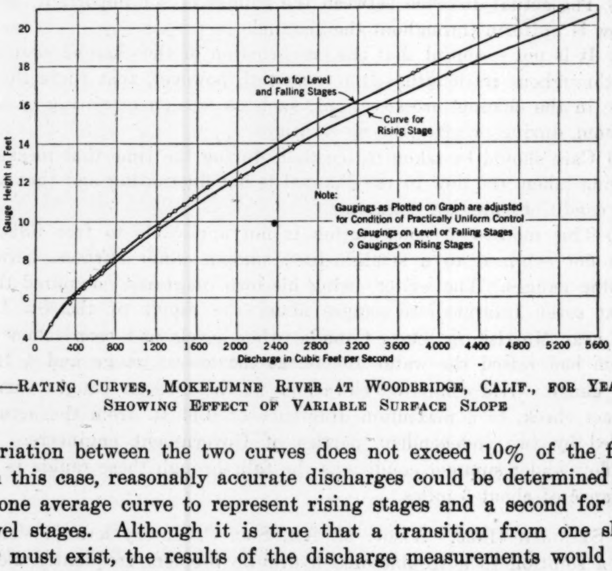


FIG. 10.—RATING CURVES, MOKELUMNE RIVER AT WOODBRIDGE, CALIF., FOR YEAR 1927, SHOWING EFFECT OF VARIABLE SURFACE SLOPE

The variation between the two curves does not exceed 10% of the flow, so that, in this case, reasonably accurate discharges could be determined by the use of one average curve to represent rising stages and a second for falling and level stages. Although it is true that a transition from one slope to another must exist, the results of the discharge measurements would appear to indicate that such transition periods are of brief duration.

RAY E. MACKENZIE,[13] ASSOC. M. AM. SOC. C. E. (by letter).—The author has afforded a good opportunity for discussion on the subject of back-water effect at a measuring station. While he has simplified the problem of determining the discharge at a station subject to back-water, further simplification is desirable. The form taken should be that which best suits the immediate purpose. If this is the reduction of gauge heights to daily discharge figures, there can be no better arrangement than the table. This must result from a graph, as all rating tables do, and will consume considerable time in preparation. However, it must be remembered that the task of reduction to discharge is the main one, and it is here that simplicity, speed, accuracy, and facility for checking are of paramount importance. The rating table is the common and accepted means of reduction, and, therefore, a simple form of it should be the more logical method of transfer.

---

[13] Associate Hydro-Elec. Engr., U. S. Engr. Office, St. Louis, Mo.

The use of separate tables, one for the "normal" discharge and the other giving values of $\sqrt{z}$ would produce the desired result from a single multiplication, but two interpolations might be necessary. A tabular representation of Fig. 7, with columns headed "Fall, in feet," and lines to read "Elevation below Chicago City Datum," would condense the data into a single table. With attention given to the probable accuracy of the data, the table could be made so that hasty interpolation could be accomplished to the percentage of accuracy required.

Numerous applications have been made of the principles involved in the paper. For some time the writer has used the principle in the reverse condition, that is, for back-water computation. The problem is the familiar one

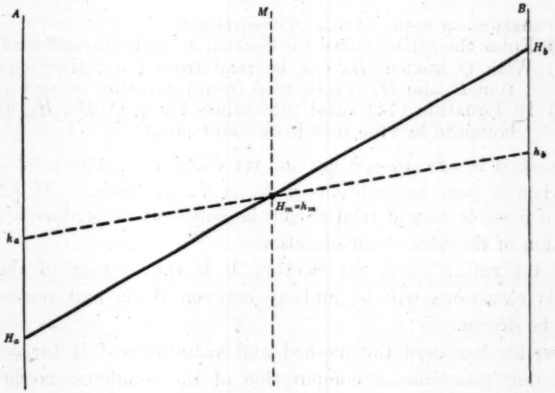


FIG. 11.

with back-water above a dam. The discharge to consider is known and the solution is for the back-water elevations at various points up stream.

Natural flood elevations and discharges are determined from field surveys with attention paid to the elevation and discharge at the time of the survey and to the probable point of zero flow. Often a satisfactory rating curve can be made and extended from as few as three points on full logarithmic paper.

A study of sections that form major and minor controls for various stages of flow will materially help the drawing of the curves and their extension to the elevations of probable back-water. Separate rating curves are drawn for each section forming an end of a reach. While these should be made so as to show properly the stage of zero flow (or approximately), it is very helpful in studying the curves and their extensions to draw the rating curves on a common scale of elevation. On such a graph the intersection of curves will show a level slope between the sections represented.

In Fig. 11, let F from Equation (2) equal $H_b - H_a$; in Equation (3) let $F_x = h_b - h_a$, and $q = Q_x$. Then, if Equation (12),

$$Q = K\,(H_b - H_a)^{0.5} \quad\text{.....................(12)}$$

is used as the formula for natural flow, and Equation (13),

$$q = k\,(h_b - h_a)^{0.5} \quad\text{.....................(13)}$$

for back-water flow, and, furthermore, if h_m is made equal to H_m, then k equals K, and the author's Equation (5) becomes,

$$h_b - h_a = (H_b - H_a)\,\frac{q^2}{Q^2} \quad\text{...................(14)}$$

In back-water computations, q is known and the point, h_a, is determined from the computed rise or back-water at the dam or obstruction. Then, if the assumption can be made that $h_m = H_m$, a working basis will be found to proceed with the back-water computation. The order follows:

(1) Assume a value of h_m that equals H_m.
(2) Enter the rating curve for Section M with H_m and read Q.
(3) With Q known, H_b can be read from the rating curve for Section B, and H_a can be read from the rating curve for Section A.
(4) In Equation (14) substitute values for q, Q, H_b, H_a, and h_a; then compute h_b (the new back-water point).

The method is, of course, a cut-and-try one, but it gives good approximate results when a poor assumption of $h_m = H_m$ is made. If refinement is required a possible second trial should be sufficient to produce a satisfactory computation of the back-water elevation.

Since the rating curve for Section M is the average of the B and A curves, its elevations will be midway between them, and the curve for M need not be drawn.

The writer has used the method and variations of it for several years. He finds that sometimes a computation of the roughness coefficients for a few of the natural and back-water elevations is helpful in confirming the computation. He believes that the method, where it can be used, enables a useful and practical solution of back-water computation.

MURRAY BLANCHARD,[14] M. AM. SOC. C. E. (by letter).—The writer is gratified and pleased with the discussions of his paper. They contain suggestions to simplify the procedure and indicate numerous uses that have been made of the principles and theories involved. The arguments relate to the different features or steps in the derivation of the discharge diagram and its uses, and will be treated separately.

The Base Curve.—Mr. Macphail proposes a substitute curve of K, referred to stage and discharge, to be used as a base from which, by Equation (2), are obtained the curves of the discharge diagram (Fig. 7). The base curve is derived from Equation (2) transposed to,

---

[14] Cons. Engr., Chicago, Ill.

$$K = \frac{Q}{F^{0.5}} \dots\dots\dots\dots\dots\dots\dots\dots\dots (15)$$

General Pillsbury proposes the same curve in different terms. His is a curve of 1-ft. fall (Fig. 9), obtained by making F unity in Equation (4), whence,

$$Q_x = \frac{Q}{F^{0.5}} \dots\dots\dots\dots\dots\dots\dots\dots\dots (16)$$

and $K = Q_x$.

Mr. Perry uses the same relation (Equation (2)), as the others and obtains a diagram of K curves referred to discharge and fall. These are, in effect, a series of base curves, but the procedure is lengthened by the necessity of determining the valves of (A), (C), and (R), in order to obtain a discharge corresponding to a given stage and slope.

Mr. Matheson does not state, but in correspondence with the writer he has revealed, that his method, developed about 1915, makes use of the 1-ft. fall curve (Fig. 9), and he calls it the "unit-fall" method. His suggestion to plot the resulting unit-fall discharges derived from each observation, in order to note any inconsistencies, is not considered by the writer as a necessary step because at any elevation all the observations, if they gave true discharges, would reduce to the same quantity. Comparison of these figures will enable one to detect any inconsistencies and will permit the elimination of abnormal observations. The arithmetic mean will give the best determination of a point on the base curve. The points plotted to determine the curve of Fig. 9, as well as the points for the base curve, Fig. 6, are groups or arithmetic means.

The K and the unit-fall base curves are the same as the writer's 1.0-ft. curve of fall in Fig. 7 and, as a slightly shorter operation, he would suggest the substitution of Fig. 9 for Fig. 7. The resulting discharge diagram will not be effected.

The Chezy Formula.—The references of Mr. Perry to the Kutter formula do not appear to be pertinent to the method adopted in the derivation of the discharge diagram for the reasons stated in the paper, that, "for a single stage under consideration the factors, (A), (C), (R), and (L), are constant" and are eliminated in the derivation of Equation (4) or Equation (5) from Equations (2) and (3). If Mr. Perry chooses to use his diagram (Fig. 8), the writer would suggest, for the determination of C, the Manning formula or the Kutter formula with the s terms omitted, which gives practically the same results, as proposed[15] by H. W. King, M. Am. Soc. C. E.

In the Chezy formula, $V = C\sqrt{RS}$, the factors, V, C, R, and S, are the means for the reach under consideration. Because the discharge is often measured at an end station, the velocity or hydraulic radius at this end station is frequently used as if it were the mean for the reach. Such an approximation will ordinarily give satisfactory results, as in Mr. Jones' method of correcting discharge for changing stage.

---

[15] "Hydraulics," by King and Wisler, 1922 Edition, p. 190, and "Handbook of Hydraulics," by Horace W. King, Second Edition, 1929, p. 261.

Normal Discharge.—This is a term of general usage that serves a useful purpose in the investigation of certain stream flows; but it is not for the general application that is sometimes given to it. It has been explained[16] as the discharge at a stage which would be caused by a slope that is the average of all the slopes that obtained for the period of record. Such a slope may never occur at the stage under consideration, especially if this is a flood stage. Such discharges should be labeled "normal," and the period of record and average slope used should be noted. Mr. Jones states, in reference to the Hall-Pierce method, which is based on Equations (9) and (10), how the true discharge was obtained by multiplying the 'normal' discharge by the $\sqrt{z}$ factor taken from the table." This "true" discharge is the one corresponding to the stage and slope of an observation, and the "normal" discharge curve from which it is obtained, is, in effect, a "base" discharge curve. There may be similarly obtained for the same stage, a large range of "true" discharges, depending on the various slopes that occur. Thus, the Hall-Pierce method is similar to that of the writer and is also applicable on streams to which the rating curve applies as well as where a range of slopes occurs at the various stages. The situation on the Tennessee River, at Chattanooga, which the "normal" stage method was designed to meet, is one in which the discharge diagram can be used to obviate the necessity of approximations.

Reference to Other Than Mean Stage.—The method of Mr. King, to which Mr. Jones refers, and that of Mr. Crandall, wherein discharges are referred to the stage at one station instead of the mean elevation of the reach, are approximate methods, but with a considerable field of adaptability. The writer has similarly used discharge diagrams, referred to a station at the end of a reach, to advantage in studying flood flows of the Lower Illinois River. The approximation comes in the assumption that the stage at other than the mean station varies as the mean stage. This assumption does not hold in a case in which there is artificial regulation above and below a reach, and hence it does not always follow that, for a single (other than mean station) stage under consideration, the factors (means for the reach), (A), (C), (R), and (L), are constant.

Application to Back-Water Computations.— Mr. Mackenzie has made application of the stage-slope-discharge relation to the determination of the back-water curve. He applies it to the condition of a stream in which there is only one discharge corresponding to a stage. A calculation can be made that would be applicable to all open channels, by a "cut-and-try" method with the discharge diagram. Thus, referring to the Chicago Sanitary District Canal, for which there is no established rating curve—and upon which the experiments were made that were used in this paper to demonstrate the derivation of the discharge diagram—the following is an example, using Mr. Mackenzie's notation and Figs. 11 and 7.

---

[16] *Water Supply Paper No. 345,* U. S. Geological Survey.

The problem is as follows: While flowing 8 000 cu. ft. per sec. in the reach from Willow Springs (B) to Lemont (A), the water surface is raised to — 4.85 at (A). What will be the resulting water surface elevation at (B)? The solution is:

	First trial	Second trial
Assume $F_x =$	1.1	1.0
From Fig. 7, or Table 3, $h_m =$	—5.04	—4.35
$\frac{F_x}{2} =$	0.55	0.50
$h_m - \frac{F_x}{2} = h_b =$	—5.59 (too low)	—4.85 (O. K.)

From the second trial, which checks for 8 000 cu. ft. per sec., the assumed fall, added to h_a, gives — 3.85, the required elevation.

TABLE 3.—Rating Table for Discharge Diagram (Fig. 7)

Fall, in feet	Discharge at Stage (Elevation Below Chicago City Datum), in Cubic Feet per Second							
	—0.90	—1.59	—2.28	—2.97	—3.66	—4.35	—5.04	—5.73
0.1......	3 580	3 260	3 020	2 830	2 670	2 530	2 410	2 300
0.2......	5 060	4 620	4 280	4 000	3 780	3 580	3 420	3 260
0.3......	6 200	5 660	5 240	4 900	4 620	4 380	4 180	4 000
0.4......	7 260	6 540	6 050	5 660	5 340	5 060	4 830	4 620
0.5......	8 000	7 310	6 760	6 320	5 960	5 660	5 400	5 140
0.6......	8 770	8 000	7 400	6 940	6 540	6 200	5 900	5 660
0.7......	9 460	8 650	8 000	7 480	7 050	6 700	6 380	6 120
0.8......	10 110	9 240	8 540	8 000	7 550	7 160	6 820	6 540
0.9......	10 750	9 800	9 150	8 480	8 000	7 600	7 230	6 920
1.0......	11 300	10 300	9 640	8 950	8 440	8 000	7 620	7 310
1.1......	11 890	10 900	10 000	9 380	8 850	8 400	8 000	7 650
1.2......	....	11 300	10 480	9 800	9 250	8 750	8 350	8 000
1.3......	....	11 700	10 900	10 200	9 600	9 110	8 700	8 340
1.4......	....	....	11 300	10 600	9 970	9 470	9 020	8 650
1.5......	....	....	....	10 960	10 320	9 810	9 350	8 950
1.6......	....	....	....	11 300	10 660	10 120	9 650	9 250
1.7......	....	....	....	....	11 060	10 400	9 950	9 540
1.8......	....	....	....	....	11 300	10 700	10 220	9 800
1.9......	....	....	....	....	....	11 000	10 500	10 080
2.0......	....	....	....	....	....	11 300	10 800	10 320
2.1......	....	....	....	....	....	....	11 080	10 600
2.2......	....	....	....	....	....	....	11 300	10 820
2.3......	....	....	....	....	....	....	11 580	11 200
2.4......	....	....	....	....	....	....	....	11 300

Rating Table.—As stated by Mr. Mackenzie a rating table may be desirable. In Table 3, note that, in each stage column, the discharges, 8 000 cu. ft. per sec., are the points where the base curve (Fig. 6) intersects the "fall" lines at 0.5 ft. to 1.2 ft. The stages thus obtained, — 0.90 to — 5.73, were used as convenient elevations along which to transpose the discharge-stage-slope relation. If Fig. 9 is used instead of Fig. 6 for a base curve it would be more convenient to use the even foot stages, — 1.0 to — 6.0.

Discharge Diagram.—From the variety of practical uses that can be made of the discharge diagram it would appear, generally, to be worth the effort to develop. General Pillsbury has suggested that the base curve (Fig. 9), is sufficient and that, instead of developing the discharge diagram, he would calculate from the curve, for given data, the discharges as required. The other extreme is taken by Mr. Mackenzie who would construct the discharge diagram and from it make a rating table. The writer concurs with him that "the form taken should be that which best suits the immediate purpose." From the Chicago Sanitary District Canal experiments the writer produced,[17] several years ago, a diagram of discharge referred to the elevations of the end stations. Mr. Matheson has developed a stage-slope-discharge relation of this form by the use of unit-fall base curves and has made extensive application. His suggestions as to precautions to be taken in making discharge measurements for this purpose are pertinent.

---

[17] "Hydraulics of Chicago Sanitary District's Main Channel," *Journal,* Western Soc. of Engrs., September, 1920, opp. p. 506.

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
TRANSACTIONS

Paper No. 1808

LEGAL PROBLEMS INVOLVED IN ESTABLISHING SET-BACK LINES[1]

BY CLIFTON WILLIAMS,[2] ESQ.

WITH DISCUSSION BY MESSRS. J. M. ALBERS, J. E. WILLOUGHBY, HORACE H. SEARS, GEORGE H. HERROLD, CLIFFORD C. MUHS, H. B. COOLEY, WALTER C. SADLER, H. J. MCFARLAN, DONALD M. BAKER, AND E. G. WALKER.

SYNOPSIS

When fire, flood, disease, or other calamities threaten the people of a community, National, State, and personal property may be sacrificed without compensation. This paper calls attention to a new idea, based on the age-old function of sovereignty; namely, that if narrow streets in a community become so dangerous as to require widening for the preservation of public health and safety, the acquirement of property for such widening is properly a function of sovereignty.

If the Courts could be convinced by engineers that street-widening problems are indeed functions of sovereignty, the tremendous financial problems encountered at present would be solved. Set-back lines could be established on narrow streets by giving due notice that in ten, twenty, or thirty years from that date the property thus segregated would revert to public ownership. In the meantime, the private owner would have time to make the necessary adjustments to meet the requirements of such a law.

INTRODUCTION

The term, "set-back lines", as used in this paper, refers to those established for the purpose of future widening of streets for traffic and not to the so-called set-back lines employed in zoning which were established to mark bounds beyond which buildings shall not be constructed.

Later, the so-called zoning lines or building lines are mentioned in connection with some Supreme Court decisions which established the validity of such lines, without compensation to the abutting owner. The set-back line for street widening is only one step farther and is as clearly within the field of governmental sovereignty as the other form.

---

[1] Presented at the meeting of the City Planning Division, Milwaukee, Wis., July 11, 1929.

[2] Special Asst. City Attorney, Milwaukee, Wis.

The main purpose of this paper is to help engineers realize that the Courts are not trifling with this subject and that lawyers are not trifling with the Courts when they maintain that the street-widening problems of America are just as much problems of sovereignty as fire-fighting, quarantine to prevent the spread of disease, the abatement of nuisances, the struggle to prevent the spread of crime, etc.

POLICE POWER AND SET-BACK LINES

This effort is prompted by repeated remarks made to the writer by city planning engineers and others who are devoting the best years of their lives to this work, to the effect that they are "in a fog" in attempting to understand the legal somersaults that are apparently going on throughout the United States. Even law students do not understand the term, "police power", and they do not understand clearly how there can be a phase of sovereignty involved in this problem, which is older than any constitutional provision on the subject.

If this legal problem is solved along historical lines and if the Courts are finally convinced that the tremendous street-widening problems of this country are problems of sovereignty, and if the decisions are quite uniformly announced to that effect, then the great financial problem that is involved is also solved.

In other words, if the Courts would uniformly hold that the great American problem of widening streets to take care of present and future traffic needs is a problem of sovereignty, then as a matter of sovereignty the streets and highways can be widened the same as buildings can be destroyed to prevent the spread of fire or the same as a high-salaried man can be quarantined to prevent the spread of disease. All these things are done under a recognized governmental power which is older than any Constitution. Under such circumstances the property that it is necessary to acquire, has not been taken for a public use under any clause of any State or Federal Constitution and it is not necessary to pay compensation to the abutting owner.

The mere statement of the proposition creates resistance and the writer proposes not only to show how historically sound it is; but to show how close some of the Courts are to a correct solution of the problem.

No space will be devoted to a statement of the necessity of widening streets and highways throughout this broad land. Engineers are more familiar with that phase of the question than the writer, who will just take it for granted. The enormous financial problem involved also requires no further discussion other than a re-statement of the fact that the correct solution of the legal problem at the same time solves the financial problem. Of course, the financial and legal problem is twofold because now it is not only necessary to re-build the old "horse-and-buggy" streets into automobile highways, but it is also necessary to plan for future requirements, along the streets which are being laid out, with the tremendous growth of self-propelled vehicles in mind.

THE ENGINEER IN RELATION TO LEGISLATION CONCERNING SET-BACK LINES

Without any reflection on engineers the writer knows from personal talks with them that quite generally they are not familiar with the Constitution of the State in which they practice. Ordinarily, the engineer realizes somewhat vaguely that there is a clause in his State Constitution as well as in the Federal Constitution to the effect that private property shall not be taken for a public use without compensation. The engineer is quite thoroughly convinced that the widening of a street and the establishment of a set-back line involves the taking of property from an abutting owner for a public use and, for that reason, compensation is necessary. Then he gasps at the billions and millions of dollars that are required to do what ought to be done to the present "horse-and-buggy" streets in order to convert them into automobile highways.

Engineers can do a great service by compiling and broadcasting throughout the land an estimate of the billions of dollars which it would take to put the highways of the United States in a safe condition for automobile traffic. A few years ago the writer drew the first ordinance for regulating the heights of buildings in Milwaukee, Wis. Twenty years previously (1910) the regulation of building heights in this country was practically unknown. Legislators were still adhering to the old principle that a man had the right to do with his own property what he pleased as long as he did not reach over on to his neighbor's property and commit some active trespass. The Courts had already recognized that he might reach over actively either by creating offensive odors or a nerve-racking noise; but up to the time of the Milwaukee ordinance no one had admitted that the regulation of building heights was a function of sovereignty. At about that time American engineers offered the statement that there were enough people on Lower Manhattan Island at ten o'clock in the morning to fill the streets to the top of the fourth floor windows if, for any reason, the people should rush into the street at the same moment. That statement startled the country and the Courts began to realize that the regulation of building heights was in fact a function of sovereignty. Since then they have held that the municipal units have the right to say that building heights shall only reach certain limits for the simple purpose of preventing more people from getting into a district than can be accommodated in the streets.

What the writer wishes to emphasize is that if the engineering societies of this country will inform the Courts of the billions of dollars that are required at the present time to put the "horse-and-buggy" streets in a safe condition for automobile traffic and, at the same time, tell them of the tremendous loss of life and damage to property that is constantly taking place, due to the fact that such streets are not safe for automobile traffic, these organizations will thereby do much to cause the Courts to realize that the solution of this great traffic problem is a function of sovereignty.

There is nothing new or novel in this suggestion. The writer does not intend to do any violence to the clause in any Constitution which, in round phrases, states that private property shall not be taken for a public use without compensation. All he suggests is that the Courts, through the engineers, should be made to realize that the great traffic problem of this country to-day is as much a problem of sovereignty as fire-fighting, quarantine to prevent the spread of disease, the peremptory destruction of nuisances, entry upon private land to construct levees to prevent great rivers from destroying property, and the various other phases of recognized sovereignty.

The writer knows from personal experience that engineers and city planners have had more to do with the establishment of the legality of the building heights regulations and zoning laws than the legal profession—in that engineers and city planners furnished the facts, figures, etc., and, in some instances, even submitted non-technical briefs to the Courts. Furthermore, engineers can do more than lawyers in convincing the Courts that the present tremendous traffic problem (which is at the bottom of the requirement of set-back lines to street widening) is a phase of sovereignty.

The lawyer will simply declare that it is a struggle between the police power and the Constitutions of the various States. The term, "police power", is a very unfortunate one. It suggests bluecoats, brass buttons, and billy-sticks, or something of that kind, when, in fact, it means the power of government or sovereignty, or the power of a people to preserve itself.

PRIVATE PROPERTY FOR PUBLIC USE WITHOUT COMPENSATION

Mere words are often misleading unless the reader is able to go back of them, into history, and know the fundamental principles. Consider the Federal Constitution and the Constitutions of all the States. They contain a composite provision that private property shall not be taken for public use without just compensation. The word, "property", offers no difficulty. Some difficulty may be encountered with the word, "taken", but it is necessary to trace the meanings even farther back. Every one knows that even before the establishment of any American colony, Governments were taking property as a phase of sovereignty without compensation, and they are still doing it. For illustration may be cited: Quarantine; the abatement of public nuisances; the destruction of contaminated food; the destruction of valuable buildings to prevent the spread of fire; the confiscation of gambling devices; and the peremptory right to enter upon private land and repair levees, which is quite common throughout the Southern States.

Many other illustrations of sovereignty might be cited, most of which are older in principle than any Constitution in this country. These powers were extended to the very existence of sovereignty in its simplest form. They were all recognized before America was discovered as being phases of the power of government. When the Fifth Amendment to the Federal Constitution was written, which stated plainly that private property shall not be taken for a public use without compensation, no one had in mind the prevention or stopping of the further exercise of this recognized phase of sovereignty. The Constitution of the State of Wisconsin was written in 1848. Section 13 of Article 1 reads as follows: "The property of no person shall be taken for public use without just compensation therefor." Every one will admit that in those words there was no intention to abolish quarantine; there was no intention to prevent the destruction of buildings to stop the spread of fire; and there was no intention to abolish the power of abating nuisances. The Wisconsin Courts have recently ruled that the sovereignty of the State may say to a man that he may not put a certain kind of a building in a residence district (Carter *vs.* Harper, 182 Wis. 148); that he cannot construct his building beyond a certain height in a certain district; and that he cannot build within 15 ft. of the street on either side of a corner lot (Hayes *vs.* Hoffman, 192 Wis. 63).

If a person can be quarantined thus causing him to lose his salary without compensation, or if a Government can destroy his property to prevent the spread of fire, or if his land can be seized without compensation for the purpose of constructing a levee to prevent the spread of a river, then it should be possible to convince the Courts that the United States is confronted with a very dangerous traffic problem, the solution of which is necessary to existence. After that it is not a long step for the Courts to state that this traffic problem can be solved as a phase of sovereignty, and that the acquisition of the necessary property is not a prohibited taking within the constitutional provisions, any more than the other confiscations.

The Courts have already held, and are now uniformly holding that, as a phase of so-called police power or as an element of sovereignty, a State may prevent a property owner from building above a certain line or beyond a certain distance from the street line. It is not asking them to go much farther to require that they recognize that the present traffic problems and the necessity for street widening are indeed phases of sovereignty, and that the execution thereof can be carried out without compensation, the same as quarantine or the destruction of property for fire-fighting purposes.

Largely because engineers expect the law to be an exact science, they look upon the growth of such problems in the Courts with suspicion. They find certain words in a Constitution and expect to go out and erect a series of legal decisions with those words in the same manner as they use a plumb-bob to erect a perpendicular. The writer would urge that engineers be patient with the Courts and not lose their respect for them even if they do find them about-facing in a proposition of this kind. It is only natural that twenty-five years ago judges did not state that the automobile traffic was a problem of such seriousness that it was a phase of sovereignty, because then it was not such a problem. Is it now not a fact that the problem has become so nationwide, that the loss of life and destruction of property are so tremendous, and that the present effort to use the old-style streets for self-propelled vehicles is creating so many difficulties and hazards that, in fact, the problem has become one of sovereignty? In widening streets and providing for the solution of the problem, the power of government must support the undertaking as, in the past, it has supported the effort to combat disease and the spread of fire.

CITATION OF LEGAL DECISIONS IN WISCONSIN

In the case of Hayes *vs.* Hoffman (192 Wis. 63), decided in 1927, the validity of the set-back line was established under a zoning ordinance. There is only a short step from that zoning set-back line to one that would result in the widening of those very streets for traffic purposes. The defendant owned a vacant lot on the northwest corner of a street intersection and the plaintiff was a neighbor across the street on the southwest corner. At that time, Milwaukee had a zoning ordinance which had not yet been interpreted by the Courts; it required a 15-ft. set-back line in that residence district. City officials had construed the ordinance to mean that the 15-ft. set-back line was applicable only to the front of the property and not to the side; in other words, that it would be applicable in that case on the street passing between the two properties. The defendant began building operations under that interpretation of the law. The plaintiff lived across the street and at the end of 31st Street so that if the defendant built within the 15-ft. line along the side street he would obstruct the view of the plaintiff. The latter enjoined the building operations on the ground that, by building to the 15-ft. line along the side street, the defendant obstructed his view, and the Supreme Court held that the 15-ft. set-back line applied to both sides. This left the defendant a piece of ground that was too small for building anything of value, but the City of Milwaukee paid him nothing, because it was established in Wisconsin that the zoning ordinance is an element of sovereignty.

In other words, it was enacted under the so-called misnomer, "police power". If, in this case, the defendant was unable to build on this 15-ft. strip on either side, has not his property been taken? And if that restriction is placed there for air, light, and sanitation, or for any other reason which can be written into a zoning ordinance, then was not his property taken for a public purpose? Nevertheless, there was no compensation, because the so-called "taking" was under a phase of sovereignty, which is older than any Constitution of this land. The defendant in this case was merely in the same position in which he would have been if buildings on these 15-ft. strips had been dynamited to prevent the spread of fire. He had no more claim against the public for compensation than if he had been quarantined away from a $25 000 job for two months to prevent the spread of smallpox.

This case can be used to define a line of demarcation between "the taking for a public use" as mentioned in the various Constitutions which requires compensation, and "the taking" in the case of Hayes *vs.* Hoffman which was undeniably a confiscation, but which comes under the head of sovereignty and does not require compensation. It is admitted that the owner cannot put any buildings on either one of the 15-ft. strips. He can run a lawn mower over both of them. He might have flower beds in them, but he cannot even build a dog house on either one of them. If the City permits automobiles to run over either one of these 15-ft. strips, the City apparently would have to pay the owner if it was done under any municipal recognition, although if automobiles gradually began driving over both the strips and kept it up for twenty years, the highway will be established without compensation. If the City should go in and run a lawn mower over these 15-ft. strips, or put in flower beds, there might be a claim that the City had made a park out of them and for that reason would have to pay compensation. The question is that narrow. The City in this very case prevents the owner from building over these 15-ft. strips so that air currents can pass up and down, so rays of sunlight may pass in, and so that the view of the street is improved; and this is all done without compensation. It is a short step to the claim that the City may permit automobiles to pass over the 15-ft. strips the same as light rays or air currents. The automobile traffic in either of the streets may be so tremendous that the City, in the exercise of its sovereignty, may class the problem as one that is just as dangerous to the people as any of the others.

Several years ago the Wisconsin Legislature was bold enough to state in one of these laws[3] that a set-back line as a notice of future street widening could be established without any compensation to the abutting owner.

The set-back line may be established without compensation; its establishment is not considered equivalent to the taking of property, although it does provide that the owner who builds over the line does so at his peril and cannot recover any damages for the building beyond the line. The validity of this statute has not been litigated in the Courts although it has been on the statute books for several years. The establishment of the set-back line has many elements of fairness, and it is not as harsh as quarantine, the destruction of buildings to prevent the spread of fire, or the destruction of farms to prevent the spread of a river. Another law recognizes the necessity of paying compensation to the abutting owner because the other law was drafted a great many years ago as a part of the charter of the City of Milwaukee, and in that age when the seriousness of the automobile traffic problem had not yet been realized.

The history of the world shows that sovereignty knows no limits. When put to the ultimate test it recognizes no property rights. The self-preservation of a people is a law unto itself. A written Constitution is an agreement between the then existing units of people who make it a sovereignty concerning the exercise of the phases of government mentioned therein, and there is not in existence a single Constitution which contains an abdication of all phases of sovereignty not mentioned therein.

A set-back line which serves to notify a property owner that within a reasonable time in the future it is apparent to the authorities that the governmental sovereignty will be called upon to expand a street for the solution of this great threatening problem, is much fairer to the property owner than the sudden destruction of his building to prevent the spread of fire or the peremptory and sudden quarantine of his body to prevent the spread of disease. The cry that generally goes up in the United States is that this is the richest country in the world and that the Government should pay and pay well for these strips of land necessary to convert the out-of-date streets into automobile traffic highways. This country may be the richest nation in the world; it may have the richest group of States in the world; but this problem is not a National one, although it is nation-wide, and it is not a State problem, although it exists in every State in the Union. It is a problem that must be financed ordinarily by the local municipal units, such as towns, cities, and villages, and it is a well-known fact that such municipal units are habitually financially embarrassed.

The writer has had some experiences with street widening that have changed his views considerably concerning the harshness that might be apparent at the first mention of the necessity of taking over this problem as a phase of sovereignty. In Milwaukee there have been many instances in which the remnants of property left over after street widening, were worth several times the original value of the entire lot before the widennig. Admittedly, there will be cases of hardship when the Courts are brought to realize that the necessity of street widening is a case of sovereignty (which means it will be done without compensation for the land taken); but there are hardships in every phase of the contact between the private citizen and the administration of public duties. In his twenty years of experience in municipal law the writer has been forced to recognize that even taxes are unequal; that the jury duty burden does not fall equally upon the public; and that the support of the public schools cannot be made equal. In fact, not one single burden in the public field can be said to fall equally upon the people. The statement that there may be hardships in the proposed solution of the problem does not condemn that solution. For each case of apparent injustice and hardship where there may be a set-back line in a necessary widening of the street without compensation to the abutting owner, there will be dozens of cases in which the widening of the street will create enormous additional property values. The advantage of the set-back line is that it can be so slowly and gradually established as a street line as to remove most of the injustice. It might even be provided in such a law that, if there must be a "taking" within five years of the establishment of the set-back line, there will be compensation; but that there should be no compensation after a 5-year notice of the establishment of the line. On some streets the advance notice might even be extended to ten years.

---

[3] The Wisconsin statute which claims that the set-back line can be established without any compensation to the abutting owner is Sub-Divisions (10) and (11) of Section 62.23, of the General Statutes.

CASES OF THE EXERCISE OF GOVERNMENTAL SOVEREIGNTY

Suppose the City of Milwaukee should serve notice on an owner that five years from date his house would be blown up to prevent the spread of a dangerous fire which it is known would happen. That would give the owner plenty of time to move and re-adjust himself. And, again, suppose the City of Milwaukee should say to a man that he would be quarantined in five years for three weeks to prevent the spread of a smallpox epidemic which the municipal authorities know is going to occur. The comparison does not need to be followed further. If it is recognized that the correct solution of this great traffic problem is a phase of sovereignty, then this set-back law provides for instrumentalities for removing the hardships which are incident to the very cases in which the Government does exercise its sovereignty without any compensation whatever.

As the United States continues to grow and its problems become more complex, a great many of the old ideas which have been instilled into the minds of its people will have to be discarded. People seem to like to say that "a man's house is his castle". They seem to enjoy saying that this is a country of liberty and a man has absolute ownership of his property. What is ownership? Nothing is more mythical than a title. Title is nothing but a gift of sovereignty. If title is claimed to a piece of real estate in America to-day, what is the nature of that claim? It is merely an evidence that some one took the land away from the Indians as a matter of military strength, or by some other method of that kind, and that as a matter of sovereignty there is, at the beginning of the chain of title, a patent from the Government. That title is no stronger than the sovereignty under which citizens are living.

The very existence of the title depends on sovereignty. In fact, the title is simply an instrument of sovereignty, because the Government with its Courts is what makes the title to land good, so that, after all, there is not much hardship in permitting sovereignty to deal with that title or to restrict it so as to comply with the requirements that spring up in the self-preservation of a great people. For instance, consider the statement that people like to "roll under their tongues" to the effect that "a man's house is his castle". When two castles are built close together things begin to happen. For instance, one castle owner cannot dig down on the property line so as to disturb and cause the soil of the neighboring castle owner to fall in. As a result the easement of lateral support has grown up. One castle owner cannot run a tannery on his property so as to offend the nose of his neighbor. Things gradually developed so that he was not allowed to create other nuisances, such as disturbing noise. When a castle owner lived all alone he could throw his garbage out on the surface of the ground, but when other castles commenced to be built in the neighborhood he found that his so-called liberties were being restricted. As the castles continued to spring up, their owners found that they could not build so close to the property line as to shut out light and air; and in the great cities buildings cannot be constructed so as to attract more people to the district than can be accommodated safely. It is not going very far to state that, since it is recognized that a castle owner must give up many of his so-called rights, such as in the case of zoning and building height restrictions, he will likewise have to give up land so that streets may be expanded to solve one of the greatest problems that has ever confronted any country.

CONCLUSION

American Courts will soon hold uniformly that the solution to the traffic problem is a phase of sovereignty and that private rights shall give way, as they must, to fire-fighting, abatement of nuisances, prevention of crime, the operation of levees, and to those other forms of sovereignty that are happening every day. When that time comes the public will accept it the same as they accepted zoning, building-heights regulations, billboard ordinances, health regulations, ordinances confiscating contaminated food, laws confiscating gambling devices, laws permitting the restraining of floods, and other recognized phases which are accepted without question and without any compensation whatever to the people who are being called upon to submit to the exercise of the sovereignty.

DISCUSSION

J. M. Albers,[4] **Esq.**—It appears from a study of the Court decisions affecting zoning, and particularly from some of its special phases, that the legality of regulations creating building set-back lines is established. This statement can be accepted without undertaking a lengthy discussion of it, because supporting decisions may be found all the way from the United States Supreme Court down to the Lower Courts.

In the Chicago, Ill., Region, city engineers and planners are especially interested in the legality of set-back line legislation from the standpoint of its possible application to street widening. They are well aware of the contention that such an application of this principle encroaches upon the realm of eminent domain in that it may constitute a taking of private property for public use without compensation. There seem to be plenty of advocates on both sides of the case of police power *versus* eminent domain when street widening is being discussed. Is not an owner deprived of the use of his property by a zoning set-back line just as much as he is by a set-back line for future widening? Is not the establishment of a building set-back line on a future wide street just as much in the interest of public health, safety, convenience, and welfare as any other set-back line? The preservation of these things is the objective of police power. The actual taking of land can be left until the physical work of widening is to be undertaken. In the last few years the application of the police power has found a much greater field of usefulness in zoning legislation, and the large number of zoned communities in the United States bear witness to the success and propriety of this form of regulation. Practically all the existing zoning ordinances contain set-back provisions, and these are established in the interest of public health, convenience, and welfare.

Streets, adequate as to location, width, point of origin, and destination are of tremendous importance to proper civic development. Public safety and welfare are perhaps, to-day, more dependent upon the proper provision of streets than upon any other single function of government. The rapid growth of urban communities is continually demanding streets wider than those originally laid out. The larger the community the more difficult it is to secure the wider rights of way. The cost of condemnation is in many cases a greater burden than the community is able to bear and, therefore, these highly necessary improvements are delayed and sometimes abandoned to the detriment of the entire community.

At the time when he published the plan for the City of Chicago, the late Daniel H. Burnham recognized the advantage of set-back lines for future street widening and recommended that the city acquire sufficient front-yard space to prevent future encroachment. If the necessity for wider streets is admitted and, at the same time, if the fact that the cost of condemnation will prevent much necessary improvement also is admitted, then city planners must seek further for a more economic method of accomplishing this work. The establishment of set-back lines now, under the police power, is the only practical method by which the widening of many traffic arteries can be secured in the future when greater width will assuredly be required. It is painless, economical, and has the decided advantage of making necessary a comprehensive plan for future street improvements and civic development.

---

[4] Sub-Div. and Zoning Engr., Chicago Regional Planning Assoc., Chicago, Ill.

The speaker believes that sufficient foundation has been laid to warrant some procedure along this line. The United States Supreme Court has ruled (Gorieb *vs.* Fox) that,

> "It is hard to see any controlling difference between regulations which require the lot owner to leave open spaces at the sides and rear of his house and a regulation which requires him to set his building back a reasonable distance from the street".

The Cleveland, Ohio, set-back ordinance has been sustained as a proper exercise of the police power, as was that of the Town of Windsor, Conn. In the last-mentioned case the Court decided that such legislation was not in violation of constitutional rights. The Courts of Wisconsin, California, Ohio, and several other States have all sustained set-back regulations. In the case of Kaufman *vs.* City of Akron, Ohio, the Court declared that the burden is upon the plaintiff; it is for him to show that set-back lines bear no substantial relation to public health, convenience, and welfare, before they can be declared invalid. In another Ohio case (Pritz *vs.* Messer) the Court ruled that the validity of the set-back law must be viewed in the light of probable future development. The speaker does not believe that any of the outstanding set-back decisions bear specifically on the use of set-back lines for street widening, but an array of opinion is being built up which, it is hoped, will finally sustain it. The construction of costly buildings in the bed of new streets must be prevented if there is to be any hope of progress toward adequate street systems. It appears that the regulation of the use of property to protect a major street plan is acceptable as to purpose, but there is a difference of opinion as to whether or not this should be accomplished through the police power or the right of eminent domain.

An examination of the Court decisions to date indicates that set-back laws are rapidly being sustained by the high Courts of the land as a proper exercise of the police power, and that resort does not have to be made to the power of eminent domain with its attendant prohibitive expense. If such legislation is worked out by the governing body on the basis of a well-defined policy, after a careful study of the entire area affected, and after due notice and hearing so as to avoid all charges of discrimination, arbitrariness, and unreasonableness, the Courts will not interfere with such legislation. Such enactments will speedily justify their own existence, and there is a growing sentiment in favor of them in the Courts as well as in public opinion. When such legislation is predicated upon public health, safety, convenience, and welfare, it is brought within the scope of a proper exercise of police power.

In the Chicago Region many communities are now protecting future street widenings by the present establishment of set-back lines for this purpose.

Some of these communities have passed ordinances creating such lines on specific streets, others are obtaining wider streets through agreement between the property owners and the city and village government. Still others have included the major street system with its required set-backs as a part of the zone ordinance. This latter seems to be the best method in that the city then has the opportunity to use the machinery set up in the Board of Zoning Appeals and this body is in a position to adjust those difficulties which are bound to arise in so comprehensive a piece of legislation.

J. E. WILLOUGHBY,[5] M. AM. SOC. C. E. (by letter).—One purpose of the written Federal Constitution is to limit the powers to be exercised over the people by Federal authority. That particular sovereignty advocated by the author is denied by the Federal Constitution, but (in the original draft) not to the extent which the debate in the various States (particularly Virginia) on its adoption developed was necessary for the protection of the people. Hence, ten amendments were introduced immediately after the Constitution had become effective and, in due course, they were adopted. Of these, the Fifth Amendment had particular reference to protection of private property against seizure for public purposes. This protection was further strengthened by the later adoption of the Fourteenth Amendment.

What the author is suggesting is a change in the Federal Constitution to be procured through such propaganda as will induce the Courts to issue decisions to the effect that private property, if taken under the guise of the police power, is not a taking such as is prohibited by the Fifth and Fourteenth Amendments. It may be assumed that the author is of the opinion that his purpose cannot be attained by repeal of the Fifth and Fourteenth Amendments although such a repeal probably would authorize public authority to take private property for public uses without compensation and without due process of law. The writer is not in sympathy with the suggestion. The exercise of the police power is properly used to further the comfort and safety of the people. That comfort and safety can be obtained without confiscation of private property.

The arguments advanced by the author as warranting a Court decision for the extension of the police power to include the taking of private property without compensation are not sound in that proper distinction is not made as between the principle which permits the lawful limitation of the height of a building and denies as unlawful the taking of an area of land occupied by the building. Every land owner of the fee of a part of the surface of the earth has the right to use it for lawful purposes, provided that such use does not take away in any particular the property rights of an adjoining owner. Among such property rights are sunlight and air. Whenever the height of a building destroys the right of another to full enjoyment of sunlight and air beyond the area covered by the building, then the height of that building can be fixed by law so as to remove the invasion of such a full enjoyment. The regulation of height of a building is not the taking of the property of the owner thereof, but is a prevention of the owner of the building from taking the property of another; which "another" may be the user of the public streets. The quarantine of a man with a contagious disease is not the taking of that man's property in form of wages, but is the prevention of that man from taking away the right of his neighbor to have freedom from infection.

---

[5] Chf. Engr., A. C. L. R. R., Wilmington, N. C.

The municipal authorities under the police power can properly decree a set-back line for building construction, and can forbid the use of the area between the set-back line and the edge of the street under the lawful principle of preventation of the lot owner from using his property to the detriment of others. So long as the municipality refrains from using as municipal property any part of the area between the street line and the set-back line, no property is taken; the rights of the adjoining others are protected, and nothing more. The actual conversion of such area to street purposes is the taking of private property for public uses which the Federal Constitution now forbids. The rights which develop from long-continued use are conditioned on another principle of law—open and notorious possession and use presume title in the holder and the user. Such is not a taking under the police power.

The fact that property needed for street widening is valuable and may cost the public much money, can never be a warrant for seizure without compensation. Possibly such was the thought and practice of Attila.

HORACE H. SEARS,[*] M. AM. SOC. C. E. (by letter).—The inclusion of "set-back lines" in future street-widening zone laws is worthy of consideration for cities where this future need is anticipated. The police power exercised as an incident to present zoning legislation has been limited by Court decisions to future, as distinct from past, activities by the owners of vested property rights both real and personal.

State and Federal Constitutions assure the owner of real property rights that no retroaction laws shall be passed. The claim concerning the effect of destroying vested corner-lot rights in the larger cities by legislating arbitrary destruction of improved property values through the exercise of police powers incident to new zoning laws, as proposed in this paper, is fallacious. The attempted deduction of the legality of thus interfering with vested rights of property owners as being in line with fire and flood prevention, involves the same fallacy as attempting to prove that a dog is a horse for the reason that a dog is an animal and a horse is an animal, hence, they are equal.

It is here submitted that the selection of the decided case of Hayes *et al v.* Hoffman (192 Wis. 63), and the same case (211 N. W. 271, decided in 1927), is not "on-all-fours" with the argument stated by the author for the following reasons stated in a syllabus of that opinion; that is, the Lower Court, upon trial, found that the proposed apartment building owned by the defendant would violate the provisions of the zoning ordinance in overstepping established lines on all four sides of the property. Furthermore, the area to be occupied by the apartment house as well as the area occupied by the defendant's frame house at the rear of the premises, was greater than the maximum permissible area allowed under the terms of the ordinance.

---

[*] Cons. Engr. and Attorney-at-Law, Hastings-upon-Hudson, N. Y.

In its finding, the Court states:

> "It is our own conclusion that the ordinance is, in the respects here considered, a reasonable, valid, and constitutional enactment. It is appreciated that there are other provisions of the ordinance the validity of which may be the subject of future challenge. It is to be understood that no opinion is expressed with reference to any features of the ordinance except such as are herein treated."

The Appellate Court further states (192 Wis. 63) concerning the failure of defendant to plead his constitutional rights:

> "It is the duty of a person who claims that an ordinance invades rights guaranteed by the Constitution to raise that question at his earliest opportunity. Taking part in a proceeding which fixes his liability under the ordinance without raising any question as to constitutionality of the ordinance is a waiver of the right to raise that question subsequently.
>
> "The plaintiffs were given an opportunity to meet this issue (as to constitutionality) in the Court below. The record was made up without reference to that claim made here for the first time. We cannot entertain it."

No attempt will be here made to discuss the author's suggestion under the heading, "The Engineer in Relation to Legislation Concerning Set-Back Lines", or that there is need for engineers to inform Courts as to the enormous cost of putting city streets which are congested by motor traffic in safe condition; nor for engineers to inform the Courts that street relief sought in "set-back lines" is "as much a problem of sovereignty as fire-fighting, quarantine, flood relief, nuisances, etc."

The exercise of the right of eminent domain and the use of recent legislation in street widening through the medium of excess condemnation gives far more assurance of stability to property values and hence to collection of taxes; and if these recognized remedies for the exercise of the sovereign power of the State are properly conducted, the financial relief will be had which Mr. Williams indicates is much needed.

Zoning laws directed to future possible taking of realty for street widening, leave values inert, and the improvement of city street frontage in a stagnant condition. The decided cases are numerous which uphold the contracts that a person makes with the State, or its agencies, such as a municipal corporation. Built-up blocks with narrow streets which require widening, come within the classification of vested rights, the disturbance of which by reason of new legislation under the guise of "exercise of the police power" invites costly and prolonged litigation.

The exercise of the police power and the interference with vested rights of realty owners in connection with zoning laws and city ordinances is discussed by the Court at great length in the recently decided suit of Jones *v.* City of Los Angeles (Cal. Supreme Court, December 31, 1930).[7] The opinion is replete with citations of decided cases and the rules of law controlling the use of the zoning laws, police powers, and the retroactive effect incident thereto. Said the Court, in part, in its concluding statements:

> "No general test can be derived from these or other cases, all of which reiterate the well-worn doctrine that each one must be decided on its particular facts. But we think that there is a decided difference between the ordinance

---

[7] Reported in the *U. S. Daily*, February 2, 1931, p. 8.

which operates to limit the future use of property, no matter how great the impairment of its value, and one which requires the discontinuance of an existing use. * * *

"If the city desires to abolish the non-conforming use, this may be a legitimate object of the police power, but the means of its exercise must not include the destruction of the property interest with compensation. The words of Mr. Justice Holmes in Penn Coal Co. *v.* Mahon, 260 U. S. 393, 413, are very much in point; 'As applied to this case the statute is admitted to destroy previously existing rights in property and contract. The question is whether the police power can be stretched so far.'

"Even where the destruction of private property is warranted by the vital public necessity, it is sometimes the legislative practice to compensate the injured owner. Such was the legislative intent in the eradication of bovine tuberculosis by the destruction of diseased cattle (see Patrick *v.* Riley, 79 Cal. Dec. 379).

"Our conclusion is that where, as here, a retroactive ordinance causes substantial injury and the prohibited business is not a nuisance, the ordinance is to that extent an unreasonable and unjustifiable exercise of the police power."

George H. Herrold,[8] M. Am. Soc. C. E. (by letter).—Establishing set-back lines to create front yards is a common procedure under zoning. They are not difficult to establish because the people like set-backs in residential districts. It is done under the legal fiction of providing light and air and of promoting health and general welfare. Taking a bath is theoretically a matter of personal hygiene, but practically a bath is taken because it makes one feel good; a person likes it. Home owners like set-backs.

Set-back lines for the purpose of widening streets come under a different legal fiction. It is said to be a taking of land for public use without compensation. Mr. Williams' contention is that both are within the field of Government sovereignty; that if set-back lines can be established to create front yards for light, air, health, etc., they can be established for the purpose of widening streets because of congestion; that if limit can be established for the height to which a building can be built by zoning, the ground area used can be limited also. Following this same line of thought, the writer would suggest that, as the speed of vehicles is now limited in congested districts in the interest of safety, it might also be possible to determine and limit the number of vehicles and people that would be permitted to use a given street in the interest of safety.

The street-widening problems of the cities of the United States are staggering. Costs of street widening range from $1 000 000 to $9 000 000 per mile, omitting such projects as Michigan Boulevard and the Wacker Drive, in Chicago, Ill., and the Fairmont Parkway, in Philadelphia, Pa., which cost far more than this. These costs are due to methods of procedure in condemnations. These methods make possible the building up of excessive values for the property to be taken. The values are so great that they absorb the assessments for benefits, legal fees, and other items, and make the citizens at large (through bond issues) pay a profit to the owner. Later, the owner also receives the benefit of the increases in value that always follow a street widening. This method cannot go on.

---

[8] Managing Director and Engr., The City Planning Board, St. Paul, Minn.

The City of St. Paul, Minn., has a retail street, 60 ft. wide between property lines, 40 ft. of pavement between curbs, 10-ft. walks, and a double-track car line occupying 19 ft. of the 40-ft. roadway. Real estate values on this street are the highest in the city. Congestion is intolerable and has been so for years; no more people or vehicles use the street to-day than ten years ago; they cannot because there is no room. Three department stores on this street have aided their own situation by arcading, each placing a 16-ft. walk in their building parallel to the street sidewalk. The Planning Board made a study of the cost of widening this street. It found that if by agreement the property owners would widen the street to 86 ft., each paying his own costs, cutting back his building, putting in a new front, remodeling the interior, building a new 15-ft. sidewalk and paying for widening the pavement 8 ft. in front of his property to create a 56-ft. roadway, the actual cash outlay in round figures would be $1 000 000. The owner would be reimbursed in a few years by the increased value of his property and increased rentals due to his property having greater accessibility on the widened street.

If this street was widened by the City under the power of eminent domain, assessing benefits and damages, the cost would amount to $6 500 000. This is the cost of the present procedure under methods which are common practice where land and buildings are acquired under condemnation proceedings.

Referring again to Mr. Williams' interesting discussion on the use of sovereignty: It takes a long time to change an established legal precedent. This, therefore, seems to the writer to be the proper time to call attention to a proposal made by C. E. Grunsky, Past-President, Am. Soc. C. E., before the City Planning Division of the Society a few years ago. He suggested establishing a building line for widening the street, condemning the strip of land lying between this building line and the street line separate and distinct from the buildings, paying the owners for this land, but compelling each owner to pay the city 6% interest on the amount paid him as long as his building stood on the land. When the owner sets his building back to the new line, he would stop paying interest. In the meantime, the city would have been paying, say, 4½% on the bonds which it floated to pay the owners for the land.

The City of St. Paul has a provision in its charter for carrying out the first part of the procedure, that is, the condemnation of land separate and distinct from the buildings thereon. The objection to using this method is that the owner of the building then has the use of the land for which the city has paid until such time as the Council may take physical possession, and if the City takes possession before the owner has seen fit to set his building back, it must then pay for the building. Mr. Grunsky has thought this idea through to a more logical conclusion, and his proposal is worthy of more consideration than has yet been given it.

CLIFFORD C. MUHS,[*] M. AM. SOC. C. E. (by letter).—The usual legal bias seems to be that the earth was created to demonstrate the beauties of the Illinois statutes. The viewpoint expressed in Mr. Williams' paper is so different from this that it is decidedly refreshing.

---

[*] (Muhs & Co.), Chicago, Ill.

Obviously, the more industrialized (which must also apparently mean the more thickly settled and migratory) a people become, the more surveillance is required to prevent irresponsible parties from taking advantage of their comparative obscurity. Among nomadic peoples anarchy can, and often does, exist as an ideal system. Among pastoral peoples, some faint beginnings of rules of conduct are discernible although there may be nothing more ponderable than public opinion to enforce them.

As civilization progresses, the code of conduct must increase in scope and complexity with a corresponding increase in economic life. It must be extended so as to protect the average person from the ruthless, irresponsible, or obdurate individual whom public opinion cannot now effectively curb.

Legislators, who have more frequently been chosen for their dynamic personalities (real or assumed), have been quick to recognize this trend. If their ability and familiarity with the ordinary economics of life had equalled their zeal, there might now have existed a fairly workable code in spite of the ability of organized minorities to advance their own interest in law at the expense of that of the public.

In the judicial branch of Government the case is different because the candidates are selected for entirely opposite characteristics to such an extent that they might be designated the "high priests of the objectors". To the utter negation of all progress, these objectors seem to abide by the slogan, "Whatever is is right. What has not been done cannot be done." Such men would say that justice is immutable and what was just in Lycurgus' day was just now, ignoring what justice might be essentially.

With such an attitude, legal procedure must always lag a century behind that of its contemporaries, thus becoming indeed the check upon Government it is often proclaimed. Such an apposite as that taken by Mr. Williams is, therefore, the more worthy of lay support, and of such support undoubtedly the most effective should be that of the engineer. The engineer is as little concerned with the past as the lawyer is preoccupied with it, and his hypothesis is that "anything can be accomplished."

In special assessment work the writer has had ample occasion to view the chronic objector in action, even when his interests are being advanced by the project to which he objects and when the compensation in money and enhancement far outweighs the loss.

If then the Bench can be induced to view, with less sympathetic eye, those who see in obstructionist tactics merely another chance to advance their own interests at the expense of others, it will be a consummation devoutly to be wished.

H. B. Cooley,[10] Assoc. M. Am. Soc. C. E. (by letter).—The tendency of the Courts to treat the establishment of set-back lines as a function of the police power of the municipality may be traced through a series of decisions of the Supreme Court of the State of Louisiana.

---

[10] New Orleans, La.

In 1915 (Clavo *v.* City of New Orleans, 136 La. 480, 67 So. 338), and, again, in 1917 (State *ex rel* Blaise *v.* City of New Orleans, 142 La. 73, 76 So. 244), the Supreme Court of Louisiana held that the Charter of the City of New Orleans did not give the Municipal Council the authority to enact a so-called zoning ordinance. The sections relied upon were those which granted the usual police power to a municipality. The following year the Legislature passed an Act enabling cities of more than 50 000 inhabitants to make such laws to regulate the kind, style, and manner of constructing buildings on certain streets and to permit or prohibit the establishment of business within designated limits. The scope of this Act was enlarged and its provisions were extended to all municipalities, regardless of size, by the Constitution of 1921.

Shortly afterward, a case reached the Supreme Court, which originally had been brought under the ordinance declared unconstitutional in the Clavo Case, but which also came under a later ordinance passed after the Act of 1918. On this occasion, the Court held that,

"If the municipal council had not enacted Ordinance No. 6689 but had relied upon Ordinance No. 5599, we might, in respect for the expressions of the legislature and of the Constitutional Convention, and in the light of recent decisions of the Supreme Court of the United States and of several State courts of last resort, depart from the rulings made in Clavo's Case and repeated in Blaise's Case, and maintain that Ordinance No. 5599 was within the authority granted in the City's Charter and was a valid exercise of the police power".

While it was not necessary for the Court to over-rule its previous decisions, at the same time, it shows clearly a much broader view on the subject of the police power as related to zoning laws. (State *ex rel* Civillo *v.* City of New Orleans, 154 La. 271, 98 So. 440.)

In 1928, the question of a set-back ordinance was raised. This ordinance prohibited the City Engineer from issuing a permit for the erection of a building within 15 ft. of the property line of St. Claude Avenue, one of the principal thoroughfares of New Orleans, between certain streets. The Lower Court declared the ordinance unconstitutional, because the power to enact such a regulation was part of the power of eminent domain, and, therefore, could not be exercised without payment of adequate compensation to the owner for the taking of his property for public use or benefit. The Supreme Court in over-ruling the decision of the Lower Court held that a set-back ordinance does not constitute a taking of private property for public purpose under the power of eminent domain, but, like the so-called zoning ordinances, is merely an exercise of the ordinary police power. (Sampere *v.* City of New Orleans, 166 La. 776, 117 So. 827.)

In the last case the Court cited Gorieb *v.* Fox (274 U. S. 603). A city ordinance of Roanoke, Va., created set-back lines to be at least as far from the street as the line occupied by 60% of the existing houses in the block. In this case the term, "block", denotes that portion on the same side of the street as the proposed building and bounded by nearest intersecting streets to the right and left of the building site. The Supreme Court of the United States held that there was no difference between ordinances prescribing the height of buildings to be erected and the area to be left open for light and air and in aid of fire protection, which have been held valid under the Federal Constitution, and ordinances which required an owner to set back his house a reasonable distance from the street. (In this case the set-back was equal to 34 ft.).

In two recent zoning cases (State *ex rel* Dema Realty Co. *v.* McDonald *et al* 168 La. 172, 121 So. 613; and State *ex rel* Dema Realty Co. *v.* Jacoby 168 La. 752, 123 So. 314), the Court upheld an ordinance which required business to liquidate and move out of the zoned area within one year, as it did not deny defendants of property without due process of law.

In view of the tendency shown by the cases cited, it does not seem a very "far cry" to the time when, as Mr. Williams suggests, such set-backs may be created. Owners of buildings erected over the set-back line will then be given a limited time to move back the proper distance and after that period the property will be used by the municipality under its "police power", for the purpose of handling the ever-increasing and, consequently, more dangerous traffic problem.

WALTER C. SADLER,[11] M. AM. SOC. C. E. (by letter).—This paper is an exceedingly interesting presentation of a proposal to extend the police power of the State to the acquisition of property for the widening of congested thoroughfares. The present discussion will be confined to two aspects of this problem: First, the legal issues involved; and, second, the desirability of such a doctrine.

It is quite clear that the police power has never been used in this type of condemnation of land, or property rights, because all cases of such power bear some element of emergency, where time of action is of the essence. In the case of removal of oyster-bed stream obstruction to stop an epidemic, it is imperative that the obstruction be removed at once without recourse to the slower alternative of applying a suitable chemical to the water. In the case of fire, it is necessary to destroy the intermediate block at once, to prevent the spread of the conflagration; and this is also true in the case of the destruction of gambling devices, to prevent the spread of this vice. Each overt act is taken to prevent a greater harm to the public, with immediate action essential. The nature of the remedy suggested by Mr. Williams bears no such time element of emergency. In fact, the suggested program involves from five to twenty years for securing the proposed result. Surely that period of time is sufficient in which to find some other engineering solution of the problem, such as the construction of an elevated highway, subway, or detour. In other words, the police power of the State is an emergency device with no proper application to a long-term project.

The only justification for the exercise of this power lies in the preservation of public life. Surely it could not be applied to the case of "public convenience", or the doctrine could then be extended to grade-crossing eliminations, etc. If the doctrine is to be applied to the preservation of life, wherein lies the emergency of a remedy twenty years in the future?

---

[11] Associate Prof., Civ. Eng., Univ. of Michigan, Ann Arbor, Mich.

The case of levee work in the South appears to bear out the doctrine suggested by Mr. Williams, but even this group of cases does not seem applicable, for the land there is held under the Napoleonic Code. In that territory, a man holds the property with the understanding that his rights in land are limited in this respect. In the present case of set-back lines, the abutting owners on the congested thoroughfare purchased their property secure in the knowledge that their rights would not be taken without compensation except in emergency cases. The situations are easily distinguishable.

The "zoning case" cited presents a particularly interesting situation. It is a clear case of a taking of property. In fact, it is just as much a taking as though the State had established an easement for highway purposes, the owner retaining the fee. However, the results obtained are due to the peculiar shape of the ground, and its location, and the precedent does not seem to be a very strong one upon which to justify the taking of extensive abutting property rights. Certainly the zoning of 15 ft. on the lawn of a lot in the center of a block would not be suggested as justification for the taking without compensation of that 15 ft. for road purposes.

It is the writer's understanding that Mr. Williams is not trying to establish his doctrine as existing law, but merely as a near step to existing law. The present discussion is offered merely to question the proximity of his doctrine to existing law. It is felt that the taking of property without compensation is quite a long stride.

It would seem appropriate to question the very desirability of the taking of land without compensation. It is certainly shocking to the American concept of property rights. The inviolable nature of property is accepted throughout the land, and the average American understands the essence of the protection. In fact, it is so thoroughly embedded in the American concept of government that should the Courts adopt Mr. Williams' theory and interpret the law as authorizing the taking of this property without compensation, every law-making body in the United States would come to the aid of the abutting land owner.

Mr. Williams has suggested that the problem is an extensive one, involving enormous sums of money, and offers this as a partial justification for the plan. For every square foot of land acquired there must have been an equal amount lost by some individual. The expense to the community is scarcely a justification. In one place it was stated that the cost must be borne by the municipality; that, in the final analysis, the problem is not one of State or Federal concern. The Courts have stated that grade-separation problems are distinctly of State concern. The law itself takes cognizance of this situation. For instance, statutes in New York provide that the State pay 49% of the expenditure and the county 1% in the grade-separation project within a city. Other States contribute in varying amounts. The same relationship should apply to the widening of existing streets used as State trunk lines. It is certainly logical in view of the amount of automobile weight taxes and gasoline taxes, which go directly into the State Treasury. It would appear that the Engineering Profession might do better to sponsor a re-apportionment of the expenditure of these enormous State taxes, than to endeavor to secure legal sanction to the establishment of the doctrine of "taking of property without compensation".

Mr. Williams should certainly be commended for his able and interesting presentation of a proposed solution to a serious transportation problem.

H. J. McFARLAN,[13] ASSOC. M. AM. SOC. C. E. (by letter).—In this paper Mr. Williams is making an appeal for engineers to lend their influence in convincing the Courts to accept his "new idea, based on the age-old functions of sovereignty". If engineers have and are to maintain any influence it is only because they may have some ability in making an adequate analysis and a reasonable solution of the problems that come to them. It is the purpose of this discussion to consider further the function of sovereignty, to refer in a general way to some of the cases cited by Mr. Williams, to suggest a different point of view, and to submit an argument in its defense.

There can be little question about the right or power of sovereignty to do whatever is necessary for the public good; but there can be many questions as to what constitutes the public good. That the function of sovereignty extends back beyond the time of any formulated expression, is true. In fact, it might be called a "natural" function, first exercised unconsciously when men began to live together, and first exhibited when the power of the group was exercised coercively.

That the widening of streets is a matter of serious public concern is not denied. This admission is also a concession that this problem is a function of sovereignty. More than that, it is agreed that it is a function of sovereignty (and a necessary one) to solve all serious public problems, and that these are rapidly increasing in modern society. Little more than a century ago highway construction and maintenance were not recognized as of public importance. To-day, sovereignty is exercised in varying manner and degree ranging from actual ownership and control of highways to regulation and quasi-regulation of numerous other public utilities.

A change in the nature of vital functions can be foreseen in the future. Many now considered as private, will be recognized as public. If industry is speeded to the point of producing nervous wrecks, sovereignty will intervene. The beginning phase of this is already appearing in safety laws, the regulations of working hours for women, the establishment of food laws, and numerous other types of regulatory measures. If existing economic devices do not or cannot be made to assure the means of livelihood to the masses, sovereignty will re-adjust the devices. These are implications that must be drawn if one is to consider seriously the functions of sovereignty. These are things that the engineer should be concerned about because, unless there is some careful and precise thought exerted, the adjustments are likely to be made in a bungling way. Sovereignty is a "natural" power and Nature has little concern for economy, as men conceive it. If some measure of human economy is to be secured it is by directing sovereignty in such a way that the predictable effect will be desirable. It is the business of engineers working in the physical field to direct forces to the accomplishment of desirable ends. The attempt to interest engineers in getting an understanding of the problems of sovereignty is laudable.

---

[13] Asst. Prof. of Surveying, Univ. of Michigan, Ann Arbor, Mich.

The legal attitude is to seek authority in the shape of precedent to justify a conclusion. Cases in which sovereignty confiscates rather than condemns are cited in support of confiscation for street-widening purposes. It would seem that the analogies between cases cited and the proposed confiscation are not entirely sound. The Fourteenth Amendment to the Constitution of the United States is a formulation of a principle that was recognized before constitutions, as being necessary to the continuance of law. The demand on the part of laws for socially acceptable conduct is not in contravention of that principle. Neither can it be expected that by law one can be protected against the vicissitudes of Nature. It is true that as society has become more complex there has been a growing tendency toward restricting the use of private property. If an increase in social complexity makes it necessary seriously to restrict, or to confiscate, as a common thing, sovereignty may readjust values to the extent that all rights in property may become social.

The taking of property for street widening would seem a step in this direction. It does not seem analogous to the prevention of fire, flood, or disease, and even seems somewhat different from zoning. The precedents cited are of two classes. One is of the emergency type due to a condition caused by what is known in law as "an act of God." The other is a demand for socially acceptable conduct. There is great necessity when a building has to be destroyed to save the spreading of fire. The right of a quarantined man to expose the community is frowned upon socially. The right to build a factory in a residence district is also denied. As will be argued subsequently, social insurance for the unfortunate ones suffering loss might not be unreasonable. If a set-back line is established before property is developed, this is of the nature of zoning. This, in effect, is done now through the power exercised by planning boards. No one arguing for zoning would be so bold as to suggest the destruction of factories to provide a residence district. The demand for socially acceptable conduct is vastly different from upsetting values that have accrued through legitimate use. The proposed confiscation is not necessary because of an accident of Nature, it is not a demand for the regulation of conduct (in an active sense), and it is fraught with dangers because the "when" and "where" of its being practiced is within the control of men. This latter fact also makes it different under the "equal rights" law from all the cases cited, except perhaps zoning, and that is not retroactive, so to speak, against improvement.

The business of the engineer is to conceive the desirable and then to set about to find ways and means of accomplishment. He does not ignore prejudice and precedent, because an account of these must be taken in making his evaluations. Neither does he accept what has been done as authority for new endeavors. To him the worth of an idea is dependent upon the effect it will have. This is what is known in the terminology of modern writers on the philosophy of science as the operational point of view. This is the point of view that has speeded the production of the marvelous creations of modern science. It is responsible for deliberate invention in contrast to accidental discovery.

It is true that an engineer works in the so-called field of the exact sciences. It does not follow, however, that the operational point of view cannot be held when dealing with problems that have to do with human reactions. Indeed, the difference between the two fields of endeavor is due probably only to the difference in precision with which the data are obtainable. One frequently reads or hears an appeal made for the engineer to lend his aid to the solution of some grave problem. If he is to have an effectiveness comparable with what he has had in the solution of his own problems, it will be because he carries over his scientific type of thinking to this other field. It is not conceivable that he would be likely to be more successful with lawyers' law, sociologists' sociology, or economists' economy than men who have made these studies their life work. It is within the realm of imagination, however, to think that the engineer—because he is active in a field that demands that the worth of an idea be tested by the effect it has—may be able to lend valuable aid to those engaged in the solution of social problems. Unless he is cognizant of the philosophy of science, he is likely to lend his support upon recommendation of the specialists in the other fields, and thus bring no more influence to the solution of the various problems than any other citizen.

The burden of the argument in favor of confiscation has been carried by a citation of precedent, aimed at persuading that the solution is in a realization of the fact that it is a problem of sovereignty. It seems like a begging of the question to assume that confiscation is the only remedy that sovereignty suggests.

The engineer, when called upon to attempt the solution of modern traffic problems, must consider all factors of the situation. He may doubt the wisdom of widening as a solution. He certainly can foresee that there are limits of the extent to which widening may be carried out. He must think about the feasibility of decentralization and its economic consequences. Granted that confiscation might expedite the widening programs, the engineer realizes that this might not be unmitigated good. There are limits beyond which almost any developmental change cannot be made to economic advantage. Congestion in crowded districts is already leading in a natural way to the development of new business areas in many cities. Therefore, engineers who are aware of their social responsibilities should not be misled into developing an enthusiasm for a point of view which apparently considers only one phase of the problem. The necessity for confiscation is predicated upon the fact that taxation and the management of municipal units (where the bulk of the burden would fall) are such that the resources for compensation under condemnation are not available. Possibilities for the exercise of sovereignty by means of taxation are without limit, except that taxes be made so great as to absorb all private values in property. A remedy for the municipal financial situation might be suggested as a solution of the problem under discussion.

It seems that "in the long run" the practice of condemnation would prove to be more equitable. In effect, this practice involves an insurance element. All citizens unite through the taxation system and agree to compensate those who have located where society later decides they are in the way. It would seem that an extension of the idea of compensating those who are caused to suffer loss for the public good by fire, water, or disease might be in the direction of justice. The question of accrued values occurring as a result of widening is another and broader one. Undoubtedly, many properties adjacent to widening appreciate in value without suffering any loss. The entire question of rights in property is involved. An unlimited extension of the idea of confiscation would lead to a reformulation of the principles which guarantee security in the possession and use of property. This unlimited extension of confiscation may be as logical a step as that from quarantine to street widening.

It is necessary to stop and consider desirability by making an estimate of effect. When land is taken for street widening there is a loss of that land for other uses. The question is whether that loss shall be borne by the freeholder or by the social group that needs the wider street. Confiscation would not remove the financial problems; it would merely shift the burden. Faulty taxation or abuses under existing practice do not necessarily argue for confiscation. Perhaps good arguments can be given, but it would seem that the author of this paper has not presented them. Until the effect of the proposed solution is injected into the argument and until engineers are convinced that all other possible solutions are by comparison not as good in effect, it should not be expected that they will respond to the appeal made to them.

Donald M. Baker,[13] M. Am. Soc. C. E. (by letter).—The theory advanced by Mr. Williams is interesting, but if it were generally adopted it might establish a dangerous precedent; and, through an extension of the same theory to similar subjects, it might lead to rather alarming consequences. The theory advocated is that narrow streets in the present age of intensive operation of motor-driven vehicles are dangerous to public health and safety and that the governing agency should be able, by giving proper notice to the owners of abutting property of its intention, to take sufficient of this abutting property, at some future date, to make a highway of a width adequate to satisfy traffic demands, this taking to be without compensation to the owners.

Narrow streets in themselves are not dangerous to public safety. On the contrary, due to their very narrowness, traffic is deterred from using them. Wider streets attract more and faster traffic; in fact, they invite traffic of a nature which is more likely to become dangerous to public safety through accidents than that occurring on narrow streets. As to public health, there are data pointing to the fact that on narrow streets lined with tall buildings the health of the users thereof, and particularly that of the occupants of buildings on adjoining property, may be somewhat affected by carbon monoxide (CO) given off from the exhausts of automobiles. Evidence on this point, however, is not conclusive. Narrow streets do cause a clogging of vehicular circulation, but this affects primarily the values of the adjacent property, and it alone suffers. Without adequate opportunities for circulation, activity moves elsewhere and values depreciate.

---

[13] Cons. Engr., Los Angeles, Calif.

If the principle advocated by the author were generally adopted by the Courts it might lead to a wholesale taking of property for public purposes without compensation. Consider, for example, the case of a badly congested neighborhood where there is urgent need for a playground. Such need might be just as severe from the standpoint of public health and safety as the need for wider streets. Under the principle advanced, the governmental agency could state that at a date five or ten years in the future it proposed under its sovereign powers to take property necessary for this playground from the owners thereof, without compensation. In the meantime, the owners must adjust themselves to the inevitable, being warned that any improvements or alterations made upon the property would be made at their own risk. It would be a simple matter in time to extend this principle to the acquisition of all property needed for public purposes.

The illustrations of the exercise of sovereignty, cited by the author, are emergency cases for which no other remedy is available. When buildings in the path of a fire are dynamited to stop the approach of a widespread conflagration, in all probability they would have been consumed by the fire in any event. The exercise of the right of quarantine is the only present feasible way of stopping an epidemic. It is an emergency measure. Property as represented by gambling devices may be seized because, in general, such devices are owned in violation of the law, and the owner may not come into Court with clean hands to reclaim them. The extermination of nuisances falls in a different category. If the nuisance existed before the surrounding development was made, the promoters of this later development would be forewarned of it and, in some cases, would have no remedy. If the nuisance commenced after the surrounding area was developed, the party causing the nuisance was forewarned and instituted it at his own peril.

The only reason for the establishment of set-back lines for future street widening is that it would be uneconomic to make the widening at the time the set-back lines were proposed. Either traffic needs are not sufficient at present to require it, or its cost would be such that the property assessed for the widening, be it a district or the entire city, could not afford to pay the resulting price. In other words, the benefits resulting from the widening would not equal the cost thereof. In the first instance, it is a perfectly proper procedure, which should be supported by the Courts, to establish lines beyond which no owner may build and expect to obtain future damages for the destruction of that portion of his building extending beyond the line. This procedure is a wise one and prevents the public from being subjected to excessive and unwarranted damages when widening occurs. However, if the public is not in a position to pay for the land at the time the set-back lines are established, the owners should be compensated for the land taken, at such time as the project becomes feasible and economical.

The writer also disagrees with Mr. Williams as to the general enhancement in values which result from a street widening or opening. It is true that such enhancement in value does occur in some instances where a portion of a parcel of property has been condemned, but this is rather the exception than the general rule. In many cases the widening and improving of a street with the resultant increase in traffic causes an actual detriment to the owners of abutting property, with the benefits, if any, accruing to property on side streets, which streets achieve greater accessibility without being injured by increased traffic.

City planners have established through investigation a definite relationship between business frontage and supporting population. With the present density of motor-vehicle traffic it is absolutely impossible to fill up the frontage on all the major traffic arteries with business. It is only in locations where business is justified and warranted that a resulting enhancement in values will occur. The widening of a quiet residential street into a heavily traveled artery will ruin such street for single-family residential use unless the property abutting thereon is developed into rather large holdings where the residences can be well set back from the property line, masked by shrubbery, hedges, etc., and thus screened from resulting confusion, noise, and dust.

Furthermore, such traffic arteries are not desirable for multiple residential or apartment-house purposes. Resulting noise, dust, etc., occasioned by heavy traffic makes the construction of these buildings—unless they too can be screened effectively from the traffic—an economic impossibility. The general effect of an extensive street-widening and improvement program, taking into consideration all the property involved, is a "freezing" of a considerable proportion of the frontage thereon. It cannot all be used for commercial purposes. In many cases, it is unsuitable for multiple residential or apartment structures, and entirely unsuitable for single-family residential uses. This has been the actual experience in Los Angeles, Calif., during the period from 1927 to 1930.

There is no question but that traffic congestion caused by the rapid increase of motor vehicles requires drastic remedies, but these remedies lie more along the lines of proper planning and budgeting of the program of street widening, proper methods to be developed for the allocation of costs, more aggressive opposition on the part of city authorities toward unduly high awards in condemnation proceedings, and greater realization on the part of the Courts that excessive awards will prevent the ultimate achievement of a satisfactory and necessary arterial traffic plan, rather than in the adoption of the principle suggested by the author. Acquirement of adequate traffic capacity solely at the expense of abutting property is not just, because usually not the abutting property, but the entire surrounding district, is benefited, and the district should contribute toward the benefits received. If these benefits do not justify the cost, then the project is uneconomical and should not be attempted.

E. G. Walker,[14] M. Am. Soc. C. E. (by letter).—The new idea offered in this paper involves rather drastic action. There can be no question of the correctness of the basis upon which the author ultimately relies; namely, that, in an emergency, the community as a whole has the right to interfere with private rights and to upset them completely if necessary in the interests of the people. However, in stretching his analogies of fire-fighting and quarantine to make them the bases of proposals for a drastic alteration of the existing principles of dealing with street widening, the author is not completely logical.

---

[14] (Maxted & Knott), London, England.

Conflagrations that make it necessary to enter private land and destroy buildings and other property thereon, and epidemics that require the isolation of all individuals coming from an infected area, are both subjects which are but partly under human control. Drastic curtailment of individual liberty entailed by remedial measures to which the author refers in his paper are brought about for that reason.

On the other hand, the regulation of traffic is under the entire control of the authorities appointed to deal with it. There is, therefore, no more reason for expropriating private property in one place than there is in another. If a particular road becomes too congested the traffic can be diverted to another one, or new roads can be built through less congested areas to by-pass traffic which is not proceeding to a destination in the area. The logical outcome of the author's suggestion is that any land required for road development could be expropriated without compensation. It would then follow that any property required for any purpose—which any State or local government that happened to be in power at the moment might consider to be in the interests of the community—could also be expropriated without compensation.

There is certainly much to be said against the huge and frequently inflated values which are assessed upon property taken for street widening in congested areas. There is room for considerable improvement in making the assessments. The argument that has been advanced frequently, is that these values belong in great part to the community because, but for the activity of the community, central sites would not reach such high values. Attempts have been made in the past to appropriate to the community such proportion of the increase of value of property sites as could be claimed to be due to general communal development. These have failed primarily because of the impossibility of assessing any value for a site independently of the existence and activities of the occupants of surrounding sites.

The problem of taking land for road widening is full of difficulties and can be solved only by having reasonable regard to the interests of the individual as well as to those of the community as a whole. The suggestion that property should be taken without any compensation whatsoever is much too drastic. It is as unfair in one direction as the influences which frequently are brought to bear to enhance values for compensation, are in the other direction.

correctness of the basis upon which the author ultimately relies, namely, that in an emergency the community as a whole has the right to interfere with private rights and to meet them completely if necessary, in the interests of the people. However, in sketching his analogies of fire-fighting and quarantine to make them the basis of proposals for a drastic alteration of the existing principles of dealing with street widening, the author is not completely logical.

Conflagrations that make it necessary to enter private land and destroy buildings and other property thereon, and epidemics that require the isolation of individuals coming from an infected area, are both subjects which are but partly under human control. Drastic curtailment of individual liberty entailed by remedial measures to which the author refers is [illegible] brought about for that reason.

On the other hand, the regulation of traffic [illegible] is under the entire control of the authorities appointed to deal with it. There is, therefore, no more reason for appropriating private property in one case than there is in another. If a particular road becomes too congested, the traffic can be diverted to another one, or new roads can be built through less congested areas to by-pass traffic which is not proceeding to a destination in the area. The logical outcome of the author's suggestion is that any land required for road development could be expropriated without compensation. It would then follow that any property required for any purpose which any State or local government that happened to be in power at the moment might consider to be in the interests of the community could also be expropriated without compensation.

There is certainly much to be said against the high and frequently inflated values which are assessed upon property taken for street widening in congested areas. There is room for considerable improvement in making the assessments. The argument that has been advanced frequently is that these values belong in great part to the community, because, but for the activity of the community, central sites would not reach such high values. Attempts have been made in the past to appropriate to the community such a portion of the increase of value of property sites as could be claimed to be due to general communal development. These have failed primarily because of the impossibility of assessing any value increase independently of the character and activities of the occupants of surrounding sites.

The problem of taking land for road widening is fraught with difficulties and these can be solved only by having reasonable regard to the interests of the individual as well as to those of the community as a whole. The suggestion that property should be taken without any compensation whatever is much too drastic. It is as unfair in one direction as the [illegible] which frequently are brought to bear to enhance values for compensation are in the other direction.

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
TRANSACTIONS

Paper No. 1809

THE EIGHT-MILE CASCADE TUNNEL GREAT NORTHERN RAILWAY

A SYMPOSIUM[1]

PAGE

Part I.—Preliminary Studies and Results of Improving Cascade Crossing.
By D. J. Kerr, M. Am. Soc. C. E. 917

Part II.—Surveys, Construction Methods, and a Comparison of Routes.
By Frederick Mears, M. Am. Soc. C. E. 926

Part III.—Construction Plans and Methods.
By J. C. Baxter, M. Am. Soc. C. E. 950

With Discussion by Messrs. R. H. Keays, J. G. Sullivan, and Frederick Mears.

---

[1] Published in February, 1931, *Proceedings*.

SYNOPSIS

This Symposium constitutes the final professional record of the construction of the 7.79-mile Cascade Tunnel by the Great Northern Railway Company. It is presented in three parts, each part being written by an official of the Great Northern Railway Company, or of A. Guthrie & Co., Inc., the contractors on this work.

Part I, by Mr. Kerr, is an outline of the history of the Great Northern Railway line over the Cascade Mountains in the State of Washington. It also contains a discussion of the reasons leading to the improvement of this mountain crossing.

Part II, by Colonel Mears, contains a description of the surveying methods and instruments used in constructing the tunnel. It presents the general plan of attack, the construction standards, and a description of the line improvement on the east and west slopes of the Cascade Mountains; and it gives a comparison of the physical characteristics of the new and old routes.

Part III, by Mr. Baxter, with two appendices by H. J. King, M. Am. Soc. C. E., is a study of the plans made for the tunnel and the detailed problems affecting methods of construction.

PART I.—PRELIMINARY STUDIES AND RESULTS OF IMPROVING CASCADE CROSSING

BY D. J. KERR,[2] M. AM. SOC. C. E.

The Great Northern Railway, while not the most recently built transcontinental line, was only opened through to the Pacific Coast in January, 1893. It was in December, 1889, that John F. Stevens, Past-President and Hon. M. Am. Soc. C. E., explored the Marias Pass in the Rocky Mountains and thus opened up a short and low summit route for a line not far south of the Canadian border. Mr. Stevens did not have the satisfaction of staying to complete the location survey over the Rockies because his success there resulted in the late James J. Hill, F. Am. Soc. C. E., sending him to the Cascade Range to determine the route of the Great Northern Railway across that barrier.

In 1890, there was little information available about the Cascade Mountain passes in Northern Washington, and the slopes were heavily timbered. The route to the sea by way of the Fraser River in British Columbia and by way of the Columbia River along the boundary between Washington and Oregon had been known to white men for nearly a hundred years. The Northern Pacific transcontinental line, opened in 1887, followed the Valley of the Yakima River (see Fig. 1) from its confluence with the Columbia, near Pasco, Wash., and crossed the Cascade Range through the Stampede Tunnel to the head-waters of Green River, and thence down the Green River Valley to the Pacific Coast.

Mr. Hill, as he had done across Dakota and Montana, wanted the line across the State of Washington to open up new territory, and Mr. Stevens proceeded to make a thorough examination of the Cascade Range between the Northern Pacific crossing and the Canadian boundary, a distance "as the crow flies" of 125 miles. A reference to the map, Fig. 1, will make the problem and the description clearer.

The result of Mr. Stevens' work was the selection of a pass which connected the head-waters of Nason Creek, a tributary of the Wenatchee River, with Tye River, a tributary of the Skykomish River. The Wenatchee River joins the Columbia River near Wenatchee, Wash., while the Skykomish flows into the Snohomish which enters the sea at Everett, Wash. The route, therefore, was fixed from Spokane, Wash., *via* Wenatchee, the new pass (since known as Stevens Pass), and Everett; thence, 35 miles along the seashore, to Seattle, Wash.

The pass had an elevation of 4 059 ft. The approach from the east was through the narrow Tumwater Canyon of the Wenatchee River and on the west side the slopes fell away so rapidly as to necessitate considerable development to obtain a reasonable grade line. Mr. Stevens selected 2.2% as the maximum grade of the permanent line which was to be located down hill on each side from the portals of a projected tunnel, 2.7 miles long. The summit was to be at the east portal of this tunnel at an elevation of 3 383 ft.

---

[2] Asst. to Vice-Pres., Operating Dept., G. N. Ry., St. Paul, Minn.

The Great Northern Railway Company had no land grant west of Minnesota, and it was necessary to develop traffic quickly to justify the construction of the extension. The opening of the through line could not be delayed awaiting the completion of a tunnel nearly three miles long and, therefore, a switchback route with 4% grades was located over the mountain across the open pass and connecting with the permanent line near the portals. This switchback route served to handle the through traffic of the Railway Company from January, 1893, until December, 1900, when the 2.7-mile Cascade Tunnel was completed. As a result the summit elevation was reduced from 4 059 to 3 383 ft., and all grades heavier than 2.2% were eliminated. The profile of the line from Wenatchee to Skykomish, Wash., is shown in Fig. 2.

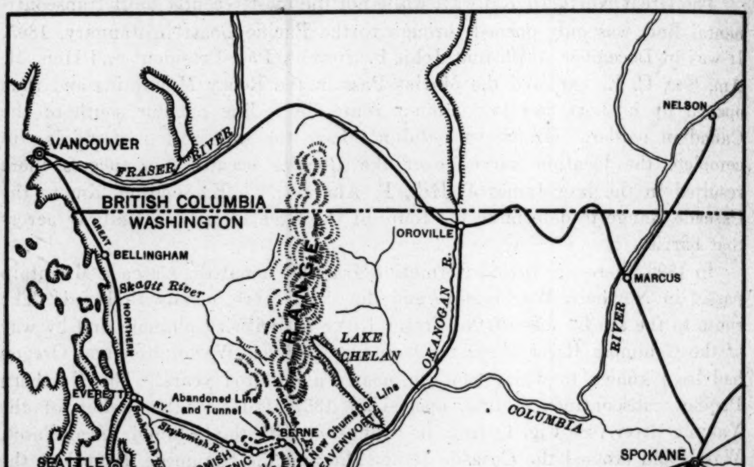

NELSON
VANCOUVER
FRASER RIVER
BRITISH COLUMBIA
WASHINGTON
OROVILLE
GREAT
BELLINGHAM
Skagit River
NORTHERN
MARCUS
RANGE
LAKE CHELAN
OKANOGAN R.
RIVER
EVERETT
RY.
Abandoned Line and Tunnel
Skykomish R.
BERNE
New Chumstick Line
LEAVENWORTH
COLUMBIA
SPOKANE
SEATTLE
SKYKOMISH
SCENIC
New Cascade Tunnel
Abandoned Line
WENATCHEE
NORTHERN
RY.
GREAT
TACOMA
Green R.
Snoqualmie Tunnel
Stampede Tunnel
OLYMPIA
COLUMBIA
CHICAGO MILWAUKEE ST. PAUL
PACIFIC
RY.
NORTHERN
CHICAGO
MILWAUKEE
ST. PAUL
PACIFIC
CASCADE
NORTHERN
SEATTLE
SNAKE
RIVER
YAKIMA RIVER
PACIFIC
RY.
PASCO
WASHINGTON
OREGON
PORTLAND
WISHRAM
SPOKANE
PACIFIC
COLUMBIA
RIVER
UNION
PORTLAND
RY.

FIG. 1.—PLAN OF THE CASCADE RANGE, SHOWING RAILROAD ROUTES.

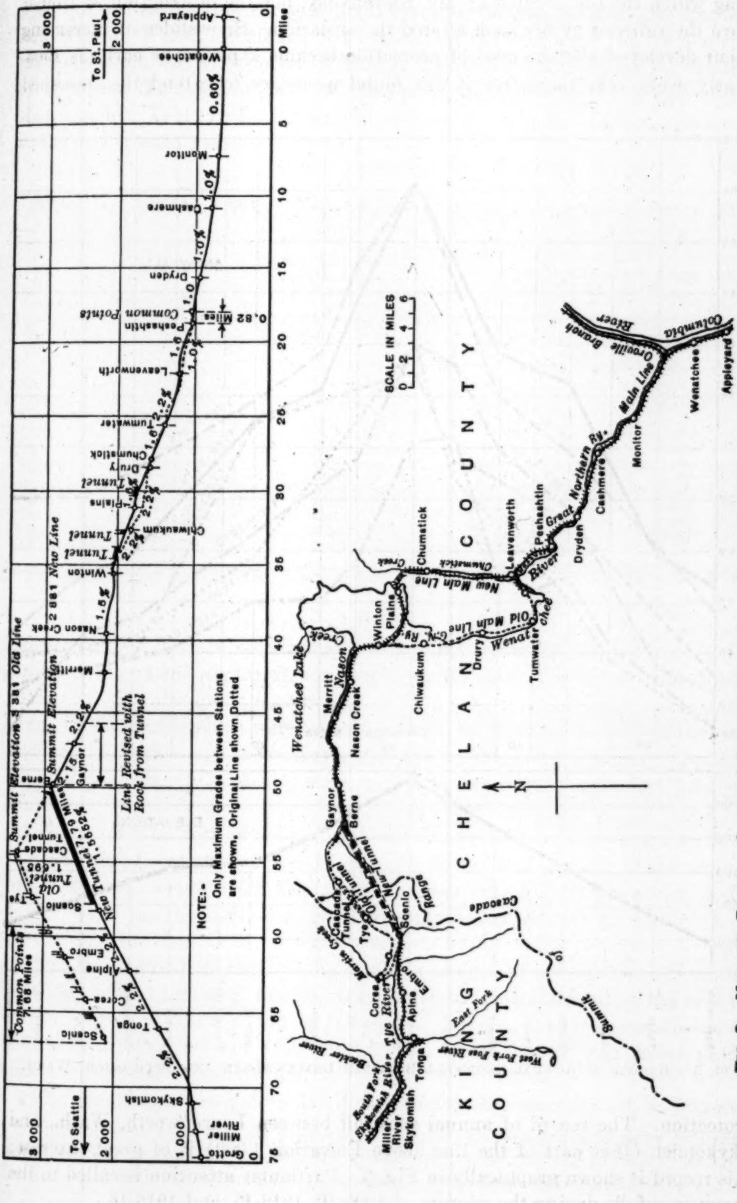

Summit Elevation 3 381 Old Line
Summit Elevation 2 881 New Line
To Seattle
To St. Paul
3 000
2 000
1 000
0
Common Points 7.68 Miles
0.82 Miles Common Points
Line Revised with Rock from Tunnel
New Tunnel 7.79 Miles 1.5662%
Old Tunnel 1.695
NOTE:- Only Maximum Grades between Stations are shown. Original Line shown Dotted
Grotto
Miller River
Skykomish
Scenic
Tonga
Cores
Alpine
Embro
Tye
Cascade Tunnel
Berne
Gaynor
Merritt
Nason Creek
Winton
Tunnel
Chiwaukum
Plains
Drury
Chumstick
Tumwater
Leavenworth
Peshastin
Dryden
Cashmere
Monitor
Wenatchee
Appleyard
2.2%
1.5%
1.6%
1.0%
0.60%
75
70
65
60
55
50
45
40
35
30
25
20
15
10
5
0 Miles
SCALE IN MILES
0 2 4 6
N
KING COUNTY
CHELAN COUNTY
Cascade
Summit of
Range
Wenatchee Lake
Nason Creek
Chumstick Creek
New Main Line
Old Main Line G.N.Ry.
Wenatchee River
Great Northern Ry.
Main Line
Oroville Branch
Columbia River
Tye River
Beckler River
South Fork Skykomish River
Martin Creek
East Fork
West Fork Foss River
Old Tunnel
New Tunnel

FIG. 2.—MAP AND PROFILE, SHOWING OLD AND NEW MAIN LINES BETWEEN APPLEYARD AND SKYKOMISH, WASH

There was little evidence of snowslide trouble on the heavily timbered slopes along which the line located by Mr. Stevens lay, but the destruction of timber above the railroad by fire soon altered the situation. Snowslides of increasing extent developed and the need of protection became apparent as early as 1903. Nearly every year thereafter it was found necessary to extend the snowshed protection. The record of annual snowfall between Leavenworth, Wash., and Skykomish (that part of the line above Elevation 1 000) is of great interest; this record is shown graphically in Fig. 3. Particular attention is called to the heavy snowfalls during the winters of 1909-10, 1912-13, and 1915-16.

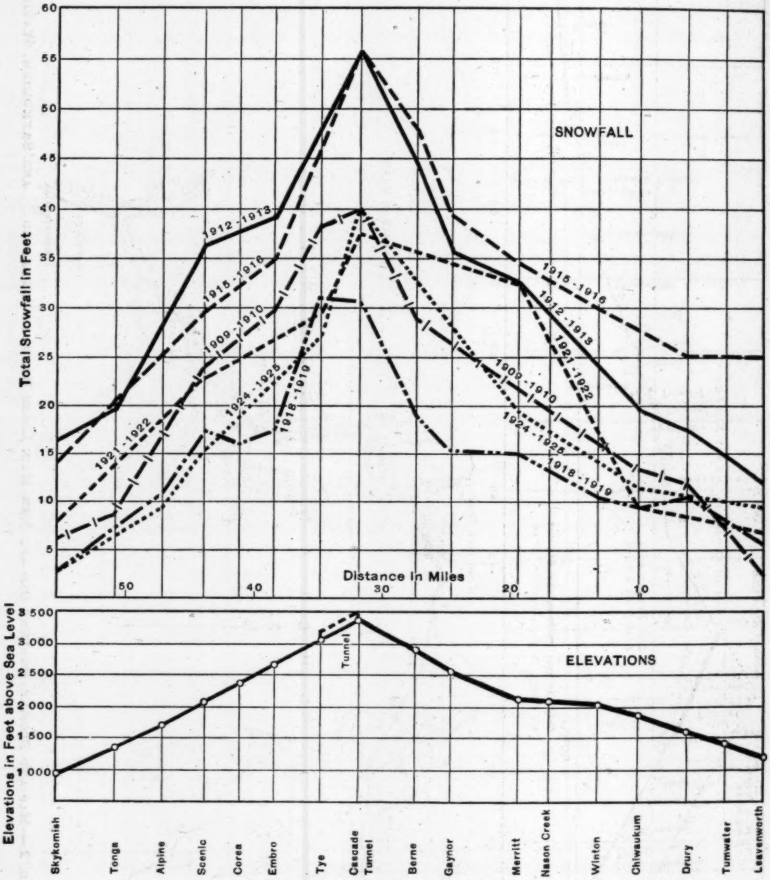


FIG. 3.—RECORD OF ANNUAL SNOWFALL BETWEEN LEAVENWORTH AND SKYKOMISH, WASH.

The sections of the line exposed to snowslide menace were rather limited and consisted of two districts—one on each slope. On the east slope the trouble was confined to the line through the Tumwater Canyon between Leavenworth and Winton, Wash., a distance of 13 miles; on the west slope the trouble zone extended from the west portal of the old tunnel to Scenic, Wash., a distance of 12 miles. The rapidity with which this menace developed and extended is shown best in Table 1 which gives the number of linear feet of track protected at the end of each year.

TABLE 1.—LINEAR FEET OF LINE PROTECTED AT THE CLOSE OF EACH YEAR

Year	TUMWATER CANYON, LEAVENWORTH TO WINTON	CASCADE MOUNTAIN CROSSING, MERRITT TO SCENIC	
	Sheds	Sheds	Tunnels
1903	.....	3 894	15 595
1905	.....	4 833	15 595
1907	.....	7 044	15 595
1909	.....	7 638	15 595
1911	1 429	14 766	15 595
1913	2 022	25 877	15 595
1915	2 022	26 221	16 816
1917	8 401	32 033	19 332
1919	8 401	32 412	19 332
1921	8 401	32 420	19 332
1923	8 401	32 313	19 332
1925	8 401	32 322	19 332

The earlier snowsheds constructed before timber and labor costs had advanced as they have in recent years, were built for about $70 per lin. ft. for single-track and $83 per lin. ft. for double-track sheds. Those constructed with concrete back-walls and timber roofs cost $175 per lin. ft. of double-track shed. Many of the roof timbers were 24 in. by 12 in. by 42 ft., and such timbers are now high in price and increasingly difficult to obtain. At present prices, a double-track concrete back-wall shed, with a timber roof, costs almost $210 per lin. ft.

The increasing cost of replacement and the constantly increasing length of line requiring protection caused considerable thought to be given to the finding of a route that would avoid the snowslide areas entirely. Studies were made from time to time by the late A. H. Hogeland, M. Am. Soc. C. E., and by O. S. Bowen, M. Am. Soc. C. E. A most complete study of the problem was made by Mr. E. J. Beard in 1917, which not only compared the relative economies of a long tunnel line with the existing line from the standpoint of the cost of maintenance and construction of snow-sheds, but also from the standpoint of the transportation savings from the shorter, lower summit and easier grade line, coupled with electrification.

The conclusions reached by Mr. Beard in 1917 are summarized, as follows:

"That the present line in Tumwater Canyon, Leavenworth to near Winton, and from Merritt to Skykomish, be abandoned, substituting therefor changes in line by which all snowsheds are avoided, providing either a 1.4% or a 1%

ruling grade and involving either a 14½ or a 17¼-mile tunnel, as chosen, and estimated to cost, including the changes, electrification, Leavenworth to Delta, necessary equipment, and 100-lb. rails, $25 000 000 or $27 000 000.

"This, because the present line, if retained, will have to be electrified some day, when the cost, plus the great expense that must be incurred account of snowsheds, existing and additional, would be $21 000 000, an investment that would make the property for years less profitable than it is at present; *per contra* the undertaking first above will, upon completion, increase net corporate income over that with the present line, electrified, about $350 000 per year, and the advantageous difference will increase each year.

"That neither the present line, steam operated or electrified, nor the undertaking first above should be double-tracked, because the surplus train capacity of either is enough for the increase in traffic of so many years in the future that no money whatever should be spent for, or in anticipation of, double-tracking.

"The present snow-sheds, with the minimum of additions that circumstances will permit—to be built of timber—be maintained at the least practicable cost until the undertaking first above is completed.

"That every effort be made and consistent expense be incurred to complete the undertaking first above in from 4½ to 5 years instead of the estimated 6½ to 7½ years, thereby reducing the estimates above by from $1 500 000 to $2 000 000."

Unfortunately, the intervention of the World War forced the project into the background and at the conclusion of the war neither the traffic nor the financial condition warranted the consideration of a large project. The full effect of the Panama Canal on transcontinental rail traffic was only felt as ship tonnage became plentiful in 1919 and 1920.

Discussion of the project was revived in 1921, and new estimates were made from that time until 1925. These were based largely on the 14-mile tunnel line recommended by Mr. Beard. By that time costs had changed materially and it was evident that the project would involve an expenditure in excess of $30 000 000. Such an outlay could not be justified economically at that time and, having reached the conclusion that a long tunnel would not be built, the Operating Department of the Railway Company began the improvement of operation over the existing line.

In earlier days trains were moved eastbound from Skykomish by coal-burning locomotives with the result that the atmosphere in the old tunnel (which was on a 1.69% grade against eastbound traffic) was intolerable. In 1909, a hydro-electric development was made in the Tumwater Canyon, a short distance above Leavenworth, and plans were begun for electrifying the tunnel.

The Tumwater Plant consisted of three 2 500-kw. units and power was transmitted to the tunnel at 33 000-volts, 3-phase, 25 cycles. The overhead trolley installation consisted of two 4-0 wires, carrying 6 600 volts, and extended about 3½ miles from Cascade Tunnel Station through the tunnel to Tye, Wash. Four 3-phase locomotives were purchased which had constant speed of 15 miles per hour, that being the only running speed available. The plan of operation was to have a train of about 2 000 tons leave Skykomish with a steam road engine and two steam pushers. At Tye (the west portal of the old tunnel), the train was cut in two and, with the road engine, was taken through to Cascade Tunnel Station (the east portal of the old tunnel) by electric locomotives.

The first move made in 1925 to improve operation was to re-arrange the electric locomotives so that the traction motors could be connected in cascade giving the locomotive when so connected a speed of 7.5 miles per hour instead of 15 miles per hour. This change permitted taking passenger trains through the tunnel at 15 miles per hour as before and freight trains in one section at 7.5 miles per hour. In this way the full power available at Tumwater (the only source) was used, and the tractive effort of all four locomotives was made useful for freight trains.

The success of this change prompted the Company to consider extending the electrification down the west slope to Skykomish, with the idea of using electric pushers throughout. The plan was to obtain two electric locomotives which, with steam engine working, would move a train from Skykomish to Tye on a 2.2% grade and which could haul the train and the steam road engine (not working) from Tye to the Cascade Tunnel through the old tunnel on the 1.69% grade.

The limited supply of power available at the Railway Company's Tumwater Plant and the need of variable-speed locomotives to work with steam road engines resulted in the selection of the motor generator type of locomotive with 11 000-volt, single-phase, 25-cycle trolley, and 600-volt, direct-current traction motors. Two such double-cab locomotives (each having a 1 500-kw. motor generator set) were ordered in 1925, and work had already been started on the electrification of the old line, when serious consideration was given again to the long tunnel plan.

The success of the operation of freight trains in one section through the old tunnel, the improvement in steam operation (which had permitted running freight trains through from Seattle and Everett to Wenatchee in less than the 16-hour crew limit), and the failure of the through freight traffic to show a material increase, made it apparent that savings in transportation expense alone could not justify any long tunnel line and that if such a project were to be warranted economically it must be the shortest tunnel which would permit the abandonment of all the line on the west slope subject to the snowslide menace.

A study made by Frederick Mears, M. Am. Soc. C. E., showed that this could be accomplished by a tunnel 7¾ miles long, having its east portal at Berne, Wash., and its west portal at Scenic. In contrast to the 14 or 17-mile tunnels recommended by Mr. Beard, this plan did not eliminate the 2.2% grade, but left in use the five miles of such grade against westbound traffic between Merritt, Wash., and Berne and eleven miles against eastbound traffic between Skykomish and Scenic.

Mr. Stevens was called upon to go over the ground that was so familiar to him and as a result of an exhaustive study in the summer of 1925, he urged the immediate approval of the 8-mile tunnel line as the only project that could be justified. He pointed out that there was much unwarranted prejudice against pusher districts and that with electrification the former objection to sections of 2.2% grade was largely eliminated, especially when such grades were comparatively short. Accordingly, the Board of Directors of the Railway

Company reached the conclusion in November, 1925, that the 7.79-mile tunnel should be built.

As a distinct and separate project on the east side of the mountains, a new line (the Chumstick Line) was projected from Peshastin (near Leavenworth) to Winton, Wash., a distance of 17 miles. It followed the open Chumstick Valley, eliminated all 2.2% grades east of Merritt, and provided a line with 3° maximum curvature, entirely free from the snowslide menace to which the old line in the Tumwater Canyon was exposed.

In addition to the major improvement made by the construction of the Chumstick Line on the east slope, numerous short sections of the old line between the west end of the Chumstick Line at Winton and the east portal of the new tunnel at Berne, were improved in alignment and grade.

When the decision to build the new tunnel was reached in November, 1925, the age of the existing snowsheds made it extremely important, if heavy renewals were to be avoided, that the new route be made ready for use before the snowslide season of 1928-29. To avoid delay, the firm of A. Guthrie & Co., Inc., of St. Paul, Minn., was instructed to proceed with the work on a fixed fee basis—a plan which later was extended to cover the construction of the Chumstick Line.

The need of an adequate supply of power for the driving of the new tunnel made it advisable to turn over the Company's power station at Tumwater to the Puget Sound Power and Light Company and to contract with that Company for power to be supplied from its widespread system. Having such power available brought with it the decision to electrify the entire district from Wenatchee to Skykomish (75 miles).

These four pieces of work (1) the 7.79-mile Cascade Tunnel; (2) the Chumstick Line; (3) line revisions between Winton and Berne; and (4) the electrification from Wenatchee to Skykomish, constituted a complete program for the improvement of the Cascade crossing.

The first, in addition to the improved alignment and lower summit obtained, permitted the abandonment of all the line on the west slope subject to snowslides; the second, which also improved alignment and grade, resulted in the discontinuance of operation over all that part of the line on the east slope subject to slides; the third resulted in a marked improvement in grade and curvature; and the fourth was a necessary step following the decision to build the long tunnel and to avoid too frequent locomotive changing.

The completion of the program has put the Cascade crossing on an equal footing with the remainder of the main line in character, freedom from interruption, and freight-train tonnage that may be handled.

A comparison of the old and new line from the east end of the Chumstick revision, at Peshastin, to the west end of the tunnel revision at Scenic, is given in Table 2.

The electrification program, together with the change in line, grade, and rise and fall indicated in Table 2, has permitted the operation of 5 000-ton freight trains eastbound out of Skykomish, using a three-cab road locomotive and a two-cab helper. These helpers are operated generally between Skykomish and Merritt—the foot of the 2.2% grade on the east slope—thus avoiding

any stop at the summit at Berne. Westbound, in ordinary operation, the three-cab locomotive takes a 3 500-ton train from Wenatchee to Merritt, where a one or two-cab helper, whichever may be available, is used to Berne and remains in the train on the down-hill slope to Skykomish. As compared with the steam operation prior to any electrification on the slope west of the old tunnel, the train tonnage eastbound has been increased from 2 400 to 5 000. The average time for a freight train between Skykomish and Berne has been reduced 3½ hours and the gross ton-miles per train hour between Skykomish and Wenatchee have been increased from 11 520 to 70 000. The traffic on this part of the line is prevailingly eastward, substantially in the proportion of 5 000 to 3 000 tons.

TABLE 2.—Comparison of Old and New Lines

	Old line.	New line.	Favorable to new line.
Length, in miles	49.32	40.90	8.88
Length of tangent track, in miles	26.86	27.66	0.80
Length of curved track, in miles	22.92	13.24	9.68
Total rise eastbound, in feet	1 353.70	898.92	454.78
Total rise westbound, in feet	2 342.15	1 887.37	454.78
Summit elevation, in feet	3 382.00	2 881.00	501.00
Length of snowsheds, in feet	40 153.00	841.00	39 312.00
Length of tunnels, in feet	19 620.50	48 600.40	28 979.90*
Maximum degree of curvature	10° 00′	10° 00′	....
Total degree of curvature	5 617° 34′	2 059° 06′	3 558° 28′

* Unfavorable to the new line.

The various steps in the improvement of the railroad between Wenatchee and Skykomish have so overlapped that it is impossible to divide the savings resulting from the entire project into the part due to electrification and the part due to the improved alignment, lower summit, shorter mileage, etc. These savings in the aggregate are not more than sufficient to pay interest on the entire investment. The Railway Company must look for the justification of the project to the benefit which the system as a whole has received through the elimination of a weak link in its transportation chain, with consequent increased traffic resulting from service that is absolutely dependable, quicker, and, in the case of passenger travel, extremely comfortable and pleasant.

PART II.—SURVEYS, CONSTRUCTION METHODS, AND A COMPARISON OF ROUTES

BY FREDERICK MEARS,[3] M. AM. SOC. C. E.

Surveying parties were put into the field on the west slopes of the Cascades in September, 1925, to locate a line that would meet the general requirements of the problem, which called for reduction in curvature, saving in distance, and permanent protection of the line against snowslides. From a study of the United States Geological Survey maps of the Cascade Range, and other engineering data available, it was apparent that a tunnel approximately eight miles in length would meet these requirments, with east and west portal near Berne and Scenic, respectively.

The line adopted provided for a single-track tunnel, 7.79 miles in length, on tangent, with 0.74 mile of new line approaching from the east and 1.54 miles from the west.

West Slope.—The locating party on the west slope made a detailed survey of the country in the Tye River Valley above Scenic, to determine suitable portal sites and proper gradients for the west approach, two portal sites in particular being developed, as shown by Fig. 4. Close topography was taken and soil and rock conditions were noted. The existence of a 0.99% grade on the main line, throughout the length of the old Scenic passing track, made it possible to raise the grade of the new line without exceeding the maximum 2.2% grade, so that 1½ miles of new embankment could be constructed on the floor of the Tye River Valley leading up to the face of the mountain.

The Cascade Range is in a region of high precipitation, particularly in the fall and winter months. Several hot springs were known to exist on the mountain slopes above Scenic, and every precaution had to be taken to locate the portal and approaches to avoid menace from snowslides, as well as to secure proper drainage.

The south slope of a ridge, in a bend of the Tye River, affording sufficient cover to "portal under" was finally selected as the west portal site, as shown in Fig. 5. The approaches were free from snowslides, and the Tye River provided good drainage for surface and underground water. It was known that the westerly ½ mile of the new tunnel would lie in a somewhat broken formation, requiring heavy timbering while in course of construction, but it was impossible to better this ground condition and, at the same time, meet other requirements.

East Slope.—On the east side of the Cascade Range, a portal site was located, as shown in Fig. 6, which met every requirement. A solid rock face of a mountain slope was selected for the portal, which could be reached from the operated line by the construction of less than a mile of new line, on good grade and alignment. This site was free from snowslides or drainage problems.

---

[3] Asst. Chf. Engr., G. N. Ry., Seattle, Wash.

Open Traverse.—As soon as the east and west portal sites were decided upon, a preliminary survey was made to connect the portals. This was an open traverse, about 10 miles long, following the operated line. (See Fig. 4.) The traverse showed that the tunnel would be on approximately a 1.6% grade, descending from east to west, and that the open approaches, at either end of the tunnel, could be built on a 1.5% grade, to provide for passing tracks of suitable length near the portals.

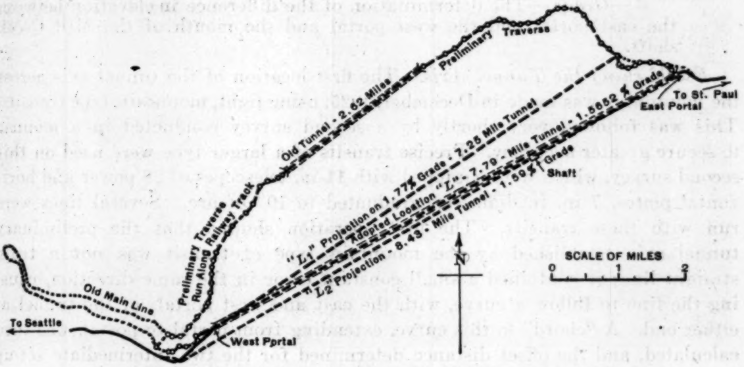

Preliminary Traverse
Old Tunnel - 2.62 Miles
7.25 Mile Tunnel
Mile Tunnel - 1.5652 % Grade
To St. Paul
East Portal
"L1" Projection on 1.77% Grade
Adopted Location "L" - 7.79 - Mile Tunnel
Shaft
1.59% Grade
"L2" Projection - 8.43 - Mile Tunnel
Preliminary Traverse Run Along Railway Track
Old Main Line
To Seattle
West Portal
N
SCALE OF MILES
0
1
2

FIG. 4.—MAP SHOWING PRELIMINARY SURVEYS FOR NEW TUNNEL THROUGH THE CASCADE RANGE.

The preliminary traverse was extended up the valley of Mill Creek, and detailed surveys were made which developed that an intermediate shaft site could be located in the Mill Creek Valley, which would have the proper relation to the east and west portals and to the sub-grade of the proposed tunnel.

The floor of the valley of Mill Creek, where it is crossed by the axis of the tunnel (see Fig 7), was found to be about 622 ft. above the proposed tunnel sub-grade, at a point 2.41 miles west of the east portal. A shaft site, therefore, was laid out on the west slope of the valley, which will be referred to later. The relative location of this shaft is indicated on Fig. 8.

Second Survey.—Before establishing the tunnel tangent on the ground, a second survey was made, using more precise methods, but following very closely the first open traverse.

The azimuth of the long tunnel, calculated from this second traverse, was thrown across the four mountain ridges intervening between the portals. A light mountain type of transit was used in this field work and the error of closure between this first location of the tunnel axis and the open traverse, which followed the heavy curves of the operated line, was about 6 ft. The line thrown across the mountain tops was accepted as the preliminary tunnel axis, and the 8 by 9-ft. pioneer tunnel at the west portal, and the 10 by 10-ft. center heading at the east portal, were started by reference to it.

PRECISE SURVEYS OF THE LONG TUNNEL

The field work for the "definite location" of the New Cascade Tunnel, including the precise location of the intermediate shaft, in the rugged mountains of the Cascade Range, was an interesting problem, in which the following fundamental steps were involved:

1.—Alignment.—The establishment of the tunnel axis across the mountain tops between portals, and the accurate projection of this line into the Valley of Mill Creek at the shaft site.

2.—Distance.—The measurement of the tunnel axis and the determination of the exact position of the Mill Creek shaft with respect to the east and west portals.

3.—Grade.—The determination of the difference in elevation between the east portal and the west portal and the mouth of the Mill Creek shaft.

Establishing the Tunnel Axis.—The first location of the tunnel axis across the mountains was made in December, 1925, using light, mountain-type transits. This was followed very shortly by a second survey conducted in a manner to secure greater accuracy. Precise transits of a larger type were used on this second survey, which were equipped with 11-in. telescopes of 28 power and horizontal plates, 7 in. in diameter, graduated to 10″ of arc. Several lines were run with these transits. The first operation showed that the preliminary tunnel axis established by the mountain type of transit was not a truly straight line; it contained a small constant error in the same direction, causing the line to follow a curve, with the east and west portals of the tunnel at either end. A "chord" to this curve, extending from portal to portal, was then calculated, and the offset distance determined for the two intermediate set-up points, at "Cowboy" Station and at "Big Chief" Station. (See Fig. 9.)

These two stations were then used to form a base line, from which the axis of the tunnel was produced toward the east portal and toward the west portal. Several trials were necessary before determining the exact points on Cowboy and Big Chief Mountains.

When the line had been refined by the precise transits to a degree such that it was considered safe to construct concrete monuments at the several observation points (seven in number, see Fig. 9), a high-power telescope direction instrument was used in the final adjustment of the line. Fig. 10 illustrates the type of instruments used for various parts of this work. As an ordinary tripod would not give the required stability, the precise transit was mounted on a 5-ft. section of 6-in. iron pipe, set vertically in concrete at each of the observation stations; it was equipped with a 22-in. telescope with two eye-pieces, one of 30 power and one of 45 power. The plate or horizontal circle has a diameter of 10 in. which, by use of a micrometer attachment, can be read directly to 2″ of arc. It is provided with a precise striding level, the graduations of which have a value of 5″ of arc. The instrument, without the bronze base on which it rests, weighs 41.5 lb., and with the base, 78.5 lb.; the detail of mounting, at the several observation stations, is shown in Fig. 11. About 18 in. of the lower end of the pipe was embedded in concrete, and openings, 3 in. in width and 7 in. in height, were cut in the pipe, just above the surface of the concrete, to permit the use of a plumb-bob in centering the instrument over the "point". Brass tacks, fixed in wooden hubs, were used at first to mark the points, but when the line was finally adjusted, the wooden hubs were replaced with brass plates set in concrete, on which the exact "point" was marked by a punch.

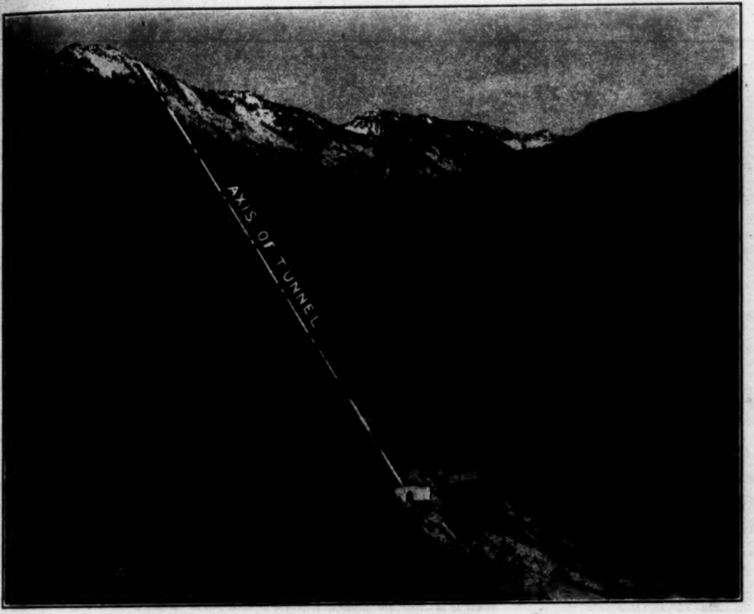

AXIS OF TUNNEL

Fig. 5.—West Portal of the New Cascade Tunnel.

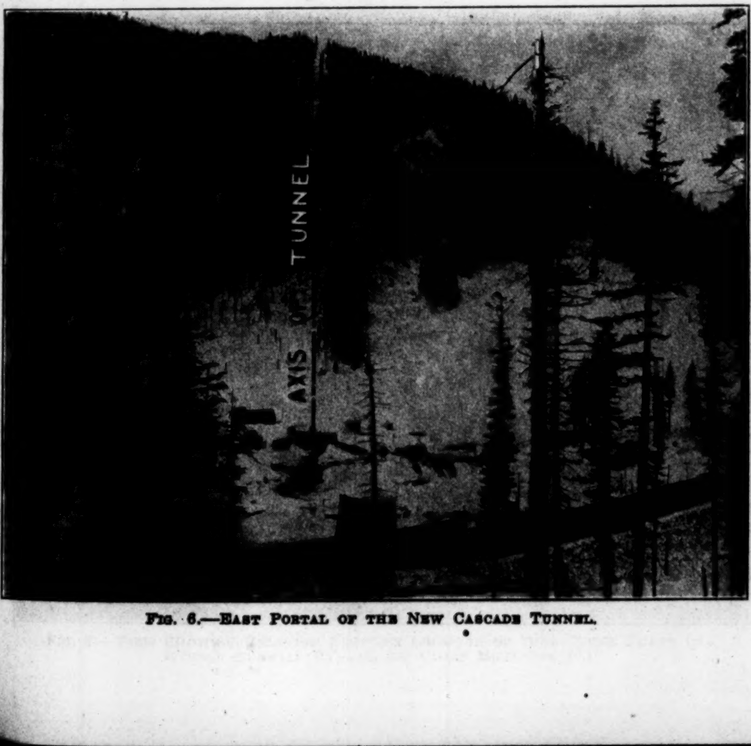

AXIS OF TUNNEL

Fig. 6.—East Portal of the New Cascade Tunnel.

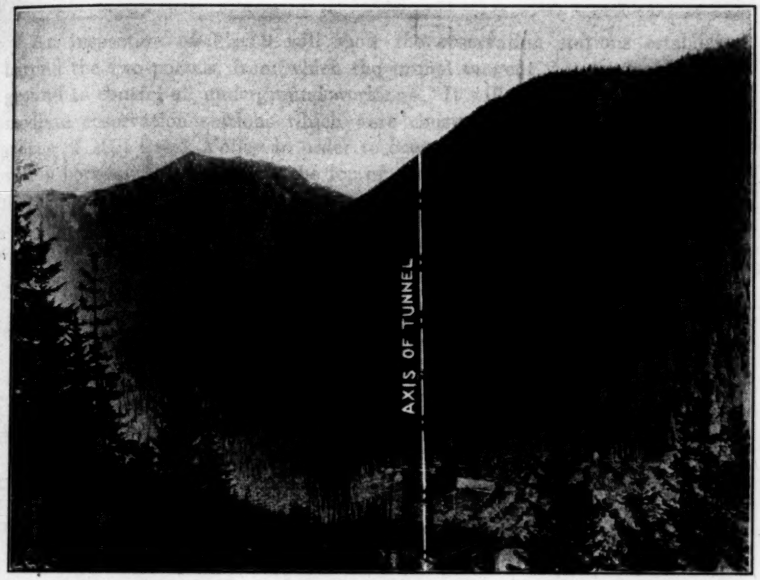


FIG. 7.—VIEW SHOWING FLOOR OF MILL CREEK VALLEY IN THE DIRECTION OF THE CASCADE TUNNEL.

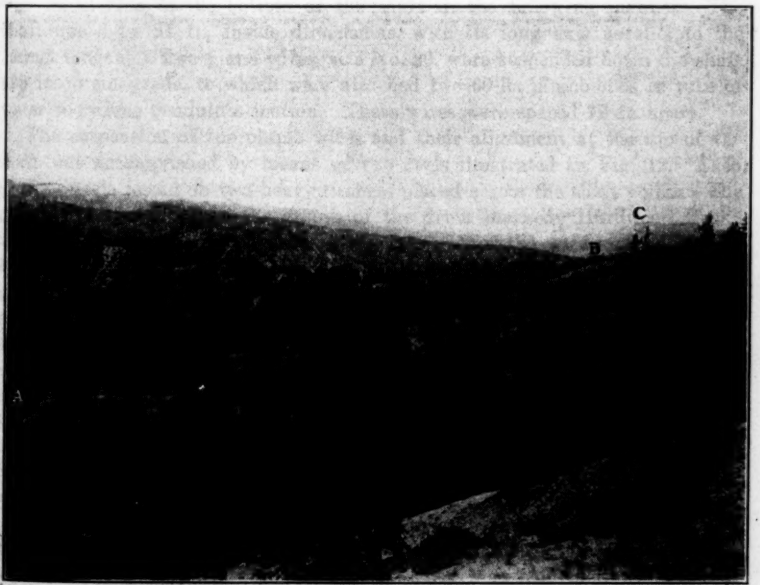


FIG. 8.—VIEW SHOWING RELATION BETWEEN LOCATION OF MILL CREEK SHAFT (A), WINTON TUNNEL (B), AND BIG CHIEF MOUNTAIN (C).

An inspection of Fig. 9 will show the observation stations established beyond the two portals, from which the tunnel tangent was produced underground to control all underground workings. It will also show the two intermediate observation stations which were constructed on the east and west slopes of Mill Creek Valley in order to overcome the great difference in elevation between Big Chief and the top of the Mill Creek shaft, in the location of the shaft site.

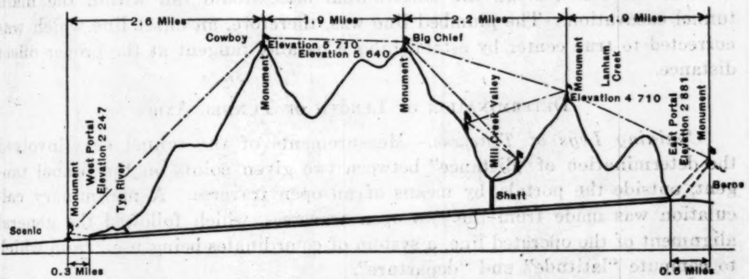

2.6 Miles
1.9 Miles
2.2 Miles
1.9 Miles
Cowboy
Elevation 5 710
Elevation 5 640
Big Chief
Monument
Monument
Monument
Lanham Creek
Elevation 4 710
Mill Creek Valley
East Portal Elevation 2 881
Monument
Monument
West Portal Elevation 2 247
Tye River
Shaft
Berne
Scenic
0.3 Miles
0.6 Miles

FIG. 9.—LONGITUDINAL SECTION, SHOWING OBSERVATION POINTS ON AXIS OF LONG TUNNEL, CASCADE RANGE.

Projection of Tunnel Axis from Top of Mill Creek Shaft.—The tunnel axis established on the surface of Mill Creek Valley, across the mouth of the shaft, was transferred to the bottom of the shaft in the following manner: The shaft was 8 by 24 ft., inside dimensions, with its long axis parallel to the tunnel tangent. Two piano wires, size No. 20, were suspended down the shaft 622 ft. to sub-grade, to which were attached two 60-lb. plumb-bobs in tubs of water to reduce pendulum motion. These wires were spaced 19 ft. apart.

The suspension of the plumb wires and their alignment at the top of the shaft was accomplished by means of two reels illustrated in Fig. 12. Each of these reels rested on two heavy timbers placed across the shaft collar. The piano wire was coiled in the groove of the drum marked "Hardwood Reel", and a small weight was attached to the free end of the wire and allowed to hang a short distance into the shaft, the wire being made to pass in the V-groove of the rod marked, "Rod 1 in. round by 1 ft. 4 in." Inspection of the diagram will show that the hanging wire can be seen through the frame from either end, so that the frame could be shifted into position by sighting on the wire with a surface transit, set up in proper relation to the tunnel axis.

The final adjustment was made by means of the grooved rod shown in Fig. 12. The nuts through which this rod turns, were bedded firmly in the wood, one of them being stripped of the thread and machined smoothly to act as a bearing. By turning the rod one way or the other, the position of the wire could be shifted very slowly in comparison with the turning of the rod.

When the small weights were lowered to the bottom of the shaft and replaced by the heavy plumb-bobs, these were brought to the required elevation by turning the reels with the cranks. They were held there by means of the pawls engaging the ratchet wheels as shown in Fig. 12. Any additional adjustment of the wires that might be required at the top after the heavy bobs had been attached, could still be easily made by turning the grooved rods.

Two transits were set up at the bottom of the shaft, on opposite sides of the plumb lines in the main tunnel excavation, and each transit was moved into exact line with the two suspended wires. The telescopes were then reversed to throw the line a greater distance from the bottom of the shaft, which was located "off center" from the main tunnel in such manner that 2 ft. of the shaft, along the longitudinal axis, would fall within the main tunnel excavation. The plumbed line was, therefore, an offset line which was corrected to true center by establishing a parallel tangent at the proper offset distance.

DETERMINATION OF LENGTH OF TUNNEL AXIS

Chaining Legs of Traverse.—Measurements of the tunnel axis involved the determination of "distance" between two given points on the tunnel tangent, outside the portals, by means of an open traverse. A preliminary calculation was made from the two open traverses, which followed the general alignment of the operated line, a system of co-ordinates being used from which to compute "latitude" and "departure".

To determine "distance" in the most precise manner, an open traverse, consisting of four courses, was laid out on the ground and measured, beginning at a point on the tunnel axis produced east of the east portal, and ending at a point in the axis east of the west portal. (See Fig. 13.) One of the courses was laid through the old Cascade Tunnel, and produced eastward for some distance beyond the east portal of the old tunnel.

In laying out the courses through the timber, wide "cuttings" were made in order to secure unobstructed vision between the observation points at either end of each leg of the traverse. These "cuttings" extended for miles through fir, spruce, and hemlock, many of the trees being more than 3 ft. in diameter and 150 ft. in height. The actual measurement of the different "legs" of the traverse was done in the following manner: Substantial stakes were driven in the ground at proper intervals; a brass brad was placed in the top according to transit alignment, and the elevation of the top of the stakes determined by spirit leveling. The actual, or "slope" distance between tack points on the stakes was measured with steel tapes, a precise record being kept of the temperature and the tension on the tape at the time of measurement.

Two tapes were used on the survey; one, 500 ft. long, was used in measuring the old Cascade Tunnel, 2.63 miles in length, through which a part of one of the sides of the traverse was located; the other tape, 300 ft. long, was used entirely in the open.

These tapes were calibrated in Seattle by comparison with U. S. Government Standard, at Government Locks, Lake Washington Canal, at intervals of 100 ft. In addition to the calibration, experiments were conducted to test the standard formulas for lengthening due to tension and for shortening due to sag, with the tape in horizontal and inclined positions. These tests were made for the full length tape, and for 100-ft. fractional parts of it. From the data thus obtained, in order to facilitate the reduction of notes in the office,

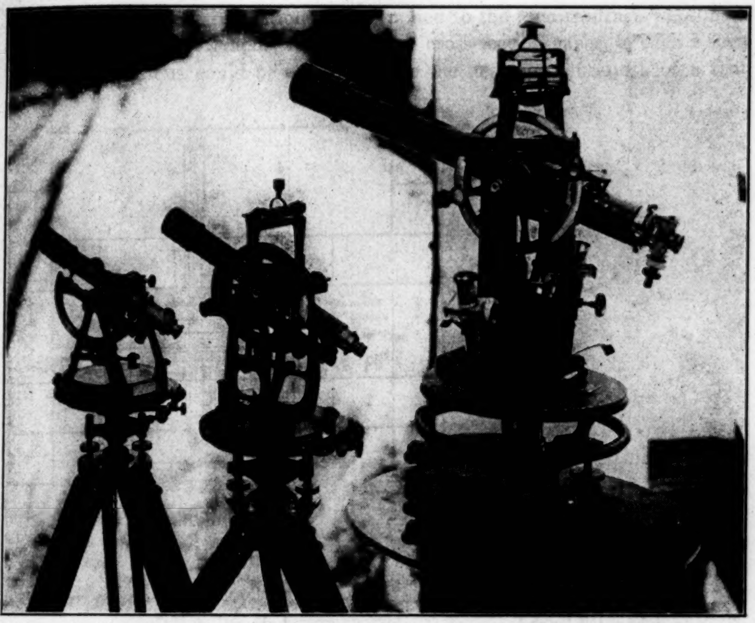


FIG. 10.—TYPES OF INSTRUMENTS USED ON SURVEYS FOR THE NEW CASCADE TUNNEL.

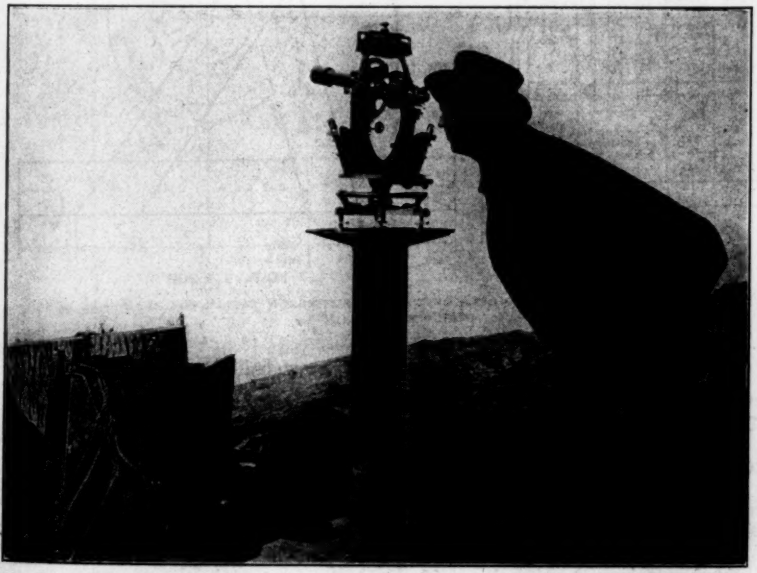


FIG. 11.—VIEW OF THEODOLITE MOUNTED AT ONE OF THE OBSERVATION STATIONS.

graphs of the various corrections to be applied to the tape readings were made for each tape. These graphs were drawn on cross-section paper, to such a scale that all corrections could be easily read to the nearest thousandth of a foot.

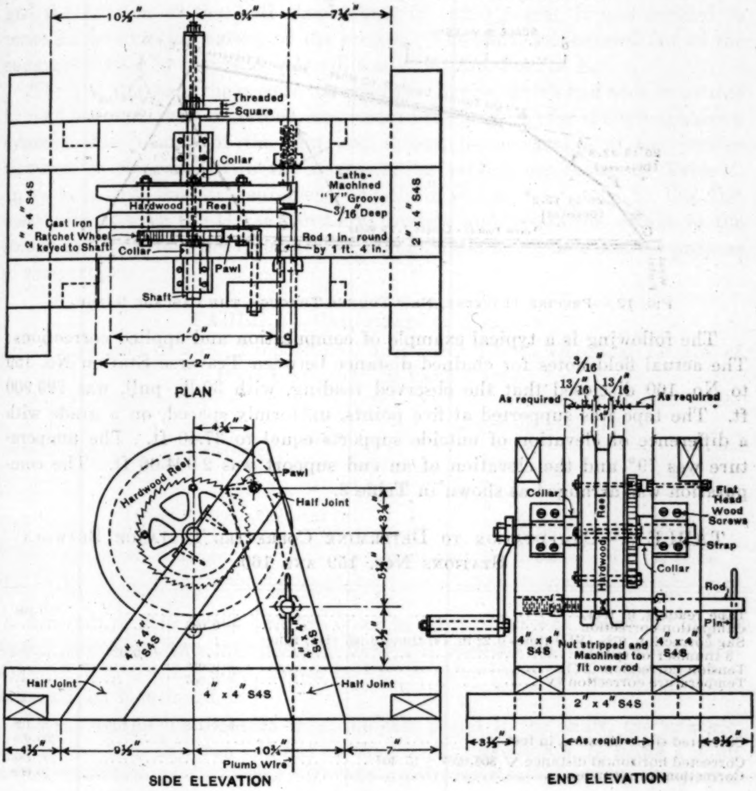

10½"
8¾"
7¾"
Threaded
Square
Collar
Lathe-Machined "V" Groove 3/16" Deep
2" x 4" S4S
Hardwood Reel
Cast Iron Ratchet Wheel keyed to Shaft
Collar
Rod 1 in. round by 1 ft. 4 in.
Pawl
Shaft
2" x 4" S4S
1'0"
1'2"
PLAN
4¾"
Hardwood Reel
Pawl
Half Joint
3¼"
5¼"
4½"
4" x 4" S4S
4" x 4" S4S
Half Joint
4" x 4" S4S
Half Joint
4½"
9½"
10¾"
7"
Plumb Wire
SIDE ELEVATION
3/16"
13/16"
13/16"
As required
As required
Collar
Hardwood Reel
Flat Head Wood Screws
Strap
Collar
Rod
Pin
4" x 4" S4S
Nut stripped and Machined to fit over rod
4" x 4" S4S
2" x 4" S4S
3½"
As required
3½"
END ELEVATION

FIG. 12.—REEL FOR PLUMB WIRES FOR PROJECTING AXIS OF LONG TUNNEL AT BOTTOM OF MILL CREEK SHAFT.

The corrections applied were, as follows:

I.—A sag correction graph for the 300-ft. tape supported on level supports at the 0-ft. and 300-ft. marks, with the pull varying, in 10-lb. units, to 60 lb.

II.—A sag correction graph for the 500-ft. tape—conditions similar to those outlined for Correction I.

III.—A correction graph to apply to results taken from Corrections I and II, where tapes are supported by more than two level supports.

IV.—Graphs showing variation in elongation due to variation in pull.

V.—Graphs showing correction for temperature.

VI.—Graphs for corrections required in order to reduce measurements to a datum plane of 2 247 ft. above sea level, this datum being the elevation of the sub-grade at the west portal.

VII.—A graph for applying corrections for sag when the supports are not level, but on a uniform grade.

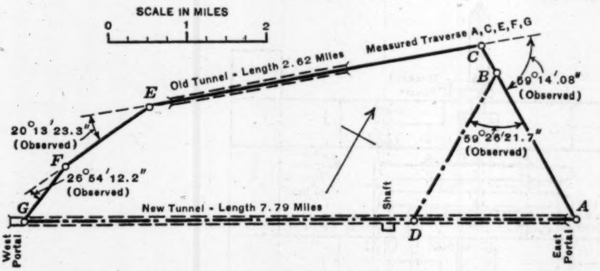


FIG. 13.—PRECISE TRAVERSE, NEW TUNNEL THROUGH THE CASCADE RANGE.

The following is a typical example of computation and applied corrections: The actual field notes for chained distance between Traverse Station No. 159 to No. 160 disclosed that the observed reading, with 30-lb. pull, was 299.900 ft. The tape was supported at five points, uniformly spaced, on a grade with a difference of elevation of outside supports equal to 37.40 ft. The temperature was 79° and the elevation of an end support was 2 904.06 ft. The computation was arranged as shown in Table 3.

TABLE 3.—COMPUTATION TO DETERMINE CORRECTED DISTANCE BETWEEN STATIONS NOS. 159 AND 160

Tape reading, in feet	....	....	299.900
Calibration correction	+ 0.033	....	....
Sag correction, 97% (VII) times 0.22 in. (I) times 0.037 (III) times 3 (number of span)	....	− 0.025	....
Tension correction (IV)	+ 0.053	....	....
Temperature correction (V)	+ 0.067	....	....
	+ 0.153	− 0.025 =	+ 0.128
Corrected slope distance, in feet	....	....	300.028
Corrected horizontal distance, $\sqrt{300.028^2 - 37.40^2}$	....	....	297.688
Correction for altitude	....	....	− 0.009
True horizontal distance at elevation datum, 2 247 ft., in feet	....	....	297.679

Observing Angles.—The precise transits, used in establishing the tunnel axis, were also used to observe the five angles of the precise traverse. The angles were observed in three sets of repetitions by two observers, each repetition consisting of three readings.

An exact record made of each angle reading was entered in the field book. The lengths of the courses were computed from these various field observations, curvature of the earth being disregarded, because the error involved in assuming the surface of the pentagon to be plane instead of spherical, was found to be less than 0.1 ft. for the longest side.

The short distances from Observation Stations *A* and *G*, Fig. 13, to the east and west portals, were measured by the same method used in measuring the sides of the precise traverse.

Triangulation Survey.—As a further check on the distance between portals and the relation of the Mill Creek shaft to either portal, it was decided to make a triangulation survey of the project. The work was carried out in the summer of 1926 by Henry B. Alvord, Assoc. M. Am. Soc. C. E.

The first course of the precise traverse near Berne, which had been measured by methods previously described, was used as the base line for the triangulation system. The results of the first and second measurements of the precise traverse, as compared with the triangulation survey, are shown in Table 4. In determining final measurement, one-third weight was given to the first measurement over the Great Northern traverse and two-thirds weight to the second measurement, the result of the triangulation being used for the purpose of comparison.

TABLE 4.—Comparison of Results

(H. B. Alvord's Results Proportioned to Conform with Observed Length of Base Line, *A C*, in Each Case)

Measurement	Observer	Length of Lines, in Feet		
		A D	*C E*	*A N*
First	Great Northern Engineers	....	25 062 681	43 752.65
	H. B. Alvord	11 402.921	25 064.144	43 751.02
Second	Great Northern Engineers	11 400.358	25 061.449	43 750.22
	H. B. Alvord	11 401.451	25 060.913	43 745.38

Location of Mill Creek Shaft.—The 8 by 24-ft. shaft was "lined in" on the tunnel axis from two intermediate theodolite stations on the east and west slopes of Mill Creek Valley, shown in Fig. 9. The plumbing of the shaft has already been mentioned.

To determine the distance from the east portal to the shaft, the triangle, *A-B-D* (see Fig. 13), was laid out on the ground, the station, *B*, being a point on the base line, *A-C*. Station *D* was a point accurately set on the tunnel axis. The solution of the triangle, *A-B-D*, gave the length of the side, *A-D*, to which was added the measured distance from Station *D* to the mouth of the shaft.

Determination of Grade.—The west slope of the Cascades is everywhere much steeper than the east slope, the rise to the summit of the range being made in a shorter distance. In order to overcome this difference in natural ground slopes, the summit of the tunnel grade was placed at the east portal, which provided a longer distance on the west slope in which to make the climb to the summit, than would have been possible with the summit at some intermediate point between the portals. By following this plan, the same ruling grade was established over the engine district for eastbound and westbound movements.

A uniform grade of 1.565% was laid through the tunnel, descending from east to west. A long vertical curve of 0.10 variation per station connected the tunnel grade tangent with the 1.665% grade, ascending from east to west, on the new approach tracks to Berne.

When the elevation of the top of the shaft had been determined in its proper relation to the east and west portals, the sub-grade of the main tunnel was set 622 ft. below the top of the shaft. A steel tape was used to carry the measurement down the vertical side walls of the shaft.

Table 5 shows the comparison between the old and new line, between Berne and Scenic.

TABLE 5.—Comparisons Between the Old and New Lines Between Berne and Scenic

Items	Old line	New line	Favorable to new line
Length, in miles	17.67	9.99	7.68
Summit elevation, in feet (sub-grade)	3 382	2 881	501
Total rise westward, in feet	546	45	501
Total fall westward, in feet	1 325	824	501
Snowsheds, total length, in miles	6.04	0.00	6.04
Bridges, total length, in miles,	0.23	0.04	0.19
Tunnels, total length, in miles	3.66	7.79	(4.13)*
Maximum curve	10°00′	6°00′	4°00′
Total curvature	2 128°	187°	1 941°
Maximum grade, percentage	2.2%	2.2%	....
Tunnel grade, percentage	1.695%	1.565%	0.13%

* Unfavorable to new line.

General Plan of Attack

The final plan for the construction of the Simplon Tunnel in the Swiss Alps was devised by the Jura-Simplon Railway Company in 1893, and was submitted, by an arrangement with the Swiss and Italian Governments, to a commission of experts, made up of Guiseppe Columbo, an engineer of Milan, Italy; Francis Fox, of London, England; and C. J. Wagner, of Vienna, Austria, State Inspector of Railways, formerly one of the engineers of the Arlberg Tunnel.

The plan provided for the driving of a secondary gallery, or pioneer tunnel, parallel to the main tunnel and 55.75 ft. from it. This pioneer tunnel was to be used to drain off underground water and to convey fresh air and cold water to the working faces. The plan had special merit in that it made it possible to enlarge the auxiliary gallery to a full-sized tunnel for double-track operation, whenever traffic conditions warranted such expenditure.

In formulating the plan for driving the New Cascade Tunnel, considerable study was given to the method of construction used on the Simplon Tunnel, and, as a result, it was decided to drive a pioneer tunnel between Mill Creek and the west portal.

The Mill Creek shaft divided the tunnel into two segments, the easterly portion being 2.41 miles in length, and the westerly portion, 5.38 miles. A study of time schedules showed that the section of tunnel between the east portal and Mill Creek could be completed within three years by following the center-heading method, without the aid of a pioneer tunnel, but that the section of tunnel between Mill Creek and the west portal would require the use of an auxiliary haulage-way.

Accordingly, a pioneer tunnel was laid out between the shaft and the west portal, 66 ft. to the south of, and parallel with, the center line of the main tunnel, corresponding to the 55.75-ft. offset distance used in the Simplon Tunnel. (See Fig. 14.) It was considered that this interval provided sufficient distance away from the main tunnel to avoid disturbance to the rock formation without being too far away to complicate the construction of cross-cuts and the movement of spoil trains and construction supplies, back and forth between the pioneer and the main tunnel. In the event of future double track, the pioneer tunnel could be enlarged for second track, as was done in the Simplon Tunnel, or it could be used as a haulage-way from which to attack the new tunnel at intermediate points, as might be required. Likewise, the pioneer tunnel made permanent provision for taking care of the drainage from the main tunnel.

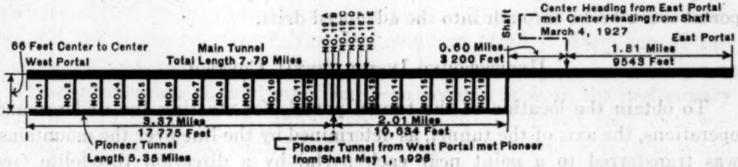

Center Heading from East Portal met Center Heading from Shaft March 4, 1927
Shaft
66 Feet Center to Center
West Portal
Main Tunnel
Total Length 7.79 Miles
NO.12A
NO.12B
NO.12C
NO.12D
NO.12E
NO.12F
0.60 Miles
3 200 Feet
East Portal
1.81 Miles
9543 Feet
NO.1
NO.2
NO.3
NO.4
NO.5
NO.6
NO.7
NO.8
NO.9
NO.10
NO.11
NO.12
NO.13
NO.14
NO.15
NO.16
NO.17
NO.18
3.37 Miles
17 775 Feet
2.01 Miles
10,634 Feet
Pioneer Tunnel
Length 5.35 Miles
Pioneer Tunnel from West Portal met Pioneer from Shaft May 1, 1928

FIG. 14.—PLAN OF MAIN TUNNEL, PIONEER TUNNEL, AND CROSS-CUTS.

CONTROLLING FEATURES OF PROJECT

The controlling features of the project, which made it necessary to attack the work simultaneously at several intermediate points between portals, were as follows:

(1) Length of tunnel, 7.79 miles.
(2) Size of tunnel, single-track (see Fig. 15).
(3) Grade, 1.565%, descending east to west.
(4) Tunnel to be lined with concrete throughout.
(5) Possibility of encountering heavy volume of water.
(6) Time for completion, 3 years.

In formulating a plan of attack, every consideration had to be given the water problem, with the possibility of encountering water in such quantities that drifting operations, down grade, might be brought to a standstill.

The tunnel grade descended from east to west, and at the time of starting construction it was not known how successful the tunneling operations would be, if driven down grade. Judging from the rainfall in the winter months throughout the Cascade Range, with its numerous mountain streams and springs, it was reasonable to expect water-bearing rock formation in all primary penetration. These uncertain factors, together with the speed requirements of the time schedule, were responsible for the decision reached to sink the Mill Creek shaft and drive a pioneer tunnel west of the shaft. The shaft opened up several new points of attack, and in case water in any considerable quantity was met between the east portal and the shaft, drifting operations could be continued up grade from the bottom of the shaft, the water encountered being drained by gravity into large sumps, and then lifted to the top of the shaft by electrically driven force pumps. Similarly, the pioneer tunnel, in advancing eastward (up grade) from the west portal, made it possible to drive the several cross-cuts to the main tunnel up grade, so that water from the enlargement operations would have gravity flow to the west portal.

Tye River Incline.—The surface indications at the west portal were of such a character that it was practically certain that soft ground would be encountered for a distance of about 1 000 ft. To avoid the delay which would result from excavating through this heavy ground, it was decided to drive a 30° incline adit, 251 ft. long, from the Tye River Valley to the pioneer tunnel, intersecting the pioneer center line and grade at a point 2 279 ft. from the west portal. Subsequent developments justified this move, as the material encountered in the pioneer tunnel, west of the adit, was of such a character that the advanced pioneer drifting crews, served from the Tye River incline, had penetrated to a point 5 703 ft. from the west portal by the time the west portal crews broke through into the advanced drift.

UNDERGROUND INSTRUMENTAL CONTROL

To obtain the location of the tunnel portals for starting the underground operations, the axis of the tunnel, as determined by the line over the mountains, was transferred to a point near each portal by a direction theodolite (see Fig. 9).

In the case of the east portal, the line was thrown down from the theodolite set up on the mountain east of the east portal (near Berne), using the monument east of Mill Creek as a foresight. A transit was then set up over the "point" near the east portal, and the monument east of Berne was used as a back-sight to produce the line into the tunnel. At the west portal, corresponding operations were carried out and, in addition, a base line was established to control the pioneer tunnel. An offset line, 66 ft. parallel to the main tunnel axis, was laid out on the south side of the tunnel tangent, and this base line was used to project the line into the pioneer tunnel. In the case of the underground workings near the Mill Creek shaft, the 19-ft. base line, plumbed down from the top of the shaft, was used to carry the line forward toward the east and west portals.

All the advanced workings in the main tunnel between the west portal and the Mill Creek shaft, were controlled from the pioneer tunnel, both as to line and levels. As soon as the underground operations had advanced any considerable distance, more precise methods of establishing line and grade were used. In following up the main enlargement from the east and the west portals, the large direction theodolite, mounted on a 6-in. iron pipe support, was used to give the final line. On account of the heavy smoke, long sights were, as a rule, impossible. Leveling was done with levels having 24-in. telescopes, 4-ft. Philadelphia rods being used, where required.

In driving the 10 by 10-ft. center heading between the east portal and the Mill Creek shaft, under the instrumental controls previously described, the drifting crews met on March 4, 1927, at a point 3 200 ft. east of the shaft, 9 543 ft. west of the east portal. The error of closure was 0.23 ft. for "line", 0.2 ft. for "grade", and 1.9 ft. for "distance".

In driving the 8 by 9-ft. pioneer tunnel eastward from the west portal, and westward from the shaft, the two crews came together on May 1, 1928, at a point 17 775 ft. from the west portal. The error of closure was 0.64 ft. for "line", 0.78 ft. for "grade", and 1.0 ft. for "distance". (See Fig. 14.) This junction of the two control lines in the pioneer tunnel, well in advance of enlargement operations in the main tunnel, provided a ready means of checking back over the offset lines, connecting the pioneer with the main tunnel, so that a perfect tangent was established in the main tunnel, before the concrete lining was placed.

GENERAL STANDARDS OF CONSTRUCTION

Tunnel Section.—The internal width of the tunnel, between the concrete side walls, was determined from Great Northern tunnel standards which provided for a 16 ft. 0 in. width on tangent track. The internal vertical clearance did not follow the Great Northern standard of 22 ft. from top of rail to center of roof arch.

The electrification of the tunnel had been decided upon in the preliminary studies and in designing the tunnel section, a three-centered roof arch was provided. An 11 000-volt contact system was specified for present installation but, in planning the electrification, it had been decided to provide space for insulation against a future 22 000-volt trolley line and, in order to avoid the necessity of increasing the height of the roof arch, to provide for the longer insulators which would be required for the higher voltage, special plans were made for attaching the overhead trolley system to the roof of the tunnel, which reduced the distance between the trolley and the tunnel roof. Circular domes, 5 ft. 0 in. in diameter and 1 ft. 2 in. deep, were provided at the center of the concrete roof arch, at 75 ft. 0 in. intervals, from which the trolley insulators were suspended by means of a bronze insert, set accurately in place during the process of pouring the concrete lining.

The use of these domes and the three-centered roof arch was economical in that for a given height of trolley wire, as shown by Fig. 15, it resulted in reduction in quantity of rock excavation and diminished height of concrete side walls throughout the length of the tunnel.

The external horizontal and vertical axes of the tunnel section are variable according to the nature of the rock and the amount of excavation necessary to provide proper thickness of concrete lining. (See Fig. 15.) As a general rule, the following vertical and horizontal dimensions were used in the process of enlarging to full tunnel section:

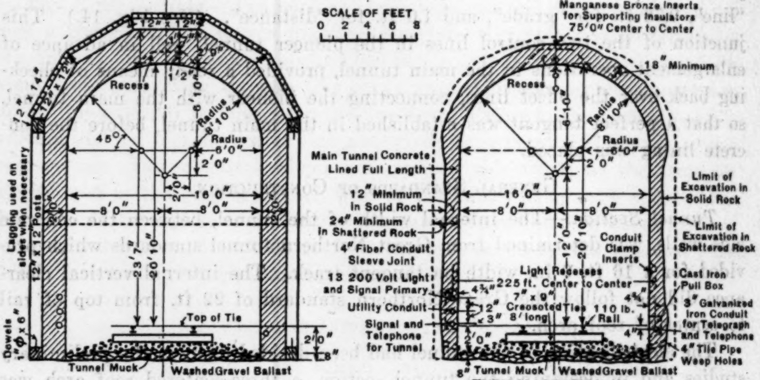
Hard rock, requiring no timbering, 18 ft. 0 in. width by 25 ft. 0 in. above tunnel sub-grade.

Soft rock, requiring no timbering, 20 ft. 0 in. width by 25 ft. 6 in. above tunnel sub-grade.

Earth or rock, requiring timber lining, 22 ft. 0 in. width by 27 ft. 6 in. above tunnel sub-grade.

In placing the 12 by 12-in. timber lining, the timber sets were spaced, longitudinally, according to the character of the earth or rock.

Track.—Track is laid with Great Northern section, 110-lb. rail, and 7½ by 10-in. steel tie-plates, weighing 13.05 lb. each; 3 200 creosoted ties, 7 by 9 in. by 8 ft. long, are used per mile of track; 21 in. of washed gravel ballast was placed under the ties after the tunnel floor was built and leveled off to sub-grade with broken rock.


12"x12"
12"x14"
12"x12"
Recess
45°
1'2"
2'9"
8'10"
Radius 8'10"
Radius 6'0"
2'0"
2'0"
16'0"
8'0"
8'0"
2'0"
2'0"
23'7"
20'10"
Lagging used on sides when necessary
12"x12" Posts
Dowels 1"Φ x 6"
Top of Tie
2'0"
8"
Tunnel Muck
Washed Gravel Ballast
SCALE OF FEET
0 2 4 6 8
Manganese Bronze Inserts for Supporting Insulators 75'0" Center to Center
18" Minimum
Recess
6'10"
Radius 8'10"
Radius 6'0"
2'0"
Main Tunnel Concrete Lined Full Length
16'0"
8'0"
8'0"
12" Minimum In Solid Rock
24" Minimum In Shattered Rock
Limit of Excavation in Solid Rock
Limit of Excavation in Shattered Rock
12'0"
20'10"
Conduit Clamp Inserts
4" Fiber Conduit Sleeve Joint
13 200 Volt Light and Signal Primary
Light Recesses 225 ft. Center to Center
Cast Iron Pull Box
4¾"
4¾"
7"x9" Creosoted Ties 8' long
110 lb. Rail
12"
3"
Utility Conduit
3" Galvanized Iron Conduit for Telegraph and Telephone
Signal and Telephone for Tunnel
2'0"
4"
4" Tile Pipe Weep Holes
8"
Tunnel Muck
Washed Gravel Ballast

TYPICAL TIMBERED SECTION

TYPICAL SECTION IN SELF SUPPORTING ROCK (LOOKING EAST)

FIG. 15.—TYPICAL SECTIONS, NEW TUNNEL THROUGH THE CASCADE RANGE.

Drainage System.—The quantity of water encountered in underground operations differed greatly throughout the tunnel, and the permanent drainage system was designed to meet these varying conditions so as to provide ample capacity without unnecessary expense.

The east half of the tunnel was driven through comparatively dry formation, so that from the east portal westward for 3.85 miles, water, entering the tunnel from 4-in. weep-holes through the concrete lining along both sides of track, was carried in side ditches. In this distance, there was not a sufficient quantity of water to cause any trouble. At a point 3.85 miles from the east portal the accumulation of water was turned into the pioneer tunnel by means of a 12-in. pipe, embedded in the south wall of the concrete tunnel lining.

From this easterly cross-drain westerly, for a distance of 2.38 miles, the tunnel passed through a heavy, water-bearing formation, which required that cross-drains to the pioneer tunnel be constructed about every 1 500 ft. The pioneer tunnel, therefore, became the permanent artery of the drainage system, into which all water entering the main tunnel between the east portal and the most westerly cross-drain, a distance of 6.23 miles, was diverted. No cross-drains from the main tunnel to the pioneer tunnel were found necessary, from a point 1.56 miles east of the west portal to the west portal, because no heavy, water-bearing formation was encountered in this westerly section.

The bottom of the Mill Creek shaft was filled with concrete to a point 10 ft. above the top of the concrete tunnel arch. A 6-in. pipe was placed in the concrete, discharging into the south side ditch of the main tunnel. The Mill Creek shaft was then completely back-filled with the broken rock, removed in the process of sinking the shaft.

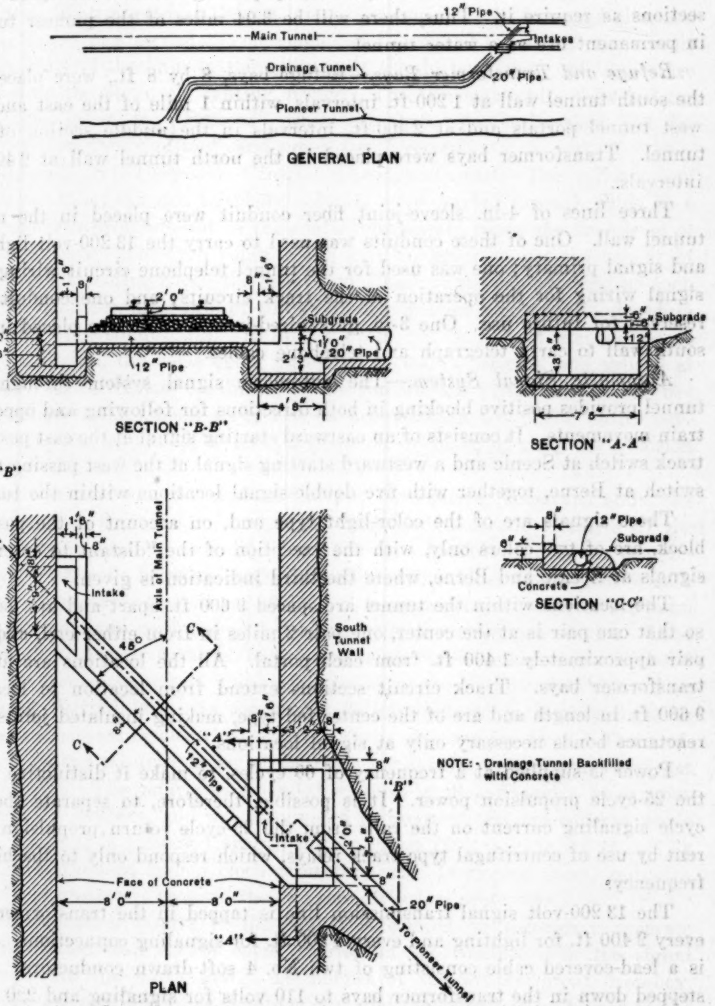

12" Pipe
Main Tunnel
Intakes
Drainage Tunnel
20" Pipe
Pioneer Tunnel
GENERAL PLAN
1'6"
8"
2'0"
8"
1'6"
Subgrade
1'0"
1'6"
1'0"
2'3"
20" Pipe
12" Pipe
4'8"
SECTION "B-B"
6"
6"
Subgrade
12"
3'8"
7'6"
SECTION "A-A"
"B"
1'6"
8"
1'9"
8"
Intake
Axis of Main Tunnel
North Tunnel Wall
C
45°
South Tunnel Wall
8"
12" Pipe
C
"A"
8"
1'6"
3'2"
8"
8"
"B"
NOTE: - Drainage Tunnel Backfilled with Concrete
Intake
2'0"
5'0"
8"
Face of Concrete
8'0"
8'0"
20" Pipe
"A"
To Pioneer Tunnel
PLAN
8"
12" Pipe
6"
Subgrade
Concrete
SECTION "C-C"

Fig. 16.—Plan Showing Method of Diverting Water from Side Ditches of Main Tunnel to Pioneer Tunnel.

Fig. 16 shows the typical plan used to divert water from the main to the pioneer tunnel. The size of the cross-drain pipe was dependent upon the flow of water, 20-in. pipe being used in most cases. Thirteen permanent cross-

drains were built and these continue in service. At present, there are about 5 000 gal. per min. flowing out of the pioneer tunnel at the west portal, and steps are now being taken to maintain the pioneer tunnel, west of the first cross-drain (3.85 miles from east portal), by placing concrete lining in such sections as require it. Thus, there will be 3.94 miles of the pioneer tunnel in permanent use as a water tunnel.

Refuge and Transformer Bays.—Refuge bays, 8 by 8 ft., were placed in the south tunnel wall at 1 200-ft. intervals, within 1 mile of the east and the west tunnel portals and at 2 400-ft. intervals in the middle section of the tunnel. Transformer bays were placed in the north tunnel wall at 2 400-ft. intervals.

Three lines of 4-in. sleeve-joint fiber conduit were placed in the north tunnel wall. One of these conduits was used to carry the 13 200-volt lighting and signal primary; one was used for the tunnel telephone circuit wiring and signal wiring for the operation of the track circuits; and one conduit was reserved for future use. One 3-in. galvanized-iron conduit was placed in the south wall to carry telegraph and telephone cables.

Automatic Signal System.—The automatic signal system through the tunnel provides positive blocking in both directions for following and opposing train movements. It consists of an eastward starting signal at the east passing-track switch at Scenic and a westward starting signal at the west passing-track switch at Berne, together with five double-signal locations within the tunnel.

These signals are of the color-light type and, on account of the positive block, are of two colors only, with the exception of the "distant to entering" signals at Scenic and Berne, where the third indication is given.

The locations within the tunnel are spaced 9 600 ft. apart and are located so that one pair is at the center, one pair 2 miles in from either end, and one pair approximately 1 400 ft. from each portal. All the locations are at the transformer bays. Track circuit sections extend from location to location 9 600 ft. in length and are of the center-fed type, making insulated joints and reactance bonds necessary only at signal locations.

Power is supplied at a frequency of 60 cycles, to make it distinctive from the 25-cycle propulsion power. It is possible, therefore, to separate the 60-cycle signaling current on the rails from the 25-cycle return propulsion current by use of centrifugal type track relays, which respond only to the higher frequency.

The 13 200-volt signal transmission line is tapped in the transformer bay every 2 400 ft. for lighting and every 4 800 ft. for signaling connections. This is a lead-covered cable consisting of two No. 4 soft-drawn conductors. It is stepped down in the transformer bays to 110 volts for signaling and 220 volts for lighting. The cable is entirely sectionalized at the center bay and at the 2-mile bays. The track reactor bonds, with a capacity of 500 amperes per rail, which make the track continuous for propulsion current, but permit it to be cut into insulated sections for signaling current, are located in the transformer bays at the signal locations. The control circuits are carried in rubber-covered wire through a fiber conduit in the tunnel wall and alternating-current line relays are used throughout.

The light signals are bolted directly to the tunnel wall so that their lowest point is at the level of the top of the rail, without encroaching on the side clearance. They are 8-volt 10-watt lamps, lighting directly from the alternating current, with a second or emergency light arranged to cut into the circuit automatically if the bulb in the green unit should burn out. All lights are operated on the approach lighting system. The two-color units show red for "Stop" and green for "Proceed", while the three-color units show red for "Stop", yellow for "Caution", and green for "Proceed".

Telegraph and Telephone Circuits Through Tunnel.—The railway and commercial telegraph and telephone circuits were carried through cable in a 3-in. galvanized-iron conduit in the south tunnel wall, below the floor level of the refuge bays. The iron conduit was used in order to get the benefit of its magnetic shielding effect, the usual lead sheath of the cable having no such magnetic qualities, although, of course, it does possess electrostatic shielding properties. The conduit run was broken every 600 ft. by cast-iron pull-boxes of special design, the cable being pulled in in 600-ft. lengths, and spliced at each box. The electrical continuity of the conduit was preserved past the box joints by spot-welded bond wires carried around each box. The entire conduit system was grounded, to provide effectual drainage of stray high-tension current.

The cable used was of American Railway Association, Telegraph and Telephone Section, Type H, 14 pairs, with the added dielectric strength of the double-paper insulation, providing for 2 000-volts breakdown between conductors, and 3 500 volts between conductor and sheath. All conductors were both paired and quadded, and all are 13 B. & S. gauge. Three quads, or six pairs of conductors, were equipped with loading coils, for telephone service. Load coils are located every 6 000 ft., or at every tenth pull-box, where an additional niche was cast in the tunnel lining of sufficient size to accommodate the coil on the top of the pull-box. As the load coils are enclosed in iron casings, no further mechanical or electrical protection was required for them. The loading coils reduce the impedance of the cable conductors to approximately that of open wire. The cable was extended at the portals sufficiently to run out even 6 000-ft. sections, and was terminated in fireproof tile and concrete cable houses, where it is protected against high voltages from outside the tunnel by two sets of standard carbon block protectors, with 14-ampere fuses, and with a set of gas protectors between the two sets of carbon protectors.

Communication locally in the tunnel is carried on through a separate rubber-insulated lead sheath cable, carried in the conduit with the signal secondaries in the north wall. Telephones of the weather-proof, iron-box, mine type are provided in each refuge bay, and the circuit is continued to the depot office near each portal, where necessary switching apparatus is provided to connect such tunnel circuit to the dispatcher's or other telephone circuits, as may be required. Extra conductors are also provided in this cable to take care of the dispatcher's telephone and to provide necessary test conductors, should the main cable be damaged beyond immediate repair. Each telephone on the tunnel circuit is operated through an insulating transformer, to insure operation under adverse inductive conditions without danger to the user.

Tunnel Lighting.—Lighting recesses were placed at 112½-ft. intervals, "staggered", throughout the length of the tunnel. A plan was developed to utilize the 3/0 conductor No. 2 cable, which was used to furnish power for tunnel construction, as a means of providing for flood lighting at any point throughout the tunnel. These cables are attached to the side walls of the tunnel in such a manner that any part of the track can be lighted for the benefit of track gangs.

Trolley.—The 11 000-volt contact system throughout the tunnel is of simple catenary construction, consisting of a 350 000-cir. mils, stranded, copper messenger from which is suspended, by flexible loop hangers, a single 4/0 hard-drawn copper contact wire. The messenger is supported at 75-ft. intervals by a three-unit suspension type insulator and the entire contact system is sectionalized with an air gap and anchored at spacings of approximately 1 mile. Suspension type insulators were used throughout, both for messenger suspension and for strain insulators at the air gaps, with a voltage rating of 60 000.

LINE AND GRADE IMPROVEMENTS, EAST SLOPE OF CASCADES

Coincident with the construction of the New Cascade Tunnel, and the electrification of the main line, a comprehensive program of grade and curve reduction and elimination of snowsheds was carried out on the east slope of the Cascade Range between Peshastin (a station 19 miles west of Wenatchee) and Berne, near the east portal of the new tunnel. (See Fig. 2.)

The Chumstick Line.—The important part of this line revision was the construction of the new Chumstick Line from Peshastin to Winton, 16.92 miles, at a cost of nearly $5 000 000. The building of the Chumstick Line, so-called from the name of the creek valley which it follows for 7 miles, had been under consideration for a number of years prior to the construction of the New Cascade Tunnel.

In 1921, surveys were made for various routes *via* the Chumstick Valley, to avoid the Tumwater Canyon of the Wenatchee River. Most of these investigations contemplated turning off the old line near the Town of Leavenworth. The line that was constructed was a more radical improvement, in that better standards were used, with maximum curvatures of 3°, and a maximum grade of 1.6 per cent. In order to adhere to these standards, it was necessary to construct a spur line, 8 860 ft. long, to Leavenworth, to serve that community with carload freight.

The old main line followed the water grade of the Wenatchee River from its mouth to a point about 2 miles east of Winton, as shown in Fig. 2. The fall of the Wenatchee River between Leavenworth and its confluence with the Columbia River, is only 23 ft. per mile, but above Leavenworth, as far as Chiwaukum, the river passed through the Tumwater Canyon in a succession of rapids, where the rate of fall, in certain sections, is 100 ft. per mile. The old line was benched in on the north bank through this canyon, which required the use of 2.2% grades. For some distance through the canyon, the old track was protected against snow and rock slides from the upper slope by timber sheds, a large percentage of which required renewal in 1928 and 1929. Of these sheds, 8 401 lin. ft. were already in place, and to protect the line adequately through the Tumwater Canyon, it was considered necessary to build approximately 4 000 ft. of new sheds. The large snowshed program confronting the Railroad Company in the near future was a deciding factor in favor of the new route, but there were other advantages to be gained by the construction of the Chumstick Line, which may be briefly listed as in Table 6.

TABLE 6.—ADVANTAGES OF CHUMSTICK LINE COMPARED WITH OLD LINE, PESHASTIN TO WINTON (16.92 MILES)

Item	Old line	New line	Saving
Length	18.08 miles	16.92 miles	1.16 miles
Curvature	2 039°-52′	622°-08′	1 377°-44′
Maximum curve	9°-30′	3°-00′	6°-30′
Number of curves	77	24	58
Maximum grade	2.2%	1.6%	0.6%
Length of snowsheds	8 401 ft.	....	8 401 ft.
Length of tunnels	....	7 448 ft.	....

General Route.—Chumstick Line.—The new Chumstick Line turns off from the old line at Peshastin, and follows comparatively level bench lands until the Chumstick Valley is reached. The line then turns up this valley for about 7 miles, with very good general alignment; it penetrates a ridge, separating the Upper Chumstick Valley from the Wenatchee River Valley, by a tunnel 2 601 ft. long, and crosses the Wenatchee River ½ mile from the west portal of the tunnel on a 360-ft. steel bridge, at a point about 5 miles above the old line crossing, near Chiwaukum. For the next 2 miles, the line is laid along the slopes above the Wenatchee River, and follows Dead Horse Canyon for 1 mile, to a tunnel, 4 059 ft. long, through the ridge to Winton, where it connects with the old grade.

The maximum grade on this line is 1.6%, against westbound traffic, and is compensated for curvature. The first section of the 1.6% grade is 3 miles long, and the second section, 3½ miles long, and, except for these two stretches, the maximum grade is 1.5%, compensated.

Minor Line Revisions.—In addition to the new Chumstick Line, numerous short sections of the old line between Winton and Berne, a distance of 14 miles, were improved in alignment and grade. From Winton westward to Nason Creek (3 miles), a radical line change was made at a cost of about $500 000, which crossed and recrossed the old line, eliminating considerable curvature and improving the grade. There was a false summit on the old line at Winton which was approached by 2.2% grades westbound and 1% grades eastbound. This summit was lowered, and the sag to the westward was raised so as to provide a 1% westbound and a 0.6% eastbound grade.

Several other small line changes, in which a number of sharp curves were eliminated or reduced, were made on the east slope of the Cascades, by the distribution of 254 109 cu. yd. of tunnel spoil in a distance of 7 miles.

PART III.—CONSTRUCTION PLANS AND METHODS

BY J. C. BAXTER,[4] M. AM. SOC. C. E.

As stated in Part I, the snowshed problem on the west slope of the Cascade Range early brought under consideration, by the Chief Engineers of the Railway Company, routes avoiding the snowslide area. All these routes embodied long tunnels through the Cascade Range and in the later examination of these various projects, studies were made by the contractor (A. Guthrie & Co., Inc.), of the probable time required for driving the tunnels.

These studies were made with the primary thought of rapid progress:

First.—To save interest during construction.

Second.—Because additional savings could be made during the last year and possibly during the last two years of the construction period by not spending any money on replacement of snowsheds.

Before proceeding with a detailed time study of any of the various tunnel projects, an examination was made of the methods that had been used for driving railway and mine tunnels, and of the probable drainage requirements of a long tunnel through the Cascade Range. From this examination the following statements were developed:

(*a*) The highest rate of progress in a long railway tunnel, as represented by total length divided by total time, had been made in the Simplon Tunnel, completed in 1906, and driven from an outside working gallery, or pioneer drift.

(*b*) The next highest rate of progress had been made in the Rogers Pass or Connaught Tunnel on the Canadian Pacific Railway in British Columbia, which was completed in 1916 and was probably the first example in the Western Hemisphere of the use of the Simplon outside gallery, or pioneer drift.

(*c*) The lowest cost per cubic yard for heading excavation obtained by the contractor had been made in a double-track tunnel through shale in the Rocky Mountains on the Great Northern Railway, where working galleries on each side of the tunnel at the wall-plate elevation had been driven its entire length before proceeding with the heading excavation, which was then carried out from cross-cuts connecting the two working galleries, or pioneer drifts.

(*d*) The advantages of an outside working gallery could be obtained, under certain conditions, by the use of a shaft, as exemplified by the Ozark Tunnel, in Illinois, on the Illinois Central Railway.

(*e*) Where, as in the case of the Simplon Tunnel, the pioneer drift was afterward enlarged to a second railway tunnel, a pronounced saving in cost accrued, in addition to the increases in speed, and it was possible that a long tunnel through the Cascade Range might eventually require double track, or at least the construction of a passing track at its mid-length.

From a study of mining methods it appeared that most low-cost coal mining operations were conducted from working galleries or drifts. A study of the probable drainage requirements indicated that:

---

[4] Contr. and Engr. (Carey, Baxter & Kennedy), New York, N. Y., formerly Vice-Pres., A. Guthrie & Co., Inc.

(1) Practically all long tunnels driven in the past had developed large flows of water.
(2) The rocks of the Cascade Range, on account of the heavy annual precipitation, might be expected to carry water in large quantities.
(3) An outside working gallery, or pioneer drift, would facilitate the handling of water during construction, and provide an auxiliary drainage gallery into which any permanent flows of water could be diverted.

In consideration of the foregoing it was decided to assume a pioneer drift, or vertical shaft, where either were applicable, in making studies of the probable time that would be required to drive any of the projected tunnels.

These studies indicated that the tunnel on the route finally selected could be driven in three years. To arrive at this time the following plan had been outlined:

(*a*) The line of the tunnel was crossed at a point about one-third the distance from the east end by the Valley of Mill Creek, the tunnel grade at this crossing being only 622 ft. below the floor of the valley. A shaft at this point would cut the tunnel into two portions: An eastern section, 2.41 miles long, and a western section, 5.38 miles long; thereby reducing the continuous length of tunnel to be attacked to the length of the western section; that is, 5.38 miles.
(*b*) The Western section if attacked from both ends (through the west portal and the Mill Creek shaft), could be completed in three years, using a pioneer drift and center heading and removing all the enlargement muck from one face if the ground was favorable to enlargement from a center heading. If the ground was not favorable to such an operation, then the section could be completed in three years by as many secondary enlargement operations served through the pioneer drift as were necessary.
(*c*) The eastern section was obviously one that could be completed in three years in many different ways. However, it was almost certain the tunnel would require concrete lining throughout. There was no suitable aggregate for concrete near the Mill Creek shaft, which was two miles from the operated line of the railroad. The plan on this section was to speed it up by driving a center heading from both the east portal and the Mill Creek shaft, enlarging this section entirely from the east portal while securing access to the center heading from the Mill Creek shaft instead of a pioneer drift. In this way it would be possible to complete the eastern section early enough so that aggregate for the concrete lining of that section and of the section driven west from the Mill Creek shaft, could be delivered by rail at the east portal, which was practically on the line of the existing railroad.
(*d*) The ground from *A* to *B*, Fig. 17, consisted of the remains of an old slide, and it was certain that the penetration of this distance would be very slow. Therefore, an inclined shaft was proposed at *B* in the Valley of the Tye River, in order that the pioneer drift might be advanced continuously during the period that would be required to drive through from the west portal to *B*.

A time-distance chart based on this general plan is shown in Fig. 18, on which has been superimposed the actual progress in the various operations. After decision to construct the tunnel had been reached, this general plan was adopted and the tunnel successfully completed. However, this success was due to the flexibility of the general plan adopted, in that it permitted wide variations of procedure where necessary to meet changing ground and water conditions. Sudden and great changes in ground and water conditions have been typical of all long tunnel operations, and the Cascade Tunnel ran true to type, but, as mentioned, the basic plan of operation lent itself readily to meeting these contingencies.

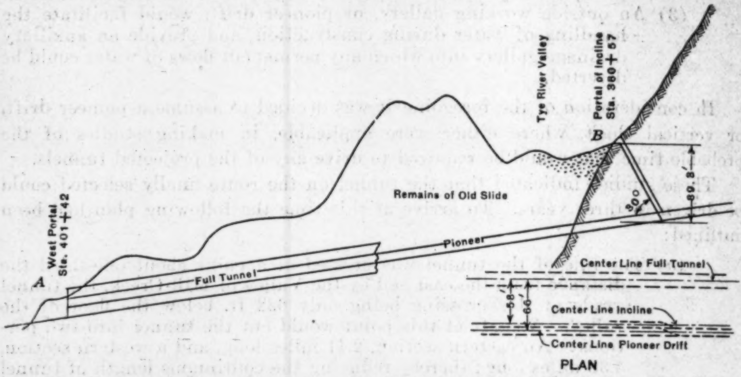

Tye River Valley
Portal of Incline Sta. 380+57
B
80.8′
300
Remains of Old Slide
West Portal Sta. 401+42
A
Pioneer
Full Tunnel
Center Line Full Tunnel
58′
66′
Center Line Incline
Center Line Pioneer Drift
PLAN

FIG. 17.—DIAGRAM SHOWING LOCATION OF TYE RIVER INCLINE AND REMNANTS OF OLD SLIDE.

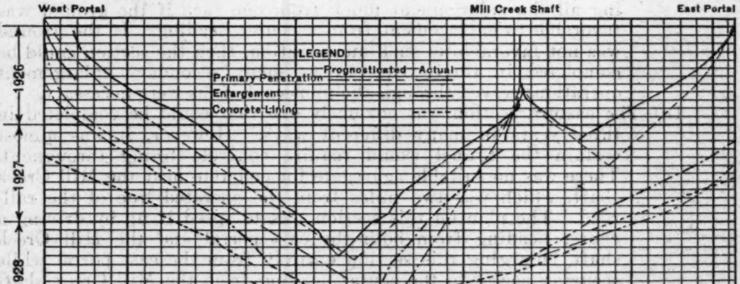


FIG. 18.—TIME-DISTANCE PROGRESS CHART, SHOWING ACTUAL COMPARED WITH PROGNOSTICATED PROGRESS, CASCADE TUNNEL.

To meet the various ground conditions encountered, seven general methods of attack were utilized, with combinations of some of them for short distances. These methods of attack and the conditions governing their use, were:

1.—Center heading, followed by enlargement (shown in Fig. 19). This was the basic method in the Cascade Tunnel plans. However, it only produces the best results where the ground is such that a full section of the tunnel will stand without timbering.

2.—Center heading, followed by top heading and bench (shown in Fig. 20). This method was used in ground where, after driving the center heading, it became apparent that the full tunnel section would not stand without timbering.

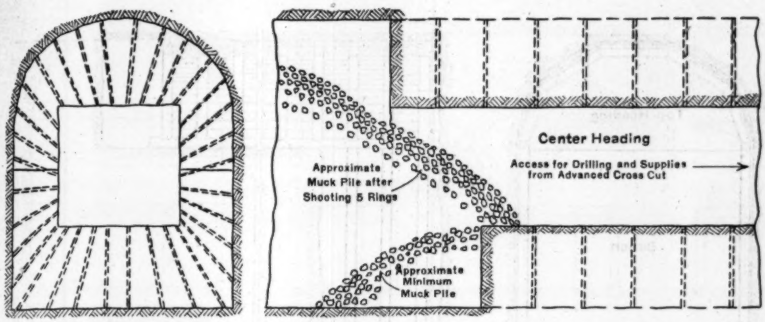

Approximate
Muck Pile after
Shooting 5 Rings
Center Heading
Access for Drilling and Supplies
from Advanced Cross Cut
Approximate
Minimum
Muck Pile

FIG. 19.—DIAGRAM SHOWING METHOD OF ENLARGING TO FULL TUNNEL FROM CENTER HEADING.

3.—Top heading and bench (shown in Fig. 21). This method is the usual so-called "American Method" and was used in ground which, from indications in the pioneer drift, obviously would not stand without timbering.

4.—Wall-plate drifts, followed by top heading and bench (shown in Fig. 22). This is a modification of the method described under Method 3 and was used in ground too soft to stand for a full top heading.

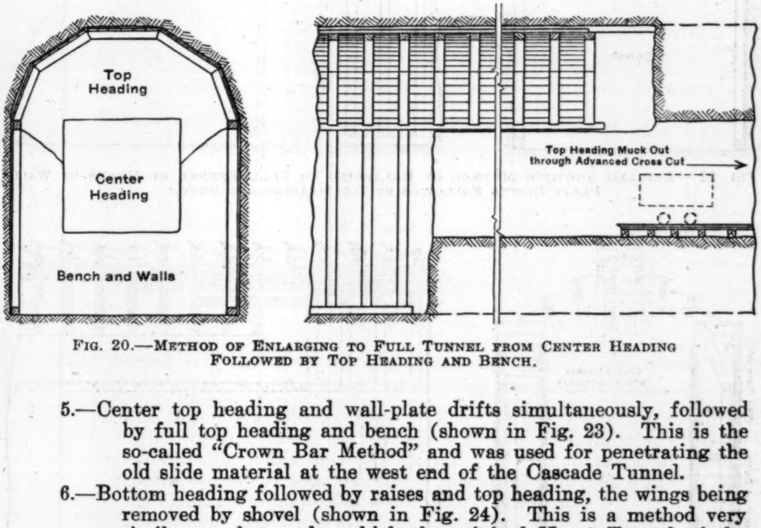

Top
Heading
Center
Heading
Bench and Walls
Top Heading Muck Out
through Advanced Cross Cut

FIG. 20.—METHOD OF ENLARGING TO FULL TUNNEL FROM CENTER HEADING FOLLOWED BY TOP HEADING AND BENCH.

5.—Center top heading and wall-plate drifts simultaneously, followed by full top heading and bench (shown in Fig. 23). This is the so-called "Crown Bar Method" and was used for penetrating the old slide material at the west end of the Cascade Tunnel.

6.—Bottom heading followed by raises and top heading, the wings being removed by shovel (shown in Fig. 24). This is a method very similar to that under which the original No. 1 Tunnel of the Simplon was driven, and was made use of in the Cascade Tunnel in order to expedite completion by introducing hand-tunneling operations through several of the cross-cuts after the pioneer was holed through.

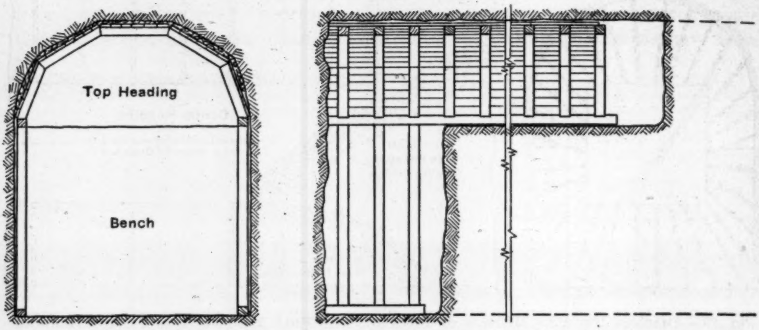

Top Heading
Bench

FIG. 21.—DIAGRAM SHOWING TOP HEADING AND BENCH METHOD OF DRIVING FULL TUNNEL.

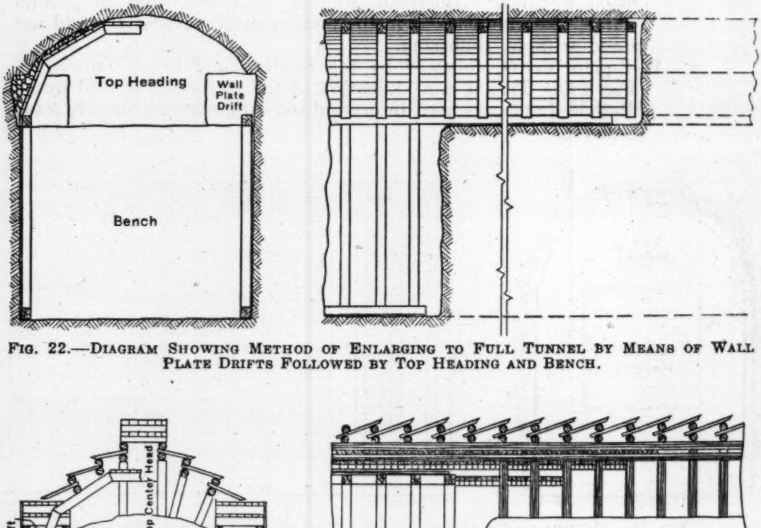

Top Heading
Wall Plate Drift
Bench

FIG. 22.—DIAGRAM SHOWING METHOD OF ENLARGING TO FULL TUNNEL BY MEANS OF WALL PLATE DRIFTS FOLLOWED BY TOP HEADING AND BENCH.

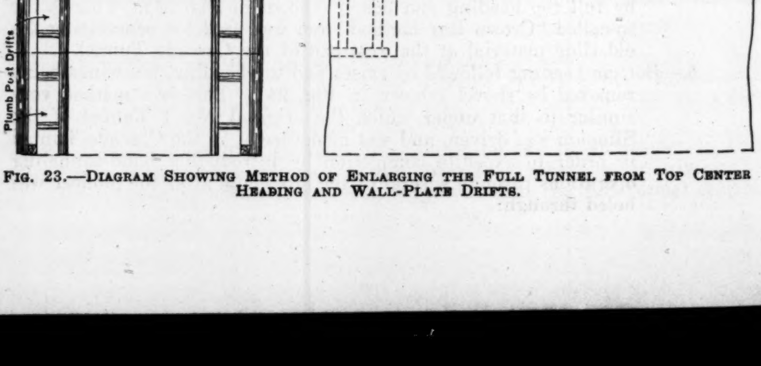

Top Center Head
Breast Boards where necessary
Wall Plate Drift
Plumb Post Drifts

FIG. 23.—DIAGRAM SHOWING METHOD OF ENLARGING THE FULL TUNNEL FROM TOP CENTER HEADING AND WALL-PLATE DRIFTS.

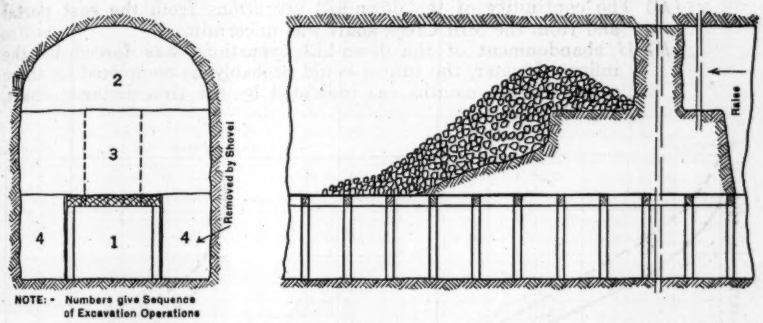


FIG. 24.—DIAGRAM SHOWING METHOD OF ENLARGING TO FULL TUNNEL FROM BOTTOM CENTER HEADING BY TOP HEADING AND RAISES.

7.—Full tunnel face (shown in Fig. 25). This is a method seldom used in tunneling, but was adopted in the Cascade Tunnel in order to expedite completion by providing additional mechanical loading places, reached through cross-cuts from the pioneer drift in advance of the main enlargement operation. The procedure is to shoot 1*A* first and 1*B* immediately thereafter. The independent round is then drilled and shot as soon as ready.

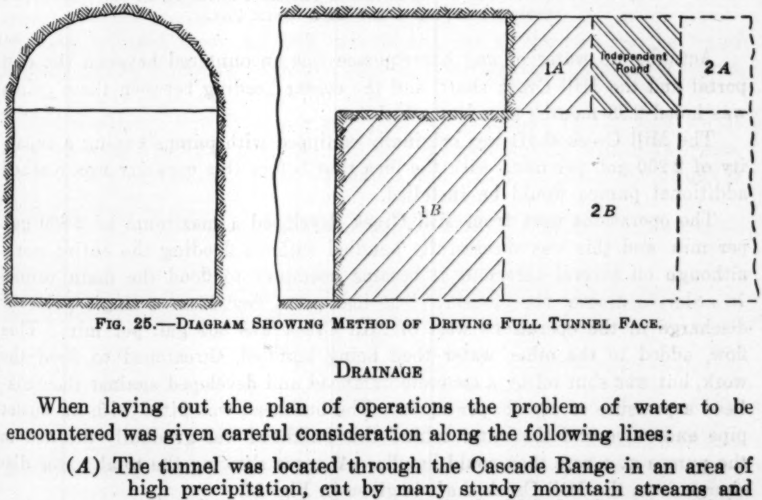


FIG. 25.—DIAGRAM SHOWING METHOD OF DRIVING FULL TUNNEL FACE.

DRAINAGE

When laying out the plan of operations the problem of water to be encountered was given careful consideration along the following lines:

(*A*) The tunnel was located through the Cascade Range in an area of high precipitation, cut by many sturdy mountain streams and containing many springs, several of which in the near vicinity were warm. It was, therefore, almost certain that water in large but unknown quantities would be encountered.

(*B*) The tunnel grade was all in one direction, down hill from the east portal.

(*C*) Water in any quantity would slow up the operation, but in large quantities (from 3 000 to 5 000 gal. per min.) would probably cause the abandonment of an operation being carried out down hill.

(*D*) The continuity of the down-hill operations from the east portal and from the Mill Creek shaft was uncertain.

(*E*) If abandonment of the down-hill operations was forced by the inflow of water, the tunnel could probably be completed in three years and six months (as indicated by the time-distance chart, Fig. 26).

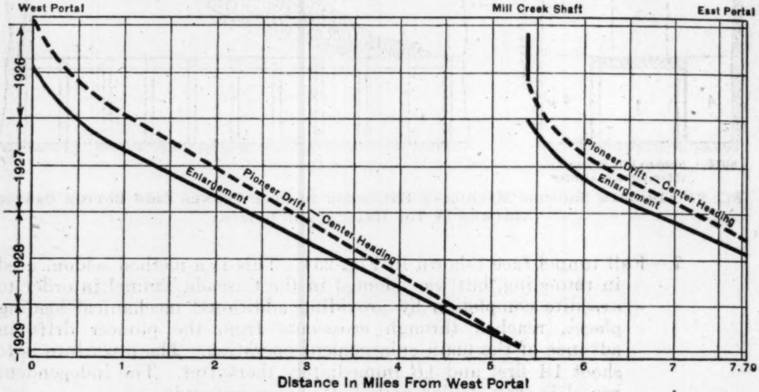


FIG. 26.—TIME-DISTANCE CHART SHOWING PROBABLE PROGRESS BASED ON UPGRADE OPERATION FROM WEST PORTAL AND MILL CREEK SHAFT.

Actually no water of any consequence was encountered between the east portal and the Mill Creek shaft, and the center heading between these points was holed 2.93 months ahead of schedule.

The Mill Creek shaft was originally equipped with pumps having a capacity of 2 200 gal. per min., with the idea that before this capacity was reached additional pumps would be installed.

The operations west from Mill Creek developed a maximum of 2 800 gal. per min. and this was successfully handled without flooding the entire work, although on several occasions it became necessary to flood the main tunnel in order to protect the pioneer. The maximum flow from a single point of discharge in the operations west of Mill Creek was 800 gal. per min. This flow, added to the other water then being handled, threatened to flood the work, but was shut off by a concrete bulkhead and developed against the bulkhead a pressure of 100 lb. per sq. in. The bulkhead was fitted with an outlet pipe and valve and the water behind this bulkhead was gradually drained to the pumps at a rate they could handle. A curve showing the total water discharge from the Mill Creek shaft is given in Fig. 27.

Fortunately, the worst water conditions were encountered in the operations being driven up hill from the west portal where the water could flow out by gravity and where a maximum discharge of 10 300 gal. per min. was developed. Practically all the water encountered was ground-water filling fissures in the rock. These fissures were of all sizes—from hair-cracks to 6 or 8 in. across. All these fissures acted in the same manner: A maximum flow when first encountered; a radical decrease in the flow in the next few days (sometimes

hours), with a gradual decrease over weeks and months, at times drying up entirely.

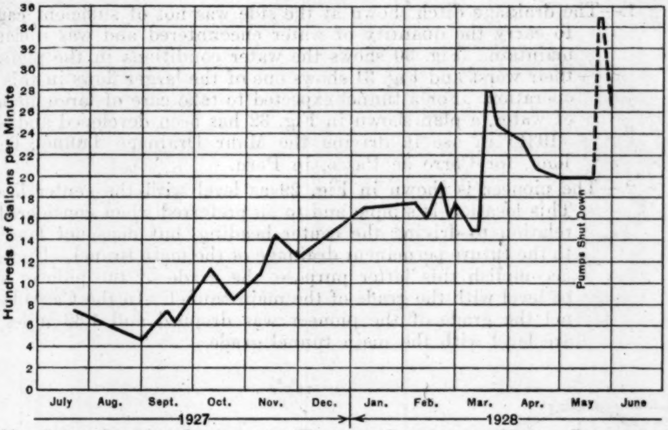


FIG. 27.—CHART SHOWING QUANTITY OF WATER IN GALLONS PER MINUTE PUMPED AT MILL CREEK SHAFT. BROKEN LINE INDICATES FLOW TO WEST PORTAL.

Very high pressures were occasionally encountered in drilling, but were never measured accurately; for instance, a 12-ft. drill steel, 1¼ in. in diameter, when released from the drill, was forced out of the hole by the water pressure, the steel passing back over the drill carriage and injuring a nipper who was about 15 ft. back from the face.

The maximum flow from a single point of discharge in the west portal operations was 5 000 gal. per min., but this dropped rapidly within 24 hours. A curve showing the total water discharge from the west portal operations is given in Fig. 28.

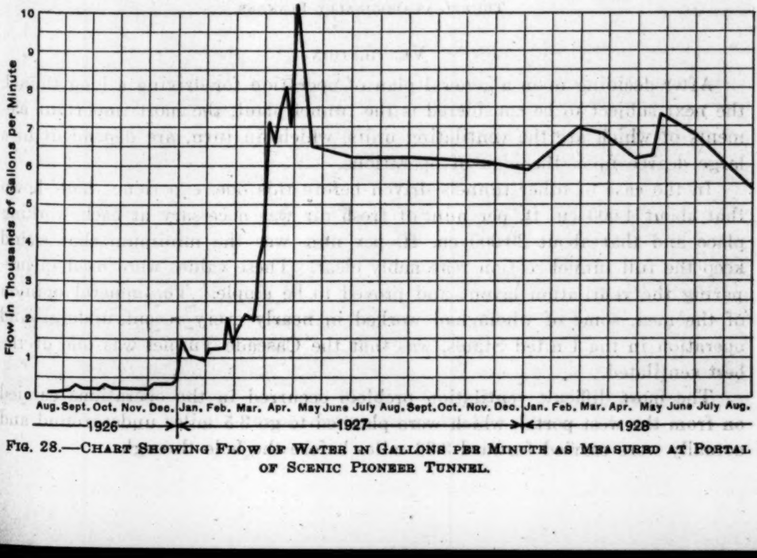


FIG. 28.—CHART SHOWING FLOW OF WATER IN GALLONS PER MINUTE AS MEASURED AT PORTAL OF SCENIC PIONEER TUNNEL.

The original plan of the pioneer tunnel is shown in Fig. 29. This was wrong in two respects:

1.—The drainage ditch shown at the side was not of sufficient capacity to carry the quantity of water encountered and was difficult to maintain. Fig. 30 shows the water conditions in the pioneer at their worst and Fig. 31 shows one of the larger flows in this same operation. For a tunnel expected to take care of large quantities of water, a plan shown in Fig. 32 has been developed and is now (1931) in use in driving the Mahr Drainage Tunnel, 6 miles long, for Cerro de Pasco, in Peru.

2.—The pioneer is shown in Fig. 29 as level with the center heading. This location is simple and to be preferred when considered with relation to driving the center heading, but does not lend itself to the future permanent drainage of the main tunnel. In order to accomplish this latter purpose, the grade of the pioneer should be level with the grade of the main tunnel. In the Cascade Tunnel the grade of the pioneer was dropped and 1.32 miles of it are level with the main tunnel grade.

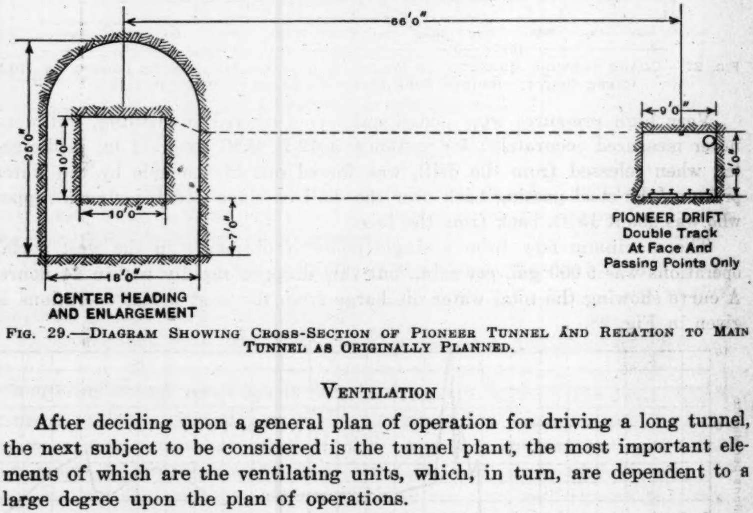


FIG. 29.—DIAGRAM SHOWING CROSS-SECTION OF PIONEER TUNNEL AND RELATION TO MAIN TUNNEL AS ORIGINALLY PLANNED.

VENTILATION

After deciding upon a general plan of operation for driving a long tunnel, the next subject to be considered is the tunnel plant, the most important elements of which are the ventilating units, which, in turn, are dependent to a large degree upon the plan of operations.

In the case of other tunnels driven before this one, experience had shown that about 4 000 cu. ft. per min. of fresh air was necessary at each working place and that about 20 000 cu. ft. per min. was the minimum that would keep the full tunnel section reasonably clear. These values were used in preparing the ventilation layout and proved to be ample. The general opinion of the men, some of whom had worked in nearly every recent underground operation in the United States, was that the Cascade Tunnel was one of the best ventilated.

The most difficult ventilation problem occurred in the operations carried on from the west portal, which were planned to go 3.5 miles underground and actually were carried forward 3.36 miles before the hole through.

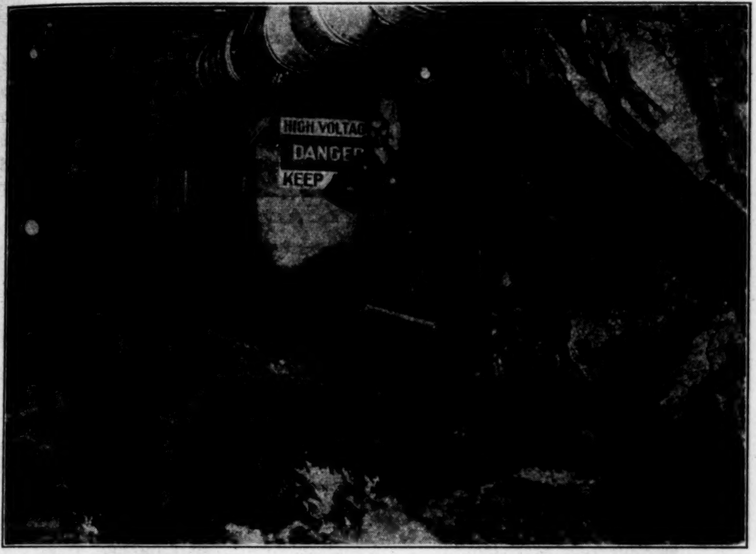

HIGH VOLTAG
DANGER
KEEP

FIG. 30.—WATER IN PIONEER TUNNEL NEAR THE SCENIC END, WEST PORTAL.

FIG. 31.—WATER IN MILL CREEK PIONEER TUNNEL.

The plan adopted at the west portal was to bulkhead the pioneer and force air into it with low-pressure fans in volumes ranging from 20 000 to 30 000 cu. ft. per min., the air passing through the pioneer to some advanced cross-cut where a bulkhead was to be erected, diverting the air current through the cross-cut into the main tunnel and thence out to the portal. The cross-cuts between the portal and the advanced bulkhead in the pioneer were to be closed. This plan provided the primary ventilation.

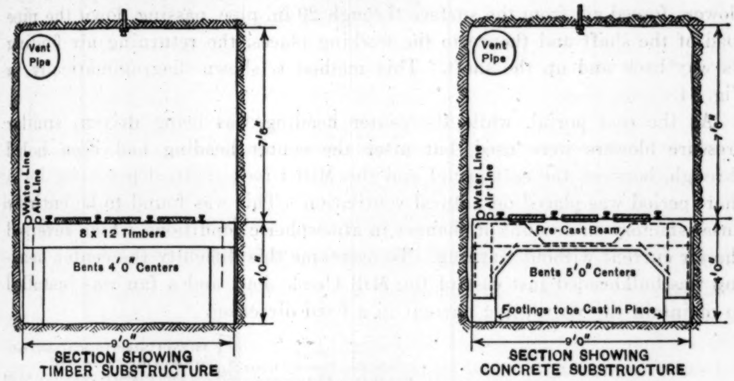

Vent Pipe
Water Line
Air Line
8 x 12 Timbers
Bents 4'0" Centers
7'6"
1'0"
9'0"
SECTION SHOWING TIMBER SUBSTRUCTURE
Vent Pipe
Water Line
Air Line
Pre-Cast Beam
Bents 5'0" Centers
Footings to be Cast in Place
7'1"
1'0"
9'0"
SECTION SHOWING CONCRETE SUBSTRUCTURE

FIG. 32.—DIAGRAMS SHOWING CROSS-SECTION OF DRIFT DESIGNED TO CARRY LARGE FLOW OF WATER.

At an advanced bulkhead in the pioneer was to be installed a high-pressure fan, capable of delivering 12 000 cu. ft. per min. against a static head of 20 to 30 in. of water, taking air from the primary ventilation current and delivering it through a 20-in., 18-gauge, riveted and soldered ventilating pipe to the various advance heading operations, at the rate of about 4 000 cu. ft. per min. for each working place. Furthermore, at the advance bulkhead an equalizing fan was to be installed to pass the returning air through the bulkhead back into the primary ventilating current. This plan is shown diagrammatically in Fig. 33 and worked perfectly, with the following exceptions:

(*a*) The use of an advance bulkhead and the equalizing fan was found unnecessary. The primary ventilating current was simply allowed to find its way through the only cross-cut remaining open from the pioneer to the main tunnel and the high-pressure fan was set at this cross-cut, drawing fresh air from the primary

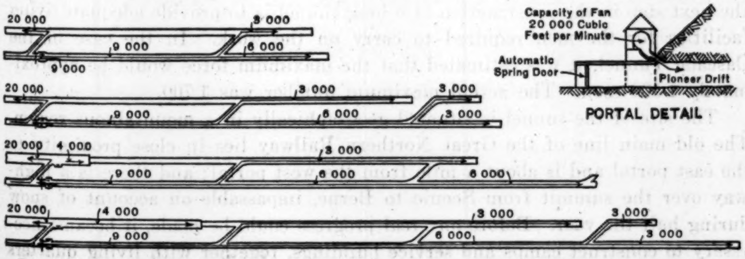

20 000
3 000
9 000
6 000
9 000
20 000
3 000
3 000
9 000
6 000
3 000
20 000
4 000
3 000
9 000
6 000
6 000
20 000
4 000
3 000
3 000
9 000
6 000
3 000
Capacity of Fan 20 000 Cubic Feet per Minute
Automatic Spring Door
Pioneer Drift
PORTAL DETAIL

FIG. 33.—DIAGRAM SHOWING VENTILATION SCHEME USED IN WEST PORTAL OPERATIONS.

current at a point portal-ward from the cross-cut, the returning air finding its way back into the primary current.

(*b*) The high-pressure fans that were available, while delivering their rated capacities against the rated static heads, were too lightly constructed for the service and required heavy repairs. This is a detail, however, that can easily be corrected by the fan manufacturers in future installations of this character.

At the operations east and west from the Mill Creek shaft common pressure blowers forced air from the surface through 20-in. pipe, passing down the pipe road of the shaft and thence to the working places, the returning air finding its way back and up the shaft. This method is shown diagrammatically in Fig. 34.

At the east portal, while the center heading was being driven, similar pressure blowers were used, but after the center heading had been holed through, between the east portal and the Mill Creek shaft, dependence for a short period was placed on natural ventilation. This was found to be entirely unsatisfactory on account of changes in atmospheric conditions, which reversed the air current without warning. To overcome this difficulty the center heading was bulkheaded just east of the Mill Creek shaft and a fan was installed to maintain the ventilating current in a fixed direction.

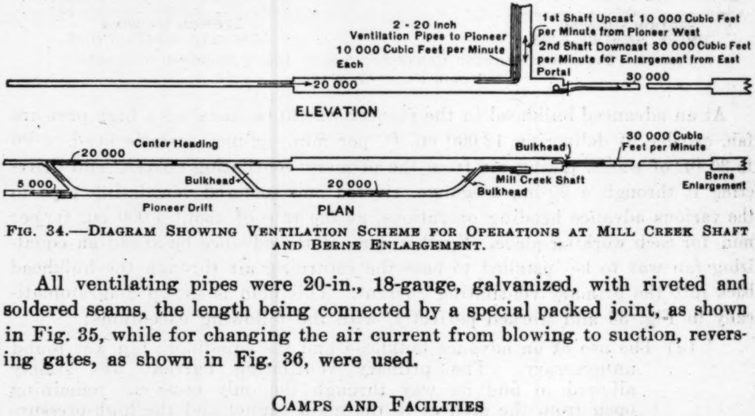

2 - 20 Inch Ventilation Pipes to Pioneer 10 000 Cubic Feet per Minute Each
1st Shaft Upcast 10 000 Cubic Feet per Minute from Pioneer West
2nd Shaft Downcast 30 000 Cubic Feet per Minute for Enlargement from East Portal
20 000
30 000
ELEVATION
Center Heading
20 000
5 000
Bulkhead
Pioneer Drift
20 000
PLAN
Bulkhead
Mill Creek Shaft
Bulkhead
30 000 Cubic Feet per Minute
Berne Enlargement

FIG. 34.—DIAGRAM SHOWING VENTILATION SCHEME FOR OPERATIONS AT MILL CREEK SHAFT AND BERNE ENLARGEMENT.

All ventilating pipes were 20-in., 18-gauge, galvanized, with riveted and soldered seams, the length being connected by a special packed joint, as shown in Fig. 35, while for changing the air current from blowing to suction, reversing gates, as shown in Fig. 36, were used.

CAMPS AND FACILITIES

After having decided upon a plan of attack and a method of ventilation, the next step in the construction of a long tunnel is to provide adequate living facilities for the men required to carry on the work. In the case of the Cascade Tunnel, it was estimated that the maximum force would be approximately 1 500 men. The actual maximum number was 1 700.

The site of the tunnel is situated geographically in a mountainous region. The old main line of the Great Northern Railway lies in close proximity to the east portal and is about a mile from the west portal; and there is a highway over the summit from Scenic to Berne, impassable on account of snow during half the year. Before any real progress could be made, it became necessary to construct camps and service buildings, together with living quarters for married employees, at all points of attack. It was also necessary to construct a road, about 3 miles long, from Berne on the main line of the railroad to the Mill Creek shaft.

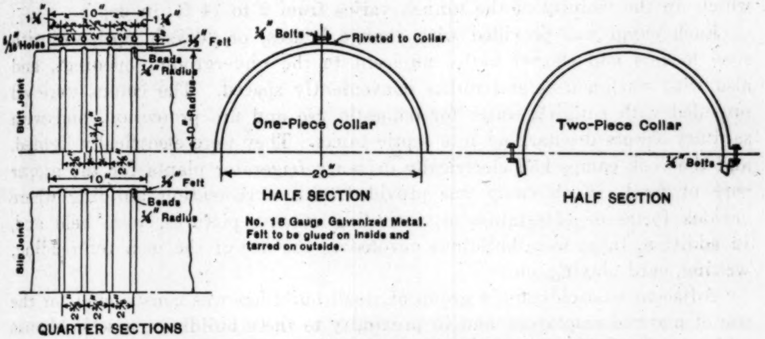

1¼"
10"
1¼"
5/16" Holes
½" Felt
Beads
¼" Radius
10" Radius
Butt Joint
1 3/16"
10"
½" Felt
Beads
¼" Radius
Slip Joint
QUARTER SECTIONS
¼" Bolts
Riveted to Collar
One-Piece Collar
20"
HALF SECTION
No. 18 Gauge Galvanized Metal.
Felt to be glued on inside and tarred on outside.
Two-Piece Collar
¼" Bolts
HALF SECTION

FIG. 35.—PIPE JOINTS ON VENTILATION SYSTEM, CASCADE TUNNEL.

The difficulty of constructing these camps and the road will be appreciated from the statement that timber standing, in some instances, 50 000 ft. b. m. to the acre had to be cleared and disposed of before the work could proceed. Fig. 37 is a view of a portion of the west portal camp and shows the stand of timber clearly. Fig. 38 shows the camp and family quarters, at Berne.

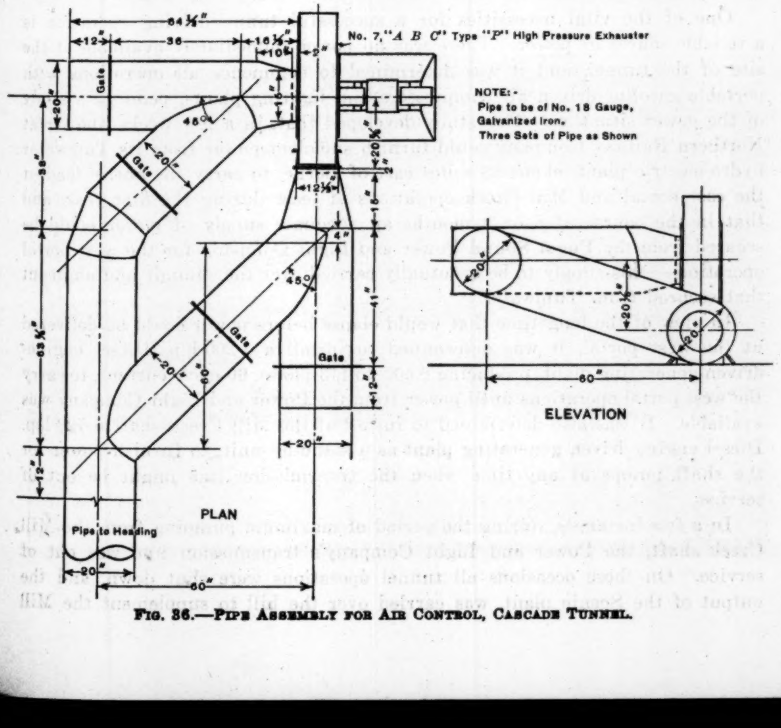

64⅛"
12"
36
16⅛"
10"
2"
No. 7, "A B C" Type "P" High Pressure Exhauster
Gate
20"
15"
45°
20⅛"
20"
Gate
38"
2"
12½"
15"
4"
45°
41"
Gate
20"
60"
Gate
12"
63⅝"
12"
20"
PLAN
Pipe to Heading
20"
60"
NOTE:-
Pipe to be of No. 18 Gauge,
Galvanized Iron,
Three Sets of Pipe as Shown
20"
20⅛"
20"
60"
ELEVATION

FIG. 36.—PIPE ASSEMBLY FOR AIR CONTROL, CASCADE TUNNEL.

The buildings in all the camps were erected 6 to 10 ft. above the ground and connected with sidewalks for two reasons: (1) To avoid the tremendous expense of grubbing the natural surface; and (2) to aid in the disposal of snow which, in the vicinity of the tunnel, varies from 2 to 14 ft. in depth.

Each camp was provided with change houses, or "drys", equipped with steel lockers and shower baths, adjacent to the underground openings, and also with wash-houses and toilets conveniently spaced. The camps were all provided with running water for domestic use and fire protection, and with sanitary sewers discharging into septic tanks. They were electrically lighted, and the cook camps had electrically driven refrigerator plants for the proper care of food. Each camp was provided with a recreation building where various forms of entertainment, including moving pictures, were held and, in addition, there were buildings devoted to the use of the men for reading, writing, card playing, etc.

Adjacent to each camp a group of small buildings was constructed for the use of married employees, and in proximity to these buildings, a school-house and school teachers' dwelling. Each camp was provided with first-aid stations and an emergency hospital, presided over by a resident physician. Fig. 39 shows the general type of camp construction; Fig. 40 the type of toilet and change houses; and Fig. 41 shows the type of quarters provided for married employees.

POWER

One of the vital necessities for a successful tunnel-driving operation is a reliable source of power. There was no power immediately available at the site of the tunnel, and it was determined to commence all operations with portable gasoline-driven air compressors and lighting plants, pending a study of the power situation. This study developed that, in a few weeks, the Great Northern Railway Company could furnish sufficient power from its Tumwater hydro-electric plant, about 25 miles east of Berne, to carry the entire load of the east portal and Mill Creek operations at least during the first year, and that in the course of 8 or 9 months an adequate supply of power could be secured from the Puget Sound Power and Light Company for the west portal operations—this supply to be eventually carried over the summit and augment that secured from Tumwater.

In view of the long time that would elapse before power could be delivered at the west portal, it was determined to install a 1 200-h.p. Diesel engine-driven generating plant, producing 2 300-volt, 3-phase, 60-cycle current, to carry the west portal operations until power from the Power and Light Company was available. It was also determined to install at the Mill Creek shaft a 720-h.p. Diesel engine-driven generating plant as a stand-by unit, to furnish power for the shaft pumps at any time when the transmission line might be out of service.

In a few instances, during the period of maximum pumping from the Mill Creek shaft, the Power and Light Company's transmission line was out of service. On these occasions all tunnel operations were shut down, and the output of the Scenic plant, was carried over the hill to supplement the Mill

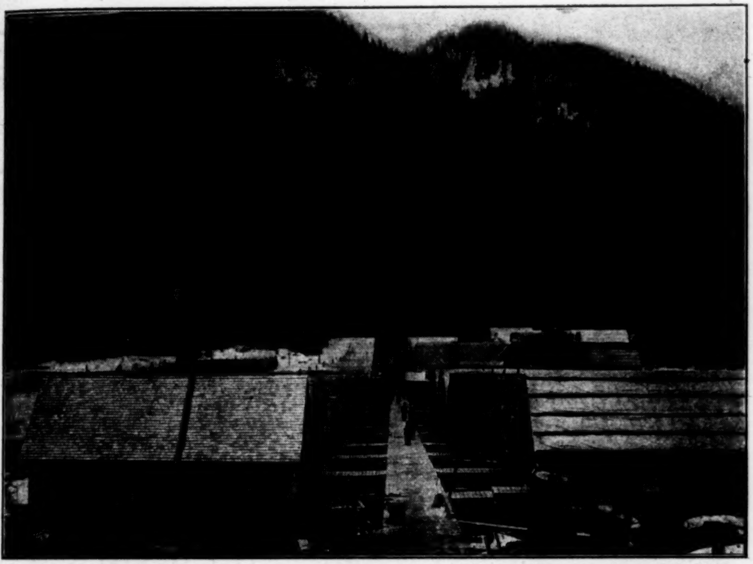


FIG. 37.—VIEW OF CONTRACTOR'S CAMP AT WEST PORTAL, SCENIC, WASH., APRIL 30, 1927.

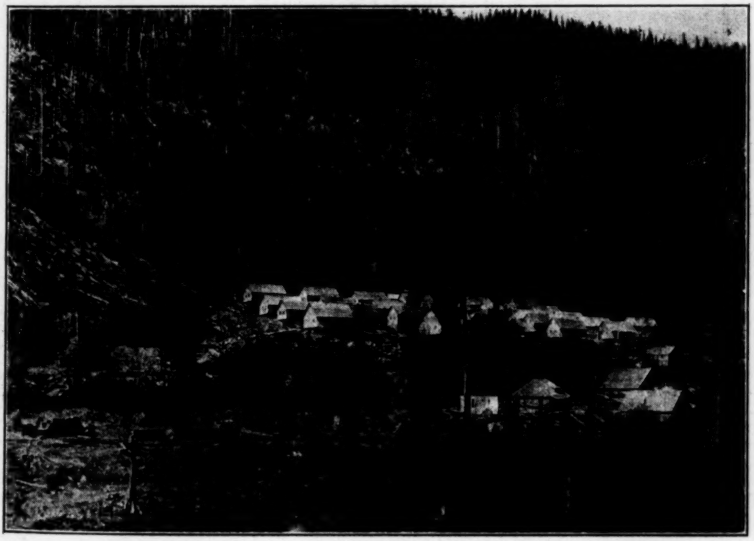
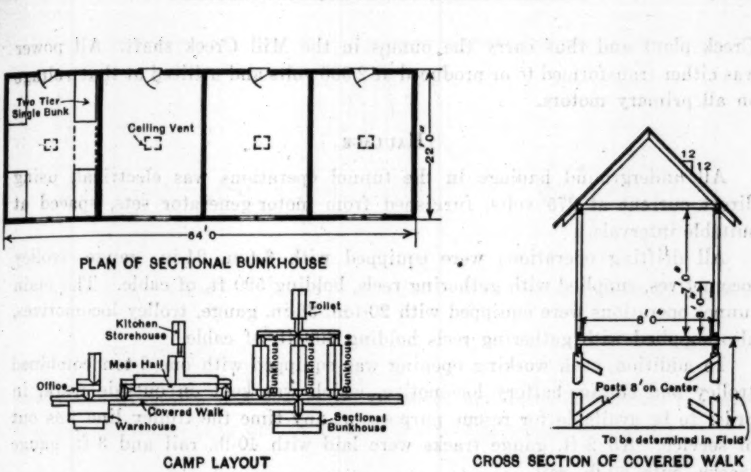


FIG. 38.—CONTRACTOR'S CAMP AND NEW FAMILY QUARTERS, BERNE, WASH.

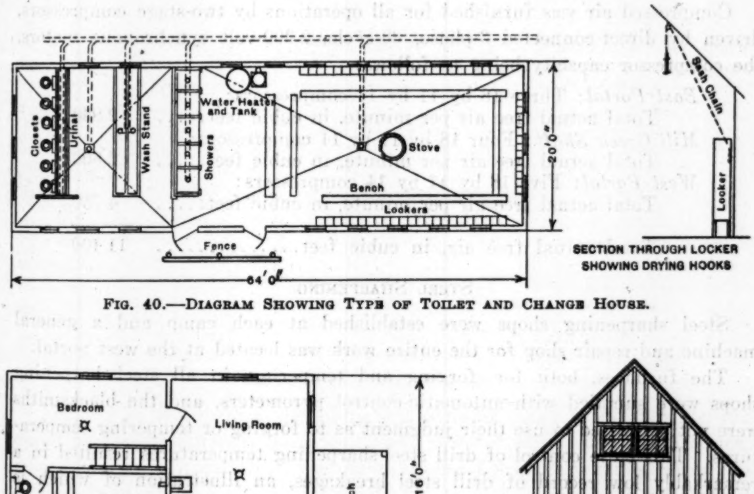

Two Tier
Single Bunk
Ceiling Vent
22'0"
64'0
12
12
8'0"
8'0"
Toilet
Kitchen
Storehouse
Mess Hall
Office
Bunkhouse
Bunkhouse
Bunkhouse
Covered Walk
Warehouse
Sectional
Bunkhouse
Posts 8' on Center
To be determined in Field

PLAN OF SECTIONAL BUNKHOUSE

CAMP LAYOUT

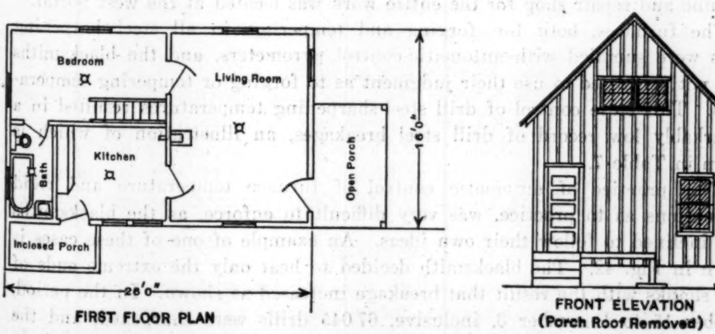
CROSS SECTION OF COVERED WALK

FIG. 39.—DIAGRAM SHOWING GENERAL TYPE OF CAMP CONSTRUCTION, CASCADE TUNNEL.


Closets
Urinal
Wash Stand
Showers
Water Heater
and Tank
Stove
Bench
Lockers
Fence
20'0"
64'0"
Wash Chain
Locker

SECTION THROUGH LOCKER
SHOWING DRYING HOOKS

FIG. 40.—DIAGRAM SHOWING TYPE OF TOILET AND CHANGE HOUSE.


Bedroom
Living Room
Kitchen
Bath
Open Porch
Inclosed Porch
16'0"
26'0"

FIRST FLOOR PLAN

FRONT ELEVATION
(Porch Roof Removed)

FIG. 41.—DIAGRAM SHOWING TYPE OF QUARTERS PROVIDED FOR MARRIED EMPLOYEES.

Creek plant and thus carry the pumps in the Mill Creek shaft. All power was either transformed to or produced at 2 300 volts and utilized at that voltage on all primary motors.

HAULAGE

All underground haulage in the tunnel operations was electrical, using direct current at 275 volts, furnished from motor-generator sets, spaced at suitable intervals.

All drifting operations were equipped with 6-ton, 24-in. gauge, trolley locomotives, supplied with gathering reels, holding 500 ft. of cable. The main tunnel operations were equipped with 20-ton, 36-in. gauge, trolley locomotives, also supplied with gathering reels holding 500 ft. of cable.

In addition, each working opening was equipped with one 6-ton combined trolley and storage-battery locomotive, which was kept on outside work, in order to be available for rescue purposes at any time the trolley line was out of service. All 2-ft. gauge tracks were laid with 40-lb. rail and 3-ft. gauge tracks, with 60-lb. rail.

COMPRESSORS

Compressed air was furnished for all operations by two-stage compressors, driven by direct-connected 3-phase, 60-cycle, 2 300-volt synchronous motors, the compressor capacity being as follows:

East Portal: Three 18 by 11 by 14 compressors:	
Total actual free air per minute, in cubic feet.....	2 850
Mill Creek Shaft: Four 18 by 11 by 14 compressors:	
Total actual free air per minute, in cubic feet.....	3 800
West Portal: Five 18 by 11 by 14 compressors:	
Total actual free air per minute, in cubic feet.....	4 750
Total actual free air, in cubic feet.............	11 400

STEEL SHARPENING

Steel sharpening shops were established at each camp and a general machine and repair shop for the entire work was located at the west portal.

The furnaces, both for forging and tempering, in all steel-sharpening shops were supplied with automatic-control pyrometers, and the blacksmiths were not permitted to use their judgment as to forging or tempering temperatures. This close control of drill steel-sharpening temperatures resulted in a remarkably low record of drill steel breakages, an illustration of which is shown in Table 7.

This practice of automatic control of furnace temperature and rigid instructions as to practice, was very difficult to enforce, as the blacksmiths were inclined to follow their own ideas. An example of one of these cases is shown in Fig. 42. The blacksmith decided to heat only the extreme ends of drill shanks with the result that breakage increased as shown. In the period, October 15 to December 3, inclusive, 67 045 drills were sharpened, and the number broken during the same time was 1 672. The total average breakage was 2.49%, as compared with the value of 0.59% given in Table 7.

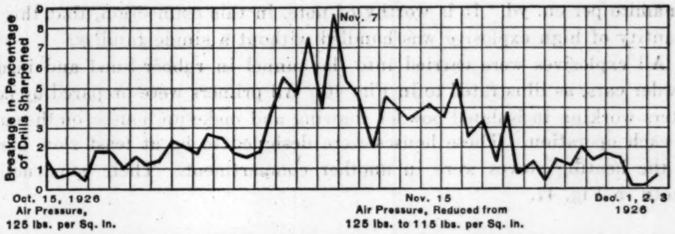


FIG. 42.—GRAPHIC RECORD OF BROKEN DRILL STEEL AT WEST PORTAL OPERATIONS, SHOWING RESULT OF SLIGHT DEPARTURE FROM PRESCRIBED METHODS.

Drill steel was hauled into the working places in special cars provided with compartments, as shown in Fig. 43, and a full supply of steel for one round was loaded in one car, taken in, used, and brought back to the steel shop for refitting. (For details of drill sharpening see Appendix A.)

TABLE 7.—RECORD OF BROKEN DRILL STEEL AT MILL CREEK SHAFT OPERATION, NEW CASCADE TUNNEL, SHOWING RESULTS THAT MAY BE OBTAINED WITH PRECISE REGULATION OF FORGING AND TEMPERING TEMPERATURES

Year and month	Number sharpened	Number broken	Broken drills in percentage of total sharpened
September, 1926	6 959	10	0.14
October, 1926	19 046	6	0.03
November, 1926	28 656	166	0.58
December, 1926	32 691	66	0.20
January, 1927	36 399	116	0.31
February, 1927	31 015	319	1.03
March, 1927	42 560	312	0.73
April, 1927	48 169	462	0.96
May, 1927	52 372	400	0.76
June, 1927	60 550	440	0.73
July, 1927	60 209	258	0.43
August, 1927	62 535	277	0.44
Total	481 161	2 832	0.59

DRILLS AND DRILLING LAYOUTS

Drilling in the various operations was done by many types of drills, produced by several different manufacturers. Great credit is due the American manufacturers of rock drills, who have produced what is perhaps, when considered from all angles, the most effective machine known to present-day construction. Fig. 44 is a view of drilling operations. Typical drilling and loading diagrams, which are largely self-explanatory, are shown in Fig. 45.

EXPLOSIVES

The next item of importance in tunneling operations, after drilling, is a proper explosive. In the Cascade Tunnel gelatine dynamite varying from 60 to 80% in strength, was used, in a total of 4 197 877 lb. In the tunnel 994 178 cu. yd. of rock were excavated, requiring an average of 4.2 lb. of dynamite per cu. yd. It is worthy of note, in this connection, that this large quantity of high explosive was handled without a single fatality.

All explosives were carried into the tunnel in rubber-lined and insulated powder cars, as illustrated to in Fig. 46. All primers were prepared by powder fitters working in isolated powder thawing and make-up houses, on the outside, at each operation. These houses were designed to be at least slow burning, as the heating stoves were in another compartment. Their construction is shown in Fig. 47.

MUCKING

Because of the restricted space available the work of removing broken rock (commonly called "mucking") is the most difficult operation to equip properly with mechanical appliances. With modern rock drills and explosives, little difficulty need be expected with breaking the ground, but under present conditions hand labor for "mucking" on a long tunnel is not to be considered and loading machines that will work within the cross-section areas of drifts and full-sized railway tunnels are highly specialized machines.

All "mucking" in drifts was done by machines, such as illustrated in Figs. 48 and 49. The best record of a machine of this type is the loading of 32 cu. yd. of hard, coarsely broken granite in 1½ hours, including the time occupied in cleaning up the fly rock.

The "mucking" in the enlargement was done by rebuilt railway-type shovels operated by compressed air. The longitudinal and cross-sections of the general arrangement of these shovels are shown in Fig. 50. These were originally steam shovels mounted on two 4-wheel railroad trucks, which were removed and replaced with caterpillar treads. Their boilers were removed and replaced with air receivers.

The shovel engines were of the slide-valve type and in order to prevent freezing at the exhaust, the air receivers were equipped with eighteen direct-current, 220-volt, immersion heating units connected at 100 volts to prevent destruction by overheating. Each receiver was also equipped with an indicating thermometer, the indicating element being mounted in front of the shovel operator, together with a switch that controlled the heating elements, in order that the temperature could be kept within practical limits.

CARS AND SWITCHING

The success of all mucking operations in tunnels and drifts depends upon the speed with which cars can be switched behind the mucking machine. In the Cascade Tunnel the switching was accomplished by transferring the empty cars from one track to another with car transfer hoists. On the model of shovel used in the main tunnel these car transfer hoists were constructed on the rear of the shovel, as shown in Fig. 50, the switching operation being shown in plan diagrammatically in Fig. 51. For use in the drifts where mucking machines (Fig. 48) were used, special car transfer hoists, running on their own trucks, were designed and built. The design of these car trans-

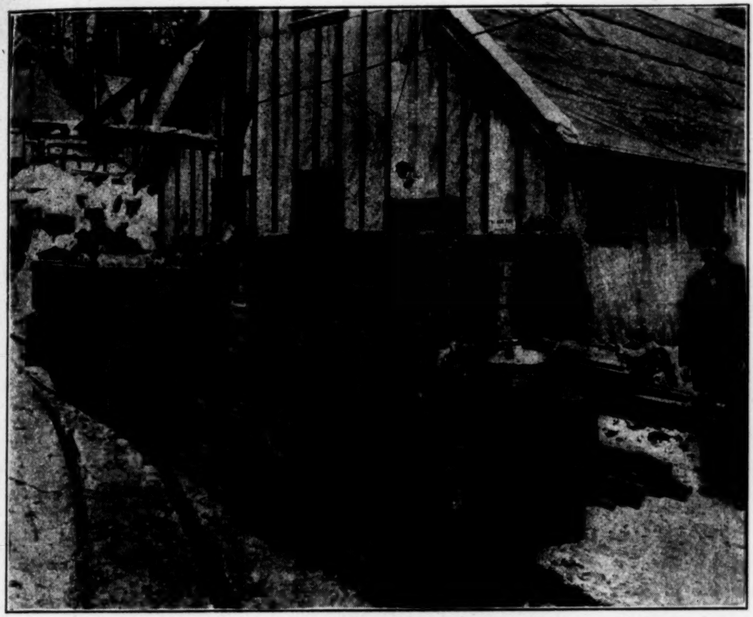


FIG. 43.—DRILL CAR READY FOR ACTION.

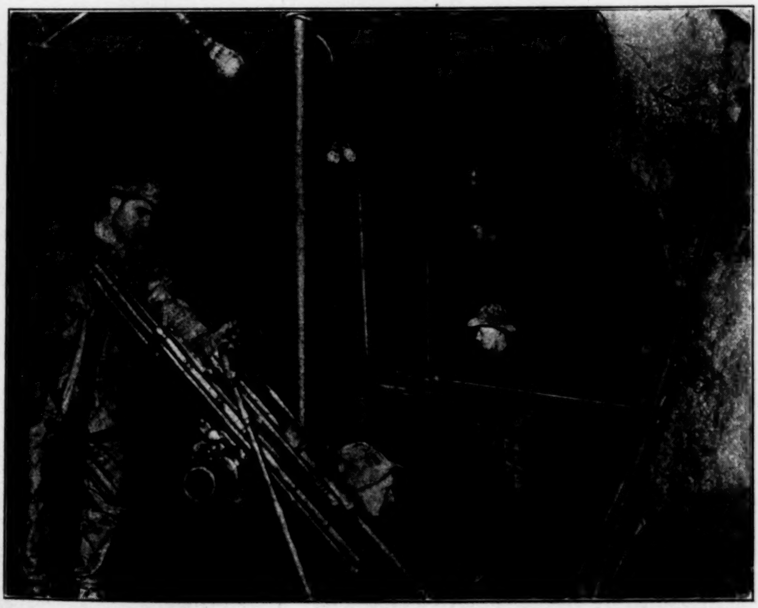
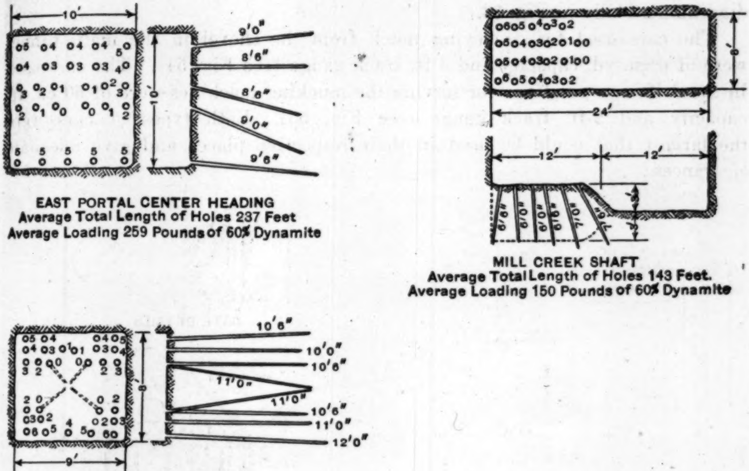


FIG. 44.—RING DRILLS IN THE CENTER HEADING, CASCADE TUNNEL.


10'
9'0"
8'6"
8'6"
9'0"
9'6"
10'
8'
24'
12'
12'
8'6"
8'0"
9'0"
9'6"
10'6"
10'0"
10'6"
11'0"
11'0"
10'6"
11'0"
12'0"
8'
9'

EAST PORTAL CENTER HEADING
Average Total Length of Holes 237 Feet
Average Loading 259 Pounds of 60% Dynamite

MILL CREEK SHAFT
Average Total Length of Holes 143 Feet.
Average Loading 150 Pounds of 60% Dynamite

TYE RIVER INCLINE
Average Total Length of Holes 313 Feet
Average Loading 265 Pounds of 60% Dynamite

FIG. 45.—DRILLING AND LOADING DIAGRAMS.

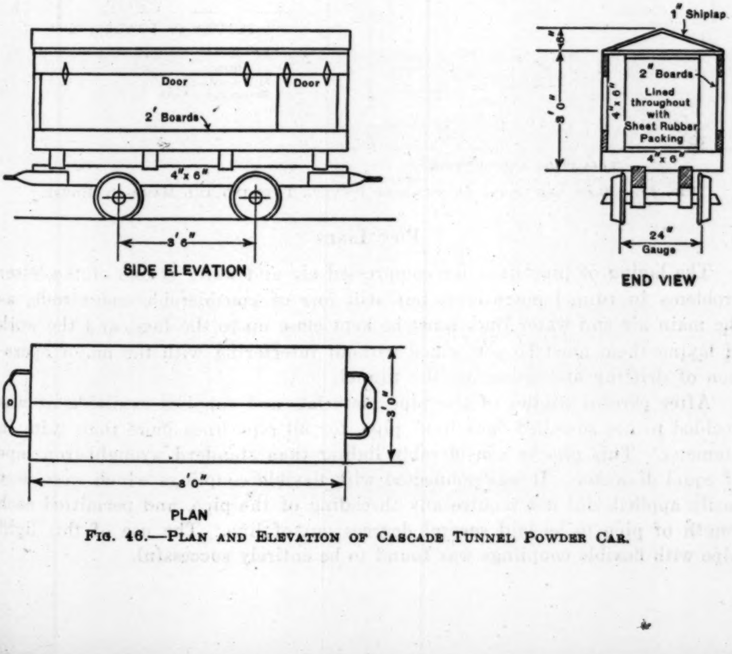
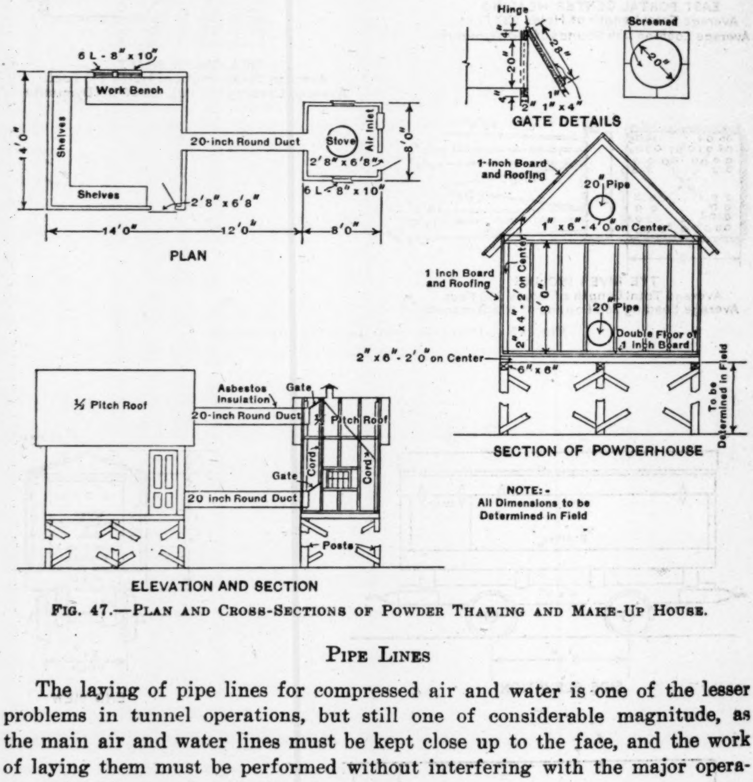

Door
Door
2' Boards
4"x 6"
3'6"
SIDE ELEVATION
1" Shiplap
2" Boards
Lined throughout with Sheet Rubber Packing
4"x 6"
4"x 6"
8'0"
24" Gauge
END VIEW
3'0"
8'0"
PLAN

FIG. 46.—PLAN AND ELEVATION OF CASCADE TUNNEL POWDER CAR.

fer hoists is shown in Fig. 52 and the switching layout is shown in plan diagrammatically in Fig. 53.

The cars used for conveying muck from the shovel in the main tunnel were of 6 cu. yd. capacity and 3-ft. track gauge (see Fig. 54). The cars used in the drifting operations for serving the mucking machines were of 50 cu. ft. capacity and 2-ft. track gauge (see Fig. 55). Both types of cars were the largest that could be used in their respective places and give adequate clearances.


6 L - 8" x 10"
Work Bench
Shelves
Shelves
14'0"
20-inch Round Duct
Stove
Air Inlet
2'8" x 6'8"
2'8" x 6'8"
6 L - 8" x 10"
8'0"
14'0"
12'0"
8'0"
PLAN
Hinge
4"
20"
4"
2"
28"
1"
1" x 4"
Screened
20"
GATE DETAILS
1-inch Board and Roofing
20" Pipe
1" x 6" - 4'0" on Center
1 inch Board and Roofing
2" x 4" - 2'0" on Center
8'0"
20" Pipe
Double Floor of 1 inch Board
2" x 6" - 2'0" on Center
6" x 6"
To be Determined in Field
SECTION OF POWDERHOUSE
½ Pitch Roof
Asbestos Insulation
Gate
20-inch Round Duct
½ Pitch Roof
Cord
Cord
Gate
20 inch Round Duct
Posts
ELEVATION AND SECTION

NOTE:-
All Dimensions to be Determined in Field

FIG. 47.—PLAN AND CROSS-SECTIONS OF POWDER THAWING AND MAKE-UP HOUSE.

PIPE LINES

The laying of pipe lines for compressed air and water is one of the lesser problems in tunnel operations, but still one of considerable magnitude, as the main air and water lines must be kept close up to the face, and the work of laying them must be performed without interfering with the major operation of drifting and enlarging the tunnel.

After careful studies of the pipe materials and supplies available, it was decided to use so-called "gas line" pipe, for all pipe lines more than 2 in. in diameter. This pipe is considerably lighter than standard wrought-iron pipe of equal diameter. It was connected with flexible couplings which were very easily applied, did not require any threading of the pipe, and permitted each length of pipe to be laid several degrees out of line. The use of this light pipe with flexible couplings was found to be entirely successful.

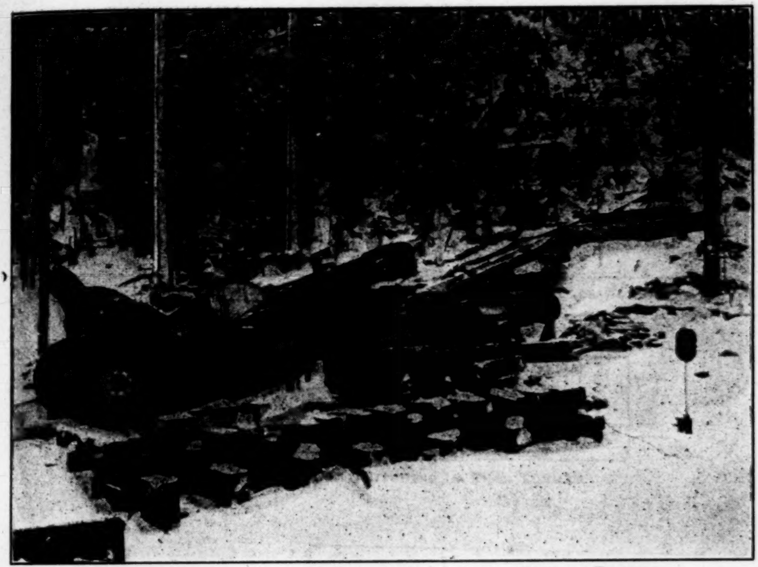


Fig. 48.—View of Mucking Shovel Used on the Cascade Tunnel.

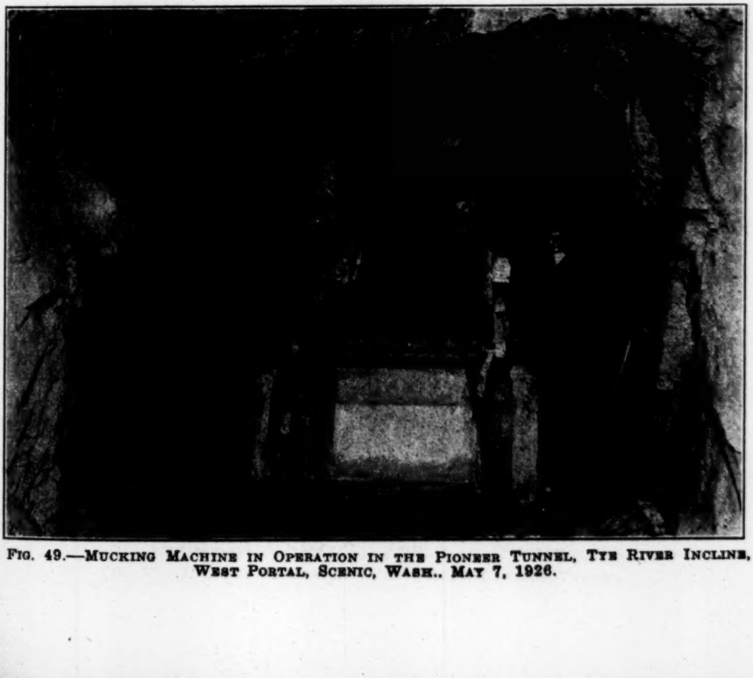


Fig. 49.—Mucking Machine in Operation in the Pioneer Tunnel, Tye River Incline, West Portal, Scenic, Wash., May 7, 1926.

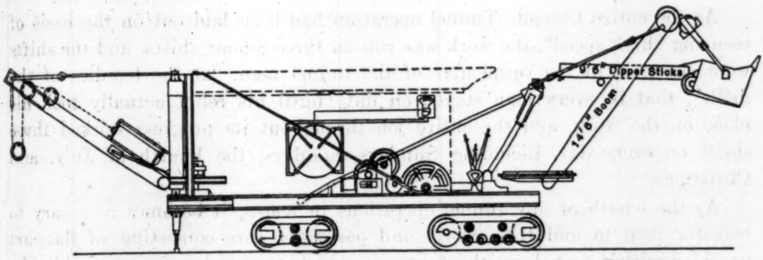

9'6" Dipper Sticks
14'6" Boom

FIG. 50.—EXAMPLE OF RAILWAY TYPE OF SHOVEL REMODELED FOR TUNNELING.

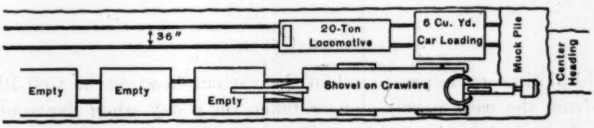

20-Ton Locomotive
6 Cu. Yd. Car Loading
Muck Pile
Center Heading
36"
Empty
Empty
Empty
Shovel on Crawlers

FIG. 51.—DIAGRAM OF SWITCHING IN FULL TUNNEL.

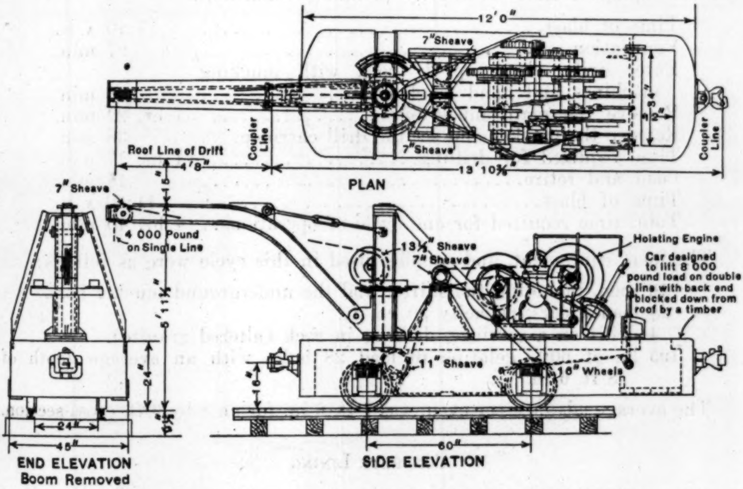

12'0"
7" Sheave
7" Sheave
34"
12"
Coupler Line
Coupler Line
Roof Line of Drift
4'8"
13'10⅝"
PLAN
7" Sheave
4 000 Pound on Single Line
13½" Sheave
7" Sheave
Hoisting Engine
Car designed to lift 8 000 pound load on double line with back and blocked down from roof by a timber
5'11¼"
24"
6"
11" Sheave
16" Wheels
24"
45"
60"
END ELEVATION
Boom Removed
SIDE ELEVATION

FIG. 52.—NEW CASCADE TUNNEL: DERRICK CAR, GENERAL ASSEMBLY.

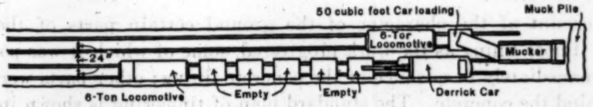

50 cubic foot Car loading
Muck Pile
6-Ton Locomotive
Mucker
24"
6-Ton Locomotive
Empty
Empty
Derrick Car

FIG. 53.—DIAGRAM OF SWITCHING IN DRIFTS, CASCADE TUNNEL.

WORKING TIME

As the entire Cascade Tunnel operation had been laid out on the basis of securing "high speed", the work was run on three 8-hour shifts, and the shifts were changed, in the vernacular of the tunnel man, "at the handles of the drills", that is, every man stayed on duty until his relief actually took his place on the work, and the entire job throughout its progress worked three shifts on every day, including Sundays, holidays, the Fourth of July, and Christmas.

As the length of any tunnel operations increases, it becomes necessary to transfer men to and from work, and passenger cars consisting of flat-cars provided with a seat down the center on which two ranks of men could ride, sitting back to back, were constructed and used. These cars are shown in Fig. 56.

TIME CYCLE

The ideal time cycle per round in the various headings is well illustrated by that from the west portal pioneer taken on a day when five rounds were pulled and mucked in three shifts. The following tabulation of one cycle of operations at the Tye River heading on August 25, 1926, represents what was accomplished under ordinarily favorable conditions.

Time of blast	7:10 A. M.
Ventilation	27 min.
Enter drift, clean up fly rock with mucking machines and hand muckers	38 min.
Remove the main muck pile	1 hr. 22 min.
Remove the mucker and set the drill carriage	38 min.
Time required for drilling	1 hr. 17 min.
Load and retire	18 min.
Time of blast	11:50 A. M.
Total time required for one cycle of operations	4 hr. 40 min.

The men, equipment, and supplies used in this cycle were as follows:

16 men, including the shifter and the underground mucker boss.
28 loaded cars, capacity 50 cu. ft.
4 drills on a carriage, drilling in rock (altered granite).
195 lb. of 60% gelatine to load 28 holes with an average depth of 8 ft. 6 in.

The average advance per cycle was 7 ft. 6 in. for an 8 by 9-ft. cross-section.

CONCRETE LINING

When planning the construction of the Cascade Tunnel it was assumed that concrete lining would be required throughout its length; this assumption proved to be correct.

On account of the character of the ground certain parts of the tunnel required timber lining as the work progressed, some of which it was possible to remove immediately in advance of the concrete lining; other parts were left in place behind the concrete. The standard plan of timbering is shown in Fig. 57 and the standard section of the concrete lining, in Fig. 58.

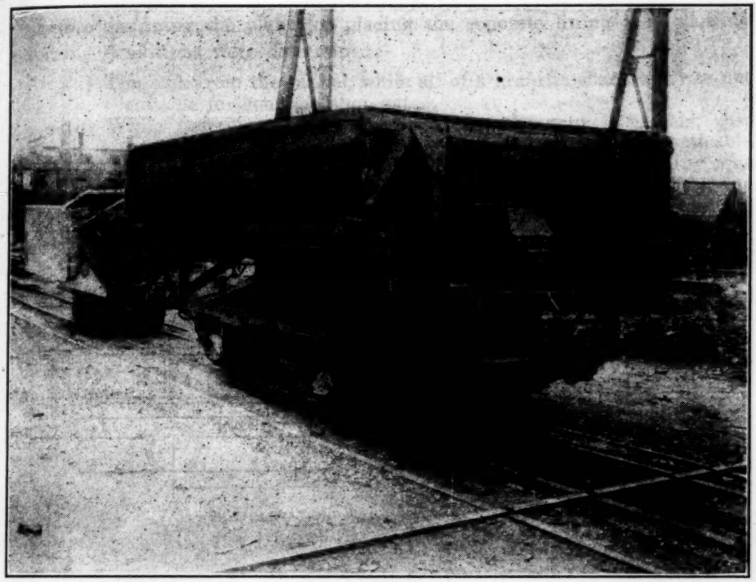


FIG. 54.—SIX-YARD CARS USED FOR CONVEYING MUCK FROM THE SHOVEL IN THE MAIN TUNNEL.

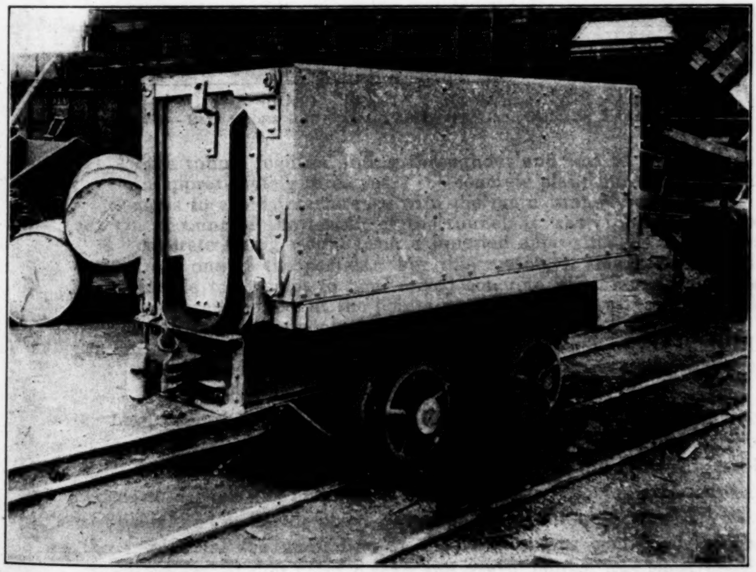


FIG. 55.—CARS USED IN DRIFTING OPERATIONS, CASCADE TUNNEL.

Before designing the plant for placing the concrete lining the following governing conditions were determined:

(*a*) The rock from the tunnel, while all of a granitic character, was not suitable for concrete aggregate.

(*b*) While there were three openings into the tunnel, namely, east portal, Mill Creek shaft, and west portal, it was impracticable to pour concrete from Mill Creek shaft as this opening was approximately three miles from the railroad. Consequently, the entire concreting operations would have to be carried on from the east and west portals.

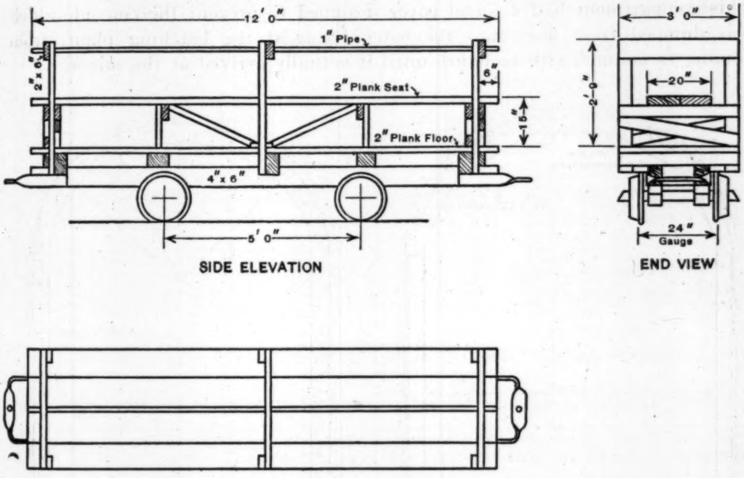


FIG. 56.—ELEVATIONS AND PLAN, PASSENGER CAR, CASCADE TUNNEL.

(*c*) As the tunnel required lining throughout and was to be completed in approximately three years the concrete plant must be designed so as to avoid interference with the main tunnel operations.

(*d*) On account of the length of the tunnel, it was probable that the concrete lining would form a junction approximately four miles from one of the portals. For this reason, it was deemed inadvisable to attempt to haul mixed concrete, the alternative being to operate mixers at the points where concrete was being poured, hauling the aggregate into the tunnel dry.

(*e*) On account of the conduits required in both walls of the tunnel it was deemed necessary to pour the side walls in two lifts to expedite the placing of these conduits.

With these conditions in mind the Great Northern Railway Company equipped its ballast pit at Reiter, Wash., to produce concrete aggregate from glacial drift, that is, natural washed sand, washed gravel, and crushed gravel, and arranged to deliver this material in standard bottom hopper railroad cars at the east portal and at Scenic about 1½ miles west of the west portal.

At the east portal the main line of the railway was contiguous to the tunnel operations and at that point a sand and gravel bunker was constructed,

into which the railway cars unloaded by gravity. A similar bunker was constructed for the west portal, but as the main-line railway at the west end was 1½ miles from the tunnel portal the sand and gravel were rehandled over a 3-ft. gauge railroad to a second bunker at the tunnel portal. Both the sand and gravel bunkers were equipped with batching devices, using scale beams for sand batchers and volumetric batchers for the gravel. The batchers were discharged into batch boxes, four of which were carried on 3-ft. gauge flat cars. Each batch box was made with a vertical partition creating a compartment just large enough to hold the required quantity of cement for one batch. This compartment had a metal cover designed to prevent the cement, which was dumped from bags into the batch boxes at the batching plant, from coming in contact with moisture until it actually arrived at the mixer.

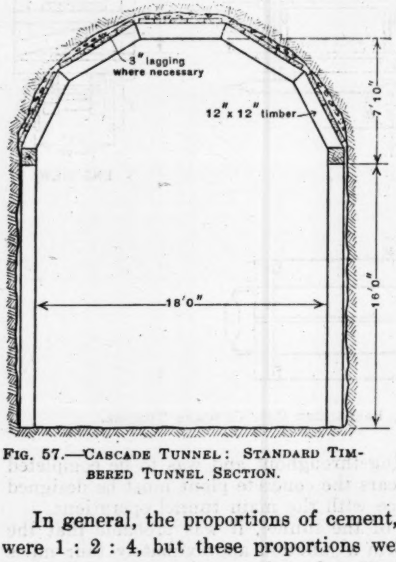


FIG. 57.—CASCADE TUNNEL: STANDARD TIMBERED TUNNEL SECTION.

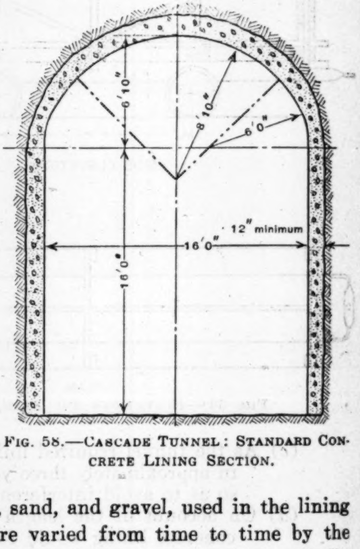


FIG. 58.—CASCADE TUNNEL: STANDARD CONCRETE LINING SECTION.

In general, the proportions of cement, sand, and gravel, used in the lining were 1 : 2 : 4, but these proportions were varied from time to time by the engineers to meet the varying characteristics of the aggregate. Practically all concrete had varying percentages of diatomaceous material added to improve its flowing qualities and the water-tightness of the lining.

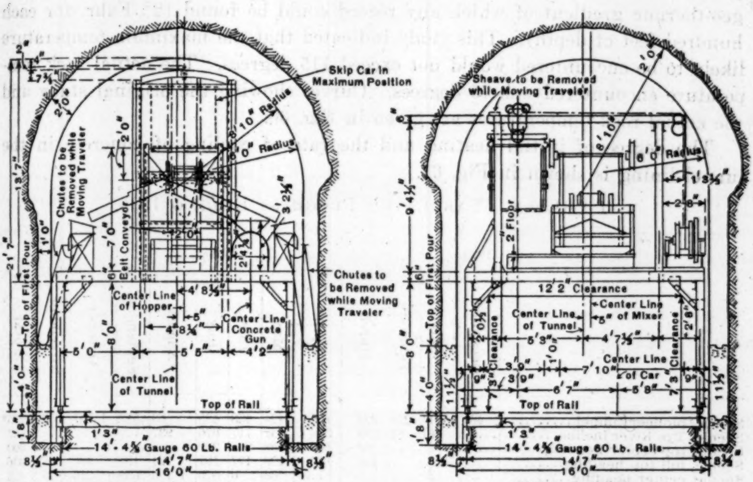
A traveling steel form was designed[5] which permitted pouring of the side walls in two lifts and which completed 37½ ft. of tunnel at each move. The travelers for this form carried (in addition to the forms) hoisting apparatus, a 14-cu. ft. concrete mixer and air placer, together with various chutes and elevators, and permitted pouring of the lining up to the springing line of the arch by gravity, the arch itself being placed pneumatically. Cross-section drawings of these forms and their travelers are shown in Fig. 59.

Eight form travelers with their necessary equipment were used, three being operated from the west portal and five from the east portal.

---

[5] By and with the assistance of Blaw-Knox Co.

In laying out the concrete schedule for lining it was assumed that the forms should remain in place for 48 hours after pouring the last concrete in them. As the work progressed, this time was gradually shortened until movements of forms were made in 12 hours after the placing of the last concrete in that particular section. On numerous occasions, however, forms were struck in 10 hours and the records show the shortest time in which a form was struck to have been 8 hours after the final placing of the concrete. In no instance was there any evidence of damage to the concrete. For a detailed discussion of air placing of concrete in the arch ring, see Appendix B.


Skip Car in Maximum Position
10″ Radius
6′0″ Radius
Chutes to be Removed while Moving Traveler
Belt Conveyor
Center Line of Hopper
Center Line Concrete Gun
Center Line of Tunnel
Top of Rail
Top of First Pour
14′-4⅜ Gauge 60 Lb. Rails
14′7″
16′0″
8½″
Sheave to be Removed while Moving Traveler
6′0″ Radius
2″ Floor
12′2″ Clearance
Center Line of Mixer
Center Line of Tunnel
Center Line of Car
Clearance
Top of Rail
14′-4⅜ Gauge 60 Lb. Rails
14′7″
16′0″

SECTION THROUGH THE CHUTES

END ELEVATION

FIG. 59.—SECTION AND END ELEVATION OF TRAVELER, CASCADE TUNNEL.

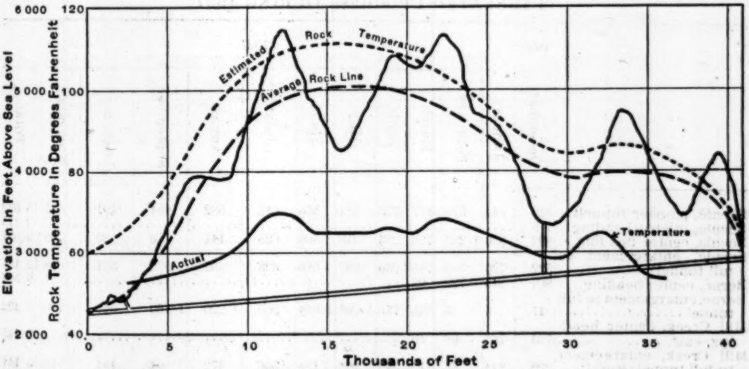


FIG. 60.—CURVES SHOWING RELATION OF ACTUAL ROCK TEMPERATURES TO ESTIMATED ROCK TEMPERATURES, NEW CASCADE TUNNEL.

Miscellaneous Data

Before proceeding with the Cascade Tunnel a study was made of the possible rock temperatures that might be encountered, using the very highest geo-thermic gradient of which any record could be found (2° Fahr. for each hundred feet of depth). This study indicated that the maximum temperature likely to be encountered would not exceed 115 degrees. The actual rock temperature encountered was 75 degrees. Curves showing the original study and the actual rock temperatures are given in Fig. 60.

The degree of initial heating and the rate of cooling of concrete in the tunnel lining is shown in Fig. 61.

TABLE 8.—Tabulated Progress During 1926

	1925	1926											
	December	January	February	March	April	May	June	July	August	September	October	November	Total
Scenic, pioneer tunnel	40	190	224	226	100	424	458	485	63	512	1 157	952	8 275*
Scenic, Tye River incline	......	40	82	178	376	651	717	676	937	....	......	....	3 657
Scenic, top center heading	......	....	....	193	162	158	205	282	...	....	......	....	1 000
Scenic, full top heading	......	....	....	100	142	129	177	191	224	195	496	373	2 036
Scenic, center heading	......	....	....	......	......	56	0	570	1 196	659	216	926	3 623
Scenic, enlargement to full tunnel	......	....	....		......	....	....	....	415	348	451	597	1 811
Berne, center heading	14	111	335	476	488	657	709	758	665	954	893	938	6 998
Berne, enlargement to full tunnel	......	...		......	......	....		....	......	....	42	62	104
Mill Creek, shaft	......	....	61	106½	122½	78	158	133	......	....	......	....	659
Mill Creek, center heading, east	......	....	....	......	......	....	....	....	......	159	707	648	1 514
Mill Creek, enlargement to full tunnel, west	......	....	....	......	......	....	....	....	......	114	144	200	458
Mill Creek, pioneer tunnel	......	....	....	....	......	....	....	....	......	....	66	620	686

Tabulated Progress During 1927

	1926	1927											
	December	January	February	March	April	May	June	July	August	September	October	November	Total
Scenic, pioneer tunnel	819	844	579	477	273	414	556	647	582	454	491	621	15 032
Scenic, full top heading	179	....	.....	....	....	......	......	......	......	......	......	.....	2 215
Scenic, center heading	992	990	1 222	723	283	218	606	625	144	0	20	105	9 551
Scenic, enlargement to full tunnel†	532	720	700	786	930	697	586	623	792	575	351	1 048	10 151
Berne, center heading	865	845	723	93	....	......	......	......	......	......	......	.....	9 524
Berne, enlargement to full tunnel	47	0	0	343	776	1 008	1 064	900	1 220	1 160	1 040	760	8 422
Mill Creek, center heading, east	793	765	63	10	....	......	......	......	......	......	......	.....	3 145
Mill Creek, enlargement to full tunnel, west‡	220	244	83	112	548	696	188	556	472	564	484	516	5 141
Mill Creek, pioneer tunnel	745	752	708	607	854	625	756	579	630	319	236	410	7 907
Mill Creek center heading, west	17	0	663	624	405	796	613	714	545	768	526	527	6 198
Scenic, concrete lining	108½	0	0	0	0	945	1 163	1 161½	1 350	1 387½	787½	0	6 903
Berne, concrete lining	.....	.....	.....	....	....	......	......	356½	1 162½	1 425	1 500	1 500	5 944

* Represents total of pioneer tunnel and incline.

† Includes enlargement at Nos. 5 and 8.

‡ Does not include 69 ft. east of shaft.

TABLE 8.—(*Continued*)

TABULATED PROGRESS DURING 1928

	1927	1928											
	December	January	February	March	April	May	June	July	August	September	October	November	Total
Scenic, pioneer tunnel	558	543	541	522	537	7	......	......	......	......	......	......	17 740
Scenic, center heading	482	481	517	541	287	0	......	......	......	......	......	......	11 859
Scenic, bottom heading	......	14	100	68	0	187	235	488	308	......	......	......	1 400
Scenic, enlargement to full tunnel[§]	839	532	668	396	388	480	821	1 075	443	398	762	765	17 718
Berne, enlargement to full tunnel	476	660	878	742	776	720	......	......	......	......	......	......	12 674
Mill Creek, enlargement to full tunnel, west[‖]	512	480	368	438	412	409	243	136	303	679	814	726	10 789
Mill Creek, pioneer tunnel	708	499	647	442	325	......	......	......	......	......	......	......	10 528
Mill Creek, center heading, west	318	54	42	259	408	107	436	524	415	......	......	......	8 861
Mill Creek, bottom heading	......	......	......	......	......	253	383	84	291	......	......	......	1 011
Scenic, concrete lining	750	1 143½	1 106½	675	1 050	712½	0	37½	901	712	976	1 311	16 278
Berne, concrete lining	1 500	1 049	0	976½	1 199½	1 237½	1 201½	2 137½	2 101	2 174	1 124	1 687½	22 332

[§] Includes enlargement at Nos. 10, 10*B*, 11, 12, 12*A*, and 12*B*, and an allowance of 60 ft. for top-heading enlargement between Nos. 14 and 15.

[‖] Includes 69 ft. east of shaft and enlargement at Nos. 14 and 15.

In Figs. 62 and 63 are shown the total man-days as compared with cubic yards of excavation and cubic yards of concrete, respectively.

The progress in all the various operations is shown in Table 8. The wage scale is shown in Table 9.

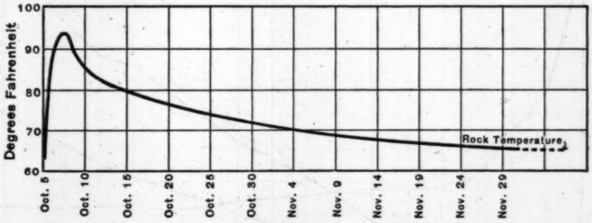


FIG. 61.—CURVE SHOWING CONCRETE TEMPERATURES FROM TIME OF POUR TO TIME IT ASSUMED TEMPERATURE OF ADJACENT ROCK.

CONCLUSION

The Contractors were instructed to proceed with the construction of the Cascade Tunnel on Thanksgiving Day, November 26, 1925. Clearing and the construction of camps were commenced immediately and had advanced so far by December 14, that the approach cut for the west portal of the pioneer tunnel was started on that date. The approach cut for the center heading at the east portal was started on December 29, 1925, and the sinking of Mill Creek shaft on January 30, 1926.

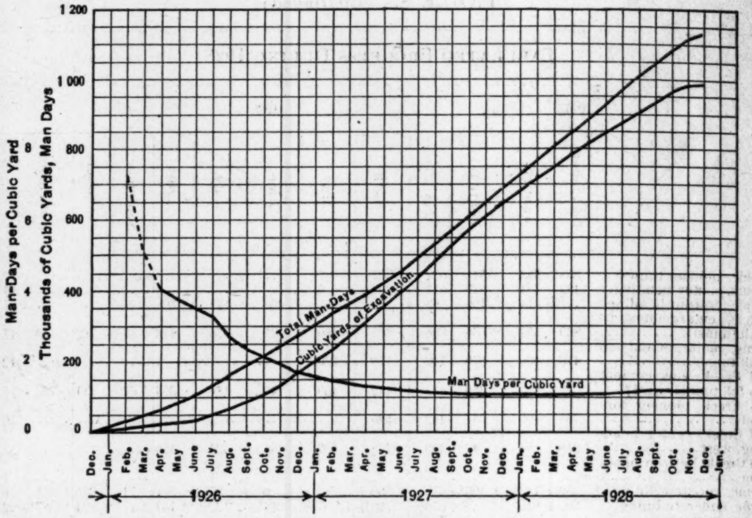


FIG. 62.—RELATION BETWEEN MAN-DAYS EMPLOYED ON EXCAVATION AND ACTUAL NUMBER OF CUBIC YARDS MOVED.

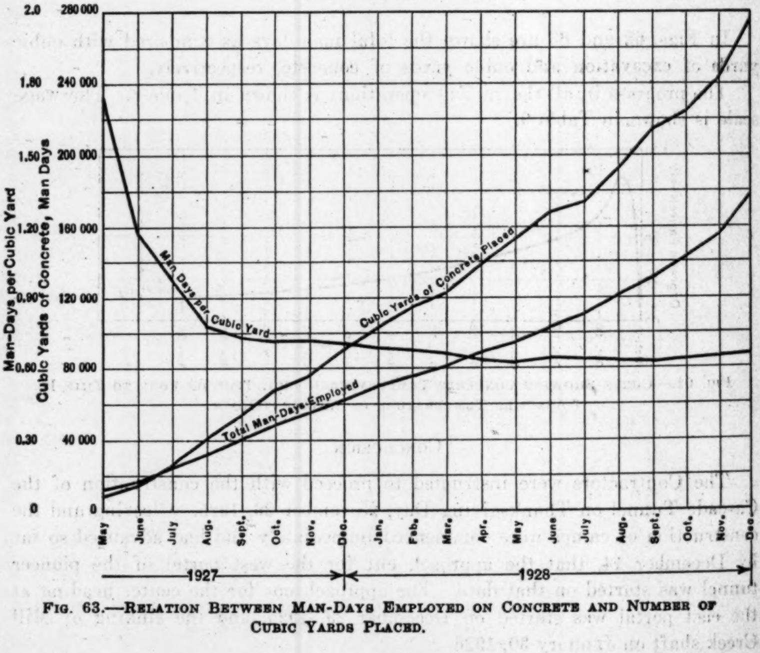


FIG. 63.—RELATION BETWEEN MAN-DAYS EMPLOYED ON CONCRETE AND NUMBER OF CUBIC YARDS PLACED.

TABLE 9.—WAGES PAID BY CONTRACTOR IN THE 8-MILE CASCADE TUNNEL, APRIL 1, 1926

Accountant, Camp	$150 or $175 and board*
Axeman	$0.56¼
Baker	$90 or $125 and board
Barber (Headquarters)	$125 and board + 10% of business
Blacksmith	$0.80
Blacksmith, helper	$0.55 or $0.60
Brakeman	$0.60
Bull Cook, Foreman	$75 and board
Bull Cook	$70 and board
Cageman	$0.60
Camp flunky	$60 and board
Camp flunky, Head	$65 and board
Carpenter	$0.75
Chief Clerk (Headquarters)	$200 and board
Clerk	$100, or $125 and board
Cook	$125, or $200 and board
Cook, Assistant	$75, or $125 and board
Deputy Sheriff	$150 and board
Dishwasher	$60 and board
Draftsman (Headquarters)	$150 and board
Driller	$0.80
Driller, helper	$0.65
Drill doctor	$0.80
Drill sharpener	$0.80
Electrician	$0.80
Electrician, helper	$0.60
Faller	$0.62½
Fireman	$0.62½
Foreman, Carpenter	$1.00
Foreman, Clearing	$0.80-1.00
Foreman, Concrete	$0.75-1.00
Foreman, General	$250 and board
Foreman, Labor	$0.75
Foreman, Mechanical	$1.00
Foreman, Rigger	$1.00
Foreman, Steam Shovel	$1.75 and board (outside)
Handyman, Surface	$0.60
Handyman, Underground	$0.70
Helper, Kitchen	$65, or $75 and board
Hoist Operator	$0.80
Inspector, Safety (Headquarters)	$200 and board
Labor Agent	$150
Laborer, Surface	$0.50
Laborer, Underground	$0.60
Locomotive Operator	$0.75
Mechanic, Master (Headquarters)	$300 and board
Mechanic	$0.80
Mucker Operator	$0.80
Plant Operator	$0.75
Powder Fitter	$0.80
Pumpman	$0.60
Rigger	$0.75
Saw Filer*	$0.68¾-0.75†
Shifter	$0.90
Stenographer	$100, or $125 and board
Storekeeper, General (Headquarters)	$150, or $175 and board
Storekeeper, Assistant (Headquarters)	$100, or $125 and board
Teamster	$75 and board
Timberman	$0.75
Timekeeper	$100, or $125 and board
Top Man (shaft only)	$0.60
Tractor Operator, 2-Ton	$125 and board
Tractor Operator, 10-Ton	$150 and board
Truck Driver	$100 and board
Team (rented)	$30, or $40 and board
Walker	$200, or $250 and board
Watchman	$60 and board
Waitress	$45 and board

---

* Board and sleeping accommodation = $1.35 per day. Men receiving rates, including board, were paid an allowance of $25 per month when boarding at home.

† This classification expired April 30, 1926.

The center heading, between the east portal and Mill Creek shaft, was holed through March 4, 1927, eighty-eight days ahead of schedule. The elapsed time on this operation was 430 days, during which 12 733 ft. of 10 by 10-ft. drift was excavated from two faces at an average rate of 14.8 ft. per day at each face, attaining a maximum month's progress at one face of 954 ft.

The pioneer tunnel between the west portal and Mill Creek shaft was holed through May 1, 1928, the final shot being fired from Washington, D. C., by President Coolidge. This operation was completed fifteen days ahead of schedule in an elapsed time of 867 days, during which 28 268 ft. of 8 by 9-ft. drift was driven from two faces, together with 21 cross-cuts, each 100 ft. long, or a total length of 30 368 ft., at an average rate of 17.5 ft. per day at each face, attaining a maximum month's progress at one face of 1 157 ft.

The enlargement between the east portal and Mill Creek shaft was completed 150 days ahead of schedule in an elapsed time of 452 days, during which 12 733 ft. of tunnel was enlarged to full section from one face, at an average rate per day of 28.2 ft., attaining a maximum month's progress at one face of 1 220 ft.

The enlargement between the west portal and Mill Creek shaft was completed December 8, 1929, eight days behind schedule, thus completing the excavation for the Cascade Tunnel, from which was removed 994 178 cu. yd. of rock.

Concrete lining was commenced from the west portal, April 27, 1927, and from the east portal, July 22, 1927, and was completed December 24, 1928, sixteen days after the completion of the excavation. The entire length of the tunnel was lined, a distance of 41 152 ft., at an average rate of 75.5 ft. per day, attaining a maximum month's progress of 3 002 ft. The total quantity of concrete placed in lining the tunnel was 275 218 cu. yd.

Track-laying and ballasting were commenced December 25, 1928, and the first scheduled train ran through the tunnel from east to west on January 12, 1929, 3 years and 47 days after the Contractors had been instructed to proceed with the work. From the date of instructions to proceed, to the date of the first scheduled train, the Cascade Tunnel was completed at an average rate of 36 ft. per day.

The opening of the tunnel was attended by a notable group of statesmen, railway executives, and engineers. To commemorate the occasion a remarkable radio broadcasting hook-up was arranged through thirty-two stations, over which speeches were made by President Hoover, from Washington; W. W. Atterbury, M. Am. Soc. C. E., President of the Pennsylvania Railroad, from New York; and J. B. Campbell, of the Interstate Commerce Commission, from Washington. When the first passenger train entered the tunnel the fact was announced by Ralph Budd, M. Am. Soc. C. E., President of the Great Northern Railway Company, speaking from the train itself. Thus was placed in service the longest railroad tunnel in the Western Hemisphere and the fifth longest railroad tunnel in the world.

APPENDIX A

DRILL-STEEL SHARPENING PRACTISE AT WEST PORTAL, NEW CASCADE TUNNEL

By H. J. King,[*] M. Am. Soc. C. E.

Description of Shop and Equipment.—The steel shop at the west portal was located between the 24-in. gauge service track leading to the pioneer tunnel and the 36-in. gauge track serving the main tunnel. A 24-in. gauge track leading from a switch in the pioneer track extended completely through the shop and beyond it into the machine and blacksmith shop located in the same building. The shop room, 32 ft. wide and 60 ft. long, was arranged with a view to minimizing the labor of handling steel in and out of the building and, at the same time, to give ample room for storage of bar stock and of made-up steel.

The equipment consisted of the following (see Figs. 64 and 65):

- Three oil furnaces, equipped with pyrometer control, thermo-couplers, control-boxes, and magnetic indicators.
- Two driller sharpeners.
- One pedestal grinder.
- One 250-lb. anvil.
- One 3 000-gal. steel oil-storage tank (elevated).
- One 1 500-gal. steel tank mounted on 3-ft. gauge, 6-yd. car truck (for transporting oil).
- One 3 by 2 by 4 duplex pump (for handling fuel oil from tank car to storage tank).
- Four platform trucks.
- Storage racks for made-up steel.
- Storage racks for bar stock (under shop).
- Two quenching tanks (wood) for water (bits).
- One quenching tank (steel) for oil (shanks).

Cutting-Bars.—Bars of 1¼-in. round, hollow steel, containing 0.78 to 0.85% of carbon, in lengths averaging 22 ft., were used for all primary drilling. A 1-in. hexagon steel of the same carbon content was used for stoper and jack-hammer steel. Bars were cut into the desired lengths by nicking them on opposite sides with a track chisel or cold cut. Then they were laid across the hub of an old 6-yd. car wheel and struck a smart blow with a 16-lb. hammer. Ragged ends of the bars on the shank end were then ground down to a square striking surface and properly chamfered.

Forging and Tempering New Bits and Shanks.—For forging shanks the pyrometer control was set at from 1 900° to 2 000° Fahr. About thirty bars of the same length were placed in the furnace and slowly brought to this temperature. Especial care was taken to avoid "soaking" the steel at high temperature. Only two or three steels were allowed to reach forging temperature at the same time, as oxidation takes place rapidly above 1 500° to 1 600° Fahr., with very harmful effects on the steel.

---

[*] Supt., A. Guthrie & Co., Inc., St. Paul, Minn.

After the bars were heated they were removed from the end of the combustion chamber farthest from the burner, placed in the sharpening machine, and upset sufficiently to give a bulb of metal large enough to form the shanks. The actual forging took place at temperatures between 1 800° and 1 900° Fahr. The steel must not be worked below a cherry red heat.

When the entire lot of bars had been "bulbed" they were replaced in the furnace and heated as before. The bulb die was removed from the sharpener and the shanking die substituted, after which the bulbs were forged to exact shank dimensions. The same methods and precautions were used for this heat as for the preceding one.

After the shanks were forged they were allowed to cool to well below a black heat. They were then reheated in the furnace with the pyrometer control set at 1 600° Fahr. and the hearth opening as wide as possible. The entire shank up to a point at least 2 in. beyond the lugs was brought to a temperature of 1 500° Fahr., tested on the magnetic indicator to prove they had been heated past the point of decalesence, and quenched in the oil bath.

Bits were forged in exactly the same manner except for the substitution of appropriate dies and dollies. The process of hardening the bits differed from the treatment of the shanks only in a slightly lower temperature (pyrometer set at 1 550° to 1 600° Fahr., with a 2-in. hearth opening). The steels attained a temperature of about 1 450° Fahr., and were quenched in clear, cold, running water.

Reconditioning Steel.—A round of steel, as it was returned from the tunnel to the shop for re-sharpening, consisted of from 24 to 36 sets of steel, varying in length by 2 ft., from 3 ft. to 13 ft. long, or 144 to 216 bars. These were examined both for broken bits and shanks, and if any were found, they were set aside for measurement and for cutting down to shorter lengths. Bent steel was similarly taken from the car for straightening. The remainder of the carload was placed on two platform trucks, each length being separated by wooden strips placed between layers. The trucks were wheeled to Nos. 1 and 3 Furnaces and sharpened. As steels were sharpened they were replaced on empty trucks and, subsequently, taken to No. 2 Furnace, reheated, and tempered. As truck loads of reconditioned steel accumulated, they were wheeled to the storage racks to await loading on the steel cars for further use underground.

As drill steel has a limited life and can be expected to begin breaking rapidly as the life limit is approached, it is imperative, in order to avoid concentrated breakage, that new steel be introduced into the stock regularly. Consequently, it was the practice to make up a small number of new drills in the longer lengths each day. Breakage, particularly shank breakage, is more pronounced in the longer lengths in steel of the same age. This is caused by the more serious effect of machines being out of line when using longer steel. In the comparatively easy drilling rock in the Cascade Tunnel very little bit breakage was experienced. As crystallization almost always occurred immediately at or below the shanks long before much change took place in the structure of the steel forming the body of the bar, the longer ones were cut down as they broke at the shank, to the next shorter length. Con-

Fig. 64.—Drill Sharpeners, West Portal, Scenic, Wash.

Fig. 65.—Drill Sharpeners and Furnace, Scenic, Wash.

sequently, as time went on large quantities of the shorter lengths accumulated, particularly starters and 5-ft. steels. By the time a steel had been successfully cut down from 11 or 13 ft. to 5 or 3 ft., most of its useful life was gone. Consequently, actual breakage in the shorter lengths was greatest.

Where automatic heat control is used, as it was on the Cascade Tunnel, only two causes contribute greatly to premature breakage of steel. The first of these is the action of machines running out of line. Breakage from this cause cannot be attributed to faulty shop practice, and the remedy for it lies solely in the hands of those responsible for the operation of the drills. The second cause is due to using a drill with a pitted hammer face, or a face so badly worn down as to permit part or all of the hammer blow to be taken up by the lugs instead of the end of the steel. This is the largest single cause of premature shank breakage. An examination readily shows when this condition is present, and it is the function of the steel shop to remedy it by reshanking.

Selection of Personnel.—Experience at the Cascade Tunnel indicates that the old-time blacksmith or steel sharpener is often thoroughly saturated with worn-out theories and prejudice against any method or theory other than the one in which he believes. Consequently, it was found better to break in young, intelligent men with sufficient educational background to enable them to understand the theory of heat treatment.

APPENDIX B

PNEUMATIC CONCRETE PLACING PRACTISE IN THE NEW CASCADE TUNNEL

By H. J. King,[6] M. Am. Soc. C. E.

Description of Apparatus.—The pneumatic concrete placing machine used on the Cascade Tunnel (see Figs. 66 and 67) consists essentially of a horizontal steel cylinder, provided with an air-operated drop door through which the machine is charged, and an outlet opening recessed below the bottom of the forward end of the cylinder connected by means of flanges to the discharge line. A steel worm keyed to a heavy shaft is mounted in bearings in the ends of the cylinder at a sufficient height to permit the edge of the worm barely to clear the steel liner which protects the bottom side of the cylinder from abrasion. The worm is driven by a small motor mounted near the rear (or operator's) end, the speed being regulated by a train of gears in an oil-filled transmission case between the motor and the cylinder. The air inlet is situated at the rear end of the machine between the worm-bearing and the floor of the cylinder. A small air cylinder actuates the charging door, and the control of air supply to the cylinder is effected by a quick-acting valve which, together with the motor control, is mounted at the operator's end of the machine.

---

[6] Supt., A. Guthrie & Co., Inc., St. Paul, Minn.

Operation—First, the charging door is opened by the application of air to the door cylinder, and the proper volume of previously mixed concrete is allowed to flow into the cylinder. Before shutting the door, it is necessary to remove any concrete that may adhere to the upper side so as to prevent the escape of air between the door and the round rubber gasket ring upon which it is seated. This is best done by tapping the main air supply line and connecting a short length of ½-in. air hose fitted with a trigger nozzle. A quick circular motion around the edge of the door cleans it effectively with little or no loss of time. The door is then closed by reversing the piston travel in the cylinder. The worm motor is then started. After the worm has made two or three revolutions, the air-control valve is opened. As the air pressure builds up inside the cylinder, the concrete is moved forward by the worm and is finally forced through the outlet into the discharge line. No hard and fast rule can be set for the length of operation of the worm beyond stating that "the shorter the better". When they feel the movement of concrete start in the discharge line, some operators stop the worm for a few seconds, and then give it a turn or two just before shutting off the air. It should be borne in mind that the function of the worm is to deliver the concrete to the discharge recess in sufficient quantity to keep the recess full, and thus prevent the excessive use of air.

In theory, if the concrete is delivered to the discharge recess at exactly the same rate at which it is forced through the discharge line, it should move out as a solid plug. Owing to the fact that as the batch begins to leave the cylinder, the rate of delivery lessens, and also that there is a considerable loss of air through the concrete, the batch tends to separate as it moves along the discharge line, its velocity increasing as the dispersion becomes greater. It should be thoroughly impressed on all operators that they should try for minimum dispersion and that the continued feeding of air to the cylinder after the bulk of the batch is discharged is the principal cause of wear in discharge hose, bends, and nipples. It is the last of the batch which, moving at a very high velocity, impinges upon the walls of the discharge line, particularly at the bends which causes a large part of the wear. There is probably no class of construction equipment in the operation of which greater dependence must be placed upon the "feel" of the machine than this type of concrete placing machine. Minute differences in the quantity of water in the mix, and slight variations in degree of "harshness" of the aggregate, exert a profound influence on the operation of the machine; consequently, every effort should be made to provide a uniform mix. The addition of diatomaceous material to harsh aggregate materially helps in the operation of the machine, but experience in the Cascade Tunnel indicated that the excessive use of this material involved a penalty in the increased difficulty in making the mixture "stand" in the key of the arch. However, its moderate use with all but the smoothest aggregates is recommended.

Provided suitable aggregate is used, with proper measures to overcome any harshness, concrete can be placed with this machine dry enough to satisfy the most drastic specifications. The disadvantage of trying to handle sloppy concrete in the top of a tunnel arch generally results in a dry mix being used by preference.

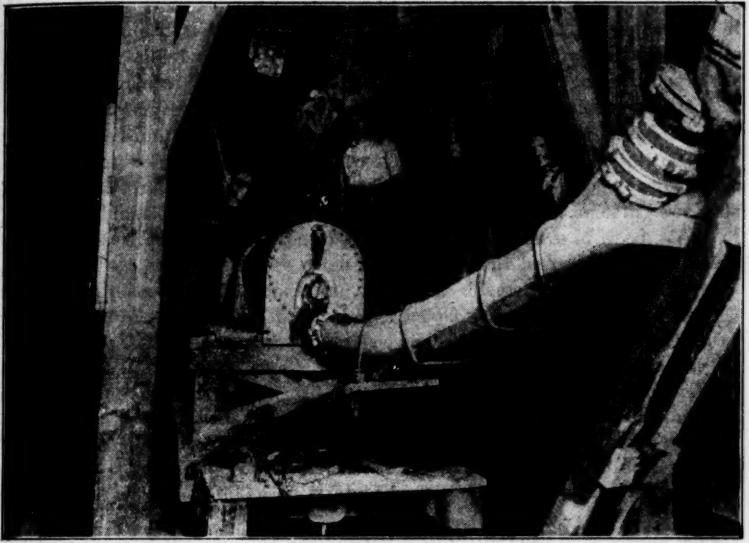


Fig. 66.—View of Pneumatic Concreting Machine Used on Cascade Tunnel Work.

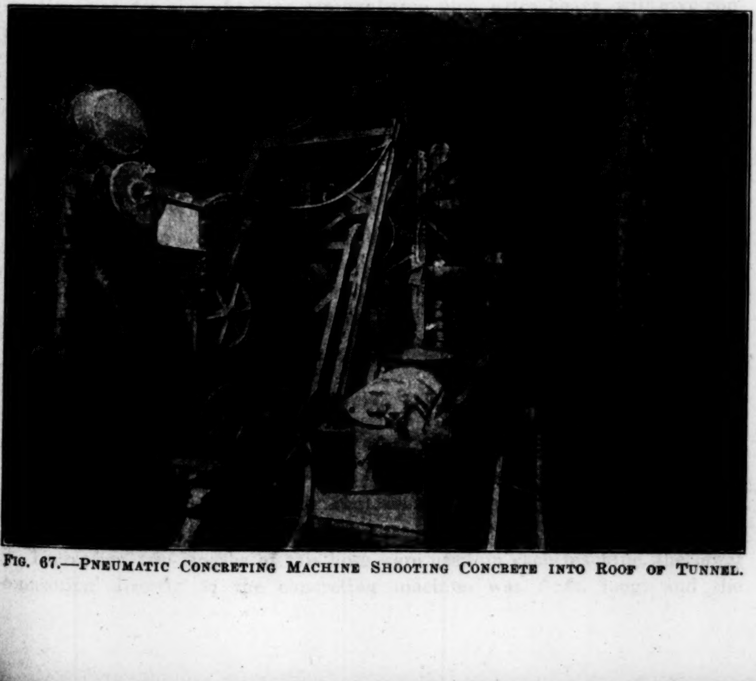


Fig. 67.—Pneumatic Concreting Machine Shooting Concrete into Roof of Tunnel.

[illegible]

[illegible]

Aggregate—With concrete machines of this type, having a 6-in. outlet, the aggregate should not exceed 2 in. in greatest dimension. If the percentage of fractured material is great this should be reduced to 1½ in. Slabby pebbles and thin elongated pieces of rock are very difficult to pass through the machine, and aggregate containing very many of these should be avoided.

Fairly well-rounded, well-graded washed gravel should be procured if available, even at a considerably higher cost. The loss of time resulting from "plugs" in the discharge line is lessened materially, if not altogether avoided, by the combination of this class of coarse aggregate with smooth, workable sand. Great care should be taken to prevent the small pieces of wood, pieces of gunny sacking, etc. (which are often used to prevent leaks in railroad cars) from being placed in the concreting machine. These will invariably cause plugs. This also holds true for pieces of caked cement.

Discharge Lines—As a general rule, avoid abrupt changes in direction of the discharge line. Elbows should never be used. The more nearly straight the discharge line is, the greater will be the ease and certainty of operation, and the wear on the line is infinitely less. If changes in direction are unavoidable use long sweeping bends. Wear is greatest on the bottom of straight runs and on the concave side of bends; consequently, to insure uniform wear, a quarter turn should be taken on straight pipe occasionally and on hose bends after every pour of, say, 100 yd.

The makers of the machine recommend a specially built wire-wound rubber hose for discharge line. Experience at the Cascade Tunnel has proved beyond question that, for straight runs, wrought-iron pipe, extra heavy, will give considerably better service. Continued observation has indicated that the standard wire-wound hose gave reasonably good service until the soft, line-rubber, interior tubing was worn through. After this occurred wear was extremely rapid and a blow-out occurred within about an hour. When the wear penetrated to the spiral wire reinforcing it was only a question of a few batches until a plug occurred at the point of greatest wear. This was caused by the wire cutting in two, thus separating from the cotton hose wall and snarling inside the tube. With this in mind, it occurred to the writer that since no blow-outs had been observed, which could be traced to lack of wall strength, the wire reinforcing should be abandoned and its cost put into additional soft, line-rubber tubing. Consequently, a hose of the following characteristics was specially made up at a price considerably below that of the wire-wound product:

Weight, cotton fabric	32 oz.
Number of plies	8
Thickness of special abrasion-resisting rubber tubing	¾ in.
Length	5 ft. and 10 ft.
Diameter, internal	6 in.
Cost (f. o. b. Scenic) of one 10-ft. length, including nipples and clamps	$101.50

Thereafter, two lengths of this hose were used on each machine, the lower connecting directly to the concreting machine was 5 ft. long, and the upper connecting to the inclined discharge pipe was 10 ft. long. This combination gave the necessary flexibility of maneuver to that section of the line immediately above the form itself.

Due to the greater dispersion of material at the upper bend, greater wear was experienced at this point. Data were kept on the performance of a number of these hoses, the following being a representative and average test:

Date placed in service	September 22.
Date blown out	November 12.
Yards placed through hose	2 181
Hose[7] cost per yard	$0.0465

The lower bends were approximately 75% longer, although the angle of bend was the same.

A large number of 30° wrought-iron pipe bends were used as substitutes for the hose with greatly varying results. With extra heavy pipe, 5 ft. long, evenly bent its entire length, the wear of six pairs of bends gave the following average:

Average number of yards, upper bend	425
Average number of yards, lower bend	721
Cost per bend complete with flanges	$21.00
Cost per yard, upper bend	$0.0505

Two methods of patching worn-out bends were developed. The first consisted of welding heavy patches of steel over the worn spots. These patches when placed in service gave approximately the same results as new bends, but the practice was discontinued because after an indeterminate amount of wear the heavy patches blew off with grave danger to the crew working about them.

The second method was an outgrowth of the first and consisted of clamping several thicknesses of used placing-hose over the worn spot with heavy steel straps. This worked very successfully and as many as 750 cu. yd. were placed through the bends before the patches wore out. This method is effective in making a quick repair while pouring without removing the bend.

Patched bends, however, are the potential cause of plugs. In spite of a slight difference in cost in favor of the pipe bends (when patches are used), the fact that plugged discharge lines are much more infrequent with hose bends is the weightiest argument in their favor. Plugs in 99 cases out of 100 occur at bends. With the pipe all that can be done is to disassemble the line and clean it out. With the hose a few well-directed blows with a wooden maul at the point of obstruction will almost invariably release the plug. (This is not so effective with the stiff wire-wound hose). Another advantage of the flexible hose is the fact that thin spots can be located by feeling the hose, whereas there is no way of telling how badly worn an iron pipe is without getting at the inside.

Flanges.—Instead of using standard six or eight-hole companion flanges a slotted four-hole flange with attached swinging bolts was developed. These materially cut the time consumed in removing short lengths of pipe as the concrete advanced toward the forward bulkhead and prevented the loss of bolts and nuts.

---

[7] Hose given one-quarter turn after every 125 yd.

Care of Machine.—Although the number of moving parts in the machine are few these few need constant care to prevent undue wear. The standard worm furnished with a new machine is too light for constant service, the fin being made of ¼-in. boiler-plate spiraled around the hub and riveted to it through brackets. At the start of the Cascade Tunnel work these worms were a source of constant and annoying delays. (It is a 4 to 6-hour job to replace a worm.) In several cases the fin was torn completely off the hub by thin pieces of rock that jammed between the lower edge of the fin and the cylinder liner. This difficulty was remedied by the use of a specially designed worm cast of special abrasion-resisting steel with the fin ½ in. thick at the edge and ¾ in. thick at the junction with the hub. None of these cast worms broke and only two replacements were required in any machine because of wear. After the edge of the fin wears back more than an inch, the delivery of material to the outlet recess is so slow as to affect progress materially.

No trouble is usually had with the bearing (closed end) at the forward end of the cylinder, but frequent inspections are required to avoid trouble with the bearing at the motor end. The packing in the gland is quickly cut out by the air pressure forcing thin grout along the shaft. Lubrication must be attended to hourly by turning up the grease cup on the bearing. If this bearing and the corresponding bearing of the transmission case are not looked after the grout will work along the shaft and enter the transmission case. Two instances of the transmission case being almost filled by grout mixed with oil came under the writer's observation. In one case the grout had set so hard that the gear train was frozen in it. Due to the dust which is the usual accompaniment of concrete operations the motor requires more than ordinary care. When replacing the worm it is a very good idea to examine the liner plate and if it is materially worn to replace it at the same time.

Air Supply.—The results of experiments to determine the air requirements of this pneumatic concrete placer may be tabulated, as follows:

Number of batches poured	56
Size of batch, in cubic yards	0.565
Total number of cubic yards poured	31.64
Number of charges of machine	98
Amount per charge, in cubic yards	0.32
Quantity of air used, in cubic yards per minute	1 600
Length of time that air is used for each charge of the concreting machine, average in seconds	14
Amount of air used per charge of machine, in cubic feet	373
Air in cubic feet per cubic yard of concrete placed	1 134
Pressure, in pounds per square inch	105
Length of 6-in. placing hose, in feet	12
Length of 6-in. placing pipe, in feet	16
Combined length, in feet	28

With an air pressure of 105 lb. as used in this test, consumption would probably be a maximum. Experience at the Cascade Tunnel indicated that a compressor of much smaller capacity can be utilized if a good-sized air receiver is mounted near the concreting machine. Since the demand for air occurs at regular intervals it is practicable to build up a supply while the air-

control valves are closed. High pressure is not essential to the successful operation of the machine; in fact, with adequate reservoir capacity the best results are had at a pressure of from 70 to 80 lb., and the apparatus will function very well with pressure at low as 60 lb.

***Safety Precautions.*—It should be remembered that the apparatus is periodically under rather high internal pressures and should be treated with the same respect as a steam boiler of equivalent pressures. Connections should be examined and tightened periodically (they have a habit of loosening from vibration), and flange threads should be inspected for wear. This is particularly important over the forms because the puddlers usually work in constricted quarters and have little chance to escape from bursting pipes.**

When hose is used in the discharge line in proximity to wires carrying electric current the iron pipe near the discharge end of the line should be well grounded, thus avoiding the possibility of electrocuting workmen in contact with the discharge line above the forms.

DISCUSSION

R. H. KEAYS,[*] M. AM. SOC. C. E. (by letter).—The writer has been much interested in this Symposium, especially Part III, "Construction Plans and Methods." He only regrets that the description is not much longer, because there are no doubt many important features that might be elaborated.

The railroad authorities are to be congratulated that in the very beginning it was decided to line the Cascade Tunnel completely with concrete and not take chances with a timbered tunnel. Under some circumstances even the initial cost of a concrete lining might be less than that of timber.

The construction of this long railroad tunnel, by the use of a so-called pioneer drift (formerly known as the "two-tunnel system") supplies another example of the application of this method and well illustrates its advantages. Reference has been made to the method in connection with the Simplon, Rogers Pass, and Moffat Tunnels, the Simplon, of course, far antedating that of the other two. As adapted to these tunnels, however, the method shows some variations in detail, and the reasons for its adoption in each case may reveal some important differences.

It will be interesting, even at this late date, to review some of these reasons in the case of the Simplon. Going even farther back in history to the case of the St. Gotthard Tunnel (double track) it will be found that the highest rock temperature recorded was 30.7° cent., under a rock cover of 1 700 m. On the Simplon, the greatest rock cover was 2 200 m. (5 577 ft.) and, calculating from experience at the St. Gotthard, the highest temperature to be expected was 42° cent. On the other hand, to insure efficient service from the workmen in the tunnel, it was considered important to bring the air temperature in the Simplon Tunnel down to 25° cent.

At the time the construction of the Simplon was undertaken, it was realized that two tracks were necessary, and it was merely a problem of building one double-track tunnel, or two single-track tunnels. It was not necessary to start the second single-track tunnel, however, until the first was completed if a passing track were provided temporarily in the first tunnel. Therefore, the two-tunnel plan was adopted.

The only method considered to cool the tunnel was by ventilation with large quantities of air. For the supply of fresh air without regard to cooling, the total requirements were calculated at less than 5.0 cu. m. per sec. (176.6 cu. ft. per sec.), but to cool down to 25° cent., it might be necessary to provide for no less than 35 cu. m. per sec. (74 500 cu. ft. per min.).

Computations indicated that the total cost of piping such a quantity of air into the tunnel would be the staggering sum of approximately $7 000 000. Under these circumstances scarcely any calculations were necessary to determine that the best way to provide for the ventilation was to adopt the two-tunnel plan.

---

[*] Chf. Engr., Ulen & Co., Athens, Greece.

The Simplon Tunnel was constructed before the days of modern air drills. A bottom-heading method was used, the heading being drilled with rotary hydraulic drills. The tunnel enlargement drilling was entirely by hand methods. The lining is stone masonry.

As a matter of fact the rock temperatures found were much higher than calculated, amounting as a maximum to 55.4° cent. This, of course, made conditions intolerable, and it was necessary to supplement the cooling effect of the air by the use of large quantities of cooling water. An insulated water main, 25.3 cm. (10 in.) in diameter, was installed therefore in the pioneer tunnel. Through this pipe was pumped a maximum of 79 liters per sec. (2.8 cu. ft. per sec.). In addition to this pipe, there were two smaller insulated water mains for the conveyance of high-pressure water to the headings for use in the drills.

The water from the large pipe was used mainly to supply sprinkling nozzles installed in the main tunnel, between the last cross-cut and the enlargement operations. By far, the greatest number of men were employed in the enlargement, and it was necessary to protect them. These nozzles blocked the transportation to the heading of the main tunnel, so that it was necessary for all heading transportation to cross over to the pioneer tunnel at the cross-cut nearest the heading. The maximum water temperature at the nozzles was 17° cent.

Some of the cold water, however, was conducted to the headings and was used for sprinkling the walls and wetting down the outside of the ventilating pipes leading to the headings. The ventilation layout for the headings was similar to that described by Mr. Baxter for the Cascade Tunnel.

The foregoing information in regard to the Simplon Tunnel has been discussed in an interesting manner by Professor C. Andreae.[9]

Before dismissing the subject, the writer wishes to express his curiosity as to the reasons, in the case of the Cascade Tunnel, for estimating rock temperatures as high as 110° Fahr. If the Simplon Tunnel had any influence in this decision, it should be remarked that as far as rock temperatures are concerned Alpine tunnels seem to be in a class by themselves and of these, the Simplon was much the hottest, even allowing for its greater depth. Rock temperatures in mountain tunnels may have something to do with the age of the mountains, the Alps being of relatively recent origin.

The question of the quantity of air to be provided for ventilating purposes in an ordinary rock heading merits a few remarks. In many cases a large nominal quantity is provided for ventilation, until it is found that, due to light-weight pipe and leaky joints, only a small portion of the air reaches the end of the pipe; and then, of course, the advanced end of the pipe is not at the heading where the men are working, but is usually 100 ft., or more farther back. It is exceptional, therefore, for any of the low-pressure air to reach the heading. On the other hand there is always a supply of high-pressure air at the headings. During drilling operations there can be nothing in the heading but air from the high-pressure mains and during blasting operations the high-pressure mains are usually more or less wide open "to blow out the smoke". The function of the low-pressure air, therefore, is merely to hurry the smoke and bad air out of the tunnel after it has passed the end of the ventilating pipe.

---

[9] "Der Baulänger tieflingender Gebirgstunnel," Berlin, 1926.

In this connection, the writer knows of one important tunnel, in which a peculiar notion was stoutly maintained, probably by all the tunnel men. It was noted that after blasting (with high-pressure air discharging close to the heading and with the low-pressure system in operation at the same time), there seemed to be a wall of smoke maintained on the heading side of the end of the ventilating pipe and distant therefrom, say, 15 or 20 ft. The idea prevailed, therefore, that the operation of the low-pressure system held the smoke back and would not let it come out. After that the men actually went so far as to stop the ventilator during blasting, until all the smoke had been driven past the end of the low-pressure pipe by the high-pressure air from the heading. This was merely another case of things not always being what they seem. The true explanation is that the jet of air from the ventilator pipe instantly diluted all the smoke in range of the jet in the proportion of, say, 25 to 1, or more, so that the air in this zone seemed quite clear.

A variation of this idea is seen in the case of the tunnel superintendent who always had the high-pressure air line tapped with 1-in. nozzles at regular intervals all the way from the heading to the portal. All these nozzles were carefully pointed toward the portal and, after blasting, they were all opened with the idea of driving the smoke out of the heading. Then, there was the case of the superintendent who insisted on putting on a reverse bend at the heading end of the ventilator pipe, so that the jet would point toward the portal. In this case the only result was that he introduced so much back pressure that he practically stopped the air entirely.

J. G. SULLIVAN,[10] M. AM. SOC. C. E. (by letter).—Discussion usually implies some difference of opinion, but in the case of this Symposium the writer has only words of praise for the engineers and contractors who so rapidly and successfully carried this work to a satisfactory completion, especially in view of the fact that they had to contend with considerable quantities of water in a tunnel with a gradient entirely in one direction. Beginning in 1892, the writer had charge, as Assistant Engineer, of the construction of that portion of the abandoned line from the west end of the Scenic side-track to a point about half way to Tye. (See Fig. 2.) He has, therefore, taken a somewhat personal interest in the progress and outcome of this project.

It was gratifying to note that the contractors followed the plan of driving a center heading and radial drilling (see Fig. 19). When this plan was adopted for the first time, about 1916, in the construction of the Connaught Double-Track Tunnel on the Canadian Pacific Railway, it was rather severely criticized by some engineers of New York City.[11] Where rock is reasonably stable, this method is much more economical in the construction of a double-track tunnel than it would be in a single-track tunnel.

---

[10] Pres., Sullivan, Kipp and Chace, Winnipeg, Man., Canada.

Temperature.—The actual temperatures of rock in the Connaught Tunnel were practically the same as those shown in Fig. 60, although the distance from the summit of the mountain to the grade of the tunnel was about twice as great as that in the Cascade Tunnel. Before the Connaught Tunnel was started it was predicted by some engineers that serious trouble would be encountered from heat. They went so far as to predict an increase of about 1° Fahr. per 100 ft. of depth.

Double-Track vs. Single-Track.—The writer has the impression that the Great Northern Railway Company might have constructed a double-track tunnel. Using the center-heading and radial-drilling system of construction in rock that is reasonably stable, a double-track tunnel can be constructed at a cost not exceeding 125% of the cost of a single-track tunnel.

Since there is already a pioneer tunnel built for about 68% of the distance and considering that time in the construction of the second tunnel would not be an important factor, the second might be constructed for about 60% of the cost of the first tunnel. If these assumptions are approximately correct, and if the second track would be required within 15 or 20 years, depending on the cost of money, it would have been economical to have constructed a double-track tunnel in the first place; that is, of course, with the understanding that the nature of the rock was such that it would stand without support during construction. In the construction of the Connaught Tunnel it was found that, on account of the increased space, the time required for enlarging and lining would not be appreciably increased.

FREDERICK MEARS,[12] M. AM. SOC. C. E. (by letter).—The writer has noted Mr. Sullivan's remarks in regard to double-track *versus* single-track tunnel. A study of the present and probable future traffic over the Cascade Division of the Great Northern Railway showed that a double-track tunnel would not be warranted, particularly in view of the increased average speed of all trains over this district, brought about by electrification of 71 miles of main line—Wenatchee to Skykomish.

The extra cost of driving a double-track tunnel, as compared with a single-track tunnel, would have been far greater than the proportional difference in the cross-sectional area of the two tunnels. The nature of the rock was such that more heavy timbering, in advance of concrete lining, would have been required in a double-track tunnel than was used in the single-track tunnel, the side walls of which, in solid formation, were only 18 ft., face to face. Furthermore, the time limitation of three years would not have permitted the construction of a double-track tunnel in one operation. If it ever becomes necessary to construct a second tunnel to handle increased traffic, the pioneer tunnel, already built for 5.38 miles, could be used to good advantage in driving the second bore.

---

[11] *Engineering News*, January 11, 1917, p. 77.

[12] Asst. Chf. Engr., G. N. Ry., Seattle, Wash.

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
TRANSACTIONS

Paper No. 1810

MEASURING IRRIGATION DELIVERIES IN THE PANJAB[1]

By E. S. Lindley,[2] M. Am. Soc. C. E.

Synopsis

The paper describes the conditions of Indian canals, as affecting the problem of distributing the supplies received in the manner desired. It describes the devices available for effecting such distribution with a minimum of manual control, mentions their principal qualities, and refers to the records of investigations directed to their improvement. An outline is given of the theories on which to base design in using such devices to form a complete scheme of automatic distribution; and, finally, their practical application is discussed, in so far as they are found possible under present limitations of knowledge and invention.

The method and means of measuring and apportioning irrigation deliveries, that are in course of development in India, and especially in the Panjab, are described. The writer, however, has not the local knowledge needed to judge whether, and how, these could be applied to American conditions.

General Conditions

The irrigation canals of India are built, owned, and managed by the Government, which sells the water to the actual cultivators of the land served. The water is paid for at fixed acreage rates levied on the fields actually cropped on canal water in any season. The reason for this is that hitherto it has been impracticable to measure the water delivered to a holding, or even to a group of holdings, while field-to-field assessment was already in force for

---

Note.—The Special Committee on Irrigation Hydraulics selected the subject of "Irrigation Deliveries" as one of ten for study and research. This paper was submitted to the Committee by the author, and the Committee recommended its publication (see Progress Report of the Committee, *Proceedings*, Am. Soc. C. E., March, 1929, Society Affairs, p. 97).

[1] Published in February, 1931, *Proceedings*.

[2] Superintending Engr., Indian Public Works Dept., Panjab Irrig. Branch, Wotton-under-Edge, Gloucester, England.

the rent or land revenue levied by the Government as the primary owner of all land.

Water rights are not fixed by volume, or by a rate of flow demandable at will. For each group of holdings a rate of flow is calculated from its area and other conditions. The canal system is designed to deliver these flows to all holdings when it is running full; when the supply is insufficient to fill the canal, the water is shared proportionately. For a variety of reasons, some local and some general, the simplest and best method of sharing has been found to be to keep each subsidiary channel running full or closed in short turns, for the correct proportion of the entire time.

The channels of a canal system fall into three classes: (1) Watercourses (farmers' ditches) serving the holdings that can be grouped together on one such channel, into not too large a unit; (2) distributaries and minors, feeding such watercourses; and (3) main canals and branches, feeding distributaries, but not watercourses. The distinction between Class (1) and Classes (2) and (3), is that the former is managed and maintained by the farmers, subject to the duty to keep the watercourses in good condition. The distinction between the second and third classes, both being managed and maintained by the Government, is that Class (2) channels are run full only to feed their watercourses correctly and automatically; while those of Class (3) may be run at any discharge, with the off-taking discharges controlled according to plan.

The supplies of the more important canals of the Panjab are secured by weirs; thus, they can be run up to their capacity as long as so much water is available in the river; but there is no storage for supplies in excess of momentary requirements. The canal systems vary in length up to 200 miles, and water takes as long as five days from the head to reach the last fields. (See Fig. 1.) Some canals have no weirs, so that they have to utilize the supply entering them from seasonal river levels. Many canals outside the Panjab are fed from storage of the monsoon flow of rivers which are almost dry for part of the year. This paper deals principally with the conditions of a large perennial canal of the Panjab. Fig. 1 shows the degree to which the Panjab plains are covered by canal irrigation. The areas commanded by "perennial" (weir-fed) canals are shown hatched; those commanded by "inundation" canals (fed only by natural rises of river level) are dotted.

The heads of main canals, branches, and main distributaries necessarily contain mechanism for the manual control of the supplies passing. The criterion in control has hitherto been satisfaction of the demand of those channels that were intended to receive supplies at any time. The correctness of these demands has been checked by periodical discharge observations at the heads of channels, and at the boundaries of the management districts. This method of check has become increasingly unsatisfactory as development of the irrigated area has caught up with the supply available. Deficiencies are due to the unavoidable errors of observation, considerable "personal factors", and accumulation or scour of silt which changes the rating curve. Progress is being made with the provision of meters to eliminate discharge observations. The task of calibrating existing works by individuals, or of designing standard calibrated works, is of old standing; the present tendency is to provide "semi-modular weirs or flumes" on the lines described subsequently and often by only a slight modification of existing works.

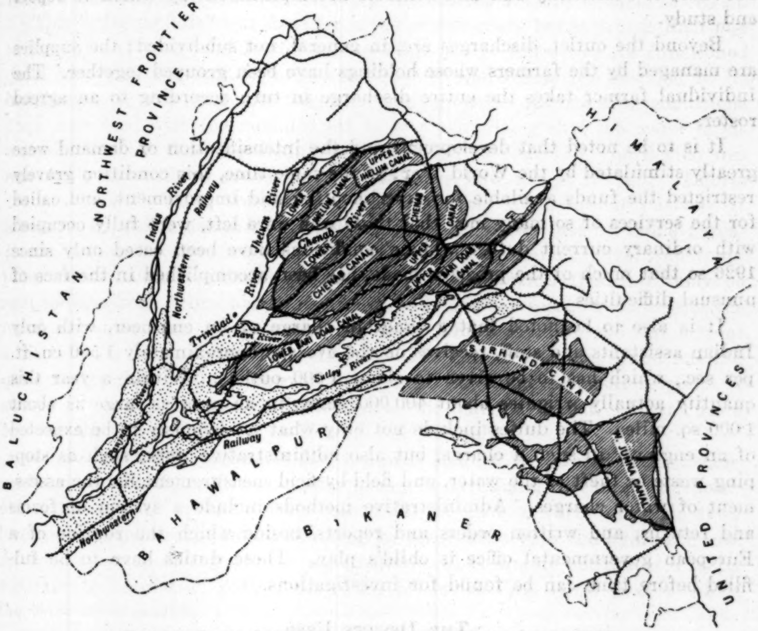

NORTHWEST FRONTIER PROVINCE
BALUCHISTAN
HIMALAYA
UNITED PROVINCES
BAHAWALPUR
BIKANER
Indus River
Northwestern Railway
Jhelum River
Chenab River
Ravi River
Sutlej River
Trinidad
UPPER JHELUM CANAL
LOWER JHELUM CANAL
CHENAB CANAL
LOWER CHENAB CANAL
UPPER CHENAB CANAL
UPPER BARI DOAB CANAL
LOWER BARI DOAB CANAL
SIRHIND CANAL
W. JUMNA CANAL
Panjnad River
Railway
Northwestern

FIG. 1.—CANAL-IRRIGATED AREA IN THE PANJAB AS OF 1922.

The heads of minor distributaries were also controlled by hand, but it has now become fairly general to effect distribution at such points automatically, by pairs or groups of semi-modular flumes.

The heads of watercourses, or "outlets", were originally mere open cuts in the distributary bank. Several decades ago pipes and culverts in the bank were substituted for these cuts; and it is instructive to note that at the time this substitution met as much opposition as is now offered by some to more modern ideas. The discharge of such outlets was dependent on the water level, both on the distributary and the watercourse side. While the former was relatively constant, especially under a policy of running a full supply or nothing, the latter was liable to considerable variation, according to the level of the field being irrigated, and to the state of neglect or clearance of the water course, depending on keenness of demand. The heads available did not make it generally possible to eliminate the effect of water level on the watercourse side by setting the outlets with a free fall. The first suggestion for a device with which hydraulic principles might be used to provide a virtual free fall was made about 1908, and, at present, automatic distribution is being rapidly improved by extending and improving the use of such devices.

The question is no longer "whether", but "how". The "how" is still in the stage of trial and intense discussion, both as to general policy and details; but there is sufficiently full and concrete accomplishment by which to report and study.

Beyond the outlet, discharges are, in general, not subdivided; the supplies are managed by the farmers whose holdings have been grouped together. The individual farmer takes the entire discharge in turn according to an agreed roster.

It is to be noted that development and the intensification of demand were greatly stimulated by the World War; at the same time, this condition gravely restricted the funds available for experimenting and improvement, and called for the services of so many men that those who were left, were fully occupied with ordinary current duties. These conditions have been eased only since 1926 so that much of the progress made, has been accomplished in the face of unusual difficulties.

It is also to be noted that a canal in charge of an engineer, with only Indian assistants and staff, receives on the average approximately 1 500 cu. ft. per sec., which has to be given to about 1 500 outlets. Within a year this quantity actually irrigates about 400 000 acres, in an area as large as about 1 000 sq. miles. The duties include not only what is obviously to be expected of an engineer in such a charge, but also administrative work, such as stopping waste or theft of the water, and field-by-field measurement for the assessment of water charges. Administrative methods include a system of forms and returns, and written orders and reports, beside which the routine of a European governmental office is child's play. These duties have to be fulfilled before time can be found for investigations.

THE DEVICES USED

The term, "module", as used in this paper, is defined as a device designed to deliver a constant discharge, independent of fluctuations of water level or pressure on either the supply or the delivery side.[3] The first to develop any of the devices that have proved of practical importance was the late Mr. R. G. Kennedy, of the Panjab Irrigation Department, whose investigation of channels that just carried their silt charges without scouring, is widely known. He first devised three types of "modules,"[4] for big, medium, and small discharges, covering the entire range of requirements. Of these, the medium and small types involved regulation by balanced cylinders suspended from a weigh-beam, and have not proved suitable for practical work. The largest consisted of a gate swinging on a horizontal axle, with a counterweight linked to an extension of its upper end and running on a guide-rail laid to a curve calculated and confirmed by experiment; and the basin under the bottom of the gate was also formed to a longitudinal curve. Any given difference of up-stream and down-stream water levels brought the gate to some position predetermined by the counterweight and the curve of its guide-rail, and the basin graduated the aperture at each position so that the desired discharge would pass. The device was made in several sizes, and each could be set to furnish the discharge desired, by giving the correct amount of counterweight.

---

[3] Panjab Eng. Congress (1923), p. 45.

[4] Panjab Irrigation Branch *Paper No. 12* (blue), of 1906, "On the Distribution of Water for Irrigation by Measurement".

This "gate module" was used for a few distributary heads, which then had to be equipped with modules for special reasons. It seems to have functioned quite satisfactorily, but when it had removed the difficulties that led to its installation, it was gradually forgotten and ignored. At one site, conditions (not concerned with the module) led to so much silting that the water levels rose above those that the module was intended to control. The device was removed because those concerned did not know how to put it right. Efforts are now (1931) being concentrated on regulating discharge at outlets; at minor heads, needs are more simply and probably better met by flume furcations; at distributary heads, similar flumes to meter manually controlled supplies seem to satisfy present requirements. This device, therefore, may be said to have been "born before its time", and thus far seems to have met the usual consequences. Fig. 2 shows a pair of flumes dividing the tail flow of a distributary between two form ditches. The one in the foreground has not quite a free fall, but the flow at the tail of the flume is unsubmerged. A standing wave is formed a little below the narrow throat. Distribution between the two is the same as if both had free falls.

A "gauge outlet", devised by Mr. Kennedy in 1908, is the first of the "semimodules", which differ from modules proper in rendering discharge independent of water levels on the down-stream side, the natural variation with water levels on the up-stream side continuing in accordance with the hydraulic law. As the down-stream variation is the greater, and is largely controlled directly contrary to regulation policy, this class of device does more than one-half the work of a module.

This gauge outlet[5] may be described as a modified Venturi meter. The modification consists in providing free admission of air all around the throat, so that static pressure in the jet at that point cannot fall below zero. There is the usual recovery of kinetic energy in the expanding down-stream section, so that zero pressure is possible at the throat when that is lower than the delivery water level. It will be understood without lengthy explanation, that the discharge is independent of down-stream levels as long as the difference of head is sufficient to draw the throat pressure down to throat level; if it is sufficient to draw it lower, air bubbles will be attracted into the stream, and the excess head will be wasted. The discharge then depends only on the depth of submergence of the throat by the supply level. The outlet is not adjustable, but the desired discharge can be given by setting it with the necessary submergence, subject to the maximum allowed by the difference of head available.

The outlet has been widely used. It has brought about a considerable improvement in distribution, and blazed the way for most of the further work that has been done on this subject in India. After it had shown that the needed improvements were possible, however, certain defects gradually became apparent.

---

[5] Panjab Irrigation Branch *Paper No. 12* (brown), "Results of Tests of Mr. Kennedy's Gauge Outlet"; together with addenda on its improvement, protection, and application.

From the beginning, the most serious defect has been that it is perfectly easy to stop the air supply to the throat, in order to increase the supply received. In the absence of any public opinion among the Indian peasantry, it is difficult to deal with this interference, and impossible to stop it. Some means of meeting this defect have been devised, but the outlet has fallen completely out of use in the Provinces with which the writer is in touch, and he has heard nothing of any actual use of these newer models.

Other defects are the natural result of the textbook theory that dictates the measuring of the head which causes velocity, down to the center of the orifice through which the water flows. To make it theoretically an orifice discharging freely into air, Mr. Kennedy provided an enlarged section in the throat in such a way that a supply of air surrounds the jet. He seems to have assumed that the top and bottom of the throat would be aerated simultaneously; and he claimed that the appearance of air bubbles in the discharge was a certain and simple indication that the device was discharging as designed. In unpublished tests the writer has found that, between the condition in which discharge was fully dependent on difference of water level with no air bubbles passing, and that in which it was fully independent of downstream level with air bubbling freely, there was an intermediate condition with discharge less than the theoretical, but with bubbles appearing.

A further result of following this "center of orifice" theory involved no practical defect, but is of interest. Mr. Kennedy calculated the coefficient of discharge from observed discharges and submersions measured from the center of the orifice as zero point. He was impressed by the fact that the values of the resulting coefficient were so low, and that the coefficient decreased with decreasing head. The writer plotted $\surd^2$ against water level, and found that if the zero point were taken at almost two-thirds of the height of the orifice, the coefficient was constant down to quite small submersions, and that it had a reasonable value for the bellmouth used. Another instance in which the writer was able to point out that the "center of orifice" theory had been misapplied, was the calibration of the sluices of the Assuan Dam.[6] The same error had also been found in investigating relatively short pipes discharging freely into air.[7]

Both these two last points would be avoided by making the orifice rectangular, or at least aerating only a flat top, as in the standing wave orifice semi-module mentioned subsequently in this paper.

A defect that is not yet generally felt, is that the discharge can only be adjusted by means of the depth of submersion, which should be fixed by other considerations to be discussed later. That defect is overcome by another form devised by Mr. Kennedy.[8] In this form the throat which constitutes the

---

[6] *Minutes of Proceedings*, Inst. C. E., 1921, Paper No. 4350, pp. 50 and 71.

[7] Ministry of Public Works, Egypt, Research Work, Delta Barrage, "The Discharge of Pipes", by A. Deane Butcher.

[8] "The Irrigmeter: An Irrigation Distributor and Meter, as Specially Adapted to American Methods of Distribution," by R. G. Kennedy, 1918, Colonist Press, Victoria, B. C., Canada.

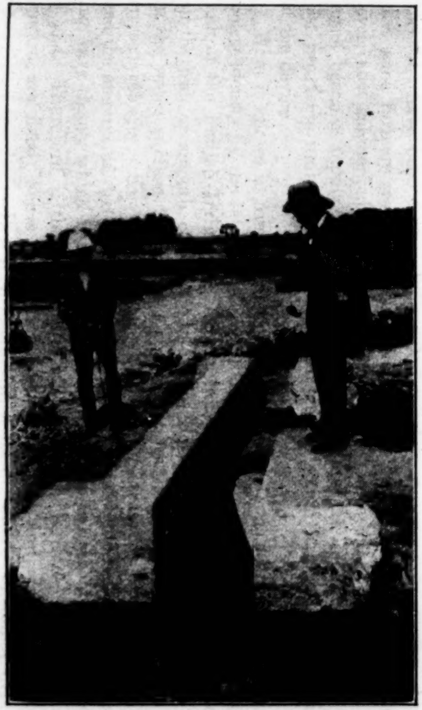


FIG. 2.—FLUMES DIVIDING THE TAIL FLOW OF A DISTRIBUTARY BETWEEN TWO FARM DITCHES.

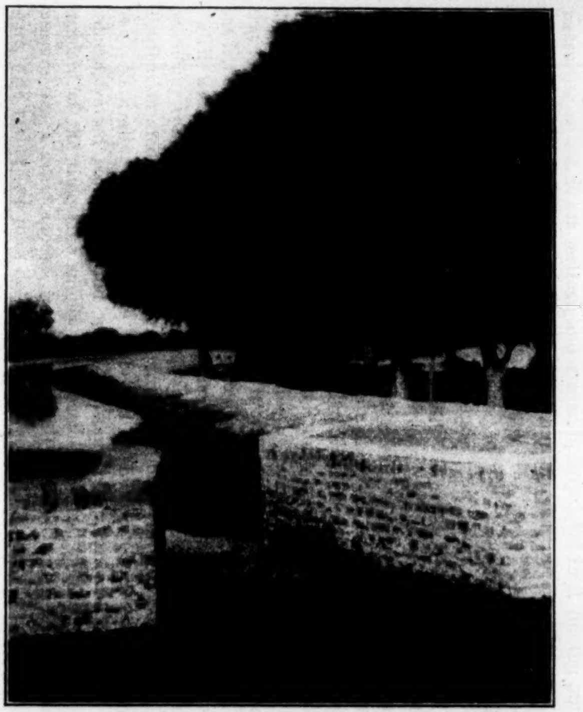


FIG. 3.—A STANDING WAVE FLUME.

[illegible]

[illegible]

measuring orifice, is set vertical, and a cone is suspended in it for the purpose of adjusting the area available for flow. This form has not been brought into practical use, within the writer's knowledge.

In about 1906, Mr. A. S. Gibb, formerly of the Panjab Irrigation Department, devised what is believed to be the only true module in which nothing moves but the water.[9] A flume of rectangular section follows a circular arc. The flow, therefore, must be part of a free vortex, and the surface of the designed discharge can be calculated. Vanes are set across the flume, leaving the designed discharge free, but skimming off any excess and turning it toward the center and causing turmoil. This module must be set unsubmerged by delivery levels; the supply water level must be high enough to give the designed velocity, and must not exceed this by more than the amount of head that the vanes provided can waste. It is not made adjustable for discharge, but is designed for discharges varying by small steps.

The use that has been made of this module is, as far as the writer is aware, practically confined to complete regulation of one long channel in which the distribution among watercourses gave great trouble with ordinary culvert outlets. Naturally, the cultivators who had their former excessive supplies reduced, raised objections and made constant complaints. Judged by the standard of the time (the supply that reached the tail of the channel), the measure was a great success in that the tail received rather more than its share, with rather less than designed discharge admitted at the head; and the same is still the case when, for a number of years, the local engineers have not had to give the channel special attention.

However, the fact as stated indicates that some outlets receive less than their share. The earlier forms of the module were found to be slightly incorrect, but have been replaced by a later form. A recent inspection showed that some farmers were apparently getting too little water because delivery levels now submerged the modules; and others were apparently getting too much because supply levels had now risen above the modular limit. The last two facts indicate slight disadvantages to weigh against the simplicity of the device.

What is known in America as the "Venturi flume" was originated earlier and independently by the late Dr. A. A. Stoddard and the late Mr. W. B. Harvey, both of the Panjab Irrigation Department. By introducing a smooth hump on the bed, or smooth contractions of the sides of a regular channel, a dip in the water surface is produced equivalent to the head converted into increased velocity. Knowing the areas of the up-stream and throat sections, and the amount of this dip, the discharge can be calculated; but by the standing wave theory which is now widely known, V^2, at the throat, cannot exceed $g \times d$. Therefore, if the contraction is sufficient, conditions from the throat upward are fixed; below it, a standing wave forms and wastes any excess energy, and the device is a semi-module with its discharge constantly dependent on the up-stream water level only.

---

[9] Panjab Irrigation Branch *Paper No. 13*, "Gibb's Module; Report on Its Working on the Shahkote Distributary.

In the Panjab, this device was first introduced to distribute supply among the outlets grouped at the tails of distributaries. These supplies are even now variable; therefore, the first need is to distribute them proportionately, in spite of differences between, and fluctuations of, delivery water levels. This need would not be met at all by the two-gauge Venturi flume, and the one-gauge standing-wave flume has been the one developed. Fig. 3 illustrates a standing wave flume used for automatic semi-modular division of supply between two small branches. The flume shown in the center is discharging about 30 cu. ft. per sec., with an up-stream depth of about 2.5 ft. With an afflux of less than 6 in., it is discharging as if it had a free fall. "Modularity" with small working heads was the first desideratum, and as the difference between ordinary and extreme efficiency meant only a difference of about 0.1 ft. in maximum permissible delivery level, even this demand was mild. Accuracy of discharge was not at first very important because the discharges dealt with had been subject to 100% variation and a reduction of this to, say, 20%, satisfied all concerned. Both have become much more important since the device has been used on a much larger scale. The *prima facie* simplicity of elementary theory is found to be complicated by points that this theory ignores; and much important work has been done to perfect the device.[10]

The simultaneous investigation of the device in America has concerned itself more with the two-gauge Venturi flume. Recently, advance seems to have led more to the one-gauge flume.[11] It seems to the writer that the ample resources available for American research would achieve even more valuable results, if a study were also made of what engineers in India have accomplished with very cramped resources.

When Messrs. Stoddard and Harvey originated their flume, they also devised an orifice gauge outlet based on the same principle. In this, a bellmouth leads to a rectangular orifice; instead of the closed expanding pipe of the outlet as developed by Mr. Kennedy, this continues as an open flume the bed and sides of which carry on those of the orifice. Under appropriate conditions, a standing wave forms in the flume, leaving the orifice free to discharge into air with a supported jet.

The outlet has the advantage of extreme simplicity; but under many conditions a relatively large working head is necessary for the standing wave to form. This led Messrs. Stoddard and Harvey to concentrate on an attempt to make smaller heads sufficient. These attempts produced some extremely ingenious ideas, but achieved no practical success. In spite of this limitation, the outlet[12] had a considerable field of utility in this simple form, and has been used successfully. A further considerable advantage is that the outlet is adjustable, by making the roof of the bellmouth and orifice slide up and down the sides. A special form was also constructed,[13] which diminished the working heads needed by using the outlet with shallow submergence, for proportional distribution of supplies. The relative advantages of that and other distributions are discussed in this paper.

---

[10] Unpublished investigations of E. S. Crump, Panjab Irrig. Dept., Panjab Eng. Congress, 1927, "Report on Flume Experiments on the Sirhind Canal," by A. G. C. Fane; Bombay Public Works Dept., *Technical Paper No. 15*, "Notes on Standing Wave Flumes and Flume Meter Falls," by C. C. Inglis; and Panjab Eng. Congress, 1925, "Canal Falls and Their Use as Meters," by E. S. Lindley, M. Am. Soc. C. E.

[11] "The Improved Venturi Flume," by Ralph L. Parshall, Assoc. M. Am. Soc. C. E., *Transactions*, Am. Soc. C. E., Vol. 89 (1926), p. 841.

[12] Panjab Irrigation *Nomograms*, Pl. 45*a*, and the pamphlet of the manufacturers.

After considerable further work by other engineers, the problem of saving head seems now to have been solved by Mr. W. Kirkpatrick, of the Bombay Public Works Department. He sets vanes across the flume of the outlet, with their lower edges on an incline rising from the orifice. These help to support the standing wave, and the orifice is free with a satisfactorily small working head.[14]

A number of devices have not been mentioned which, like Kennedy's two smaller modules, have never found application and seem unsuited to irrigation conditions in the field. The same applies to devices that are really only minor modifications of the originals described. The literature that has been quoted in references contains brief descriptions and references to any published, more complete descriptions.

Only one further group of devices calls for mention here. One is a module intended for outlets, which is now used to regulate a constant supply for sand filters. It is of the type in which a piston "floats" on the difference of head across a fixed orifice, and as it moves it adjusts a second orifice so as to waste more or less head and keep the former head constant. It has been tried in the Bombay Presidency and is understood to have proved unsuccessful because of the long streamers of weed that grow in the water and the colloidal mud which accumulates on the guide-rod for the floating piston. It was applied to the outlets of a small distributary in the Panjab and this installation has never been properly examined since, but on one occasion the writer had the opportunity to go over the channel and found distribution still satisfactory.

A second device is a module intended for large discharges. It is a Venturi meter provided with a butterfly valve a little down stream from the throat. The valve is actuated by reversible mechanism controlled by the Venturi head, set to be stationary at the designed head, to close with greater head, or open with smaller heads. A few were installed for trial in Bombay, but the writer has not had any certain report of their working.

The Venturi meter has been used for some very large discharges in Southern India and is believed to be satisfactory. In the Panjab its field is filled by the flumes described herein.

METHODS OF USING THE DEVICES

At first, "moduling" was done for outlets individually; but methods of collective use have since been developed, so that the supply admitted at the head of a canal is automatically distributed to the outlets on it. At first thought, it might appear that if modules were fitted to the head of the channel and to all outlets, this result would be simply secured; but no positive module has yet been definitely shown to be so reliable and satisfactory as to justify the cost. Even if that were the case, fluctuations would still have to be faced, as, for example, when cultivators close their outlets for silt clearance without notice, or when channels have to be overloaded temporarily while a breach is repaired.

---

[13] Panjab Irrigation Branch *Paper No. 26*, 1922, "Moduling of Irrigation Channels," by E. S. Crump.

[14] Bombay Public Works Dept., *Technical Paper No. 12*, "Jamrao Semi-Modules," by W. Kirkpatrick.

The ideal aim for a plan of "moduling" a distributary may lie anywhere between two extremes: One is to give each outlet and the distributary the full designed supply, and to close the distributary as required; the other is to give each outlet its correct share of the supply passing at the moment. The former is theoretically possible; but for practical reasons referred to previously, it is accepted by all as necessary to fit the tail group of outlets with flumes in order to distribute proportionately the supply reaching them. With the devices available, the latter ideal is theoretically possible only by means of a pair of flumes at each outlet, distributing the supply proportionately between the outlet and the distributary. As this would involve a small drop of distributary supply level at each outlet, it is not really practicable. What is practicable and practised is to assume that the supply level corresponding to a given discharge is constant, and to fit each outlet with a semi-module intended to take off the proper proportion of supply as long as this holds good. As the assumption in practice fails to a varying extent, this latter ideal is not fully possible; but before proceeding to more detailed discussion of the two extremes and of intermediate possibilities, it is necessary to state the theory of the subject that has developed.

THEORY OF GROUP DISTRIBUTION

This theory is based on the idea of proportionality, which is most simply explained in algebraic form.

When giving each outlet a fixed proportion of the supply, if D is the designed supply of a distributary and if nD is the designed supply of the outlet, when $D \pm d$ is the actual supply in a distributary at any moment, then the outlet receives $n\ (D \pm d)$ gal. of water per hour. It is always necessary to provide that the outlet shall receive nD gal., when D gal. are available; but fluctuations can be otherwise dealt with, and if the outlet receives $n\ (D \pm rd)$ gal. of water per hour, the fraction, r, is by definition the "proportionality".

It has been found that the discharge of silt-formed channels in the Panjab varies almost exactly as the $\frac{5}{3}$ power of the depth.[15] If an outlet is a flume in which the discharge varies as the $\frac{3}{2}$ power of the submergence, set with its sill at one-tenth the designed supply depth above the distributary bed, with the width such as to give the correct discharge at the designed supply, the curves for proportional and actual discharge would be tangential; and since the two exponents, $\frac{5}{3}$ and $\frac{3}{2}$, are not very dissimilar, the two curves do not diverge enough to call for practical objection; that is, such a flume is a semi-module giving sensibly proportionate distribution as long as the rating curve of the distributary at the point does not change.

---

[15] Panjab Eng. Congress, 1924, *Paper 89*, Paragraph 5, *et seq.*

It has also been shown mathematically, that an orifice semi-module discharges proportionally when the submergence of its zero point is three-tenths the designed supply depth.[16] However, in this case the two exponents, $\frac{5}{3}$ and $\frac{1}{2}$, are dissimilar, involving opposite curvature.[17] They are only tangential at a point, and away from it they diverge rapidly. Such an orifice is, therefore, "proportional" to a much more limited degree; but that objection is partly overcome by the device developed by Mr. Crump.[18]

Expansion of this mathematical treatment results in the conclusion that, for a flume the submergence of which is the fraction, h, of the supply depth, the proportionality, r, equals $\frac{0.9}{h}$, and for an orifice, r equals $\frac{0.3}{h}$. These formulas give the measure of the degree to which the performance of a semi-module may be made to approach that of a module by setting it deeper.

If the outlets of a length of distributary are fitted with modules, all the variation greater or less than the designed supply reaches the tail, since none of the outlets receives more or less than before. On the other hand, if all the outlets received proportional supplies, the percentage variations at the head and at the tail would be equal. Between these extremes, the use of semi-modules set with proportionality of less than 1.0 magnifies the percentage variation of supply from point to point down the channel; and the mathematical treatment referred to gives the measure of this magnification. The mathematical assumptions must not be overlooked, namely, that the variations are small, and that the outlets are infinite in number and of infinitesimal discharge. Let,

r = the proportionality of the outlets used.
S = the ratio of supply at the head of the reach to that at the tail.
V = the ratio of percentage variation from the designed supply at the head, to the percentage variation resulting at the tail.

Then,

$$V = S^{r-1} \qquad (1)$$

Subject to the assumptions made, it is then possible to calculate and design a system with semi-modules for outlets and furcations of channels with a desired distribution not only of the designed supply, but also of variations from the designed supply.[19] For example, if it is necessary to be prepared for each distributary to admit, in an emergency, an excess of $2\frac{1}{2}\%$ at the head, while up to 20% excess can be carried at the tails without danger, and it is desired to use orifice semi-modules set near bed level (proportionality = 0.3), it will suffice to fit flumes that are set for proportional distribution to tail outlets that receive one-twentieth of the head discharge.

---

[16] Panjab Eng. Congress, 1917, *Paper 36*, p. 137f.
[17] Panjab Eng. Congress, 1923, *Paper 80*, Fig. 1.
[18] Panjab Irrigation Branch, *Paper No. 26*, 1922, "Moduling of Irrigation Channels".
[19] Panjab Irrigation *Nomograms*, Plate 28b.

PRACTICE OF GROUP DISTRIBUTION

If positive modules are used, it suffices to determine the proportion of tail outlets that must be fitted for proportional distribution, to form a sufficient safety valve. For want of a sufficiently satisfactory form of positive module, their use has nowhere yet become general. For some conditions such modules are very desirable, and even the nearest substitutes, are rather unsatisfactory. For example, in the Deccan, weed growth is apt to be so heavy that the supply level in a channel designed to run 3 ft. deep may rise by 3 ft. For other conditions, such modules are useless; as, for example, in inundation canals the supplies of which are not secured by river weirs, but depend on the natural rise and fall of the river. In such canals the best use possible must be made of the widely fluctuating supplies, and, therefore, they must have rather flexible distribution.

Proportional distribution has most adherents in the Panjab at the present time. This is probably in large part because designing is very simple and progress always takes the simpler steps first. It is found satisfactory that these measures effect great improvement on distribution by uncontrolled culvert outlets, and the limitations involved in the assumptions made to simplify theory are not realized. For example, the fluctuations of supply level due to silting cause departures from designed distribution which are greater than need be incurred. It is very easy, although illegal, for cultivators to steal quite a quantity of water by obstructing the distributary below their outlets.

Theoretical ideals being thus unrealizable, an intermediate policy is advocated as the best practicable course. This involves distribution, as nearly constant as possible, to the maximum number of outlets. It is effected by fitting the maximum number of outlets with orifice semi-modules set as deep as possible, confining proportional distribution to tail clusters, and using positive modules only where fluctuations of supply level are too great to allow of compromise.

This policy is naturally the subject of controversy in opposite directions with the advocates of the two ideal policies quoted. Positive moduling is likely to win for some conditions if and when types of modules are developed which compete with semi-modules in certain essentials. They must be cheap; simple and robust of construction and action; sufficiently accurate in discharge; susceptible of adjustment by the engineer, but not of undetected adjustment by the cultivator; they must work with water containing weeds and mud, and must not be deranged by flotsam and jetsam; they should require only a small minimum working head, and should have such range that they are easy to install and will cover actual fluctuations of supply level. It is also desirable that they show the discharge they are set for, and indicate (like the bubbling of a Kennedy outlet) when they are working. When such modules are produced, the field they can best cover can only be found in actual practice; for example, on a Panjab perennial canal the present tail clusters may still prove to be sufficient as safety valves. On the other hand, it may be necessary either to fit a few more outlets for proportional distribution, or retain a larger number with sub-proportional distribution.

For other conditions the final solution is likely to remain that of maintaining nearer proportional distribution, in spite of the realization that the practicable version of the theoretical ideal involves disadvantages to be avoided as far as possible. It is argued that, even on a Panjab perennial canal, fluctuations of supply are such that any approach to rigid distribution endangers the tails; where relatively rigid distribution has been used, this has been disproved for present conditions; and present conditions include much more variation of the supplies received than obtain as metering and moduling are extended. There is also a measure for more effective use of the tail safety valves to be applied.

If a distributary furcates into two minors, one large and the other small, both fitted for sub-proportional distribution, but with equal proportional tail clusters; and if the furcation into minors is also fitted with flumes for proportional distribution, the tail of the larger minor will experience much larger fluctuations than that of the smaller. A method has been shown[20] in which the proportionality of distribution between minors is adjusted so as to give equal fluctuations at both tails, enabling each safety valve to be fully used.

---

[20] Panjab Eng. Congress, 1923, p. 67, Paragraph 84 *et seq.*; and Panjab Irrigation *Nomograms*, Pl. 28*b*.

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
TRANSACTIONS

Paper No. 1811

SURVEYS FOR SWIFT RIVER RESERVOIR OF THE BOSTON METROPOLITAN WATER SUPPLY[1]

BY N. LEROY HAMMOND,[2] M. AM. SOC. C. E.

WITH DISCUSSION BY MESSRS. C. B. BREED, C. L. HALL, FRANK C. SELLNOW, AND N. LEROY HAMMOND.

SYNOPSIS

It is the purpose of this paper to outline, briefly, the methods pursued in the work of real estate and topographic surveys preparatory to the construction of the Swift River Reservoir.

The methods adopted are similar to those used on other large projects, with modifications to meet local conditions of valuation and time. On this work, the aerial survey has been used as a guide to the ground survey rather than in the final preparation of maps.

INTRODUCTION

The water-shed of the new Swift River Reservoir for the Metropolitan Water District of Boston, Mass., embraces an area of about 186 sq. miles. Of this, about 88 sq. miles will be required for reservoir purposes, such an area including land within and extending approximately ½ mile from the proposed reservoir. The area of operation involves practically all of the Towns of Enfield, Greenwich, and Prescott, Mass., and parts of at least seven other towns.

The Swift River is formed by three main branches (see Fig. 1), the East, Middle, and West Branches, so-called. The water-shed of the river is characterized by a series of high ridges running generally parallel in a northerly and southerly direction. The middle and upper part of the main valley is occupied by a number of rugged, mountain-like rocky hills which rise 300 to 500 ft. above the valley floor. The valley is sparsely settled, having had a population in 1920 of about 20 persons per sq. mile, who lived mainly in the small villages in the valleys of the principal streams. Small areas in the valley, however, were devoted to agricultural use, the remainder consisting for the most part of woodland varying from scrub growth to mature timber.

---

[1] Presented at the meeting of the Surveying and Mapping Division, Boston, Mass., October 10, 1929.

[2] Div. Engr., Metropolitan Dist. Water Supply Comm., Enfield, Mass.

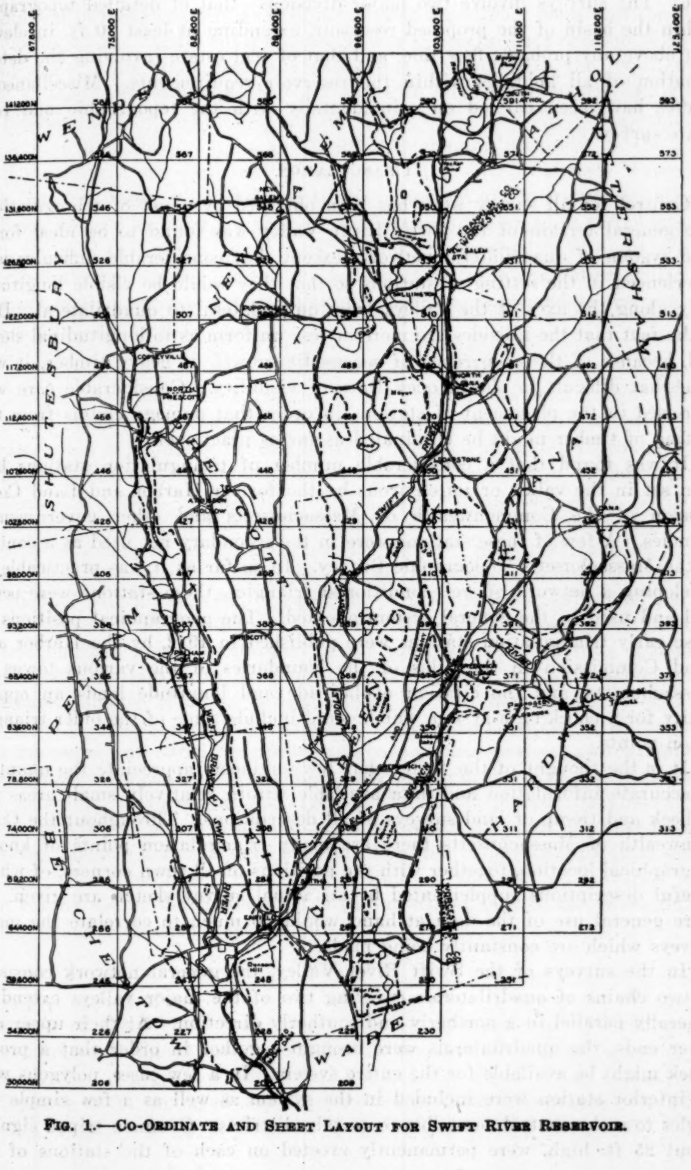

67,800 E
75,000 E
82,200 E
89,400 E
96,600 E
103,800 E
111,000 E
118,200 E
125,400 E
141,200 N
136,400 N
131,600 N
126,800 N
122,000 N
117,200 N
112,400 N
107,600 N
102,800 N
98,000 N
93,200 N
88,400 N
83,600 N
78,800 N
74,000 N
69,200 N
64,400 N
59,600 N
54,800 N
50,000 N
WENDELL
SHUTESBURY
PELHAM
NEW SALEM
ATHOL
PETERSHAM
HARDWICK
GREENWICH
WARE
SWIFT RIVER

FIG. 1.—CO-ORDINATE AND SHEET LAYOUT FOR SWIFT RIVER RESERVOIR.

Work of surveying the large area involved was started in the late fall of 1926. The surveys involve two major divisions—that of detailed topography within the basin of the proposed reservoir, extending at least 10 ft. in elevation above any probable flow line, and that of real estate involving the determination of all holdings within the reservoir requirements. Miscellaneous studies have been carried on coincidentally with the topographic and real estate surveys.

TRIANGULATION

Control for all survey work has been obtained by a net of triangulation. The general terrain of the Swift River Valley was found to be ideal for a good system of quadrilateral network, except that considerable difficulty was experienced in the setting of stations so that they would be visible longitudinally along the axes of the valleys from quadrilateral to quadrilateral. Due to the fact that the hillsides are more or less uniform as to longitudinal slope, and, because of the occurrence of spasmodic growth of heavy timber, it was somewhat difficult to obtain open lines of visibility. Considerable care was exercised in the placement of stations in order that damage claims from the cutting of timber might be eliminated as far as practicable.

It was found that a considerable number of triangulation stations had been set in the valley or its environs by the former Harbor and Land Commission of the Commonwealth of Massachusetts and other governmental agencies. A few of these stations were in the secondary net used as a control of the Massachusetts Topographic Survey. In so far as it was practicable in developing a network of well-conditioned triangles, these stations were occupied and used in the general system adopted. The geographical positions of these early triangulation stations were published in 1912, by the Harbor and Land Commission in the atlas of the boundaries of the various towns of Massachusetts, and the citation of latitude and longitude lends an opportunity for a check on a system which could include some of the older triangulation points.

It is the thought of the writer that few engineers appreciate the quantity of accurate information and data available within relatively small areas for a check and tie-up of land surveys of all descriptions. Throughout the Commonwealth of Massachusetts there are many triangulation points of known geographical location, together with the positions of all town corners, of which careful descriptions supplemented by an actual survey sketch are given. A more general use of the data at hand would do much to correlate the many surveys which are constantly being made.

In the surveys of the Swift River Valley, the general network consisted of two chains of quadrilaterals covering two of the major valleys extending generally parallel in a northerly and southerly direction. At their upper and lower ends, the quadrilaterals were brought together in order that a proper check might be available for the entire system. In a few cases, polygons with an interior station were included in the system as well as a few simple triangles to make control accessible to certain districts. Common tripod signals, about 25 ft. high, were permanently erected on each of the stations of the system. The signals were constructed so as to permit setting the transit directly under the signal mast. Angles were measured by an engineer's transit graduated to minutes. Six repetitions of each angle were made and the error of closure of the figures of the network was generally kept considerably less than 8 sec. The triangulation network was laid out with the requirement that no angle should be less than 27° and with triangles as nearly equilateral as possible.

A base line about 1 mile in length was chained precisely by the slope method, using a standardized Chicago tape with spring balance. From the base line the system was developed to the major net of the triangulation. By utilizing the old existing triangulation points into the network of the system, with the published locations of these stations, it was possible to apply several checks to the new system. From the main network of triangulation, called the primary system, secondary triangulation stations were located as the needs of the detailed survey developed.

CO-ORDINATES

In order to facilitate the location of any point in the area, a plane system of co-ordinates was adopted with axes of reference sufficiently removed from the area so as to eliminate the possibility of minus co-ordinates, all co-ordinates being north and east of the locus of the system. The point of tangency to the surface of the earth of the plane of reference was assumed as an arbitrary point about midway of the area of operation. All vertical co-ordinates are parallel to a tangent to the meridian, 72° 18′ 00″, west of Greenwich, at its intersection with north latitude, 42° 22′ 30″, and all horizontal co-ordinates are parallel with a tangent to the parallel of north latitude, 42° 22′ 30″, at its intersection with the meridian, 72° 18′ 00″. Knowing the geographical position of triangulation points established in the town boundary surveys, it became possible to translate these positions to co-ordinates in the assumed plane of reference, thereby affording several checks on the entire system.[3] For all triangulation points, co-ordinates were carefully calculated and direct azimuth was computed and listed for the convenience of field parties. All surveys of every nature were run on a true azimuth derived from the triangulation system, and individual traverse stations were co-ordinated to the general system. As the triangulation points are established at relatively short distances apart, traverses on true azimuth were run into the area to be surveyed from either the primary or secondary triangulation legs. Practically all traverses have been run by use of the stadia, calculated and closed back to the triangulation system or other checked traverses.

TOPOGRAPHY

A precise line of levels based on the datum plane in use at the Wachusett Reservoir of the Metropolitan Water District was run into the valley and permanent bench-marks were established throughout the area of operation for the control of topographic and future construction data. Only a small part of the area to be surveyed was in open country, and topography was taken entirely by use of the transit and stadia board. Levels have been carried from the precise bench-marks by vertical angles and stadia board, all levels being checked back to either the precise bench-marks or to series of differential lines run by use of the dumpy level. The topographic survey covers about 28 000 acres and sufficient detail is taken for the plotting of contours at 5-ft. intervals, at a scale of 1 in. = 200 ft.

---

[3] See "Geodesy," by George L. Hosmer, M. Am. Soc. C. E., 1919, p. 175.

In view of the fact that real estate survey requirements covered an area of great extent, involving the legal status of a vast number of parcels, the topographic surveys were carried on independent of real estate, except that the traverses established have been used for both the topographic and real estate work. However, the topographic survey included the location of all possible lines of real estate occupation to facilitate the actual real estate survey. The customary features of topographic information have been located, such as highways, watercourses, buildings, and cultural development.

REAL ESTATE

Under its Legislative Enabling Act, the Commission may purchase land or acquire it by right of eminent domain. To date (1931), the latter privilege has not been exercised and property has been acquired by title in fee. It is probable that ultimately a general taking line will be laid down accurately and all property within the area described thereby will be acquired. A considerable number of years elapsed in the preparation and final enactment of the Legislative Act creating the future Swift River Reservoir and most property owners within the area affected were cognizant of the need of their holdings, resulting in the early listing of much of the property for sale. Upon the listing of property its location was approximately determined by the field office, in order that need for its purchase might be determined. The property was then appraised by agents of the Commission, and steps were taken for its purchase.

Prior to the actual transfer of ownership, the title of the property was examined by the Legal Department, and if metes and bounds or acreage appeared obscure or inaccurate, surveys were requested of the Engineering Office. The largest percentage of the holdings in the valley was in either farm or woodland, more or less undeveloped, and it soon became evident that, in most cases, the survey of boundary lines and corners could be made by the stadia method to meet most requirements satisfactorily. It should be borne in mind that the major part of all surveys lies within a probable general taking line, and individual acquisitions will ultimately border other acquisitions, so that considerable economy in field work may be exercised by the proper use of the stadia method of survey. In the actual surveys, a rigid requirement was enforced in that at least two independent locations were required of each property corner where such corners are located as side shots from a main traverse and are not included in the traverse calculations. Such a method serves as a check in plotting.

Plots were made to a scale of 1 in. = 200 ft. All traverses are checked back to themselves or other checked traverses, lines being run on a true azimuth derived in the beginning from the triangulation net. As often happens in undeveloped country, such as that encountered on this work, it has been found that exact property lines are more or less obscure even to the knowledge of the owner and it becomes necessary to examine, carefully, deed and title of abutting owners in order that all property in the vicinity of the surveyed parcel may be satisfied.

Title examination by the Legal Department was not confined to properties under immediate option for acquisition. It embraced all property which might be required. A systematic search of all titles in one of the towns was inaugurated and practically completed within one year. Four copy sheets of title abstracts were reduced by photostatic process to one letter-size sheet and given to the Engineering Office for its use. Thus, the Office had the advantage of being able to keep up to date with the title work and considerable economy resulted in the ability to survey considerable tracts of land in a systematic manner, with the result that to date (1931) about 58 000 acres of real estate have been surveyed in anticipation of request for such survey, or in the ultimate taking of all property.

Together with the preparation of maps for the actual acquisition of real estate by deed, an analysis of the property was made by the Engineering Office; that is, a review of all lines was written, with its relation to the deed held by the owner, as well as to the deed held by the abutting owner. The writer has found that assistants acquire a much broader outlook on real estate survey requirements by being obliged actually to write out their analyses for review by the Legal Department. A suggested description for possible deed conveyance also accompanied the map of the property when it was returned. This requirement has the tendency to make certain that necessary bounds, lines of occupation, distances, etc., appear properly on the map before it is passed to the Legal Department. The requirement of map analysis of deeds, and suggested description appeared at first to be somewhat burdensome to an engineering force, but as such analyses would be required mentally in the process of a survey, the actual writing of the story soon became a comparatively simple matter. It was found extremely valuable not only in the training of the personnel to a proper real estate survey, but also in the placing on record of the reason for each bound, line of occupation, or map line, embodied in the final map.

AERIAL SURVEYS

Fortunately, prior to any actual work of intensive survey in the Swift River Valley, a contract had been made for an aerial survey of that part of the valley covered by the proposed future operation. The contract called for the photographing of about 100 sq. miles of area and the delivery of so-called contact prints. Photography from the aircraft was at an elevation of about 12 000 ft. above sea level, resulting in contact prints of about 7 by 9 in., with an average scale of approximately 1 in. = 1 100 ft. The contract also provided for enlarged prints to a scale of 1 in. = 400 ft. Because of the natural topography of the Swift River Valley with its series of high ridges, the individual contact prints were of areas with elevations varying in many cases as much as 500 ft., or more. With the resultant camera distortion, it was found extremely difficult to determine a suitable control scale for reproduction to a scale of 1 in. = 400 ft.

A variety of scales existed between various points of the same print, due to the considerable differences in ground elevation, with resultant photographic distortion. It was decided that, except by the utilization of considerable ground survey, the enlarged prints were of little comparative value; therefore, few contact prints were enlarged. In fairness to this method of survey, it may be stated that by considerable study for the proper placing of ground control, ratios could have been deduced so that a reasonably accurate mosaic could have been developed. For the purpose to be served it appeared that the contact prints in themselves were sufficiently clear and capable of being pantographed as the ground survey developed a control for the needs of the work. The contact prints were in more or less constant use from the time of their receipt and were extremely valuable in many ways.

Value of Contact Prints.—An inspection of aerial contact prints indicates, by lines of occupation or by variation in timber growth, the possibility of proper location in the field. In the cases of three of the towns affected by the Swift River Reservoir such lines were pantographed to a skeleton map of the town. With these data on the map a perambulation of the town was made in company with its assessors to determine the approximate ownership of lands within its limits. Lines which had no significance as property lines were crossed out in the field and a somewhat crude map of property ownership within the town was developed. Such maps have been of considerable advantage to title examiners in the tracing of ownership titles. The maps were executed simply as an aid to an accurate ground survey of the area.

Considerable use was made of the contact prints in the location of difficult lines and corners. Many times a difference of a few years' growth of timber would not be apparent on the ground, but the line of demarcation was shown on the aerial photograph, and data were taken from the contact print indicating azimuth and distance for a field location of the desired lines or corners. The contact prints were useful during the process of checking the mapping of topographic detail, as any error in field work or of interpretation of notes was quite evident.

CEMETERIES

Within the area needed for reservoir purposes were several cemeteries which required removal outside the reservoir limits. Each cemetery was surveyed, including a location of all monuments and markers, together with the exact location of the indication of all burials. Copies were made of inscriptions on all stones and so far as possible data were gathered as to actual lot ownership. A census of burials in the cemeteries indicated the existence of more than 5 000 interments.

A card index system was developed covering the data of each interment, together with information concerning lot ownership. In many cases the latter data are more or less obscure due to the absence of old town records; but as actual removals are made, public interest will make known much of the missing history of actual interments and of interested parties. To date (1931), four hundred removals have been made to lots outside the reservoir area, owned by parties at interest. No general work of removal has as yet been undertaken. A new cemetery is to be established by the Commission outside the reservoir area for the re-interment of remains to be moved. Such a cemetery has not yet been located.

MAPPING

All record plots of topography and real estate were made to a scale of 1 in. = 200 ft. The entire area was laid out to a network of standard sheets with definite assignments of sheet numbers to areas included within set co-ordinates. The standard sheets have an inside border dimension of 24 by 36 in., with no overlapping of data to adjoining sheets. Original plots of topographical and real estate data were made on the same sheet, consisting of white paper mounted on cloth, standard titles with graphical scale having been printed on the sheets before delivery to the field office. The sheets are oriented with their margins parallel to the co-ordinate lines previously described. Convenient working co-ordinates were carefully laid down on the sheets and as triangulation and traverse stations were calculated with their proper co-ordinate positions, they may be easily plotted. Detailed data were plotted by means of a paper protracter with an arm, or by a drafting machine. As all data were taken on the true azimuth which corresponds to the vertical co-ordinate lines, the matter of plotting was extremely simple.

Two sets of tracings were developed from the record maps, one showing topographic features and the other, real estate holdings. These tracings were of the same size as the record map with no overlap of data, one to the other. Standard titles with graphical scale were printed on cut tracing cloth before use by the field office, thus making for uniformity in the finished result.

With the early assignment of sheet numbers to areas included within definite co-ordinate limits, it became easily possible to control for reference, the location of any data taken. Each triangulation station was given a number which corresponded to that of the record sheet on which it would occur, and all data taken in the field were correlated to their proper sheet numbers. Loose-leaf note-books were used and the data taken were grouped together to form books that had numbers similar to those of the record sheet on which the information might be found. This system simplified the matter of keeping in order the information secured. If a field party was assigned to a definite task, it could (by locating the sheet number of the area in which it was interested) immediately secure all available data taken previously in that particular area and could then reduce its own work to a minimum.

All real estate parcels were numbered first with the key number of the sheet on which the largest part of its area occurred and then with a second number that was one of consecutive numbers on each individual sheet. On an abutting sheet, a portion of the parcel occurring on that sheet took the same number as that on the sheet on which its largest area occurred. Thus, there was no duplication of parcel numbers in the entire area and the location of any parcel of land could be quickly determined if its sheet number had been assigned.

Distances on real estate maps were scaled from the original record map, except in the case of small or village areas where the field survey distance was entered. Areas for the most part were determined by planimeter, or were calculated by scaling from the record map. Each sheet had an absolute theoretical area included within its limit, and in the determination of individual areas it was made certain that the sum total of all areas equalled the theoretical area of the sheet.

PHOTOGRAPHY

All photographic work, including the developing and printing of photographs was executed by the field office. Photographs were of standard size (5 in. by 7 in.) and for record purposes they were mounted on white cloth. Photographs were made of all buildings or other objects about which information was desired for record of real estate value. Each cemetery lot occurring within the area subject to removal was photographed to make a record of monuments and markers in existence in each individual lot.

Photographs were numbered consecutively as taken, but, for purposes of identification, a small blackboard was placed in the view. On the board the name of the property owner and the date and number of the photograph were written in white chalk. The record sheet number on which the survey of the building appeared was also shown on the board, thus tying the location down to a comparatively small area. A small white or black arrow, depending on the ground color, was placed in front of the board in the direction of the north point as determined by a pocket compass. A small sketch was drawn in the lower left-hand corner of the negative, showing the position of the camera and objects in the photograph, with relation to highways or other general features in the vicinity of the objects photographed.

PERSONNEL

All the work of surveying and mapping, as far as the Swift River Valley is concerned, was undertaken from one field office with a personnel varying from a maximum of forty-five men to an existing field engineering force of twenty men. The work is being done by the Metropolitan District Water Supply Commission, with Headquarters Engineering Offices in Boston, Mass., all work being under the direction of Frank E. Winsor, M. Am. Soc. C. E., as Chief Engineer.

DISCUSSION

C. B. Breed,[4] M. Am. Soc. C. E.—This paper describes a surveying project unique in its performance, although quite common in its general aspects. It involves procuring a great deal of detail, the accuracy of which is well controlled where needed, yet "sketchy", where a low degree of precision is sufficient. It is a problem in surveying economics. In such problems good judgment based upon considerable experience is required in order that the survey-dollar may be reflected properly in the resulting data or plans. It is very easy to spend too much money on unimportant parts of the survey and too little on essentials. In fact, the opinion of well qualified surveyors may properly differ as to the most practical assembly of surveying methods to apply in order to reach a balanced economic result.

While the opinion of the speaker differs somewhat from that of the author in a few instances, it should be borne in mind that Mr. Hammond was on the site and that the speaker has not been on this project; the author has seen conditions at first hand while the speaker has viewed them through Mr. Hammond's lucid description of the conditions he encountered.

The control of the entire survey appears to have been logically laid out. It was properly joined with the triangulation stations established by the previous Massachusetts State Commissions. Mr. Hammond's comment that many valuable data of this kind are available and in usable form cannot be emphasized too forcefully. Many surveyors would gladly make use of such data, but, unfortunately, private projects too often have to be started before the surveying party has had time to connect it with the Government base of levels or triangulation control. In State work of the character described by Mr. Hammond, it is more readily done than in private projects, because more time is usually permitted for preliminaries.

Accuracy of Instrument Work.—It is somewhat surprising to learn that a 1′ transit was used in the triangulation system. The idea of using a 10″ transit may be abandoned because of its weight, but a 30″ or a 20″ transit of the same weight as a 1′ instrument may readily be obtained. By using a 20″ transit, the adopted precision could have been reached more quickly; or, with the same number of repetitions of angles, a higher precision could have been reached for the same cost.

It is so easy to obtain "closures" of much less than 8″ that it seems a pity that such a well-planned system as that described by the author, could not have been surveyed with greater precision. The time required for reconnaissance and signal building is the same in either case. The additional few minutes required to obtain the angle within 2 or 3″ is negligible. Had this been done, the results obviously would be of greater value to future surveys in that general district.

It seems a little unusual to measure the base lines with an ordinary steel tape and to determine and apply temperature corrections throughout the work, when an invar tape could have been used and the temperature corrections omitted. Of course, the invar tape is more brittle and requires more care in handling in rough country. It seems doubtful, however, whether lines measured by the ordinary tape as described would give a better base than could have been obtained by using the old State triangulation distances or by computing bases from the geographical position of those old State monuments which were properly joined with the system.

---

[4] Prof. of Railway and Highway Eng., Mass. Inst. Tech.; Cons. Engr., Boston, Mass.

The application of the plane co-ordinate system for the location of all points is especially adapted to the ready replacing of lost points in a district where so much cutting of timber and general change in immediate surface of the terrain is to be made.

Transit and Stadia Versus Plane-Table.—While the control of the topographical map by triangulation and by precise levels is excellent, the speaker questions the advisability of using transit and stadia for detail topography. In the open country the plane-table has distinct advantages over the stadia and transit methods. Even in wooded countries, the speaker prefers the plane-table for economy and for other reasons.

The number of points that have to be located is much smaller; the shape of the terrain ought always to be sketched while it is before the eyes, because no amount of experience in office plotting and in drawing contours based on many plotted points in the office can represent so faithfully the shape of the ground, as the work of a good topographer who sketches the contours in the field and governs their accuracy by the position and elevation of a few intelligently selected points; and, lastly, the cost is less for a given accuracy of plan.

The Cruising and Sketching Method.—In dense woods, such as those described by Mr. Hammond, the speaker has found that the following cruising and sketching method gives satisfactory results for much of the topographic work of the character he has had to produce. Stadia lines are run by plane-table and alidade through clearings in the woods at intervals of, say, 500 to 5 000 ft., preferably parallel to each other; and contours are sketched on either side of these cleared lines for a short distance into the dense woods. The topographer then "cruises" the country between these stadia lines by running compass lines (usually at right angles to the stadia lines) at, say, 100 to 300-ft. intervals. He starts from one stadia line with the elevation determined by stadia level and runs at right angles through the woods to the other line. He sketches contours on a small hand-sketching board as he goes, using a hand level or a clinometer for determining elevations and for checking these elevations at the other stadia line ahead. Then he returns on the next range to the first stadia line, obtaining all the topography over the route which he has "cruised".

Those data are transcribed directly on to the plane-table sheet which has been left on the first cleared stadia line. He then proceeds to the next pair of ranges, running south, say, on one and back north on the other, checking out at each time on his levels, on his distance, and on his position up and down on the stadia line as he reaches the clearing where that line is run. This method gives an independent control on all the topography. It provides a check on all the cruising work, and it is the most economical method of which the speaker knows for obtaining fairly accurate contours in heavily wooded districts. The transcription of the contours drawn on the sketching board on to the plane-table discloses any apparent error, which should be corrected immediately in the field.

Cemetery Surveys.—The survey of cemeteries also can be readily accomplished by the plane-table, using taped measurements, in the following economical manner. First, select a system of triangulation, the vertices of the triangles being the outer corners of stones marking cemetery lot corners. These triangles can be, say, 400 to 800 ft. on a side. Measure these distances with a tape. Plot them on the plane-table sheets by use of the beam compass and scale. Use these points for the control of the detailed survey to follow.

The plane-table is set up at the corner of one of these stone bounds and oriented by sighting on another triangulation point. Several other existing, easily recognized, monuments are then cut in. The plane-table is next taken to another triangulation point and these same points are cut in and thus located on the plan. This does not require sending a man to the monument for the sight.

An iron rod three times the size of an ordinary lining pin is driven in the ground at the plane-table station. This rod is driven through the tape ring. The tapeman measures the distance from this pin to the various lot corner markers and the plane-table cuts in the direction with the alidade. He then scales on each line the distance called off by the tapeman, draws the boundary lines, and records the lot, its serial number, and the family name. Two men are all that is needed in this field party. They can survey a great number of lots in a day in this manner. It is well not to extend the measurements from the plane-table beyond, say, 80 to 100 ft. Then, the plane-table should be moved to another point which has been located from the last point. At any time the plane tabler can cut in his postion from the many monuments which appear on his sheet, and thus check his position. In fact, it is to his advantage when he can cut in three or four characteristic monuments within his sheet limits.

The next step is to measure with the tape the four sides of each lot and the two diagonals. These are recorded to the nearest 0.1 ft. on the plane-table sheet, the scale of which is 1 in. = 20 ft.

Office Work.—On a rainy day the individual lots are plotted on large cards forming a card catalog. The upper half of the card is cross-sectioned and the lower part is in lines and columns. In the upper part the lot is plotted to the scale of 4 ft. to the inch, or 8 ft. to the inch. These are plotted from the four dimensions and the two diagonals which were recorded in the field. In the lower portion of the card the tabulation of names, dates of birth and death, etc., are recorded. These are copied from the cemetery records.

The card is then taken into the field, and at each individual lot the headstones, footstones, and monuments are located by tape measurements and ranges. Each grave is identified from the list of interments in the lower portion of the card and from the wording of markers, and if there are interments that cannot be found that are indicated in the list, or graves that appear on the lot which are not in the list, further investigation is made to determine who the the person is that is buried at that spot or whether or not there is any interment.

The result of this kind of a survey is a plan of the entire cemetery on the scale of 1 in. = 20 ft. which, of course, can be reduced to any desired scale. It shows all the roads, paths, lots, and the water system. This is supplemented by a card catalog plan of every individual lot that shows every physical thing upon it. It records each interment by a number in the tabulation and by an actual drawing of the shape of the lot on the upper portion of the card, each interment being given a number corresponding to the number in the tabulation.

The advantages of this method are: (1) It is controlled by the measured triangulation, and a detail of every lot is drawn while that lot is before one's eyes; (2) it requires only two men in the field party; (3) it is economical; and (4) it presents plenty of work for rainy weather as well as for fair days.

The Aerial Survey.—In the property line surveys for real estate plans, aerial maps (or contact prints) were used on the Swift River surveys for two appropriate purposes, namely (*a*), reconnoissance; and (*b*), filling in, or identifying, details.

These contact prints form a good check on the position of physical features located by the stadia and transit notes when they are plotted in the office, in that the finished map should (except for distortion due to difference in elevation) appear like the contact prints in so far as fence lines and the location of buildings, roads, and other details of flat topography, are concerned. Of course, they do not form any check on contour locations except that where a steep face of a ledge may occur or a brook is located the contact prints should give valuable data.

In aerial work of this kind, the speaker has found it advantageous to place cheese-cloth markers at points which he wanted to identify readily in the contact prints, where the object was one that probably would not show up well on the print. These places can readily be marked by 3-ft. strips of cheese-cloth laid at right angles over these points, or in the form of a circle around the points. The little white crosses or circles show up clearly in the photographs.

Identifying Property Lines.—The analysis of evidence of property lines is an excellent practice. In wood lots these should properly show the position of blazes marking property limits; and, where different blazed lines are encountered on the same general property line, the apparent age of the blazes should be determined by cutting into the trees and counting the annual rings, and these facts should be noted on the analysis so that the line adopted may be supported by the strongest evidence found.

While the edge of timber clearings, which usually show up clearly in contact prints, is some evidence of property lines, it should be supported by other evidence as a rule, because it is not uncommon to find that when timber has been cleared an encroachment on adjacent property has been carelessly made, for which damage may or may not have been adjusted.

As stated at the beginning of this discussion, the problem is one of surveying economics. The speaker has tried to keep in mind the cost of the surveys and to suggest such modifications, as, in his judgment, would give either better results for a given outlay or would have given as accurate and complete results for less money.

C. L. HALL,[5] M. AM. SOC. C. E. (by letter).—It is noted that Mr. Hammond adopted the equivalent of the Army grid system of co-ordinates for his control, yet retained true azimuths. The writer's experience with the use of a system of rectilinear co-ordinates for surveys of small areas has led him to believe that it is much simpler to use grid azimuth when rectilinear co-ordinates are available. In this system, the true azimuth of any line at the point of tangency of the plane of projection becomes the grid azimuth for all lines drawn parallel thereto everywhere on the map; and hence the angle which any straight line on the earth's surface makes with grid north is constant for the entire length of the line. It is very little trouble to calculate grid azimuths between points the rectilinear co-ordinates of which are known because the cosine of the angle between the line joining the two points, and grid north, can be determined at once by noting the proper relation between the differences of their x and y- co-ordinates, respectively.

No advantage appears to accrue from the use of true azimuths, except possibly that lines in legal documents can be more readily described in ordinary surveyor's language. However, no one expects exact accuracy in the azimuths of boundary lines. Admittedly, differences in true azimuth over an area 11 miles from east to west will not be great, but if no grid azimuth is established field surveyors will be unnecessarily annoyed whenever they run control traverses.

Under "Aerial Surveys," Mr. Hammond indicates that he could see no use in aerial photographs enlarged from 1:13 200 to 1:4 800. This agrees with the writer's experience. For general checking, an enlarged aerial photograph may sometimes be more convenient (although seldom more economical) than a pantographed enlargement; but as enlargement always reduces clarity, it will never be as satisfactory as the original print, in studying the country. If surveys are being made instrumentally on a scale of 1:2 400, neither an aerial photograph nor an enlargement thereof will appreciably reduce the cost of field topography, except as an aerial view permits better planning. The scale of the aerial view, thus used, is relatively unimportant.

FRANK C. SELLNOW,[6] ASSOC. M. SOC. C. E. (by letter).—Although technical literature is quite replete with the procedure that has been successfully followed in planning and constructing water supply and other major hydraulic undertakings, little seems to have been published on the methods of surveying, mapping, and acquiring the real estate of reservoir sites as these methods have tended to become standardized. Perhaps it has been thought that surveying is too old an art to require further exposition as applied to reservoir sites. That this is a mistake is evidenced by the fact that all too often in the past—through too great a subordination of this work to other seemingly more important features of the project—no well-considered program was mapped out, the data obtained were not collected with an economy of effort, and the results did not meet expectations or fulfill requirements.

---

[5] Lt.-Col., Corps of Engrs., U. S. A., Cincinnati, Ohio.

[6] Div. Engr., State Water Policy Comm., Newark, N. J.

Reservoir sites in the eastern part of the United States have many common characteristics. All, naturally, are surrounded by relief which will range from hills to mountains; the valleys are usually fertile land, cultivated, and of fairly high value, and there will be some utilities, both public and private. Community life will usually center in one or more villages in which, also, are occasionally found industries requiring large quantities of water. The upland will usually be wooded and, compared with the bottom-lands, of low value. The area within the taking line will seldom be less than 5 sq. miles, or more than 100 sq. miles, depending upon the gradient of the stream and the size of the development. With this similarity in the physical features, standardization of surveying and mapping methods, with such slight modifications as are necessary to meet local conditions, would seem to be invited.

Due, perhaps, to the low average value of the land, maps, if any exist, are almost invariably of low-grade accuracy, or are published on too small a scale to be of value in the design of water-works structures, relocation of utilities, or the acquisition of the real estate. The topographical maps of the United States Geological Survey, supplemented by reconnaissance, will usually suffice to determine the feasibility of the project, but before detailed plans can be made a topographic map, preferably to a scale of not less than 1 in. equals 200 ft., must be available. Enlargements of the topographical maps of the United States Geological Survey will only have the effect of accentuating the inaccuracies inherent in drafting these or any small scale maps and will not produce a map which will fulfill requirements.

Simultaneously with the topographical survey, or at least well in advance of the flooding of the reservoir, a cadastral survey must be made for the purpose of acquiring all the real estate. Fortunately, the locus approaches the ideal for carrying out both these surveys with a high order of accuracy and, at the same time, at very reasonable costs. The manner in which the Swift River Reservoir surveys were conducted demonstrates an orderly and efficient procedure for collecting the necessary data. It would be interesting and useful information if the cost of the Swift River Reservoir survey and the surveys of other reservoirs—classified, say, into triangulation, traverse, real estate, and topography—were published. On a unit of area basis, there should be no great variance in cost.

If the terrain is not too heavily wooded, it can be stated confidently that the control for both surveys can be established more economically by triangulation than by precise traverse. Considerable thought and study will have to be devoted to locating the triangulation stations to obtain intervisibility, to avoid excessive cutting, and to keep within a 30° angular limitation. In the reconnaissance, the use of an aerial map should be a very useful adjunct. To avoid the displacement or destruction of stations by utility relocations or construction activities, they should be located at least 50 ft. higher in elevation than the future flow line. They should be preserved by monumentation and properly referenced, so that their usefulness will be continued should it be found desirable in later years to increase the real estate holdings or to make improvements. The surest reference for the control system is to tie, by traverse or triangulation, the origin of the local system of plane co-ordinates to one or more of the control stations on the North American datum. This will have the advantage not only of perpetuating the survey, but of eliminating all astronomical observations, and making it possible to orient and locate properly the survey data on any Government or State maps.

In establishing the triangulation control for the Swift River Reservoir survey, the author was fortunate in that the township corners were marked and their geographical positions known. These geographical positions, transposed into rectangular co-ordinates, furnished, as stated in the paper, several checks to the new system. This condition of frequently located monuments, the geographical co-ordinates of which are known, will not ordinarily obtain in most of the Eastern States, and without these checks a greater refinement in the measurement of the angles would be required to secure the same accuracy. Two sets of pointings—each consisting of six measures of the angle and six measures of the explement, with an instrument graduated to 20″—would be the least number of angular measurements deemed necessary for this class of work.

Although the several steps in establishing the control and in making the cadastral and topographical surveys of reservoir sites may be performed in much the same way, the degree of required accuracy will depend upon valuations as between two reservoir sites or between different parts of the same reservoir. In general, the higher the valuations the greater the accuracy demanded. In any event, the standard adopted for the cadastral work will meet all the requirements of the topographical map. It is believed that the following specifications for accuracy for maps plotted to a scale not greater than 200 ft. to the inch, will meet the most exacting requirements, whether distances are to be mathematically reduced or scaled: (*a*) Triangulation, 1 part in 20 000; (*b*) traverses, the stations of which are to be used for locating property corners, or as the starting and closing points of stadia surveys, 1 part in 10 000; and (*c*) horizontal distances between well-defined points as located by stadia, to scale accurately within 3 ft.

Engineers, for the most part, now recognize that aerial photography will aid and supplement the ground survey of any large tracts. The author made commendable use of this comparatively new art, but it is questionable whether he availed himself of its full possibilities. The aerial survey made was uncontrolled. Control could have been furnished by the valuation maps of the Boston and Albany Railroad Company, providing they contain the required data; if not, then a moderate amount of ground control would have been necessary, the expense of which, it seems safe to state, would have been more than offset by the benefits. It is believed that controlled mosaics to a scale of 200 ft. to the inch, on sheets to correspond to the co-ordinate layout, could have been obtained at not more than four times the cost of the contact prints. Mosaics in this form would have obviated pantographing and furnished, by direct comparison, a check on all property surveys. As most aerial survey companies will guarantee an accuracy within 0.25% in distances of 5 000 ft., or more, topographical features within areas surrounded by natural monuments (such as river, roads, etc., located by ground method), could have been transferred to the line map with the assurance that their positions would have been at least within the limitations of the stadia method. Contours would have to be obtained by the usual field methods.

N. LeRoy Hammond,[7] M. Am. Soc. C. E. (by letter).—For surveys of the extent covered by the Swift River Reservoir Project, Professor Breed has well expressed the necessity of properly applying the yardstick to methods to be followed when he writes, "it is very easy to spend too much money on unimportant parts of the survey and too little on essentials."

Professor Breed correctly states that greater accuracy in triangulation could have been obtained by a 30″ or 20″ transit, rather than by the 1′ transit actually used. On this particular project, instruments were selected which would be satisfactory for all phases of the entire project, including all necessary surveys and construction layouts. Because of the numerous checks afforded by existing State triangulation points, it was decided that the 1′ transit for triangulation would give satisfactory results.

While distances were computed in the triangulation system from the geographical positions of the State monuments used, it was deemed necessary to have an independent base line measured, in order to make certain that no error had been made in the selection of the older points. Had entire dependence been placed on the base-line measurement, it is probably true that the use of an invar tape would have been better.

The writer is in complete agreement with Professor Breed as to the possible use of the plane-table in filling in topographic features, and he believes that much of the Swift River Reservoir work could have been done with economy by the plane-table about as described by him. The writer has completed many surveys by this method, but feels that the make-up of personnel has much to do with its economical application. It is to be regretted that more engineers are not properly acquainted with the plane-table. In one survey with which the writer was connected, the party had started its detail by ordinary stadia methods when it was changed to the plane-table. After a week, the party chief was convinced that better progress could be made by the previous stadia method, but after a month he and his party were completely converted to the plane-table method over the ordinary stadia methods for the particular problem in hand.

Colonel Hall has indicated that the use of the true azimuth in a system of plane co-ordinates would involve considerable work for adjustment from point to point. The writer's statement in his paper that all lines were run on a true azimuth was incorrect, as lines were actually run from the grid azimuth derived from the true azimuth at the origin of the grid system. Colonel Hall has very clearly stated the advantage of this system.

In his discussion Mr. Sellnow has shown a broad vision of the general problem of reservoir survey. He has emphasized the necessity of properly tying in the origin of the local system of plane co-ordinates in order that the entire system may be referenced at any time to future use and extended mapping. In the Swift River Reservoir work, tie was made by a citation of the geographical position of the origin of the system.

---

[7] Div. Engr., Metropolitan Dist. Water Supply Comm., Enfield, Mass.

If the control is well established with an accuracy of 1 part in 20 000, as cited by Mr. Sellnow, it appears to the writer that no hard-and-fast general rules can be fixed as to the accuracy to be secured in the details of the survey, but that value of real estate, capacity of reservoir basin, construction layout details, and other local factors influence largely the degree of precision which may be deemed necessary and sufficient for each survey, and that the engineer must decide upon his degree of precision as local conditions warrant, in order that the desired results may be secured with the greatest economy.

The aerial surveys of the Swift River Reservoir area were run in 1926, and it is probable that considerable advance has been made in this art since that time. The writer found variations as great as 25% in the scale of different portions of individual contact prints due to difference in elevation of ground surfaces and probably to a tilting of the camera by gusts of wind, etc. An effort was made to secure enlargements of contact prints to a scale of 1 in. equals 400 ft., but it was deemed more economical to make enlargements by means of a pantograph from the contact prints direct, and use the results as a basis of comparison with ground surveys. This decision may have been brought about by a lack of knowledge of proper methods to be adopted in enlarging contact prints.

While companies engaged in aerial photography may guarantee an accuracy, as stated by Mr. Sellnow, it appears to the writer that this guaranty must depend upon the ground control, and, in cases similar to the Swift River Reservoir area, when such ground control is secured, the use of the aerial survey has passed and necessary ground surveys have become well established. The writer, however, is still of the opinion that the aerial survey is a decided asset to the work of the ground survey and its expense is well justified in many ways.

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
TRANSACTIONS

Paper No. 1812

RUN-OFF—RATIONAL RUN-OFF FORMULAS[1]

BY R. L. GREGORY[2] AND C. E. ARNOLD[3] ASSOCIATE MEMBERS, AM. SOC. C. E.

WITH DISCUSSION BY MESSRS. LEROY K. SHERMAN, FRANCIS BATES, JOHN W. RAYMOND, JR., REGINALD A. RYVES, G. S. TAPLEY, W. I. HICKS, CARL H. REEVES, JOHN M. KEMMERER, LEONARD L. LONGACRE, G. H. HICKOX, DONALD M. BAKER, ALBERT R. ARLEDGE, CLARENCE S. JARVIS, C. E. GRUNSKY, RALPH W. POWELL, MERRILL M. BERNARD, CHARLES H. LEE, AND R. L. GREGORY AND C. E. ARNOLD.

SYNOPSIS

A discussion of methods of estimating storm-water run-off, statements, conclusions and comparisons, weighing of errors, advantages and disadvantages, and suggestions for improvement, form an important part of this paper.

Logically, the paper is divided into two parts. Part I contains an analysis of the factors involved; it presents conclusions, and proposes changes in the present methods of estimating run-off. Part II contains a series of derivations, introducing and advocating new factors and formulas. This part of the paper also contains practical examples and suggestions for the guidance of the designer.

PART I.—ANALYSIS OF VARIABLE FACTORS

INTRODUCTION

Many engineers are engaged in the design of culverts, bridges, storm drains, dams, and various structures, in which the first, and probably the most difficult, problem encountered is that of estimating the maximum storm, or flood, water flow to be expected from a given drainage area.

Being indeterminate, the problem is made more difficult by lack of observation and the present uncertainties in available data for the determination of values for the many varying factors that contribute to flow. Individual judgment must always play a large part in selecting values for these factors, and, consequently, some variation in results may be expected, regardless of the method used.

---

[1] Published in April, 1931, *Proceedings*.

[2] Chf. Insp., Los Angeles County, Storm Drain Div., Los Angeles, Calif.

[3] Los Angeles, Calif.

Nevertheless, it should be the duty of all engineers engaged in the estimation of run-off, not only to strive for the more accurate determination of values for the various factors involved, but also for better methods of applying values when determined.

The profession owes much to the engineers who evolved the various empirical formulas, which were the natural fore-runners of the more recent, so-called, rational method. One purpose of this paper, however, is to show why the use of these empirical formulas is not now to be recommended.

The rational method recognizes the element of time in the computation of storm flow and, although there is much room for difference in assumptions, causing disagreement in results, it is considered by engineers to offer the most reasonable approach to accuracy. This method in some form is, therefore, rapidly supplanting the use of empirical formulas. The present users of the rational method may be placed in two main classes: First, those who follow the detail method as outlined subsequently; and second, those who attempt to arrive at a time of concentration for the water-shed either by a comparison with tests on areas of somewhat similar characteristics, or, by roughly estimating the average velocity of the main channel of flow.

The detail method is slow and tedious, and the writers believe it to be erroneous under the customary assumptions. On the other hand, in attempting to determine average velocities for the purpose of obtaining the time of concentration, and, finally, the intensity of rainfall, by either dividing a large water-shed into only a few parts, or, by comparing them with other areas of somewhat similar characteristics, results may be found to be no more accurate than those obtained by the empirical formulas.

There is no uniformity in the conception of methods for deriving the various factors involved in empirical formulas, and in assumptions made in the present use of the rational method. If a detail process had existed, which could have been used as a criterion of accuracy, the writers would have needed on much of their work, a rational formula for obtaining a reasonable result for a water-shed, without "tabling" the area in detail. Many times the engineer must know the probable maximum run-off at one or more points on a water-shed, as, for instance, in designing a bridge, culvert, or dam, or a section of drain with large tributary area. There are also many occasions for the use of such formulas for preliminary and comparative estimates when the engineer is not justified in taking the time and going to the expense of making a long, detailed calculation of the run-off. There should be a rational method of arriving at a close approximation of the run-off from a given area by a more direct use of the several factors involved. The writers, therefore, have made the following analysis, resulting in formulas which may be used for run-off computations. It is also hoped that the paper will present a clearer and more uniform conception of the rational method; and that the inadequacy of the empirical formulas for general run-off computations will be made evident.

It should be stated here that no attempt has been made to set up a "mill" into which the various factors could be thrown, a crank turned, and a perfect result obtained. The computation of run-off will always require the best judgment of experienced engineers, and the data in this paper are submitted only with the hope that their intelligent use may be a time-saver in guiding the judgment of the engineer to a rational result.

NOTATION

Throughout this work the writers have endeavored to concentrate on original analysis, using previously published information only when necessary for clarity. The reader, therefore, is assumed to have a certain familiarity with existing methods for computing run-off. A complete list of the notation introduced in Part II of this paper is arranged in Appendix I. The notation for Part I occurs in the body of the paper as the analysis progresses.

EMPIRICAL FORMULAS

Various empirical formulas for estimating storm flow have been presented from time to time, to the Engineering Profession, and not a few of the leading cities use one of these formulas in estimating storm flow. Many engineers also use one or more of the formulas for estimating, checking, and comparing with local data, the run-off for the larger flood-control projects.

Presentation of the best known formulas by Metcalf and Eddy includes a clear account of their derivation, as well as a comparison of results obtained by their application. A wide difference in results is shown,[4] but this is not surprising when it is noted that no one formula, in the process of its derivation, includes all the varying factors contributing to the result sought.

It is not necessary to dwell at length upon these formulas, but it should be interesting to observe that, because of three considerations, the very nature of their construction precludes any possibility of allowing for the difference in flow, on areas of the same acreage and slope of the main channel of flow. These considerations are:

(1) Shape of area both as to external boundary and internal arrangement of drainage courses leading to the point of concentration;
(2) Condition and shape of the main channel of flow; and
(3) In addition to Considerations (1) and (2), the initial or inlet time cannot be allowed to vary in the empirical formulas, due to their construction.

A better conception of the effect of these three considerations, and others, is given subsequently in this paper. (See, "Inadequacy of Empirical Formulas".)

THE RATIONAL METHOD

The Fundamental Relation.—The relation between run-off and rainfall is, of course, correctly stated by the expression,

$$Q = C\,i\,A \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

in which, Q, C, i, and A are as defined in Appendix I.

Attempts by engineers to solve this equation have evolved the so-called rational method. This method is based on the assumption that the maximum rate of run-off from any water-shed for a given rainfall intensity occurs when all parts of the area are contributing. To fulfill this condition, it follows that the rain must continue a length of time equal to, or exceeding, the time it takes water to travel from the most remote portion of the water-shed to its outlet. This is known as the time of concentration for the water-shed.

---

[4] "American Sewerage Practice," by Metcalf and Eddy, Vol. I, First Edition, p. 235.

Through the study of rainfall records, certain relations have been found to exist between intensity and time. It has been generally shown that a smaller average intensity may be expected for a longer period of time than for a short one. Probably the most common form of this relation in use for estimating run-off is,

$$i = \frac{K}{t^e} \qquad (2)$$

in which, K is a constant, and i is the expected average intensity for the time period, t.

Equation (1) is, therefore, more a statement of a fundamental relation than a formula. The factor, i, represents not merely a constant to be readily obtained, but depends for its value on the time of concentration of the water-shed, which, in turn, is determined by the average velocity of the water in traveling from the most remote section of the area to its outlet. This velocity is necessarily a function not only of the slope, shape, and condition, of the channel through which the water must run in traversing the water-shed, but also of the quantity of water itself, which varies due to the shape of the area and its manner of concentration.

Rainfall Curves.—In this paper the reader is assumed to be familiar with the general methods used in making rainfall curves. Selection of the proper curve is controlled not only by the economic features of the particular problem, but also by the locality of the water-shed. This part of run-off study, however, is not intangible, nor indeterminate, since the curves may be made from actual rainfall records. Many engineers avail themselves of the valuable records kept by the United States Weather Bureau in the particular section in which their problems occur.

Although the rainfall curves, as used, are based on tangible rainfall records, there is much room for the play of individual judgment in their making. While it is recognized that accurate records may be had for a definite period of years, it is also true that for another equal period of years, even in the same locality, the intensities for similar time periods may differ widely. Two engineers, working separately on the same rainfall records for a given period of years, may draw different types of curves, since it is difficult to determine whether a given point from observed data belongs on the curve representing the values for a certain expected frequency, or on a curve of a different frequency. Wide differences in the types of rainfall curves in use may be attributed to this latitude afforded the judgment.

Use of Rainfall Curve.—More important to this work is the intelligent use of the rainfall curve, which requires a knowledge of its make-up. It will be assumed herein that the ordinate of the curve represents the average rainfall intensity, i, expressed in inches per hour, that may be expected to last for the corresponding time, t, represented by the abscissa; or, stating it in another way, a water-shed with a time of concentration, t, may be expected to receive a rainfall with an average intensity, for the entire time, of i in. per hour.

Thus far, no mention has been made of how that part of the storm, which is expected to last for the time, t, is supposed to conduct itself. It may have an average intensity of i in. per hour, yet begin intensely and end lightly;

it may be intense in the middle of the time period and light at either end; or, it may begin lightly and end intensely.

In their study of rainfall records, the writers and others have found that an individual storm may begin or end in any manner, no definite general tendency as regards the order of greatest intensity having been observed.

The rainfall curves generally in use (including the construction of the various expected frequency periods, as 10, 15, 25, etc., years), are built on the basis of an average intensity for a given period, such intensity being determined from records of storms of various types as regards the relative intensity within a given time period. It is clear, therefore, that because a curve may have the form of Equation (2), there is no reason to assume that an individual storm will follow the law of this curve. In using curves so constructed the type of storm for a given time of concentration should be considered as independent of the law of the curves. To construct curves, the use of which is to imply that the type of storm to be expected for a given time of concentration is one whose variation in intensity for that time follows a definite law, would require the selection of data for the various time intervals from storms the rates of precipitation of which actually followed the particular law. The probable frequency periods would depend necessarily on the type of storm selected. This would be impracticable; and it would seem that the method now followed in making curves offers a rational attack, since the type of storm actually used approaches one of uniform intensity for the time of concentration.

This conception should be concurred in by those who pursue the detail process of the rational method in storm drain estimates, in which it is the evident intention to assume that the intensity is uniform over the entire watershed throughout the time of concentration. Such assumption, no doubt, would conform to that of C. E. Ramser, M. Am. Soc. C. E., of the U. S. Bureau of Public Roads, who states;[5]

> "The maximum rate of run-off, therefore, would result from a rainfall of maximum uniform intensity, continuing for a time equal to or exceeding the time of concentration."

The assumption of a uniform, or constant, intensity for a given time of concentration, affording the most feasible means of building rainfall curves to forecast the probable intensities and frequencies of storms, does not, of course, preclude the possible occurrence of precipitation varying in intensity within a given period of time; but a storm, following any definite law (in regard to the relative intensity within the time period) other than that of the average storm used in constructing the rainfall curves, may be expected to have a different frequency of occurrence. Hence, the 10-year, 15-year, 25-year, etc., frequency periods of the curves, as ordinarily constructed, cannot properly be used for forecasting storms of varying types.

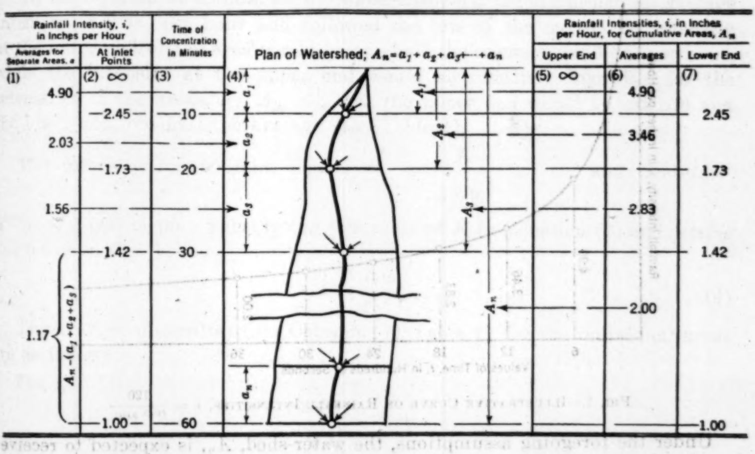
The Detail Method.—In solving by the detail method, it is first necessary to determine the time it takes the water to flow from the most remote portion of the first small area, or nucleus, to its point of inlet to the main channel of flow. For this time the intensity, *i*, is determined from the rainfall curve in use, the value, Q, is calculated from Equation (1) and its time of flow to the next inlet point is determined. The time of concentration for the total area contributing at or above this second inlet point equals the concentration time for the first area plus the time of flow in the drain to the second inlet point. The rate of run-off, Q, is then calculated for the second inlet point, using the new time which gives a decreased intensity for the entire area contributing at, or above, the second inlet point, etc., at all inlet points until the entire water-shed is traversed.

---

[5] "Run-Off from Small Agricultural Areas," reprinted from *Journal of Agricultural Research*, Vol. 34, No. 9, p. 798.

Merely for the purpose of illustrating the use of the rainfall curve as generally practiced in the detail method, a simple water-shed is chosen. In Table 1 the points enclosed with circles represent inlet points on the main drainage channel, which is designated by the heavy line.

TABLE 1.—COMPUTATION TABLE FOR STUDY OF RAINFALL INTENSITY BY MEANS OF RAINFALL CURVE


Rainfall Intensity, i, in Inches per Hour
Averages for Separate Areas, a
At Inlet Points
Time of Concentration in Minutes
Plan of Watershed; $A_n = a_1 + a_2 + a_3 + \cdots + a_n$
Rainfall Intensities, i, in Inches per Hour, for Cumulative Areas, A_n
Upper End
Averages
Lower End
(1)
(2) ∞
(3)
(4)
(5) ∞
(6)
(7)
4.90
2.45
10
2.03
1.73
20
1.56
1.42
30
1.17
$A_n-(a_1+a_2+a_3)$
1.00
60
a_1
a_2
a_3
a_n
A_1
A_2
A_3
A_n
4.90
3.46
2.83
2.00
2.45
1.73
1.42
1.00

Let the total areas contributing water above the successive inlet points equal A_1, A_2, A_3, etc.; times of concentration, t_1, t_2, t_3, etc.; and corresponding average intensities, i_1, i_2, i_3, etc., respectively.

Let the individual areas contributing water to the successive inlet points be represented by a_1, a_2, a_3, etc. (shown on left of Table 1). Then, let i'_1, i'_2, i'_3, etc., equal the average intensity of rainfall on the areas a_1, a_2, a_3, etc., at the respective times, t'_1, t'_2, t'_3, etc. For this simple water-shed it is assumed that the time of concentration of the individual areas is approximately equal to the differences in time between inlet points on the main line, or, that the intensities, i'_1, i'_2, i'_3, etc., are due to the time periods, 0 to t_1, t_1 to t_2, t_2 to t_3, etc., respectively.

Considering Area A_n, Table 1, and following strictly the assumption of the rational method, that the maximum flow occurs when the storm lasts as long, or longer, than the time required for the water to travel from the most remote portion of the water-shed to its outlet, enabling all parts of the area to contribute simultaneously, then, the run-off from Area a_1, produced by Intensity i'_1, occurring at Time t'_1 will reach the outlet when Area a_n is contributing, due to Intensity i'_n falling at Time t'_n.

Likewise, the run-offs from Areas a_2, a_3, etc., produced by Intensities i'_2, i'_3, etc., occurring at Times t'_2, t'_3, etc., respectively, will reach the outlet when Area a_n is contributing, due to Intensity i'_n falling at Time t'_n. The total discharge at the outlet of Area A_n will equal the sum of the discharges from the individual areas composing the water-shed.

Now, solely for the purpose of illustrating the present use of the rainfall curve in the detail method, it will be assumed that the run-off coefficient is constant throughout, say, equal to one. Any form of Equation (2) may be selected. For example, let $e = 0.5$ and (when t is expressed in seconds) let $K = 120$. Values of average rainfall intensities, corresponding to successive abscissas of time, may be determined and plotted as ordinates in Fig. 1.

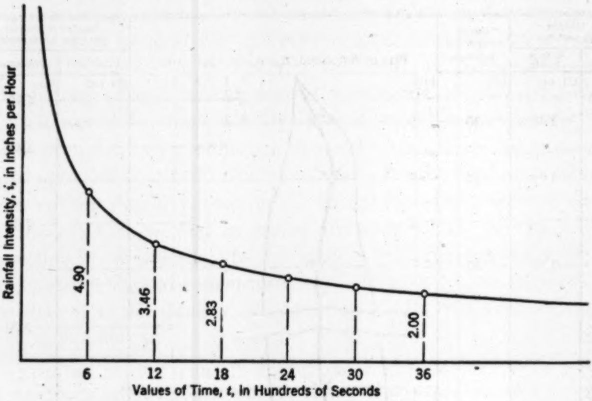


FIG. 1.—ILLUSTRATIVE CURVE OF RAINFALL INTENSITIES, $i = \dfrac{120}{t^{0.5}\text{ sec.}}$

Under the foregoing assumptions, the water-shed, A_n, is expected to receive an average intensity of 2 in. per hour for the time of 60 min. If this general detail method is followed, the time, 60 min., is determined by velocities, produced by the use of the intensities, 4.90, 3.46, 2.83, ... 2.0, for the areas, A_1, A_2, A_3, ... A_n, respectively.

Now, if the storm is assumed to have a uniform intensity for the period of time, t_n then, $i_1 = i_2 = i_3, \ldots = i_n$, and the velocities that produce the time, t_n, in the computations, will be the result of a uniform, or constant, intensity for the time of the water-shed.

Thus, although it is the evident intention to assume a uniform intensity over the entire area throughout the time of concentration, the process followed in the detail method automatically assumes the equivalent of a variable intensity during the period.

The particular type of storm thus automatically assumed, not only depends on the physical characteristics, and the shape and manner of concentration

of the water-shed, but also on the type of rainfall curve in use. When the area is shaped so that the time of concentration increases proportionately with the contributing area, the type of storm automatically assumed follows the law of the curve in use.

Assume, for the purpose of illustration, a rainfall curve of the type,

$$i = \frac{K}{t^{0.5}} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3)$$

The average value of the ordinate, i, may be found as follows: When $y = f(x)$, or $i = f(t)$, the expression for the area bounded by the curve, the outer ordinate, and the co-ordinate lines, is $\int f(x)\,dx = \int K\,t^{-0.5}\,dt = \frac{K\,t^{0.5}}{0.5} = 2\,K\,t^{0.5}$. Then, $2\,K\,t^{0.5}$ divided by the abscissa, t, $= \frac{2\,K\,t^{0.5}}{t} = \frac{2\,K}{t^{0.5}}$. Therefore, the average ordinate equals twice the end ordinate.

If, for a period of 60 min. on the water-shed, A_n, the storm had an average intensity of 2 in. per hour and followed the law of the curve, the intensity at the lower end of the area would be one-half the average, or, 1 in. per hour, while the intensity at the upper end would be infinite. Likewise, for the intensities of the areas, A_1, A_2, A_3, etc., the upper end would be infinite and the lower end, one-half the average (see Table 1).

The intensity for 60 min. equals $R_1 = \frac{K'}{3\,600^{\frac{1}{2}}} = 1$, and therefore, $K' = \sqrt{3\,600} = 60$. Substituting this value of K in Equation (2) and letting $\epsilon = 0.5$ as in Fig. 1; then,

$$i = \frac{60}{t^{0.5}} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (4)$$

The average intensities (see Column (1) Table 1) for the individual areas are as follows:

For a_1,

$$\frac{2\,K'}{\sqrt{t_1}} = 4.90$$

For a_2,

$$\frac{\left(2\,K'\,t_2^{\frac{1}{2}} - 2\,K'\,t_1^{\frac{1}{2}}\right)}{(t_2 - t_1)} = 2.03$$

For a_3,

$$\frac{\left(2\,K'\,t_3^{\frac{1}{2}} - 2\,K'\,t_2^{\frac{1}{2}}\right)}{(t_3 - t_2)} = 1.56$$

and, for $A_n - (a_1 + a_2 + a_3)$,

$$\frac{\left(2\,K'\,t_n^{\frac{1}{2}} - 2\,K'\,t_3^{\frac{1}{2}}\right)}{(t_n - t_3)} = 1.17$$

Assume any values for the individual areas in Table 1 that will vary directly as the time, such as $a_1 = 100$, $a_2 = 100$, $a_3 = 100$, and $A_n - (a_1 + a_2 + a_3) = 300$ acres.

In Equation (1) let $C = 1$; then, the "Q-collecting" at the outlet of the water-shed would be as shown in Columns (2) and (3) of Table 2, with proper values of i and a substituted for each successive case. The average intensity would be $\frac{1\,200}{600} = 2$ in. per hour, which is the value used in the detail method. However, this process will produce the same time of concentration as the detail method, when and only when the area varies directly with the time.

When the area increases at a faster rate than time, the results are as illustrated in Columns (4) and (5) of Table 2. In this case, 1 900 acres × 2.00 in. per hour = 3 800 cu. ft. per sec., which is greater than the rate of run-off obtained by assuming that the storm obeyed the law expressed by the curve in use. Similarly, (Columns (6) and (7)), 200 acres × 2.00 in. per hour = 400 cu. ft. per sec., which is less than the rate of run-off obtained by assuming that the storm obeyed the law expressed by Equation (3).

TABLE 2.—Rates of Run-Off, Q, for Three Types of Water-Shed

Intensity of rainfall, in inches per hour	Area, a, and Time, t, Increasing Proportionately		Area, a Increasing Faster than Time, t		Time, t, Increasing Faster than Area, a	
	a, in acres	Q, in cubic feet per second	a	Q	a	Q
(1)	(2)	(3)	(4)	(5)	(6)	(7)
4.90	100	490	100	490	100	490
2.03	100	203	300	609	50	101.5
1.56	100	156	500	780	30	46.8
1.17	300	351	1 000	1 170	20	23.4
Total......	600	1 200	1 900	3 049	200	661.7

Conclusions.—The foregoing observations lead to four major conclusions as follows:

1.—For a constant value of the run-off coefficient, and for the special type of water-shed assumed in Table 1, three facts are noted:

(*a*) When the time of concentration increases proportionately with the contributing area, the type of storm assumed follows the law of the rainfall curve in use.

(*b*) When the area increases at a faster rate than the time, the run-off obtained by the detail method is greater than that obtained when the type of storm is assumed to follow the law of the curve in use.

(*c*) When the area increases at a slower rate than the time of concentration, the run-off, Q, obtained by the detail method is less than that obtained when the type of storm is assumed to follow the law of the curve in use.

2.—In all cases, the detail method automatically assumes the equivalent of a storm of varying intensity, causing a smaller time of concentration because of a greater average velocity of flow (and, consequently, a greater value of Q) than would be obtained by assuming a storm of uniform (or constant) intensity for the time of the water-shed. The use of the rainfall curve is, therefore, inconsistent with its making, in that while the curve represents a certain period of expected frequency based on types of storms occurring in any variety of ways (averaging in effect a uniform intensity for a given period of time), it is really used to accommodate a system of tabling which automatically assumes a different type of storm for every type of rainfall curve and water-shed.

3.—Consider a storm that varies in intensity within the time of concentration, but is evenly distributed over the entire area at any moment. If the run-off coefficient is constant throughout the water-shed, the summation of discharges from the individual areas is the same as that due to a rainfall with a uniform rate of precipitation (equal to the average of the preceding rates) over the water-shed for the time period, when (and only when) the contributing area increases proportionately with the time.

4.—It is apparent, therefore, that neither the assumption of a uniform intensity over the water-shed for the time of concentration, nor its approach through the medium of the detail method, affords, in all cases, the absolute maximum run-off.

Statement.—It is manifestly the intention of those who use the detail rational method to assume that the expected storm is one of uniform intensity over the entire area throughout the time of concentration. The afore-mentioned inconsistency, affording the variable factor of safety, is due to the use of velocities that do not obtain under the assumption. Practically, however, this factor of safety is not of major importance, being usually only a small percentage. Practical visualization of its probable magnitude may be had from a comparison of the values of the factor, P, in Table 3 with those in Table 4, raised to the exponent indicated in Equations (28) and (29), given subsequently. The values shown in Table 3 are based on the assumption that the type of storm is one of uniform intensity over the area throughout the time of concentration; while those in Table 4 resulted from the use of the rainfall curve, of the type, $i = \frac{K}{t^{0.5}}$, as ordinarily practiced in the detail rational method. It should be mentioned that this factor of safety is varied somewhat by the type of rainfall curve; it may be somewhat greater for a given water-shed, for instance, when e in Equation (2) is 0.7 than when e is 0.5 as in Equation (3).

In order to arrive at a greater run-off than that rendered by the assumption of a uniform intensity for the area throughout the time of concentration, it would be necessary to assume a different manner of variation in intensity, within the period, for every water-shed. The peak, or rather the peaks, of the storm, for the period of concentration, would then be so arranged that the contributions from the individual areas composing the water-shed might

render the greater run-off. Intensities are relative, however, and there is no limiting peak, either in height or duration. There is, then, no absolute maximum for a given water-shed; and, although it is possible to arrive at a greater run-off than that to be expected from the rational assumption, the impracticability of predicting the probable frequency of occurrence of storms particularly selected for every type of water-shed, the difficulty in determining the effect on the run-off coefficient, and the variability of the time of concentration, due to all physical characteristics of a water-shed and changes in intensity, complicate the problem to a measure almost beyond practical solution.

TABLE 3.—VALUES OF THE FACTOR, P, FOR STORMS OF UNIFORM INTENSITY

		$P' = 2.0$			$P' = 1.0$			$P' = 0.5$			$P' = 0.3$			$P' = 0.1$		
		Concentration			Concentration			Concentration			Concentration			Concentration		
$\frac{A}{A'}$	$\frac{L}{W}$	Delayed	Medium	Advanced	Delayed	Medium	Advanced	Delayed	Medium	Advanced	Delayed	Medium	Advanced	Delayed	Medium	Advanced
16	2	0.76	0.79	0.87	0.59	0.64	0.69	0.44	0.46	0.48	0.30	0.31	0.32	0.11	0.11	0.12
	3	0.75	0.80	0.88	0.60	0.65	0.70	0.46	0.48	0.51	0.34	0.35	0.37	0.13	0.14	0.14
	4	0.74	0.81	0.89	0.61	0.66	0.71	0.48	0.50	0.53	0.36	0.37	0.39	0.14	0.15	0.15
	8	0.72	0.78	0.89	0.64	0.67	0.72	0.53	0.55	0.56	0.42	0.44	0.46	0.20	0.21	0.21
	16	0.72	0.76	0.79	0.65	0.68	0.73	0.56	0.57	0.58	0.48	0.49	0.50	0.27	0.28	0.28
64	2	0.62	0.67	0.72	0.55	0.58	0.63	0.43	0.45	0.48	0.33	0.35	0.37	0.16	0.16	0.17
	3	0.63	0.67	0.73	0.56	0.59	0.64	0.45	0.48	0.50	0.36	0.38	0.40	0.18	0.19	0.19
	4	0.63	0.68	0.78	0.57	0.60	0.65	0.47	0.50	0.52	0.38	0.40	0.42	0.20	0.21	0.22
	8	0.64	0.69	0.74	0.60	0.63	0.66	0.52	0.54	0.57	0.44	0.46	0.48	0.25	0.26	0.27
	16	0.66	0.69	0.72	0.62	0.65	0.67	0.56	0.58	0.60	0.50	0.51	0.52	0.31	0.32	0.32
256	2	0.55	0.58	0.68	0.50	0.52	0.57	0.42	0.44	0.47	0.35	0.37	0.38	0.18	0.20	0.20
	3	0.55	0.59	0.64	0.51	0.54	0.58	0.45	0.47	0.50	0.38	0.40	0.42	0.22	0.22	0.23
	4	0.57	0.60	0.65	0.52	0.55	0.59	0.47	0.50	0.52	0.40	0.42	0.44	0.24	0.25	0.26
	8	0.58	0.63	0.67	0.56	0.60	0.61	0.51	0.54	0.57	0.46	0.48	0.49	0.29	0.30	0.31
	16	0.62	0.65	0.68	0.60	0.61	0.62	0.56	0.58	0.61	0.51	0.52	0.53	0.35	0.36	0.37
4 096	2	0.48	0.51	0.55	0.47	0.47	0.53	0.42	0.44	0.47	0.36	0.40	0.41	0.24	0.27	0.27
	3	0.49	0.52	0.56	0.48	0.50	0.54	0.44	0.47	0.50	0.41	0.43	0.44	0.29	0.29	0.32
	4	0.51	0.53	0.57	0.49	0.51	0.55	0.46	0.49	0.52	0.43	0.46	0.46	0.32	0.32	0.35
	8	0.54	0.58	0.62	0.53	0.56	0.58	0.51	0.54	0.57	0.49	0.51	0.53	0.38	0.39	0.41
	16	0.59	0.62	0.64	0.58	0.58	0.60	0.56	0.59	0.62	0.53	0.54	0.56	0.42	0.43	0.44
262 144	2	0.45	0.47	0.50	0.45	0.45	0.50	0.42	0.44	0.47	0.39	0.43	0.44	0.32	0.34	0.34
	3	0.46	0.48	0.51	0.46	0.48	0.51	0.44	0.47	0.49	0.42	0.45	0.48	0.37	0.38	0.40
	4	0.48	0.49	0.52	0.47	0.49	0.52	0.46	0.49	0.51	0.44	0.48	0.50	0.40	0.42	0.44
	8	0.52	0.55	0.57	0.51	0.54	0.56	0.50	0.53	0.57	0.50	0.53	0.55	0.47	0.48	0.49
	16	0.57	0.60	0.62	0.56	0.57	0.58	0.56	0.59	0.62	0.54	0.55	0.57	0.50	0.51	0.52

Assuming a uniform intensity for the entire area throughout the time of concentration enables the engineer to forecast with reasonable accuracy the probable frequency of this average storm. The solution of the run-off problem by means of rainfall curves, showing the relation between time and intensity for various periods of expected frequency, therefore, may be adopted as the standard basis for economical design. It may reasonably be considered that the possible arrangement of intensities, within the time of concentration, to render a greater run-off, constitutes a very rare occurrence, which (in special cases only) may be taken care of by a special factor of safety.

TABLE 4.—VALUES OF THE FACTOR, P, FROM USE OF RAINFALL CURVE OF THE TYPE, $i = \frac{K}{t^{0.5}}$

$\frac{A}{A'}$	$\frac{L}{W}$	$P' = 2.0$			$P' = 0.9$			$P' = 0.5$			$P' = 0.3$			$P' = 0.1$		
		Concentration			Concentration			Concentration			Concentration			Concentration		
		Delayed	Medium	Advanced	Delayed	Medium	Advanced	Delayed	Medium	Advanced	Delayed	Medium	Advanced	Delayed	Medium	Advanced
16	2	0.88	0.92	0.99	0.63	0.67	0.72	0.46	0.49	0.52	0.32	0.35	0.38	0.12	0.13	0.14
	3	0.86	0.92	0.99	0.65	0.70	0.74	0.49	0.52	0.55	0.35	0.38	0.41	0.15	0.15	0.16
	4	0.85	0.91	0.99	0.67	0.71	0.75	0.52	0.54	0.57	0.37	0.40	0.43	0.17	0.17	0.18
	8	0.83	0.90	0.96	0.70	0.74	0.78	0.57	0.60	0.63	0.44	0.47	0.50	0.24	0.24	0.25
	16	0.82	0.89	0.94	0.73	0.76	0.80	0.62	0.65	0.68	0.51	0.54	0.56	0.31	0.32	0.33
64	2	0.72	0.76	0.82	0.58	0.62	0.66	0.46	0.49	0.52	0.34	0.37	0.40	0.16	0.17	0.18
	3	0.73	0.77	0.83	0.60	0.65	0.69	0.49	0.52	0.55	0.39	0.41	0.43	0.19	0.19	0.20
	4	0.73	0.77	0.83	0.62	0.66	0.70	0.52	0.55	0.57	0.41	0.44	0.46	0.21	0.21	0.22
	8	0.75	0.79	0.84	0.66	0.70	0.74	0.58	0.61	0.63	0.47	0.50	0.52	0.27	0.28	0.28
	16	0.76	0.81	0.85	0.70	0.73	0.77	0.63	0.66	0.69	0.52	0.56	0.58	0.33	0.34	0.35
256	2	0.62	0.66	0.72	0.55	0.58	0.62	0.46	0.49	0.52	0.37	0.40	0.43	0.21	0.22	0.23
	3	0.63	0.67	0.73	0.57	0.60	0.64	0.50	0.52	0.55	0.41	0.44	0.46	0.23	0.25	0.26
	4	0.64	0.68	0.73	0.58	0.62	0.66	0.52	0.54	0.57	0.43	0.46	0.48	0.25	0.27	0.29
	8	0.68	0.72	0.76	0.64	0.68	0.72	0.58	0.61	0.63	0.49	0.52	0.55	0.32	0.33	0.34
	16	0.72	0.76	0.79	0.68	0.72	0.75	0.63	0.66	0.69	0.55	0.58	0.61	0.38	0.40	0.41
4 096	2	0.54	0.59	0.63	0.52	0.55	0.58	0.47	0.49	0.52	0.41	0.44	0.47	0.28	0.29	0.30
	3	0.56	0.60	0.64	0.54	0.57	0.60	0.50	0.52	0.55	0.44	0.47	0.50	0.32	0.32	0.34
	4	0.58	0.62	0.64	0.55	0.58	0.62	0.63	0.55	0.57	0.47	0.50	0.52	0.34	0.35	0.37
	8	0.63	0.67	0.71	0.61	0.65	0.69	0.58	0.61	0.63	0.52	0.56	0.59	0.39	0.41	0.43
	16	0.68	0.72	0.76	0.66	0.70	0.73	0.63	0.66	0.69	0.58	0.61	0.64	0.44	0.47	0.50
262 144	2	0.50	0.54	0.58	0.50	0.53	0.56	0.47	0.49	0.52	0.44	0.47	0.51	0.36	0.38	0.41
	3	0.52	0.55	0.59	0.52	0.55	0.58	0.50	0.53	0.56	0.47	0.50	0.54	0.41	0.43	0.46
	4	0.54	0.57	0.59	0.53	0.56	0.60	0.53	0.55	0.58	0.50	0.53	0.56	0.43	0.46	0.49
	8	0.60	0.63	0.66	0.59	0.63	0.67	0.58	0.61	0.64	0.55	0.59	0.62	0.48	0.51	0.54
	16	0.65	0.69	0.72	0.64	0.68	0.71	0.64	0.67	0.70	0.61	0.64	0.67	0.52	0.57	0.61

Factor C, The Run-Off Coefficient.—Among the afore-mentioned uncertainties in available data for the estimation of storm flow, the run-off coefficient, Factor C, probably offers the widest range of values for water-sheds with similar characteristics calculated to affect this factor. Much has been written concerning this factor, and it is not here considered necessary to review all the many characteristics of a water-shed that determine its value. The writers, however, do wish to stress one main difference in assumptions for the variation in the value of the run-off coefficient as made by those who use the rational method; that is, C is assumed to vary in one of the following three ways (the type of territory as regards physical characteristics, irregularities of surface, etc., being the same):

(1) C increases as the time increases.
(2) C remains constant as the time increases.
(3) C decreases as the time increases.

Although many examples could be shown, it should suffice, for the purpose of comparison, to give one typical example of each of these assumptions.

Assumption (1) is exemplified by Table 5 which was used in design by the City of Hamilton, Ont., Canada.

TABLE 5.—Run-Off Coefficients Used for Design Purposes in Hamilton, Ontario, Canada

Duration of rainfall, in minutes	Run-Off Coefficients		
	Pervious surface	Outlying district	Central business
5	0.00	0.50	0.75
10	0.05	0.60	0.80
15	0.10	0.75	0.85
20	0.15	0.80	0.90
25	0.20	0.825	0.95
30	0.25	0.85	0.95
35	0.30	0.865	0.95
40	0.35	0.883	0.95
45	0.40	0.90	0.95
50	0.45	0.917	0.95
55	0.50	0.93	0.95
60	0.55	0.95	1.00

Table 6 used by the City of Washington, D. C., may serve as an example of Assumption (2).

TABLE 6.—Run-Off Coefficients Used for Design Purposes in Washington, D. C.

Characteristics	Constant run-off coefficient
Extreme suburban areas with 20 to 40% parking and widely detached houses...	0.35
Suburban areas with widely detached houses....................................	0.45
Suburban areas with widely detached houses....................................	0.55
Areas with 50% attached and 50% detached houses....................................	0.65
Areas closely built up....................................	0.75
Business areas....................................	0.85

An example to illustrate Assumption (3) has been given* by C. E. Grunsky, Past-President, Am. Soc. C. E., and need not be reproduced for present purposes. It should be evident to the reader, without further elaboration, that these assumptions for the value of the run-off coefficient for use in the fundamental relation, $Q = C\,i\,A$ (or its equivalent in other letter designations), may lead to widely different run-off quantities for a given water-shed, no matter how carefully and how accurately the calculations may be carried out for a given case.

On account of the many varying conditions and characteristics of watersheds, accurate measurement of the run-off coefficient is very difficult to accomplish. Much of the available experimental information is of little value, and, no doubt, improper analysis of some of the more valuable results is often made. The writers at this point wish to urge a more unified effort on the part of the profession not only toward a thorough analysis of present available data, but also future experimentation along lines following a preconceived plan.

---

* *Transactions*, Am. Soc. C. E., Vol. LXV (1909), p. 325.

Pursuant to this thought, the writers offer the following analysis with the hope that the conclusions drawn will be helpful in future consideration of Factor *C*.

Among those who have contributed to the information on run-off, probably no one man has done more and better work than the late Emil Kuichling, M. Am. Soc. C. E. **Many of those who base their computations on Assumption (1), probably have been guided by the following five conclusions drawn** by Mr. Kuichling[7]:

5.—"The percentage of the rainfall discharged from any given drainage area is nearly constant for all rains of considerable intensity, and lasting equal periods of time * * *.

6.—"The said percentage varies directly with the urban development of the district; or, in other words, with the amount of impervious surface thereon * * *.

7.—"The said percentage increases rapidly, and directly or uniformly, with the duration of the maximum intensity of the rainfall, until a period is reached which is equal to the time required for the concentration of the drainage waters from the entire tributary area at the point of observation; but if the rainfall continues at the same intensity for a longer period, the said percentage will continue to increase for the additional interval of time at a much smaller rate than previously. This circumstance is manifestly attributable to the fact that the permeable surface is gradually becoming saturated and is beginning to shed some of the water falling upon it; or, in other words, the proportion of impervious surface slowly increases with the duration of the rainfall.

8.—"The said percentage becomes larger when a moderate rain has immediately preceded a heavy shower, thereby partially saturating the permeable territory and correspondingly increasing the extent of impervious surface.

9.—"The sewer discharge varies promptly with all appreciable fluctuations in the intensity of the rainfall, * * *."

There can be no doubt as to the correctness of Conclusions (6), (7), (8), and (9), but it is probable that some users of the rational method have misconstrued Conclusion (5) (and perhaps Conclusion (7)), or taken it too literally, in forming conclusions regarding the variation of the run-off coefficient.

For any water-shed not completely saturated from the beginning, and having constant physical characteristics throughout which are calculated to **affect the run-off coefficient, the following conclusions will hold:**

10.—Because of an increasing wetness (causing a higher percentage of imperviousness) the run-off coefficient for the successive individual areas composing the water-shed, increases with the time.

11.—For the intensity decreasing with the time, the run-off coefficient for the successive individual areas composing the water-shed, will decrease with the time.

12.—For all rainfalls of considerable uniform intensity on any given watershed (the previous degree of saturation and physical characteristics remaining constant for the separate precipitations), the run-off coefficient for a rain of given intensity will be higher than that for one of less intensity, and conversely.

Referring again to Table 1, assume a simple water-shed with constant physical characteristics throughout. Let the individual areas contributing

---

[7] *Transactions*, Am. Soc. C. E., Vol. XX (1889), p. 37.

water to the successive inlet points be represented by a_1, a_2, a_3, etc. (see Table 1). Let i'_1, i'_2, i'_3, etc., equal the average intensities falling on the areas, a_1, a_2, a_3, etc., at the respective times, t'_1, t'_2, t'_3, etc. Let C'_1, C'_2, C'_3, etc., represent the respective run-off coefficients.

Let the total areas contributing water above the successive inlet points equal A_1, A_2, A_3, etc. Let C_1, C_2, C_3, etc., equal run-off coefficients; t_1, t_2, t_3, etc., equal successive times of concentration; and i_1, i_2, i_3, etc., equal corresponding average intensities for A_1, A_2, A_3, etc., respectively.

Let r = rate of retention, in cubic feet per second per acre. It should be understood that retention represents that proportion of the falling water which does not run off because of evaporation, absorption, and storage, or,

$$r = i - C\,i = i\,(1 - C) \qquad (5)$$

Hence, let the average rates of retention for the areas: a_1, a_2, a_3, ... $a_n = r'_1, r'_2, r'_3, \ldots r'_n$, respectively; and for $A_1, A_2, A_3, \ldots A_n = r_1, r_2, r_3, \ldots r_n$, respectively.

Conclusion (10) may be best illustrated by assuming a constant intensity, i, for any water-shed, as A_n. Then $(ia_1 - r'_1a_1)$, $(ia_2 - r'_2a_2)$, $(ia_3 - r'_3a_3)$, etc., represent run-off quantities, in cubic feet per second, for the areas, a_1, a_2, a_3, etc. From Equation (1), $Q = C\,i\,a$, or,

$$Q'_1 = C'_1\,i\,a_1;\; Q'_2 = C'_2\,i\,a_2;\; Q'_3 = C'_3\,i\,a_3;\; \ldots Q'_n = C'_n\,i\,a_n \qquad (6)$$

Therefore, equating, $C'_1\,i\,a_1 = i\,a_1 - r'_1\,a_1 \ldots C'_n\,i\,a_n = i\,a_n - r'_n\,a_n$, or,

$$C'_1 = 1 - \frac{r'_1}{i} \ldots C'_n = 1 - \frac{r'_n}{i} \qquad (7)$$

Likewise, it may be inferred that, considering the areas, $A_1, A_2, A_3, \ldots A_n$,

$$C_1 = 1 - \frac{r_1}{i} \ldots C_n = 1 - \frac{r_n}{i} \qquad (8)$$

Consider the water-shed, A_n, and follow the assumption of the rational method that the maximum flow occurs when the storm lasts as long as, or longer than, the time required for the water to travel from the most remote part of the water-shed to its outlet. This would enable all parts of the water-shed to contribute simultaneously; that is, run-off from Area a_1, produced by Intensity i, occurring at Time t'_1, will reach the outlet when Area a_n is contributing, due to Intensity i, falling at Time t'_n.

Likewise, the run-offs from Area a_2, a_3, etc., occurring at Times t'_2, t'_3, etc., respectively, will reach the outlet when Area a_n is contributing, due to Intensity i, falling at Time t'_n.

The rainfall that produced the run-offs from a_1, a_2, a_3, etc., lasted for the time, t'_n; but at the time run-off occurred the rain had lasted for a time, t'_1, on Area a_1, t'_2 on Area a_2, t'_3 on Area a_3, etc.

But, $t'_1 < t'_2 < t'_3, \ldots < t'_n$, and unless the water-shed was completely saturated before the rain occurred, $r'_1 > r'_2 > r'_3 \ldots > r'_n$. Therefore, $C'_1 < C'_2 < C'_3, \ldots < C'_n$, and the same process of reasoning would show that $C_1 < C_2 < C_3 \ldots < C_n$. It should be conceded, therefore, that Conclusion (10) is correct.

Conclusion (11) as applied to the rational method may be best illustrated by assuming a constant rate of retention for the individual areas composing any water-shed, such as A_n.

Then, the logic in the case of Conclusion (10) may be applied to Conclusion (11), substituting i' for i and r for r'. The result will be similar to Equation (8),

$$C_1 = 1 - \frac{r}{i_1}; \ldots C_n = 1 - \frac{r}{i_n} \ldots\ldots\ldots\ldots\ldots\ldots\ldots (9)$$

By inspection, it may be seen that, when $i'_1 > i'_2 > i'_3 \ldots > i'_n$, then $C'_1 > C'_2 > C'_3 \ldots > C'_n$.

For a practical visualization of the detail method the rainfall curve in Fig. 1 may be used with values of time and average intensities for Areas A_1, A_2, A_3, etc., as shown in Column (6) of Table 1. Assuming a constant value of r, say, 1.00 cu. ft. per sec. per acre and substituting in Equation (9), the values of intensities given in Table 1 (Column (6)): $C_1 = 0.80$; $C_2 = 0.71$; $C_3 = 0.65$; ... $C_n = 0.50$.

By a somewhat similar process of reasoning, it may be shown that when r varies directly as i a constant value of C results.

Referring again to the foregoing Conclusion (5), (by Mr. Kuichling), five assumptions may be made for illustrative purposes as follows:

(4) A given drainage area;
(5) A rain of considerable intensity, say 3 in. per hour;
(6) A rain of considerable intensity, say 6 in. per hour;
(7) Equal time periods; and,
(8) A constant value of C, say, 0.50.

Then, in Equation (9), $0.50 = 1 - \frac{r}{3.00}$, and $r = 1.50$ cu. ft. per sec. per acre. When $i = 6.00$ and $C = 0.50$, $r = 3.00$ cu. ft. per sec. per acre.

Stating the foregoing Conclusion (5) by Mr. Kuichling in words would be the equivalent of saying that for a given drainage area the capacity for retention, in cubic feet per second per acre, for a rain of 6 in. per hour is double that for a rain of 3 in. per hour; or, generally, that the capacity for retention of a given area varies directly as the intensity.

It is customary in the rational method (and properly so) to assume a certain existing wetness or degree of saturation, of a given area due to previous rains. This existing degree of saturation will always be one of the main determining factors in considering this rate of retention, and, consequently, the value of Factor C, including its variation with the time. Although the requirements of any particular problem must always tax the judgment of the individual engineer, it may be possible, in a future study of the run-off coefficient (through a more unified effort of the profession), to establish some kind of standard scale of previous degree of saturation to be used as a guide to this judgment.

Reverting to the consideration of Conclusions (11) and (12), the rate of retention (assuming a certain degree of previous wetness), may be subdivided into three main component parts, namely, evaporation, absorption, and storage.

There is no reason to assume any material difference in evaporation, in cubic feet per second per acre, from a given area due to two separate rains of considerable but different intensities. Likewise, there is no reason to assume a materially greater rate of absorption, expressed in cubic feet per second per acre, on a given area with a certain previous saturation from a rain of any considerable intensity than from one of less intensity severe enough to produce general run-off.

Considering storage, expressed in cubic feet per second per acre, a given area may be assumed to have a certain degree of previous saturation. For example, assume that on such an area a rain of considerable intensity, say, 3 in. per hour occurred. It is reasonable to suppose that, in addition to evaporation and absorption, a certain quantity of water is lost to run-off for the time of concentration by storage in irregularities of surface, vegetation (if any), etc. Before producing general run-off the water must overcome these factors. If, at another time, a rain of greater intensity, say, 6 in. per hour, should fall upon the same area with the same degree of previous wetness, it would have like storage irregularities, etc., to overcome before producing general run-off.

It is, therefore, not reasonable to assume that the storage capacity of a given area (in cubic feet per second per acre), which is in condition to shed its water, is twice as great for a rain of 6 in. per hour as that for one of 3 in. per hour; or, generally, that the capacity for storage of a given area varies directly as the intensity.

The foregoing analysis points to the correctness of Conclusions (11) and (12). Experimental proof showing the correctness of Conclusions (11) and (12) is not entirely lacking[8], although much work is to be done before comprehensive values of the run-off coefficient due to changing intensities, can be established for use in the rational method.

The range in intensities encountered in run-off studies is considerable, and the present tendency of many users of the detail rational method to increase Factor C as the time increases (as shown in current publications and textbooks), without also allowing for the fluctuation in C for changing intensities, seems entirely unjustified.

Conclusions.—Based on the foregoing analysis the writers make the following conclusions, which (it is hoped) may be of some use as guides to a proper selection of Factor C (the characteristics calculated to produce run-off and the previous degree of saturation are assumed to be constant for the individual areas forming the water-shed):

13.—For the type of storm with a uniform (or constant) intensity of rainfall over the entire area throughout the time of concentration of any water-shed, as A_n, with individual areas, $a_1 + a_2 + a_3$, etc. (see Table 1), Conclusion (10) as previously stated, is correct.

The increase of the run-off coefficient ($C_1 < C_2 < C_3$) depends on the previous degree of wetness, as well as the physical characteristics of the watershed.

---

[8] Technical Reports, Miami Conservancy Dist., Pt. VIII, "Rainfall and Run-Off in the Miami Valley," by Ivan E. Houk, M. Am. Soc. C. E.

14.—When the expected storm for the time of concentration of any water-shed, as A_n, is of the type that decreases in intensity with the time, there is a tendency toward a balance in Factor C (see Conclusions (10), (11) and (12)). For average rates of retention, on the individual areas, a_1, a_2, a_3, ... a_n, or the areas, A_1, A_2, A_3, ... A_n, varying directly with the respective intensities, then $C'_1 = C'_2 = C'_3 \ldots = C'_n$, or $C_1 = C_2 = C_3 \ldots = C_n$.

Since it is manifestly the intention of the users of the detail process of the rational method to assume a storm of uniform intensity over the entire area throughout the time of concentration, the variable factor of safety (mentioned herein under the heading, "Use of Rainfall Curve"), may be considered as erring from the assumption due to the use of fictitious velocities. In this case Conclusion (13) applies in estimating the run-off from any water-shed, A_n.

However, while the run-off coefficients, C'_1, C'_2, C'_3, etc., correctly apply (under the assumption), to the areas, a_1, a_2, a_3, etc., in the process of estimating the "run-off collecting" at the outlet of the water-shed, A_n, they do not represent the proper values for use in the economical design of the drainage system. For design purposes Areas A_1, A_2, A_3, etc., must be considered as separate water-sheds. For instance, Area A_1, because of its shorter time of concentration, may be expected to receive a higher intensity than Area A_n, and, consequently, a higher run-off coefficient than the value used for it in estimating the run-off for the entire area.

Granting the correctness of Conclusions (10), (11), and (12), therefore, and permitting the use of velocities resulting from the detail method, the users of the customary tabling process, striving for economical design throughout the water-shed, should consider both the magnitude and duration of intensity on the contributing area.

Suggestion (a).—In regard to the application of intensities the rational method treats a given drainage area as a separate water-shed above each inlet point in succession on the main channel of flow, the smaller area receiving a higher uniform intensity throughout its time of concentration than the larger, of which it is a part. Those who use a constant run-off coefficient for a given type of territory (Assumption (2)), automatically assume, therefore, whether intentionally or otherwise, that, as progress is being made on a given rainfall curve, the average rate of retention on each successive water-shed varies directly as the intensity used; that is, that a balance is set up in the coefficient, an increase due to a greater average degree of saturation balancing the decrease due to a lower average intensity.

Stated in another way, the adoption of a constant coefficient (with respect to the time of concentration) for use with a certain rainfall curve, automatically assumes that degree of previous wetness of a given water-shed which will render a balance between the effects of increasing wetness and lowering intensity. Although the rate of retention may vary in different degrees for areas of dissimilar physical characteristics and shape, and the types of rainfall curves differ somewhat, in view of the extremely complicated nature of the problem and the necessary conflicts to be expected in experimental data, it may be the better part of wisdom at present to set up tentative tables on this basis to be used on the smaller water-sheds and the initial portions of the larger areas (say, the portion contributing during the first 60 to 120 min. of the time of concentration), on which it is desirable to assume an unsaturated surface condition at the outset of the maximum run-off producing intensity.

The making of such tables, among other considerations, would involve the selection of the best average shape; and would necessarily provide different sets of constants, as warranted by the change in average intensity due to frequency periods or location. The 60-min. ordinate could be made the criterion.

The use of such tables would entail the assignment by the designer of values of the coefficient to individual portions of the water-shed on the basis of physical characteristics, the column representing the frequency period of the curve in use having first been determined.

Suggestion (*b*).—For the smaller water-sheds and the portions of the larger areas (say, the portion contributing during the first 60 to 120 min. of the time of concentration), it may be feasible in the light of future experimentation to devise a set of curves (or tables) for elementary areas providing for both the effect of time and intensity on the various soils, degrees of perviousness, etc. It may be reasoned that the use of such curves (for instance, in the detail process) would require the estimation of an average value of the coefficient for the entire area above each inlet as new values of the intensity were selected from the rainfall curve.

Comment.—The uncorrected results of the tabling processes will, in any case, include the automatic error (or factor of safety above the assumption of a uniform intensity over the entire area throughout the time of concentration) noted under the heading, "Use of Rainfall Curve". Where provision is made for consideration of the effect of both time and intensity (as outlined under Suggestions (*a*) and (*b*), for water-sheds comparatively dry at the onset of the maximum run-off producing intensity), a similar error will be automatically included due to the use of an average value of the coefficient for the successive water-sheds above each inlet point, instead of the actual values for the individual areas. The magnitude of this latter error (for water-sheds of uniform characteristics throughout) will depend on the water-shed and the degree of previous wetness assumed, and will be smaller on the average, than that of the former. Both errors are on the side of safety, and, together (considering the approximate nature of the problem), are not of very serious consequence.

It should be evident that, for general use, either in the detail process, or in any method for estimating run-off, a table of values for the run-off coefficient should provide for changing intensities as well as for varying times.

PART II.—RATIONAL RUN-OFF FORMULAS

DERIVATION OF FORMULAS

In Equation (2) let $t = \frac{L}{v}$, then,

$$i = K \frac{v^e}{L^e} \qquad (10)$$

Substituting this value of i in Equation (1),

$$Q = C\,A\,K \frac{v^e}{L^e} \qquad (11)$$

Let $R_H = \frac{X}{H}$, as, for example, $R_{0.5} = \frac{X}{0.5}$, etc. When $i = \frac{K}{t^e}$, $R_H = \frac{K}{(3\,600\,H)^e}$, the time being expressed in seconds; or,

$$K = R_H\,(3\,600\,H)^e \qquad (12)$$

Velocity may be expressed by the Chezy formula,

$$v = c\sqrt{r\,s} \qquad (13)$$

Substituting these values of K and v in Equation (11),

$$Q = (3\,600\,H)^e\,C\,A\,R_H \left(c\sqrt{r}\right)^e \frac{s^{\frac{e}{2}}}{L^e} \qquad (14)$$

Comparisons.—It should be of interest here to digress temporarily, and convert Equation (14) into the form of the ordinary empirical formula.

If S = fall in feet per 1 000 ft. of channel, or $s = \frac{S}{1\,000}$; if the exponent, e, is given the value of 0.5; and if $R_H = R_1$ (the intensity in inches per hour for a period of 1 hour), then Equation (14) becomes,

$$Q = 10.67\,C\,A\,R_1 \left(c\sqrt{r}\right)^{0.5} \frac{\sqrt[4]{S}}{\sqrt{L}} \qquad (15)$$

The ordinary empirical formula takes no account of the shape of the drainage area; it is usually customary to assume (which later will be shown to be erroneous) that,

$$L = K'\sqrt{A} \qquad (16)$$

or some constant times $\sqrt{A}$. Assuming, for the purpose of illustration, that Equation (16) is correct, and substituting in Equation (15),

$$Q = 10.67\,K'' \left(c\sqrt{r}\right)^{0.5} C\,R_1 \sqrt[4]{S\,A^3} \qquad (17)$$

It is interesting here to observe that Equation (17) results in the well-known Bürkli-Ziegler formula when the expression, $10.67\,K'' \left(c\sqrt{r}\right)^{0.5} C$, is considered as a single run-off coefficient. The Hawksley formula may be reduced to similar terms for exponents of S and A, as shown[*] by the late Charles E. Gregory, M. Am. Soc. C. E. Since these and other empirical formulas neglect the shape of the drainage area, do not vary the fluctuating value of the average velocity, or its function $\left(c\sqrt{r}\right)^e$, and make no allowance for a variation in the nature of the rainfall curve, results accurate enough for general use cannot be reasonably expected from their application.

---

[*] *Transactions*, Am. Soc. C. E., Vol. LVIII (1907), p. 461.

The value of L is independent of A, not only since the area may be any shape in so far as the exterior boundary is concerned, but also because the water may concentrate to the main channel in an infinite number of ways. To assume, then, that $L = K'\sqrt{A}$, not only presupposes a constant exterior shape of area, but also a constant law of concentration within the area, and is, of course, erroneous.

Introducing Factor F.—It follows, then, that in order to approach accuracy through the medium of a formula, the value of L should be retained, and the effect of velocity in the main channel of flow should be allowed to vary.

In Equation (14), the expression representing varying velocity, as far as shape and condition of the main channel of flow are concerned, is $\left(c\sqrt{r}\right)^e$. At the present time, Kutter's formula for c in $c\sqrt{r\,s}$ is probably used more widely than any other; and, on account of its general adaptability to all shapes and conditions of channel, it affords a feasible basis for a variable velocity factor in run-off formulas. Since the velocity varies inversely as the time, the velocity factor, $c\sqrt{r}$, in a run-off formula, carries the exponent of the time, e. Due to the construction of rainfall curves this exponent varies in practice from about 0.4 to 0.6, the average probably being approximately 0.5.

In order to visualize more easily the effect of the fluctuating value of $c\sqrt{r}$ on run-off, the writers made many listings from "Hydraulic and Excavation Tables", published by the United States Reclamation Service. The process consisted of assuming first, a definite shape of channel, such as a circular pipe, or a closed rectangular box with a constant ratio of width to height. For a definite shape of channel on a given slope and Kutter's n, the hydraulic radius was allowed to vary, the corresponding velocity was selected from the hydraulic tables, and the values of Q, area, $c\sqrt{r}$, and $\left(c\sqrt{r}\right)^{0.5}$, were calculated.

It was found that for any one definite shape of channel on a given slope and Kutter's n, $\left(c\sqrt{r}\right)^{0.5}$, divided by $Q^{\frac{1}{8}}$, remained practically constant for all values of $c\sqrt{r}$ greater than 50 to 60, or,

$$\left(c\sqrt{r}\right)^{0.5} = N\,Q^{\frac{1}{8}} \qquad (18)$$

in which, N represents a constant. More recent experimentation has shown that Kutter's formula is somewhat in error for very small sections. In selecting values of N (obtained by solving Equation (18)), choose from the Hydraulic Tables of the Reclamation Service, hydraulic radii rendering values of $c\sqrt{r}$ greater than 60; velocities may then be calculated, for the smaller sections, which are closer on the average to recent experimentation than those given by the use of Kutter's formula. In any case Equation (18) offers a reasonably accurate basis for obtaining a variable velocity factor, representing the shape and condition of the main channel of flow, for run-off formulas. The method of arriving at values of N and F is shown in more detail in the Appendix II.

As shown in Equation (14), $Q = (3\,600\ H)^e\ CAR_H\ (c\sqrt{r})\ \frac{s^{\frac{e}{2}}}{L^e}$; but, hydraulically, $(c\sqrt{r})^{0.5} = N\ Q^{\frac{1}{8}}$, or $c\sqrt{r} = N^2\ Q^{0.25}$. It must be remembered that the term, $c\sqrt{r}$, occurring in Equation (14), represents the average value of $c\sqrt{r}$ as determined by the average velocity of the water in running from the most remote part of the water-shed to its outlet. In this equation then, let $c\sqrt{r} = P\ N^2\ Q^{0.25}$, or,

$$(c\sqrt{r})^e = P^e\ N^{2e}\ Q^{\frac{e}{4}} \ldots\ldots\ldots\ldots\ldots\ldots\ldots (19)$$

Methods of arriving at values of P are discussed subsequently in this paper.

Substituting $s = \frac{S}{1\,000}$, and the value of $(c\sqrt{r})^e$ shown in Equation (19), into Equation (14),

$$Q = \frac{(3\,600\ H)^e}{(1\,000)^{\frac{e}{2}}}\ CAR_H\ P^e\ N^{2e}\ Q^{\frac{e}{4}}\ \frac{S^{\frac{e}{2}}}{L^e} \ldots\ldots\ldots\ldots\ldots (20)$$

From Equation (18), $N = \frac{(c\sqrt{r})^{0.5}}{Q^{\frac{1}{8}}}$, that is, $N \propto \frac{1}{Q^{\frac{1}{8}}}$; but Q = area $\times\ (c\sqrt{r\,s})$. Therefore, Q, for a given channel, may be considered to vary directly as the square root of s, since the value of c varies only slightly with a change in slope. Then, it may be written that $Q \propto S^{\frac{1}{2}}$; and, since $N \propto \frac{1}{Q^{\frac{1}{8}}}$, $N \propto \frac{1}{S^{\frac{1}{16}}}$, or, $N = \frac{F}{S^{\frac{1}{16}}}$, in which, F represents a constant (see Appendix II).

Then,

$$N^{2e} = \frac{F^{2e}}{S^{\frac{e}{8}}} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (21)$$

Substituting this value of N^{2e} in Equation (20),

$$Q = \frac{(3\,600\ H)^e}{(1\,000)^{\frac{e}{2}}}\ CAR_H\ P^e\ Q^{\frac{e}{4}}\ F^{2e}\ \frac{S^{\frac{3e}{8}}}{L^e} \ldots\ldots\ldots\ldots\ldots (22)$$

$$Q^{1-\frac{e}{4}} = \frac{(3\,600\ H^e)}{(1\,000)^{0.5e}}\ CAR_H\ P^e\ F^{2e}\ \frac{S^{\frac{3e}{8}}}{L^e} \ldots\ldots\ldots\ldots\ldots (23)$$

or, if the exponent, $\frac{1}{4-e} = g$:

$$Q = \frac{(3\,600\ H)^{4eg}}{(1\,000)^{2eg}} \left(\frac{P}{L}\right)^{4eg} (CAR_H)^{4g}\ F^{8eg}\ S^{1.5eg} \ldots\ldots\ldots\ldots (24)$$

Equation (24) is a general expression for the value of Q with seven main factors: $C, A, R_H, S, P, F,$ and L. Factor C, of course, should be determined from experimental information, while A, S, and L must be derived from topographical maps or surveys.

Factor F varies with the shape and condition of the main channel of flow. Values for this factor may be selected from Table 11 in Appendix II. Factor P may be selected from Tables 3 and 4. The method used for the derivation of the values in these tables is discussed in another part of this paper.

Factor R_H is necessarily involved with H and e. The combination renders flexibility in the choice of the time period for observation of rainfall data, the type of rainfall curve, and storms of various expected frequency periods. Thus, for the smaller areas, it may be found advisable to choose 1 hour as the guiding period for rainfall observation, in which case, $H = 1$ hour, and R_H is represented by R_1. For the larger areas circumstances may make more feasible the adoption of longer time periods, as $H = 6, 12, \ldots 24$, with $R_H = R_6$, R_{12}, or R_{24}, as the case may be. Thus, when $R_H = \frac{X}{H}$, the expected rainfall may be $4\frac{1}{2}$ in. for a period of 6 hours. The average expected rainfall intensity expressed in inches per hour, for a 6-hour period, equals $R_H = \frac{X}{H} = \frac{4.5}{6.0} = 0.75$. The exponent, e, of the time in Equation (2) determines the type of rainfall curve.

Inadequacy of Empirical Formulas

At this point the reader may gain, by comparison, a more thorough comprehension of the inadequacy of the leading formulas of the empirical type, the construction of which offers opportunity for variation only in Factors C, A, R, and S. This comparison may be clarified by five statements as follows:

(*a*) Factor R_H of Equation (24) is represented in the empirical formula by R. The factor, R, in the empirical formula is usually assumed to represent the expected average rainfall intensity, in inches per hour, for a 1-hour period. This is, of course, a specialization of R_H.

(*b*) The exponent, e, of Equation (24) is necessarily given a constant value in any one empirical formula, thus rendering inflexibility in the choice of the type of the rainfall curve.

(*c*) Factor P of Equation (24), is, in any one empirical formula, given a constant value. The reader may gain some conception of the range of variation in this factor by a study of Tables 3 and 4.

(*d*) In the same manner, Factor F of Equation (24) is represented in any one empirical formula by some constant. Opportunity for measuring the effect on run-off, due to variation in shape and condition of the main channel of flow, is thereby rendered impossible. (See Table 11, Appendix II.)

(*e*) In the empirical formulas, in which the length of the main channel of flow, L, does not appear, the effect on run-off due to the shape of the area cannot be measured. The absence of L entails the assumption that it equals some constant times A to an exponent, as $L = K' \sqrt{A}$, thereby precluding the possibility of weighing the effect on run-off caused by varying the shape of the water-shed either in external boundary or manner of internal concentration.

REDUCTION OF EQUATION (24)

This equation reduces to simpler terms, once the required data on rainfall are known. For example, should the rainfall data for a given locality point to the use of a curve following the equation, $i = \frac{K}{t^{0.6}}$, and should it be found advisable to use a 12-hour time period, then $H = 12$ and $e = 0.6$.

Substituting these values, Equation (24) reduces to:

$$Q = 163.5 \left(\frac{P}{L}\right)^{0.706} (CAR_{12})^{1.176} F^{1.412} S^{0.265} \ldots\ldots\ldots\ldots (25)$$

Thus, when H and e are once adopted, Equation (24) reduces to the seven main factors with definite exponents, prefixed with a constant. Where a formula of the type of Equation (25) is constantly in use, it probably would be found advisable to reduce it to four variables and solve by nomograph in two parts, such that $\left(\frac{P}{L}\right)^{0.706} \times (CAR_H)^{1.176}$ forms one nomograph, and $F^{1.412} S^{0.265}$, another. The formula also lends itself to quick solution by log-log slide-rule.

To find the time of concentration compute the value of i from Equation (1). For this purpose, Q is determined from Equation (24) reduced. The time, t, may then be read from a curve of Equation (2). If there is no diagram, t, in minutes, may be computed from Equation (2) by substituting for K its value in Equation (12) and dividing the result by 60; thus,

$$t = \frac{60\, H\, R_H^{\frac{1}{e}}}{i^{\frac{1}{e}}} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (26)$$

Special Reduction of Equation (24).—It is realized that occasions may arise necessitating the estimation of run-off in territories where adequate rainfall records are not available. Even in localities where available records vary somewhat from average conditions, it may be found unnecessary, or economically inadvisable, to go into great refinement in the making of rainfall curves.

Mr. Grunsky[10] and others advocate the use of a rainfall curve of the form of Equation (3). Moreover, the writers have found that it applies very well to conditions in Southern California, as, for example, it conforms very closely to the curves now used by the City of Los Angeles, the Los Angeles County Flood Control Department, and the Los Angeles County Storm Drain Division. The writers, therefore, have developed the following special reduction with a twofold purpose in mind, namely, (*a*), to furnish a formula which will be approximately applicable for general use; and (*b*), one which may very readily be used for making quick transitions to other types of curves as subsequently recommended.

In Equation (24) if $e = 0.5$, $g = 0.2857$, and,

$$Q = 15\, H^{0.5714} \left(\frac{P}{L}\right)^{0.5714} (CAR_H\, F)^{1.1429} S^{0.2148} \ldots\ldots\ldots\ldots (27)$$

---

[10] *Transactions*, Am. Soc. C. E., Vol. LXXXV (1922), p. 67.

and since the expression, $\left(\frac{P}{L}\right)^{0.5714} = \left(\sqrt{\frac{P}{L}}\right)^{1.1429}$, Equation (27) may be written:

$$Q = 15\ H^{0.5714}\ (C A R_H F B)^{1.1429}\ S^{0.2143} \qquad (28)$$

in which, $\sqrt{\frac{P}{L}} = B$, which may readily be obtained by slide-rule.

Equation (28) reduces to two variables if the time period, H, is adopted, and the R_H value is determined from the relation, $R_H = \frac{X}{H}$. Hence, when $H = 1$ hour, Equation (28) reduces to:

$$Q = 15\ (C A R_1 F B)^{1.1429}\ S^{0.2143} \qquad (29)$$

Whenever the adopted rainfall curve varies materially from Equation (3), use may be made of Equation (28) to give an approximate value of Q to be corrected as follows:

Substitute the values of Q, C, and A, of Equation (28) in Equation (1) and solve for i. Then obtain the time, t, in minutes, from a diagram of Equation (3), or by solving Equation (26). Using this value of t, obtain a corresponding value of i from the curve in use. With this value of i solve Equation (1) for a corrected Q, which will be a close approximation to the quantity of run-off to be expected from the use of the adopted rainfall curve.

METHOD OF ARRIVING AT VALUES OF FACTOR *P*

Factor P has been defined as the ratio of the average velocity of flow to velocity at the outlet of the water-shed, based on average values of S and F or $P = \frac{v}{V}$. Upon inspection of Equation (24), reduced to its seven main factors—C, A, R_H, S, P, F, and L—it may be readily seen that when average values for a given water-shed are assigned to Factors S, F, C, and R_H (these factors then assume the rôle of constants in the formula), the variation in Factor P is determined by the remaining two factors, A and L. The fluctuation in Factor P, therefore, depends on the shape of the drainage area, both as to external boundary and internal concentration.

It is evident, of course, that P may have an infinite number of values. Since, in any rational method of estimating run-off, inlet time must be considered, the value of P is further complicated. Factor $P = \frac{v}{V}$; but $v = \frac{L}{t}$, in which, t = time of concentration of the water-shed, including the initial, or inlet, time of the first small area, or nucleus.

Before attempting to devise a table of values for Factor P it was necessary to investigate both the use of the rainfall curve and the run-off coefficient (particularly as regards its variation with the time). The reader at this point is assumed to be familiar with the conclusions under headings, "Use of Rainfall Curve," and "Factor C, The Run-Off Coefficient." For a given area it may be reasoned that:

(*a*) The use of a factor, C, increasing with the time would give a value of P somewhat lower than a constant C.

(*b*) The use of a factor, C, decreasing with the time would give a value of P somewhat higher than a constant C.

(*c*) The type of storm assumed for a given water-shed has a certain effect on the expected run-off. This effect influences not only the run-off coefficient, but also Factor P. If, for a given area, it is assumed that the storm for the time of concentration decreases in intensity with the time, a somewhat higher value of P results than that for the assumption that the storm is one of uniform intensity throughout.

The effects on Factor P of the foregoing considerations under Paragraphs (*a*), (*b*), and (*c*), are, however, not of major importance, since (except for extreme conditions) the final run-off is varied only a few per cent. one way or the other. The latitude for variation in the run-off result, due to errors in judgment in choosing values for Factor C, R_H, and F, is large in comparison; but, on account of the many varying conditions entering into the estimation of run-off by any method, the engineer, as far as practicable, should be able to visualize all limitations or advantages of any particular formula or system.

Estimation of run-off inevitably depends to a great extent on individual judgment. The writers, in evolving formulas, are merely attempting to aid this judgment in a quick solution. In the face of so many variations it would be highly impracticable to attempt a mathematical solution of P for the purpose of making the tables of values for use in the formulas. It is possible, however, to establish workable tables by trial method, that should be far superior at least to the use of a constant (single) value of this factor, as is automatically done when an empirical formula is used. An average, or constant, value of C was used in preparing Tables 3 and 4. The values in Table 3 are based on the assumption of a storm of uniform (or constant) intensity for the time of concentration. Hence, a constant intensity throughout was used on any one area type shown in Fig. 2. Table 3, therefore, should be used when the assumption is that the expected storm is one of uniform intensity for the time of concentration. Referring to Conclusion (13) under "Factor C, The Run-Off Coefficient" (Part I), it may be seen that the use of this table carries with it a small factor of safety, which is comparable with, but smaller than, that automatically assumed in the usual detail process of the rational method (see "Statement" following Conclusion (4)); and its use, rather than being objectionable, is to be commended, especially when the possible variation in types of storms is taken into consideration.

The values in Table 4 were evolved through the use of a rainfall curve of the type of Equation (3), in the manner ordinarily used in the detail process of the rational method. This table, therefore, involves the small factor of safety resulting from the use of Equation (3) as ordinarily practiced in the detail rational method.

Values of P in Tables 3 and 4 may be used in any formula reduced from Equation (24). In preparing Tables 3 and 4, the procedure was as follows:

1.—Areas, shown on Fig. 2, were chosen for the purpose of establishing practical limits of shape, so that interpolations could be made for values of P.

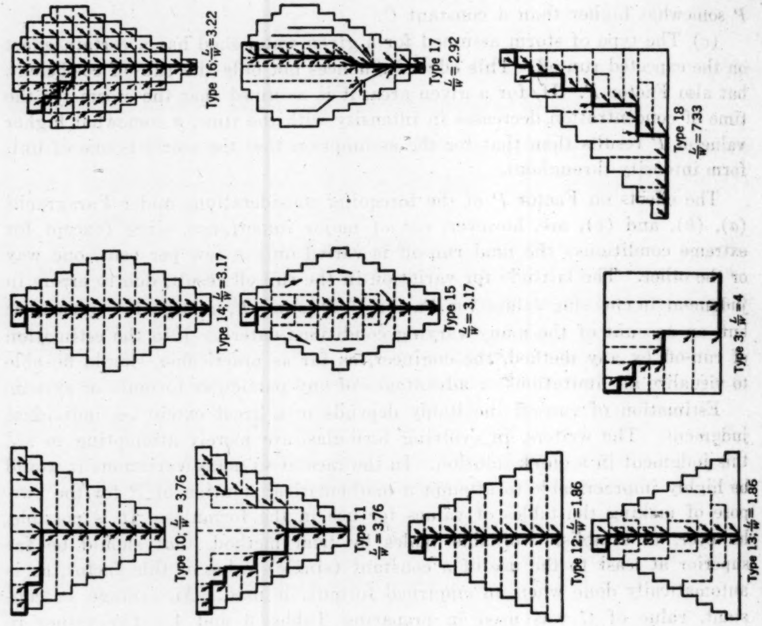

Type 16, $\frac{L}{W}=3.22$
Type 17 $\frac{L}{W}=2.92$
Type 18 $\frac{L}{W}=7.33$
Type 14, $\frac{L}{W}=3.17$
Type 15 $\frac{L}{W}=3.17$
Type 3, $\frac{L}{W}=4$
Type 10, $\frac{L}{W}=3.76$
Type 11 $\frac{L}{W}=3.76$
Type 12, $\frac{L}{W}=1.86$
Type 13, $\frac{L}{W}=1.86$
Type 8 $\frac{L}{W}=18.00$
Type 9 $\frac{L}{W}=17.00$
Type 2, $\frac{L}{W}=2.25$
Type 7 $\frac{L}{W}=11.20$
Type 1, $\frac{L}{W}=2.25$
Type 6 $\frac{L}{W}=11.20$
Type 4 $\frac{L}{W}=5.06$
Type 5 $\frac{L}{W}=6.32$

FIG. 2.—TYPICAL AREAS AND THEIR MANNER OF CONCENTRATION

2.—In making Table 3, a constant intensity and an average value for each of the factors, S, C, and F, were used on any one area type.

3.—In the case of Table 4, a value (any practical value) for each of the factors, S, C, R_H, and F, was chosen, the same values being used on all areas.

4.—The areas were subdivided for concentrating the water to the main channel of flow (shown on Fig. 2) anticipating, as far as practicable, actualities to be met in drainage practice. For purposes of interpolation the manner of concentration along the main channel of flow was designated advanced, medium, or delayed, concentration. What constitutes the three types of concentration may best be ascertained by studying Tables 3 and 4, and Figs. 2 and 4. Extreme delay or advancement was avoided.

5.—It was necessary to assume a beginning nucleus, or initial area. For the purpose of striking the best average a square nucleus, with the value of $\frac{L'}{W'} = \frac{L'}{\frac{43\,560\,A'}{L'}} = \frac{(L')^2}{43\,560\,A'} = 4$. For the purpose of providing a sufficient range of interpolation for values of P to meet the expected need, varying values of P', ranging from 0.1 to 2.0, were used on the various area types. The value of $P_1 = \frac{v'}{V'}$, is to be corrected for use in Tables 3 and 4 through reference to Fig. 3, giving P'. Thus, in Tables 3 and 4,

$$P' = f\,P_1 = f\,\frac{v'}{V'} \quad \text{.....................} (30)$$

in which, f is a correction factor read from Fig. 3.

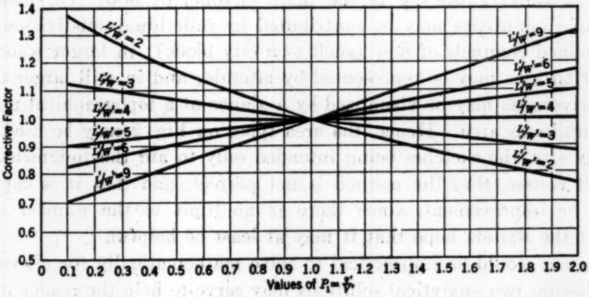


FIG. 3.—DIAGRAM TO DETERMINE CORRECTION FACTOR IN EQUATION (30).

For areas of the same shape and manner of concentration the value of P is the same (irrespective of the size of the area) for a given assumption of P' on the nucleus. However, in order to have the same manner of concentration, the nucleus of each area must be of the same shape and must vary in size with the area of the water-shed. It follows, then, that when the size and shape of the nucleus remain the same in all areas of similar shape and manner of concentration, the value of P changes somewhat with the area.

FIG. 2.—TYPICAL AREAS AND THEIR MANNER OF CONCENTRATION

In Tables 3 and 4 a constant nucleus was used; hence, the variation in P with the area divided by area of nucleus as shown.

The ratio, $\frac{L}{W}$, in these tables was adopted as a practical means of helping to define the change in area shape when used in conjunction with the typical areas shown in Fig. 2, which are also intended for guides as to the manner of internal concentration. The ratio, W, equals area of water-shed, in square feet, divided by L. Hence,

$$\frac{L}{W} = \frac{L^2}{43\,560\,A} \quad \text{(31)}$$

Fig. 4 will be found useful in the determination of delayed, medium, or advanced concentration. This diagram is intended to be used as an aid in determining the manner of concentration. The numbered points refer to the area types shown on Fig. 2. The relative positions of the various points for a given value of P' were determined by using values of P for abscissas (not shown on the diagram to attain compactness), and $\frac{L}{W}$ for ordinates.

P' having been determined, and the area type on Fig. 2 (most nearly representing the area being studied) selected, enter Fig. 4 under the value of P' and note the position of the area type with respect to the concentration lines marked "Delayed", "Medium", and "Advanced"; then enter Table 3 or Table 4 under the heading of the concentration chosen on Fig. 4.

The manner of concentration of a given water-shed, designated in this work, delayed, medium, or advanced, is, of course, determined by the relative quantities of water gathering to the main channel of flow. In small areas these relative quantities may be contributed by individual subdivisions without well-defined channels of flow (such as a city block); in larger water-sheds these contributions may be represented by laterals; and in still larger ones by creeks, or rivers, as may be visualized by a glance at a topographical map of a large mountainous area. Hence, the area types on Fig. 2 may be used for an area of any size, the sketches being intended only to aid the judgment. It is realized, of course, that the method is not perfect, and that it is capable of expansion or improvement, since there is no limit to the number of area shapes; yet the writers hope that it may at least be helpful.

Although it would be impractical to solve mathematically for all values of P, the following two analytical solutions may serve to help the reader in forming a conception of the factor's usefulness.

Let Fig. 5(*a*) and Fig. 5(*b*) represent water-sheds of triangular and rectangular shapes, as shown, each subjected to uniform intensities, the water being assumed to concentrate uniformly to the main channels of flow, designated by the heavy lines. The run-off coefficient is assumed to be constant, and the velocities (combining Equations (13) and (18)) in the main channels of flow are determined by,

$$v = N^2 \, Q^{\frac{1}{4}} \, s^{\frac{1}{2}} \quad \text{(32)}$$

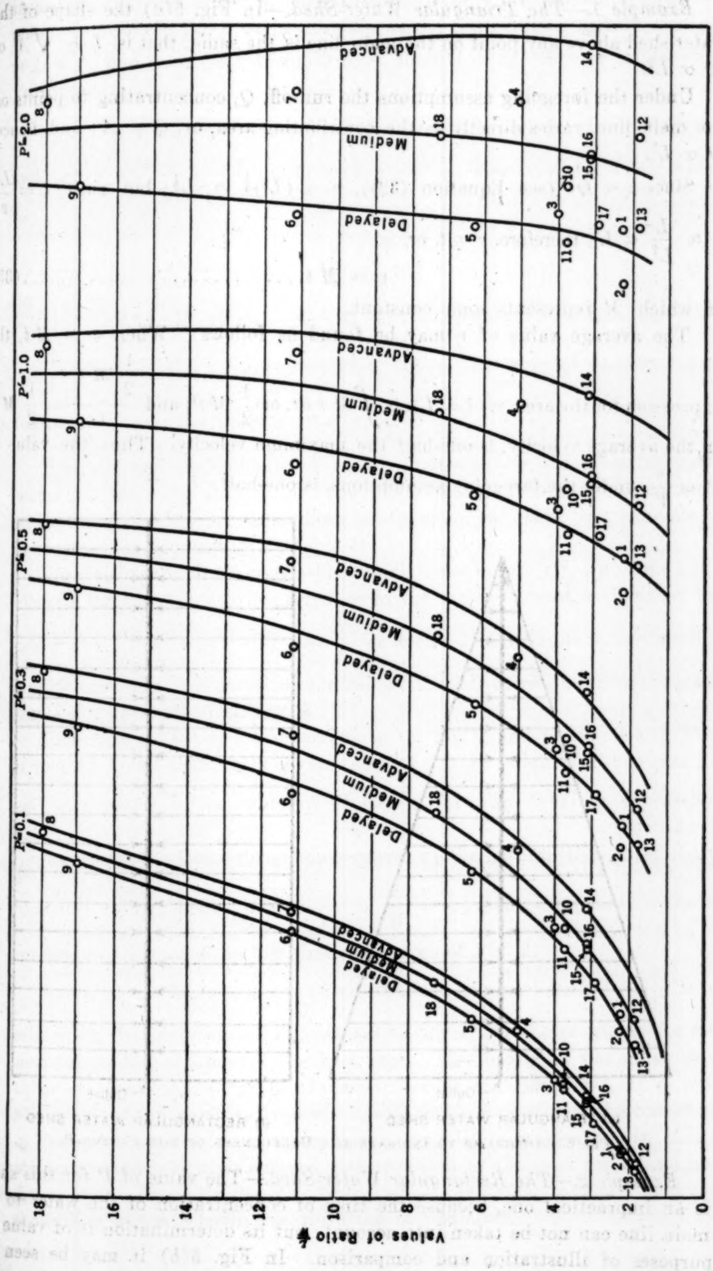


FIG. 4.—GUIDE LINES TO AID IN DETERMINATION OF THE MANNER OF CONCENTRATION

Example 1.—The Triangular Water-Shed.—In Fig. 5(*a*) the shape of the water-shed above any point on the main line is the same; that is, $L \propto \sqrt{A}$, or $A \propto L^2$.

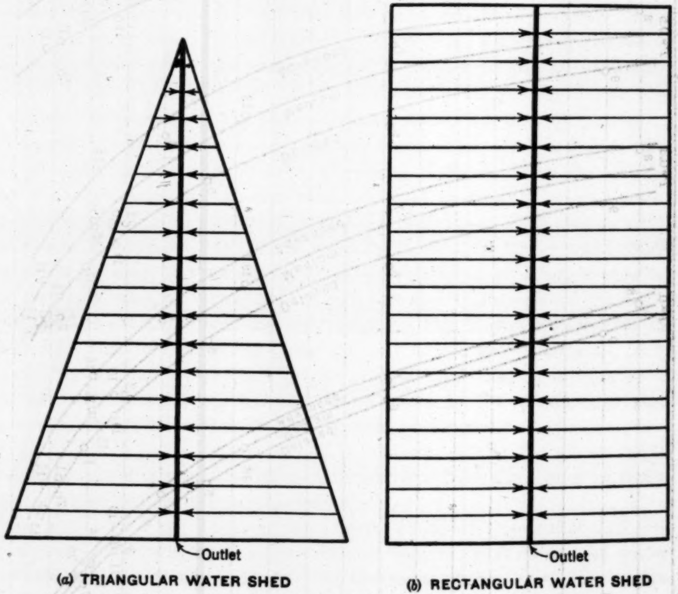
Under the foregoing assumptions the run-off, Q, concentrating to points on the main line, varies directly as the contributing area, or, $Q \propto A$; and, hence, $Q \propto L^2$.

Since $v \propto Q^{\frac{1}{4}}$ (see Equation (32)), $v \propto (L^2)^{\frac{1}{4}} \propto L^{\frac{1}{2}}$; but since $t = \frac{L}{v}$, $t \propto \frac{L}{L^{\frac{1}{2}}} \propto L^{\frac{1}{2}}$, therefore, $v \propto t$, or,

$$v = M\,t \qquad (33)$$

in which, M represents some constant.

The average value of v may be found as follows: When $v = M\,t$, the expression for the area, $v\,t\ (= L)$, is $\int M\,t\,dt$, or $\frac{1}{2} M\,t^2$; and $\frac{\frac{1}{2} M\,t^2}{t} = \frac{1}{2} M\,t$, or the average velocity, is one-half the maximum velocity. Thus, the value of $P = \frac{v}{V}$, under the foregoing assumptions, is one-half.



(*a*) TRIANGULAR WATER SHED (*b*) RECTANGULAR WATER SHED

FIG. 5.—DIAGRAMS TO INDICATE THE USEFULNESS OF THE FACTOR P

Example 2.—The Rectangular Water-Shed.—The value of P for this shape is an impractical one, because the time of concentration of the water to the main line can not be taken into account; but its determination is of value for purposes of illustration and comparison. In Fig. 5(*b*) it may be seen by inspection that the length, L (measured from the top of the water-shed to any point on the main line), varies directly as the contributing area to that point, or $L \propto A$; and since (as before), $Q \propto A$, then $Q \propto L$. Since $v \propto Q^{\frac{1}{4}}$, $v \propto L^{\frac{1}{4}}$ Furthermore,

$$t = \frac{L}{v} \dots\dots\dots\dots\dots\dots\dots\dots\dots (34)$$

and, hence, $t \propto \dfrac{L}{L^{\frac{1}{4}}} \propto L^{\frac{3}{4}}$, or, $L \propto t^{\frac{4}{3}}$; therefore, since $v \propto L^{\frac{1}{4}}$, $v \propto \left(t^{\frac{4}{3}}\right)^{\frac{1}{4}} \propto t^{\frac{1}{3}}$, or,

$$v = M'\, t^{\frac{1}{3}} \dots\dots\dots\dots\dots\dots\dots\dots\dots (35)$$

Therefore, $\int M'\, t^{\frac{1}{3}}\, dt = \dfrac{3}{4}\, M'\, t^{\frac{4}{3}} =$ area, $v\, t\ (= L)$, and $\dfrac{\frac{3}{4}\, M'\, t^{\frac{4}{3}}}{t} = \dfrac{3}{4}\, M'\, t^{\frac{1}{3}} =$ the average velocity. Thus, the average velocity is three-fourths the maximum velocity, or $P = \dfrac{v}{V} = \dfrac{3}{4}$.

Remarks and Deductions.—Fig. 5(a) represents a closed water-shed, the water concentrating uniformly to the main channel of flow, and, since the shape of the water-shed is the same above points along the main line, the value of P is constant, as deduced in Example 1.

Fig. 5(b) represents a water-shed which is not closed, since no time has been allowed for the water to concentrate to the main channel, designated by the heavy line. The value of $P = \dfrac{3}{4}$, as deduced in Example 2, is, therefore, of significance only in illustrating the values in Table 3, evolved from the practical limiting shapes, shown in Fig. 2.

Referring to Types 1 to 9, inclusive, of Fig. 2 and to Fig. 4 and Table 3, it may be noted that, for a given value of P', the value of P becomes higher as $\dfrac{L}{W}$ increases. The value of $P = \dfrac{3}{4}$ (for Fig. 5(b)) is approached as $\dfrac{L}{W}$ increases, the initial delay being caused by the natural division of the water in concentrating; hence, the choice of $\dfrac{L}{W}$ as a guiding influence in the determination of Factor P for the limiting shapes of Fig. 2.

Referring to Table 3, it is apparent that the column headed by $\dfrac{A}{A'}$ is necessitated by the inclusion of varying values for P', or the provision for assignment of an initial time for the inlet area. This provision was made in order to create possibilities for the broadest use of the formulas. It is evident, however, that for many water-sheds, especially the larger ones, the consideration of the inlet area as a separate entity, may be unnecessary. In this case, the water-shed is neither delayed, nor advanced by the inlet areas, and its condition at the outset of the maximum run-off producing intensity is ready for the falling water to begin concentrating immediately.

When the inlet area is not considered separately, the slope, S, of the water-shed must be based on the total slope from the most remote area (including the nucleus) to the outlet, and the value of $\frac{A}{A'}$ in Table 3 must be considered as infinite.

As $\frac{A}{A'}$ increases (see Table 3), the values of P decrease under the columns headed by the higher values of P' and increase for the lower values of P'. The natural convergence of these values as $\frac{A}{A'}$ approaches infinity is approximately to the column headed by $P' = 0.5$.

For all practical purposes, therefore, the values of P (in Tables 3 and 4) shown opposite $\frac{A}{A'} = 262\ 144$, and in the column headed by $P' = 0.5$, may be used when the formulas are applied without separate consideration of the inlet area, or nucleus.

THE INLET AREA, OR NUCLEUS

In any rational method of estimating run-off special consideration in many cases should be given to the choice of the beginning area of the water-shed. The time of concentration for the water-shed includes the time required for the water to gather in the first small area, or nucleus, and travel to a defined channel of flow. This beginning area is a small water-shed and its time of concentration is the time required for the water to travel from the most remote portion to its outlet. The average velocity of the nucleus may be defined as the distance which the water must travel from the most remote portion to the outlet, divided by its time of concentration. This average velocity, in the same small nucleus (such as a city block), may consist of flow over grass plots, drive-ways, and gutters; in rural areas, it will be retarded by vegetation, or increased by barrenness.

It is customary for those who use the detail process to assume an inlet time, which is based on field observations. Reliable data on inlet time are scarce, and more experimentation is needed.

For some water-sheds the run-off formulas herein may be applied without separate consideration of the nucleus. For all water-sheds the formulas may be used to approximate the expected value of Q and to estimate times of concentration without separate consideration of the nucleus.

When the surface of the water-shed is in a saturated condition at the onset of the maximum run-off producing intensity, the inlet time of concentration for a high intensity will be shorter than that for lesser ones, due only to the greater quantity of water flowing. If the surface is not saturated the time of concentration for the greater intensities will be shorter than for lesser ones, due both to the greater quantity of water flowing and to a shorter period required for saturating the surface sufficiently for run-off to occur.

In any method of computing run-off great care should be used in choosing the main channel of flow on which the time of concentration for the water-shed so largely depends. The run-off, Q, sought for a given water-shed is the maximum that may be had for definite assumptions on rainfall. For some areas, with extreme contrasts in values of characteristics governing the run-off coefficient, and slope, shape, and condition of the main channel of flow, a part of a given water-shed may be found to give a greater run-off than the whole.

Experience is of value in the choice of the main channel of flow; but, even so, it may sometimes be found necessary to determine it by trial.

The initial area, or nucleus, is a part of the water-shed (and the length, L, of the water-shed includes L' of the nucleus). Consequently, the choice of the nucleus plays no small part in the determination of the maximum run-off. A good conception of the influence of the nucleus may be had from a study of Tables 3 and 4.

AVERAGE SLOPE

Heretofore, in using formulas for the estimation of run-off, the writers have been unable to determine a practical method for calculating the average slope. Of course, the slope should be that of the main channel of flow, that is, the channel which determines the time of concentration of the water-shed.

The average hydraulic slope is not the average of the various slopes weighted by the individual lengths, the sum being divided by the total length as, $S = \frac{1}{l}\,(L_1\,S_1 + L_2\,S_2 + L_3\,S_3 \text{ etc.})$, in which, l equals the sum, $L_1 + L_2 + L_3$, etc. By inspection it may be seen that this average slope is identical with $S = \frac{\text{total fall}}{\text{total length}}$, which would be easily obtained.

Average hydraulic slope depends on the velocities induced by the individual slopes. The average velocity, therefore, will be induced by the true average hydraulic slope. Since the average velocity equals the total length divided by the time, or $v = \frac{l}{t}$, it may be seen that the true average hydraulic slope is that which, with a given quantity of water flowing through length, l, will cause the same total length of time to elapse as that consumed in traveling the slopes, S_1, S_2, S_3, etc.

In Equation (32), $v = N^2\,Q^{\frac{1}{4}}\,\sqrt{s}$, or $v \propto N^2\,\sqrt{S}\,Q^{\frac{1}{4}}$. Since $N = \frac{F}{S^{\frac{1}{16}}}$ (according to Equation (21)), $v \propto F^2\,S^{\frac{3}{8}}\,Q^{\frac{1}{4}}$; but the time, t, varies as length divided by velocity, or $t \propto \frac{l}{v} \propto \frac{l}{F^2\,S^{\frac{3}{8}}\,Q^{\frac{1}{4}}}$. It follows, therefore, that if the average hydraulic slope is to cause the same lapse of time as that consumed by the water in traversing the individual slopes, S_1, S_2, S_3, etc.,

$\frac{1}{S_1^{\frac{3}{8}}}$, $\frac{1}{S_2^{\frac{3}{8}}}$, $\frac{1}{S_3^{\frac{3}{8}}}$, etc., should be weighted by $\frac{L_1}{F_1^2\,Q_1^{\frac{1}{4}}}$, $\frac{L_2}{F_2^2\,Q_2^{\frac{1}{4}}}$, $\frac{L_3}{F_3^2\,Q_3^{\frac{1}{4}}}$, respectively. For example,

$$\frac{1}{S^{\frac{3}{8}}} = \frac{\dfrac{L_1}{F_1^2\,Q_1^{\frac{1}{4}}\,S_1^{\frac{3}{8}}} + \dfrac{L_2}{F_2^2\,Q_2^{\frac{1}{4}}\,S_2^{\frac{3}{8}}} + \text{, etc.}}{\dfrac{L_1}{F_1^2\,Q_1^{\frac{1}{4}}} + \dfrac{L_2}{F_2^2\,Q_2^{\frac{1}{4}}} + \text{, etc.}}$$

or, inverting,

$$S^{\frac{3}{8}} = \frac{\dfrac{L_1}{F_1^2\,Q_1^{\frac{1}{4}}} + \dfrac{L_2}{F_2^2\,Q_2^{\frac{1}{4}}} + \text{, etc.}}{\dfrac{L_1}{F_1^2\,Q_1^{\frac{1}{4}}\,S_1^{\frac{3}{8}}} + \dfrac{L_2}{F_2^2\,Q_2^{\frac{1}{4}}\,S_2^{\frac{3}{8}}} + \text{, etc.}} \quad \text{.........(36)}$$

Thus, $S^{\frac{3}{8}}$, and then S, the average slope, may be solved.

Example 3.—F and Q Constant.—Either s or S may be used throughout; F and Q, being constant, drop them from Equation (36) and assume $L_1 = 1\,848$ ft., with $S_1 = 90$, and $L_2 = 8\,316$ ft., with $S_2 = 0.9$ then (computations by slide-rule) substituting these values in Equation (36), $S^{\frac{3}{8}} = 1.131$; and $S = 1.39$ ft. per 1 000 ft.

A check on this answer is as follows: Assume that $Q = 200$ cu. ft. per sec., and that $F = 6.63$, then by Equations (21) and (32),

$$v = N^2 \sqrt{s}\ Q^{\frac{1}{4}} = \frac{F^2}{S^{\frac{1}{8}}} \sqrt{s}\ Q^{\frac{1}{4}} \quad \text{..................(37)}$$

Substituting the assumed values in Equation (37), $v_1 = 28.3$ ft. per sec.; and $t_1 = \frac{L_1}{v_1} = \frac{1848.0}{28.3} = 65$ sec.

Similarly (see Equation (37)), $v_2 = \frac{44.0}{0.988} \times 0.03 \times 3.76 = 5.0$ ft. per sec.; $t_2 = \frac{8\,316}{5.0} = 1\,655$ sec.; and $t = t_1 + t_2 = 65 + 1\,655 = 1\,720$ sec. (actual time).

Using the average slope, $S = 1.39$, in Equation (37), $v = 5.91$ ft. per sec.; and $t = \frac{l}{v} = \frac{10\,164}{5.91} = 1\,720$ sec., which checks the time on individual slopes.

Example 4.—F Constant and Q Varying.—In this case, only F drops from Equation (36). Assume that $L_1 = 1\,848$ ft., with $S_1 = 90$ and $Q_1 = 10$ cu. ft. per sec., and $L_2 = 8\,316$ ft., with $S_2 = 0.9$ and $Q_2 = 656$ cu. ft. per sec. Then, substituting in Equation (36) as before, $S^{\frac{3}{8}} = 1.41$, and $S = 2.50$ ft. per 1 000 ft.

As a check, assume that $F = 6.63$, then, by Equation (37): $v_1 = \frac{44.0}{1.755} \times 0.3 \times 1.776 = 13.36$ ft. per sec., and $t_1 = \frac{1\,848}{13.36} = 138$ sec. Similarly, (see Equation (37)), $v_2 = 6.76$ ft. per sec., and $t_2 = 1\,230$ sec. As in Example 3, $t = t_1 + t_2 = 138 + 1\,230 = 1\,368$ sec. (actual time).

Using the average slope, $S = 2.50$ in Equation (37), $v_1 = 3.49$ ft. per sec., and $t_1 = \frac{1\,848}{3.49} = 530$ sec. Likewise, $v_2 = 9.94$ ft. per sec., and $t_2 = \frac{8\,316}{9.94} = 837$ sec. As before, $t = t_1 + t_2 = 530 + 837 = 1\,367$ sec., which checks the total time on the individual slopes.

Example 5.—F and Q Varying.—In this case, Equation (36) is used directly. For instance, assume that $L_1 = 9\,240$ ft., with $S_1 = 90$, $F_1 = 5.0$, and $Q_1 = 200$ cu. ft. per sec., and $L_2 = 7\,392$ ft., with $S_2 = 0.9$, $F_2 = 7.0$, and $Q_2 = 800$ cu. ft. per sec.

Substituting the proper values in Equation (36), $S^{\frac{3}{8}} = 2.65$ and $S = 13.52$ ft. per 1 000 ft. A check on this average value of S may be made as in Examples 3 and 4.

Practical Use of Method.—The foregoing method of estimating the true average hydraulic slope will be found useful, no doubt, in general hydraulic work. It will be found especially useful in run-off formulas when the slope of the main line of flow of the water-shed changes materially.

In the run-off formulas the exponent of the slope is, of course, determined by the type of rainfall curve. For instance, when the curve is of the type of Equation (2), and e has the value 0.4, 0.5, or 0.6, there results approximately $S^{0\cdot167}$, $S^{0\cdot214}$, $S^{0\cdot265}$, respectively, for use in the run-off formulas. On account of the smallness of these exponents in the average formula, extreme refinement in estimating the average slope is unnecessary. However, the extreme differences in slopes of sections of the main channel, encountered in many water-sheds, require a rational approach to the true average slope. The refinement resorted to should depend both on these differences and on the requirements of the particular problem at hand. The water-shed may be divided into two, or more, parts, according to the judgment of the engineer.

When the expected storm is assumed to have a uniform (or constant) intensity for the time of concentration, Q varies directly as the product, CA. It may be seen, therefore, that, for this assumption, the product, CA, may be substituted for Q in Equation (36).

Products $C_1\,A_1$, $C_2\,A_2$, $C_3\,A_3$, etc., represent the average relative Q_1, Q_2, Q_3, etc., contributing to individual slopes, S_1, S_2, S_3, etc., respectively. Since the relative Q, in the foregoing method for estimating the average slope (see Equation (36)), carries the exponent of $\frac{1}{4}$, its value (or product, CA) may be roughly approximated without affecting the final run-off result materially.

Substitution of $S^{\frac{1}{2}}$ for $S^{\frac{3}{8}}$ in Equation (36) renders a fairly close approximation for the average slope. The resulting slope is a little smaller than that obtained by using $S^{\frac{3}{8}}$.

AVERAGE VALUE OF F

Having observed the foregoing method for estimating the average slope, it should be evident that the following equation may be used in arriving at the average value of F, for use with the average slope (substituting $C A$ for the relative Q):

$$F^2 = \frac{\dfrac{L_1}{(C_1 A_1)^{\frac{1}{4}}} + \dfrac{L_2}{(C_2 A_2)^{\frac{1}{4}}} + \text{etc.}}{\dfrac{L_1}{(C_1 A_1)^{\frac{1}{4}} F_1^2} + \dfrac{L_2}{(C_2 A_2)^{\frac{1}{4}} F_2^2} + \text{etc.}} \qquad \ldots\ldots\ldots\ldots (38)$$

AVERAGE VALUE OF C

The proper assignment of values to the run-off coefficient requires an intimate knowledge of all the various factors affecting run-off as well as a fund of experimental information from which to draw. The refinement to which the engineer should resort in establishing its value depends both on the particular problem confronting him and the reliability of available information.

Values for Factor C can be secured only through experimentation by obtaining field measurements for A, i, and Q; and solving by Equation (1). Some of the information presented in current textbooks and past and present publications has been procured on this basis; various tables of values for C (especially for use in storm drain work) are listed. However, much of the available information is of a conflicting nature, due in some cases, no doubt, to unreasonable conceptions, and in others to conclusions drawn from insufficient data concerning the many varying causes of fluctuation.

There is, no doubt, much unpublished information possessed by many of the larger flood-control organizations and others, that is used privately, but which is of potential value to the profession generally. Some of these data are issued from time to time in local publications, while many are retained for aid in the solution of local problems, another portion being still in the experimental stage.

Liberal contributions of this information, coupled with a systematic collection by the Society of existing available data, could be made the first step in a more thorough analysis of Factor C with the view of establishing tentative tables of general value to the profession.

Such tables could not be final; experimentation will be continually needed in the future; but a more concerted effort of the profession in analyzing data on the run-off coefficient would not only be of temporary worth, but its results should point the way to better experiments. To this general fund of information the reader is referred for values of the run-off coefficient. Reference is also made to the conclusions herein under the heading, "Factor C, The Run-off Coefficient".

At present, average values of C for a given water-shed are arrived at by many through comparisons with experimental values obtained on areas of somewhat similar characteristics and size. When a water-shed is considered in parts the average, C, is, of course, the weighted average of separate divisions, for example, $C = \frac{a_1 C_1 + a_2 C_2 \ldots . + a_n C_n}{a_1 + a_2 + a_3 \ldots . + a_n}$, in which, $a_1, a_2, a_3, \ldots . a_n$, represent individual areas composing the water-shed.

When information on the variation of C with time and intensity is possessed, it may be considered advisable to divide the area into approximate time divisions. For areas very irregular in regard to the factors, F and S, the divisions for Factor C may be correlated with those for the other factors.

It should be evident that the increase of the run-off coefficient with the time (as at present practiced by many of the users of the rational method) can be predicated only on the assumption of a water-shed, the surface of which is not in a saturated condition when the maximum run-off producing intensity occurs. The assumption of a saturated surface condition is, of course, conducive to the maximum run-off. The question naturally arises, when may the surface be considered saturated? Much good information on this point has been given by Ivan E. Houk, M. Am. Soc. C. E. In this connection the chapter of his report entitled, "Rainfall and Run-Off Rates on Saturated Soils",[11] is of particular interest.

It is also worthy of note that proper assumptions on degree of previous saturation pursuant to the use of the rational method cannot be made without due consideration of the manner of making the rainfall curve the use of which is contemplated. The intensity-time-frequency curves in present use (as evidenced by current publications and textbooks) are platted from data selected from portions of storms, in such manner that their use presupposes that precipitation may be expected to occur both before and after the run-off producing intensity represented by a given ordinate.

It seems reasonable, therefore, that in attempting to estimate the probable maximum run-off for the wet season of the year (at which time the storm predicted may be expected to follow closely on the heels of previous rains) through the use of such curves, a saturated surface condition should be assumed for the usual water-shed, the surface of which is composed of ordinary soil, or combinations of soil and impervious materials.

Analysis of Experimental Data.—Equation (39), following, is derived for the purpose of facilitating the analysis of experimental data, leading to the compilation of tables (or diagrams) of Factor C.

For evolving the formula, retention is considered as subdivided into three main parts, namely, evaporation, absorption, and storage.

Evaporation, which herein is meant to include transpiration, is not a great consideration in the process of estimating instantaneous run-off, and usually could be neglected without serious error. The loss to run-off through absorption is large when the soil is dry, and is considerable for saturated soils. Storage, as used in the formula, is intended to represent the loss to run-off of the total precipitation falling in depressions, cavities, etc. Thus, the term includes percolation, when the precipitation is lost in this manner.

---

[11] "Rainfall and Run-Off in the Miami Valley," by Ivan E. Houk, Technical Repts., Miami Conservancy Dist., Pt. VIII, p. 119.

It is significant that the combined rates of evaporation and absorption may be represented in the formula by a single symbol, since they are not altered materially by a change in intensity. The rate of storage, on the other hand, will be greater for an increased intensity. Since all practical areas contain some storage capacity, this fact (in addition to the consideration of evaporation), explains why experimental results never show unity for Factor C, even for a saturated impervious surface.

For a given area, let m = percentage of precipitation, i, that is being stored. Then $m\,i\,A$ = rate of storage, in cubic feet per second, and $i\,A - m\,i\,A$ = rate of run-off, having deducted storage alone, in cubic feet per second. Of the remaining water, evaporation and absorption are taking toll.

Let z = rate of absorption plus rate of evaporation, in cubic feet per second per acre. Then $z\,A - m\,z\,A$ = cubic feet per second being absorbed and evaporated; and, therefore, $(i\,A - m\,i\,A) - (z\,A - m\,z\,A) = i\,A - z\,A + m\,z\,A - m\,i\,A$ = actual run-off, in cubic feet per second.

Substituting this expression for Q in Equation (1), there results: $C\,i\,A = i\,A - z\,A + m\,z\,A - m\,i\,A$, and,

$$C = 1 - \frac{z - m\,z + m\,i}{i} = \left(1 - \frac{z}{i}\right)(1 - m) \qquad (39)$$

In connection with Equation (39), reference should be made to Fig. 6. The curves thereon not only transform the very valuable experiments of Mr. Houk into a form suitable for use in the rational method, but also tend to illustrate the usefulness of Equation (39).

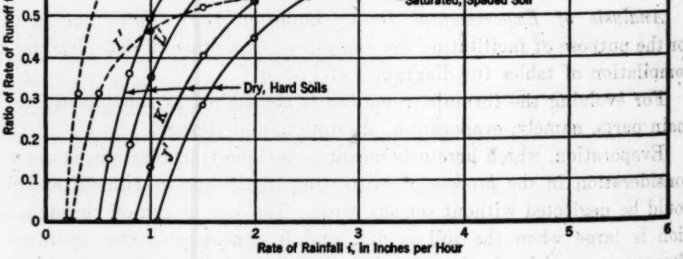


FIG. 6.—VALUES OF C FROM EXPERIMENTATION ON ELEMENTARY AREAS

The intercept on the abscissa axis (or the value of intensity when Factor $C = 0$) is represented by the letter, b, in Mr. Houk's report, and by z in Equation (39). There is no letter designation in the Houk report to indicate the percentage of storage, m, of the formula.

Values of m and z for the curves, H', I', J', K', L', and M', respectively, are shown in Table 7.

TABLE 7.—VALUES OF m AND z FOR CURVES, H', I', J', K', L' AND M' OF FIG. 6

Curves	m	z
H'	0.066	0.20
I'	0.375	0.25
J'	0.048	1.05
K'	0.048	0.87
L'	0.0	0.67
M'	0.0	0.52

Calculations for the values in the line, H', will illustrate. From the straight-line equation, $y = 1.07\,x + 0.20$ (for Line H of the report), derive the value of $\frac{x}{y} = \frac{y - 0.20}{1.07\,y} = \left(1 - \frac{0.20}{y}\right)\frac{1}{1.07}$. Since $\frac{x}{y}$ is the equivalent of Factor C of Equation (39), $\frac{1}{1.07} = 1 - m$, and, $m = 0.066$ (see Table 7).

It should be observed (see Table 7), that some storage, as defined in assumptions for the derivation of Equation (39), is recorded (although the areas represented were very small) for all the curves with the exception of L' and M'.

The values of m for Curves H', J', K', L', and M' are small because the soil of the areas experimented upon was packed. It seems that the extremely high value for m of Curve I' was caused by spading the soil, thus permitting a certain percentage of the precipitation (regardless of its intensity) to enter cavities directly. In such case, the water due to a certain intensity and time of sprinkling would merely fill the cavities. Additions above this quantity would still be lost because of the nature of the subsoil and the head developed in the cavities.

SUGGESTIONS FOR USE OF FORMULAS

Method No. 1.—The first method involves seven distinct steps, as follows:

1.—Decide on the type of rainfall curve, and reduce Equation (24) accordingly:

(*a*) In the absence of rainfall data and for approximate estimates, use Equation (28) or Equation (29).

(*b*) Establish H.

(*c*) Determine R_H.

2.—Determine A and locate the main channel of flow (if necessary do this by trial, using approximate values for nucleus, C, F, S, P, etc.). Establish L, and $\frac{L}{W}$.

3.—Determine average values for Factors C, F, and S, for the water-shed, being guided by admonitions in the preceding parts of the paper and judgment concerning the requirements of the particular problem.

4.—Determine C', A', L', $\frac{L'}{W'}$, and $\frac{A}{A'}$; assign a time, t', to A', and calculate Q' from $Q' = C' i' A'$. (When the use of Table 3 is contemplated, or when the assumption is that the storm is of uniform intensity for the time of concentration of the water-shed, i' should be the approximate intensity of the water-shed, and t' should be based on this intensity.

When the use of Table 4 is contemplated, or when it is desired to take advantage of the factor of safety rendered by the use of a rainfall curve of the type of Equation (3) in the manner ordinarily employed in the detail process of the rational method, $i' = \frac{K}{(t')^{0.5}} = R_H \frac{(60\ H)^{0.5}}{(t')^{0.5}}$ (for t', in minutes).

5.—Obtain $v' = \frac{L'}{t'}$, for t', in seconds, and V' = velocity of Q', based on average values of F and S for the water-shed, $= \frac{F^2}{S^{\frac{1}{8}}} s^{\frac{1}{2}} (Q')^{\frac{1}{4}}$. Determine $\frac{v'}{V'}$ and correct this value of P_1 in Fig. 3, giving P'.

6.—With the corrected value of P' enter Table 3, or Table 4, in the proper columns for $\frac{L}{W}$ and $\frac{A}{A'}$, being guided by Fig. 2 and by guide lines for the manner of concentration in Fig. 4, finding P.

7.—Substitute the values of Factors C, A, R_H, S, P, F, and L in the reduced formula and solve for Q. It should be remembered that:

(*a*) Length L includes L' of the nucleus.
(*b*) The average, C, includes consideration of C' of the nucleus.
(*c*) The average slope for use in formulas is the average slope of the main channel of flow, exclusive of the nucleus.
(*d*) The average, F, is the average value of F for the main channel of flow, exclusive of the nucleus.

This method offers opportunity for varying the effect of the inlet area, or nucleus, in much the same manner as may be followed in the detail process of the rational method. As has been shown, the assignment of a value for P' on a given nucleus is the equivalent of allowing so much inlet, or initial, time. The method will be found valuable, depending on the water-shed and the purpose in view. For instance, it may be considered advisable to allow a certain lapse of time to prepare a regular water-shed to begin shedding the water by allowing for storage in small irregularities of surface, leaves, vegetation, etc. (this, however, is a special consideration), or, again, it may be of advantage to use Method No. 1, when the types and slopes of channels through which the inlet areas contribute, differ radically from that of the main channel of flow.

Method No. 2.—It should be borne in mind, however, that Method No. 1, is not the only manner in which the formulas may be found useful. Broadly speaking, they may be used in the same way as the ordinary empirical formulas, with the added advantage of permitting the engineer to allow for variables, which the use of existing formulas prohibits (see the heading, "Inadequacy of Empirical Formulas").

When used in this manner the length, L, should be the full distance from the most remote part of the water-shed to its outlet, and Factors S and F should represent average values for the full length.

Furthermore, Factor P should be chosen from the column of Table 3 headed by $P' = 0.5$, and under the section where $\frac{A}{A'} = 262\,144$ (approaching infinity, see the statement concluding the heading, "Method of Arriving at Values of Factor P"). The value of $\frac{L}{W}$ for the water-shed, and the manner of concentration, should be considered, of course, in arriving at the value of P.

This method is short, but, if proper judgment is exercised in selecting values of the various factors of the formula used, it affords a means of arriving at a quick approximate Q and, consequently, the time of concentration of a given water-shed for the assumptions. Having computed Q, the intensity is found from Equation (1), and the time of concentration is determined by referring the intensity to the rainfall curve on which the formula used is based.

Indeed, this method may be used in all ordinary cases without recourse to Method No. 1, admitting a small error, which is less on the average than the automatic error of the detail process mentioned under the heading, "Use of Rainfall Curve" (see, also, "Examples: Remarks on Example 10").

After the time has been found, simply add or deduct the difference in inlet time, referring the new time to the rainfall curve for the intensity to be used in Equation (1).

It should be evident that the first solution by this method entails the assumption that the water is ready to begin concentrating immediately, the time of the inlet area, or nucleus, being contained in the total time of the water-shed as rendered by average values of the various factors of the formula for the entire area. Any increase (or decrease) in the inlet time above (or below) this assumption may be applied as suggested herein.

Special Case.—Attention should be called to a special case sometimes encountered in the rational method of computing run-off. In mountainous territory it is often found that the higher altitudes of a given water-shed may reasonably be expected to receive a greater intensity for an assumed period of frequency than the lower portions. In attacking the solution of this problem the territory may be divided into zones, to each of which is assigned the probable R_H that may be expected to occur. Thus, where R_1 represents the probable average intensity over any one zone throughout a period of 1 hour (for a water-shed divided into three parts), the zone of highest altitude may be assigned a value, $R_1 = 3$ in., the middle zone, $R_1 = 2.50$ in., and the lowermost zone, $R_1 = 2$ in. per hour.

In the detail process of the rational method the problem is sometimes approached as follows: On the diagram of rainfall curves, that curve representing the assigned intensity requirements (such as $R_1 = 3$ in.), is chosen; this curve is followed until all the individual areas of the first zone have contributed; then transfer is made to the required curve for the next zone, such as $R_1 = 2.50$ in. per hour (without making reduction at the point of transition), the second curve being used as the first. In this manner the entire water-shed is covered. The method is an approach to a reasonable solution for the conditions assumed. The method results in the use of an average rainfall curve for the several zones.

In using the formulas and methods herein outlined the average value of R_H may be obtained for the water-shed through weighting the individual zone values of R_H by the respective zone areas. The average value of R_H is to be used, of course, with the rainfall curve to which it belongs.

Other Coefficients.—It is unfortunate that more information is not available concerning the distribution of rainfall especially over the larger areas. It is possible that more accurate information on this point could be utilized through three possibilities, as follows:

(*a*) Alteration of the frequency periods (or values for a given frequency) of the time-intensity-frequency curves in present use.
(*b*) Coefficients of distribution.
(*c*) The use of only a part of a given drainage area instead of the whole.

The writers believe that it is customary at present among American engineers to use rainfall curves as if the intensity were uniform over the entire area throughout the time of concentration. This practice (in the absence of more complete information on the distribution of rainfall) is based, no doubt, on the common observation that rains of lighter intensity and longer duration (required for consideration of the larger areas) are likely to be steadier and of wider distribution than those rendering higher rates of precipitation, which are more generally applicable to the smaller areas.

The time-intensity-frequency curves in present use are usually developed from the records of individual gauges. In this case it would be unwise to assume that the intensity designated by a certain ordinate on a curve represents the rate of precipitation at the eye or focus of the storm; or that it should be used to indicate the maximum rate within an area on which the intensity is unequally distributed at a given moment.

Coefficients based on this assumption, therefore, should not be used as direct multiple factors in reducing the run-off results that may be obtained through the use of the curves developed from the records of individual gauges.

As the writers view the problem the following procedure (in the light of present and future information) should be followed:

1.—Efforts should be made to ascertain whether or not the intensities rendered by curves in present use may reasonably be applied over the entire area.

2.—If such intensities can not be applied over the entire area, then endeavor should be made to apply the information through the foregoing suggested possibilities, (*a*), (*b*), or (*c*).

The "coefficient of retention" is accounted for in the run-off coefficient as used by American engineers. (See the heading, "Factor *C*, The Run-Off Coefficient".)

The "coefficient of retardation" is defined as the ratio of the contributing area to the total drainage area when the maximum discharge is caused by a storm which lasts for a lesser length of time than the time of concentration of the total drainage area. This possibility should be taken into account, especially for some areas, very irregular in regard to shape and physical characteristics.

Instead of using a coefficient, however, the problem should be solved by trial, use being made of that portion of the area rendering the maximum. The formulas herein (usually through Method No. 2), may be used advantageously for this purpose. Other writers have discussed this phase of the rational method.

Remarks.—The outstanding feature of the rational method is its conception of the time element in the estimation of maximum storm flows, affording, as it does, through the medium of its assumptions, a reasonable basis for predicting the probable intensities and frequencies of storms. It should be emphasized, however, that the solution of the run-off problem, although approached more easily through the special assumptions of the rational method, is still indeterminate. This statement may be understood more easily when it is recognized that there is no conceivable way of arriving at a perfect solution under the assumptions, because it is necessary to approach accuracy by trial.

The detail method is only an approach. For instance, at the beginning of the process, it is necessary to assign a time for the concentration of the initial area. This time, among other considerations, depends on the intensity. Since the assumption is that for a given water-shed the intensity shall be uniform over the entire area throughout its time of concentration, and, since its value cannot be known in advance, the assignment of the inlet time may be made only as an approach to accuracy under the assumption. Likewise, in the assignment of the run-off coefficient both to the initial area and the remainder of the water-shed (since its value depends on the intensity), accuracy can only be approached by trial. Therefore, it has not been the common practice of users of the detail rational method to consider the effect of intensity on either the inlet time or the value of the run-off coefficient.

Intelligent use of the formulas, requires not only a familiarity with the foregoing derivations and assumptions, but also a broad understanding of the run-off problem. The foregoing work, it is hoped, may serve as an aid in the solution of a difficult (and somewhat neglected) problem.

EXAMPLES

The following examples are intended to illustrate the application of the formulas.

Example 6.—Assume that for a given locality the intensity curve of the type, $i = \frac{K}{t^{0.5}}$ (Equation (3)), is considered adaptable, that, $R_H = R_1$, and that the time period, H, is 1 hour. For this case, Equation (29) may be used. Let $R_1 = 2.0$ = the expected rainfall intensity, in inches per hour, for a 1-hour period.

The study of a given area reveals that the water-shed area is somewhat similar in shape and manner of concentration to Type 18, Fig. 2. The dimensional data are: $A = 1\,294$ acres; $L = 20\,328$ ft.; $S = 10$ ft. per 1 000 ft.; and $C = 0.3$.

The main channel of flow is assumed to be a pipe with Kutter's $n = 0.013$. Hence, from Table 11 (Appendix II), $F = 6.63$.

Examination of the nucleus (or initial area) reveals that: $A' = 5$ acres; $L' = 700$ ft.; and $C' = 0.2$.

The following factors are estimated: $\frac{A}{A'} = 259$; $\frac{L}{W} = \frac{(20\,328)^2}{43\,560 \times 1\,294} = 7.3$; $\frac{L'}{W'} = \frac{(700)^2}{43\,560 \times 5} = 2.25$.

If the use of Table 4 is contemplated (or if it is desired to take advantage of the small factor of safety rendered by the use of a rainfall curve, Equation (3), as ordinarily followed in the detail process of the rational method), and a study of the nucleus calls for an assigned time, $t' = 10$ min., for this assumption; then,

$$Q' = C'\,i'\,A' = C'\,A'\,\frac{7.75\,R_H\,\sqrt{H}}{\sqrt{t'}} = \frac{0.2 \times 7.75 \times 2.0}{\sqrt{10}} \times 5.0$$
$$= 4.9 \text{ cu. ft. per sec.}$$

If that type of storm based on a uniform, or constant, intensity for the time of concentration had been assumed, it would have been necessary to base the assignment of t' on this assumption, using the approximate intensity for the water-shed; also, $Q' = C'i'A'$, wherein i' is the approximate intensity of the water-shed.

For the velocity ratio:

$$v' = \frac{L'}{t'} = \frac{700}{10 \times 60} = 1.17 \text{ ft. per sec.}$$

$$V' = \frac{F^2}{S^{\frac{1}{8}}}\sqrt{s}\,(Q')^{\frac{1}{4}} = \frac{(6.63)^2}{(10)^{\frac{1}{8}}} \times \sqrt{0.01}\,(4.9)^{\frac{1}{4}} = 4.87 \text{ ft. per sec.}$$

and,

$$\frac{v'}{V'} = \frac{1.17}{4.87} = 0.24 = P_1$$

Entering Fig. 3 with $\frac{L'}{W} = 2.25$, and $P_1 = 0.24$,

$$P' = 1.24 \times P_1 = 1.24 \times 0.24 = 0.3$$

Entering Table 4, with $P' = 0.3$, $\frac{A}{A'} = 259$, and $\frac{L}{W} = 7.3$, and guided by Figs. 2 and 4 to medium concentration for area, Type 18, $P = 0.51$. Hence,

$$\sqrt{\frac{P}{L}} = \sqrt{\frac{0.51}{20\,328}} = 0.005 = B.$$

Substituting values for the various factors in Equation (29),

$$Q = 15\,(0.3 \times 1\,294 \times 2.0 \times 6.63 \times 0.005)^{1.1429}\,(10)^{0.2143}$$
$$= 1\,006 \text{ cu. ft. per sec.}$$

The intensity and time of concentration for the foregoing assumptions may be found from Equation (1), thus, $i = \dfrac{1\,006}{1\,294 \times 0.3} = 2.59$, and from Equation (26), $t = \dfrac{60\,(2.0)^2}{(2.59)^2} = 35.7$ min.

***Example 7.*—Assume that information on rainfall and the consideration of larger areas point to the use of a longer time period, or $H = 6$ hours; then, $R_H = R_6$. Again, assume that the rainfall curve, Equation (3), is adaptable. In this case Equation (28) may be used, or, substituting $H = 6$,**

$$Q = 41.7\,(C\,A\,R_6\,F\,B)^{1.1429}\,S^{0.2143} \ldots\ldots\ldots\ldots\ldots\ldots (40)$$

In $R_H = \dfrac{X}{H}$, or, $R_6 = \dfrac{X}{6}$, assume that $X = 4.8$ = the expected inches of rain for a period of 6 hours; then, $R_6 = \dfrac{4.8}{6} = 0.8$ in. per hour.

From a study of a given area it develops that the shape and manner of concentration are approximately midway between Types 16 and 17, Fig. 2. Furthermore, $A = 500$ sq. miles $= 320\,000$ acres; $A' = 8$ acres; $L = 223\,600$ ft.; $L' = 1\,200$ ft.; $S = 5$ ft. per 1 000 ft.; $C = 0.2$; $C' = 0.25$; and $F = 5.7$. By estimate, $\dfrac{A}{A'} = \dfrac{320\,000}{8} = 40\,000$; $\dfrac{L}{W} = \dfrac{(223\,600)^2}{43\,560 \times 320\,000} = 3.6$; and $\dfrac{L'}{W'} = \dfrac{(1\,200)^2}{43\,560 \times 8} = 4.0$.

The use of Table 3 is contemplated. Assign an approximate average intensity for the water-shed, of 0.75 in. per hour. Examination of the nucleus points to the assignment of 15 min. for its time of concentration. Then, following Equation (1),

$$Q' = C'\,i'\,A' = 1.5 \text{ cu. ft. per sec.}$$

$$v' = \frac{L'}{t'} = 1.33 \text{ ft. per sec.}$$

$$V' = \frac{F^2}{S^{\frac{1}{8}}}\sqrt{s}\,(Q')^{\frac{1}{4}} = 2.08 \text{ ft. per sec.}$$

and,

$$\frac{v'}{V'} = 0.64 = P_1 = P'$$

(no correction is necessary since $\dfrac{L'}{W'}$ of the nucleus is approximately 4).

Entering Table 3 with $P' = 0.64$, $\dfrac{A}{A'} = 40\,000$; and $\dfrac{L}{W} = 3.6$, guided by Figs. 2 and 4, to approximately midway between medium and delayed concentration, P is found equal to 0.48 and $\sqrt{\dfrac{P}{L}} = 0.001465 = B$. Substituting values for the various factors in Equation (40),

$$Q = 41.7\,(427.5)^{1.1429}\,(5)^{0.2143} = 59\,740 \text{ cu. ft. per sec.}$$

The intensity by Equation (1) and the time of concentration by Equation (26) are, $i = 0.933$, and $t = 4.4$ hours, respectively.

Example 8.—Let the rainfall curve for a given locality be represented by $i = \frac{K}{t^{0.6}}$. Assume that $R_H = R_1$, or that $H =$ a 1-hour period. For this purpose Equation (24) reduces to:

$$Q = 28.3 \left(\frac{P}{L}\right)^{0.7059} (C\,A\,R_1)^{1.1765}\,F^{1.4118}\,S^{0.2647} \tag{41}$$

Now, assume that the values for the various factors are the same as those used in Example 6; then, by Equation (41) $Q = 1\,069$ cu. ft. per sec.

Example 9.—Let the rainfall curve be the same as in Example 8 and let $R_H = R_6$ and $H = 6$; then, Equation (24) reduces to:

$$Q = 100.2 \left(\frac{P}{L}\right)^{0.7059} (C\,A\,R_6)^{1.1765}\,F^{1.4118}\,S^{0.2647} \tag{42}$$

Substituting in Equation (42) the same factors used in Example 7, $Q = 62\,003$ cu. ft. per sec.

Remarks Concerning Examples 6, 7, 8, and 9.—The run-off, Q $(= 1\,069)$, for Example 8, may be approximated by referring the time, 35.7 min. (solved by the use of Equation (29)), to the rainfall curve,

$$i = \frac{K}{t^{0.6}} \tag{43}$$

thus obtaining a new intensity for use in Equation (1).

In the absence of the curve, Equation (43), the intensity may be calculated directly from Equation (43) by substituting for K its value when $H = 1$ hour, and $R_H = R_1$; thus,

$$i = \frac{(60)^{0.6}}{t^{0.6}} \tag{44}$$

This results in the value, $i = \frac{23.33}{35.7^{0.6}} = 2.73$. By Equation (1), the corrected run-off equals $0.3 \times 2.73 \times 1\,294 = 1\,060$ cu. ft. per sec. The percentage of accuracy is expressed by $\frac{1\,060}{1\,069} = 99.2$.

Likewise, the run-off, $Q = 62\,003$, for Example 9 may be approximated by referring the time, 264 min. (solved by the use of Equation (28)), to the curve of Equation (43) with the result that $i = 0.964$.

By Equation (1) the corrected run-off,

$$Q = 0.2 \times 0.964 \times 320\,000 = 61\,696 \text{ cu. ft. per sec.}$$

In this case the percentage of accuracy is $\frac{61\,696}{62\,003} = 99.6$.

The result to be expected from a given problem will not always be as close as the foregoing percentages indicate, since the shape, size, and manner of concentration of the area, will have some effect. However, when the adopted rainfall curve does not vary widely, the foregoing method may be used to give a quick approximate result, accurate enough for most practical purposes, for variations in the value of exponent, e, Equation (2), as encountered in common practice.

Similar transitions may be made, of course, to curves of other types, such as,

$$i = \frac{a}{b + t} \quad \text{.............................} (45)$$

Example 10.—This example is shown for the purposes of illustrating the hydraulic construction of the formulas and making certain comparisons with the usual detail method.

Fig. 7 was selected from a topographical map and represents a small water-shed (rather thoroughly divided into individual areas by streets), with contour intervals of 5 ft. For comparative purposes it is necessary to place the tabling process and the formulas on the same basis, hence, the following assumptions are made arbitrarily:

(*a*) The rainfall curve is of the type of Equation (3), with $R_H = R_1 = 1.25$ in. per hour for a 1-hour period. Since $R_1 = \frac{K}{(60)^{0.5}}$,

$$K = (60)^{0.5} \times 1.25 = 9.69$$

or,

$$i = \frac{9.69}{t^{0.5}} \quad \text{..............................} (46)$$

(*b*) Circular pipe (Kutter's $n = 0.013$) is used throughout the main, or time-giving, channel of flow.

(*c*) The run-off coefficient, C, is equal to 0.4 throughout.

Area No. 1 of Fig. 7 is chosen as the initial area for the tabling process shown in condensed form in Table 8. It is further assumed that the inlet Area No. 1 (a small water-shed) concentrates in the manner shown for Type 3, Fig. 2. For the initial area: $A = 8$ acres; $L = 1\,200$ ft.; $S = 3$ (for a fall of 3.6 ft. in 1 200 ft.); $C = 0.4$; $R_1 = 1.25$; and P (from Table 4, making use of "Suggestions for Use of Formulas, Method No. 2,") $= 0.53$. Using Equation (29) (due to the foregoing assumption for the rainfall curve): $B = \sqrt{\frac{P}{L}} = 0.021$,

and,

$$Q = 15\ (0.4 \times 8 \times 1.25 \times 6.63 \times 0.021)^{1.1429}\ (3)^{0.2143} = 9.8 \text{ cu. ft. per sec.}$$

By Equation (1) then, $i = 3.06$, and by Equation (26),

$$t = \frac{60 \times 1.25^{+2}}{3.06^2} = 10 \text{ min.}$$

For the initial area, 10 min. represents a value that would be found for its time of concentration (on the foregoing assumptions) if it were tabled down from an infinitesimal beginning by the detail process. That value is used to

TABLE 8.—CONDENSED ILLUSTRATION OF THE TABLING PROCESS TO ACCOMPANY EXAMPLE 10

Water-Shed No.	Area drained	Individual area, in acres	Total area, in acres, (A)	Coefficient of run-off (C)	Run-off, $Q = C\,i\,A$	Grade of drain (s)	Time, in minutes (t)	Intensity $i = \frac{9.69}{t^{0.5}}$	Size of pipe, in inches	Velocity, in feet per second	Length, in feet	Total length, in feet †	Total fall, in feet †
	1	8.0	8.0	0.4	9.8	0.00395	10.00	3.07	21	4.07	530		
	2	6.7	14.7	0.4	16.4	0.00395	12.17	2.78	26	4.76	700		
	3	16.5	31.2	0.4	31.6	0.00395	14.62	2.53	33	5.64	490		
	4	14.0	45.2	0.4	43.8	0.00468	16.07	2.42	36	6.52	360		
	5	8.1	53.3	0.4	50.1	0.00468	16.99	2.35	38	6.78	330		
	6	8.4	61.7	0.4	56.8	0.00468	17.80	2.30	39	6.90	375		
	7	8.8	70.5	0.4	63.2	0.00468	18.71	2.24	41	7.14	375		
	8	9.8	80.3	0.4	70.3	0.00468	19.58	2.19	43	7.38	375		
	9	10.5	90.8	0.4	77.7	0.00468	20.43	2.14	44	7.50	385		
	10	10.7	101.5	0.4	85.3	0.00331	21.28	2.10	49	6.79	1 300		
	11	108.6	210.1	0.4	164.6	0.00331	24.48	1.96	62	7.97	1 000		
	12	41.1	251.2	0.4	188.9	0.00331	26.57	1.88	65	8.22	675		
	13	75.2	326.4	0.4	239.0	0.00411	27.94	1.83	69	9.53	335		
	14	5.1	331.5	0.4	240.0	0.00411	28.52	1.81	69	9.53	335		
	15	5.5	337.0	0.4	242.6	0.00411	29.11	1.80	69	9.53	320		
	16	3.5	340.5	0.4	242.4	0.00411	29.67	1.78	69	9.53	320		
	17	75.3	315.8	0.4	292.7	0.00411	30.23	1.76	74	9.98	325		
	18	1.8	417.6	0.4	292.3*	0.00411	30.77	1.75	74	9.98	320		
	19	1.5	419.1	0.4	289.9*	0.00411	31.30	1.73	74	9.98	275		
	20	1.5	420.6	0.4	289.3*	0.00411	31.76	1.72	74	9.98	685		
	21–22	93.9 and 5.2	519.7	0.4	351.4	0.00411	32.91	1.69	79	10.42	355		
	23	4.9	524.6	0.4	350.4*	0.00411	33.48	1.67	79	10.42	320		
	24–25	7.2 and 19.7	551.5	0.4	366.2	0.00411	33.99	1.66	80	10.50	345		
	26–27	4.8 and 10.1	566.4	0.4	373.9	0.00411	34.44	1.65	81	10.59	660		
	28–29	232.7 and 260.0	1 060 0	0.4	691.9	0.00411	35.47	1.63	102	12.30	335		
	30–31	1.6 and 9.2	1 070.8	0.4	693.8	0.00411	35.93	1.62	102	12.30	270		
1.	1 to 31	....	1 070.8	0.4	693.8	....	35.93	....	....	......	......	11 825	47.19
	32–33	5.6 and 8.0	1 084.4	0.4	698.4	0.00411	36.29	1.61	102	12.30	265		
	34–35	8.2 and 7.8	1 100.4	6.4	704.3	0.00411	36.65	1.60	103	12.38	295		
	36–37	8.6 and 8.9	1 117.9	0.4	711.0	0.00411	37.05	1.59	103	12.38	325		
	38–39	9.8 and 9.5	1 137.2	6.4	718.7	0.00411	37.49	1.58	104	12.46	320		
	40–41	9.8 and 9.5	1 156.5	0.4	726.3	0.00411	37.92	1.57	104	12.46	325		
	42–43	9.7 and 9.9	1 176.1	0.4	733.8	0.00411	38.35	1.56	104	12.46	320		
	44–45	9.6 and 9.5	1 195.2	0.4	745.8	0.00411	38.78	1.56	105	12.53	350		
	46	10.6	1 205.8	0.4	747.6	0.00411	39.25	1.55	105	12.53	335		
	47–48	9.2 and 114.0	1 329.0	0.4	818.7	0.00411	39.69	1.54	109	12.84	295		
	49–50	8.4 and 4.4	1 341.8	0.4	821.2	0.00411	40.07	1.53	109	12.84	280		
	51–52	8.3 and 4.1	1 354.2	0.4	823.4	0.00411	40.44	1.52	109	12.84	320		
	53–54	8.5 and 4.3	1 367.0	0.4	831.1	0.00411	40.85	1.52	109	12.84	285		
	55–56	8.6 and 22.0	1 397.6	0.4	844.1	0.00368	41.22	1.51	112	12.36	295		
	57–58	9.2 and 8.8	1 415.6	0.4	849.3	0.00368	41.62	1.50	113	12.42	290		
	59–60	8.5 and 6.9	1 431.0	0.4	858.6	0.00368	42.01	1.50	113	12.42	360		
	61–62	10.6 and 5.7	1 447.3	0.4	862.6	0.00368	42.49	1.49	113	12.42	360		
	63	109.4	....	.....	....	....	......	....	....	......	......		
	64	96.3	1 653.0	0.4	978.6	0.00324	42.98	1.48	122	12.08	295		
2.	1 to 64	....	1 653.0	0.4	978.6	....	42.98	....	....	......	......	17 115	68.37
	65	9.8	1 662.8	0.4	977.7*	0.00324	43.38	1.47	122	12.07	290		
	66	8.8	1 671.6	0.4	976.2*	0.00324	43.78	1.46	122	12.06	310		
	67–68	8.5 and 8.7	1 688.8	0.4	986.2	0.00324	44.21	1.46	123	12.09	290		
	69–70	2.2 and 8.6	1 699.6	0.4	985.7*	0.00324	44.61	1.45	123	12.09	300		
	71–72	1.4 and 8.8	1 709.8	0.4	984.8*	0.00324	45.02	1.44	123	12.09	320		
	73	9.2	1 719.0	0.4	990.1	0.00324	45.47	1.44	123	12.09	330		
	74	9.2	1 728.2	0.4	988.6*	0.00324	45.92	1.43	123	12.09	340		
	75	9.2	1 737.4	0.4	986.9*	0.00324	46.39	1.42	123	12.09	210		
	76–77	4.9 and 145.5	1 887.8	0.4	1 072.2	0.00324	46.68	1.42	126	12.35	325		
	78	6.7	1 894.5	0.4	1 068.5*	0.00324	47.12	1.41	126	12.33	325		
	79–80	4.7 and 20.0	1 919.2	0.4	1 082.5	0.00324	47.56	1.41	127	12.36	850		
	81–82	5.9 and 14.8	1 939.9	0.4	1 078.6*	0.00324	48 70	1.39	127	12.35	1 175		
	83	61.1	2 001.0	0.4	1 096.5	0.00280	50.29	1.37	131	11.78	200		
	84	125.3	2 126.3	0.4	1 156.7	0.00280	50.57	1.36	134	11.90	330		
3.	1 to 84	....	2 126.3	0.4	1 156.7	....	50.57	....	....	......	......	22 675	83.33

* The greatest Q above was used.

† Excluding initial area.

TABLE 8.—(*Continued.*)

Water-Shed No.	Area drained	Individual area, in acres	Total area, in acres (A)	Coefficient of run-off (C)	Run-off, $Q = C\,i\,A$	Grade of drain (s)	Time, in minutes (t)	Intensity, $i = \frac{9.69}{t^{0.5}}$	Size of pipe, in inches	Velocity, in feet per second	Length, in feet	Total length, in feet†	Total fall, in feet†
	85	4.5	2 130.8	0.4	1 159.1	0.00280	51.03	1.36	184	11.90	320		
	86-87	2.6 and 4.0	2 137.4	0.4	1 154.3*	0.00280	51.48	1.35	184	11.90	330		
	88	2.7	2 140.1	0.4	1 151.3*	0.00280	51.94	1.35	134	11.90	330		
	89	2.7	2 142.8	0.4	1 148.5*	0.00280	52.41	1.34	184	11.90	90		
	90	29.0	2 171.8	0.4	1 161.5	0.00280	52.53	1.34	135	11.95	210		
	91	2.7	2 174.5	0.4	1 159.4*	0.00280	52.82	1.33	185	11.95	120		
	92	6.5	2 181.0	0.4	1 161.2*	0.00280	52.99	1.33	185	11.95	195		
	93	3.3	2 184.3	0.4	1 160.3*	0.00280	53.26	1.33	135	11.95	165		
	94	7.1	2 191.4	0.4	1 161.5	0.00280	53.49	1.33	135	11.95	150		
	95	3.0	2 194.4	0.4	1 160.5*	0.00280	53.70	1.32	185	11.95	235		
	96-97	41.2 and 3.5	2 239.1	0.4	1 180.4	0.00231	54.03	1.32	140	11.13	775		
	98	85.2	2 324.3	0.4	1 212.3	0.00231	55.19	1.30	142	11.20	1 480		
	99	83.3	2 407.6	0.4	1 231.7	0.00338	57.39	1.28	132	12.95	870		
	100	28.6	2 436.2	0.4	1 234.7	0.00338	58.51	1.27	133	12.95	325		
	101	10.1	2 446.3	0.4	1 234.9	0.00338	58.93	1.26	133	12.95	1 025		
	102-103	35.1 and 151.6	2 633.0	0.4	1 314.4	0.00515	60.25	1.25	126	15.45	1 300		
4.	1 to 103	....	2 633.0	0.4	1 314.4	....	60.25	....	....	......	......	29 625	105.22

* The greatest Q above was used.

† Excluding initial area.

start the tabling process of Table 8, using Fig. 7. The four water-sheds selected are numbered 1, 2, 3, and 4 on Table 8 and are represented, respectively, by the total areas above Points 1, 2, 3, and 4, of Fig. 7. The velocities in Table 8 were calculated by the use of Kutter's formula.

Comparison of Times of Concentration and Run-off by Different Methods.— The time and run-off, determined by five processes, are arranged for comparison in Table 9. The explanation of this table may be itemized in the form of eight remarks, as follows:

(1) Values in Columns (2) and (3) of Table 9 were selected from Table 8.

(2) Those in Columns (4) and (5) were calculated by the use of Equation (29). The computations for Water-Shed No. 2 (Fig. 7) will illustrate the process.

The following were selected from Table 8: $A = 1\,653$ acres; $L = 17\,115 + 1\,200$ (length in initial area $= 1\,200$ ft.) $= 18\,315$ ft.; $S = \frac{\text{Total fall}}{\text{Total length}} = \frac{68.40 + 3.60}{18\,315} = 3.93$ ft. per 1 000 ft.; $P = \frac{v}{V}$; $v = \frac{L}{t} = \frac{18\,315}{60 \times 42.98} = 7.1$; V (using Kutter's formula, $n = 0.013$, for circular pipe, $S = 3.93$, and $Q = 978.5$) $= 13.1$; $P = \frac{7.1}{13.1} = 0.54$; and $\sqrt{\frac{P}{L}} = 0.00543 = B$. Solving,

$$Q = 15\,(0.4 \times 1\,653 \times 1.25 \times 6.63 \times 0.00543)^{1.1429}\,(3.93)^{0.2143}$$
$$= 971.5 \text{ cu. ft. per sec.}$$

Again, substituting correct values in Equations (1) and (26), respectively, $i = 1.469$ and $t = 43.4$ min.

(3) Columns (6) and (7), Table 9 were calculated by Equation (29) using Table 3, and following Method No. 2. All factors for use in Equation

TABLE 9.—COMPARISON OF TIMES OF CONCENTRATION AND RUN-OFF FOR VARIOUS PROCESSES OF DETERMINATION

Water-Shed No. (see Fig. 7)	FROM TABLE 8, USING 10 MINUTES INITIAL TIME		FORMULA, USING TABLE 4, METHOD 2		FORMULA, USING TABLE 8, METHOD 2		Ratio	TABLING, USING 20 MINUTES INITIAL TIME		FORMULA, USING TABLE 4, METHOD 1		Ratio
	Time t_a, in minutes	Run-off, Q_a, in cubic feet per second	t_b	Q_b	t_c	Q_c	$\frac{Q_b}{Q_c}$	t_1	Q_1	t_2	Q_2	$\frac{\sqrt{t_1}}{\sqrt{t_a+10}}$
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)
1	35.9	694	35.9	692	41.3	646	1.07	47.6	600	48.0	599	1.02
2	43.0	979	43.4	972	51.0	897	1.08	54.9	866	55.8	857	1.02
3	50.6	1 157	51.3	1 149	60.8	1 056	1.09	62.5	1 043	63.3	1 035	1.02
4	60.3	1 314	60.5	1 311	72.0	1 202	1.09	72.4	1 200	73.4	1 190	1.02

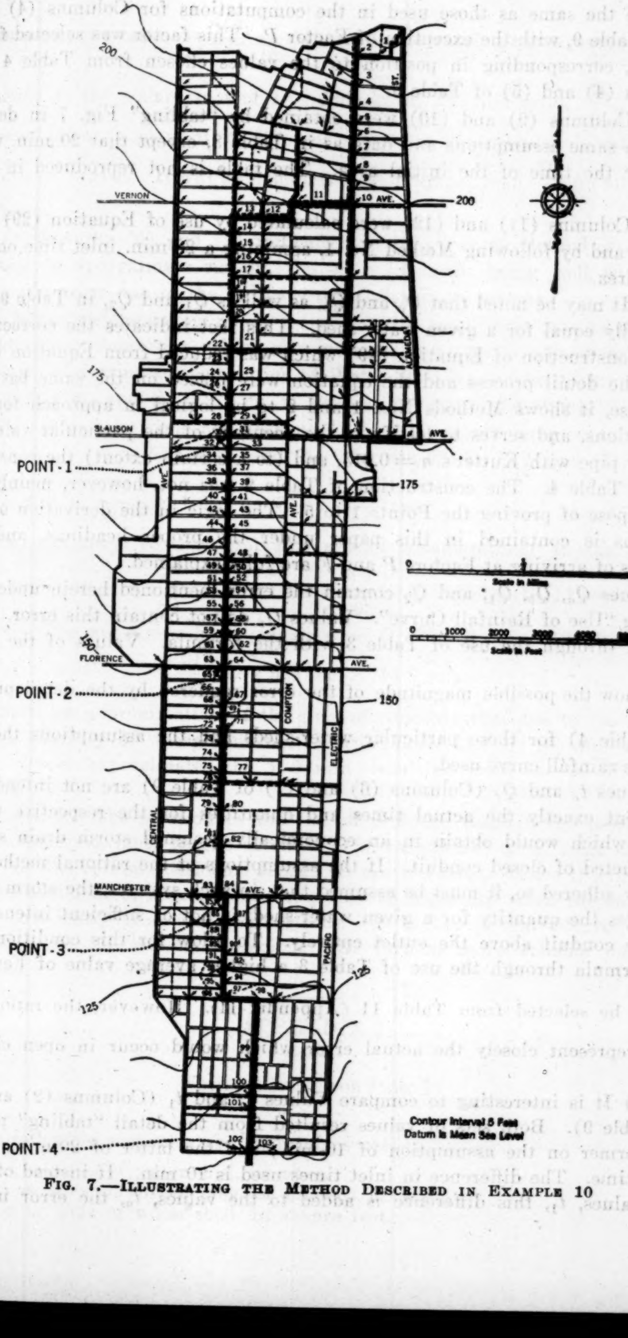

200
VERNON
10 AVE.
200
175
SLAUSON
AVE.
POINT-1
175
ALAMEDA
AVE.
FLORENCE
AVE.
150
POINT-2
150
COMPTON
ELECTRIC
CENTRAL
MANCHESTER
AVE.
PACIFIC
POINT-3
125
125
POINT-4
0 1 2
Scale in Miles
0 1000 2000 3000 4000 5000
Scale in Feet
Contour Interval 5 Feet
Datum is Mean Sea Level

FIG. 7.—ILLUSTRATING THE METHOD DESCRIBED IN EXAMPLE 10

(29) are the same as those used in the computations for Columns (4) and (5) of Table 9, with the exception of Factor P. This factor was selected from Table 3, corresponding in position to the values chosen from Table 4 for Columns (4) and (5) of Table 9.

(4) Columns (9) and (10) were obtained by "tabling" Fig. 7 in detail, with the same assumptions and data as in Table 8, except that 20 min. were used for the time of the initial area. The table is not reproduced in this paper.

(5) Columns (11) and (12) were calculated by use of Equation (29) and Table 4 and by following Method No. 1, assuming a 20-min. inlet time on the initial area.

(6) It may be noted that Q_a and Q_b, as well as Q_1 and Q_2, in Table 9, are practically equal for a given water-shed. This fact indicates the correctness of the construction of Equation (29) which was reduced from Equation (24). Since the detail process and the equation were placed on the same basis in each case, it shows Methods Nos. 1 and 2 to be logical in approach for the assumptions, and serves to testify to the accuracy of the particular value of 6.63 for pipe with Kutter's $n = 0.013$, and (to a certain extent) the construction of Table 4. The construction of Table 9 was not, however, mainly for the purpose of proving the Points 1 to 5. The logic in the derivation of the formulas is contained in this paper under the proper headings, and the methods of arriving at Factors P and F are fully explained.

Values Q_a, Q_b, Q_1, and Q_2 contain the error mentioned herein under the heading "Use of Rainfall Curve". Values Q_c do not contain this error, being evolved through the use of Table 3 with the formula. Values of the ratio, $\frac{Q_b}{Q_c}$, show the possible magnitude of the error rendered by the detail process (or Table 4) for these particular water-sheds and the assumptions therefor and the rainfall curve used.

Values t_c and Q_c (Columns (6) and (7) of Table 9) are not intended to represent exactly the actual times and quantities for the respective water-sheds, which would obtain in an economically designed storm drain system constructed of closed conduit. If the assumptions of the rational method are strictly adhered to, it must be assumed that, in such systems, the storm which produces the quantity for a given water-shed, is not of sufficient intensity to fill the conduit above the outlet entirely. To allow for this condition with the formula through the use of Table 3 a higher average value of Factor F should be selected from Table 11 (Appendix II). However, the ratio, $\frac{Q_b}{Q_c}$, does represent closely the actual error which would occur in open channel work.

(7) It is interesting to compare Values t_a and t_1 (Columns (2) and (9) of Table 9). Both sets of values resulted from the detail "tabling" process, the former on the assumption of 10-min., and the latter of 20-min., initial inlet time. The difference in inlet times used is 10 min. If instead of using the values, t_1, this difference is added to the values, t_a, the error in Q is

measured by the ratio, $\frac{\sqrt{t_1}}{\sqrt{t_a + 10}}$ (since, for the rainfall curve used, Q varies inversely as the square root of t). This error, like the ratio, $\frac{Q_b}{Q_c}$, may be expected to vary somewhat for differences in inlet times assumed, water-sheds, and rainfall curves. It will be smaller (on the average) than $\frac{Q_b}{Q_c}$ for all ordinary additions.

(8) Remarks (6) and (7) indicate that in all cases Method No. 2 may be used to approximate run-off, the error in result being well within the automatic error due to the ordinary detail "tabling" process.

A better understanding of the reason for this small error may be had from the following illustration, which also indicates how greater refinement may be attained. Referring to the solution of Water-Shed No. 2 (see "Remark (2)"), $t_b = 43.4$ min.; and $Q_b = 972$ cu. ft. per sec. (10-min. inlet time).

If 20-min. inlet time has been used, the time would have been, approximately, $t_b + 10$, or 53.4 min., and the total run-off, by Equation (1), will be

$Q = 0.4 \times \frac{9.69}{(53.4)^{0.5}} \times 1\,653 = 877$ cu. ft. per sec.

The maximum velocity (for this second consideration) on the average slope, and F, is: $V = \frac{F^2}{S^{\frac{1}{8}}} s^{\frac{1}{2}} Q^{\frac{1}{4}} = Q = \frac{(6.63)^2}{(3.93)^{\frac{1}{8}}} (0.00393)^{\frac{1}{2}} (877)^{\frac{1}{4}} = 12.65$ ft. per sec. The average velocity equals $v = P\,V = 0.54 \times 12.65 = 6.83$. Finally, $t = \frac{L}{v} = \frac{18\,315}{6.83} = 2\,682$ sec. $= 44.7$ min.

This is an approximation of the time of concentration due to the smaller average quantity of water running. Furthermore, 44.7 + 10 min. = 54.7, which closely approximates the 54.9 min. resulting from the 20-min. inlet "tabling".

Reminder.—The solution of the run-off problem, through the medium of any assumptions, must be one of trial. The assignment of an inlet time (if used) and the run-off coefficient depend on the intensity. Since the intensity must be anticipated, a second solution may be necessary when the intensity varies sufficiently from the assumptions. It is hoped that future experimentation on the run-off coefficient will furnish more reliable data for selection.

APPENDIX I

Nomenclature for Part II

The following nomenclature, in brief, is used:

A = area of water-shed, in acres.
A' = area of initial area, or nucleus, in acres.
a = area of water-shed, in square feet.

$B = \sqrt{\frac{P}{L}}$.

C = **coefficient representing the ratio of the rate of run-off to the rate of rainfall.**

C' = **coefficient of run-off for initial area, or nucleus.**

c = constant in Chezy formula, $v = c\sqrt{r\,s}$.

e = **a positive fractional exponent of time, t, in Equation (2).**

f = **correction factor (see Fig. 3).**

F = **constant, depending on shape and condition of main channel of flow;** $F = N\,S^{\frac{1}{16}}$.

H = a given period of rain, in hours, $R_H = \frac{X}{H}$.

i = **intensity of rainfall, in cubic feet per second per acre, or, approximately, in inches per hour.**

i' = **average intensity on initial area, or nucleus.**

J = constant $= \frac{N}{r^{\frac{1}{4}}}$.

K = **constant in Equation (2)** $= i\,t^e$.

K' = **constant.**

K'' = **constant.**

l = **any length, in feet.**

L_1, L_2, L_3, **etc. = subdivisions of** l.

L = **length, in feet, which water must traverse in running from the most remote portion of the water-shed to its outlet.**

L' = **length, in feet, which water must traverse in running from the most remote portion of the nucleus to its outlet.**

M **and** M' = **constants.**

m = **percentage of precipitation, i, being stored.**

N = constant $= \frac{\left(c\sqrt{r}\right)^{\frac{1}{2}}}{Q^{\frac{1}{8}}} = \frac{F}{S^{\frac{1}{16}}}$.

P = **constant, depending on shape of area of water-shed and its** manner of concentration, $P = \frac{v}{V}$.

$P_1 = \frac{v'}{V'}$.

P' = **constant, depending on beginning nucleus, = corrected** P_1.

Q = **rate of run-off, in cubic feet per second.**

Q' = **rate of run-off from nucleus, in cubic feet per second.**

R_H = **average rainfall intensity, in inches per hour, for a period of** H hours, $R_H = \frac{X}{H}$.

r = hydraulic radius in $v = c\sqrt{r\,s}$; also, rate of retention.

s = **slope, or fall, in feet per foot, in length of main channel of flow.**

S = **fall, in feet per 1 000 ft. of main channel of flow;** $S = 1\,000\,s$.

t = **time of concentration of water-shed = time required for the water to travel from the most remote portion of the water-shed to its outlet.**

t' = **time of concentration of nucleus.**

v = **average velocity of water, in feet per second, in traversing the** length, L. $v = \frac{L}{t}$, **or, hydraulically,** $v = c\sqrt{r\,s}$.

v' = average velocity of water, in feet per second, in traversing the length, L'. $v' = \frac{L'}{t'}$.

V = velocity of Q at outlet of water-shed, based on average values of F and S for the water-shed.

V' = velocity of Q' based on average values of F and S for the water-shed.

$W = \frac{a}{L} = \frac{43\,560\,A}{L}$.

$W' = \frac{43\,560\,A'}{L'}$.

X = inches of rain for a period of H hours; $R_H = \frac{X}{H}$.

s = rate of absorption plus rate of evaporation, in cubic feet per second per acre.

APPENDIX II

METHOD OF CALCULATING VALUES OF N AND F

Begin with, say, a 14 by 7-ft., open, rectangular channel with an area of 98 sq. ft., and an hydraulic radius of 3.50. Next, select from any reliable set of hydraulic tables (showing velocities for varying values of the hydraulic radius, Kutter's n, and the slope) the velocity for an hydraulic radius of 3.50 in the column of $s = 0.01$ (or $S = 10$), with Kutter's $n = 0.013$. This will be $V = 26.25$. Calculate $Q = 98 \times 26.25 = 2\,572.5$.

For $s = 0.01$, $c\sqrt{r} = 10\,V = 262.5$, and $\left(c\sqrt{r}\right)^{\frac{1}{2}} = 16.20$. Substituting this value and the value, $Q = 2\,572.5$, in Equation (18), $N = 6.07$. This factor may be calculated for varying values of Kutter's n in the foregoing manner and placed on a line opposite an open rectangular channel, as in Table 10. This table was prepared by slide-rule computations for the purpose of illustrating the method.

It was shown that Equation (18) applies for a given shape and condition of channel. Having determined N for the 2 by 1 (width by height) open conduit, its value may be calculated for other shapes (for the small change in the hydraulic radius due to shape alone) by letting $N \propto r^{\frac{1}{4}}$. Thus, having assumed an area of 98 sq. ft for a 2 by 1 open conduit, calculate the relative r and $r^{\frac{1}{4}}$ for various shapes of channels using an area of 98 sq. ft., and arrange them in tabular form for reference. Then, since $N \propto r^{\frac{1}{4}}$, $N = J\,r^{\frac{1}{4}}$, or,

$$J = \frac{N}{r^{\frac{1}{4}}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (47)$$

the corresponding values of N for any shape of channel may be calculated, and the table expanded. For instance (using $N = 6.07$), $J = \frac{N}{r^{\frac{1}{4}}} = \frac{6.07}{1.366} = 4.44$.

Then, selecting any shape, such as circular pipe, with $r^{\frac{1}{4}} = 1.292$, $N = J\,r^{\frac{1}{4}} = 4.44 \times 1.292 = 5.74$.

The value of $J = 4.44$ may be used for calculating N for other shapes under Kutter's $n = 0.013$. In like manner the value of J obtained through the use of N, under other values of Kutter's n, may be used for calculating N for various shapes.

TABLE 10.—Illustration of Method of Calculating Values of N

Values of Kutter's n		.....	.....	0.011	0.013	0.015	0.020	0.025	0.030	0.035
Relative r, $r^{\frac{1}{4}}$ (area 98)		r	$r^{\frac{1}{4}}$	N	N	N	N	N	N	N
Pipe		2.793	1.292	6.15	5.74	5.39	4.83	4.49	4.21	3.99
Single box (closed)	1 by 1	2.475	1.254	5.97	5.57	5.24	4.68	4.36	4.09	3.87
	2 by 1	2.333	1.236	5.89	5.49	5.15	4.61	4.30	4.03	3.81
	etc.	.....	.....	....	....	....	....	....	....	....
Double box (closed)	1 by 1	1.750	1.150	5.47	5.11	4.80	4.29	4.00	3.75	3.55
	2 by 1	1.650	1.133	5.39	5.04	4.73	4.23	3.94	3.69	3.50
	etc.	.....	.....	....	....	....	....	....	....	....
Triple box (closed)	1 by 1	1.428	1.093	5.20	4.86	4.56	4.08	3.80	3.56	3.38
	2 by 1	1.347	1.076	5.12	4.78	4.49	4.02	3.74	3.50	3.32
	etc.	.....	.....	....	....	....	....	....	....	....
Open rectangular channel	2 by 1	3.50	1.366	6.50	6.07	5.70	5.10	4.75	4.45	4.22
	3 by 1	3.425	1.359	6.47	6.04	5.67	5.07	4.73	4.43	4.20
	1 by 1 4 by 1	3.305	1.347	6.41	5.98	5.63	5.03	4.68	4.39	4.16
	etc.	.....	.....	....	....	....	....	....	....	....
	30 by 1	1.695	1.140	5.43	5.07	4.76	4.26	3.96	3.71	3.52

Having calculated the values of N for $S = 10$, values of F may be calculated. It was shown, in introducing Equation (21), that,

$$N = \frac{F}{S^{\frac{1}{16}}} \quad \ldots\ldots (48)$$

Therefore, using $S = 10$, $N = \dfrac{F}{(10)^{\frac{1}{16}}}$, or $F = N\,(10)^{\frac{1}{16}}$.

Thus, having found the values of N under an assumed S, the factor, F, may be readily calculated and arranged as shown in Table 11. This table, showing values of F for channels of varying shape and condition, may be used in its present form, or may be expanded for use in general hydraulic work.

For N in Equation (18) substitute its value in Equation (48) and simplify, thus,

$$\sqrt{r} = \frac{F^2}{S^{\frac{1}{8}}}\,Q^{\frac{1}{4}} \quad \ldots\ldots (49)$$

In turn, substituting this value of $c\,\sqrt{r}$ in Equation (13): $v = c\,\sqrt{rs} = \dfrac{F^2}{S^{\frac{1}{8}}}\,s^{\frac{1}{2}}\,Q^{\frac{1}{4}} = 0.03162\,F^2\,S^{\frac{3}{8}}\,Q^{\frac{1}{4}}$, or,

$$v = 0.4217\,F^2\,s^{\frac{3}{8}}\,Q^{\frac{1}{4}} \quad \ldots\ldots (50)$$

Thus, for a given run-off, Q, the approximate velocity may readily be computed for any definite shape with an assigned Kutter's n.

Since the velocity varies directly as F^2, Table 11 may also be used as a handy guide to the efficiency of relative shapes for a given Kutter's n.

Statement.—Heretofore, it has not been customary to allow for variation in the shape and condition of the main channel of flow in run-off formulas. The writers, however, have considered it well worth while to do so. In order to devise a practical working table of variations it was necessary to select, carefully, the velocity formula which could be used most advantageously for the purpose. The Kutter formula for c in $c\sqrt{r\,s}$ has been, and is at present, used very widely in hydraulic computations. It was chosen because of its general application to all shapes and conditions of channels. The exponential form for the combination of the Kutter and Chezy formulas (Equation (50)) affords a very practical means of arriving at a rational time of concentration (which is based on average velocities) of a given water-shed. Considering the inexact nature of the run-off problem, it is close enough for all practical purposes.

TABLE 11.—VALUES OF FACTOR F

Kutter's n			0.011	0.013	0.015	0.020	0.025	0.030	0.035	0.100
CLOSED SECTIONS										
Pipe			7.10	6.63	6.22	5.57	5.19	4.86	4.61	....
Closed semi-circular			6.85	6.38	6.00	5.36	5.00	4.68	4.44	....
Closed semi-elliptical			7.05	6.57	6.16	5.53	5.14	4.82	4.56	....
Flattened horseshoe			6.99	6.52	6.11	5.49	5.10	4.79	4.53	....
Rectangular (width by height)	Single	1 by 2	6.79	6.34	5.95	5.32	4.96	4.65	4.42	....
		1 by 1	6.89	6.43	6.05	5.40	5.04	4.73	4.47	....
		2 by 1	6.79	6.34	5.95	5.32	4.96	4.65	4.42	....
		3 by 1	6.65	6.21	5.83	5.21	4.86	4.55	4.32	....
		4 by 1	6.51	6.08	5.72	5.11	4.77	4.46	4.23	....
	Double	1 by 2	6.22	5.82	5.46	4.88	4.55	4.26	4.04	....
		1 by 1	6.32	5.90	5.54	4.96	4.62	4.33	4.10	....
		2 by 1	6.22	5.82	5.46	4.88	4.55	4.26	4.04	....
		3 by 1	6.10	5.68	5.34	4.77	4.46	4.17	3.95	....
	Triple	1 by 2	5.91	5.52	5.18	4.68	4.32	4.04	3.84	....
		1 by 1	6.00	5.61	5.27	4.71	4.39	4.11	3.90	....
		2 by 1	5.91	5.52	5.18	4.68	4.32	4.04	3.84	....
		3 by 1	5.80	5.42	5.08	4.55	4.24	3.97	3.77	....
OPEN SECTIONS										
Open semi-circular			7.74	7.21	6.77	6.05	5.64	5.28	5.01	3.38
Rectangular (width by height)	Single	1 by 2	7.10	6.64	6.24	5.58	5.20	4.86	4.61	3.11
		1 by 1	7.40	6.91	6.50	5.81	5.40	5.07	4.80	3.23
		2 by 1	7.51	7.01	6.58	5.89	5.48	5.14	4.87	3.28
		3 by 1	7.47	6.98	6.55	5.86	5.46	5.12	4.85	3.26
		4 by 1	7.40	6.91	6.50	5.81	5.40	5.07	4.80	3.23
		6 by 1	7.24	6.77	6.35	5.68	5.29	4.95	4.70	3.16
		8 by 1	7.10	6.64	6.24	5.58	5.20	4.86	4.61	3.11
		10 by 1	6.99	6.51	6.12	5.47	5.09	4.78	4.53	3.05
		12 by 1	6.87	6.42	6.03	5.39	5.02	4.70	4.46	3.00
		20 by 1	6.52	6.11	5.73	5.12	4.77	4.47	4.24	2.85
		30 by 1	6.27	5.86	5.50	4.92	4.57	4.28	4.06	2.74
		100 by 1	5.45	5.08	4.76	4.25	3.97	3.73	3.53	2.38

TABLE 11.—(*Continued*)

Kutter's *n*		0.011	0.013	0.015	0.020	0.025	0.030	0.035	0.100
OPEN SECTIONS WITH SIDE SLOPES									
Side slopes	Bottom width by height								
½ : 1	V-shape	7.31	6.82	6.40	5.72	5.33	5.00	4.74	3.19
	1 by 1	7.64	7.12	6.68	5.99	5.58	5.23	4.95	3.34
	2 by 1	7.61	7.10	6.66	5.96	5.56	5.20	4.94	3.32
	4 by 1	7.45	6.95	6.52	5.83	5.43	5.09	4.83	3.26
	8 by 1	7.12	6.64	6.23	5.58	5.19	4.87	4.62	3.11
	16 by 1	6.68	6.23	5.85	5.24	4.89	4.57	4.34	2.92
	30 by 1	6.23	5.86	5.50	4.91	4.58	4.29	4.07	2.75
	100 by 1	5.45	5.08	4.77	4.25	3.97	3.72	3.53	2.38
1 : 1	V-shape	7.51	7.00	6.58	5.87	5.48	5.13	4.87	3.28
	1 by 1	7.60	7.08	6.64	5.95	5.54	5.19	4.92	3.32
	2 by 1	7.54	7.02	6.60	5.90	5.50	5.15	4.89	3.30
	4 by 1	7.37	6.87	6.45	5.76	5.37	5.04	4.77	3.22
	8 by 1	7.07	6.57	6.17	5.53	5.15	4.83	4.58	3.08
	16 by 1	6.68	6.21	5.83	5.23	4.87	4.55	4.32	2.91
	30 by 1	6.25	5.81	5.47	4.89	4.55	4.27	4.05	2.72
	100 by 1	5.45	5.07	4.76	4.25	3.97	3.73	3.53	2.38
2 : 1	V-shape	7.31	6.82	6.40	5.72	5.33	5.00	4.74	3.19
	1 by 1	7.31	6.82	6.40	5.72	5.33	5.00	4.74	3.19
	2 by 1	7.27	6.78	6.36	5.70	5.30	4.97	4.72	3.18
	4 by 1	7.14	6.65	6.25	5.59	5.21	4.88	4.64	3.12
	8 by 1	6.90	6.44	6.05	5.40	5.04	4.71	4.48	3.03
	16 by 1	6.58	6.12	5.75	5.14	4.80	4.49	4.27	2.88
	30 by 1	6.20	5.76	5.41	4.85	4.52	4.23	4.01	2.70
	100 by 1	5.42	5.06	4.75	4.24	3.95	3.70	3.52	2.37
3 : 1	V-shape	7.04	6.58	6.17	5.51	5.14	4.83	4.57	3.08
	1 by 1	7.04	6.58	6.17	5.51	5.14	4.83	4.57	3.08
	2 by 1	7.01	6.53	6.14	5.49	5.12	4.79	4.54	3.06
	4 by 1	6.95	6.45	6.07	5.42	5.06	4.74	4.50	3.03
	8 by 1	6.75	6.29	5.90	5.28	4.92	4.61	4.38	2.96
	16 by 1	6.49	6.04	5.67	5.07	4.73	4.48	4.20	2.83
	30 by 1	6.14	5.72	5.38	4.81	4.49	4.20	3.99	2.68
	100 by 1	5.40	5.03	4.72	4.22	3.93	3.69	3.50	2.36

Some more recent experimentation has been done, which indicates that Kutter's formula is somewhat in error for certain shapes of section. The Scobey formula for concrete pipe flowing full is one of the best known. This formula is:

$$Q = 0.00546\, C_s\, d^{2.625}\, H^{0.5} \qquad (51)$$

or,

$$v = C_s\, d^{0.625}\, H^{0.5} \qquad (52)$$

in which, Q is the discharge, in cubic feet per second; v, the velocity, in feet per second; C_s, Scobey's coefficient; d, the diameter, in inches; and H, the friction head per 1 000 ft.

For comparative purposes it may be of interest to reduce the Scobey formula, showing the velocity in terms of C_s, H, and Q. From Equation (51),

$$d^{2.625} = \frac{Q}{0.00546\, C_s\, H^{0.5}}, \text{ and,}$$

$$d^{0.625} = \frac{Q^{0.2381}}{0.2892\, C_s^{0.2381}\, H^{0.119}} \qquad (53)$$

Substituting Equation (53) in Equation (52),

$$v = 3.459\ C_s^{0.7619}\ H^{0.381}\ Q^{0.2381} \ldots\ldots\ldots\ldots\ldots\ldots\ldots (54)$$

Using the nomenclature of this paper, and replacing H in Equation (54) by S,

$$v = 3.459\ C_s^{0.7619}\ S^{0.381}\ Q^{0.2381} \ldots\ldots\ldots\ldots\ldots\ldots\ldots (55)$$

Comparing Equation (55) (which is the Scobey formula reduced to like terms) with the exponential form of the Kutter-Chezy formula (Equation (50)), it may be seen that the exponents of S and Q do not differ greatly.

This difference will be of less consequence in its effect on run-off calculations, as may be shown by substituting the value for v, shown in Equation (55), in the formula, and following the same method of reduction as outlined in connection with Equation (14):

$$Q = (3\,600\ H)^e\ C\ A\ R_H \frac{v^e}{L^e} \ldots\ldots\ldots\ldots\ldots\ldots\ldots (56)$$

Factor H in Equation (56) is not to be confused with the H in the Scobey formula (Equation (51)), which was replaced by S.

For v in Equation (56) substitute Pv, in which, v is given in Equation (55); thus

$$Q = (3\,600\ H)^e\ C\ A\ R_H \frac{[P\ (3.459\ C_s^{0.7619}\ Q^{0.2381}\ S^{0.381})]^e}{L^e}$$

or,

$$Q = (12\,452.4\ H)^{eq} \left(\frac{P}{L}\right)^{eq} (C\ A\ R_H)^q\ C_s^{0.7619\,eq}\ S^{0.381\,eq} \ldots\ldots (57)$$

in which, q has been substituted for the exponent, $\frac{1}{1 - 0.2381e}$, for simplicity.

Equation (57) may be compared with Equation (24). It may be reduced to a special form by letting $e = 0.5$, in which case, $q = \frac{1}{1 - 0.1191} = 1.1352$, and,

$$Q = 210.9\ H^{0.5675} \left(\frac{P}{L}\right)^{0.5675} (C\ A\ R_H)^{1.135}\ C_s^{0.4324}\ S^{0.2162} \ldots\ldots (58)$$

Substituting $\sqrt{\frac{P}{L}} = B_1$ and $\sqrt{S} = D$,

$$Q = 210.9\ H^{0.5675}\ (C\ A\ R_H\ B_1)^{1.135}\ (C_s\ D)^{0.4324} \ldots\ldots\ldots\ldots (59)$$

and when $H = 1$ hour,

$$Q = 210.9\ (C\ A\ R_1\ B_1)^{1.135}\ (C_s\ D)^{0.4324} \ldots\ldots\ldots\ldots (60)$$

Equations (58), (59), and (60) may be compared with Equations (27), (28), and (29), respectively. It may be noted that the difference in exponents is small and that a certain compensating effect takes place between the exponents for S and those for the other factors.

Equations (57), (59), and (60) may be used for run-off calculations by means of C_s values established by F. C. Scobey, M. Am. Soc. C. E. It is understood by the writers, however, that the Scobey formula for velocity is restricted to circular pipe flowing full. In this case Equations (57) to (60), inclusive, resulting from its use, are likewise restricted.

To illustrate further the direct applicability and correctness of the factors and formulas herein advocated, they are applied to the upper part of the Compton Creek water-shed, which lies immediately south of the City of Los Angeles. This area is divided into two main parts, the drainage channels of which converge near the lower end of the area. (Somewhat similar to Type 17, Fig. 2.) The western part of this area contains 16 731 acres and the eastern part, 12 361 acres.

Since the topography of both sides is similar, the West Channel, with a length of 64 600 ft., will control the time of concentration, since it is the longer.

Example 11.—Find the approximate time of concentration using a rainfall curve of the type of Equation (3).

Let $R_1 = 1.50$ in. per hour.

$S = 2.07$ ft. per 1 000 ft. from U. S. Topographical map. (Total fall, 134 ft.)

$F = 6.5$ (selected because part of the channel will be a closed rectangular box section running part full, and the remainder, pipe).

$$\frac{L}{W} = \frac{(64\ 600)^2}{43\ 560 \times 16\ 731} = 5.7.$$

Using Table 3, and Method No. 2, $P = 0.47$. Let $C = 0.4$; then $Q = 6\ 466$ cu. ft. per sec. by Equation (29); $i = 0.966$ by Equation (1); and $t = 144.7$ min. by Equation (26).

Example 12.—Find the approximate time of concentration which would result from the usual tabling process, with all data the same as Example 11 except that, $P = 0.55$ (selected from Table 4, Method No. 2); and, $F = 6.0$ (assuming all conduits running full). Then, $Q = 6\ 377$ cu. ft. per sec. by Equation (29); $i = 0.953$ by Equation (1); and, $t = 148.7$ min. by Equation (26).

Example 13.—Find the approximate time of concentration, using a rainfall curve of the type, $i = \frac{K}{t^{0.7}}$ and $C = 0.35$. All other data are the same as Example 11.

This solution could be made by the use of Equation (24) with proper exponents, using $e = 0.7$, but a close approximation may be made as follows: Using the time, 144.7 min. (Example 11), read the intensity from a curve of $i = \frac{K}{t^{0.7}}$ or calculate: $i = 0.81$ (see Equation (26)); $Q = 0.35 \times 0.81 \times 16\ 731 = 4\ 743$ cu. ft. per sec. by Equation (1); $V = 14.56$; $v = P\ V = 0.47 \times 14.56 = 6.84$; and $t = \frac{L}{v} = \frac{64\ 600}{6.84} = 157.3$ min.

The solution by means of Equation (24), with $e = 0.7$, gives $t = 160$ min. The difference between the times, 144.7 and 157.3, is due, of course, to the smaller average quantity of water running. In the same manner, approximations of the time of concentration may be found for any curve in use.

Example 14.—Find the approximate time of concentration with all data the same as Example 11 except that $C = 0.30$ and $F = 4.72$ (from Table 11, main channel, open earth, Kutter's $n = 0.035$, the bottom width is twice the depth, and the side slopes are 2 : 1.) Then, $Q = 3\ 228$ cu. ft. per sec. by Equation (29); $i = 0.643$ by Equation (1); and, $t = 326$ min. by Equation (26).

DISCUSSION

LeRoy K. Sherman,[12] M. Am. Soc. C. E. (by letter).—In Part I, the authors have presented an excellent analysis of the variable factors affecting the rates of run-off. Engineers at the present time are generally agreed that the empirical run-off formulas, based simply on a function of the drainage area, are of very limited usefulness. In Part II, all the numerous factors involved in the rational method for determining run-off have been grouped into a rational formula. In the writer's opinion this is not desirable. The rational method as a procedure is preferable to the rational formula.

Although it is somewhat lengthy, the rational method is a simple process of computation. The various steps follow in logical sequence and can be readily visualized. The relative effects of the various factors and approximations are apparent. In a formula involving seven variables these relative effects are concealed. In the design of a system of storm drains, the time spent in computation by the rational method is time well spent, and the substitution of a formula merely as a time saver is not warranted.

Certain steps in the rational method may be diagrammed or tabulated—just as the authors have done—to serve as a useful guide. The writer has prepared and used several such diagrams applicable to the rational method.[13] Other engineers have presented similar diagrams.

In Example 7, the authors have applied their formula to a drainage area of 500 sq. miles. The main stream has a length of 42 miles. If this stream has a channel of regular and uniformly increasing cross-sections, without enlargements or constrictions, then the rational method as applied to storm drains may be used.

In general, natural streams draining large areas are irregular, with numerous enlargements and constrictions. On such streams the aforesaid rational method will not apply. This is because another variable (in addition to the seven mentioned by the authors) is involved; namely, the factor of pondage. In a stream with a flat slope, say, 1.5 ft. per 1 000, this pondage factor may reduce the peak rate of run-off 50 per cent.

For large drainage areas with unimproved natural channels the writer prefers the use of a rational method for determining run-off, based on the relation of an observed hydrograph to a known rainfall on the drainage area in question. This relation has been expressed as a coefficient by the Committee on Floods of the Boston Society of Civil Engineers[14] in its formula, $Q = C_r \sqrt{A}\, R$.

Francis Bates,[15] Assoc. M. Am. Soc. C. E. (by letter).—The authors present an original method for estimating storm-water run-off—based on fundamental conceptions and rational conclusions—which amplifies and refines the methods now in general use and should prove a valuable contribution to the literature heretofore published. The formulas are of sufficient flexibility to be made applicable to most of the rainfall curves, and contain factors neglected in the empirical formulas, but which, most certainly, are pertinent. They should prove very useful for determining the probable maximum flood at selected points, and for preparing preliminary cost estimates of drainage systems. The greater value of these refinements, however, should be in connection with the reduction of field observations made for the purpose of arriving at the correct values of the ratio of run-off to current rainfall, and, perhaps, the co-relating of the so-called values of C in the various empirical formulas.

---

[12] Pres., Randolph-Perkins Co., Chicago, Ill.

[13] See, *Municipal and County Engineering*, June, 1924.

[14] *Journal*, Boston Society of Civil Engineers, September, 1930.

[15] Cons. Engr., Los Angeles, Calif.

Before the formulas and the numerical values of the new factors, P, and F, can be used with confidence, they must be subjected to close scrutiny and to comparison, as to results, with the experiences of others. It is not, however, the present purpose of the writer to examine the derivation of these formulas, nor the calculations, in detail, but rather to attempt, by their application, to show wherein may lie their greater value; and by questioning certain statements and conclusions made in the paper, to encourage the authors, and perhaps others, to clarify and enlarge upon certain phases of the presentation.

On first study of this paper, the reaction is that the subject-matter is very involved in assumptions, and the formulas appear to be complicated to a degree that would make their use difficult by the average individual, in general practice. However, on more thorough examination, it is clear that they are not so complicated once the type of curve is selected, and that they may be used in the same manner as the ordinary empirical formula when it is desired to take advantage of the extra factors submitted by the authors.

These formulas could be made of especially practicable value if they were universally used in the reduction of field observations made to determine the proper values for C. A careful choosing of the type of water-shed, of correct values of P, F, and (hydraulic) S, and their application in the formulas, are not of such moment to the practitioner as is the choice of the elusive coefficient, C. These values are, as yet, so much a matter of judgment of the individual, based on his experience with present methods, that a change might easily leave one hopelessly undecided as to the proper choice. The introduction of the factors, P and F, results in an entire elimination of the former partly automatic correction of C values, and, unless the value now is chosen with extreme care, the advantages of the introduction are nullified. For instance, under the heading, "Method of Arriving at Values of Factor P: Remarks and Deductions" (Part II), it is stated that:

"For all practical purposes, therefore, the values of P (in Tables 3 and 4), shown opposite $\frac{A}{A'} = 262\,144$, and in the column headed by $P' = 0.5$, may be used when the formulas are applied without separate consideration of the inlet area, or nucleus."

These values of P have a range of from 0.42 to 0.62, and their significance in Equation (27), being $P^{0.5714}$, the actual values would lie between 0.61 and 0.76. If a constant value were assumed, such that $P^{0.5714}$ lay midway between these extremes, say, 0.685, the maximum error (either plus or minus), would be not more than about 11 per cent. One questions whether such refinement is warranted in everyday practice, at least until such time as the C values can be chosen more accurately. When using this same Equation (27), if one happened to pick 0.25 as a proper C value, whereas the correct one might be less or greater, say, 0.20 or 0.30, the erroneous assumption would result in a discrepancy of about 29% over design, or about 19% under design, as the case might be.

The formulas are developed logically and in a manner easily understood, except with respect to the factor, P. This appears first (see Equation (19)), as a coefficient of N, an abstract number found by inspection to be practically constant for any definite shape of channel with a given slope and Kutter's n. The next mention occurs directly under "Method of Arriving at Values of Factor P," wherein P is defined as the ratio of the average velocity of flow (through the water-shed) to the velocity at the outlet. In developing the formula, N is eventually replaced, in part, by the factor, F, representing the varying shapes and conditions of channel; and, in part, by the factor, P, as being representative of the ratio of internal velocities. It is not made very clear, however, why this factor should have this numerical value, nor why this final velocity has any bearing whatsoever.

However, if the results obtained by the application of the new formulas are found to be consistent with experience, they may well be accepted, at least in principle. The writer, for one, is not ready to concede unsoundness in all the empirical formulas, but will admit, only, that engineers sometimes use them unintelligently. These basic formulas may, then, be used as "yardsticks" and by simple comparison of the old with the new, one may consider whether or not the principle of the latter is correct, not necessarily because of the theory expounded, but because of its consistency with the former, in results.

In the following, it should be kept in mind that the C of this paper is presumed to be the ratio of actual run-off to current rainfall, whereas the C of the empirical formulas (hereinafter called C'), is merely a correcting factor, inserted for the purpose of adjusting the equation to prevailing conditions. The more important of these empirical formulas in general use,[18] re-arranged so as to be in similar general form, may be stated, as follows:

Hawksley:

$$Q = C' R_1^{0.750} A^{0.750} S^{0.250} \qquad (61)$$

Adams:

$$Q = C' R_1^{0.833} A^{0.833} S^{0.083} \qquad (62)$$

Burkli-Ziegler:

$$Q = C' R_1 A^{0.750} S^{0.250} \qquad (63)$$

McMath:

$$Q = C' R_1 A^{0.800} S^{0.200} \qquad (64)$$

Parmley:

$$Q = C' R_1 A^{0.833} S^{0.250} \qquad (65)$$

---

[18] "American Sewerage Practice," by Metcalf and Eddy, First Edition, Vol. 1, p. 234.

Hering:

$$Q = C' R_1 A^{0.833} S^{0.270} \ldots\ldots (66)$$

Charles E. Gregory:

$$Q = C' R_1 A^{0.860} S^{0.186} \ldots\ldots (67)$$

and, for comparison, Equation (29) may be written:

$$Q = 15 (FB)^{1.1429} C^{1.1429} R_1^{1.1429} A^{1.1429} S^{0.2143} \ldots\ldots (68)$$

Equation (68) is a special reduction of Equation (24), with $i = \frac{K}{\sqrt{t}}$.

Equations (61) to (68) show readily that identical results could not be obtained, from the use of any two or more of these formulas, except by a proper adjustment of the C values. There is no doubt but that each of these formulas gave results entirely satisfactory to the originator; but they proved satisfactory only because he was able to apply, intelligently, the C' values peculiar to the water-sheds upon which his observations were made and to the type of storms encountered. If these particulars were generally known, in detail, the formulas could probably be reconciled, and the mass of data, heretofore obtained, could be co-related.

An approximation can be made, which will give some idea of the relative C values, as used in the formulas of this paper, and those of one of the empirical formulas. The McMath formula (Equation (64)) is probably more widely used than any one of the others. Generally, it has been found to give fairly satisfactory results for the smaller areas, say, up to 1 000 acres, under conditions of natural flow; that is, without an improved drainage system. Assume, then, that the McMath formula and the authors' Equation (29), having $i = \frac{K}{\sqrt{t}}$, each give correct results, and equate them, as follows: Let $C' = C$, of the McMath formula, and $C = C$, of Equation (29). Then:

$$C' R_1 A^{0.800} S^{0.200} = 15 C^{1.1429} R_1^{1.1429} A^{1.1429} F^{1.1429} B^{1.1429} S^{0.2143} \ldots\ldots (69)$$

and,

$$C' = 15 C^{1.1429} R_1^{0.1429} A^{0.3429} F^{1.1429} B^{1.1429} S^{0.0143} \ldots\ldots (70)$$

Assume that $R_1 = 2.00$, the value originally used in the McMath formula, and that the value of S is between 10 and 100, then $R_1^{0.1429} = 1.104$; $S^{0.0143}$ lies between 1.034 and 1.068; and, taking the approximate mean of these values, $R_1^{0.1429} S^{0.0143}$ is called 1.160.

For natural conditions of flow, a value of $F = 4.20$ is chosen (Table 11) as being fairly representative, and $F^{1.1429} = (4.20)^{1.1429} = 5.156$. Substituting these representative values, Equation (70) now may be written:

$$C' = (15) (1.160) (5.156) C^{1.1429} A^{0.3429} B^{1.1429}$$

or,

$$C' = (89.71) C^{1.1429} A^{0.3429} B^{1.1429} \ldots\ldots (71)$$

Again, with,

$A = 10$ acres, $A^{0.3429} = 2.202$;

$A = 100$ acres, $A^{0.3429} = 4.851$;

and,

$A = 1\,000$ acres, $A^{0.3429} = 10.683$;

and, for average conditions, assume that for 10 acres, $L = 1\,325$ ft.; for 100 acres, 4 220 ft.; and for 1 000 acres, 13 250 ft. In Table 3 under "$P' = 0.5$, Medium Concentration, $\frac{L}{W} = 3$", P is found equal to 0.48. This table is used rather than Table 4, because of comparison being made with the uniform assumption. Hence, values of $B^{1.1429}$ are: 0.01081, 0.00557, and 0.00290, for 10, 100, and 1 000 acres, respectively.

Then, substituting further, Equation (71) may be written for the three assumptions of acreage, as follows:

For 10 acres:

$$C' = (1.9422\, C)^{1.1429} \quad \text{(72)}$$

For 100 acres:

$$C' = (2.1700\, C)^{1.1429} \quad \text{(73)}$$

For 1 000 acres:

$$C' = (2.4458\, C)^{1.1429} \quad \text{(74)}$$

These formulas may be reduced and tabulated, for convenience of comparison, for the range of values generally used, such as is given in Table 12.

It must be remembered, however, that the comparisons in Table 12 are made on the assumption of drainage running in natural channels, or similar

TABLE 12.—COMPARATIVE VALUES OF C' OF MCMATH FORMULA AND C OF EQUATION (29), UNDER AVERAGE CONDITIONS OF NATURAL FLOW

C' of McMath	C of Equation (29)			C of Equation (29)	C' of McMath		
	10 acres	100 acres	1 000 acres		10 acres	100 acres	1 000 acres
0.10	0.069	0.062	0.055	0.10	0.154	0.174	0.200
0.15	0.098	0.088	0.078	0.15	0.244	0.277	0.318
0.20	0.126	0.113	0.100	0.20	0.339	0.385	0.442
0.25	0.153	0.137	0.122	0.25	0.438	0.497	0.570
0.30	0.180	0.161	0.143	0.30	0.539	0.612	0.702
0.35	0.206	0.184	0.163	0.35	0.643	0.730	0.837
0.40	0.231	0.207	0.183	0.40	0.749	0.851	0.975
0.45	0.256	0.229	0.203	0.45	0.857	0.973	1.116
0.50	0.281	0.251	0.223	0.50	0.967	1.098	1.259
0.55	0.305	0.273	0.242	0.55	1.078	1.224	1.403
0.60	0.329	0.295	0.262	0.60	1.191	1.352	1.550
0.65	0.353	0.316	0.281	.....	.....	.....	.....
0.70	0.377	0.337	0.299	.....	.....	.....	.....
0.75	0.400	0.358	0.318	.....	.....	.....	.....
0.80	0.424	0.379	0.336	.....	.....	.....	.....
0.85	0.447	0.400	0.355	.....	.....	.....	.....
0.90	0.470	0.420	0.373	.....	.....	.....	.....
0.95	0.492	0.441	0.391	.....	.....	.....	.....
1.00	0.515	0.461	0.409	.....	.....	.....	.....

channels, in each instance. In case the flow is to be in a system of conduits, to obtain the relative values of C' of the McMath formula, for natural channel flow, and C, of Equation (29), for improved channel flow, the appropriate F value (see Table 11) becomes a correcting factor. For instance, if the improved channel is to be circular pipe, with a value of Kutter's $n = 0.013$, the F value would be 6.63, and the relative values, as given in Table 12, would be corrected as follows:

(*a*).—With C' (of McMath) as a base: C (of Equation (29)) would be $\frac{4.20}{6.63} = 0.6335$ of that shown.

(*b*).—With C (of Equation (29)) as a base: C' (of McMath) would be $\left(\frac{6.63^{1.1429}}{4.20}\right) = \frac{8.688}{5.156} = 1.6850$ times the values shown.

TABLE 13.—COMPARATIVE VALUES OF C' OF MCMATH FORMULA, UNDER AVERAGE CONDITIONS OF NATURAL FLOW, AND C OF EQUATION (29), UNDER AVERAGE CONDITIONS OF FLOW IN CIRCULAR PIPES

C' of McMath	C of Equation (29)			C of Equation (29)	C' of McMath		
	10 acres	100 acres	1 000 acres		10 acres	100 acres	1 000 acres
0.10	0.044	0.039	0.035	0.10	0.259	0.293	0.337
0.15	0.062	0.056	0.049	0.15	0.411	0.467	0.536
0.20	0.080	0.072	0.063	0.20	0.571	0.649	0.745
0.25	0.097	0.087	0.077	0.25	0.738	0.837	0.960
0.30	0.114	0.102	0.091	0.30	0.908	1.031	1.183
0.35	0.131	0.117	0.103	0.35	1.083	1.230	1.410
0.40	0.146	0.131	0.116	0.40	1.262	1.434	1.643
0.45	0.162	0.145	0.129	0.45	1.444	1.640	1.880
0.50	0.178	0.159	0.141	0.50	1.629	1.850	2.121
0.55	0.193	0.173	0.153	0.55	1.816	2.062	2.364
0.60	0.208	0.187	0.166	0.60	2.007	2.278	2.612
0.65	0.224	0.200	0.178	.....	.....	.....	.....
0.70	0.239	0.213	0.189	.....	.....	.....	.....
0.75	0.253	0.227	0.201	.....	.....	.....	.....
0.80	0.269	0.240	0.213	.....	.....	.....	.....
0.85	0.283	0.253	0.225	.....	.....	.....	.....
0.90	0.298	0.266	0.236	.....	.....	.....	.....
0.95	0.312	0.279	0.248	.....	.....	.....	.....
1.00	0.326	0.292	0.259	.....	.....	.....	.....

For convenience of comparison, these values are also reduced, and tabulated in accompanying Table 13.

The values given in Tables 12 and 13, of course, are based on data presumed to apply to average conditions and for a particular type of rainfall curve. Those of Table 12 would be applicable to sub-areas with storm water running loose to the catch-basins, while those of Table 13 have reference to a drainage basin as a whole, with improved channels thoroughly covering the area. Similar tables could easily be prepared from any set of known conditions (or assumed conditions), in combination with any one of the empirical formulas. Using the McMath formula, with these tabulated values of C, the run-off, Q, was calculated for several of the illustrative problems given in the paper, and also for several drainage systems heretofore designed. This resulted, in each instance, in a value for Q approaching very closely that obtained either by the use of Equation (29), or by the detail method. Furthermore, for the particular conditions given, Tables 12 and 13 show C values fairly consistent with those usually assumed. Similar comparisons, made under extremely differing conditions, give equally satisfactory results. It may be presumed, therefore, that the formulas of this paper are of correct construction.

Throughout the paper, the presumption is that it would be incorrect to assume the individual storm as following the law of the rainfall curve; and yet, inconsistently, such storm is assumed to follow a definite law, that of uniform intensity throughout any one time interval, with the measure of that uniformity as the average for that period. True, an individual storm might not, and usually does not, follow any known law, with respect to sub-intensities; but the improbability is just as true in one case as in the other. It must be remembered that rainfall curves are, at best, probability curves based on past experience, and that the tabulated i-values are average values, for certain time intervals, which are likely to be equalled or exceeded not oftener than the stated term of years.

If the designer is unwilling to concede the individual storm as following the law of the rainfall curve, he has no authority to assume his i-values from that curve. The two are inseparable, and to use them otherwise would be as inconsistent as designing a wooden structure with working stresses based on those of steel. Admittedly, a condition of uniformity would not occur as frequently as certain equal averages, in which case, if the former is assumed, the design would not be for the expected frequency, but for a much longer term.

Any one storm may have sub-intensities which are increasing in value, are uniform, or are decreasing, throughout concentration time, and yet may have a certain average value for that interval, hence, apparently tending to produce a maximum run-off greater than, equal to, or less than, respectively, that estimated under the uniform assumption. Under such variation, however, the concentration time would change, the tendency being for this time interval to decrease, in the order in which the conditions are stated, and the consequent adjustment of i would tend, automatically, toward a stabilization of the Q value, in conformity with that produced by the storm which follows the law of the curve. This condition is recognized in the assignment of the time of concentration of the nucleus, by assuming the maximum intensity as occurring at the beginning of the storm. Also, it is recognized in the statement, attributed to C. E. Ramser, M. Am. Soc. C. E., under "Use of

Rainfall Curves," which is no doubt true and is probably universally accepted, namely:

"The maximum rate of run-off, therefore, would result from a rainfall of maximum uniform intensity, continuing for a time equal to or exceeding the time of concentration."

One can but interpret this statement to mean what it says: The maximum will be the greatest under the uniform assumption; and, this statement being true, the tendency, with respect to the effect of sub-intensity variation, must be as stated heretofore. Much of the confusion that exists, with reference to C, is caused by the use of i-values of average intensities, from these rainfall curves, as if they were a set of uniform intensities, and by the attempts to include with C an additional factor which must vary with the type of curve. As valuable as such a set of i-values would be, no feasible method has yet been found for compiling a curve which would give duration frequencies of uniform intensities; and engineers, therefore, must get along with what they have and make the proper corrections. All rainfall curves thus far developed give these i-values of average intensities rather than uniform, notwithstanding that the late Emil Kuichling, M. Am. Soc. C. E., as long ago as 1905, called attention to the necessity for maximum rainfall data based on uniform instead of average intensities.[11]

While the routine of the detail method requires the assumption of uniformity, this is done merely for convenience and as a practicable base from which to work. Otherwise, the conception would become so involved as to render its use impossible. Recognition of the inconsistency is given, however, through the medium of the assigned values for C. In no other circumstance could one account for the comparatively small values assigned to this coefficient, nor its relatively constant value, along with changing value of intensity. If $(1 - C)$ represents only the loss by absorption, evaporation, and retention, this loss would be rather constant in volume, per unit of time, and one could not justify this volume as being a constant percentage of a variable intensity; nor could one justify the apparent large loss to current run-off. On the other hand, if $(1 - C)$ represents not only the actual losses, as stated, but also a correcting factor, for the fictitious assumption of uniform intensity, the value of C would be relatively small, and would tend toward constancy with a variable i, in such ratio as the weight of the correction.

Undoubtedly, these values of C, as used, are not the proportions of the run-off to the actually controlling sub-intensities, but are the ratios that they bear to the average intensities, arbitrarily assumed to be uniform. The fundamental relation, $Q = C\,i\,A$, is a true expression when, and only when, the factor, i, is the one actually controlling. When the values given this factor are fictitious (and it is necessary that this be done because no better data are available), a qualifying coefficient of i could well be introduced, instead of making it a part of C, and the fundamental relation might be more properly written, $Q = C\,(ci)\,A$. In such an expression, C would have its true meaning, as generally understood and corresponding with the nomenclature of this paper; and the value of c would be 1.00 when the actual rainfall is of uniform intensity for concentration time, less than 1.00, otherwise, the actual value depending on the type of rainfall curve and other prevailing conditions.

---

[11] *Transactions*, Am. Soc. C. E., Vol. LIV (1905), p. 192.

Although this factor, c, would be a coefficient of i, it is, hereinafter, so that it will not be confusing, used in conjunction with C' (as in the empirical formulas), or $C'c = C$ (as in this discussion). With c having a maximum value of 1.00, for a rainfall of uniform intensity, probably the maximum values for curves of the type, $i = \frac{K}{t^e}$, on very small sub-areas, would follow, somewhat, the logarithmic mantissa of $(1.00 - e)$. With $e = 0.4$, 0.5, and 0.6, respectively, these maximum c-values would be log (6), log (5), and log (4), or 0.778, 0.699, and 0.602. The minimum value, for the very small sub-area, would be the ratio of the end sub-intensity to the average intensity, for the respective type of curves, or 0.60, 0.50, and 0.40.

In order to allow visualization of the foreging statements, the c-values, which equal $\frac{C}{C'}$, are given in Table 14, from the values shown in Table 12, which values were determined from purely arbitrary assumptions of average conditions. Additional values are given, to clarify statements to be made presently, which were calculated simply by extending the scope of Table 12. In the McMath formula, however, when C' is given a value greater than unity, C' is no longer representative of approximately the degree of imperviousness, but is increased for other reasons. Therefore, in Table 14, for this higher range of C'-values, c would equal $\frac{C}{1.00}$ instead of $\frac{C}{C'}$.

A review of the method of making Table 12 and Table 14, would show, the other factors remaining constant, and for a rainfall curve of type, $i = \frac{K}{\sqrt{t}}$, that:

1.—With a change of C' (of McMath):

(a) C (of Equation (29)) varies as $\sqrt[1.1429]{C'}$.

(b) Within the range of $C' = 0$ to 1.00, c varies as $\frac{\sqrt[1.1429]{C'}}{C'}$.

(c) With C' larger than 1.00, c varies as $\frac{\sqrt[1.1429]{C'}}{1.00}$.

2.—With change of area size:

(d) $c^{1.1429}$ and $C^{1.1429}$ (of Equation (29)) vary inversely as $(B^{1.1429}\ A^{0.3429})$.

3.—With change of intensity:

(e) $c^{1.1429}$ and $C^{1.1429}$ (of Equation (29)) vary inversely as $R_1^{0.1429}$.

4.—With change of slope:

(f) $c^{1.1429}$ and $C^{1.1429}$ (of Equation (29)) vary inversely as $S^{0.0143}$.

Assuming that all sub-areas, subsequent to the nucleus, have a concentration time either equal to, or less than, that of their respective contributing inlets (which must be, under the conceptions of the detail method), the controlling *i*-factor will be that occurring subsequent to the time of fall of that "remote drop" which is destined to join the "remote drop" from the nucleus; that is, if the *t* of the inlet is 60 min. and that of a certain sub-area is 20 min., the measure of the run-off from that area, in the maximum cencentration from the whole, will be the actual intensities occurring during the interval (60–20) to 60 min.; and could not be the intensities prevailing during the interval, 0 to 60 min. If, subsequent to the (60–20) moment, just cited, the actually controlling intensities are less than those of the (0 to 60)-min. period (and the form of the generally accepted rainfall curve would warrant such an assumption), the maximum simultaneous run-off concentration, from this sub-area, could only be measured by the lesser rate of fall.

The concentration time at an inlet point being the timing of the "remote drop" from the nucleus, it can be seen that the corresponding factor, *c*, would be representative of the period from zero minutes to the end of the concentration time, and, therefore, would be at its maximum value. If a sub-area, contributing at a certain inlet, had a concentration time equal to that of the

TABLE 14.—VALUES OF FACTOR c $\left(\text{WITH } i = \dfrac{K}{\sqrt{t}}\right)$

McMath C'	$c = \frac{C}{C'}$			McMath C'	$c = \frac{C}{1.00}$		
	10 acres	100 acres	1 000 acres		10 acres	100 acres	1 000 acres
0.10	0.6900	0.6200	0.5500	1.05	0.5373	0.4809	0.4267
0.15	0.6533	0.5867	0.5200	1.10	0.5597	0.5010	0.4444
0.20	0.6300	0.5650	0.5000	1.15	0.5819	0.5208	0.4620
0.25	0.6120	0.5480	0.4880	1.20	0.6039	0.5405	0.4796
0.30	0.6000	0.5367	0.4767	1.25	0.6259	0.5602	0.4970
0.35	0.5886	0.5257	0.4657	1.30	0.6477	0.5797	0.5144
0.40	0.5775	0.5175	0.4575	1.35	0.6708	0.5992	0.5316
0.45	0.5689	0.5089	0.4511	1.40	0.6911	0.6186	0.5488
0.50	0.5620	0.5020	0.4460	1.418	0.6990	......	......
0.55	0.5545	0.4964	0.4400	......	......	......	......
0.60	0.5483	0.4917	0.4367	....	......	......	......
0.65	0.5431	0.4862	0.4323	....	......	......	......
0.70	0.5386	0.4814	0.4271	....	......	......	......
0.75	0.5333	0.4773	0.4240	....	......	......	......
0.80	0.5300	0.4738	0.4200	....	......	......	......
0.85	0.5259	0.4706	0.4176	....	......	......	......
0.90	0.5222	0.4667	0.4144	....	......	......	......
0.95	0.5179	0.4642	0.4116	....	......	......	......
1.00	0.5150	0.4610	0.4090	....	......	......	......

inlet, the value of c, applying to the sub-area, would be this same maximum; but if the concentration time of the area were extremely short, the factor, c, would have a minimum value. The effect of each sub-area, on maximum simultaneous run-off from the whole, therefore, would be at its greatest when its concentration time equalled that of its tributary inlet; and at its minimum, when its t-value is only a small fraction of that of the inlet.

The relation between these two concentration times, therefore, becomes an important factor in the measure of maximum concentration. The shorter the t of the sub-area, the more frequent will be the turn-over of representative rainfall, and the controlling i factor will be that of the average intensity for the last frequency period of flow in which the "remote drop" from the area joins the "remote drop" from the nucleus. If t represents the concentration time of the sub-area, and T that of the inlet, the ratio, $\frac{T}{t}$, might be called the "turn-over frequency" of the area, and, in general terms, it may be stated, that the larger the turn-over frequency of any sub-area, the less effect that area will have in the measure of final simultaneous run-off, the C value, consequently, being smaller with such increase.

Conceding this conception, one could visualize, as one extreme, a small impervious sub-area, with $t = 1$ sec., and T at its tributary inlet quite large. In such a case, the controlling i factor would be the current rate of fall, which with a curve of type, $i = \frac{K}{\sqrt{t}}$, would be one-half the average applying to the T of the inlet, or $c = 0.50$, and $C'c = (1.00)\ (0.50) = 0.50 = C$. As the other extreme, if $\frac{T}{t} = 1.00$, the controlling i factor of the area would be the same as that of the inlet, or $c = 0.70$, and $C'c = (1.00)\ (0.70) = 0.70 = C$. The range of C values, on small impervious areas, therefore, lies between 0.50 and 0.70, corresponding to the extremes of the turn-over frequency being infinitely large and of its being unity, respectively. On the other hand, in the case of a similar small sub-area, partly pervious, these limiting values of C would be different. The area of practically instantaneous run-off could have very little, if any, loss by absorption, evaporation, or retention, thus holding the C' value approximately at 1.00 (notwithstanding the area may be mostly pervious), and $C'c$ would be 0.50, as before. If $\frac{T}{t} = 1.00$, the area would sustain such losses, C' would be less than 1.00, and $C'c$ less than 0.70 (as estimated for wholly impervious areas). If the correct C' value were 0.10, and the corresponding c value slightly less than 0.70, say, 0.69, then $C'c = C = (0.10)\ (0.69) = 0.069$.

The range of C values, on small pervious areas, therefore, lies between 0.069 (or less), and 0.50, corresponding to the extremes of turn-over frequency being unity and of its being infinitely large, respectively. As the C' value increases from zero to 1.00, the corresponding c factor decreases from 0.70 to 0.50, and the product, $C'c$ (or C), would reach its maximum when $C' = 1.00$. From the foregoing, it may be noted, also, that C' (of the McMath formula) is not a measure of imperviousness of an area, but rather a coefficient expressing the effect of the combination of the degree of imperviousness and the ratio, $\frac{T}{t}$ It has a value of 1.00 when $\frac{T}{t}$ is infinitely large; decreases to zero as $\frac{T}{t}$ reaches unity and the degree of imperviousness reaches zero; and increases to its maximum value when the area is wholly impervious and the ratio, $\frac{T}{t}$, is unity.

These statements are made, specifically, with respect to the sub-area effect on the area as a whole, with concentration times of the sub-areas, and of the inlet points, being shown as controlling factors. Exactly the same conditions would apply to the sub-areas, considered independently, the minor subdivisions of these areas being substituted, and inlet points being considered as collecting points for flow from the sub-divisions. The controlling factors, in such a case, are not so easily ascertained, because of the infinite possible variation of such minor concentrations. Resort must be had to judgment based on experience, and on the foregoing conceptions, for the correct assignment of these C values to the sub-areas. If the experiments of McMath are correctly reflected in his formula, and the average conditions he encountered were approximately those herein assumed, the C values of Table 12 would be approximately correct.

However, no pretense is made that the data in Tables 12, 13, and 14 are numerically correct, because the writer does not know the exact conditions under which the McMath observations were made. They do show, nevertheless, a remarkable consistency of trend, and of limiting values, with the foregoing analyses, and demonstrate conclusively that, not only do the detail method and the empirical formula assume the individual storm as following the law of the curve, but that this is true also of the formulas presented by the authors.

Many deductions, comparisons, and analyses of differing features could be made from the foregoing. In general, however, it may be stated that, with a rainfall curve of the type, $i = \frac{K}{\sqrt{t}}$, any area characteristic which increases the turn-over frequency of its sub-areas will decrease the value of C and *vice versa.* An increase in sub-area size (T larger), of intensity (t smaller), and of slope (t smaller), will increase the value of the ratio, $\frac{T}{t}$, in consequence of which C values will decrease. The increase in degree of imperviousness (which decreases t), would also cause a change in the same direction, except that the trend is reversed by the decreasing ratio of actual loss, by evaporation, absorption, and retention. The actual variation of C values, caused by change in the foregong relations, is entirely dependent upon the type of curve. A few simple comparisons, made with other values for e, show clearly the reason for the differences among the various empirical formulas.

When the sub-areas are considered as part of the whole, their respective C values will remain constant only when the turn-over frequency of the area, or $\frac{T}{t}$, is unity. Otherwise, C must depart from these independently determined values, and will be more, or less, according to circumstance. As each sub-area is encountered along the course of the main channel—an increase in time—the ratio, $\frac{T}{t}$, would increase, of course, although t would increase slightly because of decreasing intensity. The latter may be disregarded, however, for any change to be made in the C value, because of the changing ratio, $\frac{T}{t}$, would be based from the original values of both C and t.

Theoretically, then, with increase in time, C would be reduced in such measure as the $\frac{T}{t}$ value departed from unity. Practically, however, this trend might be nullified, or even be reversed, if increasing wetness of the area increases the real value of C'. Especially in the lower bracket of values, if the original assignment of the C' value is based solely upon the percentage of impervious surface, without regard to what the condition of the pervious parts would be when thoroughly wet, the theoretical trend would most certainly be reversed; and C would have to be increased with increase of time. The underlying reasoning back of the original assignment, with respect to perviousness, wetness, size, and relative position of the sub-area, would determine whether C should, later, be increased, remain constant, or be decreased. Probably each of the authorities, who do one or the other, are correct, basicly, founding their action on the characteristics of the watersheds to which the applications are made, on the type of rainfall curve appertaining to his locality, and on his original assignment of the C' value

The initial sub-area, or nucleus, becomes of importance in the detail method, not, primarily, because of its physical characteristics, but because of the interval chosen as its concentration time. This value of t is added to the calculated time in the drains, in order to arrive at concentration times of subsequent inlets, which, in turn, determine the respective intensities. Thus, the assumed t of the nucleus becomes a factor in the whole design, of greater weight if total time is short and of less consequence if it is large. With a curve of the type, $i = \frac{K}{\sqrt{t}}$, if 10 min. is chosen as initial inlet time, whereas 15 min. would be correct, the corresponding intensity values would be in error about 22%; and at the end of the total time of 1 hour, this error would be reduced to 5%; but this error would be nullified, except as to the nucleus, if due consideration is given to the C value change on subsequent areas. The ratio, $\frac{T}{t}$, for the latter, would be different with the erroneous assumption, and C values on these subsequent areas would change accordingly, the tendency being to offset the seeming error. Nevertheless, initial inlet time should be chosen as accurately as possible.

In their discussion in Part I, the authors observe and comment on many seeming inconsistencies in the empirical formulas, and in the assumptions of the detail method. Obviously, it would be inadvisable to comment on each of these, separately, because of the space required. Many of their queries are reconciled in the foregoing analyses, and many of the apparent inconsistencies turn out to be entirely logical. Some of the more important of these may be recalled, briefly:

In their "Introduction" the authors state that, "one purpose of this paper, however, is to show why the use of these empirical formulas is not now to be recommended," whereas, actually, their own theory may be established by the use of them.

The conclusions drawn from the illustration of the simple water-shed (see Table 1) are based upon the impossible assumption of unity for the C values, and that these values would remain constant under the stipulated conditions. The several columns of tabulated values of Q (Table 2), would hold only when the average intensities, shown for each sub-area, are considered as being uniform, and as continuing at their respective rates for the entire concentration time of the total area.

Preceding the authors' conclusions to Part I, the statement that "to increase Factor C as the time increases (as shown in current publications and textbooks), without also allowing for the fluctuation in C for changing intensities, seems entirely unjustified," cannot, itself, be justified. It has been shown, earlier herein, by using Equation (29), that the weight of i, in so far as it affects the factor, C, is $R_1^{0.1429}$, and that C varies inversely as this value.

It has been previously shown that, with N a constant for any set of conditions, $C' = NC^x$, and C, therefore, varies as $(C')^{\frac{1}{x}}$; also, since $C = C'c$, c varies as $\frac{(C')^{\frac{1}{x}}}{C'}$.

With $\frac{T}{t} = 1.00$, the ratio of actual intensity to average intensity is 1.00; and, with $\frac{T}{t}$ infinitely large, the ratio of actual intensity to average intensity is $(1.00 - e)$. The mean of these two extremes is $\left(1.00 - \frac{e}{2}\right)$, and the equation of the curve for values of C and c is, therefore, the log of this mean, and $\frac{1}{x} = \log\left(1.00 - \frac{e}{2}\right)$, and $x = \frac{1}{\log\left(1.00 - \frac{e}{2}\right)}$.

With respect to the effect of intensity on C values, the authors show that velocity varies as $Q^{\frac{1}{4}}$; hence, t^e varies as $\frac{1}{R_1^{\frac{e}{4}}}$, and the ratio, $\frac{T^e}{t^e}$, varies as $\frac{TR_1^{\frac{e}{4}}}{1}$. The equation for the curve, for the effect on C' values, is, therefore,

$$\frac{ex}{4} = \frac{e}{4\left(\log\left(1.00 - \frac{e}{2}\right)\right)} \qquad \text{(75)}$$

With a rainfall curve of the type, $i = \frac{K}{\sqrt{t}}$, the exponent for the curve of C values, or x, is, $\frac{1}{\log\left(1.00 - \frac{0.5}{2}\right)} = \frac{1}{\log 0.75} = \frac{1}{0.8750613}$; and $x = 1.14277$. The effect of intensity, on C' values, is $\frac{ex}{4} = \frac{0.5x}{4} = 0.1429$.

In a similar manner, it can be shown that the effect of the slope on C' values would be, $\left(\frac{e - e^2}{2}\right)\frac{x}{10} = 0.0143$.

These are the exponents obtained by equating the McMath formula with Equation (29) to obtain Equation (70).

The authors are most vague as to their reason for inserting the factor, P, in Equation (19), but a study of its significance in the final expression (Equation (24)) would indicate this factor to be merely a measure of the effect of the ratio, $\frac{t}{T}$, on maximum run-off. In other words, the factor, P, is a measure of the reduction of C values (as assigned to the sub-areas, independently) required by the average value of the ratio, $\frac{t}{T}$. The latter, of course, would be different for each type of water-shed (see Fig. 2) and for each type of rainfall curve; but, also, these P-values (see Table 4) would change, for any one type of water-shed, or rainfall curve, if the slope of the area were variable, or if it differed considerably from that of the main channel, and if the C-values were different on the various parts, Table 4 being calculated with the assumption of uniform slope and a constant C-value. The suggestion is made that one table might answer the purpose better, wherefrom, with average $\frac{t}{T}$ -values for guides, P-values could be selected appertaining to the more generally used types of rainfall curves.

Table 3 is mis-named, in that the value of P has no significance when the storm is of uniform intensity. Under such condition, the value of e is zero, and any value of P, with zero for its exponent, has no meaning.

The illustration of the tabling process (Table 8) is useful for demonstrating the consistency of the suggested formula, but it might be very misleading with respect to the assigned values of C. Although the slopes are fairly uniform, this factor is assumed to be the same for the extreme variation of areas of 1.5 and 108.6 acres, respectively. It might be possible, but not very probable, that these sub-areas, individually, have equal *C*-values, but each, as a part of the whole, could not possibly have such equality.

In conclusion, it may be said that the paper presents many valuable and original suggestions and that the authors are to be commended for their really fine effort. Two additional factors are introduced which are of vital importance, and their adoption can only lead to a more precise determination of real *C*-values. A word of caution must be uttered, however, against the general use of these formulas without a thorough understanding of the effect, and importance, of the *C*-value assignment. The introduction of these two new factors carries with it the necessity for a very much greater refinement in these selections, which, if not done with proper knowledge, may easily lead to results more erroneous than if they had not received consideration.

John W. Raymond, Jr.,[18] **Esq.** (by letter).—This paper discusses the problem of designing culverts, bridge openings, storm drains, spillway structures, etc., in which it is required to estimate maximum flow. The estimation of discharging capacity for bridge openings, culverts, spillway structures, and flood channels, while involving basic data very similar to those required for determining flows to be provided for in storm drains, is nevertheless a distinct problem. Since it is usually considered good engineering to provide for the maximum flow to be expected in the first case, in storm drain and combined sewer design, it is rarely practicable, financially, to provide conduits throughout a drainage area, to care for the maximum flows. Thus, the problem of storm drain or combined sewer design introduces the estimation of frequency and of balancing of damages and inconvenience resulting from occasional flooding, against the extra cost of providing conduits for the larger expected flows.

While both problems deal with structures which are continually in use, the problem of designing storm drains and combined sewers is to provide a system within the funds available and in agreement with the degree of protection required, and the rate of flow to be provided for is that expected at occasional intervals, say, on the average of once in 5, 10, or 15 years. The problem of estimating discharge capacity for culverts, bridge openings, spillway structures, etc., requires the determination of the maximum flow to be expected in more than 100 years, or perhaps even 1 000 years. Thus, the conditions for the design of the latter structures require the determination of the maximum rainfall conditions, together with the maximum run-off conditions, and the problem of storm drain and combined sewer design requires the determination of the rainfall to be expected at a given frequency and average run-off conditions, since the condition of simultaneous maximum or excessive rainfall and maximum run-off is of rare occurrence.

Considerable labor is involved in the tabling process as practiced by a number of engineers. For a system of storm drains or combined sewers to be designed by means of the Gregory and Arnold formulas, probably the most convenient way to arrange the computations would be in tabular form for the different sections of the conduit, substantially as is done in the tabling process for the rational method; and it seems to the writer that the number of columns and the labor involved would not be far different in either case.

---

[18] Asst. Engr., Metcalf & Eddy, Boston, Mass.

It is not axiomatic that the maximum flow resulting from a rain of given frequency occurs when the entire area is contributing; it is generally true, especially for areas which increase substantially as the time of flow. It may not be true where a water-shed consists of large individual areas, the main channels of which have different slopes from that of the main channel for the whole water-shed or where the development or proportion of impervious surface of the individual areas is widely different. Engineers using the tabling process in the rational method of design should investigate their design for durations shorter than the time required for the entire area to contribute, to find the maximum flow resulting from the selected frequency rainfall curve.

It is true that intense rains seldom occur at uniform rates for long durations. It is believed, however, that with a complete system of storm drains or combined sewers, consisting of closed conduits, the use of uniform intensities of rainfall gives substantially correct results, on account of the limited capacity of the conduits. This causes water precipitated during the short periods of excessive rainfall to be retarded, thus smoothing out the run-off. If the rainfall curve is constructed from records of all storms, it may be used with reliability in determining the frequency, provided the records cover a sufficient number of years.

Where the proportion of impervious surface is uniform, the run-off factor, C, for a storm beginning at a time when run-off conditions on the area are average, or when the area is not completely saturated, is known to increase with time, and when this is taken into account the different values of Ci vary less than the i values.

The detailed method as practiced by most engineers is not based on the assumption of the equivalent of a variable intensity during the period of concentration, but upon the assumption of a uniform intensity for each period, or in other words, the storm used for each part of the drain is a different storm than those used for the other parts.

The usual method of construction or derivation of rainfall curves includes all significant storms, and in a record the length of which is several times the period of frequency represented by the design curve, there will be several storms which will define the position of the frequency curve. It is incorrect, therefore, to assume that the run-off at a given point resulting from a storm of a certain frequency, is made up from rainfalls of diminishing intensities, different intensities being applied to different parts of the drainage area, since the ordinate, i, equals the total fall divided by the duration.

The authors have presented a valuable discussion of the problem of estimating storm flows. It is difficult, however, for an engineer experienced in the use of the so-called "detail rational method" to visualize the bearing on the problem of the different coefficients and constants which are suggested. If the formulas were felt to be applicable, a great deal of experience in their use would be required, to obtain reliable results. On the other hand, where the detailed rational method is used for storm drain design, an experienced engineer may select the basic data, that is, the frequency rainfall curve, degree of development, and coefficients of run-off; and he may then explain the method of computation to a less experienced assistant, who can complete the labor of design, and reliable conclusions can be reached. It does not appear to the writer that the labor involved in working out the results of the suggested formulas can be entrusted to an inexperienced assistant to the same degree.

The authors have clearly pointed out the inadequacy of empirical formulas, and their continued future use for estimating the required capacity for important drainage channels and waterways can scarcely be tolerated.

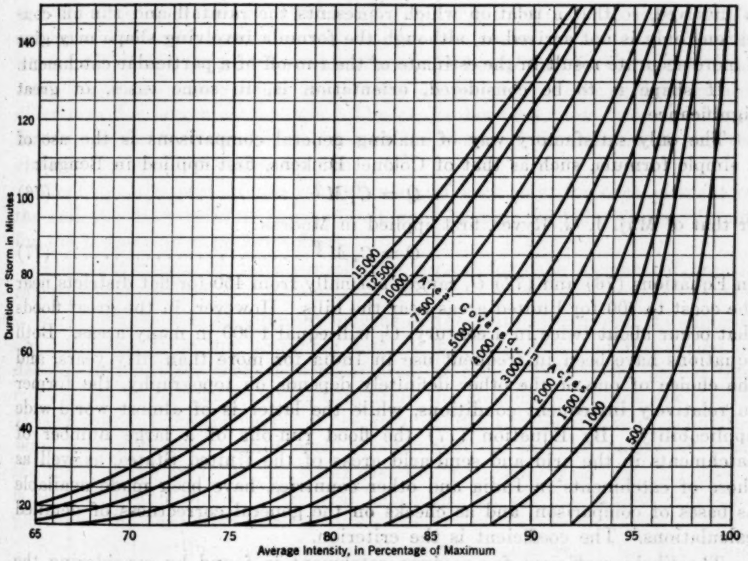


FIG. 8.—DISTRIBUTION OF RAINFALL INTENSITY WITH RESPECT TO AREA COVERED AND DURATION OF STORM FOR 15-YEAR FREQUENCY STORMS AT LOUISVILLE, KY.

Probably the "inlet time" as used in the rational method of storm drain design gives engineers the most concern because of the paucity of reliable data, and the need for more observations cannot be over-emphasized.

The distribution of intense rainfall over the area of a water-shed is an important factor. Excellent data for rainfalls of short duration have been presented[19] by Frank A. Marston, M. Am. Soc. C. E. Fig. 8, which is from the report of the Commissioners of Sewerage of Louisville, Ky., dated October 16, 1929 (and prepared from data in Mr. Marston's paper, and some subsequent data), shows the relation between the average and maximum intensities for different areas and durations.

---

[19] *Transactions*, Am. Soc. C. E., Vol. LXXXVII (1924), p. 535.

REGINALD A. RYVES,[20] ESQ. (by letter).—This paper represents a formidable mathematical effort which prompts the question, "Why seek a formula when all data and assumptions can be applied directly by a graphical statement?"

The first necessity in making calculations of the flood discharges of catchments is to have some means of comparing any catchment with any other catchment, even when they differ much in size. Comparisons by means of rates of flow per square mile are of little use, since an accompanying scale of the effect of size has to be used in order that the comparison may produce a relation which represents rainfall and run-off conditions only. On the other hand, formulas involving a consideration of the shape of the area are not satisfactory, because, in comparing two catchments, more than one variable is involved, so that a relation which represents the rainfall and run-off conditions only is not arrived at, although the formula involving shape may give a more accurate result in the estimate of the run-off of a particular catchment.

If shape is to be considered, orientation is, in some cases, of great significance.

The only satisfactory way of making general comparisons is the use of a simple formula, such as that of Colonel Dickens, first applied in Bengal:

$$Q = C_1 M^{\frac{3}{4}} \qquad \text{.......................(76)}$$

or that of Maj. J. G. Ryves, first applied in Madras:

$$Q = C_1 M^{\frac{2}{3}} \qquad \text{.......................(77)}$$

In Equations (76) and (77) C_1 ranges normally from 450 for flat districts near the coast to 700 for limited areas near the hills. However, in the great floods that occur about twice in a century, C_1 will equal 1 000 in many areas. Both equations have been in frequent use in India for more than fifty years, and the choice of one or the other definitely depends on topography, the former in relatively infrequent conditions, while the latter is of almost world-wide applicability. By Equation (77) the flood run-offs of a large number of catchments in the arid and semi-arid areas of the United States, as well as those of catchments in India and other countries, have been made available as bases of comparison, and as checks on the general correctness of detailed calculations. The coefficient is the criterion.

The likely coefficient for a given catchment is found by considering the coefficients in a number of cases in which the area is not very small or very large in comparison, and where the intensity of a short-period rainfall is about the same.

More significant data are provided when: (*a*) The topography is somewhat similar; (*b*) the area is not more than three times nor less than one-third the area under consideration; and (*c*) the intensity of rainfall for the significant period is about the same. The significant period is the period of total contribution, or the time it takes for water to flow from the part of the catchment most distant, in time, to the outlet. In some cases, however, a smaller area, eliminating salients, gives a larger run-off because of the greater rainfall intensity.

---

[20] Cons. Engr., London, England.

Such comparisons are very useful and important; but for final estimates of floods the writer always uses the most careful methods of direct calculation, by methods of his own,[21] which are quite probably also followed independently by other engineers, and may be held to represent the rational method as criticized by the authors. The procedure is as follows, the specific references being to the writer's computations made for the "Obras Contra as Seccas" Department of the Government of Brazil, in 1920-1922:

1.—Rainfall maps, made in exactly the same manner as those of annual rainfalls, are prepared, except that one is made for each day, for the appropriate period; that is, a period long enough to overlap the "period of total contribution" by about 50% before, and 50% after.

2.—The area is divided into a suitable number of portions, usually four or five in the cases under consideration, and these portions are either each a catchment in itself, approximately, or they are zones, the bounding lines of which will divide the streams which have a 4 hours' run to the outlet, from those which have an 8 hours' run to the outlet, etc. There will be greater differences for larger catchments. The choice between these two methods of forming elements of area is decided by the shape of the catchment and the direction in which the streams run.

3.—From the part of the rainfall map, of each day, for each element of the area the daily rainfall is calculated, zone by zone, and the average found, for the purpose of a general check.

4.—From curves of daily run-off, the run-off for each element of area is calculated.

5.—The velocities of different lengths of the main stream and its affluents are estimated, from any available data, or by judgment, and the period of delay is deduced for each element of area, or zone. This is the period that must elapse before the water flowing from the element will reach the outlet. It is measured from the center of the area, or, if all the areas are of about the same size, and are roughly each a catchment, it may be measured from the outlet of the area. In the latter case the "gathering time" for an element is computed, and this is a "general handicap" for all the run-off curves as plotted to the time base line.

6.—On a base line, marked off in hours to an open scale, the daily run-offs are plotted, first as horizontal lines, and then as continuous curves, bounding the same area as the horizontal line, each day; but all these curves do not start at the same point. Each is given a handicap, to the left of zero time, this handicap being, in each case, the period of delay. The longest handicap, plus the "gathering time" of that element, is the "period of total contribution."

7.—By totaling the heights of the run-off curves which are in cubic feet per second, at every hour-point for very small catchments or at longer intervals for larger catchments, the curve of total run-off is obtained.

The maximum run-off and the time of the maximum run-off are obtained from Steps 1 to 7 directly. The engineer is now able to judge, from the position of the time of maximum run-off on the base line, whether he is likely to get a greater run-off by taking a shorter period with a greater intensity of rainfall; or by taking a longer period with a less intensity of rainfall. Usually, however, all that he needs to do is to take a period that includes the day of greatest recorded rainfall, which in the case of the larger catchments, may involve the preparation of more than one map, since different stations may not have recorded their maxima in the same storm.

---

[21] *Indian Engineering*, April 29, March 6, and May 11, 1911.

As the rainfall stations in Northeast Brazil are not very close together, the writer assumed that the real maximum spot rainfall is heavier than that recorded at any station, which is a matter of importance in estimates of floods from the smaller catchments

As regards estimating the velocities of the streams, this might seem to be a weak link in the chain of reasoning, but the records of the Department, which include curves of rainfall and run-off plotted together, make it possible to find, in a number of cases, not only the time elapsing between any great intensity of rainfall and the corresponding intensity of run-off, but also the period elapsing between the beginning of a heavy rainfall on the dry catchment, and the moment when the stream at the outlet begins to flow.

When the "period of total contribution" has been found, or estimated, preliminary estimates of the maximum flood from any catchment may be made by averaging the "weighted" daily rainfalls of the stations in and around the catchment for a somewhat longer period, and from the curve thus obtained selecting the greatest average fall for any included period equal to the period of total contribution. The run-off appropriate to this fall is, at first sight, the maximum. A shorter period, however, may give as large, or a larger, run-off, although the entire area has not contributed; thus, consider an area, A, with three days as the period of total contribution. The precipitation for three successive days was 2.4, 7.9, and 2.2 in., respectively, or an average precipitation of 4.2 in. (0.35 ft.) per day. For such a rainfall a run-off coefficient of $\frac{5}{7}$ was suitable, under the conditions, giving $\frac{5}{7} \times 0.35 \times A = 0.25\ A$ cu. ft. per day, for the short period of the peak of the flood.

On the other hand, one day's rain in the part of the area nearest the outlet, where the stream velocities are greatest, will be gathered in many cases, from more than one-third (say, five-twelfths) the area. For the intense rainfall of 7.9 in. (0.658 ft.) per day, the coefficient of run-off is taken as $\frac{9}{10}$, and the run-off is, $\frac{9}{10} \times 0.658 \times \frac{5}{12} A = 0.247$ cu. ft. per day, at the peak. It is obvious that, since a period of one day gives nearly the same result as a period of three days, in this preliminary estimation, a complete diagram for the whole period is necessary.

If there are small outlying portions on the far side of the catchment, the actual result would be that the calculation for one day would give the greater flood. These special tests are often worth while. In the present connection their main significance lies in their obvious superiority to any formula and their equally obvious inferiority to the hour-by-hour graphical computation.

In some cases of direct calculation from the period of total contribution, the maximum flood is that found by calculating for an area which is the catchment less any extreme outlying portions. In the case of a long and narrow catchment it may be necessary to treat the whole of it, step by step, in the same way, because the maximum flood may be that due to a short period of heavy rainfall on quite a small proportion of the catchment, next to the outlet. The writer has found that this is especially possible in Northeastern Brazil, where the heaviest rainfalls in any day are often those due to intense falls of much less than a day compared with the average for one day.

It is clear that a formula is of small value compared with calculations based on the period of total contribution, and it is even more certain that, for final estimates, no other system of computation is in the same class with direct calculations carried out as described herein.

The accuracy of the computations depends upon the different periods of "delay" being adequately represented. Therefore, there must be division into areas, the centroids of which are at various distances from the outlet. Usually, a good method is to define, approximately, nearly whole catchments for the more distant parts of the area and part catchments for the sections nearer the outlet. The latter should be separated laterally by known or assumed water partings and, on the other two sides, by lines corresponding to those dividing the zones. In zoning, the lines are irregular, so as to give approximately the same "delay" for the entire zone. Division of the catchment into radial sectors fails to utilize the most important feature of division into elements, and is only a little more accurate than a crude whole-area computation, yet such a crude computation is more accurate than any formula, since it allows of the use of smooth curves of rainfall intensity and changing values of the run-off ratio.

The authors have made a very useful contribution to the study of the subject, but, in the writer's opinion, they have failed either to disestablish the rational method or to establish new formulas. First, however, it should be noted that their opinion that the use of empirical formulas is not now to be recommended, is unacceptable—as so expressed without qualification—to those who, like the writer, have found them, in their simplest forms, to furnish very useful comparative data, not otherwise available.

As regards the description of the detail method, this is fundamentally that of the writer, who, for preliminary calculations, or in the absence of rainfall data sufficient to draw the isoplanes for successive days, or shorter periods, uses a rainfall intensity curve. That shown in Fig. 1 is a good characteristic curve very similar to those drawn from data of storms in England, India, and Brazil. It is not, however, a fair presentment of the (detail) rational method, which the writer is defending, because rain-gauge data are used (or should be used) whenever available. Again, in Table 1, the bands, a_1, a_2 ... a_n, correspond to zones, but irregular lines based on estimates of rates of flow must give more accurate results than parallel straight lines. Such irregular lines, therefore, are drawn (or should be drawn), when the area does not lend itself to division into elements which approximate to separate catchments. The "Statement" preceding Table 3 certainly does not apply to the writer's method, by which, as nearly as possible, the rainfall of every small element and the run-off corresponding to it are given hour by hour.

There is, however, a broader and more significant consideration, which the authors have overlooked, namely, that the method really gives the nearest estimation of maximum run-off, whether it is due to the rainfall of the whole period of contribution over the entire catchment, to that of a somewhat shorter period (salients being excluded), or to that of a fraction of the period, such as one-third.

A reversion to the use of formulas, as advocated by the authors, would give, therefore, less reliable estimations, because no formula relates, as does the diagram—hour by hour in time and element by element in area—the factors of "pre-soakage," run-off in proportion to rainfall, rates of flow, and the least inaccurate representation possible (by a smooth curve) of the rainfalls of days and those of shorter periods for which it may have been measured. In any case, engineers are not likely to use to any considerable extent a method involving calculations the character of which is indicated by Appendix I.

G. S. Tapley,[22] Assoc. M. Am. Soc. C. E. (by letter).—The authors have made perhaps the most thorough analysis of the problem of run-off which has appeared to date. They have pointed out a slight fallacy in the present detail method. They have defined several factors (notably slope) which have not heretofore been clearly understood, and, with good logic, have discussed other factors. Only superficial criticism can be offered.

Probably the least easily understood feature of the analysis relates to P, the factor which introduces into the equation the effect of shape of area and manner of concentration. Apparently, P is a dimensionless factor obtained by trial solutions of sample cases. The writer would like to see a demonstration proving that two areas for which P is equal are geometrically similar as to shape of area and arrangement of the system, because this is implied.

A little confusion in notation exists where the factor, P, is introduced forming Equation (19). Possibly the authors had in mind the desirability of keeping the number of symbols at a minimum. In Equation (14), $c \sqrt{r}$ represents average conditions, whereas the symbol, Q, in the same equation represents the final value. In Equation (18) if Q still represents the final value then $c \sqrt{r}$ must also represent conditions at the final stage, since $c \sqrt{r}$ and Q are coincident factors at a given point. In Equation (19), $c \sqrt{r}$ represents average values, although Q is the final value. In this equation, P has been introduced to convert the final value of $c \sqrt{r}$ (corresponding to the final Q) to the average value.

Although the idea involved in introducing P is simple, it may not at once be obvious. Adopt the notation c', r', Q', v', and s' for elements at any point in the system; c, r, Q, and V for elements at the lower end; and c_a and r_a for average values. Then, at any point,

---

[22] With City Engr.'s Office, Los Angeles, Calif.

$$(c' \sqrt{r'})^{0.5} = N \ (Q')^{\frac{1}{8}} \ldots\ldots (78)$$

Also, $c' \sqrt{r'} \propto v'$; hence, $\frac{c' \sqrt{r'}}{c \sqrt{r}} = \frac{v'}{V}$, and,

$$c' \sqrt{r'} = \frac{v'}{V} c \sqrt{r} \ \ldots\ldots (79)$$

Therefore, at the average point,

$$c_a \sqrt{r_a} = \frac{v}{V} c \sqrt{r} = P \ c \sqrt{r} \ \ldots\ldots (80)$$

but, at the final point, it follows, from Equation (78) that,

$$(c_a \sqrt{r_a})^e = P^e \ N^{2e} \ Q^{0.25e} \ldots\ldots (81)$$

which corresponds to the authors' Equation (19).

In defining average slope as a function of time the authors have cleared up a point which has been puzzling storm drain designers. It has been the custom to consider the slope of an area as being determined by certain geometrical dimensions of it, but, since the authors have pointed it out, it is realized that an essential element of slope is time.

It is believed that the derivation of Equation (36) would be more lucid if a mathematical notation were used. As the authors have stated, $t \propto \frac{1}{F^2 S^{\frac{1}{2}} Q^{\frac{1}{4}}}$, in which the factors represent conditions at any point, S being the independent variable. That is,

$$t = u \frac{1}{F^2 S^{\frac{1}{2}} Q^{\frac{1}{4}}} \ \ldots\ldots (82)$$

Consider any particular interval, $\varDelta \ t_n$, of time:

$$\varDelta \ t_n = u \frac{L_n}{F^2_n \ S^{\frac{1}{2}}_n \ Q^{\frac{1}{4}}_n} \ldots\ldots (83)$$

The total time is,

$$T = u \sum \frac{L_n}{F^2_n \ S^{\frac{1}{2}}_n \ Q^{\frac{1}{4}}_n} = u \frac{1}{S^{\frac{1}{2}}_a} \sum \frac{L_n}{F^2_n \ Q^{\frac{1}{4}}_n} \ldots\ldots (84)$$

in which, S_a is the average slope. Hence,

$$\frac{1}{S^{\frac{1}{2}}_a} = \frac{\sum \frac{L_n}{F^2_n \ S^{\frac{1}{2}}_n \ Q^{\frac{1}{4}}_n}}{\sum \frac{L_n}{F^2_n \ Q^{\frac{1}{4}}_n}}$$

from which Equation (36) follows.

In deriving Equation (39) it is believed that a different definition of m is required in order to permit a logical following through of the development. If, for example. m is defined as the percentage of $(i - z)$ stored, a consecutive development results. Since $(i - z)$ equals the number of cubic feet per second per acre (storage plus run-off), $m \ (i - z)$ equals cubic feet per second per

acre, storage. Hence, $(i - z) - m\,(i - z)$ equals cubic feet per second per acre, run-off; or, $A\ [(i - z) - m\,(i - z)]$ equals cubic feet per second, run-off. Since, however, CiA equals cubic feet per second, run-off,

$$CiA = A\ [(i - z) - m\,(i - z)]$$

or,

$$C = \frac{(i - z) - m\,(i - z)}{i}$$

from which Equation (39) follows.

In the paragraphs following Fig. 1, and at several other points, the authors state, "* * * it is the evident intention [of users of the detail rational method] to assume a uniform intensity over the entire area throughout the time of concentration * * *." The idea involved, however, is somewhat obscure.

At any point in the system the discharge is computed from $Q = CiA = Ci\ a_1 + a_2 + \ldots a_n$; that is, each element Σ of area is multiplied by the same value of i. Hence, since the various elements of area, a_1 a_2, etc., did not receive their contributions of rainfall at simultaneous times, i must not have varied with time, and, therefore, must have been constant.

The use of uniform intensities entails the assumption that the conduit in the upper part of the system will not be flowing full. Hence, as the authors have now made apparent, the use of the detail method involves fictitious velocities.

The authors' formulas have obvious applicability as improvements over existing empirical formulas for computing flow at a point. For designing a system, however, the proposed method is not a short cut, since, of course, the conduit would have to be designed in detail from inlet point to inlet point.

W. I. HICKS,[23] ESQ. (by letter).—This paper is an exhaustive, logical, and thorough résumé and analysis of fundamental and mathematical data on the subject. It throws new light on various hitherto obscure and neglected basic principles and tends to close up several mistaken channels of thought which have been established by erroneous conceptions of the meaning of the available fundamental data.

The authors make an especially noteworthy point, that the quantities computed under the detail method may be exceeded under the circumstance of the peak of rainfall intensity becoming synchronized with a large value, CA, of some tributary area. The writer disagrees with the authors in their exclusion of the consideration of this circumstance from the computations.

The study of factors governing the selection of the coefficient of run-off, C, is valuable and will aid in the planning of experiments and analyzing data therefrom. The logic of deriving coefficients of both storage and absorption, as given in Table 7, from the experimental work of Ivan E. Houk, M. Am. Soc. C. E., is not apparent, as the coefficients are distinct from each other and must be evaluated by separate experiments.

---

[23] Asst. Engr., Storm Drain Div., Bureau of Eng., Dept. of Public Works, Los Angeles, Calif.

Comments on the detail rational method of computation regarding, in particular, the method of arriving at the time of concentration by means of varying conduit or channel velocities, bring out several possibilities, namely, that the time of concentration may be equal to the time derived in the detail method or may be much greater, dependent on the slope.

The formula derived for the run-off is built up in an ingenious manner, and is correct under the assumptions. However, some of the assumptions are not always correct, or they are difficult of application.

The assumption of uniform rainfall for the time of concentration is not acceptable for short times of concentration. Examination of storms in Los Angeles for a number of years shows a distinct tendency for the peak rainfall to occur after the middle of the storm. Approximately 70% of the precipitation for 1 hour occurs within an interval of 10 min. The single peak of precipitation will synchronize automatically with the peak of the *CA* quantities contributing and will produce a higher peak run-off than a uniform rate of rainfall. A large percentage of storm drains built by municipalities falls within the time range of about 60 min. The detail method is as much at fault as the formula method in this respect.

In the derivation of the formula, the factor, *F*, is based on the conduit running full. The rainfall is assumed to be uniform in intensity for the time of concentration. At each point of concentration above the point under consideration, the uniform intensity is of higher value, and produces a conduit of correspondingly larger capacity per unit net area. The conduit is constructed with a difference of from 3 to 6 in. between sizes. The designer ordinarily selects the higher dimension as a factor of safety; furthermore, where slopes are flat, he selects a conduit to give a minimum velocity to prevent silt deposit; this often leads to an oversize. Losses for curves and structures along the main line must be subtracted from the gross head available in the fall of the surface grades. The result is that for all conduits above the point of concentration, and under the uniform intensity assumption, the friction slope for the full conduit is flatter than the surface grade.

In areas with steep slopes, the pipe will not run full at all points, which makes the value of *F* incorrect as tabulated. Inspection of the values of *F* under partly full flow conditions indicates an increased value over that for full flow for closed conduit, and a reverse condition for open-channel sections. In the flatter terrain, where it is possible for back pressure to fill the conduit, the formula is possible of solution with the given tabular values of *F*. However, to be technically correct, it is necessary, for each point of concentration, that detail tabulation of the friction loss of each length of conduit be made, together with other head losses. An aggravated example of the latter case occurs when an existing storm drain on a flat slope (and subsequently relieved of a part of its normal tributary area by an intercepting drain) is to be examined for capacity under the reduced area. The extreme oversize of the upper part of the conduit makes the application of the formula of doubtful value. A compensating factor, which applies, practically, to all but the last-mentioned case, is the low power of *S* used in the formula. With proper judgment in selecting *S*, more correct results are obtainable.

The factor, $P\left(=\frac{v}{V}\right)$, has been tabulated by the authors by a detailed analysis of numerous types of water-sheds. While it is thus arbitrary, a number of tests of the values reveals comparatively small differences from results obtained by detailed analysis, when the value of C is the same for each computation.

The analysis of the factor, C, points to the probability of variation with time and intensity. To handle this variation in the formula, the value of CA must be estimated from a detailed study of the area. On areas containing a large percentage of pervious soil, the study will be difficult, due to the fact that the coefficient of retention varies with the rate of precipitation for any instant of time and not with the average rate of precipitation for a period of time.

The use of the formula for general municipal storm-drain design does not seem advisable. To be able to use it intelligently, the designer must thoroughly understand its derivation and ramifications, and generally must make an analysis equal to that in the detail method. The careful judgment necessary to achieve correct results is likely to be slighted in the desire for a quick answer. The authors do not recommend it for such work. For estimating work, it can be used for shortening time of computations. For bridges and short culverts, the factors of loss other than friction are relatively so much greater, that the formula can be applied for final design.

By using Method No. 2 (which consists of a trial computation assuming a conduit laid up the main channel to the farthermost ridge point), a correction for time determined by an examination of the nucleus (or initial area which has no conduits), and a recomputation with the corrected time in the equation, $Q = CiA$, a quick check can be made at controlling points of the tabling by the detail method.

The purpose of the authors, to construct a formula capable of arriving at a close approximation of the run-off from a large area, has been attained in that the formula more or less duplicates the results obtained by the detailed rational method as now practiced. As set forth in the foregoing remarks, it has certain difficulties of application which require close analysis and more careful judgment in handling than the detail method.

The writer is inclined to give the greater weight to their recommendation following Table 6, urging "a more unified effort on the part of the profession not only toward a thorough analysis of present available data, but also future experimentation along lines following a preconceived plan." Drainage engineers have about arrived at the end of their resources in theory and need data to prove or disprove it.

Carl H. Reeves,[24] M. Am. Soc. C. E. (by letter).—The writer is deeply appreciative of the great amount of work and study shown in this paper. That which stands out most prominently is the logical sequence of the analysis as shown in Part I, and the resultant derivation of the formulas in Part II, bringing to the fore that most important of all factors the determination of *C*, as given in Equation (39).

---

[24] Civ. Engr., Los Angeles County, Los Angeles, Calif.

Engineers have apparently let an innate tendency to worship precedent lead them in the line of least resistance in determining run-off. In other words, they have taken for granted all that has been said and written and have blindly proceeded to apply it to any and all cases without due regard to the facts in any particular case; and by "the facts," it is intended specifically to mention that intensity of rainfall in its relation to the run-off coefficient has been a sadly neglected quantity in many calculations. It would appear that this is the most important factor in determining the value of *C*, or the coefficient of run-off.

Equation (39), with its accompanying explanatory sentences, is so patent that it is difficult to believe that this feature has been neglected or ignored so generally through the years. The application of this one idea alone is a distinct contribution to the Engineering Profession, and will simplify and clarify all long drawn-out and complicated tabulations which have been based on a value of *C* determined only by the presumed best judgment of the designer, or, in many cases, his best guess.

Fundamentally, the rainfall curve as generally used is the "right bower" to the factor, *C*, as determined in this paper. The analysis of the rainfall curve as applied in the so-called rational method is at once both illuminating and the cause of more rational thinking along these lines. As previously mentioned, engineers have been blindly following what they thought was a use of the rainfall curve giving intensities which represented the sequence of an actual storm. It is quite plainly shown that they have not been applying intensities which followed the law of the curve in use.

The writer wishes to congratulate the authors of this paper for the simplification and clarification of the many factors affecting run-off; but, withal, it would be well not to forget the really definite determination of the factor, *C*, as shown in Equation (39). The great benefit that this paper and its deductions will mean to engineers in private practice, railroads, State highway engineers, flood-control officials, and others having to do with an economical as well as a safe design of culverts, watercourses, storm drains, etc., cannot be over-estimated.

JOHN M. KEMMERER,[26] ASSOC. M. AM. SOC. C. E. (by letter).—The writer is interested in the authors' discussion of the use of empirical formulas, which they do not recommend. The paper definitely points out the hazards in the use of such formulas and clearly demonstrates that they are little better than a "rule-of-thumb method."

Factor *C*, based upon experimental data, has always required, and will continue to require, the very best judgment of engineers. Under the heading, "Analysis of Experimental Data," combined with the conclusions, suggestions, and comments of Part I, the authors present a thorough analysis of the factor. Equation (39) should prove useful as time goes on, and as data are collected for analysis.

---

[26] Civ. and Hydr. Engr., Whittier, Calif.

The fact that the run-off coefficient increases with intensity is well illustrated in the paper by Equations (5), (6), (7), (8), and (9). Conclusions (13) and (14), in the writer's opinion, are logically drawn and should receive the attention of engineers, especially those engaged in experimental work.

The analysis leading to Equation (24), a master equation, is clear, and new factors are included, which make it flexible and generally applicable to all areas. The use of this equation and the new factors requires a greater familiarity with the subject of run-off than has been customary with engineers, but this should lead to better professional results.

Messrs. Gregory and Arnold are to be commended and congratulated for the work which they have done in connection with the preparation of their excellent paper. It is manifest that much time and careful thought have been given to the problem. The writer has often felt the need of some ready adequate method of determining run-off and, therefore, commends the authors, particularly for their painstaking work which, in his opinion, has resulted in two distinct contributions: (*a*) The most complete analysis of Factor *C* yet presented; and (*b*) an equation (Equation (24)) which includes variables not before recognized, is universally applicable, and is rational in derivation.

LEONARD L. LONGACRE,[26] ASSOC. M. AM. SOC. C. E. (by letter).—In the consideration of a new formula the first concern is whether or not the derivation is based on a logical foundation. The formula presented in this paper is derived by starting with the fundamental equation, $Q = CiA$, as used in the detail rational method. The derivation follows in logical sequence until the factors, *F* and *P*, are introduced.

There can be no question concerning the velocity in the main flow channel affecting the time of concentration and, consequently, the discharge. The velocity is affected by the shape, and type of the flow channel, and the factor, *F*, is introduced to allow for this variation. The authors have presented a novel method of determining the value of *F*.

The writer is unable to explain why $\frac{(c\sqrt{r})^{0.5}}{Q^{\frac{1}{6}}}$ should have a constant value in Equation (18). Factor *P* makes a sudden and unannounced appearance in Equation (19), and it is unfortunate that the reader must stop and search in separate portions of the paper before an idea may be gained of the authors' intention in introducing this factor.

The arrangement of the collecting channels, which are influenced by the shape of the area, determines the quantity of water flowing at any certain point in the main flow channel. The quantity flowing in the main channel affects the velocity, and, as stated, this affects the time of concentration and discharge. It is evident that Factor *P* is introduced to allow for this variation in the arrangement of the collecting channels.

The writer is somewhat confused by the discussion just preceding Equation (19). It is evident that the expression, $c\sqrt{r}$, is used first as an abstract quantity to determine the value of *N*, and then is introduced in Equation (19) as a measure of the average velocity of flow through the main channel.

---

[26] Alhambra, Calif.

With a constant slope the velocity varies directly as the term, $c\sqrt{r}$, which can be expressed as a function of the factors, P and N. In Equation (19) Factor P could then be defined as the ratio of the average quantity flowing in the main channel to the quantity at the outlet. The authors have defined the factor, P, as the ratio of the average velocity in the main channel to the velocity at the outlet. As the velocity varies with the quantity, it can be used as a measure of Factor P as well as the quantity. The method of arriving at definite values of Factor P outlined in the paper is somewhat involved with assumptions. Before a general acceptance of the values of the factor, P, listed in Tables 3 and 4, the Engineering Profession will have to confirm these values in actual use.

It is the writer's opinion that all the factors included in Equation (24) have a direct effect on the run-off, and should be considered in any run-off problem.

The authors have given an adequate discussion on the shortcomings of previous empirical formulas, and have stated forcefully the advantages of their own. The writer does not believe the refinement of computation, as suggested under "Method No. 1," is justified where seven variables are to be chosen. The errors made in the choice of values for these variables will certainly be greater than any accuracy gained by this more elaborate method. Neither does he consider that the formula will replace the detail rational method in the actual detail design of a drainage system.

Its use, as outlined under "Method No. 2," offers possibilities in conjunction with the detail method. Where it is necessary to investigate different combinations of collecting channels to determine which will give the maximum flow, or where it is advisable to investigate whether or not a part of the area may give a greater run-off than the whole, the use of the Gregory-Arnold formula will save considerable time and labor. Another use to which it may be applied is the determination of time of concentration for initial areas.

Where the discharge at certain points in the water-shed is needed for the design of some structure, such as a culvert or bridge, on a stream where no adequate hydrograph is available (and it is seldom that any satisfactory data on rainfall intensity and distribution are available for use in connection with the hydrograph), the formula used, as described under "Method No. 2," will prove a decided benefit over previous methods of computations. It is necessary however, that the proper assumption for rainfall intensity be made. If the correct values of various factors are used in each case, and the run-off coefficient and initial time are placed on the same basis, the results obtained from the formula should compare favorably with those obtained by the detail method, and with a considerable saving in time and labor. The writer has tabled several areas (about 3 000 acres) by the detail method and has compared the discharges and times with those obtained by applying the Gregory-Arnold formula, and he was somewhat surprised at the close agreement, which was within 3 per cent.

In order to save considerable time in computations the writer has prepared a nomograph (Fig. 9), for the general solution of the Gregory-

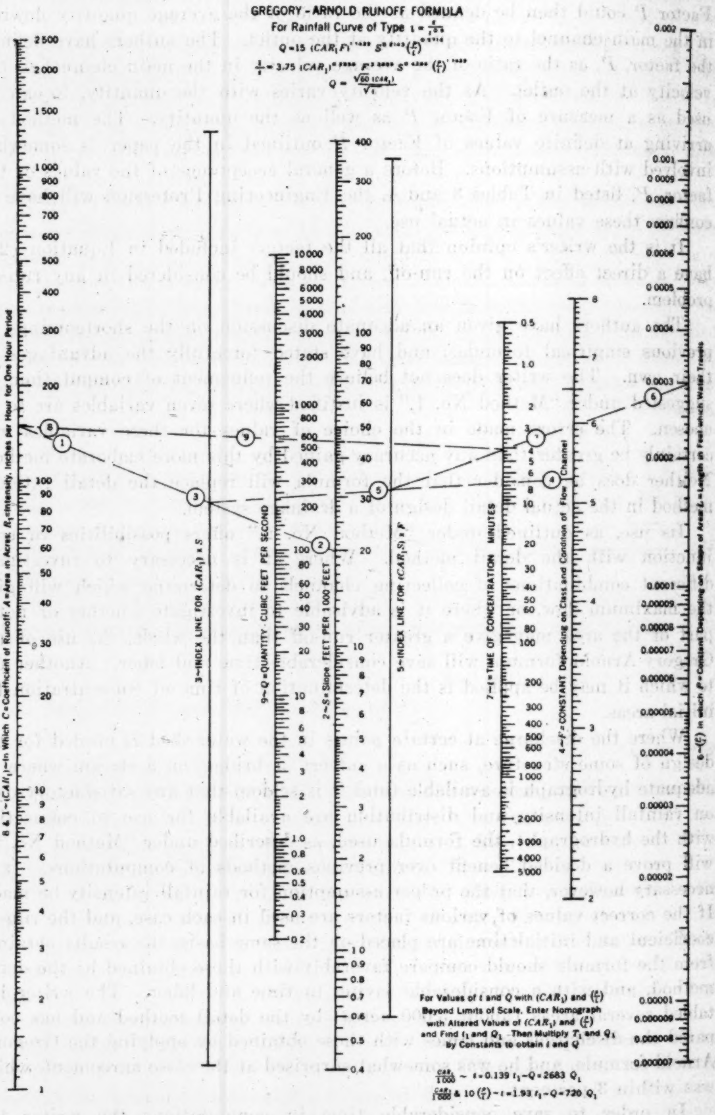

GREGORY - ARNOLD RUNOFF FORMULA
For Rainfall Curve of Type $i = \frac{a}{t^{0.5}}$
$Q = 15\,(CAR_1F)^{1.1429}\,S^{0.2143}\left(\frac{P}{\iota}\right)^{0.5714}$
$\frac{1}{t} = 3.75\,(CAR_1)^{0.2858}\,F^{2.2858}\,S^{0.4286}\left(\frac{P}{\iota}\right)^{1.1429}$
$Q = \frac{\sqrt{60}\,(CAR_1)}{\sqrt{t}}$
8 & 1–(CAR₁), in Which C=Coefficient of Runoff: A=Area in Acres: R₁=Intensity, Inches per Hour for One Hour Period
3–INDEX LINE FOR (CAR₁) × S
9–Q=QUANTITY · CUBIC FEET PER SECOND
2–S=Slope FEET PER 1 000 FEET
5–INDEX LINE FOR (CAR₁S) × F
7–t = TIME OF CONCENTRATION · MINUTES
4–F = CONSTANT Depending on Class and Condition of Flow Channel
6–(P/ι) in which P = Constant Depending on Slope of Area÷Length Fro which Waste Tra
1
2
3
4
5
6
7
8
9
For Values of t and Q with (CAR₁) and (P/ι) Beyond Limits of Scale, Enter Nomograph with Altered Values of (CAR₁) and (P/ι) and Find t₁ and Q₁. Then Multiply T₁ and Q₁ by Constants to obtain t and Q
$\frac{CAR_1}{1\,000}$ – $t = 0.139\,t_1$ – $Q = 2683\,Q_1$
$\frac{CAR_1}{1\,000}$ & $10\left(\frac{P}{\iota}\right)$ – $t = 1.93\,t_1$ – $Q = 720\,Q_1$

FIG. 9.—TIME OF CONCENTRATION AND RATE OF RUN-OFF.

Arnold formula. While it is based on a rainfall curve of the type, $i = \frac{K}{\sqrt{t}}$, one can readily be constructed for a curve of any of the general types in present use. The time of concentration has been evaluated as being the factor which is desired in most instances, and from which the quantity can easily be determined. The following example will serve to illustrate the method of use: Let, $C = 0.30$; $A = 10\,000$ acres; $R_1 = 1.23$ in. per hour; $S = 10$ ft. per 1 000 ft.; $F = 4.71$; $P = 0.55$; and $L = 35\,000$ ft. The time of concentration and discharge at the outlet are desired. The product, $CAR_1 = 3\,690$, is beyond the limit of the scale.

In the note at the bottom of the nomograph are given correction factors to apply for using $\frac{C\,A\,R_1}{1\,000}$. Then enter the CAR_1-scale with the value, 3.69, and, on a straight line connecting this value with 10 on the S-scale, locate the intersection with the index line numbered 3. From this point, on a line connecting 4.71 on the F-scale, locate the intersection with the index line numbered 5. From this point, on a line connecting 0.0000157 on the $\frac{P}{L}$-scale, read the value of 610 at the intersection with the t-scale. This value will be designated as t_1. From this point, at 610 on the t-scale, on a line connecting with 3.69 on the CAR_1-scale, read 1.16 at the intersection with the Q-scale. This value will be designated Q_1. From these values of t_1 and Q_1 the desired time of concentration and discharge are obtained by multiplying by the correction factors as given; thus, $t = 0.139 \times 610 = 85$ min.; and $Q = 2\,683 \times 1.16 = 3\,110$ cu. ft. per sec.

For some time the writer has been aware of the inadequacy of the usual practice of arbitrarily assuming a period of 5 to 15 min. as the time of concentration for the initial area used in the detail rational method. Actual observations have shown that in many cases these assumed times are grossly in error, particularly in flat country where initial times of 25 min. have been observed by the writer, and no doubt even longer inlet times occur. The Gregory-Arnold formula offers a possibility of determining the inlet time of initial areas for use in the detail rational method. The nomograph, Fig. 10, was prepared to facilitate the use of the formula. The numbers in the small circles give the sequence in which the scales are used. Factor F was given a constant value of 5.5 on the assumption that the water running from the most remote portion of the initial area to the point of concentration, would travel most of the way in a concrete gutter. Factor P was assigned a constant value of 0.50 by assuming that the average initial area is a rectangle, the length being twice the width. Another consideration in using a constant value for P is the fact that the percentage of error in the variation of P will be relatively small compared with the error which may occur in choosing the value of C. Where it is advisable to use different values for the factors, F and P, the nomograph shown in Fig. 9 may be used for finding the time of concentration for the inlet areas.

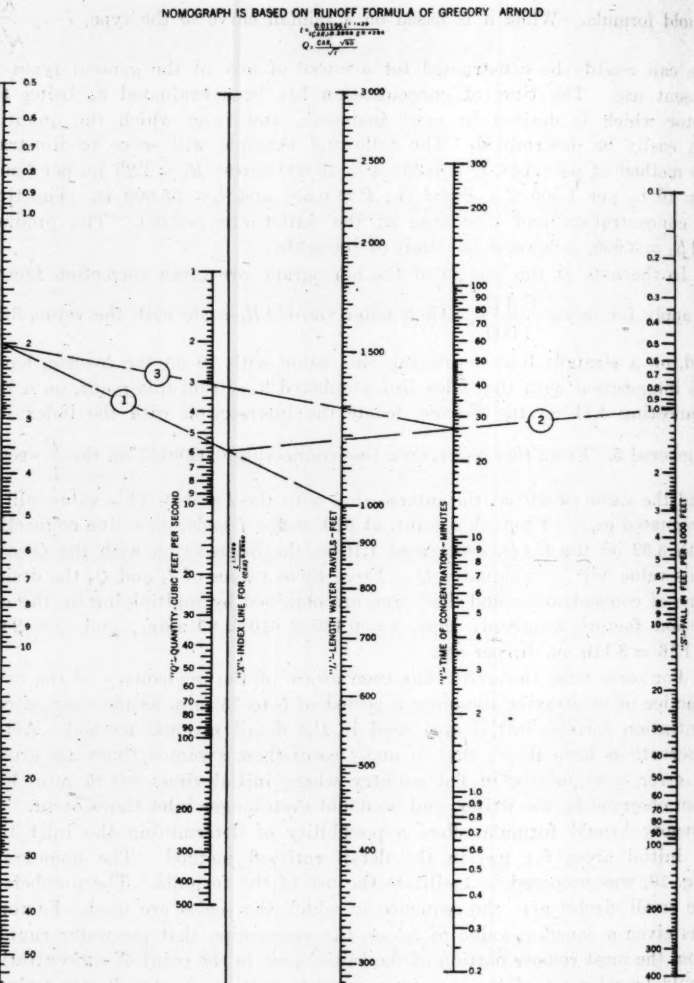


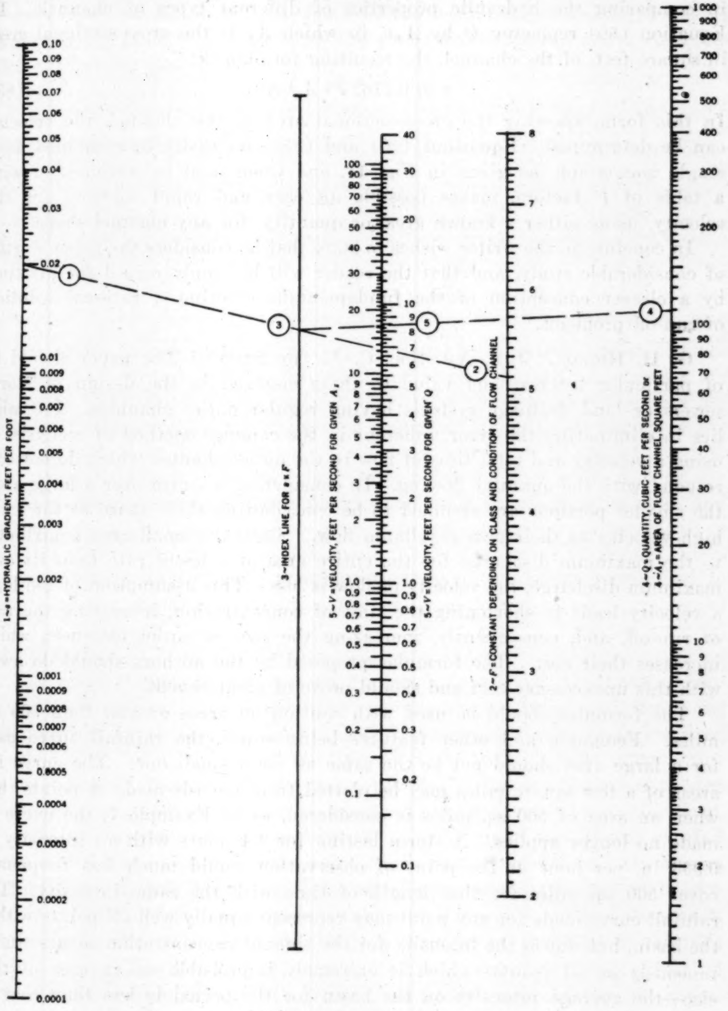
FIG. 10.—TIME OF CONCENTRATION AND QUANTITY AT INLET POINT. APPLIES ONLY WHERE $F = 5.5$; $P = 0.5$; AND $i = \frac{K}{t^{.5}}$

GREGORY ARNOLD FORMULA FOR

FLOW OF WATER IN CONDUITS & OPEN CHANNELS

$$V = 0.4217\,F^{2}\,Q^{\frac{1}{4}}\,s^{\frac{3}{8}}$$

$$V = 0.3162\,F^{\frac{8}{3}}\,A_s^{\frac{1}{3}}\,s^{\frac{1}{2}}$$



For Values of V with Q Beyond Limit of Scale Enter Nomograph with $\frac{Q}{100}$ and Multiply Value Obtained for V by 3.162 to Obtain the Correct V

FIG. 11.

Equation (50) presents a formula which upon investigation proves to give values agreeing very closely with ones obtained by the use of the Chezy-Kutter formulas when values of $c\sqrt{r}$ are greater than 50. It will prove useful in comparing the hydraulic properties of different types of channels. In Equation (50) replacing Q by $A_x v$, in which A_x is the cross-sectional area, in square feet, of the channel, the resulting formula is:

$$v = 0.3162\, F^{\frac{2}{3}}\, A_x^{\frac{1}{3}}\, S^{\frac{1}{2}} \qquad (85)$$

In this form, knowing the cross-sectional area of the channel, the velocity can be determined. Equations (50) and (85) can easily be combined on a single nomograph, as given in Fig. 11, and when used in conjunction with a table of F factors, makes possible an easy and rapid solution for the velocity, using either a known area, or quantity, for any channel shape.

In conclusion, the writer wishes to state that he considers the paper worthy of considerable study, and that the reader will be amply repaid for his time by a clearer conception of the fundamentals affecting a rational solution of run-off problems.

G. H. HICKOX,[27] JUN. AM. SOC. C. E. (by letter).—The paper should be of particular interest and value to those engaged in the design of storm sewers or land drainage systems having regular outlet channels. Its value lies in eliminating the error, inherent in the common method of analysis, of using a velocity and total time of flow in the outlet channel which do not correspond with the quantity flowing. In considering a storm over a large area, the smaller portions are assumed to be contributing their share at the same high velocity as their own maximum flow. Since the small area contributes to the maximum discharge for the entire area at a lesser rate than its own maximum discharge, the velocity of flow is less. This assumption of too high a velocity leads to shortening the time of concentration, increasing the rate of run-off, and, consequently, increasing the size of outlet channels, which increases their cost. The formulas proposed by the authors should do away with this unnecessary cost and should prove of great benefit.

The formulas should be used with caution on areas greater than 500 sq. miles. Economic and other features being equal, the rainfall curve used for a large area should not be the same as for a small one. The curve for areas of a few square miles may be plotted from records made at points; but when an area of 500 sq. miles is considered, as in Example 7, the curve so made no longer applies. A storm lasting for 4.4 hours with an intensity of 0.933 in. per hour at the point of observation would much less frequently cover 500 sq. miles for that length of time with the same intensity. The rainfall curve made for any point may represent equally well all points within the basin, but unless the intensity for the time of concentration occurs simultaneously at all points—which is extremely improbable on an area of this size—the average intensity on the basin for the period is less than that at the point. Precipitation on 500 sq. miles would necessarily be more intense at some points than at others. As the rainfall curve is made up of the maximum records at points, it follows that an estimate of discharge for the area, based on such a curve, is excessive. Table 15, prepared from data given in Part V of the Technical Reports of the Miami Conservancy District, presents coefficients by which to multiply the maximum precipitation observed within a river basin, to obtain the average over the basin (the values are probably slightly high, as it is assumed that the configuration of the storm coincides with that of the basin).

---

[27] Eng. Coll., Univ. of California, Berkeley, Calif.

TABLE 15.—RATIO OF AVERAGE RAINFALL OVER A GIVEN AREA TO MAXIMUM OBSERVED RAINFALL WITHIN THE AREA, FOR STORMS OF 1-DAY'S TO 5-DAYS' DURATION.

Drainage area, in square miles.	Coefficient.	Drainage area, in square miles.	Coefficient.
0	1.00	4 000	0.76
500	0.90	6 000	0.72
1 000	0.86	8 000	0.70
2 000	0.82	10 000	0.67

The writer is not convinced that the rational formula is ready to displace the empirical formula for the estimation of floods. Empirical formulas are subject to much abuse, being used far afield from the data on which they are based. It is the indiscriminate use of empirical formulas by engineers not familiar with their derivation and limitations, which has brought them into their present disfavor. An empirical formula for the 50-year flood, carefully based on data obtained in the vicinity where it is proposed to be used, and applied only within the range of these data, or moderately extrapolated, is probably superior to the rational formula. The element of frequency enters in connection with economic considerations, as it is seldom economical to design for the absolute maximum. The engineer is not so much concerned with the maximum flood as with that flood which may be expected once in, say, 50 years.

The rational formula assumes an average rainfall over the time of concentration and a uniform distribution over the basin. In general, floods are not produced by these conditions. They are the result of intense localized storms lasting for a fraction of the time of concentration; light, warm rains on snow-covered, frozen ground; and, occasionally, the conditions postulated by the formula. If these conditions, however, only produced the 50-year flood, it would still be impossible to evaluate it on the basis of the 50-year rainfall because of the uncertainty in the value of the run-off coefficient. On stream basins in the Middle West, the value of C may vary more than 100% for summer rainfalls of the same average intensity, due solely to the difference in saturation of the soil. Evidently, only a fraction of the 50-year rainfalls produce 50-year floods, as the soil is more often than not insufficiently saturated to produce maximum run-off. The problem is further complicated by the occurrence of floods due to intense local storms and the spring floods due to melting snow. The empirical formula, however, being based on recorded floods, automatically considers all these factors and groups them into two or three coefficients and exponents.

The Iowa River Basin above Iowa City, Iowa, may be taken as a fair example for comparison of rational and empirical formulas. Its drainage area is 3 140 sq. miles; channel length, 261 miles; and average slope, S, 0.425. The value of P may be taken as 0.65 and that of F as 4.0. The time of concentration is approximately 100 hours. Estimating the 50-year rainfall intensity from the charts[28] prepared by Merrill M. Bernard, M. Am. Soc. C. E., $i = \frac{75}{t^{0.8}}$, or, for 100 hours, $i = 0.0712$. Multiplying this by a factor taken from Table 15, $i = 0.78 \times 0.0712 = 0.0555$. The value of C for the Iowa River Basin is probably in excess of 0.35. Using these values, Equation (24) gives a discharge of 34 200 cu. ft. per sec.

F. A. Nagler, M. Am. Soc. C. E., has proposed[29] an empirical formula,

$$Q = 210 M^{\frac{2}{3}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (86)$$

for the magnitude of the 50-year flood to be expected on Iowa streams, in which, Q is the discharge, in cubic feet per second, and M is the drainage area, in square miles. By Equation (86), the discharge is 45 000 cu. ft. per sec. Floods greater than 35 000 cu. ft. per sec. have occurred three times at Iowa City since 1850, the largest being 50 000 cu. ft. per sec. The discharge given by the empirical formula is seen to fit at least as well as that by Equation (24) (which, in this case, is 24% less), and to involve much less labor.

The maximum average precipitation on the Iowa River Basin is probably about 10 in. in 4 days, or approximately 0.1 in. per hour for 100 hours. With $e = 0.8$ and $C = 0.4$, the maximum flood as given by Equation (24) is 84 400 cu. ft. per sec., a flood which is 88% greater than the 50-year flood and obviously beyond the ordinary considerations of economic design.

DONALD M. BAKER,[30] M. AM. SOC. C. E. (by letter).—The ready acceptance and use of familiar methods and formulas in design and computations, without an intelligent consideration of the limitations to which such formulas are subject, is a practice too common among engineers. A paper such as this, wherein the influences affecting the different factors embraced in the formulas used in the calculation of storm-water run-off are discussed and analyzed, is a decided service. The design of any storm drain system is a complicated process at best, involving many assumptions and a considerable degree of engineering judgment.

The master formula, Equation (24), is a considerable improvement. It goes much farther than older formulas in the inclusion of variables by the addition of terms representing changes in slope and consequent velocity of the main channel above the point of concentration, shape and roughness of this channel, and length and shape of the water-shed. It also allows greater flexibility in determining the period for the selection of the intensity and the variation in type of rainfall curve used. While it is somewhat complicated, nevertheless, in any organization in which much work of this type is done, its use can be facilitated by the development of curves or nomographs.

---

[28] See p. 604.

[29] "Survey of Iowa Floods," *Proceedings*, Iowa Eng. Soc., 1928.

[30] Cons. Engr., Los Angeles, Calif.

Of the three factors expressing the fundamental relationship between run-off and rainfall, stated by the authors in the expression, $Q = C\,i\,A$, the area, A, may be determined with a high degree of accuracy, and the intensity of rainfall, i, expressed in inches per hour (or the rate of contribution of water to the water-shed by precipitation, expressed in cubic feet per second per acre), may be ascertained with approximate accuracy in most localities where storm drain systems of any magnitude are to be constructed. Nevertheless, where the formula is to be used in the determination of storm or flood flows, in mountainous sections, or where precipitation records are lacking, much may be left to judgment or imagination.

However, it is the factor, C, which affords the opportunity for widest variation in the use of judgment. General reference to this factor in textbooks and technical papers seems to be based largely upon arbitrary assumptions or upon some previous reference and not upon results of actual experience. No one can give this subject much attention without being impressed with the paucity of experimental work carried on to establish values for this factor. Storm drain systems are expensive; a complete drainage system in a residential section, designed for a reasonable frequency of excessive precipitation, may cost \$500 to \$1 000 per acre, or \$320 000 to \$640 000 per sq. mile, or even more. The cost of such system, other things being equal, varies directly with the cost of pipes or box-drains used to carry the water, and this, in turn, varies approximately in a direct ratio with the quantity of estimated flood flow, which varies directly with the factor, C. It can readily be seen, therefore, that a variation of 10% in the value of C (say, where it was assumed as 0.45 instead of 0.40, due to lack of definite knowledge) might result in a variation in the cost of a drainage system of from \$50 to \$100 per acre, or \$\$32 000 to \$64 000 per sq. mile. In the light of this, it is surprising that more communities engaged in large programs of storm drain construction have not embarked upon a program of investigations of values for this factor. Nothing like the amount of research has been given to it as to the determination of such factors as proper values for Kutter's n, or constants in weir formulas. Such experimentation should logically be done by a governmental agency, in order that the results obtained could be made widely available. However, there does not appear to be such an agency in the Federal Government which could logically be considered as interested in this subject.

The mathematical proof which the authors offer in substantiation of their conclusions regarding variation of C with intensity, etc., is of considerable interest, and increases the confidence with which these conclusions may be used. A further feature of interest is the derivation of Equation (39), indicating the manner in which the rate of run-off varies with intensity of precipitation. In this equation, m expresses such influences as roughness of the ground surface of the water-shed, and the retarding effect of vegetal cover. Interception of precipitation by trees and brush, or its storage in a mulch of leaf mould, or in a dense grass cover, may result in the collection of a considerable quantity of precipitation, until saturation occurs and allows run-off to proceed. A few investigations along these lines have been made by experimenters interested in irrigation development and in the protection of water-sheds against fires. The quantity, z, used in Equation (39) is influenced principally by the soil type and the slope of the ground surface. During a visit, the writer was impressed by the lack of evidences of surface erosion on the floor of the Valley of Mexico as well as in Florida. Storms of high intensity and rather prolonged duration occur in these localities, but the soil appears to have such a high absorptive capacity that run-off is not evident. Some sandy soils, however, do not have this high capacity, due probably to lack of uniformity of size of grains.

The slope of the ground surface affects the rate at which the surface water concentrates in the small drainage channels and travels in them, but it also affects absorption or percolation. Equation (39) shows that as long as the rate of absorption is greater than the rate of precipitation, or when $z > i$, the factor, C, is a negative quantity, since $m < 1$ and no run-off can occur. When $i > z$, however, a surplus collects, which produces run-off. With flat slopes, velocity of run-off is decreased, depth of run-off is increased, and the resultant head of water on the soil surface also is increased. Using the same nomenclature as the authors,

$$Q = k\,(i - z') \quad \dots\dots\dots\dots\dots\dots\dots\dots (87)$$

in which, k = a constant, and z' = maximum unit rate of absorption by soil without superimposed head, constant for any given soil and surface condition; but Q varies as vd.

Let v = velocity; d = depth of water flowing over soil; and k' = a constant; then,

$$Q = k'\ vd = k'\ dc\ r^{\frac{1}{2}}\ s^{\frac{1}{2}} \quad \dots\dots\dots\dots\dots\dots\dots\dots (88)$$

Since $r = d$, approximately, Equation (88) reduces to,

$$Q = k'\ d^{\frac{3}{2}}\ c\, s^{\frac{1}{2}} \quad \dots\dots\dots\dots\dots\dots\dots\dots (89)$$

Therefore, $k\,(i - z') = k'\ d^{\frac{3}{2}}\ c\ s^{\frac{1}{2}}$, and,

$$d = \left(\frac{k\,(i - z')}{k'\ c}\right)^{\frac{2}{3}} \frac{1}{s^{\frac{1}{3}}} \quad \dots\dots\dots\dots\dots\dots\dots\dots (90)$$

Let z'' = rate of percolation from run-off, and k'' = a constant. Then, since the rate of percolation also varies as the superimposed head,

$$z'' = k''\ d = k''\left(\frac{i - z'}{c}\right)^{\frac{2}{3}} \frac{1}{s^{\frac{1}{3}}} \quad \dots\dots\dots\dots\dots\dots\dots\dots (91)$$

and, $z = z' + z''$, which shows that induced absorption, as distinguished from the natural absorption rate of the soil, varies directly with the two-thirds power of the difference between the rate of precipitation and the natural absorption rate of the soil, and also increases directly in a certain manner with the surface roughness, but increases inversely with the cube root of the slope.

In explanation of the fact that a zero value of s would make z'' approach infinity, it may be stated that this relationship only holds when the velocity of flow of the run-off follows the Chezy-Kutter formula.

ALBERT R. ARLEDGE,[31] ASSOC. M. AM. SOC. C. E. (by letter).—The authors, undoubtedly, have presented a more complete analysis of the subject with which they deal than may be found elsewhere. The paper constitutes a distinct contribution to the profession in each of the two parts into which it is logically divided.

Part I subjects existing methods of estimating the probable maximum run-off of a given area to an orderly and critical analysis, enabling the reader to visualize the many possible variations of the several factors involved. In so doing it brings to light some basic principles which seem to have been entirely neglected by those who have written about the rational method of computing run-off.

Among the hitherto neglected principles may be mentioned: (*a*) Use of the velocities in determining time in the detail method, which do not obtain under the assumptions; and (*b*) the effect on the run-off coefficient of a change in intensity.

The latter principle is by far the more important, especially for the consideration of water-sheds containing considerable pervious area. The simple nature of this phenomenon (after its able presentation by the authors), which may so vitally alter the resulting run-off, causes one to wonder why so many able writers on the method have overlooked it. Although it makes the estimation of run-off by the detail process more tedious, its effect, once understood, should certainly be recognized. The "Suggestions (*a*) and (*b*)," and "Comment" forming the conclusion of Part I, may well be heeded by engineers who find it advisable to use the detail process.

Part I, as a whole, should serve the Engineering Profession as a valuable reference for study by those of its members who contemplate the solution of maximum storm flows regardless of the method subsequently adopted.

In Part II of their paper, the authors present run-off formulas which are entirely logical in derivation, as far as the writer is able to determine. It would seem that their presentation at this time, goes far toward satisfying a need of many engineers for a quick and direct method for applying rational principles to the solution of discharges for the larger areas. In this connection attention should be called to (*a*) the paper[32] by Merrill M. Bernard, M. Am. Soc. C. E., together with subsequent discussions thereof; (*b*) treatment[33] of this subject by Harrison P. Eddy and the late Leonard Metcalf, Members, Am. Soc. C. E.; (*c*) discussion[34] by Charles H. Lee, M. Am. Soc. C. E., of the paper by Oren Reed, Assoc. M. Am. Soc. C. E.; and (*d*) writings by others in the general literature on the method.

The saving of time through use of the formulas, as compared with that of the detail process, will naturally depend not only on the nature and extent of the areas, but also on the purpose of the engineer. Generally, the time saved will vary with the size of the area above the point at which discharge is sought. It is unfair, therefore, to compare these formulas (without any qualifications) solely with the detail method of estimating storm flow for a small city drainage system, wherein the discharge is sought at every inlet point. Even for small areas (and likewise for any irregular area) the formulas should prove time savers in making trials for the determination of run-off from water-sheds, a part of which will render a greater value under the assumptions, than the whole.

---

[31] Asst. Engr., Los Angeles Bureau of Power and Light, Los Angeles, Calif.

[32] See p. 592.

[33] "American Sewerage Practice," Vol. 1, Second Edition, pp. 305-306.

[34] *Transactions*, Am. Soc. C. E., Vol. 93 (1929), p. 483.

Equation (39) deserves special mention. This formula appears to embody the culminating thoughts of the authors, in their analysis of the run-off coefficient, especially as regards future experimentation. Indeed, it would seem that the "rational method" should scarcely be called rational without possession of Equation (39), or its equivalent; and it should prove valuable in future experimentation which contemplates the use of any method for calculating discharges.

In conclusion, the writer wishes to state that the authors deserve the appreciation of the profession, not only for the great energy and thought necessarily required for the creation of their paper, but also for the actual value of the gift itself. The analysis, conclusions, and formulas therein, will be referred to in the future. The future worth of the formulas will depend largely on the amount of intelligent use to which they are subjected.

CLARENCE S. JARVIS,[85] M. AM. SOC. C. E. (by letter).—The authors have made a clear exposition of many fundamental principles and considerations related to stream flow, and have attempted a somewhat rigorous analysis, evaluation, and correlation of various factors for practical use. So much of the textual treatment is commendable and beyond challenge that it is with reluctance that the writer registers minor exceptions or divergent views.

Having deliberately and repeatedly approached the subject of run-off, together with the derivation and critical tests of the best-known formulas, with an open mind, ready to accept or to reject any and all according to practical results within the fields for which data are available, he is unable to concur in either Considerations (1), (2), and (3), under "Empirical Formulas," or Statements (*d*) and (*e*) under "Inadequacy of Empirical Formulas." Do they not actually express the summation of all the influences mentioned in the paper under discussion, based on observed behavior instead of on *a priori* reasoning? While according an important place for the profitable application of the so-called rational method of analysis, one must likewise recognize its inherent limitations, disadvantages, and defects. Where detailed information is lacking on extensive water-sheds, the empirical formulas and high-water marks of past years will doubtless continue as the main reliance for purposes of design. Before the requisite mass of data can possibly be collected regarding physiography, topography, soils, channel characteristics, and rainfall habits, there will be many occasions for measuring the stream discharge and some part of the precipitation that preceded the gauging.

"The storm causing the greatest rate of discharge in a storm sewer will usually be the maximum rain lasting a length of time equal to the time of concentration" is the original form of the statement published[36] by Anson Marston Past-President Am. Soc. C. E., in 1909, which has had great influence in guiding recent investigations. It is obvious that detailed analyses and tabulations are facilitated by the convenient assumptions that have been added as to "maximum uniform intensity" and, also, equal simultaneous distribution over large water-sheds during the time of concentration; but those assumptions are largely contrary to usual rainfall habits, especially over extensive drainage areas. Neither is the maximum rate of run-off necessarily the result of simultaneous contributions from all parts of the water-shed. It is more likely the result of an intense down-pour and concentration from a considerable portion, while much of the remaining area yields only a negligible run-off.

---

[85] Prin. Hydr. Engr., U. S. Engr. Office, War Dept., Washington, D. C.

In keeping with the name usually accorded, the rational method should exclude such an expression for rainfall intensity as appears in Equation (2), which is capable of bringing forth absurdly high intensities for very short periods of time. Evidently, a constant should be added in the denominator to make it a safe factor as in Equation (45). Adolph F. Meyer M. Am. Soc. C. E., has recommended,

$$i = \frac{A\,t}{t + B} \qquad \text{.........................(92)}$$

in which, both A and B are constants.[37] Furthermore, the introduction of N as a constant, based on a chance relation of certain variables within restricted range, may account for larger errors or irregularities than would be contemplated from Equation (18); for, in Equation (32), both the constant and the percentages are squared, and this would more than double the positive errors; for example, 1.10 × 1.10 = 1.21. Also, the varying channel sections, gradients, and velocities to be found in practically all natural drainage systems are quite contrary to the assumptions of the authors, who seem to be carrying the storm drainage conduits throughout the water-sheds under consideration. Otherwise, it is almost inconceivable that such velocities as 28.3 ft. per sec., or 19 miles per hour, would occur with the slope and discharge assumed in Example 3.

It seems to be contrary to experience that the instantaneous discharge at a given station should vary directly with the length of travel. Perhaps the total quantity bears such a relationship; but the increased time of travel affords opportunity to smooth out the flood crests, thus increasing channel storage, underflow, and period of high stage. If a linear dimension is desired instead of some function of the drainage area, the average width seems to have a more logical basis for adoption, as has been demonstrated[38] by Charles R. Pettis, M. Am. Soc. C. E.

The discharges of 1 006 and 1 069 sec-ft. from 2.02 sq. miles, as used in Examples 6 and 8, respectively, are equal to 7.1% and 7.5% on the Myers scale[39]; while the 59 740 and 62 003 sec-ft. from 500 sq. miles, as disclosed in

---

[36] Cyclopedia of Civil Engineering, Vol. VII, 1909, p. 305. "Sewers and Drains."

[37] "Elements of Hydrology, Precipitation," 1928, p. 103.

[38] "A New Theory of River Flood Flow," by Charles R. Pettis, M. Am. Soc. C. E.

[39] *Transactions*, Am. Soc. C. E., Vol. 89 (1926), p. 994.

Examples 7 and 9, represent 26.7% and 27.8%, respectively. Taking into account the time required for a reliable solution by small increments from individual areas, the many estimates, assumptions, and uncertainties involved, the acurate knowledge of the water-shed topography, geology, soil characteristics, and rainfall habits, that must be in hand for reliable results, and the involved factors and nomenclature required for the method presented in the paper, it appears that its special field for usefulness is in urban and metropolitan developments. To the writer the best of the empirical formulas seem better adapted to large areas and to the determination of run-off therefrom; and they seem to have fully as valid a claim to the descriptive adjective as has been shown for the "rational formulas" under discussion.

It is significant that the expression adopted by the Committee on Floods of the Boston Society of Civil Engineers,[40] readily reduces to a form comparable with the modified Myers formula,

$$Q = 10\,000 \times 80\% \sqrt{A} \quad \ldots\ldots\ldots\ldots\ldots\ldots (93)$$

thus ranging up to a rating of 80% for maximum floods during long periods of time.

Notwithstanding the implication so often expressed to the effect that such formulas do not take account of the shape and other variable characteristics of drainage areas, the estimated effects of such factors are readily incorporated in the percentage adopted. Thus, with ratios of length to width represented by 5, 2, and 1, the flood rating for a given water-shed and frequency of recurrence might be 50%, 60%, and 80%, respectively.

The platting of observed rainfall and run-off with time and average depths on the water-shed as co-ordinates, together with detailed studies of soil, slopes, vegetative cover, and geology along with physiography, should afford a fair index to probable stream flow from a given area, or from other water-sheds with similar characteristics, and there is a definite and important place for both the rational and the empirical methods of analysis.

C. E. GRUNSKY,[41] PAST-PRESIDENT, AM. SOC. C. E. (by letter).—As long ago as 1908 the writer attempted a mathematical demonstration of the propriety of using a rational formula for the determination of the maximum rate of run-off (maximum stream flow), based on the maximum rate of rainfall or maximum rain intensity. In the paper[42] describing that work it will be seen that as a basis for the mathematical presentation the fact was taken into account that the water which falls as rain—in any period of time that may come under consideration when the maximum rate of run-off at some selected point is to be estimated—must (at the peak of discharge) either have already passed the point in question or must still be in temporary storage: (1) As a film of water on the surface; (2) as water in conduits; (3) in watercourses; or (4) in lakes and reservoirs within the water-shed tributary to the point. Loss by evaporation in the consideration of small areas and short critical periods may be neglected or rather merged with the other factors that go to make the maximum run-off rate less than the maximum rate of fall of water upon the water-shed as rain.

---

[40] *Journal*, Boston Soc. of Civ. Engrs. (1920), Vol. VII, p. 47.

[41] Cons. Engr. (C. E. Grunsky Co.), San Francisco, Calif.

[42] "The Sewer System of San Francisco, and a Solution of the Storm-Water Flow Problem," *Transactions*, Am. Soc. C. E., Vol. LXV (1909), p. 294 *et seq.*

In the resulting analysis local records of heavy rainfall were necessarily taken into account and, of course, also the formulas for maximum rain intensity based thereon. These formulas determine the total quantity of water which may be precipitated upon any water-shed during the continuance of the rain which produces maximum stream flow.

Account was then taken of water-shed characteristics, such as extent, shape, topographic and orographic features, the degree of permeability of the surface, etc. These characteristics serve as a basis for approximating the critical time applicable to the water-shed; that is, the time within which a maximum downpour would produce maximum stream flow. It was assumed that this time would be that period which would elapse from the beginning of the rain until all essential major portions of the water-shed were contributing run-off to the stream at the point for which maximum stream flow is to be determined.

In 1930, the writer published simplified rain-intensity formulas.[43] These result in part from the substitution of the hour for the minute as the unit of time. These new formulas are intended to permit closer approach to actual fact, both for precipitation during very short time periods, ½ hour, or less, and during very long periods. The suggested refinement for very short time periods is of value mainly when a short time record must be used in determining formula constants for any locality; otherwise, it is of no particular importance.

The intensity of rainfall is never uniform throughout any time period. It is not uniform at all points in a water-shed at any particular instant. Neither is the aggregate amount of rain during any period uniformly distributed. Nevertheless, the intensity of rainfall which will naturally be introduced into any rational formula for maximum storm-water discharge is the maximum average intensity throughout the water-shed, during the time which intervenes from the beginning of a storm of the type which will produce maximum discharge to the instant when this maximum discharge occurs.

This time, called for convenience "the critical time," must be determined, as already stated, from water-shed characteristics. When the water-shed has a compact, fairly regular form without long finger-like distortions, it will approximate the time that it takes water, in quantity, to flow from the remote parts of the water-shed to the point of concentration. Ordinarily, it must be left to the good judgment of the engineer to make a fair approximation of this period. This time as well as the rain intensity being in the nature of the case approximations, it becomes evident that great refinement in computation is less important than good judgment in evaluating basic elements. Frequently, in the case of large water-sheds the maximum discharge results from rain on a fractional part only of the entire water-shed.

---

[43] *Monthly Weather Review*, October, 1930; and, *Military Engineer*, May-June, 1931; also, p. 612.

It may be noted in passing that it is inadvisable to use any of the well-known older types of formulas for maximum stream flow, which do not include rain intensity as a factor.

The writer favors his own rain intensity and run-off formulas of which the former have already been presented to the Society.[44] For convenience they are here repeated with adherence so far as practicable to the author's notation:

For $H < 0.33$ hour (20 min.):

$$X_{\text{max.}} = 0.75\,K\,H^{\frac{1}{4}} \quad \text{.......................} (94)$$

$$i_{\text{max.}} = \frac{0.75\ K}{H^{\frac{3}{4}}} \quad \text{.......................} (95)$$

For $H > 0.33$ hour and < 64 hours:

$$X_{\text{max.}} = K\,H^{\frac{1}{2}} \quad \text{.......................} (96)$$

$$i_{\text{max.}} = \frac{K}{H^{\frac{1}{2}}} \quad \text{.......................} (97)$$

For $H > 64$ hours:

$$X_{\text{max.}} = 2\,K\,H^{\frac{1}{3}} \quad \text{.......................} (98)$$

$$i_{\text{max.}} = \frac{2\ K}{H^{\frac{2}{3}}} \quad \text{.......................} (99)$$

If now the maximum rain in 1 hour be represented by R, instead of by the author's notation, $X_1\,(_{\text{max.}})$ it will appear from Equations (96) and (97) — because $K = X_1\,(_{\text{max.}}) = i_1\,(_{\text{max.}})$—that the value of K is the maximum rain in 1 hour and $K = R$. Equations (94) to (99), inclusive, may therefore be written:

For $H < 0.33$ hour:

$$X_{\text{max.}} = 0.75\,R\,H^{\frac{1}{4}} \quad \text{.......................} (100)$$

$$i_{\text{max.}} = \frac{0.75\ R}{H^{\frac{3}{4}}} \quad \text{.......................} (101)$$

For $H > 0.33$ hour and < 64 hours:

$$X_{\text{max.}} = R\,H^{\frac{1}{2}} \quad \text{.......................} (102)$$

$$i_{\text{max.}} = \frac{R}{H^{\frac{1}{2}}} \quad \text{.......................} (103)$$

For $H > 64$ hours:

$$X_{\text{max.}} = 2\,R\,H^{\frac{1}{3}} \quad \text{.......................} (104)$$

$$i_{\text{max.}} = \frac{2\ R}{H^{\frac{2}{3}}} \quad \text{.......................} (105)$$

---

[44] See p. 612.

When the value of $K = X_1\ (_{\text{max.}}) = R$ is to be determined for any area, the actual maximum precipitation on that area, during some period approximating the critical time, should be used as the basis for this determination. The necessary computation can then readily be made with the aid of the appropriate one of the three Equations (100), (102), or (104).

In any area of considerable extent the maximum average rain intensity on the entire area will be less than at single points of heavy rainfall. The maximum intensity is not coincident in time at all points of the water-shed. The precipitation of water upon a water-shed in the form of rain is at a more rapid rate than the run-off. Not all the water which falls upon a water-shed will reach the stream or other drainage conduit. Some of it sinks into the surface formation and ultimately either evaporates or remains underground so long before re-appearing in springs that its effect upon the maximum stream flow is lost.

Therefore, some modification factor with a value less than unity will have to appear in any formula for maximum stream flow if based on maximum rain intensity. Let *a* (the *a* in the writer's formulas published elsewhere) represent this modification or water-shed factor dependent upon surface characteristics and the extent of the water-shed; and let *q* represent the maximum run-off rate or maximum stream flow, in second-feet per square mile of water-shed area.

One acre-inch of rain is equivalent to $\left(\frac{43\,560}{12} =\right)$ 3 630 cu. ft. of water. Consequently, if 1 in. of rain falls on 1 acre in 1 hour (= 3 600 sec.) and if all the water should run off at the rate of the rainfall, then this rate of run-off would be $\frac{3\,630}{3\,600} = 1.0$ cu. ft. per sec. In round numbers, therefore, 1 in. of rain in 1 hour on 1 sq. mile, or 640 acres, if all ran off in 1 hour, would be equivalent to a flow of 640 sec-ft.

Not all the rain which falls on a water-shed in the critical period, even when falling on an impervious surface, will appear as run-off during this period, or in any other period of equal duration. There will be some loss, as stated previously, and there will be a greater quantity of water in transit, that is, in temporary storage, at the end of any critical period than at the beginning of the period. It will suffice for practical purposes to take this reduction in the quantity of discharge compared with the quantity of rain water at about 25% and to assume, therefore, that actual run-off per square mile in the critical period, when the rain intensity is 1 in. per hour, will be about (75% of 640 =) 480, or, in round numbers, say, 500 sec-ft. per sq. mile instead of 640.

Because the value of $X_1\ (_{\text{max.}})$, or *R*, which represents the maximum rainfall on the water-shed in 1 hour, may be substituted for the coefficient, *K*, and because *t*, expressed in hours, is now to be written for *H*, the time of concentration (the critical time), the formulas for maximum run-off per square mile will take the form:

For $t < 0.33$ hour:

$$q = \frac{375 \ \alpha \ R}{t^{\frac{3}{4}}} \quad \text{.......................(106)}$$

For $t > 0.33$ hour and < 64 hours:

$$q = \frac{500 \ \alpha \ R}{t^{\frac{1}{2}}} \quad \text{.......................(107)}$$

For $t > 64$ hours:

$$q = \frac{1\,000 \ \alpha R}{t^{\frac{2}{3}}} \quad \text{.......................(108)}$$

If A_m represents the area of the water-shed, in square miles, and $Q_{\text{max.}}$, the maximum rate of discharge, in second-feet, then:

For $t < 0.33$ hour:

$$Q_{\text{max.}} = \frac{375 \ \alpha \ R \, A_m}{t^{\frac{3}{4}}} \text{.......................(109)}$$

For $t > 0.33$ hour and < 64 hours:

$$Q_{\text{max.}} = \frac{500 \ \alpha \ R \, A_m}{t^{\frac{1}{2}}} \quad \text{.......................(110)}$$

For $t > 64$ hours:

$$Q_{\text{max.}} = \frac{1\,000 \ \alpha \ R \, A_m}{t^{\frac{2}{3}}} \quad \text{.......................(111)}$$

In some cases, the value of the coefficient, α, may be determined from known (measured or closely approximated) high rates of discharge and the rainfall characteristics which produced these rates. Generally, however, assumptions must be made relating to this value. It appears obvious that of the total quantity of water falling on a water-shed as rain, the relative quantity which evaporates, together with that which sinks into the ground and is thus prevented from contributing to the peak discharge, increases as the critical time (that is, as area) increases.

This consideration has led to the tentative adoption of the following expression for α and the selection of values for surface characteristics which may be used until something better is offered for typical localities:

$$\alpha = \frac{1}{1 + \beta \sqrt{t}} \quad \text{.......................(112)}$$

in which, β is a factor dependent on the surface conditions of the water-shed. For the present:

$\beta = 0.013$ for impervious areas.
$\beta = 0.25$ for mountains.
$\beta = 0.40$ for rolling country.
$\beta = 1.3$ for flat country (ordinary soil).
$\beta = 6.5$ for sandy regions.

These values are suggested for ordinary conditions in temperate climates. The resulting values of the coefficient, α, will be found, no doubt, too small in localities where the ground may be frozen, where it is water-logged, or where the maximum run-off occurs when heavy rains fall on snow.

The intensity of rainfall under extreme conditions is now fairly well known throughout the United States for individual stations. It is not so well known what the maximum rainfall in a limited time period may be throughout water-sheds of considerable extent. The rain intensity at single stations may quite properly be accepted as applying to small areas. For large areas the mean maximum rain intensity will be materially less than the maximum rain intensity on small areas.

For convenience, it may be assumed that for a critical time period of 1 hour, or less, the rainfall throughout the entire water-shed under consideration will be of the same intensity as at single stations.

For longer critical periods and larger water-sheds the maximum rain intensity (computed as if the rain were distributed uniformly over the area in question) will be based on the best meteorological data available. Its determination throughout any water-shed in question during the critical time, *t*, is best made from actual records of heavy rainfall and a study of its distribution to the various parts of the water-shed. Such records, however, will rarely, if ever, indicate the true possible maximum. Consequently, some correction factor greater than unity must be applied, based on a comparison of the individual station records during such a storm with the possible local maxima. It should not be assumed, however, that rain of maximum intensity will occur at all the stations at the same time, but rather that if the storm in question at controlling stations might have produced an intensity greater by some percentage, then it is proper to assume that with similar relative distribution of rain throughout the water-shed some future storm might produce the same proportionally greater amount of rain and of run-off.

Due to topographic and orographic characteristics of the water-shed, coupled with the distribution of the rain throughout its extent, it will frequently be found, as already stated, that a rainfall of maximum intensity on a part of the water-shed during a relatively short critical period will produce a greater peak discharge than that which occurs when the rain of maximum intensity for the longer critical time applicable to the entire water-shed is taken into account. Trial computations alone, in the light of rainfall data, can determine what part of the river system is to be excluded in making the estimate of maximum momentary river discharge.

The method of estimating the maximum stream flow can now be summarized briefly as follows:

(*a*) Determine the critical period for the water-shed, with a proper allowance for the time that it will take water to flow in the stream from the water-shed to the point at which maximum stream flow is to be ascertained. Express this critical time, *t*, in hours.

(*b*) Determine the value of the water-shed coefficient, α, from the surface characteristics of the water-shed by use of Equation (112).

(*c*) Determine from the rainfall records, the maximum depth of rain, expressed in inches, which has fallen upon the water-shed in any period, about *t* hours in duration, and with the aid of single station records by means of a correction factor establish the probable maximum rainfall on the water-shed in t hours. This maximum and the critical time will establish the maximum intensity for the water-shed and this, in turn, will be used in calculating the value of R ($= X_1$ ($_{\text{max.}}$)) applicable thereto. The contribution by the authors on this subject of a determination of the critical time should be found helpful.

(*d*) Use Equations (106), (107), or (108), as the case calls for, to determine the maximum flow, in second-feet per square mile, or Equations (109), (110), or (111) to determine the maximum stream flow, in second-feet, at the point under consideration.

It remains to be said that lakes and reservoirs are frequently factors which prolong the critical period and reduce the maximum stream flow. Ordinarily, their effect is fairly well taken account of in the estimate of the duration of the critical period. When, however, the point for which maximum flow is to be determined lies close below a large lake or reservoir, recourse may have to be had to a special study of the effect of the increase of storage in such a water body upon the discharge at the peak of the flood flow.

The following examples will illustrate the application of the formulas:

Example (*1*).—Let it be supposed that for a mountainous water-shed of 2 000 sq. miles, the critical time, that is, the value of t, has been approximated at 36 hours. The most severe storm of which there is record, as indicated by a number of rain gauges and by the resulting isohyets, produced an average of 4 in. of rain throughout the water-shed during this period. However, it is known that all single-station records for this storm have been, or may have been at some time, exceeded by 50 per cent. What should be assumed as the probable maximum stream flow?

Because single-station records have been exceeded it is probable that the maximum measured rain intensity throughout the water-shed should be increased in like proportion. Instead of 4 in., therefore, 6 in. should be introduced in the calculation. Equation (107) will apply.

It follows that: $i_t = \frac{6}{36} = 0.167$ in. per hour, the average for the 36 hours; and $i_t = \frac{R}{\sqrt{36}} = 0.167$. Therefore, $R = 1$ (determined in this case directly from the rain on the entire water-shed; not from any single-station record).

From Equation (112):

$$\alpha = \frac{1}{1 + 0.25\sqrt{36}} = 0.40$$

and from Equation (107):

$$q = \frac{500 \times 0.40 \times 1}{\sqrt{36}} = 33 \text{ sec-ft. per sq. mile}$$

and,

$$Q_{\text{max.}} = 2\,400 \times 33 = 80\,000 \text{ sec-ft.}$$

Example (*2*).—In a down-town area having an extent of 20 acres, or 0.0313 sq. mile, 70% of the area is impervious; the remainder is ordinary soil. The critical time is estimated at 15 min. (= 0.25 hour). The maximum rainfall in 1 hour is known to be 3 in. What will be the maximum rate of run-off from this area? Use Equation (106). Here,

$$R = 3;\ i_{\text{max.}} = \frac{0.75\,R}{0.25^{\frac{3}{4}}} = \frac{2.25}{0.354} = 6.3 \text{ in. per hour}$$

and,

$$\alpha = 0.70\ \frac{1}{1 + 0.013\sqrt{0.25}} + 0.30\ \frac{1}{1 + 1.3\sqrt{0.25}} = 0.88$$

$$q = \frac{375 \times 0.88 \times 3}{0.25^{\frac{3}{4}}} = 2\,830 \text{ sec-ft. per sq. mile}$$

and,

$$Q_{\text{max.}} = 0.0313 \times 2\,830 = 89 \text{ sec-ft.}$$

Ralph W. Powell,[45] M. Am. Soc. C. E. (by letter).—This paper has been quite fully discussed and the writer wishes to touch on only a few points. The authors derive a formula for C, the run-off coefficient. It would seem that if z is the rate of absorption plus evaporation, in cubic feet per second per acre, the total number of cubic feet per second being absorbed and evaporated would be zA, the net run-off would be $iA - miA - zA$, and C would be $1 - m - \frac{z}{i}$, instead of as in Equation (39). The authors seem to have allowed for no absorption or evaporation from the stored portion of the precipitation.

The storage here referred to is that "falling in depressions, cavities, etc.," and evidently is not meant to include "channel storage." The writer questions whether Kutter's formula, or any other formula derived from experiments on steady flow, applies to this case. It would seem that the channel must act to some extent like a retarding basin and make the time of concentration for the uniform rainfall assumed by the authors, somewhat more than that given by their method. If this is true the error, of course, would be on the side of safety.

Actual storms are not like the one assumed, however, and they do not occur simultaneously over the water-shed. There is the extreme possibility of a storm moving down a valley at the same velocity as the flood crest. This would mean that the maximum effects of the most intense rainfall in each part of the drainage area would reach the mouth simultaneously and that the maximum flow would be $Q = CiA$, in which, i would be the intensity for a very short time, t, approaching zero as the timing of the maximum rainfall at each point in the area approached the ideal just stated. This would mean values of i of the order of 10 in. per hour. It would seem, therefore, that peak run-off resulting from any given storm depends too vitally upon the velocity (including direction) of the storm across the drainage area to make worth while any refinements of methods which neglect it.

Speaking of small values of t it must be remembered that Equation (2) and all the information compiled[46] by Merrill M. Bernard, M. Am. Soc. C. E., apply only to times greater than 2 hours. For shorter times, the formula must be changed to,

---

[45] Asst. Prof. of Mechanics, Ohio State Univ., Columbus, Ohio.

[46] See pp. 618–622.

$$i = \frac{k}{(t + b)^e} \qquad \text{(113)}$$

in which, b is about 7 or 8 min.[47]

On the other hand, when the area becomes large another correction is necessary as pointed out by the authors. From the study of a small part of the data given in the Appendix of Part V of the Technical Reports of the Miami Conservancy District, the writer has derived a rough empirical correction factor of the form, $(1 - 0.015 \sqrt[3]{A_m})$, in which, A_m is the area, in squares miles. This gives values nearly the same as those in Table 15, but was derived independently. It is evident that this correction should be made whenever A_m is more than, say, 100 sq. miles.

MERRILL M. BERNARD,[48] M. AM. SOC. C. E. (by letter).—The authors' paper may be accepted as a definite step toward the goal of determinate run-off values. With it they have provided new tools with which to attack the problem of rainfall-run-off relationships.

The limitations of the rational theory must first be accepted and its field of application defined before it can become the basis of intelligent design. A principal drawback is, that the magnitude and frequency of a storm—the run-off from which brings with it the run-off of previous storms, detained in storage as snow and ice—cannot in any way become the measure of the magnitude and frequency of the resulting flood flow. The writer, therefore, would limit the application of his suggestions in the following discussion to that part of the Eastern United States south of the snow line, and to summer conditions only, for the northern section.

Too little emphasis has been laid on the frequency with which flood magnitude and duration may be expected. This factor becomes the measure of efficiency of any structure, the purpose of which is to remove or pass flood waters. Drainage structures may be divided into two general classes: First, conduits or channels, for the removal of storm run-off; and, second, spillways and important bridge openings.

In economical design the first class demands a balance between cost and capitalized damage resulting from prolonged inundation, the damage being understood to include that to private property, hazard to health, and curtailed convenience to the public.

In addition to property damage the structures of the second class involve hazard to life. In that case design must provide capacity to meet a maximum condition or, in terms of frequency, flood intensities reached or exceeded once in from 100 to 200 years. Design, based on an isolated storm intensity which is classed as "maximum" in a short-term rainfall record, could be dispensed with in favor of direct application of personal judgment to the size of the structure itself.

---

[47] "Frequency and Intensity of Excessive Rainfalls at Boston, Massachusetts," by Charles W. Sherman, M. Am. Soc. C. E., *Transactions*, Am. Soc. C. E., Vol. 95 (1931), p. 954, and the discussion by M. F. Wagnitz, M. Am. Soc. C. E., and L. C. Wilcoxen, Assoc. M. Am. Soc. C. E., p. 964.

[48] Cons. Civ. Engr., Crowley, La.

It is obvious that the frequency of the storm is not necessarily the frequency of the flood produced by it. A highly intense storm of long duration, to be expected not oftener than once in 25 years, may occur at a time following a period of little or no rainfall. Then, especially on open, pervious water-sheds, the demands of the losses to run-off are high, resulting in a low value of the coefficient, C, and a unit run-off that, were flow records available over a sufficiently long period, might be found to have a frequency of once in 5 years. Flood frequency will approach storm frequency, however, because the outstanding storms of record have produced flood flows which must be nearly as infrequent in occurrence.

Failure to advance in this natural science cannot be laid altogether to blind faith in the old and accepted empirical formulas, but in the fact, as the authors have stated, that there has been no uniformity in the conception of assumptions made in the present use of the rational method. Confusion in an understanding of its application to rural areas and stream-flow analysis has arisen where the terms in the equation, $Q = C\,i\,A$, have been given meanings applicable only in the field of storm sewer design, or where the terms have been inadequately or obscurely defined.

Particularly does there seem to be no uniformity in the understanding of the run-off coefficient, C. It is usually defined as the ratio of the rate of run-off to that of rainfall, which may take any of the following meanings:

(*a*) Total run-off to total rainfall for the entire storm period.
(*b*) Maximum rate of run-off to maximum rate of rainfall.
(*c*) Rate of run-off at time of concentration to average rate of rainfall at time of concentration.
(*d*) Maximum rate of run-off to average rate of rainfall throughout a duration period equal to time of concentration.

The rational theory assumes that the rate of travel of the flood crest is the same as that of a particle of water leaving the remote (in terms of time), portion of the water-shed. This assumption may be met in the city storm sewer system, but on the large, agricultural water-shed, it must be modified to recognize a lag between time of concentration, as established by the authors' run-off equation, or by estimated velocities and the arrival of the flood crest. The hydrologist has no interest in the rate of run-off prevailing at the end of the calculated concentration period, but in a maximum rate that will occur later. The writer believes, also, that to burden an already laborious procedure with the complications of non-uniform flow, involved in the travel of the flood crest, is out of the question. Since designers are interested only in maximum discharge and, since it is practicable to determine a concentration period, from determinate factors, representing time in transit from head-waters to concentration point, the coefficient, C, should be understood to be the ratio of maximum discharge to the average rate of rainfall intensity throughout a duration period equal to such time of concentration.

Concentration time should be understood to be a time interval, at the end of which all effective parts of the water-shed are contributing to the flow at the point of concentration. It should be conceived as a time contour, developing around the point of concentration, and progressively receding to the limits of the water-shed. This will exclude artificial reservoirs and appreciable areas of swamp, lake, or marsh, in which the run-off waters because of restricted outlet or other cause, will not reach the point under consideration until the condition of maximum flow has been reached and has passed.

While the theory is stated as a simple basic relationship, $Q = C\,i\,A$, the method of application and the procedure change continuously as the area embraced increases. Conceive of a water-shed, the outfall channel of which heads as a city storm sewer system, discharging into a local stream which, after receiving the drainage waters from an agricultural area, empties into a river which, in turn, becomes the tributary to a still larger stream. Three distinct problems are met involving cases in which run-off is produced: First, by short intense local showers; second, by a single widespread storm or a combination of such storms occurring nearly at the same time; and, third, by the melting of the winter's snow or protracted wet spells, or by their coincident occurrence, over large river basins. In each case the factors affecting the ratio of run-off to rainfall rate, except for a common name and symbol, are different in character and in the degree to which they influence that ratio.

An intense downpour of 200 min. or less, critical to the storm sewer system, becomes only one of several phases of a storm period producing run-off from the larger areas. The average rainfall intensity prevailing throughout the storm phase may not be far less than its maximum in value, while the average of an extended storm period will not approach the storm maximum; and as duration continues it will vary between much narrower limits. It is significant that the expression for rainfall intensity changes rather abruptly at the end of one or two hours. The equation,

$$i = \frac{a}{t + b} \qquad \text{.........................(114)}$$

may be taken as representing the comparatively short period of continuous downpour; and the equation,

$$i = \frac{K}{t^{n}} \qquad \text{.........................(115)}$$

for the longer durations, representing the average of intermittent rainfall throughout the storm period.

Another difference lies in the fact that individual storms may arrange themselves in many ways, peaking in the early stages, in the middle, or at the end of the storm period. An intensity curve group will give the relation between rainfall intensity, duration, and frequency for a locality, but it cannot account for the variation in average intensity at intervals throughout the individual storm.

Fig. 12 shows nine storm curves[49] plotted over the rainfall intensity curve group[50] for Central Illinois. While there is no apparent order in their early stages, they straighten out, after about 400 min., and in general follow the law of the curve group. The shape of the storm curve, materially affecting velocities, then concentration time, and, finally, the selection of the controlling rainfall intensity, is of relatively high importance to the storm sewer system, when concentration periods are short, but it reduces in importance for large areas having concentration time of many hours.

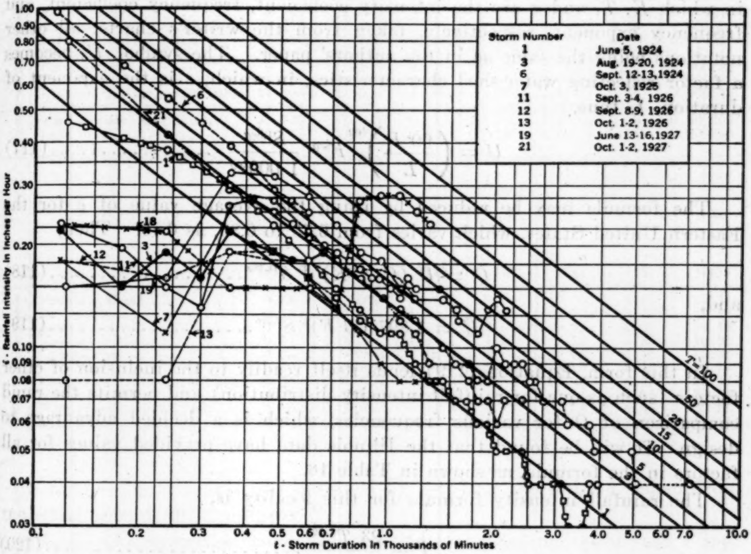


FIG. 12.—COMPARISON OF STORM CURVES WITH RAINFALL INTENSITY CURVE, GROUP FOR CENTRAL ILLINOIS $\left(i = \frac{23\,T^{0.2}}{t^{0.79}}\right)$.

There seems to the writer no justification in extending the use of the rational method, in its present state of development, to flood-flow problems covering several thousand square miles. The treatment of such cases requires a rainfall intensity distribution factor, as yet not fully developed, and the factors affected by topography and soil conditions must become carefully weighted averages. Furthermore, it is probable that the coefficient, *C*, will take an entirely new range of values.

The authors' paper, with the writer's rainfall intensity study covering the Eastern United States,[51] and data, recently gathered in detail for Central

---

[49] "Run-Off Investigations in Central Illinois," U. S. Bureau of Public Roads, and *Bulletin 232*, Univ. of Illinois Eng. Experiment Station, 1931.

[50] See p. 617 *et seq.*

[51] See p. 592, and subsequent discussions.

Illinois,[49] presents an opportunity to develop a complete example of the application of the rational theory to agricultural drainage and stream-flow problems.

For the Eastern United States and for duration periods greater than 1 hour, the general formula for Q (Equation (24)), may be expressed as:

$$Q = U\ (C\ A\ K)^{4g}\ T^{4xg} \qquad \text{(116)}$$

in which K, T, and x are the intensity coefficient, frequency coefficient, and frequency exponent, respectively, taken from the writer's charts, all other notations being the same as in the authors' paper. The symbol, U, becomes a factor reflecting water-shed characteristics, in which, e is the exponent of duration, t, thus,

$$U = \left(\frac{60\ P}{L}\right)^{4eg} F^{8eg}\ \frac{S^{1.5eg}}{1\ 000^{2eg}} \qquad \text{(117)}$$

The formula may be reduced by using the average value of e for the Eastern United States, which varies from 0.75 to 0.83, as 0.80. Then,

$$Q = U\ (C\ A\ K)^{1.25}\ T^{1.25x} \qquad \text{(118)}$$

and,

$$U = 1.897\ (B\ F)^{2}\ S^{0.375} \qquad \text{(119)}$$

In this form, Equation (119) lends itself readily to the inclusion of other factors (such as one for rainfall intensity distribution) and permits the rapid comparison of Q for various frequencies, which is a decided advantage to design. It will be found that the Illinois data have provided values for all factors in the formula, as shown in Table 16.

The rainfall intensity formula for this locality is,

$$i = \frac{23\ T^{0.30}}{t^{0.79}} \qquad \text{(120)}$$

and the run-off equation is,

$$Q = U\ (C\ A\ K)^{1.246}\ T^{0.374} \qquad \text{(121)}$$

in which,

$$U = \left(\frac{60\ P}{L}\right)^{0.984} F^{1.969}\ \frac{S^{0.369}}{29.9} \qquad \text{(122)}$$

Transposing,

$$C_a = \left(\frac{Q}{U\ (A\ K)^{1.246}\ \ T^{0.374}}\right)^{0.803} \qquad \text{(123)}$$

It will be found that, as long as T and i are in accord with the law of the curve group, the time of concentration will remain constant. The values in Columns (5), (6), (7), (8), (9), (12), (13), (14), (15), and (16), in Table 17, were obtained by the following procedure:

First.—A value of the drainage coefficient for $T = 1$ year was calculated by Equation (123);

TABLE 16.—DATA COMPILED FROM RUN-OFF INVESTIGATIONS IN CENTRAL ILLINOIS

No. (1)	Water-Shed (2)	Area, in acres (3)	Maximum channel length, in feet (4)	Channel slope, in feet per 1 000 ft. (5)	W (6)	$\frac{L}{W}$ (7)	P_1 (for uniform rainfall intensity) (8)	$\frac{60\ P_1^{0\cdot984}}{L}$ (9)
1	Fountain Head	2 982	22 700	4.65	5 720	3.975	0.49	0.00141
2	St. Joseph	3 315	17 400	2.27	8 380	1.743	0.44	0.00150
3	Lateral No. 15	7 360	41 200	0.85	7 790	5.285	0.50	0.000819
4	Kaskaskia Mutual	7 680	28 500	4.36	11 720	2.435	0.45	0.00101
5	Camp Creek	9 792	42 200	3.03	10 100	4.185	0.49	0.000784
6	Two-Mile	9 792	35 900	2.08	11 870	3.025	0.47	0.000833
7	Embarrass River, east of Tolon	17 408	50 700	2.08	14 960	3.427	0.48	0.000642
8	Kaskaskia River	36 864	90 800	1.70	17 690	5.065	0.50	0.000376
9	West Branch, Salt Creek	38 720	83 400	0.87	20 220	4.120	0.49	0.000402
10	Lake Fork	40 448	100 800	0.66	17 550	5.721	0.51	0.000346

No.	Water-Shed	P_2 (for rainfall intensity varying with $t^{0\cdot5}$) (10)	$\frac{60\ P_2^{0\cdot984}}{L}$ (11)	Side slopes (12)	Bottom to depth, ratio (13)	Kutter's n (14)	F (15)	$F^{1\cdot984}$ (16)
1	Fountain Head	0.55	0.00162	1 on 3	1 : 1	0.03	4.83	22.16
2	St. Joseph	0.48	0.00183	1 on 3	1 : 1	0.10	3.08	9.14
3	Lateral No. 15	0.57	0.000933	1 on 1	1 : 1	0.05	4.65	20.56
4	Kaskaskia Mutual	0.51	0.00120	1 on 1	1 : 1	0.10	3.32	10.60
5	Camp Creek	0.55	0.000881	1 on 2	2 : 1	0.05	4.45	18.86
6	Two-Mile	0.53	0.000994	1 on 2	2 : 1	0.10	3.18	9.74
7	Embarrass River, east of Tolon	0.54	0.000722	1 on 2	2 : 1	0.03	4.97	23.44
8	Kaskaskia River	0.57	0.000425	1 on 2	2 : 1	0.04	4.65	20.56
9	West Branch, Salt Creek	0.55	0.000451	1 on 2	2 : 1	0.04	4.65	20.56
10	Lake Fork	0.58	0.000395	1 on 2	2 : 1	0.09	3.40	11.11

Second.—The 1-year i was obtained by dividing the unit, q, taken from the storm record, by the 1-year C;

Third.—Concentration time was established by taking the 1-year i into the curve group to an intersection with the 1-year frequency curve;

Fourth.—The average rainfall intensity for the duration equal to concentration time, as determined by the first, second, and third steps, was taken from the individual storm curve;

Fifth.—This intensity, taken into the curve group with its time of duration, established its frequency; and,

Sixth.—The unit, q, was divided by the prevailing i to arrive at the actual value of the drainage coefficient, shown as C_a in Columns (9) and (16), Table 17, for each storm and water-shed.

Comparative results, using two values of the factor, P, are given in Table 17, the former, with P_1, assuming rainfall intensities as uniformly distributed, and the latter, with P_2, assuming rainfall intensities as following the curve of the type, $i = \frac{K}{t^{0\cdot5}}$.

It would appear to the writer that opportunity for a condition to occur in accordance with the use of P_2, that is, a storm that comes to a peak in

TABLE 17.—COMPARATIVE EFFECT OF P_1 AND P_2 ON OTHER FACTORS THAT AFFECT RUN-OFF

Water-Shed No. (1)	Storm No. (2)	Date (3)	Max. q (4)	t, min. (5)	(6)	T_l (7)	U (8)	C_a (9)	s (10)	C_b (11)	t, min. (12)	i (13)	T (14)	U (15)	C_a (16)	s (17)	C_b (18)
1.......	2	Oct., 1926	0.0450	179	0.300	0.5	0.001830	0.150	0.140	0.151	157	0.344	0.5	0.002105	0.133	0.186	0.133
1.......	3	June, 1924	0.1340	137	0.485	1.0	0.001830	0.276	0.012	0.277	118	0.480	0.8	0.002105	0.279	0.020	0.278
2.......	4	Oct., 1925	0.0118	730	0.128	1.1	0.000624	0.092	0.086	0.093	600	0.160	1.4	0.000756	0.074	0.120	0.073
2.......	5	Sept., 1926	0.0202	655	0.181	2.5	0.000624	0.112	0.109	0.113	525	0.203	2.1	0.000756	0.100	0.130	0.104
2.......	6	Oct., 1926	0.0299	580	0.280	8.0	0.000624	0.106	0.175	0.107	480	0.330	8.3	0.000756	0.090	0.225	0.092
3.......	7	Sept., 1926	0.0512	485	0.180	1.2	0.000529	0.284	0.000	0.284	425	0.185	0.9	0.000604	0.277	0.008	0.278
3.......	8	Oct., 1927	0.0153	670	0.082	0.2	0.000529	0.187	0.028	0.187	580	0.093	0.2	0.000604	0.165	0.040	0.165
3.......	9	Oct., 1926	0.0562	480	0.240	2.8	0.000529	0.234	0.041	0.235	420	0.240	2.0	0.000604	0.234	0.045	0.236
4.......	10	June, 1924	0.0231	510	0.238	3.1	0.000616	0.097	0.155	0.099	425	0.295	4.2	0.000733	0.079	0.212	0.081
4.......	11	Oct., 1926	0.0222	510	0.195	1.6	0.000616	0.114	0.116	0.115	430	0.202	1.3	0.000733	0.110	0.122	0.113
4.......	12	Aug., 1924	0.0285	480	0.150	0.6	0.000616	0.190	0.049	0.191	400	0.150	0.4	0.000733	0.190	0.050	0.193
5.......	13	June, 1924	0.0168	420	0.325	5.6	0.000747	0.052	0.265	0.051	375	0.360	5.6	0.000839	0.047	0.302	0.047
5.......	14	Oct., 1927	0.0228	390	0.285	2.9	0.000747	0.080	0.204	0.081	350	0.300	2.6	0.000839	0.076	0.220	0.077
5.......	15	Aug., 1924	0.0264	380	0.153	0.4	0.000747	0.173	0.059	0.174	340	0.162	0.3	0.000839	0.163	0.071	0.163
6.......	16	Sept., 1926	0.0154	940	0.155	3.7	0.000350	0.100	0.100	0.101	780	0.122	1.1	0.000423	0.126	0.068	0.128
6.......	17	July, 1927	0.0146	945	0.175	5.9	0.000350	0.083	0.122	0.086	790	0.240	10.5	0.000423	0.061	0.188	0.063
6.......	18	Oct., 1926	0.0311	790	0.130	1.4	0.000350	0.239	0.020	0.240	650	0.143	1.2	0.000423	0.217	0.038	0.213
7.......	19	June, 1924	0.0230	390	0.300	3.5	0.000658	0.077	0.219	0.077	355	0.321	3.5	0.000740	0.072	0.288	0.073
7.......	20	July, 1927	0.0437	330	0.280	1.8	0.000658	0.156	0.126	0.156	295	0.230	0.7	0.000740	0.190	0.076	0.190
7.......	21	Oct., 1926	0.0671	305	0.359	3.2	0.000658	0.187	0.120	0.189	262	0.410	3.3	0.000740	0.164	0.171	0.165
8	22	June, 1927	0.0150	750	0.125	1.0	0.000315	0.120	0.071	0.123	665	0.132	0.9	0.000357	0.114	0.080	0.114
8.......	23	July, 1927	0.0197	700	0.338	25.0	0.000315	0.058	0.269	0.058	620	0.326	15.0	0.000357	0.060	0.258	0.161
8.......	24	Oct., 1926	0.0296	640	0.140	1.0	0.000315	0.211	0.036	0.211	560	0.140	0.7	0.000357	0.211	0.039	0.209
9.......	25	June, 1924	0.0238	800	0.143	1.9	0.000262	0.166	0.059	0.167	705	0.153	1.8	0.000294	0.156	0.070	0.157
9.......	26	Sept., 1926	0.0128	950	0.162	4.6	0.000262	0.079	0.116	0.081	840	0.180	4.8	0.000294	0.071	0.135	0.072
9.......	27	Oct., 1926	0.0252	790	0.128	1.4	0.000262	0.197	0.039	0.197	700	0.143	1.4	0.000294	0.176	0.053	0.182
10.......	28	June, 1924	0.0078	2 530	0.040	0.6	0.000109	0.195	0.012	0.199	2200	0.048	0.7	0.000125	0.162	0.021	0.163
10.......	29	Oct., 1926	0.0178	2 070	0.084	4.0	0.000109	0.213	0.021	0.213	1800	0.032	4.3	0.000125	0.188	0.030	0.180

the first 20 to 60 min. of its duration and progressing down stream (thus shortening concentration time by building up high unit discharges and velocities at the head-waters), will be much more infrequent than an average condition, represented by P_1, the assumption of a uniform intensity over the water-shed above the point of concentration. Table 17 compares the effect of P_1 and P_2 on the other factors affecting run-off.

The rainfall intensity, i, the rate at which water is applied to an area as rain, is distributed as shown in Fig. 13.

The run-off, Q, is divided into that portion which will reach the point of concentration in time to contribute to maximum flow and that which is

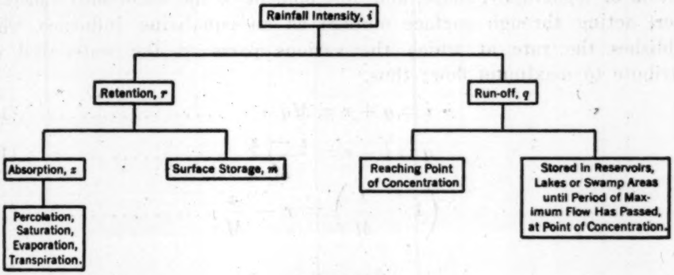

Rainfall Intensity, i
Retention, r
Run-off, q
Absorption, z
Surface Storage, m
Reaching Point of Concentration
Stored in Reservoirs, Lakes or Swamp Areas until Period of Maximum Flow Has Passed, at Point of Concentration.
Percolation, Saturation, Evaporation, Transpiration.

FIG. 13.

delayed, as storage in lakes, swamps, or reservoirs, until after the condition of maximum flow is past. The latter becomes a direct deduction from effective area.

The retention, r, is a factor embracing all other losses to run-off. It is made up of a factor, z, called absorption, and a retardation factor, m, which might be referred to as surface storage.

The factor, z, is a function of atmospheric conditions and rainfall intensities throughout the critical period of the storm and of the amount and rate of rainfall immediately preceding. It also reflects the capacity of the soil to pass water to underground storage, as percolation, or to retain it as absorption. The formula,

$$i = q + r \qquad (124)$$

is a straight-line relationship in which slope, and, therefore, the value of C, is unity. This is demonstrated by plotting i with q, and projecting the points with slope = 1, to the i-axis, where the intercepts will become the values for r.

If a point group is enveloped by a curve having a slope, M, and the points are projected with that slope to the i-axis, the intercepts, z, measuring absorption, will result; thus,

$$i = Mq + z \qquad (125)$$

$$q = \frac{1}{M}(i - z) \qquad (126)$$

and,

$$C = \frac{q}{i} = \frac{1}{M}\left(1 - \frac{z}{i}\right) \dotfill (127)$$

The factor, $\frac{1}{M}$, or $C_{\max.}$, becomes constant for, and the maximum drainage coefficient on any water-shed is established by, its topography, type of soil, vegetable cover, and degree of development. This factor, modified by the combined effects of i and z, becomes the coefficient, C, for a particular storm.

The retardation factor, m, herein classified as surface storage, is represented by the difference between unity and $C_{\max.}$ for any water-shed. It is a function of topography, shape, and development of the water-shed and is the factor, acting through surface storage as an equalizing influence, which establishes the rate at which the various parts of the water-shed will contribute to maximum flow; thus,

$$i = q + r = Mq + z \dotfill (128)$$

$$q = i - r = \frac{i - z}{M} \dotfill (129)$$

$$i\left(1 - \frac{1}{M}\right) = r - \frac{z}{M} \dotfill (130)$$

and,

$$1 - \frac{1}{M} = \frac{r}{i} - \frac{z}{Mi} \dotfill (131)$$

That m (or $1 - \frac{1}{M}$, or the ability of a water-shed to delay the delivery of run-off water to the point of concentration), is a function of the slope, M, and not a deduction factor, as z, is illustrated in the rainfall and run-off studies,[58] by Ivan E. Houk, M. Am. Soc. C. E. Bare soil, relatively incapable of retarding run-off is shown to have slope, M, very little different from 1, or to have a low value of $1 - \frac{1}{M}$, while the plots in which the soil was spaded and raked, representing a surface condition conducive to storage, showed a correspondingly high value of m.

Where water-sheds are closely grouped and where conditions are similar, a composite point group may be enveloped by a curve that will apply to the individual water-sheds, as shown in Figs. 14 and 15.

Using values of P_1,

$$C_b = 0.284\left(1 - \frac{z}{i}\right) \dotfill (132)$$

and with P_2,

$$C_b = 0.287\left(1 - \frac{z}{i}\right) \dotfill (133)$$

Values for C_b, for Central Illinois, are listed in Columns (11) and (18) of Table 17.

---

[58] Technical Reports, Miami Conservancy Dist., Pt. VIII, Fig. 24, p. 124.

Comparison is made with the rainfall-run-off studies[53] of C. E. Ramser, M. Am. Soc. C. E., in Western Tennessee. Values of $C_{max.}$ for the individual water-sheds are compared with the composite value in Fig. 15.

Rainfall-run-off measurements made by the writer at Crowley, La., are shown in Table 18. The resulting value of $C_{max.}$ is 0.52. These water-sheds, all uniformly flat, are almost identical as to soil type, which is a dense, impervious clay.

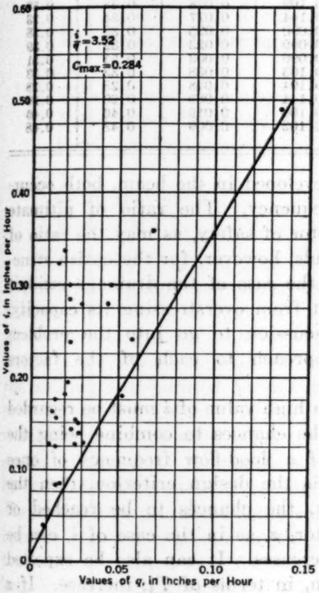


FIG. 14.

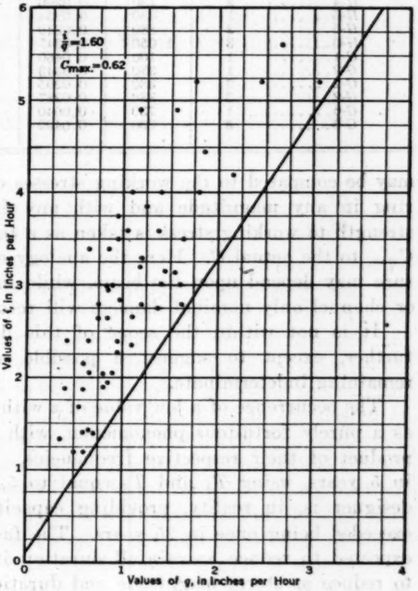


FIG. 15.

From recorded data presented in this discussion, values have been given to all factors affecting run-off, from eight storms over ten water-sheds. It has been shown that, from data made available by any reasonably accurate topographic survey (except the direct measurement of Kutter's *n* to be found in the Illinois data, which would ordinarily be assumed), all factors except *z* and $C_{max.}$ may be given determinate values. The writer has expressed *Q* in terms of storm frequency, but not flood frequency, and to that extent has fallen short of the goal of direct flood-flow determination.

The tendency in design will be to use $C_{max.}$ as the drainage coefficient, considering the margin between it, and the values actually developed as a factor of safety. This suggests an analogy between the design of hydraulic structures to remove storm run-off and the design of structural members, in that $C_{max.}$

---

[53] "Run-Off from Small Agricultural Areas," *Journal of Agricultural Research*, Vol. 34, No. 9, p. 799.

may be compared to the ultimate strength of a beam, the former being fixed by features of topography, soil, and cover of the water-shed, and the latter by its material, shape, and size. The actual C, resulting from any storm, may be compared to the working stresses developed in the beam, both occurring in any magnitude and with any frequency. The ratio of ultimate strength to working stress is taken as a factor of safety, as may the ratio of $C_{max.}$ to the actual C. Here the analogy ends, however, for the entire structure may depend upon the beam, while in the case of the drainage conduit or channel only nominal damage will result from overstressing its capacity.

TABLE 18.—RAINFALL-RUN-OFF MEASUREMENTS, AT CROWLEY, LA.

Water-Shed	Storm	t, min.	Max.q	i	z	C_a	C_b
H-4	1	730	0.0490	0.139	0.046	0.35	0.35
H-4	2	730	0.0390	0.102	0.028	0.38	0.38
H-5	1	880	0.0354	0.144	0.077	0.25	0.23
H-5	2	880	0.0340	0.090	0.025	0.38	0.38
H-6	3	1 050	0.0347	0.090	0.023	0.39	0.39
G-1	1	100	0.0460	0.090	0.003	0.51	0.51
G-2	1	380	0.0343	0.103	0.038	0.33	0.33
G-2	2	380	0.0295	0.104	0.048	0.28	0.28
G-2	3	380	0.0287	0.144	0.089	0.20	0.20
G-6	1	730	0.0640	0.138	0.016	0.46	0.46
G-6	3	730	0.0490	0.102	0.009	0.48	0.48

It is not within the scope of this discussion to go into the problem further, except to suggest a possible approach to each of the factors remaining indeterminate.

The occurrence of a low value of z with a high value of i must be regarded as a purely fortuitous phenomenon, with the chances to combine being the product of their respective frequencies. If a flood-flow frequency of once in 5 years, using T_i and T_z equal to 5, is the design criterion, then the designer is, in reality, providing capacity, the chances to be reached or exceeded being once in 25 years. The factor, z, as in the case of i, can be expected to reduce as rainfall duration increases. It can also be expected to reduce as storm magnitude and duration, in terms of T_i, increase. If z and i, in terms of T_i and T_z, are combined in Equation (127) as $\sqrt{T_i}$ and $\sqrt{T_z}$, the resulting value of C, by reducing the needlessly large safety factor involved in the use of $C_{max.}$, will produce a more balanced ratio between efficiency and economy in design and will give values of Q having an approximate flood frequency, T_f. At least, the design will have taken cognizance of the fact that the extreme values of i and z combine infrequently.

It is reasonable to expect low values of z during or immediately following an extended wet spell, disregarding, of course, winter storms on frozen watersheds. What constitutes (in terms of frequency, intensity, and duration) a storm period to produce maximum saturation is not known, but an approach to it may lie in the fact that the expression for rainfall intensity, for the Eastern United States, $i = \frac{K\,T^x}{t^n}$, may be extended to duration periods of thirty days.[84]

---

[84] See pp. 620–621, Table 7.

Apparently, the range in the value of $C_{\max.}$, for topographic and soil conditions to be found on the ordinary agricultural water-shed, is from 0.28 to 0.65. Tentative deductions from unity, C, to be added as the influences of topography, soil, and cover combine—and given as a guide to values of $C_{\max.}$ for agricultural water-sheds—are, as follows:

Topography.—	
Flat land, with average slopes of 1 ft. to 3 ft. per mile....	0.30
Rolling land, with average slopes of 15 to 20 ft. per mile..	0.20
Hilly land, with average slopes of 150 to 250 ft. per mile..	0.10
Soil.—	
Tight, impervious clay..........................	0.10
Medium combinations of clay and loam...............	0.20
Open, sandy loam..........................	0.40
Cover.—	
Cultivated lands	0.10
Woodland	0.20

CHARLES H. LEE,[55] M. AM. SOC. C. E. (by letter).—This paper represents a thorough attempt to analyze the rational method of estimating flood discharge, particularly as applied to unimproved drainage areas. The rational method was originally introduced as an improvement in estimating maximum storm run-off from small urban and suburban areas. As has been stated by the authors and pointed out by the writer,[56] it is also applicable to drainage areas of natural streams, where it has practical application in the design of waterway areas for stream-bed structures, such as culverts, bridges, spillways, overflow dams, etc. The authors have made a real contribution to the subject in their searching and logical analysis of the "detail method" of application. They have also presented some very suggestive material in Part II, but it is open to question whether as yet it is a step in advance to attempt to apply the rational method as a formula. The element of judgment is still present both in the application of the method as well as in the choice of basic values for a given drainage area. Under such conditions no formula can be devised that would be "fool proof." It is not believed that such a formula would be much of an improvement over the use of empirical formulas.

There is an important difference in the application of the rational method to the design of storm sewers or long storm drains and to stream-bed structures. To construct the former of a capacity sufficiently large to carry the maximum possible storm discharge would ordinarily be prohibitive in cost, and it is customary to design for maximum storms likely to occur once in an assumed period of years. In the case of bridges, culverts, spillways, etc., the relative cost of the waterway area is small compared to the cost of the entire structure, and it is usually economic to provide for the absolute maximum flood discharge. This fact suggests the importance of thorough consideration of factors other than rainfall intensity, which may contribute to the occurrence of a maximum when applying the rational method to unimproved drainage areas. The mention of a few instances may be in order as a part of the discussion of the authors' paper.

---

[55] Cons. Hydr. Engr., San Francisco, Calif.

[56] *Transactions*, Am. Soc. C. E., Vol. 93 (1929), p. 488.

For small drainage areas in arid or semi-arid regions subject to local storms of great intensity known as cloudbursts, special consideration should be given to the value of *C*. Soils behave quite differently in their ability to absorb surface water, depending on their initial condition of moisture. This condition may be: (*a*) At saturation with complete filling of all voids; (*b*) at field capacity with partial filling of voids after all gravitational water has drained out; (*c*) at the point (wilting coefficient) where all moisture has been removed so that vegetation wilts; or (*d*) with all except hygroscopic moisture removed. In a dry arid region during the hot summer months the quantity of hygroscopic moisture is small and there may be a high degree of desiccation of the soil to considerable depth.

For moisture conditions between field capacity and saturation and with good drainage, rainfall upon the surface is readily absorbed and passes downward through the soil column. For moisture content less than field capacity the soil retains added moisture until that capacity is reached, and progressive downward penetration occurs only as field capacity is exceeded in the soil column above. For highly desiccated soils this retarding action becomes a positive resistance to wetting, illustrated by the action of single raindrops falling upon a dusty road, or the addition of moisture to dry peat. A rainfall of great intensity, falling upon the surface of an extremely dry soil column, may thus encounter an actual resistance to absorption, producing immediate saturation at the surface and run-off, with dry soil below.

In addition to this surface tension phenomenon, there may also be resistance to penetration produced by trapped air in the soil. As the solid water represented by field capacity varies roughly from ½ in. to 1 in. per ft. of sandy to silty loam soil, it is obvious that a highly desiccated soil column of several feet might maintain this condition for the duration of the storm, with a value of *C* of nearly 1.00. On the other hand, if initially at field capacity, the same column would immediately commence the downward transmission of water absorbed at the surface with a value of *C* nearer 0.50. These considerations are especially important in a cloudburst country, such as that traversed by several of the transcontinental railways, and the long water supply conduits common in the West. The writer has also noted their importance in the coast region of Central California, where early fall rains following an excessively dry season have produced higher rates of flood run-off into storm sewers from small unimproved areas than storms of equal intensity occurring during the winter.

Among other conditions that may produce maximum flood flows exceeding those estimated by a routine application of the rational method may be mentioned frozen ground, warm rain falling upon snow, the effect of fire in destroying the vegetation mat, and the natural flow in the stream from a preceding storm at the beginning of a storm of maximum rainfall intensity. The authors considered the condition of saturation of the surface from an immediately preceding storm, but did not mention the possibility of there being appreciable flow already in the stream. The theoretical absolute maximum resulting from the latter condition is double the value computed, assuming that a second storm of equal intensity and duration followed immediately after the storm for which the computations were made. An actual instance for which the data were analyzed by the writer indicated a discharge from a small drainage area 80% greater for a second storm immediately following an earlier one of about the same intensity.[57]

These instances illustrate the importance of a broad consideration of all factors that may enter the problem where the rational method is being applied to estimate maximum flood flow as a basis for designing stream-bed structures.

R. L. GREGORY,[58] and C. E. ARNOLD,[59] ASSOCIATE MEMBERS, AM. SOC. C. E. (by letter).—Briefly, the tenor of the discussions may be summarized in four general observations, which follow:

(1) No errors in conceptions, conclusions, statements, suggestions, or analyses were proved to exist in the paper.

(2) All discussers agree that the proposed run-off formulas are rational and logical in derivation.

(3) While there is some disagreement as to the degree of usefulness of the proposed formulas, many of the discussions display a keen appreciation not only of the value of the formulas, but also of the paper as a whole.

(4) A few of the discussers profess to favor either methods of their own, or existing formulas and systems, in all cases; while others would use the proposed formulas, or a more detailed process, according to the requirements of the problem in question.

Any one, in reading the paper, should form the definite conclusion that the subject-matter therein was presented without fear, favor, or undue feeling. Severe criticisms and analyses of existing data and methods were made in the full anticipation of vigorous counter-attacks from many engineers, who had previously advanced, or openly approved, conceptions, or formulas, which the contents of the paper prove to be erroneous or inadequate.

It is, however, not surprising that analysis was not resorted to by these individuals, such generalities as, "the writer prefers," or, "in the writer's opinion," or, a brief preface to a more or less lengthy résumé of former writings, being used instead. Generalities are insufficient when dealing with fundamentals of an involved subject. The foregoing observations, (1) and (2), still hold.

On the other hand, some excellent discussions came from engineers, who, in addition to making an honest effort to understand the subject as presented, were possessed of sufficient magnanimity of spirit, sense of duty, energy, and courage, to point out (in a work, for which others were responsible) many parts of the paper, which to them seemed most pertinent and worthy of study and use. The profession owes its advancement to these individuals, and others of their kind, who, not content to confine their efforts to their own contributions, unselfishly encourage and further the development of constructive thought and methods regardless of source, making thereby fruitful use of their talents, and fulfilling the inevitable responsibilities which accompany such gifts.

---

[57] *Transactions*, Am. Soc. C. E., Vol. 93 (1929), p. 439.

[58] Chf. Insp., Los Angeles County Storm Drain Div., Los Angeles, Calif.

[59] Los Angeles, Calif.

Many and varied conflicting statements and opinions are contained in the group of discussions—some are diametrically opposed, the one to the other; some are not pertinent to the subject; others convey a lack of proper conception, with a consequent tendency to mislead; while still others are conducive to a better understanding.

After a careful reading of the discussions in conjunction with a review of their paper, the writers, in contemplating the probable state of mind of the reader who has had the fortitude to attempt a digestion of all the material submitted, can visualize a distinct tendency toward confusion. They realize that time and space will not permit discussion of all issues raised, but shall endeavor, subsequently, to cover only the more important points, and to submit material which, it is believed, will be helpful.

There is an expressed (or intimated) belief in more than one discussion that the writers intended, in their paper: (*a*) To disestablish the rational method; and (or) (*b*), to establish formulas which could cover all phases of an involved problem at one stroke and be used by the designer without any qualifications or limitations. Neither intention was, or is, possessed by the writers; nor does the paper, in any part thereof, if read thoroughly, convey the thoughts.

The writers did, painstakingly, attempt the analysis of the rational method, as practiced by many; and pointed out in no uncertain terms certain limitations, fallacious assumptions, and errors thereof, some of which were commented on and thoroughly appreciated by Mr. Reeves, and others, as shown in their discussions.

The run-off formulas, presented by the writers as a logical sequence of their analyses, rest upon the solid foundation of Equation (1). This equation is axiomatic—irrefutable—and does not belong solely to the rational method, as practiced. All run-off formulas and methods, if logical, must embody its principles. Any one of the three factors, C, i, and A, is subject to an infinite number of variations. The writers were not unmindful of these possible variations (see "Introduction," "Use of Rainfall Curve," "Conclusions" (1) to (4), inclusive, "Statement," and "Factor C, the Run-Off Coefficient," in Part I). So many re-statements of the writers' preliminary remarks (or corollaries thereof), regarding the possible variations of storms, occur in some of the discussions, that one may easily be led to believe that the writers really think that all rains follow the precise assumptions set up in the rational method.

All agree that a storm may, or may not, cover the entire water-shed throughout the time of concentration thereof; that its rates of precipitation may be more intense on one part of the area than on another; that its vortex may travel in any direction, up, down, or across; and that rainfall may even be discontinuous, or intermittent, during any time interval. In the face of so many possible variations, some assumption must be made by any one when attempting to solve a run-off problem, and, by all means, the most logical and useful assumption should be made when evolving a formula. The writers were not secretive about their choice (see sub-headings, "Use of Rainfall Curve," and "Statement" under main heading, "The Rational Method," in Part I). Observe, also, that certain limitations of the rational method are recorded (see "Remarks," under the heading, "Suggestions for Use of Formulas," in Part II). That the formulas were developed logically was admitted by practically all the discussers and may be verified by a study of the paper.

While the intensity-time-frequency curves for use with the method (see "Rainfall Curves," "Use of Rainfall Curve," and "Statement," in Part I) likewise carry inherent limitations (that is, due to construction, they set up a probable average rate of precipitation for any given time interval), they do furnish the only feasible basis yet devised for forecasting frequency periods of the average storm and correlating the time of concentration of a given water-shed (or part thereof) with an expected intensity.

Some engineers prefer to divide the water-shed into time zones for the purpose of applying varying intensities to the different sub-areas thus formed. For this purpose the special assumptions (in regard to intensities) of the rational method can be utilized as a forerunner. The proposed run-off formulas are especially adapted, since approximate times of concentration for the water-shed (or any sub-water-shed) may be found quickly through their use. For approximate purposes the solution for a water-shed may be made in one operation, if a weighted value of R_H (weighted by the sub-areas in much the same manner as outlined under the heading, "Special Case," in Part II), is obtained for use in the formulas. In the light of present information, however, the necessity for, or desirability of, so dividing the area is questionable (see "Statement," under "The Rational Method," in Part I).

It should be emphasized, however, that the use of the formulas is not limited to the strict assumption of uniformity for the intensity, but requires the consideration of an average value for it. The use of Table 3 constitutes a limit to uniformity, only in so far as the values of the factor, P, influence times of concentration.

A study of the paper should make it evident that the writers, instead of trying to disestablish the rational method, have not only presented analysis, which should help to clarify many assumptions and practices, but have contributed to its advancement into further fields of usefulness.

The run-off formulas are capable of being used with any type of rainfall curve (or group of curves), regardless of shape or frequency period (see headings, "Special Reduction of Equation (24)," and "Remarks Concerning Examples 6, 7, 8, and 9," in Part II; and refer, by all means, to "General Solution by Equation (29)," presented subsequently in this discussion). That the formulas may be logically applied to any size or type of water-shed (whether or not it happens to be a part of another area, and regardless of the shape and condition of the main channel of flow) follows naturally from their derivation. Furthermore, it should be observed that the proposed run-off formulas, are the only ones in existence, which afford: (1) A means of arriving readily and reasonably at the time of concentration of an area (or any sub-water-shed thereof), enabling, thereby, the ascertainment of the probable average intensity for any frequency period, which when necessary may be studied in turn in more detail as to distribution or effect; and (2), facilities for consideration of all factors resulting from definite assumptions.

Although their evolution is somewhat formidable, the formulas are complicated only in derivation (see "Suggestions for Use of Formulas, Method No. 2," in Part II, and the splendid discussions by Messrs. Longacre, Kemmerer, and others).

A statement, which contains a depth of meaning and is well worth repeating, was made in the excellent discussion by Mr. Arledge, as follows: "The future worth of the formulas will depend largely on the amount of intelligent use to which they are subjected." When and how to use them must be determined, to a great extent, by the ingenuity of the engineer in solving the individual problem confronting him. Their capabilities of use will increase as more extensive experimentation by the profession makes available more accurate data for selection.

It is easier, of course, to solve such formulas as, $Q = C_F \sqrt{A}\, R$; $Q = C_1 M^{\frac{3}{4}}$; $Q = 210\, M^{\frac{2}{3}}$; or, $Q = 10\,000 \times 80\% \sqrt{A}$, which were presented in the discussions by Messrs. Sherman, Ryves, Hickox, and Jarvis, respectively.

These and other similar equations have little in common with the formulas and methods proposed by the writers; direct comparisons therewith can not be made. In such formulas the run-off is approximated through a multiplication of the area (raised to some exponent) by a coefficient. Necessarily, many variables are covered by a single constant, even if the use of one of the equations be confined to a single locality. Such formulas approach closely, indeed, Equation (1), with which the writers began. The question naturally arises, why not use Equation (1) as a basis for approximations through comparisons? Why complicate it by an exponent which must change in value and meaning for the many unseen variations in missing factors?

It is true that much good information on flood flows of streams has been collected by engineers. The writers do not wish to detract from its value by any statement or act. Such collections should continue to be made and studied in the future. Hydrographs, high water marks, and other data should be obtained where possible. When a complete hydrograph, which conforms to the flood to be provided for, is available, the problem is solved—no run-off formula is needed. Seldom, if ever, is this condition found to exist in recognizable form. Proper records of precipitation for large floods are almost always lacking, and, for the smaller flows, sufficient data are seldom available. If hydrographs, together with corresponding information on precipitation, are available, there still arises the difficulty of ascertaining the flow for storms other than those which produced the recorded discharge.

The development of intensity-time-frequency curves is due, directly, to the need of other means than direct measurements, to assist in solving the run-off problem. These curves, however, could be of no avail without means of ascertaining, and correlating with their proper time intervals, times of concentration of water-sheds. Equation (1), alone, or with the area raised to an exponent, could not be of any use in making determinations of times of concentration without resorting to hypothetical reasoning.

The fundamentals of the hypotheses, on which the writers' formulas are based, are not altered by a change in the size of the channel of flow. Such arbitrary limits, as "500 sq. miles," or "artificial channels," may be recognized as applicable only when there exists a lack of proper conceptions, or ingenuity, on the part of the engineer, or inadequate available data, or combinations thereof.

The formulas, however, are not recommended to be used without the exercise of judgment. It may be necessary in some cases to alter times of concentration and resulting intensities, due to the effects of certain "restrictions and pondage." Judgment will always be needed with any method, system, or formula, but no *bona fide* assistance in the solution of the problem should be rejected.

Specific Remarks.—After rendering various opinions concerning the proposed formulas and giving a rather lengthy discourse on soil characteristics in relation to the run-off coefficient, Mr. Lee makes a number of inaccurate and misleading statements. For instance:

> "The authors considered the condition of saturation of the surface from an immediately preceding storm, but did not mention the possibility of there being appreciable flow already in the stream. The theoretical absolute maximum resulting from the latter condition is double the value computed, assuming that a second storm of equal intensity and duration followed immediately after the storm for which computations were made."

It should also be stated that the writers considered many other "conditions" than "saturation of the surface from an immediately preceding storm" (see "Factor *C*, The Run-Off Coefficient" in Part I) and that the assumptions and hypotheses, on which both the rational method and the proposed formulas are based, do not eliminate from consideration "the possibility of there being appreciable flow already in the stream." Assume, for simplicity of visualization, that the run-off coefficient is constant, and that a storm of uniform intensity occurs over the entire area throughout its time of concentration. The theoretical rate of discharge at the outlet will increase, from zero at the beginning, more or less gradually (depending on the manner of concentration) to a peak at the end of the time of concentration. Consider, further "that a second storm of equal intensity and duration" (and similarly covering the area) follows immediately after the first (that is, the rate of precipitation is continuous). The "theoretical absolute maximum" rate of discharge at the outlet will be identical with (not double) that of the peak of the first storm, and will be a continuation thereof until the end of the precipitation. Much water will be in the channels at the beginning of the second storm, increasing in quantity from zero at the remotest part of the area to a maximum at the outlet.

Of course, in an actual stream, there may be water running due to springs, seepage, or other causes. If so, provision for it may be made, but the illustration suffices to disclose the error in Mr. Lee's statement. He refers to a former writing,[60] in which occurs a listing of various conditions affecting the value of the run-off coefficient. No where in this listing, nor in the curves and diagrams of Mr. Sherman's former contribution,[61] together with accompanying recommendations for their use, is there evidence of any provision for the utilization of the principle of Conclusion 12, ("Factor *C*, The Run-off Coefficient," in Part I), one of the most important considerations of the run-off problem. With this in mind the first three paragraphs of Mr. Reeves' discussion should, indeed, be appreciated.

Mr. Sherman states that: "Although it is somewhat lengthy, the rational method is a simple process of computation * * *." True, the process has been made (in many cases) to appear so simple that long and tedious calculations were often made and supervised by individuals who understood little concerning many of the fundamentals.

The writers are appreciative of the statements contained in the discussion by Mr. Bates, to the effect that: (*a*) The writers presented an original method for estimating storm-water run-off, based on fundamental conceptions and rational conclusions; and (*b*), the formulas are flexible and contain factors heretofore neglected, and are not complicated beyond practical use.

The writers cannot agree with Mr. Bates, however, that the "greater value" of the proposed formulas lies in their use, in conjunction with the empirical formulas, to obtain "*C* values." (Turn to the five statements, (*a*), (*b*), (*c*), (*d*), and (*e*), under "Inadequacy of Empirical Formulas," in Part II.) Consider that Mr. Bates has specialized on the type of rainfall curve and on values of *F*, *P*, and *L* (the length for a given area). There is no way for any one to determine that the values given these factors correspond to those found through experimentation by McMath. Furthermore, an infinite number of "*C* values," would be possible through changes in values of the various factors, and the resulting uncertainty due to the lack of definite information in the records of experimentation leading to the compilation of any one empirical formula would be tremendous.

It should be realized that correct values of the run-off coefficient may be obtained only through direct experimentation, with all conditions known.

The substance of a few selected statements, which convey erroneous conceptions on the part of the author, is recorded in the following citations from Mr. Bates' discussion:

(*a*) The possibility of using the empirical formulas as "yardsticks," by which to measure the writers' formulas;

(*b*) The suggestion that Factor *P* be given a constant value, and the inference that the shape of the area has no significance;

(*c*) The suggestion that, in using a rainfall curve, the designer must assume that the individual storm follows the law of the curve;

(*d*) The suggestion that Table 3 is mis-named, and that the value of *P* has no significance when the storm is one of uniform intensity; and,

(*e*) The idea that the run-off coefficient could not increase due to a higher intensity because of its relation to R_1 of Equation (29).

---

[60] *Transactions*, Am. Soc. C. E., Vol. 93 (1929), p. 433.

[61] *Municipal and County Engineering*, June, 1924.

These points have been so completely covered in the original paper that more elaborate discussion seems unnecessary.

The discussion by Mr. Tapley presents in a more mathematical manner some phases of the factors, that were introduced in the paper. Some of his approaches should help to make the derivations clearer to certain readers. It was the desire of the writers throughout to give the minimum space to all developments in the paper, consistent, of course, with clarity.

It is believed that the logic in the introduction of Factor P (as developed in the paper) will be obvious to the average engineer. It should be noted, of course, that in the "Nomenclature" (Appendix I), the definition of v is given a twofold meaning; and that, while in Equation (18) it is used in the general hydraulic sense, it naturally assumes the specific rôle of the average velocity when used in the run-off formulas.

Mr. Tapley's approaches to Equations (36) and (39) differ from those of the writers. While his manner of arriving at Equation (36) may be clearer than the original to some readers, the assumptions made in his derivation of Equation (39) may be misleading.

Consider a water-shed in which, for example, there exists, scattered over the area, certain depressions or cavities, which are capable, at a given instant, of retaining the precipitation falling thereon.

Visualize a process of concentration of the water resulting from a given intensity which lasts as long, or longer, than the time of concentration of the water-shed. The discharge at the outlet at a given instant, being composed of precipitation that fell at different specific times, represents contributions from the various parts of the area. Another discharge occurring immediately before, or after, represents precipitation that fell at other specific times. Hence, the rate of run-off at the outlet is a combination of contributions, from the various parts of the water-shed, as of the specific times precipitation occurred thereon. The storage factor, m, is represented by the quotient, the rate of withdrawal due to depressions, or cavities, divided by the rate of precipitation. When the precipitation is unevenly distributed with regard to area or time, Factor m must be regarded as a percentage of the average rate of precipitation for the time of concentration.

While z is a rate, m is the percentage of a rate. The product, mi (a rate) must be regarded as a distinct loss, deductible from the rate of precipitation, i, in the process of calculating the outlet discharge for a given instant of time. If m is defined as "the percentage of $(i\text{-}z)$ stored" (as Mr. Tapley suggests), the meaning would be that the precipitation, falling in the depressions or cavities, consists of two parts—storage and absorption plus evaporation. If m is defined as a "retardation factor," as in the discussion by Mr. Bernard, the proper and full meaning is not conveyed.

The nomenclature of the paper, therefore, should be retained. It should be noted that the area, A, was used for the purpose of enabling better visualization.

For the values of the run-off coefficient, C, Professor Powell presents the expression, $iA - miA - zA$ (or $1 - m - \frac{z}{i}$). Having deducted, in miA, all the precipitation falling in cavities and depressions, he would subtract an additional amount, mzA. His expression is, of course, in error.

Mr. Grunsky states,

"It appears obvious that of the total quantity of water falling on a water-shed as rain, the relative quantity which evaporates, together with that which sinks into the ground and is thus prevented from contributing to the peak discharge, increases as the critical time (that is, as area) increases."

In order to comprehend the meaning of the statement one must visualize two water-sheds, with similar physical characteristics, one being larger and with a consequent longer time of concentration. Each must be capable of concentrating the water, else there could be no opportunity for measuring a "peak discharge." Visualize, again, a process of concentration for each water-shed, as outlined in the foregoing discussion of Factor m. Assume, for simplicity, that each water-shed receives the same intensity during the times of concentration. For the consideration of the peak rate of discharge for either water-shed "the quantity of water which evaporates, together with that which sinks into the ground," is an average rate (deductible from the rate of precipitation), and not a volume multiplied by units of time. It is an average of the rates of loss of each elementary area, as of the specific instants each such area contributes to the peak run-off. Outside the effect of loss through evaporation and percolation in drainage channels (in one area as compared with the other), the only basis for judging such relative rates, of the two water-sheds, would be the average condition of wetness of their respective elementary areas.

It is clear that for like intensities of precipitation the elementary areas of the larger water-shed, on the average, would be more thoroughly saturated (at the instants of contributing to the peak discharge), and that the average rate of loss for it would be lower (not greater) than for the smaller one. It is conceded, of course, that the smaller water-shed, because of its shorter time of concentration, may be expected to receive a higher average intensity than the larger one. Since, as the writers have shown ("Factor C, the Run-Off Coefficient," in Part I), that, other conditions being constant, the run-off coefficient is increased by an increase in intensity, Mr. Grunsky's factor, a, for the larger water-shed would tend to be less than for the smaller one. Hence, if it be assumed that the effect of a greater average wetness of the larger water-shed balances that of a higher relative intensity on the smaller one, the factor, a (being Mr. Grunsky's notation for a run-off coefficient), would be constant with respect to time.

His statement (referred to in the foregoing) does not "appear obvious" in the light of analysis. No reasonable basis for Equation (112) has been demonstrated.

Mr. Grunsky writes again of water "temporarily stored in transit" in the channels and on the ground. Other engineers have suggested the possible alteration of the times of concentration and discharges, as rendered by the writers' formulas, due to the effect of such waters "temporarily stored in transit." It should be stated that the hypotheses, on which the writers' formulas are based, not only admit of, but make imperative, the existence of water on the ground and in the channels both before and after the arrival of the peak discharge. The water in transit in the channels is not stored, but is flowing toward the outlet where it forms a rate of discharge at each instant of time, before, at, and after, the arrival of the peak rate—hence, a hydrograph. Since each such rate of discharge is a combination of contributions from various parts of the water-shed as of the specific instants precipitation occurred thereon, the quantity delivered at each instant is, fundamentally, independent of the water flowing immediately before or after, except that it is supported in position by these other quantities.

The time of concentration, considered in developing the formulas, is not based on the time of flow of a single drop of water from the remote part of the water-shed to the outlet, but rather on that of the flow of the body of water in the main, or time-rendering channel (defined in the paper), as determined by the hydraulic formula used. Kutter's formula, for instance, is based on the average velocity of the full cross-sectional area of the channel. To make material additions to, or deductions from, the times of concentration or discharges, as determined properly by the formulas, because of an imagined storage in the channels, would be a mistake. It makes no difference, fundamentally, whether the channels are large, or small, or whether they are conduits, or rough and open channels. These are hydraulic considerations that influence the choice of factors for use in the formulas.

A statement that is more or less typical of many advanced by discussers, is contained in the discussion by Mr. Hicks, that is, "in the derivation of the formula, the factor, *F*, is based on the conduit running full. * * *." The derivation of Factor *F* is, of course, independent of the run-off formulas, and the values in Table 11 do not depend on any assumptions regarding intensity distribution. While Table 11 may be incomplete to some extent, facilities are already provided (in the combined two main divisions, "Closed Sections" and "Open Sections") for the consideration of both full and partly full conduits. While confusing purely hydraulic considerations with special assumptions and methods necessitated by the detail process of the rational method, Mr. Hicks assigns, erroneously, certain limitations to the run-off formulas.

In his discussion, Mr. Hickox presents a problem, which, he states, "may be taken as a fair example for comparison of rational and empirical formulas;" that is, the solution for the run-off of the Iowa River Basin above Iowa City. Comparative run-off results are given for two groups of assumptions, which, he states, are based on the "50-year rainfall intensity" and the "maximum precipitation," respectively. Unfortunately, few data for qualifying his assumptions are given, and the details of the solution by Equation (24) are omitted.

In the solution for each set of assumptions, Mr. Hickox arbitrarily assumes 100 hours for the time of concentration. This time has no bearing on the solution by Equation (86) and is incorrect for the assumptions used in solving by Equation (24).

In the first instance, if it is known, as given by Mr. Hickox, that $i = 0.0555$, that $C = 0.35$, and that $A = 3\,140 \times 640 = 2\,009\,600$ acres, no run-off formula is needed. From Equation (1) $Q = CiA = 0.35 \times 0.0555 \times 2\,009\,600 = 39\,036$ cu. ft. per sec.

Mr. Hickox could have ascertained times of concentration for his assumptions through Equation (24). For instance, solving by the "50-year rainfall intensity" formula (and using Mr. Hickox's assumptions), $t = 117.94$ hours.

Regarding the solution by the second set of assumptions Mr. Hickox states, "the maximum average precipitation on the Iowa River Basin is probably about 10 in. in 4 days, or approximately 0.1 in. per hour for 100 hours." If the precipitation is 0.1 in. per hour for a period of 4 days (or for 100 hours), it would not necessarily be 0.1 in. per hour for the time of concentration, when based on the assumptions for the water-shed.

Assume that the average intensity is 0.1 in. per hour for 4 days (96 hours, or 5 760 min.). Using Equation (2), with $e = 0.8$ and $t = 5\,760$ min., multiply by the reduction factor, 0.78, and solve for K; then, for R_1, $R_1 = 3.005$. In Equation (24), let $e = 0.8$, $H = 1$, $R_1 = 3.005$, $C = 0.4$, and use the same values for the other factors as in the preceding solution. There results, $Q = 59\,360$ cu. ft. per sec., average $i = 0.074$ in. per hour, and $t = 103$ hours.

Note (*a*) that the time of concentration, as rendered by Equation (24), for the first group of assumptions is 117.94 hours, while that for the second set is 103 hours; and (*b*) that the use of Equation (24) automatically gives the time of concentration and corresponding intensity. Note, further, that Mr. Hickox's value for the run-off (84 400 cu. ft. per sec), resulting from the second set of assumptions, is greatly in error, and that his corresponding conclusion is likewise incorrect.

Mr. Hickox's discussion failed to recognize the following points, which should be stressed: (*a*) The formulas, instead of rendering what he terms "the absolute maximum," may be used with any frequency period as determined by the particular rainfall curve; (*b*) the assumptions and data concerning precipitation must be incorporated in the rainfall curve, not in a formula; (*c*) the formulas may be used to apply to a part of the water-shed as well as to the whole; and (*d*) one of the main purposes of the formulas is to serve as an aid in determining times of concentration and average intensities.

The discussion by Mr. Baker conveys a broad understanding, on the part of the author, and is well worth reading.

The writers are in agreement with Mr. Baker that the run-off coefficient is of vital importance to the run-off problem, even when viewed from the strictest economical standpoint; and that more and better experimentation is needed. It is also reasonable, as Mr. Baker suggests, that governmental agencies, capable of broad experimentation over extended periods of time, would be most suitable for the purpose. Moreover, when one reviews the existing conflicting data on this factor, the lack of thorough analysis (which indicates incomprehensive visualization during experimentation) is very evident. Experimentation and intelligent analysis must be co-ordinated—some capable individual must be willing to "marry the job"—if worth while results are to be obtained.

In his Equation (91), Mr. Baker treats of one of the effects of slope on the absorptive powers of soil, namely, that of the head of the water in transit. Another possible effect of a difference in slope on absorption (other influences being constant) is the disturbing action and motion of a higher velocity, as compared with the effect of a lower one, on the soil. This action would tend to balance (or, possibly, more than balance) the effect due to a difference in head.

It should be pointed out that in deriving Equation (91) Mr. Baker assumes that percolation varies directly as the superimposed head. For porous soils percolation would take place according to Darcy's law, or would vary directly as the head; but the head should be measured from the top of the water to the bottom of the soil, assuming the water has a free outlet (say, into coarse gravel). (As an example to illustrate this statement, see Fig. 4 of a paper[62] entitled "Soil Mechanics Research" by Glennon Gilboy, Jun. Am. Soc. C. E.) Thus, Mr. Baker's z'' would equal $k'' (d + d')$, in which d' equals the soil depth, and Equation (91) becomes complicated to an extent that would alter materially Mr. Baker's subsequent conclusions.

For all ordinary soils the practical effect of a difference in slope on percolation in its extent of influencing run-off is small as compared to that of other physical characteristics. An idea concerning the effect of superimposed head on percolation may be gained from a consideration of the findings in the Miami Valley by Ivan E. Houk, M. Am. Soc. C. E. He found[63] that the value of his symbol, b, for the Miami Valley, would be between 0.25 and 0.30 in. per hour for the summer and fall when the ground is saturated. Assume that the soil depth is 4 ft. to gravel deposit and that percolation is 0.3 in. per hour with no superimposed head. For a superimposed head of 6 in., the percolation would be $\frac{4.5}{4} \times 0.3$, or only 0.34 in. per hour, if it varied directly as the hydraulic head.

In actual water-sheds the water is not long in finding its way into natural or improvised channels, which form a relatively small part of the total area. The average depth of a sheet of water over a pervious area is not likely to be great for the heaviest of rains, except in very unusual circumstances. That part lying in depressions, which fail to contribute to the collection of waters forming the peak run-off at the outlet of the water-shed, is cared for by m of Equation (39).

Answers to Mr. Jarvis may be segregated into four parts, as follows:

(1) Some of the answers will be found under the heading, "General Solution by Equation (29)," in this discussion in connection with Mr. Jarvis' remarks concerning the rainfall curves of Adolph F. Meyer, M. Am. Soc. C. E. It should be noted also that the curves of Mr. Bernard belong to Equation (2), which Mr. Jarvis says "the rational method should exclude."

(2) The writers feel that Mr. Jarvis' citation concerning Equation (32) is far fetched.

---

[62] *Proceedings*, Am. Soc. C. E., October, 1931, p. 1165.

[63] "Rainfall and Run-Off in Miami Valley," Technical Repts., Pt. VIII, 1921, pp. 121–126.

(3) There is nothing in Example (3) (or any other examples of the paper) to indicate: (*a*), that the writers "seem to be carrying the storm drainage conduits throughout the water-sheds" with natural drainage systems; and (*b*), that the instantaneous discharge at a given section should vary directly with the length of travel. The high value of F (causing the velocity of 28.3) was chosen arbitrarily to illustrate a point in hydraulics, and the example is in itself no part of the solution for the run-off of a water-shed either with natural or artificial channels. In deriving their formulas, the writers used both the linear dimension, L ("some function of the drainage area"), and the "average width."

(4) Instead of selecting factors for use in examples for the purpose of indicating their judgment, the writers endeavored to illustrate methods of procedure; and ample facilities have been provided for exercise of the engineer's judgment in making such selections.

The opening paragraph of Mr. Bernard's discussion is indicative of his understanding and attitude. He has pursued a constructive course, and, doubtless, the engineer will find much of value in his contribution.

Mr. Longacre contributed a discussion that should prove of real value to many engineers. Considering the approximate nature of the run-off problem, with its many possibilities for variations in assumptions and data, the nomograph (Fig. 9) will be found accurate enough for the usual problem. As stated by Mr. Longacre, a nomograph "can readily be constructed for a curve of any of the general types in present use." This is unnecessary, however, since Fig. 9 (or any good nomograph of Equation (29)), may be used in conjunction with the simple method of transition, which the writers are introducing under the heading, "General Solution by Equation (29)," in this discussion. Mr. Longacre's Fig. 11 will also prove of value for general hydraulic purposes.

General Solution by Equation (29).—Under the headings, "Special Reduction of Equation (24)," and "Remarks Concerning Examples 6, 7, 8, and 9" (Part II), it was shown that Equations (28) and (29) could be used for the purpose of approximating closely the run-off result for any type of rainfall curve.

It should be observed that Equation (28), or Equation (29), served to obtain the time of concentration for a given area. This time, being based on the rainfall curve expressed by Equation (3), results from the application of a definite ordinate thereof (or a definite average intensity) to the water-shed. The ordinate of another curve (used to obtain the corrected Q), would cause a somewhat longer or shorter time, depending on the divergence in value of the ordinates of the two curves. The difference in time is due, therefore, to a greater or less average quantity of water running in the main (or time-giving) channel of flow.

After gaining a thorough understanding of the principle involved in making the transition, it may be visualized that the rainfall curve of the type expressed by Equation (3), together with Equation (29), is capable of being used as a scaffold work for the solution of any rainfall curve or group of curves. This method of approach differs from that illustrated in Examples 7 and 9, in that it eliminates the consideration of the variables, R_H and H, or specializes in the values, $R_H = R_1$, and $H \doteq 1$. Two examples will illustrate:

Example 15.—The solid curved line of Fig. 16 is expressed by,

$$i = \frac{235}{t + 26.3} \quad \text{(134)}$$

This curve is fairly representative of the curves of the Meyer[64] group, being arbitrarily selected from Fig. 1 of the paper[65] by Merrill M. Bernard, M. Am. Soc. C. E., entitled "Formulas for Rainfall Intensities of Long Duration."

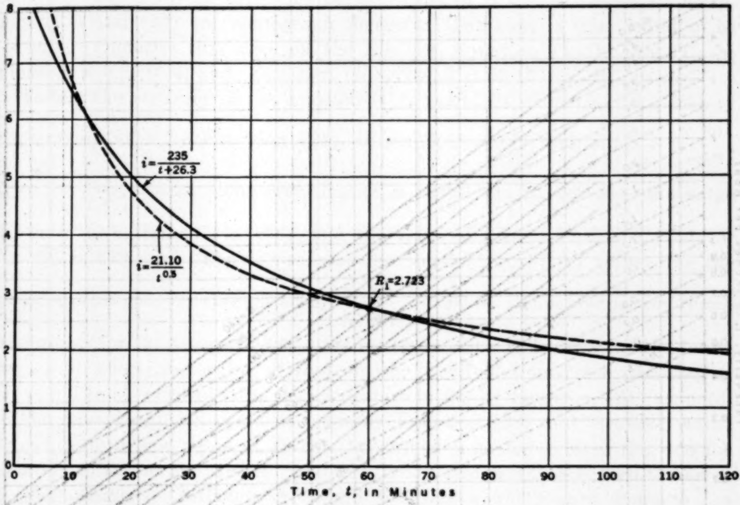


FIG. 16.

The dashed line is expressed by Equation (3) reduced. In superimposing the dashed curve the two were placed together at the 60-min. ordinate by solving Equation (134) for R_1 ($= 2.723$), with $t = 60$ min. Note that the divergence of the two types of curves is not great at any point between 5 and 120 min.

Suppose that it is intended to solve by the use of the curve expressed by Equation (134), and that the following factors have been determined: $A = 1\,000$ acres; $L = 14\,700$ ft.; $S = 10$; $F = 6.0$; $C = 0.4$; $P = 0.5$; and R_1 being, of course, 2.723.

Solve for the time of concentration by Equations (29) and (26), as illustrated by Example 8, or directly by nomograph of Equation (29) similar to that presented by Mr. Longacre.

---

[64] "Elements of Hydrology," 1928.

[65] See p. 595.

The time is found to be 28.7 min., the corresponding ordinate of Equation (134) is, $i = 4.27 +$; and (substituting in Equation (1)), $Q = 0.4 \times 4.27 + \times 1\,000 = 1\,709$ cu. ft. per sec.

Merely to test the accuracy of the solution, the problem is solved by two trials (as illustrated in Example 13), the actual Q resulting from the direct use of Equation (134) being 1 726 cu. ft. per sec. The accuracy of the "scaffold work," in this case, is within 1%, or $\frac{1\,709}{1\,726} = 0.99 +$.

Example 16.—This example illustrates the application of Equations (3) and (29) as a "scaffold work" for the solution of a group of curves.

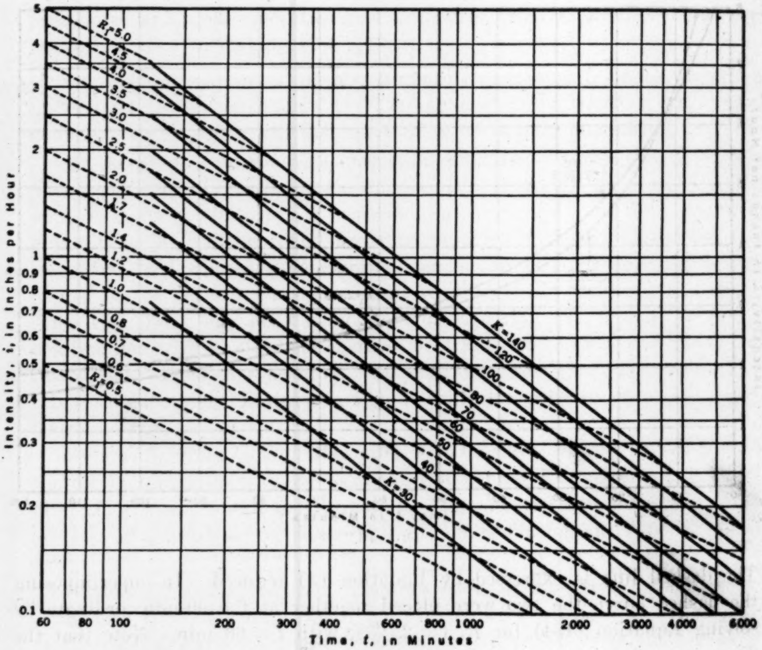


FIG. 17.

The solid and dashed lines of Fig. 17 are expressed by Equation (2) with $e = 0.77$ and 0.5, respectively, and with varying values of K, the time being in minutes. It may be noted that the value of e for the solid lines is approximately the average of the exponential values shown by Mr. Bernard in his paper, entitled "Formulas for Rainfall Intensities of Long Duration."[66] The dashed lines form the "scaffold work" of Equation (3) with varying values of K (and, consequently, R_1), t being expressed in minutes.

---

[66] See p. 592.

Assume that it is desired to solve by the equation,

$$i = \frac{80}{t^{0.77}} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (135)$$

shown on Fig. 17, the factors being (with the exception of R_1); $A = 500\,000$ acres; $L = 400\,000$ ft.; $S = 4.0$; $F = 4.5$; $C = 0.3$; and $P = 0.5$.

The solution for the time of concentration may be made by Equations (29) and (26) (or by nomograph as Fig. 9) and transition to the line representing Equation (135), when the approximate proper value of R_1 of Equation (3) is known. To ascertain the value of R_1 to use, make a rough assumption for the time of concentration of the area, say, 6 to 12 hours, or 360 to 720 min. The dashed line (of Equation (3)), which crosses the solid line, representing Equation (135), during this time period, leads to $R_1 = 2.0$. Solve by Equations (29) and (26) (or nomograph thereof). The time is thus found to be 644 min. Opposite this time on the line for Equation (135), Fig. 17, the intensity is, $i = 0.55$, and, Equation (1) gives $Q = 82\,450$ cu. ft. per sec.

To test the accuracy of the solution (this would be, of course, unnecessary in practice) solve by Equation (24), in which $e = 0.77$ and, since $i = \dfrac{80}{t^{0.77}}$, $R_1 = \dfrac{80}{(60)^{0.77}} = 3.42$. The resulting Q is 80 091 cu. ft. per sec., the value of i from Equation (1) is 0.534, and the value of t from Equation (26) (or Fig. 17) is 669 min. The accuracy of the "scaffold work" for estimating the run-off in this case is represented by $\dfrac{82\,450}{80\,091} = 1.03 -$. Thus, for a very rough assumption for the time of concentration (from 6 to 12 hours), made for the purpose of selecting a value of R_1 from the "scaffold work," the resulting Q was found to be within 3% of the accurate solution by Equation (24) reduced.

The second solution could be made by Equation (29) and "scaffold work" (by choosing a new value of R_1 for the dashed line that crosses the line for Equation (135), at the time for the first trial), which would reduce the error to insignificance. Although a second solution would always indicate the degree of accuracy, it would be unnecessary for experienced estimators when using a "scaffold work" as Fig. 17.

Two concluding observations should be made:

1.—For times of concentration, from 5 to 120 min., solutions may be made for all rainfall curves in present use, through Equation (29) and transition as outlined herein, without recourse to a "scaffold work" in graphic form; that is, by simply using the R_1 of the adopted curve.

2.—Solutions for any curve, or group of curves, may be accomplished, with a high degree of accuracy, through the use of Equation (29) and "scaffold work" of Equation (3).

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852.
TRANSACTIONS

Paper No. 1813

GRAVITY DAMS ARCHED DOWN STREAM[1]

By B. J. Lambert,[2] M. Am. Soc. C. E.

With Discussion by Messrs. Eugene Kalman, William P. Creager, T. L. E. Haug, B. F. Jakobsen, A. Floris, Floyd A. Nagler, and B. J. Lambert.

Synopsis

When a canyon site seems best adapted to a gravity type of dam there is some question as to the desirability of arching it in an up-stream direction. The writer advocates the straight gravity dam for certain sites as, for example, the St. Francis Dam, and presents further arguments to support his theory that in some situations a dam curved down stream may be preferred to the other two types.

The writer has not seen, in discussions on high, curved, gravity dams in canyons, any reference to the relation of the curvature of the dam to the slopes of the side walls. He has been interested in this problem for a number of years, particularly since the failure of the St. Francis Dam in March, 1928. Fig. 1 shows a concrete model of this dam built to a scale of $\frac{1}{240}$, or 1 in. = 20 ft., in a trough made to resemble, roughly, the valley at the site. This trough was made of planking as shown.

The model was formed in place. It had a base of about 8.25 in., a height of 10.25 in., and a cross-section similar to that of the St. Francis Dam, with the exception of the heel on the lower 20 ft. The radius of the arch was 500 ft., or, for the model, 25 in. Sections and elevations of a dam illustrating a supplementary feature are given in Fig. 2. This feature is described in another part of this paper.

Assume that the dam acts as a unit, instead of treating each foot in length as a gravity dam. In the model it is readily seen that, if the dam overturns, it will turn about a part of the toe at the deepest section. The toe along the slopes may be under no pressure when the dam starts to overturn, but the sloping ends of the dam are hanging as cantilevers on the central part of the structure. Consider that the curvature of the dam and the slopes of the canyon walls are such that the extreme ends of the dam are in front of a vertical plane passing through the toe of the highest section. Then if the dam begins to overturn the greatest pressure will come first upon these extreme ends, and the remainder of the toe will be under smaller pressures.

---

[1] Published in March, 1931, *Proceedings*.

[2] Head, Dept., Civ. Eng., State Univ. of Iowa, Iowa City, Iowa.

In the model as constructed, the toes at the ends are found to be about 3.5 in. (70 ft.) back of the toe at the deepest section. Had the slope of the canyon walls been somewhat less as shown by the dotted line in Fig. 1(*d*), the ends of the dam could easily have been in front of the toe at the deepest section, in fact, one end was in front of the toe, in the St. Francis Dam.

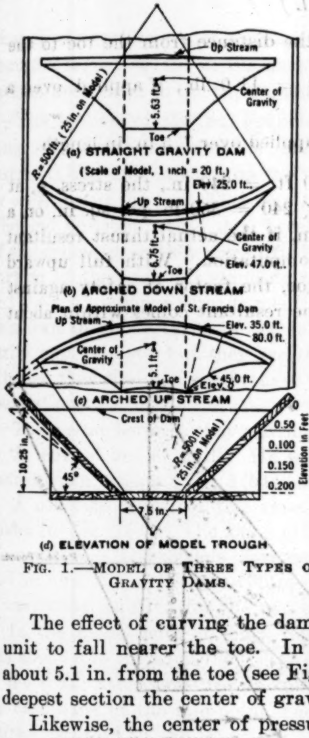

Up Stream
Center of Gravity
5.63 ft.
Toe
R=500 ft. (25 in. on Model)
(a) STRAIGHT GRAVITY DAM
(Scale of Model, 1 inch = 20 ft.)
Elev. 25.0 ft.
Up Stream
Center of Gravity
6.75 ft.
Elev. 47.0 ft.
Toe
(b) ARCHED DOWN STREAM
Plan of Approximate Model of St. Francis Dam
Up Stream
Elev. 35.0 ft.
80.0 ft.
Center of Gravity
5.1 ft.
Toe
45.0 ft.
Elev. 0
(c) ARCHED UP STREAM
Crest of Dam
0
0.50
0.100
0.150
0.200
Elevation in Feet
10.25 in.
45°
R=500 ft. (25 in. on Model)
7.5 in.
(d) ELEVATION OF MODEL TROUGH

FIG. 1.—MODEL OF THREE TYPES OF GRAVITY DAMS.

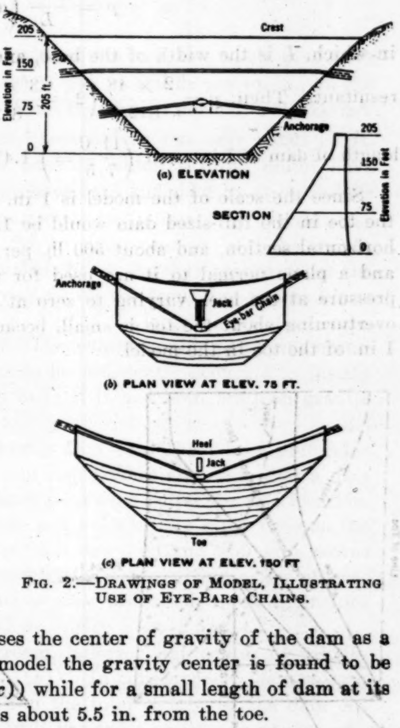

205
Crest
150
75
0
Elevation in Feet
205 ft.
Anchorage
(a) ELEVATION
205
150
75
0
Elevation in Feet
SECTION
Anchorage
Jack
Eye-bar Chain
(b) PLAN VIEW AT ELEV. 75 FT.
Heel
Jack
Toe
(c) PLAN VIEW AT ELEV. 150 FT

FIG. 2.—DRAWINGS OF MODEL, ILLUSTRATING USE OF EYE-BARS CHAINS.

The effect of curving the dam causes the center of gravity of the dam as a unit to fall nearer the toe. In the model the gravity center is found to be about 5.1 in. from the toe (see Fig. 1(*c*)) while for a small length of dam at its deepest section the center of gravity is about 5.5 in. from the toe.

Likewise, the center of pressure of water acting on the rear of the model dam is found to be about 4.14 in. above the base, and the total pressure to be about 25.7 lb. If full upward pressure is assumed at the heel, varying to zero at the toe, this will be approximately 22 lb. and will be applied slightly inside the middle third of the base. The model dam weighs approximately 48 lb.

Fig. 3 shows a cross-section of the model dam at the middle. If a center of moments is taken at the toe, it will be found that a push, p_1, of $\left(\frac{48 \times 5.1}{10.25} =\right)$ 23.8 lb. is required at the top to overturn the dam, with reservoir empty. By actual test a force of 24 lb. was required. If applied at the center of water pressure, that is, 4.14 in. above the base, the theoretical force, p, required is $\left(\frac{48 \times 5.1}{4.14} =\right)$ 59 lb. The force as actually measured was about 58 lb.

The resultant of the weight of the the dam and the water pressure is found to cut the foundation about 2.9 in. from the toe. Assuming that a section 7.5 in. wide takes all the pressure at the time of incipient overturning, the unit pressure per square inch on the model at the toe is,

$$p = \frac{2\,W}{L}\left(a - \frac{3\,a}{L}\right)$$

in which, L is the width of the base, and a, the distance from the toe to the resultant. Then, $p = \dfrac{2 \times 48}{8.25}\left(2 - \dfrac{3 \times 2.9}{8.25}\right) = 11.0$ lb., if applied over a length of dam of 1 in., and $\left(\dfrac{11.0}{7.5} =\right)$ 1.47, if applied over 7.5 in. in length.

Since the scale of the model is 1 in. = 20 ft. = 240 in., the stress, p, at the toe in the full-sized dam would be 1.47 × 240 = 350 lb. per sq. in. on a horizontal section, and about 500 lb. per sq. in. if the actual thrust resultant and a plane normal to it are used for the computation. With full upward pressure at the heel, varying to zero at the toe, the factor of safety against overturning about the toe is small, because the resultant comes within about 1 in. of the toe in the model.

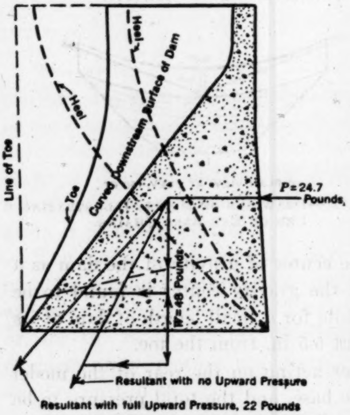

Line of Toe
Heel
Heel
Curved Downstream Surface of Dam
Toe
P=24.7 Pounds,
W=48 Pounds
Resultant with no Upward Pressure
Resultant with full Upward Pressure, 22 Pounds

FIG. 3.—SECTION AT THE CENTER, FIG. 1(*c*), ARCHED UP STREAM.

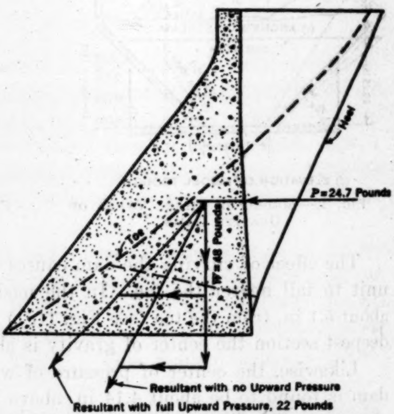

Heel
P=24.7 Pounds
Toe
W=48 Pounds
Resultant with no Upward Pressure
Resultant with full Upward Pressure, 22 Pounds

FIG. 4.—SECTION AT THE CENTER, FIG. 1(*b*), ARCHED DOWN STREAM.

The foregoing remarks indicate that if the overturning of the dam occurred about the full length of the toe, at its deep section (that is, 7.5 in. in the model, or 150 ft. in the original dam), the pressure per square inch on the toe would be only 500 lb., which would not cause failure. If the bottom of the dam along this section is not horizontal, however, but is built in a valley the greatest depth of which occurs over a short distance only, the turning might take place over a much restricted length of toe, and the pressures would increase correspondingly. For example, if in the St. Francis Dam, the toe on a section only 50 ft. long carried the full load at the time of greatest stress, the unit pressures might then be 1 500 lb. per sq. in., even with no upward pressure considered.

With upward pressure considered and with the resultant cutting the foundation 1 in. from the toe, the unit pressures as previously determined are,

$$p = \frac{2 \times 48}{8.25}\left(2 - \frac{3 \times 1}{8.25}\right) = 19 \text{ lb. per sq. in.}$$

If applied over the center section (7.5 in. wide at the toe), this would give more than 2.5 lb. per sq. in. on the model, or 2.5 × 240 = 600 lb. per sq. in., on the section horizontal to the thrust, and 850 lb. on the section normal to the thrust.

The section of the St. Francis Dam left standing was only about 90 ft. long. If this represents the length of base on which turning might have occurred, the unit stresses on the toe may have been sufficient to cause failure by crushing at the toe. This may have caused the beginning of the overturning. The end portions would then break off, due to water pressure and the weight of the concrete, as they were lifted from their base. As they turned about the portion of the base left standing the pressures on the front due to this hinge and the toe pressures already mentioned spawled off the large masses of concrete, as shown in photographs of the failure.[8] The values in this paper are necessarily approximate, but they are thought to be sufficiently accurate to justify a belief that the dam might have failed even if it had been built on granite foundations.

The writer believes that a straight gravity dam would have been preferable to the curved one as built. He will go one step farther and state that a dam arched down stream (Fig. 1(*b*)) would have been even better for this location.

A concrete model of this type of dam was built at the same time as the model in Fig. 1(*c*). The same gravity section was used and the same radius of curvature. The models weighed practically the same, within a few ounces, and an analysis of the two volumes seems to show practically equal quantities of materials required for the two conditions. (See Figs. 1 and 4.)

In the model with its back down stream, however, the center of gravity was about 6.75 in. from the toe (see Fig. 1(*b*)). In consequence, the resultant of the total water pressure on the rear and the total weight of the dam cut the base back of the mid-point, and the unit pressures under any of the conditions assumed in the foregoing example were materially reduced.

Theoretically, a force, P, of $\left(\frac{48 \times 6.75}{4.14} =\right)$ 78 lb., is required at the center of pressure to overturn this dam about the toe. In the model, about 80 lb. was required. This, compared with the water pressure of 25.7 lb. on the back of the model, gives a factor of safety of about 3, if upward pressure is neglected.

In the case of the dam with its back up stream, the factor of safety under the same conditions is about 2.8; but with an upward pressure varying from full load, *wh*, at the heel to zero at the toe, the dam with its back up stream has practically no factor of safety, while the one with its back down stream still has a factor of safety of more than 1.5.

---

[8] *Proceedings*, Am. Soc. C. E., October, 1929, Papers and Discussions, p. 2157.

It is also to be noted that if a radial section is studied, its base, in the case of a dam arched down stream, slopes up toward the toe; while in the case of the dam the arch of which is up stream, the base slopes down toward the toe. The amount of this slope varies from about 35 ft. at the deepest section to, say, 6 or 8 ft. at the ends.

It is presumed that the water pressure on a gravity dam arched up stream will tend to close any shrinkage cracks, the sections acting as voussoirs of an arch. It might seem as if the dam that was arched down stream would have a greater tendency to open up along shrinkage or construction joints than if it was curved the other way. The writer can scarcely concede even this when it is observed that the entire dam is much more stable, and any radial unit must be tipped up-hill to cause failure. The wings tend to tip against the walls of the canyon when the back of the arch is down stream, instead of tipping into the canyon as is the tendency when arched up stream.

A model of a straight gravity dam was also built (see Fig. 1(*a*)), and was found to be somewhat more stable than the dam arched up stream (Fig. 1(*b*)), and considerably less stable than the one arched down stream (Fig. 1(*c*)). In all cases, the overturning pressures were applied at the center of pressure through a bent hook and a standardized spring balance, using a horizontal pull.

The writer is strongly of the opinion that, before any definite design for a dam in a canyon is accepted, relatively large models of the actual canyon, with its actual foundation profile, and of the proposed dams and footings, should be made and studied. Had this been the practice in the past it is thought that quite likely some of the accepted designs might have been considerably modified, or the footings and abutment conditions materially changed.

It will be observed that the basis of this study is an assumption that, in general, the dam will act as a unit. Construction and shrinkage cracks cannot be eliminated, but the sections will interlock more or less, and certainly will be of large size, as illustrated by the central block of the St. Francis Dam which was left standing after the disaster.

Another suggestion that could be applied to the arched dam with its back down stream is illustrated in Fig. 2. Consider the St. Francis Dam, for example, and assume the curvature reversed. Fig. 2(*b*) shows the dam built up to a height of, say, 75 ft. At this elevation place an eye-bar suspension chain of any cross-section desired, and anchor it at each end into down-sloping tunnels in the canyon walls. After the chain is placed, and the ends are anchored, jacking pressure could be applied to a saddle at its center, and any predetermined initial stress could be maintained while concrete is being placed over the cable. As many cables or chains as desired could thus be placed at such elevations and of such cross-section as would be found most satisfactory. A small percentage of the cost of the dam thus expended in steel (preferably of high tensile strength), would increase the factor of safety both against overturning and sliding.

It seems reasonable to assume that the quantity of concrete could be reduced to such an extent as to pay for the steel and still have a safer dam. It could scarcely be said that these eye-bar cables would act as tensile reinforcement to tie the large blocks of the dam together, because the percentage of steel compared with the quantity of concrete would necessarily be small. However, picture a chain of, say, ten eye-bars, each with a section of 12 by 2 in. (240 sq. in. in all), made of a steel with an ultimate capacity of 100 000 lb. per sq. in., or more. With such a chain, requiring a force of 24 000 000 lb. to break it, one might be justified in expecting that the larger segments of a dam would resist separation for some time.

All this may seem unorthodox, but no harm and, perchance, some benefit may result from a discussion of the proposals herein outlined.

DISCUSSION

Eugene Kalman,[4] M. Am. Soc. C. E. (by letter).—This extremely interesting paper is a revelation, inasmuch as it calls attention to the fact that a gravity dam with sloping wings might be subjected to tensile stresses even if, theoretically, all the individual vertical cantilever slices were safe against such stresses. In other words, the resultants of the forces acting on the individual cantilever slices may fall within the middle third of their respective bases, and the resultant of the dam as a whole may fall outside the middle third of the base.

This is due to the effect of the sloping wings. A brief analysis of stress conditions will be given in this discussion. Consider a straight gravity dam with a constant section. The total bending moment due to water pressure and the weight of the dam with respect to the heel will be:

$$M = \frac{1}{6} H^3 (2 \gamma_c \tan^2 \beta - \gamma_w) d \quad \ldots\ldots (1)$$

in which, H equals the height of the dam; d, the width of the horizontal canyon bottom, corresponding to the height, H; γ_w, the specific weight of water; and γ_c, the specific weight of concrete. The down-stream face of the dam has a slope of 90° — β, and the up-stream face is vertical. Furthermore, let the slope of the wings equal $90° - \alpha$, and let x equal the horizontal distance of an elementary cantilever slice, dx, from the bottom of the slope of the wing toward the top. The bending moment of the cantilever slice with respect to the heel at the depth, H, may be computed as follows: The height of any cantilever slice may be expressed by the equation:

$$h = H - x \cot \alpha \quad \ldots\ldots (2)$$

The total water pressure acting on this slice is equal to:

$$p_w = \frac{1}{2} \gamma_w (H - x \cot \alpha)^2 dx \quad \ldots\ldots (3)$$

The lever arm of this force with respect to the bottom is,

$$y = H - \frac{2}{3} (H - x \cot \alpha) = \frac{1}{3} (H + 2 x \cot \alpha) \quad \ldots\ldots (4)$$

and, hence, the moment due to water pressure on the cantilever slice will be,

$$M_2 = \frac{1}{6} \gamma_w (H - x \cot \alpha)^2 (H + 2 x \cot \alpha) dx \quad \ldots\ldots (5)$$

The cantilever under consideration with a height, h (see Equation (2)), and a thickness at the base equal to,

$$b = (H - x \cot \alpha) \tan \beta \quad \ldots\ldots (6)$$

has a dead weight of,

$$w = \frac{1}{2} \gamma_c (H - x \cot \alpha)^2 \tan \beta \, dx \quad \ldots\ldots (7)$$

---

[4] Visiting Prof., Civ. Eng., California Inst. of Technology, Pasadena, Calif.

The lever arm of this force with respect to the heel line of the central portion of the dam is expressed by the equation,

$$y' = H \tan \beta - \frac{1}{3} (H - x \cot \alpha) \tan \beta = \frac{1}{3} (2 H + x \cot \alpha) \tan \beta \quad \text{..(8)}$$

Therefore, the moment due to the weight of the cantilever slice will be,

$$M_3 = \frac{1}{6} \gamma_c (H - x \cot \alpha)^2 (2 H + x \cot \alpha) \tan \beta \, dx \quad \text{.........(9)}$$

The total moment due to the forces acting upon the cantilever slice is the sum of Equations (5) and (9), that is,

$$M = \frac{1}{6} (H - x \cot \alpha)^2 [(- \gamma_w + 2 \gamma_c \tan \beta) H + (- 2 \gamma_w + \gamma_c \tan \beta) x \cot \alpha] \, dx \quad \text{................(10)}$$

The moment of all the forces acting on the two wings will be twice the moment indicated by Equations (5), (9), and (10), respectively, with x changing from 0 to $H \tan \alpha$, which is expressed by,

$$M = \frac{1}{12} (3 \gamma_c \tan \beta - 2 \gamma_w) H^4 \tan \alpha \quad \text{..............(11)}$$

The moment of all forces acting on the dam, taken about the heel line of the central part, is equal to the sum of Equations (1) and (11).

The sections of the dam are free from tensile stresses if the resultants are assumed to pass through the down-stream kern points of the base section; that is,

$$\gamma_w = \gamma_c \tan^2 \beta \quad \text{.........................(12)}$$

Hence, it is apparent that Equation (11) is always positive and that the whole dam constructed on this basis will never be in danger of overturning. However, tensile stresses are present in the central part. If β is chosen according to Equation (12) so that the resultant of the forces on all cantilever sections passes through the down-stream kern point at the base of all individual elementary cantilevers, it is obvious that the continuation of the resultant of all wing sections will fall outside the middle third of the central section, thus indicating the presence of tensile stresses in the central part of the dam when considered as a whole.

The foregoing analysis is based on the assumption that the wings will not be influenced by any load except their own weight and the water pressure. This means that there is no interaction between wings and foundation, or, in other words, that the wings are not supported by abutments at all. In such a case the single vertical cantilever elements of the wings will have a tendency to deflect as a whole directly down stream. Such deformation creates horizontal shearing stresses along the vertical cut faces of the individual cantilever slices, and these shearing stresses directed down stream, constitute an additional load acting with the water pressure on the central cantilevers, thus increasing the flexure of these longer elements and, consequently, introducing tensile stresses.

Conditions are quite different if there is real interaction between the wings and the foundation as may be expected in every practical case. The elementary wing cantilevers (Fig. 5(*b*)) may be considered identical with the upper end portions of the full central cantilevers (Fig. 5(*c*)), both from the point of view of their geometrical shape and from their loading. Therefore, it is obvious that the wing sections are subjected to smaller flexural down-stream deflections than the central section. Furthermore, there is no greater danger of sliding in the wing cantilevers than in the central cantilevers, since the unit horizontal thrust at all heights is proportional to the weight of the concrete column resting on the unit horizontal area. Under these conditions there is no reason why the wing cantilevers should begin sliding sooner than the central full-sized cantilevers since the former are subjected to smaller flexural deformations than the latter. In other words, it cannot be expected in a practical case that the wings will deflect more than the central part either because of flexure or sliding. On the other hand, this means that the wings cannot create horizontal shearing stresses directed down stream in the central part of the dam. Quite the contrary; the wings deflect less than the central part and, therefore, they help to decrease the bending in that part, and improve its stability.

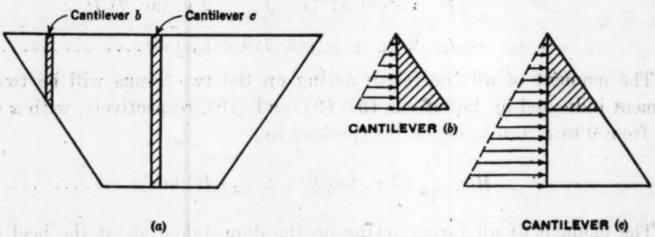


FIG. 5.

Furthermore, assuming wings that are not supported by abutments, it is clear for the reasons given by the author that the gravity dam arched down stream is more efficient than the one arched up stream. This fact is entirely consistent with good theory. It seems paradoxical to state that an arch curved down stream is more resistant than one curved up stream, but, in the present case, the latter does not refer to a string polygon in compression and the former does not refer to a string polygon in tension, since the end forces of the string polygon, which are generally determined by the end reactions at the restrained end abutments, are missing in this case. The individual horizontal arch slices are, in the present case, subjected to flexure and not to tension or compression. Hence, despite the characteristic geometric shape of the structures, they cannot be termed arches or catenaries, since there is no catenary action.

The experiments made by the author are perfectly convincing for the case of free wings. On the other hand, they do not represent actual conditions, because of the lack of structural unity between the wings and the foundation.

There is no adhesion between the concrete model and the wooden trough. Furthermore, the models were made to overturn by the application of a concentrated load at the center of gravity of the dam—that is, along its line of symmetry—while water pressure acts uniformly over the entire area of the up-stream face. The difference in the structural action resulting from the two kinds of loading is obvious. The concentrated load tends to overturn the dam by direct action at the center, while the free wings do not undergo any deformation. They simply tend to rotate about the heel of the central part of the dam. The uniform water load, on the other hand, affects the central part mostly by indirect action through the shearing stresses developed between the wings and that part, after the primary deformation of the wings has occurred down stream.

Professor Lambert is to be commended for having called attention to the different effects of arching gravity dams up stream and down stream with a view to analyzing the stress conditions in existing gravity dams located in comparatively narrow canyons. The problem as outlined, however, should not prove very important, because if the canyon is comparatively large the use of a straight gravity dam will be indicated in preference to an arched form, and if an arched gravity dam is used in a wide canyon the advantages claimed by the author are relatively insignificant due to the comparatively insignificant maximum of the wings when compared with the maximum of the central part; in fact, the shifting of the center of gravity that occurs as the result of the customary moderate arching is extremely small, regardless of which way the dam is arched, and the structural deformations that take place in a dam arched up stream will compensate entirely for such a shift.

If, on the other hand, the canyon is comparatively narrow, as in the model experiments described by Professor Lambert, the advantages of a true arched dam will outweigh those of an arched gravity dam; that is, such a site dictates the use of a structure in which the cantilever loading may be decreased liberally and the arch loading increased in proportion, by reducing the exaggerated dimensions in the lower regions of the dam such as are necessary for gravity dams either straight or slightly arched. Hence, the straight dam, or the dam arched down stream, will not be feasible. In any case, in true arched dams the position of the center of gravity has no appreciable influence on the stability of the structure.

Briefly, Professor Lambert's suggestions are valuable as applied to gravity dams located in comparatively narrow canyons; but, for such locations, gravity dam sections should not be selected.

WILLIAM P. CREAGER,[5] M. AM. SOC. C. E. (by letter).—The statements contained in the author's paper are interesting and correct, based on monolithic structures such as those used in his experiments. However, for a full-sized dam split by transverse expansion cracks or joints, it is not possible to transfer, by shear or cantilever action, the weight of the extreme ends of the dam to the central portion. In other words, whether the dam is arched up stream or down stream, each radial section must be designed independently of the others, unless, of course, in the usual type of arched gravity dam, the radius is sufficiently small to allow consideration of arch action.

---

[5] Chf. Engr., The Power Corporation of New York, Buffalo, N. Y.

In the case of a straight dam, correct practice would not allow the use of width of base at the highest part, as applying to the entire structure, in the determination of stability. The theory is no different whether the dam is arched up stream or down stream.

The author points out correctly that, for a parallel-sided site, the ends are more stable if the dam is arched down stream. This is particularly true as regards sliding, because a radial section would be required to slide up-hill. This feature, in the writer's opinion, is the only one in favor of arching the dam down stream as compared either with a straight dam or a dam arched up stream. It is believed that it is not a sufficiently desirable feature to warrant using more concrete as compared with a straight dam or voiding the possible feature of ultimate arch action before absolute failure as compared with a dam arched up stream.

In Fig. 2, the author suggests a novel steel suspension type of dam. It is assumed that this illustration is diagrammatic and that the suspension bars would actually be curved to the radius of the arch to avoid one enormous single jack. It would be interesting to know the cost of adequate anchorages for the suspension bars; it occurs to the writer that they would be quite expensive.

T. L. E. Haug,[6] M. Am. Soc. C. E. (by letter).—The author's treatment of this subject is notable for its recognition of the importance of considering the dam as a unit, rather than as merely an assemblage of thin slices or elements, of which it has been thought sufficient to treat only one as typical of the whole structure.

The results of experiments on models must be interpreted with great care because, unless the model exactly reproduces all the conditions obtaining in the full-sized structure, unwarranted conclusions may be drawn.

Professor Lambert states that the overturning load applied to each of his models was a single horizontal force at the theoretical center of pressure. In an actual dam the conditions are quite different, the water pressure being distributed over the entire up-stream face.

If the wings of an actual gravity dam are to transmit overturning loads to the central portion, they must undergo an appreciable down-stream deflection relatively to that central portion. When the wings are keyed into the canyon sides, they are thereby prevented from taking such a deflection. In other words, the wing foundations absorb the wing loads before they can be transmitted to the central portion of the structure.

As a matter of fact, since the central part of a dam is usually the greatest distance above the foundation, the crest deflection in the center, due to water load, is usually greater than that of the wings, so that part of the load on the central portion would tend to be transferred to the wings, rather than the reverse. This load transfer would be by trans-canyon beam action, which is considerably modified by the presence of contraction joints, with possibly some arch action.

---

[6] South Berkeley, Calif.

Contraction joints would prevent the dam curved down stream from acting as a unit, and the stabilizing moment computed on the basis of monolithic action would not be realized. The greater deflection of the central portion under load would tend to pull it down streamward away from the wings, thus preventing monolithic action; and it would be unsafe to attribute to such a structure greater stability than that of a straight dam having the same cross-section.

Professor Lambert's reference to the causes of failure of the St. Francis Dam appear to be well taken. In this case the water pressures on the wings actually were transmitted to the central portion, but only because the defective foundations of the wings permitted this to occur by allowing the prerequisite down-stream deflections to take place. In such a case, it is easy to show mathematically that the toe stresses in the central portion can approach the ultimate strength of the material and cause spalling, such as actually occurred in the St. Francis Dam. The central portion of the dam barely escaped being carried away; it was relieved of the extra load which caused the toe to spall off, by the breaking off of the wings. According to the writer's computations, if the dam had been straight, the central toe stresses, while high, would have been insufficient to cause spalling.

The author's clean-cut presentation of this subject demonstrates the importance of a more critical examination of these structures than they sometimes receive, especially as regards the effects of foundation conditions on stability and stress.

B. F. JAKOBSEN,[7] ESQ. (by letter).—The experimental investigations reported by the author are interesting and illustrative, and to many engineers, no doubt, more satisfactory than purely theoretical investigations. The experimental investigations are based on the dam as a continuous whole, and the results are just what theory leads one to expect. The writer considered the effect of arching a gravity dam some years ago, treating the dam as a whole, and concluded[8] that,

> "Arching in this case, therefore, means that a definite disadvantage, the reduction in stability, is exchanged for a very problematical gain due to arch action."

When a thin slice of dam is considered, as in the customary analysis, it can be readily shown that arching decreases the gravity action.[9] P. Wilhelm Werner, Assoc. M. Am. Soc. C. E., has shown the increase in stability which is obtained when the gravity dam is arched down stream instead of up stream, as is usual.[10] He applies his formulas to the Exchequer Dam and compares the stresses found with those in the dam as built, arched up stream.

---

[7] Cons. Engr., Los Angeles, Calif.

[8] "Arching Not Effective in Slightly Arched Gravity Dams," *Engineering News-Record*, June 12, 1924, p. 1020.

[9] "Notes on Arched Gravity Dams," by B. F. Jakobsen, *Transactions*, Am. Soc. C. E., Vol. 92 (1928), p. 766.

[10] *Transactions*, Am. Soc. C. E., Vol. 92 (1928), p. 784.

When considering the dam as a whole, it is necessary, as the author points out, to bear in mind that shrinkage cracks or planes of weakness may exist, so that the dam, in fact, can not act as a whole. This might be overcome, of course, by constructing proper contraction joints, or by grouting, or by both methods. When a dam rotates on its toe, it is difficult to determine with any degree of precision what the stresses are at the contact between the toe and the foundation. This would depend on the load to be carried by the toe and on the deformations of the concrete and the rock. In the case of the St. Francis Dam there were evidently definite planes of weakness, more or less vertical and radial, so that the dam did not act as a whole; and, furthermore, there appears to have been spawling at the toe of the maximum section. On the whole the safer design will generally result when thin sections of a dam are considered. The only exception to this may be where the height of the dam changes rapidly, so that the longer cantilevers lean upon the shorter, as the writer has discussed elsewhere.[11] Small dams might conceivably be considered as a whole, especially if the abutments are not too steep.

The writer does not look with favor on Professor Lambert's suggestion of anchoring the dam by steel tension members. It might be useful and acceptable where the rock is especially good and free from fractures and for a more or less temporary structure.

The tests are a welcome addition to engineering literature, and it is gratifying to see that they agree so well with theoretical deductions.

A. Floris,[12] Esq. (by letter).—In this paper the author attempts to investigate the statical behavior of gravity dams in relation to the topography of the site. This method seems to be quite logical. However, it requires that the structure be analyzed as a whole instead of by the customary practice of considering vertical slices only. This last simplification is made for the purpose of facilitating the stress analysis of gravity dams; but, nevertheless, an exact stress determination with regard to the foundation conditions of the dam is not yet available.

By proposing to consider the entire dam as acting in one piece, the author introduces additional difficulties which render even an approximate analysis of gravity dams very difficult. He restricts himself to the study of overturning movements of concrete dam models with special reference to their site, but his observations are not expressed in a precise mathematical form. In the paper, general methods are not given by which the limiting cases can be ascertained and thus the place at which the dam begins to overturn, or at which failure begins to occur due to excessive stresses, cannot be determined.

In a rather unsatisfactory manner Professor Lambert considers equilibrium conditions only, as if he was dealing with perfectly unelastic bodies. How great are the stresses and how are they distributed under the influence of the weight of the dam and the water pressure before overturning? These questions are not answered satisfactorily. Those who advocate innovations in the design and analysis of structures, must follow their ideas to the end and solve the problem completely. The study of the stability of dams by the aid of relatively large models, as suggested by Professor Lambert, are of no value, if an adequate theory of the relation between dam and site is lacking.

---

[11] *Transactions*, Am. Soc. C. E., Vol. 92 (1928), p. 835.

[12] Civ. Engr., Los Angeles, Calif.

A fair comparison between dams curved up stream and those curved down stream can be made, if the structure is considered purely as a gravity dam. Otherwise, the dam curved down stream is decidedly at a disadvantage over the one curved up stream. As dams are made from concrete, a material of low tensile strength, it follows that the ones curved down stream (which are cylinders subjected mostly to tension) are inferior to those curved up stream, stressed mainly by compression. The latter are only arches and, therefore, the author's usage of the expression, "gravity dams arched down stream," is somewhat misleading.

The standing section of the St. Francis Dam does not bear testimony in favor of either the down-stream or the up-stream forms. It indicates merely that in dams with a great radius of curvature in plan, gravity action only can be relied upon. Apparently, in the central part of the St. Francis Dam, the foundation conditions were better than in the two wings. This fact was stressed in the various reports on the failure of this dam.[13]

The suggested use of chains for the purpose of preventing gravity dams from overturning and sliding, is quite unusual and, it may be said, out of the question from purely practical considerations. Gravity dams with their great quantity of concrete, implying strength and durability, are incompatible with thin steel chains having a restricted life of usefulness. From the theoretical point of view these chains complicate the stress analysis of gravity dams, which is a fact overlooked by Professor Lambert.

As far as the writer knows, gravity dams curved down stream were first suggested[14] by P. Wilhelm Werner, Assoc. M. Am. Soc. C. E.

Floyd A. Nagler,[15] M. Am. Soc. C. E. (by letter).—The question raised by Professor Lambert with regard to the arching of gravity dams, is quite valid for dams built between narrow canyon walls with the abutments parallel to the canyon wall itself. However, the abutments of such dams would generally be seated into the canyon wall, and the dam would no longer act strictly as a gravity dam. In all such cases it would be preferable to arch the dam up stream rather than down stream, in spite of the considerations to the contrary for the true gravity type, which have been presented by the author.

Regardless of the fact that gravity dams are generally analyzed in linear-foot sections one should never fail to consider the behavior of the dam as a whole. This has received too meager emphasis in most textbooks, although it is casually mentioned by some authors. Some engineers assert that a large dam does not act as a unit, as assumed by the author, but one must answer the question as to how large a dam must be before it will fail to do so. The small models described by Professor Lambert certainly acted as a unit, and if large dams do not do so, somewhere in the range of size of masonry dams, there must be a structure so large that no shearing stress exists between adjoining sections. This is absurd. Many gravity dams known to the writer still stand because they are acting as monoliths, whereas erosion has so seriously undercut certain sections that if considered by themselves they should have failed long ago. The evidence disclosed by the St. Francis Dam disaster indicates that the shearing stress between adjacent sections of the dam must have been exceedingly great. Although gravity dams have numerous construction and expansion joints, interlocking generally exists so that the dam can function more or less as a unit. Hence, the author's contentions should stimulate serious consideration.

---

[13] "Essential Facts Concerning the Failure of the St. Francis Dam," Report of Committee of Board of Direction, *Proceedings*, Am. Soc. C. E., October, 1929, Papers and Discussions, p. 2147.

[14] *Transactions*, Am. Soc. C. E., Vol. 92 (1928), p. 784.

[15] Prof., Hydr. Eng., Univ. of Iowa, Iowa City, Iowa.

B. J. LAMBERT,[16] M. AM. SOC. C. E. (by letter).—The writer would like to make it clear that he believes that a gravity dam arched down stream can be used only in a canyon having particular features as to width of canyon, slope of walls, and height of dam. This might occur only once in a hundred or more locations, so the discussion is more or less academic. This scheme would be ridiculous if it were applied to the average canyon dam, regardless of the factors mentioned.

There also seems to be a difference of opinion as to the size of dam that ceases to act as a unit and begins to act in sections. It is true that English-speaking engineers have become accustomed, in using the pressure formula, $\frac{wh^2}{2}$, to define w as pounds per cubic foot of water and h, in feet, thus implying slices 1 ft. in width; but in the metric system, as used in France, Germany, Spain, etc., the terms, w and h, are based on the meter, in which the volume is about thirty-five times as great as that based on the foot. It is not unreasonable to suppose that the "Brobdingnagians" used a linear unit equal to ten or even one hundred times that of American units, with a corresponding volume equal to 1 000 to 1 000 000 cu. ft.

It would seem almost obvious that the sizes of the blocks into which a dam might break is a function of the thickness of the dam. If the average thickness is a few inches or a few feet, the sizes of the blocks might well be a few cubic inches or a few cubic feet; but when dealing with dams several hundred feet thick it seems reasonable to assume that the blocks or units that might be considered, would be of a much larger volume.

Where the back of a dam is constantly covered with water to a greater or less extent, the temperature changes may not be extreme through a large portion of the structure, so that the breaking into blocks is more or less limited by the shrinkage incident to the setting up of the concrete. If these contractions are under fair control, as will be attempted in the Hoover Dam, it is reasonable to suggest that possibly this dam can act practically as a unit, since it will have a maximum thickness of 650 ft., with an average of, say, 300 ft., or more, an average length of about 500 ft., and a maximum height of approximately 650 ft. If this huge plug of concrete were constructed in a canyon with smooth sloping walls, as shown in Fig. 2(*a*), there would be little doubt in the writer's mind, that the stresses on the smaller length of toe would be much in excess of those computed by the ordinary slice method. However, when one considers that, in practice, the sloping walls are roughened and also made more or less normal to the radius of the arch plan, then these modified conditions make the problem so complex that even an approximately exact analysis is impossible.

---

[16] Head, Dept., Civ. Eng., State Univ. of Iowa, Iowa City, Iowa.

It is not difficult to imagine various things that might be occurring in a structure where such immense masses, pressures, and uncertain conditions are involved, therefore, in presenting this brief paper, it was the writer's intention to set out merely another point of view, the merits of which may be dependent on many debatable factors.

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
TRANSACTIONS

Paper No. 1814

FLOW OF GROUND-WATER AS APPLIED TO DRAINAGE WELLS[1]

BY M. R. LEWIS,[2] M. AM. SOC. C. E.

WITH DISCUSSION BY MESSRS. L. STANDISH HALL, O. W. ISRAELSEN, AND M. R. LEWIS.

SYNOPSIS

Much study has been given to the capacity of wells to furnish water and oil, and, with the use of deep wells for the drainage of irrigated lands, the shape and extent of the cone of depression become of great importance.

In this paper three types of wells are discussed: (*A*) Artesian wells (not necessarily flowing), with a perforated casing extending through the water-bearing stratum; (*B*) wells in which the water-table is in the water-bearing stratum and which penetrate its full depth; and (*C*) open-bottom wells, that is, wells which just reach the water-bearing stratum.

Cases in which the zone of influence is either definite or indefinite are considered for each type; and formulas for the draw-down curves of each of these types of wells are derived.

The conclusion is drawn that if the area to be drained is more than a few hundred feet in diameter, successful drainage will depend on a general lowering of the water-table.

To secure this lowering, wells should be designed to have the greatest possible capacity with an economical lift. Such capacity may be secured most readily, where conditions are favorable, by deep wells.

INTRODUCTION

Much study has been given to the theory of the flow of ground-water and oil into wells. Practically all this study has been directed toward estimating the quantity of water or oil which might be secured from wells under conditions differing as to diameter and depth, thickness and character of water-bearing strata, and depth of draw-down. Since wells and deep-well pumps have been used for the drainage of irrigated lands, the shape and extent of the draw-down curve or cone of depression have become of primary importance.

---

[1] Published in March, 1931, *Proceedings*.

[2] Irrig. and Drainage Specialist, Oregon State Coll., and Div. of Agri. Eng., U. S. Dept. of Agriculture, Corvallis, Ore.

It is difficult, if not impossible, to determine by field studies and laboratory tests the exact capacity for the transmission of water of the water-bearing strata in any given case. In spite of this fact, however, it is well worth while to consider the effect on the draw-down curve of changes in the various controlling factors under certain assumed conditions.

The first assumption which is made in such studies is that the water-bearing strata are uniform. This will be approximately true in most cases, but far from correct in others. Since the variations in this regard may be infinite in number, and are impossible of determination, it does not seem worth while to attempt a discussion of such cases.

A second assumption, which is applied in the case of those wells that penetrate the full depth of the water-bearing stratum, is that the water pressure is transmitted without hindrance in a vertical direction and that it corresponds to static pressure in any given vertical line; in other words, that the water would rise to the same level in each of a series of piezometer tubes open to the ground-water at various elevations below the water-table in the same vertical. This assumption is probably true in regard to artesian wells that extend through a water-bearing stratum, when the retaining beds are impervious. When the water-table is in the water-bearing stratum, the lines of flow will not be horizontal, and the second assumption is not correct. However, only in the case of wells in which the draw-down is large as compared with the depth of the water-bearing strata—and even then only near the well where the slope of the water-table is great—will the error be appreciable.

In this paper the piezometric surface and water-table are assumed to be level before pumping commences. From this assumption it follows that there is no flow of the ground-water except to the well.

In the case of the drainage wells designated herein by the secondary letter, *b*, the assumption that the quantity of water reaching the water-table is uniform per unit of land area, seems to require either that the well under consideration is in the interior of a homogeneous group of wells the zones of influence of which are contiguous, or that the water-table at the boundary of the zone of influence is at the surface of the ground.

It is evident that these assumptions in themselves limit the rigorous application of the formulas to very special cases. An exact application of the laws of hydrodynamics would require consideration of the effect of inertia and of the probable inapplicability of the Darcy law to extremely high velocities. The writer has made no attempt to do this. It is believed, on the other hand, that the general relationship between the variables considered are correctly presented and have rather wide application.

The fact that the cost of power is a controlling factor in irrigation pumping makes the total extent of the draw-down in irrigation wells a matter of first consideration. In the case of drainage wells, the extent of the draw-down at the wells and throughout the drained area is of primary importance. The cost of power (except for variations in demand charges, pump and motor efficiencies, and friction losses in the piping) is proportional to the total height through which water must be lifted. This total lift is made up of the distance the water must be lifted above the static water-table, plus the total draw-down.

The former is fixed by the topography and other considerations, but is not affected by changes in the well. The total draw-down is made up of the friction in the well tube between the point where the water enters the well and the entrance to the suction pipe of the pump, plus the loss of head at the perforated well-casing or strainer, if one is used, or the entrance head into the well tube, plus the loss of head through the water-bearing strata. Under normal conditions the last item is by far the largest part of the total draw-down and is the only part considered herein. The loss of head at the strainer is known to have been an important factor in many instances.

The shape of the draw-down curve will depend on the size and character of the well and on its relation to the water-bearing strata and the source of water supply. Theoretical formulas and curves for three types of wells are given herein. For each type the effect of a local or distant water supply is taken up. Furthermore, the effect of changes in the character and thickness of the water-bearing strata, of the diameter and depth of the well, and of the quantity of water pumped, is illustrated.

NOTATION

In addition to the symbols, defined by diagrams in Figs. 1, 4 and 6, the following notation is adopted for use in this paper and its discussion:

C = a constant.
d = diameter of sand particles, in millimeters.
k = transmission coefficient in water-bearing strata.
k_1 = transmission coefficient in relatively impervious overlying stratum.
K = a factor that varies with the porosity of the material.
Q = quantity of water passing concentric surfaces of radius, r.
Q_1 = the discharge of the well.
V = velocity of a stream of water.
w = the rate of downward percolation.

In order to facilitate reference, various types of wells are identified by symbols in the paper. Briefly, the definitions of these symbols are, as follows:

Type *Aa* is an artesian well with a perforated casing that extends through the water-bearing stratum. This stratum is assumed to lie between two impervious retaining beds and contains water under such hydrostatic pressure that the water level just outside the casing will not be lowered, by pumping, below the upper edge of the stratum.

Type *Ab* is an artesian well with a perforated casing that extends through the water-bearing stratum. In this case the overlying stratum is only relatively impervious and the supply to the well is derived by vertical flow, or percolation.

Type *B* consists of wells in which the water-table is in the water-bearing stratum and which penetrate the full depth of the stratum.

Type *C* is an open-bottom well which just reaches the water-bearing stratum.

The relations between Types *Ba* and *Bb* and between Types *Ca* and *Cb* are similar to that between Types *Aa* and *Ab*.

TYPE *A*.—ARTESIAN WELLS EXTENDING THROUGH WATER-BEARING STRATUM

The case of the simplest well (Type *Aa*) has been discussed by Slichter,[3] Turneaure,[4] and others. It is that of a well penetrating the full depth of a water-bearing stratum lying between two relatively impervious retaining beds and containing water under such hydrostatic pressure that the water level in the well, or rather just outside the well-casing, will not be lowered, by pumping, below the upper edge of the water-bearing stratum. In each case these writers have assumed that all the water came into the stratum from outside the zone of influence of the well. Under such conditions, the curve of the modified piezometric surface would rise indefinitely, but at a decreasing rate, as the distance from the well is increased. The water flowing toward the well will pass through concentric cylindrical surfaces in the water-bearing stratum, having areas of $2\pi D r$ (for notation, see Fig. 1). The velocity of flow according to the well-known law of Darcy will be proportional to the slope of the hydraulic gradient. For convenience, the transmission coefficient, *k*, used by Slichter,[3] and defined as the quantity of water passed in unit time by a prism of the material of unit cross-sectional area and unit length under the pressure of a column of water of unit height—using as units, the foot and the minute—has been adopted for this paper. The velocity of a hypothetical stream of water occupying the full area of the cross-section at any distance from the well is, therefore,

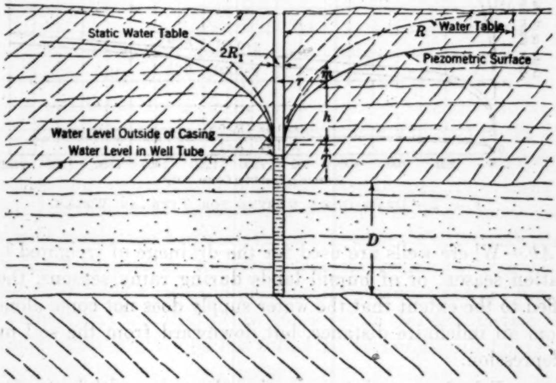


FIG. 1.—TYPE *A* WELL.

$$V = k \frac{dh}{dr} \qquad (1)$$

The quantity, *Q*, flowing toward the well, is equal to the discharge of the well, Q_1, and is constant for all values of *r;* therefore,

$$Q = Q_1 = 2\pi D r k \frac{dh}{dr} \qquad (2)$$

---

[3] "Theoretical Investigation of the Motion of Ground Waters", by C. S. Slichter, U. S. Geological Survey, Nineteenth Annual Rept., Pt. 2, p. 323, Washington, D. C., 1899.

[4] "Public Water Supplies", by F. E. Turneaure and H. L. Russell, Third Edition, p. 262, John Wiley & Sons, New York, 1927.

Integrating,

$$h = \frac{Q_1}{2\,\pi\,D\,k}\log_e r + C \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3)$$

When $r = R_1$, $h = 0$; and if $R_1 = 1$, $C = 0$. If R_1 has any other value, C will vary with changes in the values of the other constants.

On Fig. 2 the draw-down curves for certain assumed values of Q_1, D, R_1, and k are shown.

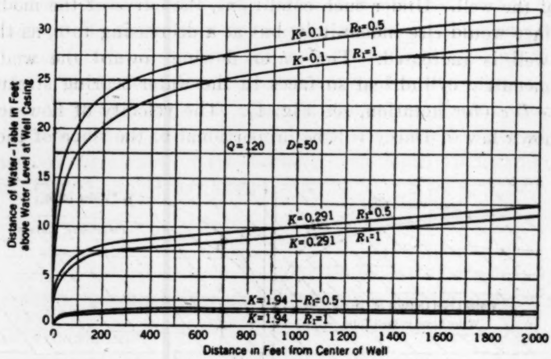


FIG. 2.—DRAW-DOWN CURVES FOR TYPE *Aa* WELLS.

Type Ab.—Where wells are used for the drainage of irrigated lands during the irrigation season, or of humid lands during rainy seasons, the conditions are modified to the extent that the water supply does not come along horizontal planes from an indefinite distance, but downward from the soil overlying the cone of depression.

When a well is first put into operation the water withdrawn from the cone of depression will most probably reach the water-bearing stratum at a greater rate per unit of land area near the well than at a greater distance. After the water-table has been lowered to an equilibrium position the supply will come entirely from the surface. The rate of downward percolation in practically all cases will be governed by surface conditions and will be substantially independent of the draw-down. For this reason a uniform rate is assumed.

If w is the rate of downward percolation, in cubic feet per minute per square foot of horizontal area, the conditions are expressed by the following formulas:

$$Q = Q_1 - w\,\pi\,r^2 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (4)$$

in which, Q_1 is the discharge of the well and Q, the variable quantity passing the concentric cylindrical surfaces of the radius, r. Substituting for Q in Equation (4), its equivalent in Equation (2):

$$Q_1 = 2\,\pi\,D\,r\,k\,\frac{dh}{dr} + w\;\pi\;r$$

$$Q_1\,dr = 2\,\pi\,D\,r\,k\,dh + w\,\pi\,r^2\,dr$$

$$dh = \frac{Q_1}{2\,\pi\,D\,r\,k}\,dr - \frac{w\,r}{2\,D\,k}\,dr$$

and,

$$h = \frac{Q_1}{2\,\pi\,D\,k}\log_e r - \frac{w}{4\,D\,k}\,r^2 + C \quad\text{..............}(5)$$

If w is assumed as any probable value; as, for example, 0.000017 (3.67 ft. in depth over the surface in 150 days), and if $R_1 = 1$ ft., the value of C will be practically zero. With other values of R_1, C will be the same in this case as in the one preceding. It will be noted that the factor, w, is somewhat analogous to the transmission coefficient, k. Its value is so small compared to the common values of k that ordinarily there will be no inconsistency in the assumption that the percolating water moves downward through the relatively impervious stratum. The zone of influence of the well in this case will be governed by the values of w and Q_1 in the following relation:

$$Q_1 = w\,\pi\,R^2 \quad\text{..........................}(6)$$

in which, R equals the radius of the zone of influence.

The water-table will stand above the piezometric surface, as both are depressed by pumping, because of the head required to force the water downward through the upper stratum. The movement will not be in an exactly vertical direction as long as the piezometric surface slopes toward the well. However, as the upper stratum is relatively much more impervious than the lower one, vertical flow may be assumed without serious error, except very close to the well where the slope of the water-table is extremely steep. Let k_1 represent the transmission coefficient for the overlying stratum; m, the height of the water-table above the piezometric surface; and T, the thickness of the more impervious material below the level of the piezometric surface at the well. Under these conditions the following formulas will hold:

$$w = \frac{m}{m + h + T}\,k_1 \quad\text{........................}(7)$$

$$m = \frac{h + T}{\dfrac{k_1}{w} - 1} \quad\text{........................}(8)$$

and,

$$m + h = \frac{h\,k_1}{k_1 - w} + \frac{w\,T}{k_1 - w} \quad\text{..................}(9)$$

Substituting in Equation (9) the value of h given in Equation (5),

$$m + h = \left(\frac{Q_1}{2\,\pi\,D\,k}\log_e r - \frac{w\,r^2}{4\,D\,k} + C\right)\frac{k_1}{k_1 - w} + \frac{w\,T}{k_1 - w} \quad\text{....}(10)$$

Equations (5) and (10) will hold only for values of r equal to, or less than, R. (See Fig. 3 for typical results.)

TYPE *B*.—WELLS IN WHICH WATER-TABLE IS IN WATER-BEARING STRATUM

Another common case (Type *Ba*) is illustrated in Fig. 4. The water-table, both under the static condition and during pumping, lies wholly within the water-bearing stratum. Turneaure[4] has shown that the formula for the draw-

down curve in this case is (adopting the notation used herein and shown in Fig. 4):

$$h^2 = \frac{Q_1}{\pi k} \log_e r + C \ldots\ldots\ldots\ldots\ldots\ldots\ldots (11)$$

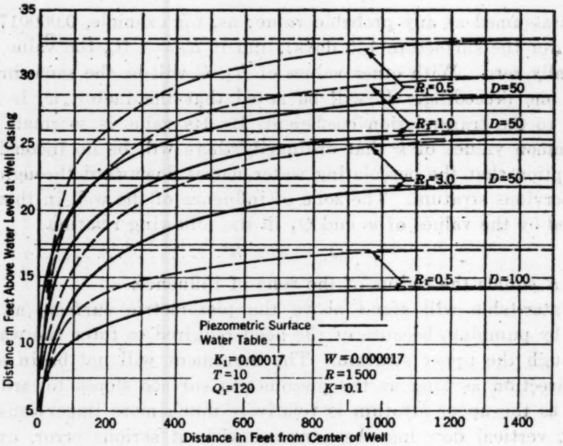


FIG. 3.—DRAW-DOWN CURVES FOR TYPE *Ab* WELLS.

In Equation (11), when $r = R_1$, $h = D$; and, if $r = R_1 = 1$, then $C = D^2$. Here, again, the value of C will vary with the other factors if R_1 does not equal 1.

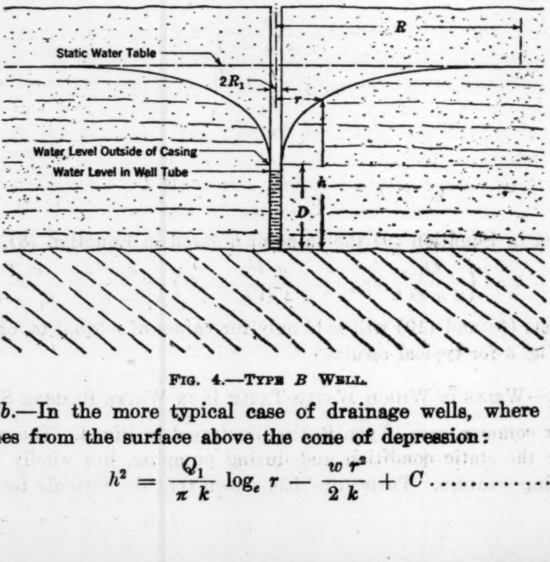

R
Static Water Table
$2R_1$
r
Water Level Outside of Casing
Water Level in Well Tube
h
D

FIG. 4.—TYPE *B* WELL.

***Type Bb.*—In the more typical case of drainage wells, where the water supply comes from the surface above the cone of depression:**

$$h^2 = \frac{Q_1}{\pi k} \log_e r - \frac{w r^2}{2 k} + C \ldots\ldots\ldots\ldots\ldots (12)$$

Again, when $r = R_1$, $h = D$; and if $r = R_1 = 1$, the second term on the right of the equation is practically zero and $C = D^2$. Fig. 5 shows some typical draw-down curves for this case.

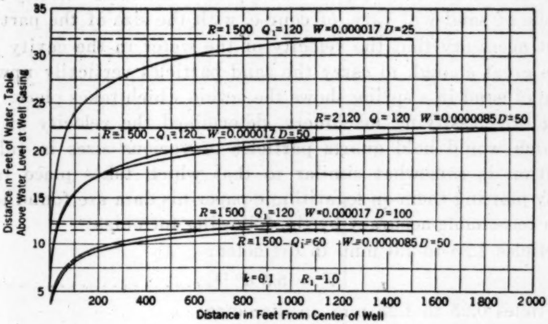


FIG. 5.—DRAW-DOWN CURVES FOR TYPE *Bb* WELLS.

TYPE *C*.—OPEN-BOTTOM WELLS DRIVEN JUST TO THE WATER-BEARING STRATUM

A third type of well (Type *Ca*) is illustrated in Fig. 6. This class includes wells which are drilled through a consolidated stratum to tap an artesian flow in a water-bearing stratum of sand or gravel. It is usual in developing wells of this type to pump out more or less sand. If the overlying bed is strong enough to stand up, the resulting cavity in the sand below will be cone-shaped with the point downward. The slope of the sides will be that of

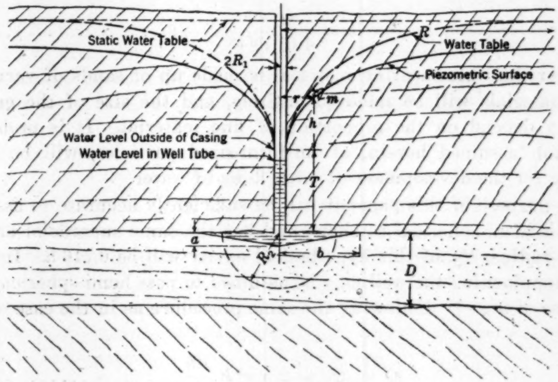

Static Water Table
2R1
R
Water Table
Piezometric Surface
r
m
h
Water Level Outside of Casing
Water Level in Well Tube
T
a
R
b
D

FIG. 6.—TYPE *C* WELL.

the angle of repose of the material when supersaturated with water. Observation of sand which is discharging water when excavated below the water-table indicates that the slope is about 1 on 12. This cavity will increase in size until the velocity of the water leaving the surface of the sand is just insufficient to lift the finer particles. In a mixed material the finer particles will be washed out, leaving the cavity lined with the coarser particles. For this reason it is not necessary in all cases to excavate a shaft of large diameter to secure the maximum efficiency of this type of well. The velocity required to lift a particle of sand will vary, of course, with the size of the particle.

It is not necessary that the velocity of the water in the cavity just above the sand be great enough to carry the sand particles vertically upward. The bubbling up of sand in a spring shows the action which takes place. Richards,[5] in studying ore-classifying machinery, determined the velocity of the rising streams which would hold quartz particles of various sizes in full "teeter". This condition is somewhat similar to that which takes place in wells or springs. By plotting them on logarithmic paper his data are found to be represented with reasonable accuracy by the following formulas:

For particles 1.25 to 7.0 mm. in diameter:

$$V = 5.5\, d^{0.75} \qquad \text{(13)}$$

For particles 0.25 to 1.25 mm. in diameter:

$$V = 5.1\, d^{1.15} \qquad \text{(14)}$$

in which, V equals the velocity of the rising stream, in feet per minute, and d is the diameter of the quartz particles, in millimeters.

The radius, b, in feet, of the cavity in the sand stratum for the two grades of sand will then be as follows:

For particles of 1.25 to 7.0 mm.:

$$b = \frac{Q_1^{\ 0.5}}{4.16\, d^{0.38}} \qquad \text{(15)}$$

For particles of 0.25 to 1.25 mm.:

$$b = \frac{Q_1^{\ 0.5}}{4.01\, d^{0.58}} \qquad \text{(16)}$$

Whenever the water-bearing material is made up of sands of various sizes, the finer materials will be removed entirely, and the size of the cavity will undoubtedly depend on the proportion of fine material. It is probable that the condition assumed herein, of uniform-sized particles, will be the most unfavorable for the development of large flows of water.

For the purpose of this paper it will be sufficiently accurate to assume that the cavity is hemi-spherical in shape and has the same superficial area as the cone. The radius, R_2, of this hypothetical cavity will be 0.716 b. In this case the water approaching the well may be assumed to pass hemi-spherical surfaces having areas of $2\,\pi\,r^2$. Following the same procedure as in the case of wells of other types,

$$Q = Q_1 = 2\,\pi\,r^2 k\ \frac{dh}{dr} \qquad \text{(17)}$$

and,

$$h = -\frac{Q_1}{2\,\pi\,k\,r} + C \qquad \text{(18)}$$

---

[5] "Development of Hindred-Settling Apparatus", by Robert H. Richards, *Transactions*, Am. Inst. Min. and Met. Engrs., Vol. 41, p. 425, 1910.

If $r = R_2$, $h = 0$,

$$C = \frac{Q_1}{2\,\pi\,k\,R_2} \qquad (19)$$

Equation (18) will hold only if the water-bearing stratum is of very great depth. If this stratum is of moderate depth, an approximate solution may be reached by using Equation (18) for values of r between 0 and D and Equation (3) for values of r greater than D.

Type Cb.—For the drainage well there will again be an approximation if the term is subtracted that accounts for water reaching the water-table from the surface; thus,

$$Q = Q_1 - w\,\pi\,r^2$$

$$Q_1 = 2\,\pi\,r^2\,k\,\frac{dh}{dr} + w\,\pi\,r^2$$

$$dh = \frac{Q_1}{2\,\pi\,k\,r^2}\,dr - \frac{w}{2\,k}\,dr$$

and,

$$h = -\frac{Q_1}{2\,\pi\,k\,r} - \frac{w\,r}{2\,k} + C \qquad (20)$$

Again, the second term on the right is negligible for small values of r and practically so for all values.

The same relation between the piezometric surface and the water-table holds in this case as in that of wells of Type *Ab*. (See Fig. 7 for typical curves.) The height of the water-table will be:

$$m + h = \left(-\frac{Q_1}{2\,\pi\,k\,r} - \frac{w\,r}{2\,k} + C\right)\frac{k_1}{k_1 - w} + \frac{w\,T}{k_1 - w} \qquad (21)$$

Another type of well which has been used in some faint-hearted attempts at drainage, and one that is fairly common in the irrigation field, is the dug well which just reaches into the water-table and is pumped to its utmost capacity. The draw-down curves for wells of this type would be quite similar to those of large diameter wells of Type *C*. The flow of water through the stratum above the level of the bottom of the well would tend, of course, to make the curve flatten out faster as r is increased.

PERMEABILITY OF WATER-BEARING STRATA

Many experimenters have worked on the problem of the rate at which soils, sands, and gravels will transmit water. Much of this study has been directed toward establishing some formula which would make possible the accurate prediction of permeability from an analysis of the physical characteristics of the material. Thus far no formula satisfactory over a wide range of materials and conditions has been found. The one devised by Slichter[6] is probably as satisfactory as any. It is as follows:

$$k = (1.1846)\,\frac{d^2}{K} \qquad (22)$$

---

[6] "Theoretical Investigation of the Motion of Ground-Waters." U. S. Geological Survey, Nineteenth Annual Rept., Pt. 2, p. 322, Wash., 1899; "The Motions of Underground Waters", U. S. Geological Survey, *Water Supply Paper* 67, Wash., 1902; "Field Measurements of the Rate of Movement of Underground Water", U. S. Geological Survey, *Water Supply Paper 140*, Wash., 1905.

in which, k is the transmission coefficient, in cubic feet per minute, per square foot of cross-section, per foot of head, through a column, 1 ft. long. The number, (1.1846), is the common logarithm of a constant factor or coefficient. Factor K varies with the porosity of the material. Equation (21) was used to determine the value of k corresponding to the given effective sizes and porosities in the computations for wells of Type C.

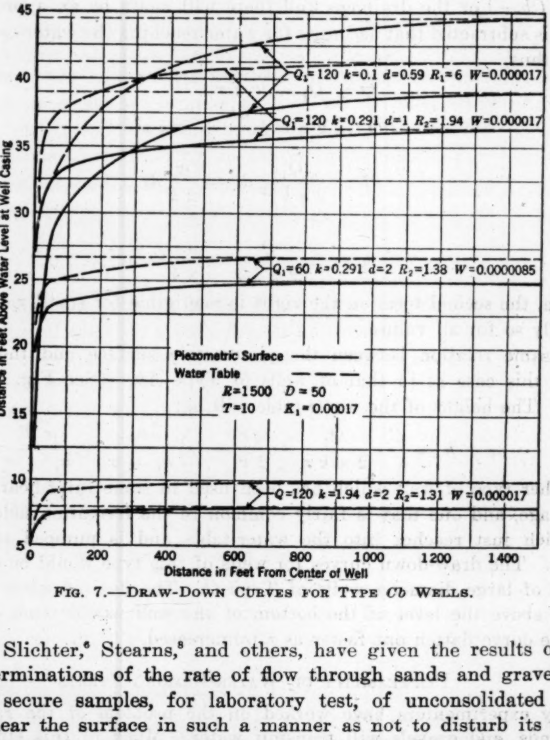


FIG. 7.—DRAW-DOWN CURVES FOR TYPE *Cb* WELLS.

Hazen,[7] Slichter,[6] Stearns,[8] and others, have given the results of experimental determinations of the rate of flow through sands and gravels. It is difficult to secure samples, for laboratory test, of unconsolidated material occurring near the surface in such a manner as not to disturb its structure and porosity. Such samples cannot be secured of the deeper-lying strata. In spite of this difficulty laboratory tests of the rate of flow seem to be much more satisfactory than the attempt to compute the rate from mechanical analyses.

Meinzer[9] refers to a method devised by Thiem, which appears to promise good results. In this method use is made of the slope of the water-table toward a well which is being pumped.

---

[7] "Some Physical Properties of Sands and Gravels with Special Reference to Their Use in Filtration", by the late Allen Hazen, M. Am. Soc. C. E., Massachusetts State Board of Health, Twenty-fourth Annual Rept., 1893.

[8] "Laboratory Tests on Physical Properties of Water-Bearing Materials", by Nora D. Stearns, U. S. Geological Survey, *Water Supply Paper 596 F.*, Wash., 1927.

[9] "Methods of Estimating Ground-Water Supplies, Pt. I" : Contribution to a Symposium, by O. E. Meinzer, Annual Technical Session, Soc. of Economic Geologists, 1928.

APPLICATION OF CURVES

The most obvious conclusion to be drawn from the curves and data shown herein is that satisfactory drainage by wells depends on the removal of sufficient water to lower the general ground-water level. This, of course, agrees with the general opinion of engineers engaged in the drainage of irrigated lands by open ditches and tile-drains. It will be noted that in none of the plotted curves in which the wells are taking all the water supplied to the ground-water in an area having a radius of 1 500 ft., is satisfactory drainage secured for more than 300 ft. from the well. In this statement "satisfactory drainage" is understood to mean a lowering of the water-table 5 ft., or more. In the case of a well discharging 360 cu. ft. per min., the radius of the circle with a 5-ft. lowering of the water-table is almost 800 ft., with a total draw-down of 31.4 ft.

Plotted curves from successful drainage wells bear out these results. Where such wells have accomplished satisfactory drainage over a considerable area there is evidence that the general level of the water-table has been affected. The plotted curves are very flat at and beyond the point where the water-table has been lowered sufficiently to provide adequate drainage. In some instances, local pot-holes may be drained by single wells which have little or no effect on the general level of the water-table.

It is evident that such a well should be designed so as to reduce the draw-down close to the well as much as possible and still maintain a sufficient depth to the water-table at the limits of the area to be drained. A comparatively shallow well appears to be most advantageous for this purpose. The quantity of water pumped must be sufficient to cause the required draw-down in the area to be drained.

Whenever that area has a radius of more than a few hundred feet, it would seem to be necessary to plan on a more or less general lowering of the water-table. To secure this effect, the wells should be designed on the basis of the maximum economic capacity.

Inspection of the formulas and of the resulting curves shows that in each class of wells the quantity of water delivered is proportional to the transmission coefficient. An increase in the depth of water-bearing strata penetrated is followed by a proportional increase of flow in Type *A* wells and deeper Type *B* wells. In shallow wells of the latter type the effect will depend on the relation between the depth of draw-down and the total depth of the well. In wells of Type *A* and Type *B* a change in the diameter of the well has a comparatively small effect on the capacity.

Note in Fig. 3 that the total draw-down of a given well, 1 ft. in diameter and penetrating 50 ft. of water-bearing strata, is 32.9 ft. Under otherwise similar conditions the draw-down of a well of six times the diameter penetrating the same depth of strata is 25.3 ft. (77% as great) while that of a well of the same diameter and penetrating twice the depth of water-bearing strata is only 17.0 ft. (52% as great). In wells of Type *C* it is obvious that as long as the cavity at the bottom of the well has a greater diameter than that of the well tube the latter will not affect the draw-down curve. In ordinary materials this means that the diameter of the well is not a factor unless it is very large. As shown in Fig. 8, open-bottom wells, even if of large diameter, will have a relatively great draw-down. These computations indicate that under uniform conditions the capacity of wells may be much more readily increased by increasing the depth of water-bearing strata penetrated than by increasing the diameter.

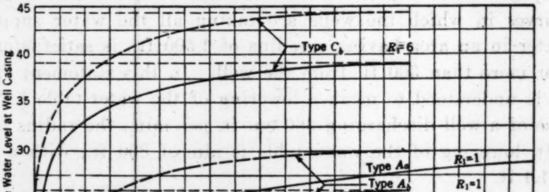


FIG. 8.—DRAW-DOWN CURVES.

When consideration is given to the fact that the strata lie normally in horizontal beds, the desirability of penetrating as deeply as possible into the zone of saturation, as long as the various water-bearing strata are interconnected, is emphasized.

ACKNOWLEDGMENTS

The author wishes to acknowledge the assistance of O. V. P. Stout, M. Am. Soc. C. E., in the preparation of this paper.

DISCUSSION

L. Standish Hall,[10] **Assoc. M. Am. Soc. C. E.** (by letter).—This paper is very timely as it offers a theoretical presentation of a subject for which real data are lacking to a large extent. It adds several formulas to the hydraulics of wells. The existing ones on this subject were unsatisfactory for general application since, as pointed out by the author, they covered only one or two special cases. The subject of the paper refers to drainage wells, but the formulas presented are of general application to wells, regardless of their use.

Although the formulas cover quite a range of cases, they are somewhat difficult of application in advance of the drilling and operation of a well, because of the difficulty in determining in advance the conditions which will be met. In fact, several other conditions relative to the flow of ground-water are possible in addition to those presented in the paper. For example, deep wells usually penetrate several water-bearing strata, the transmission coefficients of which are, in many cases, different for each stratum penetrated by the well. The Type *A* wells may penetrate several strata with downward percolation to some of the pervious beds and with no downward percolation to the remainder of the beds. A well may also belong to the Type *B* in the upper strata and to the Type *A* in the strata penetrated at greater depth. In pumping wells of this last type, the water level in the well during pumping may be drawn below the bottom of the upper strata, allowing a free discharge into the well from the strata thus exposed.

If a well should penetrate several minor water-bearing strata and be landed as an open-bottom well (Type *C*), a combination of this latter type with either Type *A* or Type *B* is possible. The formula presented by the author for the Type *C* well (Equation (20)), indicating a greater draw-down during pumping than for either the Type *A* or Type *B* well, as shown in Fig. 7, appears to be contrary to general opinion. The writer has made no actual measurements of the performance of these wells during pumping, but statements of owners indicate that the draw-down is usually small. A well not originally drilled as an open-bottom well may often evolve into this type at points where it penetrates unconsolidated strata, particularly if these strata are not cased off in the well. The writer has in mind an example of a well which was drilled originally to a depth of 194 ft. When it was first pumped, twelve wagon-loads of fine sand were discharged with the water. The present depth of the well is 141 ft., which is evidently the depth at which the main water-bearing strata were encountered. Soundings made with a 6-ft. iron rod indicated that a large cavity existed at this depth. The owner stated that pumping of the well did not lower the water level appreciably. Other similar instances could be cited.

With regard to the application of wells to the drainage of land—while they may furnish a large discharge, deep wells may not accomplish the purpose desired. The lands generally requiring drainage are at a low elevation and frequently receive the drainage from higher lands. If the deep wells penetrate sandy strata interbedded with impervious layers with the dip of the strata following the general ground slope, pumping may drain only strata outcropping on higher ground, particularly if the well is of the *A* type. Under this condition the pumping would lower only the piezometric surface and have very little effect in lowering the water-table in the territory requiring drainage.

---

[10] Chf. Hydrographer, East Bay Municipal Utility Dist., Oakland, Calif.

Even in cases where the deep wells penetrate pervious beds of sandy strata for their entire depth, they may be disadvantageous for the purposes of draining land. This is particularly true when wells that are being operated are considerable distances apart. In order to lower the water-table between the drainage wells, under these conditions, the water level in the vicinity of the well may have to be lowered excessively. This may result in the necessity of frequent irrigation of the lands near the wells, thereby partly defeating the purposes of drainage.

The shallow well that reaches just below the water-table and is pumped to its utmost capacity (which has been mentioned by the author) appears to be the most efficient for drainage. One difficulty presents itself in the case of the use of wells of this type. The yields of each individual unit may be small, thus requiring that they be spaced closely together. This necessitates many wells and the purchase of much pumping equipment, which in some cases may result in a prohibitive cost.

The use of drainage wells for lowering the water-table in the lower areas of irrigation projects is gaining in favor since it permits of the re-use of the water recovered by drainage, for irrigation of additional acreage. It also makes an irrigation system more flexible by providing an underground supply on which one may draw heavily during years of deficient surface run-off.

The subject presented by the author is one on which additional data need to be collected and available existing data need to be correlated.

O. W. ISRAELSEN,[11] M. AM. SOC. C. E. (by letter).—The study of flow of ground-water toward wells is not only of scientific interest, but also of real practical importance. Mr. Lewis is to be congratulated on the thorough and valuable analyses presented.

The term, transmission "constant," was used by Slichter, not transmission "coefficient." In defining the transmission constant or coefficient, care should be exercised to specify unit pressure head difference. That part of the author's definition ("Type of Wells") of the transmission coefficient which reads, "under the pressure of a column of water of unit height," is clearly incomplete.

In a paper entitled "The Drainage of Land Overlying Artesian Basins,"[12] by Messrs. Willard Gardner, W. W. McLaughlin, and the writer, an equation analogous to the author's Equation (3) was derived, namely:

$$h = \frac{Q_2}{\rho} \times \frac{\log_e \frac{r}{R_1}}{2\pi\, D g\, k_2} \ldots\ldots\ldots\ldots\ldots\ldots\ldots (23)$$

---

[11] Prof., Irrig. and Drainage Eng., Utah State Agri. Coll., Logan, Utah.

[12] *Soil Science*, Vol. 26, No. 1 (1928).

in which, in addition to the author's notation:

Q_s = mass of water per unit time.
ρ = density.
g = acceleration due to gravity.
$k_s = \frac{k}{g}$

The author's k has the physical dimensions of $\frac{L}{T}$. The symbol, k, is a transmission constant that has the physical dimensions of time; thus,

$$\frac{k}{g} = \frac{\frac{L}{T}}{\frac{L}{T^2}} = T.$$

In commenting on Equation (5) the author states that: "It will be noted that the factor, w, is somewhat analogous to the transmission coefficient, k." This statement could well be made more direct. As shown in Equation (7), w is proportional to the transmission coefficient of the overlying, relatively impervious material, k_1. The quantities, w, k, and k_1, are dimensionally the same.

Field studies of the cone of depression in the piezometric surface near the axis of the pump in an experimental drainage well in Utah, of the Type *Ab*, confirm in general the requirements of Equation (5).[13]

The author's Equations (10) and (21) and their applications are of special interest. It is important to note that for Type *Ab* wells "satisfactory drainage" at a given distance from a well is not only a function of h, but also of the transmission constant, k_1. Equation (8) shows that as k_1 increases, m decreases, for any given values of h, w, and T. Satisfactory drainage with the Type *b* wells is also dependent on the quantity of irrigation water applied, which influences the quantity, w.

Measurements of the piezometric surface, as influenced by pumping from a Type *Ab* well, in Utah, show significant reductions of pressure head at distances considerably more than 1 000 ft. In this case, it would seem that if k_1 were fairly large and m relatively small, satisfactory drainage may be secured at a distance from the well several times greater than 300 ft.

All the equations derived by the author, together with the diagrams illustrating their applications to typical cases, are of genuine interest to engineers who are concerned with pumping water from ground-water sources whether for drainage or for irrigation purposes.

The increasing "steepness" of the cone of depression of the piezometric surface as the axis of the well is approached, especially for water-bearing materials having a relatively low transmission coefficient, stresses the importance of research concerning design of wells with a view to flattening the cone economically and reducing the power requirements by decreasing the heavy draw-down.

---

[13] *Proceedings*, Utah Academy of Sciences, Vol. 8, p. 35 (1931).

M. R. Lewis,[14] M. Am. Soc. C. E. (by letter).—The writer is fully aware that, as Mr. Hall points out, the formulas developed in his paper are not easily applied in advance of well-drilling operations. The chief value of such theoretical studies is in securing a better understanding of the laws governing the flow of underground water. Actual drilling operations must be governed by experience and judgment. However, judgment may be improved by a better knowledge of the underlying laws.

It is quite true that underground conditions are extremely variable, and that such studies might be extended to cover many other typical cases.

Mr. Hall states that the general opinion seems to be that, under similar conditions, Type *C* wells have a smaller draw-down than either Type *A* or Type *B* wells. It occurs to the writer that this opinion is perhaps based on observations of wells with faulty screens. The formulas developed by the writer did not attempt to take into account the loss of head through screens. In the case of fine sands which are sometimes screened out by the use of fine screens, the loss of head at the screen may be very large in Type *A* or Type *B* wells. On the other hand, in Type *C* wells a large quantity of the fine sand may be pumped out of the well, thus making its effective radius comparatively large.

It is true, of course, that wells pumping from deep-seated aquifers would not be effective for drainage if these strata were separated from the uppermost water-bearing material by a truly impervious stratum. However, experience has shown that in many instances such wells are effective, even where they appear to penetrate almost impervious strata. The explanation probably is that the rate of movement downward through the comparatively impervious strata, while slow, is yet appreciable. When the very large area over which vertical downward movement may be taking place, and the gradient causing the flow, are considered, the effectiveness of such wells is readily explained.

In the paper the writer stated a conclusion that successful drainage by means of wells requires a general lowering of the water-table, unless the area to be drained is only a few hundred feet in diameter. When a general lowering of the water-table is required, the location and design of wells would be based entirely on the economics of the problem, and wells should be located and drilled to such depth as will supply water most economically. The same problem of economics applies to the depth to which the water in the well should be drawn. There are many interesting problems in this connection. Mr. Hall suggests the use of a battery of shallow wells. In some instances such batteries of wells with very low draw-down in each well may be economical. As Mr. Hall points out, the question of cost of pumping equipment is then an important factor.

Professor Israelsen is correct in his statement that the term, "transmission constant," was used by Slichter rather than the term, "transmission coefficient." He is also correct in pointing out that the definition of the transmission constant is incomplete. Perhaps it was best stated by Slichter himself,[15] as follows:

---

[14] Irrig. and Drainage Specialist, Oregon State Coll., and Div. of Agri. Eng., U. S. Dept. of Agriculture, Corvallis, Ore.

> "The constant, k, is the quantity of water that would be transmitted in unit time through a cylinder of the soil of unit length and unit cross-section under unit difference in head at the ends."

The transmission constant, k_2, suggested by Professor Israelsen, having the physical dimension of time, is numerically the same regardless of the units of force and length used in its determination. This is, unquestionably, an advantage as compared with the transmission constant, k, used by Slichter, the numerical value of which depends on the unit of length used. However, in both field and laboratory studies, units of force and length must be used. If k_2 is to be used, the appropriate value of g, the acceleration due to gravity, must be applied in determining the gradient. This fact seems to counterbalance the advantage of a constant having a single value regardless of the units of length used. Moreover, Slichter's k, or similar constants in other units, is almost universally used in the study of the flow of water through saturated materials. For these reasons the writer does not deem it wise to use the constant here represented by k_2.

As used in Equation (7), and except for the single condition that k_1 is the maximum limit of the value of w, the rate of percolation, w, and the transmission constant of the relatively impervious material, k_1, are independent constants.

As Professor Israelsen points out, the value of w is an important factor in influencing the effectiveness of drainage and the area which may be drained by a given well. It is quite probable that in the case of the Utah well mentioned, the value of w is extremely small. If this is true, the value of R would be high and, therefore, the radius of the area having a comparatively large draw-down would be large. Under such conditions a comparatively small number of wells would serve to lower the general water-table level over a large area.

---

[15] Nineteenth Annual Rept., U. S. Geological Survey, Pt. II, p. 323.

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852

TRANSACTIONS

Paper No. 1815

DESIGN OF A REINFORCED CONCRETE SKEW ARCH[1]

By Bernard L. Weiner,[2] Jun. Am. Soc. C. E.[2a]

With Discussion by Messrs. G. D. Houtman, J. Charles Rathbun, A. A. Eremin, and Bernard L. Weiner.

Synopsis

This paper consists of:

Part I.—Analysis of the Stresses in a Skew Arch: A Restatement, Approximation, and Extension of the Original Theory by J. Charles Rathbun, M. Am. Soc. C. E.; with a Numerical Example.

Part II.—Theory of Proportioning Sections; with a Numerical Example.

Part III.—Short-Cut for Computing Crown Stresses in a Symmetrical Skew "Rigid Frame" with Hinged Ends; with a Numerical Example.

The text contains a practical description of a method of designing skew arches. Parts of it are built upon the theory first presented by Professor Rathbun, with special adaptations developed from experience in the drafting-room.

Complete derivations of formulas are given for symmetrical and unsymmetrical, and for fixed and two-hinged arches. A theory for proportioning the sections is developed which is believed to be entirely new. An approximate "short-cut" for designing Rigid Frames also forms a part of the paper. Numerical examples are given to demonstrate the theory.

The writer has assumed that his readers have a working knowledge of the design of a right arch by standard methods.

Introduction

Modern traffic conditions make the construction of a skew bridge the rule rather than the exception. The past practice of locating sharp bends in a highway on each side of a bridge in order to eliminate the skew, is no longer feasible because of the high speed of the present automobile traffic. The necessity of a scientific method of designing skew arches of the solid barrel type is thus apparent.

---

[1] Published in January, 1931, *Proceedings*.

[2] New York, N. Y.

[2a] Now Assoc. M. Am. Soc. C. E.

Analysis of the Stresses in a Skew Arch.—Until recently, very little was known about the action of a skew arch. In 1924, J. Charles Rathbun, M. Am. Soc. C. E., published his theory for the analysis of skew frames.[3] This paper gave a method for computing the total moments, thrusts, and shears acting on any section of the skew frame parallel to the abutments. The theory, as presented, applied only to symmetrical skew frames fixed at the abutments. At the request of Arthur G. Hayden, M. Am. Soc. C. E., Designing Engineer of the Westchester County Park Commission, Professor Rathbun rewrote and extended his theory to include symmetrical structures hinged at the abutments. The latter also furnished a computation for the crown stresses of a hinged, rigid-frame structure for the crossing of the Hutchinson River Parkway under Old Mamaroneck Road, Westchester County, New York (span, 56 ft.; skew angle, 33°). These manuscripts were checked by or under the direction of Henry G. Babcock, M. Am. Soc. C. E., of the Design Division of the Commission. This work has never been published.

By actual experience in the drafting-room, it was found advisable to revise the form of the equations for the analysis of the skew arch, and to make certain approximations suggested by Professor Rathbun. Part I of this paper concerns itself with this problem and also completes the theory by extending it to include unsymmetrical skew frames, both fixed and hinged at the abutments. A numerical example for a two-hinged, symmetrical rigid frame is also given.

Theory of Proportioning Sections.—The theory developed by Professor Rathbun gave merely the total moments, thrusts, and shears acting on a section of the skew frame parallel to the abutments. It did not treat the problem of distribution except in a general way. Various suggestions were made as to the distribution of the stresses produced by these forces and couples, and as to the method of proportioning the sections to resist these stresses. Upon examination, these suggestions were found to be unsatisfactory and, in most cases, scientifically unsound. Several theories for proportioning the sections were compared numerically and found to give widely varying results as to the amounts and locations of the steel areas. Part II proposes a theory for stress distribution and proportioning of the sections, which has been developed by the writer, and, although not entirely exact because of certain assumptions (especially with respect to the torsion moment), it is believed to be logical.

This part of the paper first develops the theory of the distribution of the stresses produced by the various forces and couples acting on a section of the skew frame. It then discusses the method of determining the location of the maximum stresses. In the case of a right arch the steel is perpendicular to the section to be designed, which may be called the design plane. In the case of the skew arch, the steel may not be perpendicular to the section. The effect of the angle that the steel makes with the design plane upon the stresses and deformations is determined. The usual flexural formulas for designing a right reinforced concrete section, therefore, do not apply, and new formulas are developed. The application of these formulas is explained by numerical examples.

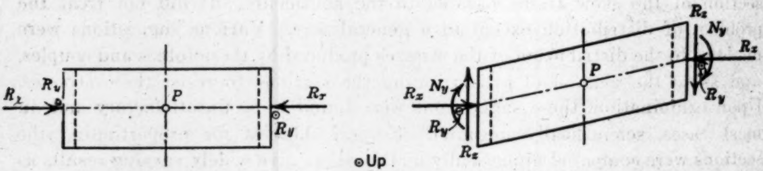
---

[3] *Transactions*, Am. Soc. C. E., Vol. LXXXVII (1924), p. 611.

Crown Stresses in a Symmetrical Skew Rigid Frame with Hinged Ends.—The Westchester County Park Commission builds very extensively a type of structure known as the "Rigid Frame", developed by Mr. Hayden. After designing several symmetrical skew rigid frames with hinged ends, the writer found that certain approximations could be made because of the special shape of the frame. These approximations reduce the numerical work of computing the total moments, thrusts, and shears to not much more than that required for a right arch. This "short-cut" does not extend, however, to the proportioning of the sections. Since the "rigid frame" is now being used by others besides the Westchester County Park Commission, this "short-cut" is presented here with an illustrative numerical example. It is possible that a similar approximation can be made for the fixed symmetrical, and for the fixed and the hinged unsymmetrical, "rigid frame", but no data are available to determine the closeness of such approximations.

PART I.—ANALYSIS OF THE STRESSES IN A SKEW FRAME

The skew frame is a statically indeterminate structure containing forces and moments which in general cannot be made parallel to a single plane. They must be analyzed, therefore, in three dimensions instead of in two as is done in the case of the right frame. The skew of the structure introduces forces and moments which either do not exist at all in a right frame or are present only in a small degree. For example, compare the two structures shown in plan in Fig. (1).



(*a*) PLAN OF A RIGHT FRAME

(*b*) PLAN OF A SKEW FRAME

FIG. 1.—COMPARISON OF FORCES ACTING IN A RIGID FRAME WITH THOSE IN A SKEW FRAME.

In Fig. 1(*a*) the vertical load, *P*, produces the horizontal reactions, R_x, which have the same line of action and, therefore, balance each other. In Fig. 1(*b*), the two reactions, R_x, do not tend to produce translation, but they form a couple which must be resisted by other reactions. This is done by the couple formed by the two horizontal shears, R_z, and by the twisting couples in both footings, N_y. Similarly, the vertical reactions, R_y, in Fig. 1(*b*), form a couple with the load, *P*, which is resisted by a couple at each footing in the vertical planes parallel to the abutments (not shown in the diagram). These couples in the footings produce unequal soil pressure between one end of the footing and the other, exactly similar to the case of a retaining wall. In the skew frame, therefore, there are three additional reaction components which do not exist in a right arch for a load placed symmetrically with respect to the spandrel walls. The numerical values of the reactions are statically indeterminate and depend upon the geometric and elastic properties of the frame. These reactions produce three additional stresses throughout the arch which do not exist in a right arch for the condition of loading described.

In analyzing a right arch it is customary to take a section of unit width. In the case of a skew arch, the structure must be considered as a whole. In doing this, it is assumed that the load is distributed over the width of the arch ring; otherwise some error is involved. In the study prepared for the Westchester County Park Commission, Professor Rathbun states that recent experiments, some conducted by himself and some by George E. Beggs, M. Am. Soc. C. E., show that in an arch the error in this assumption is negligible. As the frames investigated here are arched and not flat slabs, this assumption of distribution is allowable.

The theory presented herein differs in several minor respects from the original theory. In the analysis of a right arch, the shear and thrust deformations are usually neglected, because they are small compared with the bending deformation. Similarly, in the skew arch, two shear deformations, one thrust deformation, and the bending deformation about the radial axis at a point midway between the spandrel walls are neglected. The theory is therefore approximate, but it is sufficiently accurate for design purposes.

The system of signs used is different from that used in the original. This change is made after a thorough study of the question with the view of facilitating the use of the theory in the drafting-room. For a similar reason, some of the details of the derivations are also changed.

METHOD OF DERIVING FORMULAS

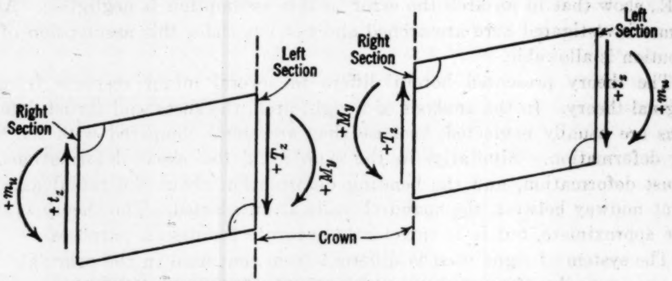
Consider the arch as fixed at the left abutment and free to move at the right abutment, that is, a curved cantilever beam fixed at the left end. When a load is applied to the arch it will deform, causing the right abutment to deflect relative to the left abutment. Now, let a force be applied at the free end so that it returns to its original position (or to any position imposed by the condition of the ends). This force must then be equal to the reaction which would be induced at the right abutment if it were prevented from deflecting under the load. The problem then becomes one of determining the amount, direction, and point of application of a force which, applied to the free end of the cantilever, will produce a deflection equal and opposite in direction to that produced by the loading.

In general, the reaction at the right abutment (which will hereafter be designated as the reaction) will be an eccentric force inclined to the plane of the abutment. This force is replaced by a force and a couple acting at the center of the abutment, and then each is resolved into its components in the directions of the three major co-ordinate axes. Thus, there are six unknown quantities for the evaluation of which six simultaneous equations are required.

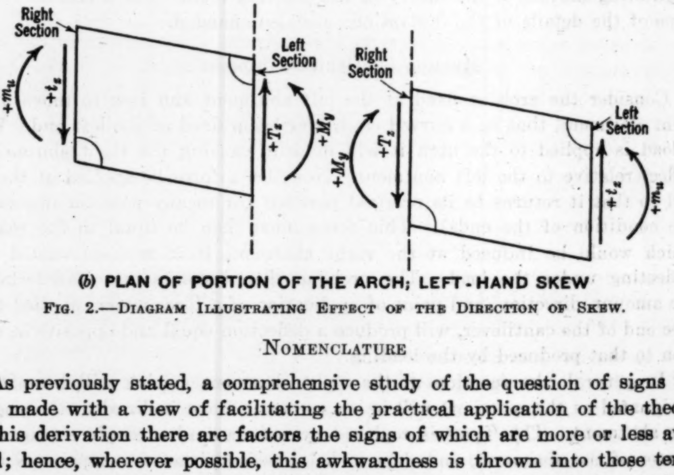
The angular deflections and linear deflections of the right abutment relative to the left abutment can each be resolved into its components in the directions of the three co-ordinate axes. These six deflection components can each be written in terms of the unknown reaction components, the external loads, and certain physical characteristics of the frame. Since the deflection due to the

loading must be equal to that due to the reaction, six equations are obtained. These apply only to the fixed arch.

For the two-hinged arch, the fact that the right abutment is hinged prevents the assumption that the angular deflections about the Z-axis are equal; and the fact that the left abutment is hinged prevents the assumption that the two deflections parallel to the Y-axis are equal. Thus, there are only four equations from statically indeterminate considerations, but two more equations can be obtained from statics. With these two additional equations, the unknown components of the reaction can be evaluated.



(a) PLAN OF PORTION OF THE ARCH; RIGHT-HAND SKEW



(b) PLAN OF PORTION OF THE ARCH; LEFT-HAND SKEW

FIG. 2.—DIAGRAM ILLUSTRATING EFFECT OF THE DIRECTION OF SKEW.

NOMENCLATURE

As previously stated, a comprehensive study of the question of signs has been made with a view of facilitating the practical application of the theory. In this derivation there are factors the signs of which are more or less awkward; hence, wherever possible, this awkwardness is thrown into those terms which appear only in the derivation, but do not occur in the final equations.

The directions of the forces and co-ordinates are all referenced to the geometric properties of the structures, rather than to directions in space. For instance, T_z, which is the horizontal shear at the crown parallel to the Z-axis, is defined as positive toward the obtuse angle of the section on which it is acting. Changing the direction of skew, changes the direction of T_z in space, but it would keep the same sign because its direction with reference to the

obtuse angle of the section would remain the same. In both Fig. 2(*a*) and Fig. 2(*b*) it should be noted that M_y produces compression at the obtuse corner of a left section and at the acute corner of a right section.

The nomenclature to be used in the derivation of the formulas is as follows:

P = any point on the neutral surface of the ring lying in that vertical plane which is half way between the spandrel walls.

x, y, z = the co-ordinates of P referred to axes through the crown for symmetrical arches, as shown in Fig. 5. The co-ordinates, x and y, are always positive, and z is positive in the direction toward the obtuse angle of the crown sections, so that when the right half of the arch is revolved about the line of symmetry to the position of the left section, the co-ordinates will be the same. For unsymmetrical arches, the origin of co-ordinates is taken at the center of the right abutment, as shown in Fig. 9.

u, v = the co-ordinates of the crown center referred to the tangent and radial planes through P and parallel to the plane, $z = 0$, u being parallel to the radial plane, and v, to the tangent plane. Their signs are fixed by the equations, $u = x \sin \phi - y \cos \phi$ and $v = x \cos \phi + y \sin \phi$, respectively.

u', v' = the u and v co-ordinates, respectively, of the center of the right abutment, and their signs are fixed by Equations (3) and (21) as given subsequently.

s = the length of an elementary parallelopiped measured on the square along the neutral surface.

h = rise of the neutral surface.

L = half span of the neutral surface measured on the square.

b = width of the ring measured parallel to the Z-axis (on the skew).

θ = skew angle.

ϵ = the tangent of the skew angle and is always positive.

ϕ = the angle of the slope of a plane tangent to the neutral surface at P and for the symmetrical arch is always positive (see Fig. 4). For the unsymmetrical arch, it is positive to the left and negative to the right (see Fig. 9).

I = moment of inertia of the area of the ring cut by a radial plane through P and parallel to the axis of the barrel (Z-axis), about a horizontal line through P and lying in the radial plane.

E = modulus of elasticity in tension and compression.

F = the factor of torsion.

λ = the ratio, $\dfrac{\text{Modulus of elasticity in compression}}{\text{Modulus of elasticity in shear}}$.

H = the horizontal load.

W = the vertical load.

x', y' = the x and y-co-ordinates of a load point (and apply only to the left half of the arch for the symmetrical arch).

T_x, T_y, T_z = the components of the crown thrust parallel, respectively, to the X-axis, Y-axis, and Z-axis. The component, T_x, is positive when it produces compression; T_y is positive away from the center of curvature (upward) on the left section, and toward the center of curvature (downward) on the right section of the crown; and T_z is positive toward the obtuse angle of the section on which it is acting.

M_x, M_y, M_z = the components of the crown moment about the X-axis, Y-axis, and Z-axis, respectively. The moment component, M_x, is positive when it tends to make the obtuse angle of the section on which it is acting move toward the center of curvature; M_y is positive when it produces compression at the obtuse angle of the left section, and tension at the obtuse angle of the right section, of the crown; and M_z is positive when it produces compression in the extrados fibers.

t_u, t_v, t_z = the components of the thrust acting on the surface cut by the radial plane through P, parallel to the u-axis, v-axis, and z-axis; t_u is positive away from the center of curvature on a left section, and toward the center of curvature on a right section; t_v is positive when it produces compression; and t_z is positive toward the obtuse angle of the section on which it is acting.

m_u, m_v, m_z = the components of the moment acting on the surface cut by a radial plane through P, about the u, v, and z-axes, respectively. The moment component, m_u, is positive when it causes compression at the obtuse angle of a left section and tension on the obtuse angle of a right section; m_v is positive when it causes the obtuse angle of the section on which it is acting to deflect toward the center of curvature, and m_z is positive when it produces compression in the extrados fibers.

R_x, R_y, R_z = the thrust components of the right reaction parallel to the X-axis, Y-axis, and Z-axis, respectively. Their signs are fixed by the relation that for $\phi = 90°$, $t_u = R_x$, $t_v = R_y$, and $t_z = R_z$; thus making R_x positive to the right; R_y, positive upward; and R_z, positive toward the obtuse angle of the right end of the structure as seen in plan.

N_x, N_y, N_z = the moment components of the right reaction about the X-axis, Y-axis, and Z-axis, respectively. Their signs are fixed by the relation that, for $\phi = 90°$, $m_u = N_x$, $m_v = N_y$, and $m_z = N_z$; thus making N_x positive when it tends to make the obtuse angle move up; N_y positive when it tends to make the obtuse angle move inward; and N_z positive when it produces compression in the extrados fibers.

$d\alpha$, $d\gamma$ = the differential angular deformation of the right face of an elementary parallelopiped relative to the left face about the v and z-axes. The positive directions of these quantities are the same as the positive directions of the moments which produce them acting on the left section; that is, m_v and m_z, respectively.

θ_x, θ_y, θ_z = the angular deflections of the center of the right abutment relative to the left abutment, about the x, y, and z-axes, respectively, and they are positive in the same directions as M_x, M_y, and M_z, acting on the left section of the crown. These symbols may have superscripts (R, H, or W), indicating whether the deflections are produced by the reactions, the horizontal loading, or the vertical loading.

Δ_x, Δ_y, Δ_z = the linear deflections of the center of the right abutment relative to the left abutment parallel to the x, y, and z-axes. The deflection, Δ_x, is positive to the right, Δ_y is positive upward, and Δ_z is positive in the same direction as the Z-axis for the right half of the arch. These symbols may also have superscripts (R, H, and W).

The factor of torsion, F, may be computed from the equation: $\frac{1}{F} = \frac{205\,\pi\,(t^2 + b^2)}{180\,b^3\,t^3}$, which may be written, $\frac{1}{F} = \frac{205\,\pi}{180}\left(\frac{1}{b^3\,t} + \frac{1}{t^3\,b}\right)$, and since b is usually large compared to t, the term, $\frac{1}{b^3\,t}$, may be neglected; hence,

$$\frac{1}{F} = \frac{205\,\pi}{180\,b\,t^3} = \frac{3.578}{b\,t^3} \qquad (1)$$

As the angular rotation of a shaft may be computed by the equation, $\alpha = \frac{T\,l}{E_s\,I_p}$, in which, α is the angular rotation; T, the torsion moment; l, the length of the shaft; E_s, the modulus of elasticity in shear; and I_p, the polar moment of inertia; so with a rectangular shaft the angular rotation may be computed from the formula, $\alpha = \frac{T\,l}{E_s\,F}$, in which, F has the value given in Equation (1). As the value of F enters quite prominently into the equations, reference is made to *Bulletin No. 3,* Faculty of Applied Science and Engineering, School of Engineering Research, University of Toronto, Toronto, Ont., Canada.

MATHEMATICAL DERIVATION OF FORMULAS

Consider the arch as fixed at the left abutment and free to move at the right abutment. Let the arch ring be cut by a series of radial planes. These planes may be considered as cutting the ring into blocks of lengths, S, and the portions enclosed between adjacent planes as acted upon and deformed by the forces on the section between the block and the right abutment. Each block is subjected to the thrust and moment at the right abutment and the known loads, and deformed by them. The resulting deformations of each block causes the right abutment to deflect relative to the left abutment and the total is the sum of the deflections due to each block deformation.

The derivation of the formulas falls into four steps. It is necessary to find:

1.—The stresses produced at the center of the block by the loads, and the reactions at the right abutment.

2.—The deformation of the block due to these stresses.

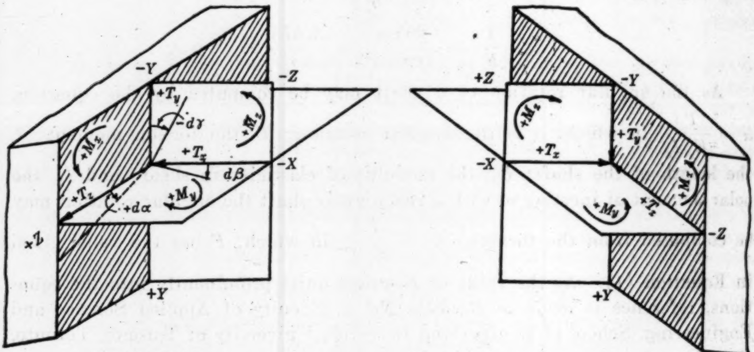
3.—The deflections of the right abutment relative to the left abutment due to these deformations.

4(*a*).—For the fixed arch, these deflections produced by the reactions are equated to the deflections produced by the loads.

4(*b*).—For the two-hinged arch, two additional equations are obtained from statics as already explained.

In the first case considered herein, the structure is symmetrical about a vertical line through the crown. For the sake of simplicity, the origin of the principal co-ordinates is taken where this line intersects the neutral surface and will be designated as the point, C. The Y-axis is vertical, the Z-axis is parallel to the axis of the barrel, and the X-axis is normal to the other two. (See Fig. 3.)

Owing to the symmetry of the structure, the final equations simplify if written in terms of the crown stresses rather than in terms of the reactions. For this and other reasons, as will be seen later, it is more convenient to find the forces acting on a block at its center, *P,* first in terms of the crown stresses and then to transform the resulting equations in terms of the reactions.


−Y
−Z
+T_y
−$d\gamma$
+M_z
+M_z
+T_x
−X
$d\beta$
+T_z
+M_y
+$d\alpha$
+Z
+Y
+Z
−Y
+M_z
+T_y
+T_x
−X
+M_z
+M_y
+T_z
−Z
+Y

LEFT SIDE

RIGHT SIDE

FIG. 3.—DIAGRAMS SHOWING POSITION OF CO-ORDINATE PLANES IN SYMMETRICAL ARCHES.

Cutting the arch in two parts by a radial plane through *C,* the half arch must be kept in equilibrium by the stresses at the crown (see Fig. 4). To find the stress at *P,* the section between the crown and *P* is taken as a free body (see Fig. 5). As it must be in equilibrium, the sum of the forces parallel to each axis and the sum of the moments about each axis must equal zero as shown in Equations (2).

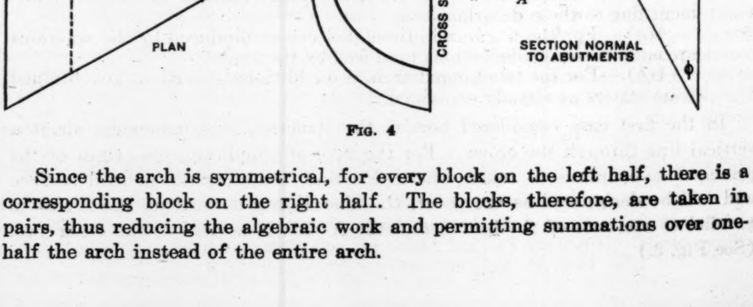

T_z
C
T_x
M_y
T_z
T_y
M_x
y
b
$x=\varepsilon z$
P
P
$\tan^{-1}\varepsilon$
PLAN
CROSS SECTION AT CROWN
T_y
x
M_z
C
T_x
s
y
P
t
A
SECTION NORMAL TO ABUTMENTS
ϕ

FIG. 4

Since the arch is symmetrical, for every block on the left half, there is a corresponding block on the right half. The blocks, therefore, are taken in pairs, thus reducing the algebraic work and permitting summations over one-half the arch instead of the entire arch.

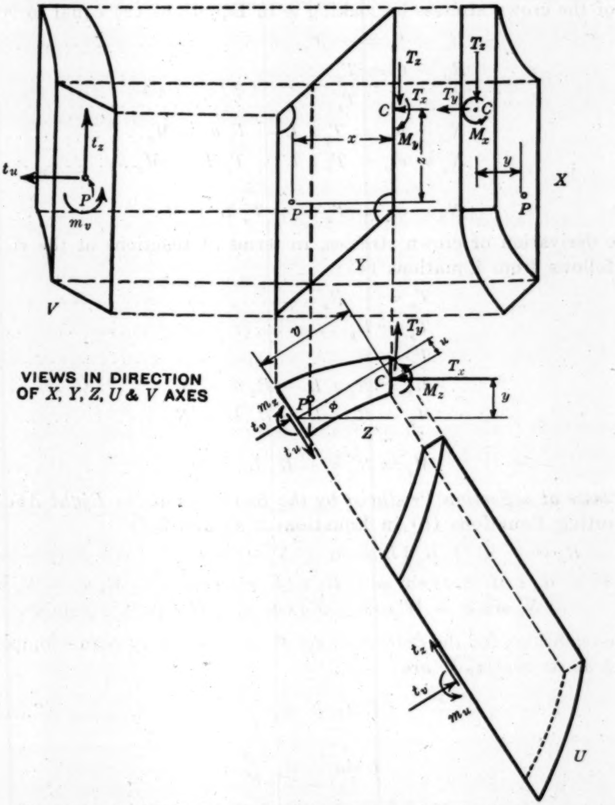
SYMMETRICAL ARCHES

Equations to determine the stresses at a section of a symmetrical arch may be written as follows. (See Fig. 5.) Note that for symmetrical arches the upper signs apply to the left half of the structure, and the lower signs to the right half. The loads, W and H, apply to the left half of the structure only.

$$t_v = T_x \cos\phi \mp T_y \sin\phi + W \sin\phi - H \cos\phi \ldots\ldots (2a)$$

$$t_u = \pm T_x \sin\phi + T_y \cos\phi - W \cos\phi - H \sin\phi \ldots\ldots (2b)$$

$$t_z = T_z \ldots\ldots (2c)$$



T_z
T_z
T_x
T_y
C
C
M_y
M_x
x
z
y
X
P
P
t_z
t_u
P
m_v
Y
V
v
u
T_y
T_x
C
M_z
y
P
φ
m_z
t_v
t_u
Z
t_z
t_v
m_u
U
VIEWS IN DIRECTION OF X, Y, Z, U & V AXES

FIG. 5.—DIAGRAM SHOWING RELATION BETWEEN CROWN STRESSES AND STRESSES AT ANY OTHER POINT.

$$m_v = T_x\, \varepsilon\, x \sin\phi \pm T_y\, \varepsilon\, x \cos\phi - T_z\, u + M_x \cos\phi \mp M_y \sin\phi$$
$$- W\, \varepsilon\, (x - x') \cos\phi - H\, \varepsilon\, (x - x') \sin\phi \ldots\ldots (2d)$$

$$m_u = \mp T_x\, \varepsilon\, x \cos\phi + T_y\, \varepsilon\, x \sin\phi \pm T_z\, v \pm M_x \sin\phi + M_y \cos\phi$$
$$- W\, \varepsilon\, (x - x') \sin\phi + H\, \varepsilon\, (x - x') \cos\phi \ldots\ldots (2e)$$

and,

$$m_z = T_x\, y \pm T_y\, x + M_z - W\,(x - x') - H\,(y - y') \ldots\ldots (2f)$$

Equations for the co-ordinates of the crown center, with respect to the point, P, of any section, may be written as follows:

$$u = x \sin\phi - y\cos\phi \tag{3a}$$

$$v = x\cos\phi + y\sin\phi \tag{3b}$$

and,

$$u' = x\sin\phi - y\cos\phi \pm L\sin\phi + h\cos\phi = u \pm L\sin\phi + h\cos\phi. \tag{3c}$$

These formulas, Equations (3), are restatements of the definitions given in the nomenclature. The reactions at the right abutment may be found in terms of the crown stresses by making ϕ in Equations (2) equal to 90°; thus,

$$R_x = t_u = - T_x \tag{4a}$$

$$R_y = t_v = T_y \tag{4b}$$

$$R_z = t_z = T_z \tag{4c}$$

$$N_x = m_u = T_y\,\varepsilon\,L - T_z\,h - M_x \tag{4d}$$

$$N_y = m_v = T_x\,\varepsilon\,L - T_z\,L + M_y \tag{4e}$$

and,

$$N_z = m_z = T_x\,h - T_y\,L + M_z \tag{4f}$$

The derivation of crown stresses, in terms of reactions at the right abutment, follows from Equations (4):

$$T_x = - R_x \tag{5a}$$

$$T_y = R_y \tag{5b}$$

$$T_z = R_z \tag{5c}$$

$$M_x = R_y\,\varepsilon\,L - R_z\,h - N_x \tag{5d}$$

$$M_y = R_x\,\varepsilon\,L + R_z\,L + N_y \tag{5e}$$

and,

$$M_z = R_x\,h + R_y\,L + N_z \tag{5f}$$

Stresses at a Section Produced by the Reactions at the Right Abutment.— Substituting Equations (5) in Equations 2(d) and 2(f):

$$m_z = R_x\,(h - y) + R_y\,(L \pm x) + N_z - W\,(x - x') - H\,(y - y') \tag{6a}$$

$$m_v = \mp R_x\,\varepsilon\,(L \pm x)\sin\phi + R_y\,\varepsilon\,(L \pm x)\cos\phi - R_z\,u' - N_x\cos\phi \mp N_y\sin\phi - W\,\varepsilon\,(x - x')\cos\phi - H\,\varepsilon\,(x - x')\sin\phi \tag{6b}$$

The equations for the deformations of an elementary parallelopiped due to stresses at its center, P, are:

$$E\,d\gamma = m_z\,\frac{s}{I} \tag{7a}$$

$$E\,d\alpha = m_v\,\frac{\lambda\,s}{F} \tag{7b}$$

There are four more deformations; that is, two shear, one thrust, and the bending deformation about the U-axis. As already explained these four deformations are small compared to $d\alpha$ and $d\gamma$, and will be neglected.

In Equations ($8A$), Equations (6) have been rewritten by taking each reaction or load in turn and making the remaining terms zero. The equations listed, therefore, are for the stresses at a section, due to each reaction component at the right abutment.

***Equations** (8A).*—**Stresses at a section due to each reaction component at the right abutment, are:**

Reaction component.	LEFT SIDE OF ARCH.		RIGHT SIDE OF ARCH.	
	m_z.	m_v.	m_z.	m_v.
R_x	$R_x (h - y)$	$-R_x \epsilon (L + x) \sin \phi$	$R_x (h - y)$	$R_x \epsilon (L - x) \sin \phi$
R_y	$R_y (L + x)$	$+R_y \epsilon (L + x) \cos \phi$	$R_y (L - x)$	$R_y \epsilon (L - x) \cos \phi$
R_z	....	$-R_z u'$	....	$-R_z u'$
N_x	....	$-N_x \cos \phi$	....	$-N_x \cos \phi$
N_y	....	$-N_y \sin \phi$	....	$+N_y \sin \phi$
N_z	N_z	....	N_z	....

Equations (8B).—Stresses at a section due to a vertical load, *W*, at a point, $x = x'$; $z = \epsilon x'$, are similarly given by:

$$\left.\begin{aligned} m_z &= - W (x - x') \\ \text{and,} \quad m_v &= - W \epsilon (x - x') \cos \phi \end{aligned}\right\} \ldots\ldots\ldots\ldots\ldots\ldots (8B)$$

Equations (8C).—Stresses at a section, due to a horizontal load, *H*, at a point, $y = y'$; $z = \epsilon x'$, are:

$$\left.\begin{aligned} m_z &= - H (y - y') \\ \text{and,} \quad m_v &= - H \epsilon (x - x') \sin \phi \end{aligned}\right\} \ldots\ldots\ldots\ldots\ldots\ldots (8C)$$

Equations (8*A*) give the moments produced by the individual reactions and, when combined, are used together with either Equations (8*B*), or Equations (8*C*), as will be shown later.

Equations (9) give the differential deflections of the right abutment relative to the left abutment, due to the deformation of a pair of corresponding elementary parallelopipeds.

Equations (9).—Differential deflections of the right abutment are:

Deflection.	LEFT SIDE.		RIGHT SIDE.		Equation No.
	Due to m_z.	Due to m_v.	Due to m_z.	Due to m_v.	
$d\theta_x$	....	$d\alpha \cos \phi$	....	$d\alpha \cos \phi$	(9*a*)
$d\theta_y$	....	$-d\alpha \sin \phi$	....	$d\alpha \sin \phi$	(9*b*)
$d\theta_z$	$d\gamma$	....	$d\gamma$	....	(9*c*)
$d\Delta_x$	$d\gamma (h - y)$	$-d\alpha \epsilon (L + x) \sin \phi$	$d\gamma (h - y)$	$d\alpha \epsilon (L - x) \sin \phi$	(9*d*)
$d\Delta_y$	$d\gamma (L + x)$	$d\alpha \epsilon (L + x) \cos \phi$	$d\gamma (L - x)$	$d\alpha \epsilon (L - x) \cos \phi$	(9*e*)
$d\Delta_z$	....	$d\alpha u'$	....	$d\alpha u'$	(9*f*)

Differential Deflections of the Right Abutment Produced by the Reactions.—Substituting Equations (7) and (8*A*) in Equations (9), Equation (9*a*), for example, is expanded into the form:

$$E \, d\theta_x{}^R = R_x \left[- \epsilon (L + x) \sin \phi \cos \phi \, \lambda \frac{s}{F} + \epsilon (L - x) \sin \phi \cos \phi \, \lambda \frac{s}{F} \right]$$

$$+ R_y \left[\epsilon (L + x) \cos^2 \phi \, \lambda \frac{s}{F} + \epsilon (L - x) \cos^2 \phi \, \lambda \frac{s}{F} \right]$$

$$+ R_z \left[- u'_L \cos \phi \, \lambda \frac{s}{F} - u'_R \cos \phi \, \lambda \frac{s}{F} \right] + N_x \left[- 2 \cos^2 \phi \, \lambda \frac{s}{F} \right]$$

$$+ N_y \left[- \sin \phi \cos \phi \, \lambda \frac{s}{F} + \sin \phi \cos \phi \, \lambda \frac{s}{F} \right] \ldots\ldots\ldots (10a)$$

The equations for the deflections corresponding to Equations (9*b*), (9*c*), etc., are similarly obtained and are not shown.

Total Deflections of the Right Abutment Produced by the Reactions (in Terms of the Crown Stresses).—Equations (11) give the total deflections of the right abutment relative to the left produced by the reactions, and are in terms of the crown stresses. They are obtained by substituting Equations (4) in Equations (10).

Making the substitutions indicated and integrating or summing up over one-half the arch:

$$E \, \theta_x{}^R = 2 \left\{ \left[\varepsilon \sum x \sin \phi \cos \phi \, \lambda \frac{s}{F} \right] T_x - \left[\sum u \cos \phi \, \lambda \frac{s}{F} \right] T_z \right.$$

$$\left. + \left[\sum \cos^2 \phi \, \lambda \frac{s}{F} \right] M_x \right\} \ldots\ldots\ldots\ldots\ldots\ldots (11a)$$

$$E \, \theta_y{}^R = 2 \left\{ - \left[\varepsilon \sum x \sin \phi \cos \phi \, \lambda \frac{s}{F} \right] T_y + \left[\sum \sin^2 \phi \, \lambda \frac{s}{F} \right] M_y \right\} . (11b)$$

$$E \, \theta_z{}^R = 2 \left\{ \left[\sum y \frac{s}{I} \right] T_x + \left[\sum \frac{s}{I} \right] M_z \right\} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (11c)$$

$$E \, \Delta_x{}^R = 2 \left\{ - \left[\sum y^2 \frac{s}{I} + \varepsilon^2 \sum x^2 \sin^2 \phi \, \lambda \frac{s}{F} \right] T_x + \left[\varepsilon \sum u \, x \sin \phi \, \lambda \frac{s}{F} \right] T_z \right.$$

$$\left. - \left[\sum y \frac{s}{I} \right] M_z - \left[\varepsilon \sum x \sin \phi \cos \phi \, \lambda \frac{s}{F} \right] M_x \right\}$$

$$+ h \, E \, \theta_z{}^R + \varepsilon \, L \, E \, \theta_y{}^R \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (11d)$$

$$E \, \Delta_y{}^R = 2 \left\{ \left[\sum x^2 \frac{s}{I} + \varepsilon^2 \sum x^2 \cos^2 \phi \, \lambda \frac{s}{F} \right] T_y \right.$$

$$\left. - \left[\varepsilon \sum x \sin \phi \cos \phi \, \lambda \frac{s}{F} \right] M_y \right\} + L \, E \, \theta_z{}^R + \varepsilon \, L \, E \, \theta_x{}^R \ldots . (11e)$$

and,

$$E \, \Delta_z{}^R = 2 \left\{ \left[\varepsilon \sum u \, x \sin \phi \, \lambda \frac{s}{F} \right] T_x - \left[\sum u^2 \lambda \frac{s}{F} \right] T_z \right.$$

$$\left. + \left[\sum u \cos \phi \, \lambda \frac{s}{F} \right] M_z \right\} - L \, E \, \theta_y{}^R + h \, E \, \theta_x{}^R \ldots\ldots (11f)$$

Total Deflections of the Right Abutment Produced by a Vertical Load at the Point, $x = x'$, $z = \varepsilon \, x'$.—Substituting Equations (7) and (8)B in Equations (9), and bearing in mind that a load deforms only that part of the structure between the left abutment and the load:

$$E \, \theta_x{}^W = \left[- \varepsilon \sum_A^P x \cos^2 \phi \, \lambda \frac{s}{F} + \varepsilon \, x' \sum_A^P \cos^2 \phi \, \lambda \frac{s}{F} \right] W \ldots\ldots\ldots (12a)$$

$$E \, \theta_y{}^W = \left[\varepsilon \sum_A^P x \sin \phi \cos \phi \, \lambda \frac{s}{F} - \varepsilon \, x' \sum_A^P \sin \phi \cos \phi \, \lambda \frac{s}{F} \right] W . (12b)$$

$$E\,\theta_z^W = \left[-\sum_A^P x\,\frac{s}{I} + x' \sum_A^P \frac{s}{I}\right] W \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (12c)$$

$$E\,\Delta_x^W = \left[\sum_A^P x\,y\,\frac{s}{I} - x' \sum_A^P y\,\frac{s}{I} + \varepsilon^2 \sum_A^P x^2 \sin\phi\cos\phi\,\lambda\,\frac{s}{F} - \varepsilon^2 x' \sum_A^P x \sin\phi\cos\phi\,\lambda\,\frac{s}{F}\right] W + h\,E\,\theta_z^W + \varepsilon\,L\,E\,\theta_y^W \ldots (12d)$$

$$E\,\Delta_y^W = \left[-\sum_A^P x^2\,\frac{s}{I} + x' \sum_A^P x\,\frac{s}{I} - \varepsilon^2 \sum_A^P x^2 \cos^2\phi\,\lambda\,\frac{s}{F} + \varepsilon^2 x' \sum_A^P x \cos^2\phi\,\lambda\,\frac{s}{F}\right] W + L\,E\,\theta_z^W + \varepsilon\,L\,E\,\theta_x^W \ldots\ldots (12e)$$

and,

$$E\,\Delta_z^W = \left[-\varepsilon \sum_A^P u\,x \cos\phi\,\lambda\,\frac{s}{F} + \varepsilon\,x' \sum_A^P u \cos\phi\,\lambda\,\frac{s}{F}\right] W + h\,E\,\theta_x^W - L\,E\,\theta_y^W \ldots\ldots\ldots\ldots\ldots\ldots (12f)$$

Total Deflections of Right Abutment Produced by Horizontal Load at the Point, $y = y'$ and $z = \varepsilon x'$.—Similarly, substituting Equations (7) and (8*C*) in Equations (9):

$$E\,\theta_x^H = \left[-\varepsilon \sum_A^P x \sin\phi\cos\phi\,\lambda\,\frac{s}{F} + \varepsilon\,x' \sum_A^P \sin\phi\cos\phi\,\lambda\,\frac{s}{F}\right] H. (13a)$$

$$E\,\theta_y^H = \left[\varepsilon \sum_A^P x \sin^2\phi\,\lambda\,\frac{s}{F} - \varepsilon\,x' \sum_A^P \sin^2\phi\,\lambda\,\frac{s}{F}\right] H \ldots\ldots\ldots\ldots (13b)$$

$$E\,\theta_z^H = \left[-\sum_A^P y\,\frac{s}{I} + y' \sum_A^P \frac{s}{I}\right] H \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (13c)$$

$$E\,\Delta_x^H = \left[\sum_A^P y^2\,\frac{s}{I} - y' \sum_A^P y\,\frac{s}{I} + \varepsilon^2 \sum_A^P x^2 \sin^2\phi\,\lambda\,\frac{s}{F} - \varepsilon^2 x' \sum_A^P x \sin^2\phi\,\lambda\,\frac{s}{F}\right] H + h\,E\,\theta_z^H + \varepsilon\,L\,E\,\theta_y^H \ldots\ldots (13d)$$

$$E\,\Delta_y^H = \left[-\sum_A^P x\,y\,\frac{s}{I} + y' \sum_A^P x\,\frac{s}{I} - \varepsilon^2 \sum_A^P x^2 \sin\phi\cos\phi\,\lambda\,\frac{s}{F} + \varepsilon^2 x' \sum_A^P x \sin\phi\cos\phi\,\lambda\,\frac{s}{F}\right] H + L\,E\,\theta_z^H + \varepsilon\,L\,E\,\theta_x^H \ldots\ldots (13e)$$

and,

$$E\,\Delta_z^H = \left[-\varepsilon \sum_A^P u\,x \sin\phi\,\lambda\,\frac{s}{F} + \varepsilon\,x' \sum_A^P u \sin\phi\,\lambda\,\frac{s}{F}\right] H - L\,E\,\theta_y^H + h\,E\,\theta_x^H \ldots\ldots\ldots\ldots\ldots\ldots (13f)$$

Symmetrical Fixed Arch

If the abutments are considered fixed, all the deflections produced by a load or by temperature expansion (or contraction) are equal and opposite to the deflections produced by the reactions. (See Equations (14).)

EQUATIONS (14).—SHOWING EQUALITY OF DEFLECTIONS PRODUCED IN A FIXED ARCH BY A LOAD OR BY TEMPERATURE.

Reactions.	For vertical load.	For horizontal load.	For temperature rise.	Equation No.
$\Delta_x^R =$	$-\Delta_x^W$	$-\Delta_x^H$	$-2\,c\,t\,L$	(14*a*)
$\Delta_z^R =$	$-\Delta_z^W$	$-\Delta_z^H$	$-2\,\epsilon\,c\,t\,L$	(14*b*)
$\theta_x^R =$	$-\theta_x^W$	$-\theta_x^H$	0	(14*c*)
$\theta_z^R =$	$-\theta_z^W$	$-\theta_z^H$	0	(14*d*)
$\Delta_y^R =$	$-\Delta_y^W$	$-\Delta_y^H$	0	(14*e*)
$\theta_y^R =$	$-\theta_y^W$	$-\theta_y^H$	0	(14*f*)

Equations (14) may be rewritten as given in Equations (15). (See page 1227.)

Substituting in Equations (15) the values previously obtained in Equations (11), (12), and (13), the final equations (in terms of the crown stresses) for the fixed symmetrical arch are obtained. These, with other final equations, are given in Appendix I, for the convenience of the designer. They are numbered Equations (16) in order to help the designer in referring back to the text when necessary.

Symmetrical Two-Hinged Arch

As in the case of a right arch, if the abutments are considered hinged,

$$\Delta_y^R + \Delta_y^H \neq 0;\ \Delta_y^R + \Delta_y^W \neq 0;\ \text{and}\ \theta_z^R + \theta_z^H \neq 0;\ \theta_z^R + \theta_z^W \neq 0$$

and two additional Equations (17) are obtained from statics which take the place of Equations (14*d*) and (14*e*).

EQUATIONS (17).—ADDITIONAL FORMULAS FOR HINGED ABUTMENTS.

	For vertical load, *W*.	For horizontal load, *H*.	For temperature rise.	Equation No.
$M_z + h\,T_x =$	$\frac{1}{2}\,(L - x')\,W$	$\frac{1}{2}\,(h - y')\,H$	0	(17*d*)
$T_y = R_y =$	$\frac{1}{2}\left(1 - \frac{x'}{L}\right)W$	$\frac{1}{2}\left(\frac{h - y'}{L}\right)H$	0	(17*e*)

Equations (14*a*), (14*b*), (14*c*), and (14*f*) still hold true; that is, these four equations with Equations (17*d*) and (17*e*), are the required six formulas and they may be rewritten as given in Equations (18). (See page 1228.)

Substituting in Equations (18) the values previously obtained in Equations (11), (12), and (13), and eliminating M_z between Equations (18*d*) and the equation obtained by expanding Equation (18*a*), the final Equations (19) (in terms of the crown stresses), for the two-hinged symmetrical arches, are obtained. These are placed in the Appendix with the fixed arch equations.

EQUATIONS (15).—OBTAINED BY RE-ARRANGING EQUATIONS (14).

	For vertical load.	For horizontal load.	For temperature rise.	Equation No.
$-\frac{1}{2} E \left(\Delta_x^R - h\,\theta_z^R - \epsilon L\,\theta_y^R\right) =$	$+\frac{1}{2} E \left(\Delta_x^W - h\,\theta_z^W - \epsilon L\,\theta_y^W\right)$	$+\frac{1}{2} E \left(\Delta_x^H - h\,\theta_z^H - \epsilon L\,\theta_y^H\right)$	$+E\,c\,t\,L$	(15*a*)
$-\frac{1}{2} E \left(\Delta_z^R + L\,\theta_y^R - h\,\theta_x^R\right) =$	$+\frac{1}{2} E \left(\Delta_z^W + L\,\theta_y^W - h\,\theta_x^W\right)$	$+\frac{1}{2} E \left(\Delta_z^H + L\,\theta_y^H - h\,\theta_x^H\right)$	$+E\,\epsilon\,c\,t\,L$	(15*b*)
$+\frac{1}{2} E\,\theta_x^R =$	$-\frac{1}{2} E\,\theta_x^W$	$-\frac{1}{2} E\,\theta_x^H$	0	(15*c*)
$+\frac{1}{2} E\,\theta_z^R =$	$-\frac{1}{2} E\,\theta_z^W$	$-\frac{1}{2} E\,\theta_z^H$	0	(15*d*)
$-\frac{1}{2} E \left(\Delta_y^R - L\,\theta_z^R - \epsilon L\,\theta_x^R\right) =$	$+\frac{1}{2} E \left(\Delta_y^W - L\,\theta_z^W - \epsilon L\,\theta_x^W\right)$	$+\frac{1}{2} E \left(\Delta_y^H - L\,\theta_z^H - \epsilon L\,\theta_x^H\right)$	0	(15*e*)
$-\frac{1}{2} E\,\theta_y^R =$	$+\frac{1}{2} E\,\theta_y^W$	$+\frac{1}{2} E\,\theta_y^H$	0	(15*f*)

EQUATIONS (18).—OBTAINED BY RE-ARRANGING EQUATIONS (14) AND (17).

	For vertical, load, W.	For horizontal load, H.	For temperature rise.	Equation No.
$-\frac{1}{2}E\left(\Delta_x{}^R-\epsilon L\theta_y{}^R\right)=$	$+\frac{1}{2}E\left(\Delta_x{}^W-\epsilon L\theta_y{}^W\right)$	$+\frac{1}{2}E\left(\Delta_x{}^H-\epsilon L\theta_y{}^H\right)$	$E\,c\,t\,L$	(18a)
$-\frac{1}{2}E\left(\Delta_x{}^R+L\theta_y{}^R-h\theta_x{}^R\right)=$	$+\frac{1}{2}E\left(\Delta_x{}^W+L\theta_y{}^W-h\theta_x{}^W\right)$	$+\frac{1}{2}E\left(\Delta_x{}^H+L\theta_y{}^H-h\theta_x{}^H\right)$	$\epsilon\,E\,c\,t\,L$	(18b)
$+\frac{1}{2}E\theta_x{}^R=$	$-\frac{1}{2}E\theta_x{}^W$	$-\frac{1}{2}E\theta_x{}^H$	0	(18c)
$M_z+h\,T_x=$	$\frac{1}{2}(L-x')\,W$	$\frac{1}{2}(h-y')\,H$	0	(18d)
$T_y=$	$\frac{1}{2}\left(1-\frac{x'}{L}\right)W$	$\frac{1}{2}\left(\frac{h-y'}{L}\right)H$	0	(18e)
$-\frac{1}{2}E\theta_y{}^R=$	$\frac{1}{2}E\theta_y{}^W$	$\frac{1}{2}E\theta_y{}^H$	0	(18f)

EXAMPLE 1.—ANALYSIS OF A TWO-HINGED SKEW FRAME

The following numerical example based on the two-hinged skew rigid frame, shown in Fig. 6, illustrates the preceding theory. Where possible, only such portions of the analysis which differ from the design of a right arch are given in detail. The work is carried through in detail to the point of obtaining influence lines, dead load, and temperature stresses. Maximum moments, shears, and thrusts are obtained in the usual manner, and the results only are given. It should be noticed, however, that in a right arch, the bending moment is most important in determining the combination of loading to produce the maximum stresses; and the thrusts are obtained for the same condition and position of loading as for the bending moment. Similarly, in a skew arch, m_z governs, and all the other stresses are obtained for the same kind of loading as is used for this moment.

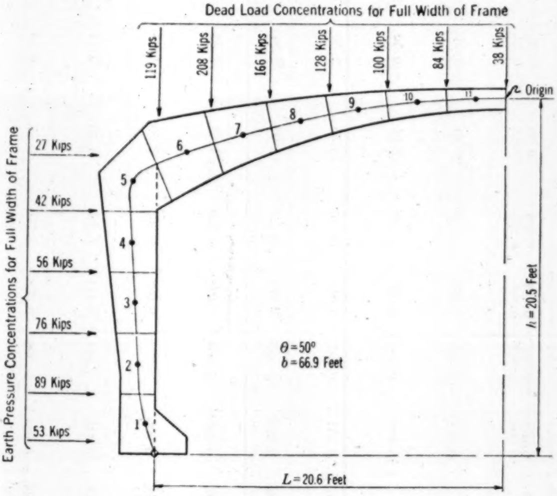

Dead Load Concentrations for Full Width of Frame
119 Kips
208 Kips
166 Kips
128 Kips
100 Kips
84 Kips
38 Kips
Origin
Earth Pressure Concentrations for Full Width of Frame
27 Kips
42 Kips
56 Kips
76 Kips
89 Kips
53 Kips
1
2
3
4
5
6
7
8
9
10
11
θ=50°
b=66.9 Feet
h=20.5 Feet
L=20.6 Feet

FIG. 6.—DESIGN SECTION (NORMAL TO ABUTMENTS).

Table 1 gives the elastic and geometric constants required for the evaluation of the coefficients of Equations (19).

The constants from Table 1 are substituted in the expressions for the coefficients given in Equations (19). (See Tables 2, 3, and 4.)

The unit load coefficients from Table 3 are multiplied by the dead load concentrations from Fig. 6, giving the dead load coefficients in Table 5. Similarly, the coefficients of Table 4 are multiplied by the earth pressure concentrations from Fig. 6, giving the earth pressure coefficients in Table 5.

From Tables 2, 3, and 5, the coefficients are substituted in Equations (19) which are then solved for the unknown reactions as shown in Table 6.

In practical design, Items 11, 15, 20, 25, 27, and 32 (Table 6) should be incorporated in a new and condensed table for the convenience of the

TABLE 1.—CONSTANTS.

Point.	Thickness, in feet.	x.	y.	$\tan \phi$.	$\sin \phi$.	$\cos \phi$.	$\frac{1}{t^3}$.	$\frac{b\,\lambda}{F}$.	$x \sin \phi$.	$y \cos \phi$.	u.				$y^2 \frac{b}{I}$.
1 2	3.10 2.17	21.10 21.67	18.74 15.30	− 3.350 −16.74	0.958 0.998	−0.286 −0.060	0.034 0.098	0.304 0.875	20.23 21.63	−5.35 −0.92	25.58 22.55				143.20 275.00
3	2.60	21.88	11.80	−16.74	0.998	−0.060	0.057	0.509	21.83	−0.71	22.54				418.20 95.20
4	3.00	22.10	8.30	−16.74	0.998	−0.060	0.037	0.330	22.05	−0.50	22.55				513.40 30.60
5	3.95	22.05	4.78	+ 1.555	0.841	0.541	0.016	0.143	18.55	2.59	15.96				544.00 4.38
6	4.19	18.92	3.10	0.329	0.313	0.950	0.014	0.125	5.92	2.94	2.98				548.38 1.61
7	3.28	15.60	2.10	0.269	0.260	0.966	0.028	0.250	4.05	2.03	2.02				549.99 1.48
8	2.52	12.20	1.28	0.2095	0.205	0.979	0.063	0.563	2.50	1.25	1.25				551.47 1.24
9	1.92	8.76	0.65	0.1495	0.148	0.989	0.141	1.260	1.30	0.64	0.66				552.71 0.71
10	1.48	5.27	0.20	0.0895	0.089	0.996	0.308	2.750	0.47	0.20	0.27				553.42 0.15
11	1.30	1.78	0.05	0.0322	0.032	1.000	0.455	4.060	0.07	0.05	0.02				553.57 0.01
															553.58

TABLE 1.—(*Continued.*)

Point.	$\sin^2\phi\,\frac{b\,\lambda}{F}$.	$x^2\sin^2\phi\,\frac{b\,\lambda}{F}$.	$u^2\,\frac{b\,\lambda}{F}$.	$u\sin\phi\,\frac{b\,\lambda}{F}$.	$u\,x\sin\phi\,\frac{b\,\lambda}{F}$.	$\frac{b}{I}$.	$x\,\frac{b}{I}$.	$x\,y\,\frac{b}{I}$.	$\cos\phi\,\frac{b\,\lambda}{F}$.	$u\cos\phi\,\frac{b\,\lambda}{F}$.
1	0.280	124.80	198.80	7.44	157.00	0.408	8.63	161.00	−0.087	−2.222
2	0.870	409.00	444.00	19.70	428.00	1.175	25.55	391.34	−0.053	−1.118
	1.150	533.80	642.80	27.14	585.00	1.583	34.18	552.34	−0.140	−3.340
3	0.506	242.00	259.00	11.47	252.00	0.684	14.99	176.74	−0.031	−0.700
	1.656	775.80	901.80	38.61	837.00	2.267	49.17	729.08	−0.171	−4.040
4	0.328	160.20	167.80	7.41	164.00	0.444	9.83	81.61	−0.020	−0.451
	1.984	936.00	1 069.60	46.02	1 001.00	2.711	59.00	810.69	−0.191	−4.491
5	0.101	49.20	36.20	1.92	42.35	0.192	4.21	20.47	0.077	1.230
	2.085	985.20	1 105.80	47.94	1 043.35	2.903	63.21	831.16	−0.114	−3.261
6	0.012	4.30	1.11		2.20	0.168	3.18	9.87	0.119	0.355
	2.097	989.50	1 106.91		1 045.55	3.071	66.39	841.03	0.005	−2.906
7	0.017	4.14	1.04		2.06	0.336	5.22	10.97	0.242	0.489
	2.114	993.64	1 107.95		1 047.61	3.407	71.61	852.00	0.247	−2.417
8	0.024	3.57	0.73		1.73	0.756	9.23	11.81	0.552	0.690
	2.138	997.21	1 108.68		1 049.34	4.163	80.84	863.81	0.799	−1.727
9	0.028	2.15	0.55		1.08	1.690	14.83	9.63	1.248	0.823
	2.166	999.36	1 109.23		1 050.42	5.853	95.67	873.44	2.047	−0.904
10	0.022	0.61	0.20		0.35	3.700	19.53	3.91	2.740	0.740
	2.188	999.97	1 109.43		1 050.77	9.553	115.20	877.35	4.787	−0.164
11	0.004	0.01	0.00		0.01	5.460	9.73	0.49	4.060	0.081
	2.192	999.98	1 109.43		1 050.78	15.013	124.93	877.84	8.847	−0.083

TABLE 1.—(*Continued.*)

Point.	$u\, x \cos\phi \frac{b\,\lambda}{F}$.	$\cos^2\phi \frac{b\,\lambda}{F}$.	$x \cos^2\phi \frac{b\,\lambda}{F}$.	$\sin\phi \cos\phi \frac{b\,\lambda}{F}$.	$x \sin\phi \cos\phi \frac{b\,\lambda}{F}$.	$x^2 \sin\phi \cos\phi \frac{b\,\lambda}{F}$.	$y \frac{b}{I}$.	$x \sin^2\phi \frac{b\,\lambda}{F}$.
1	−46.80	0.025	0.525	−0.083	−1.77	−37.40	7.65	5.91
2	−25.65	0.003	0.068	−0.053	−1.15	−24.95	18.00	18.90
3	−72.45	0.028	0.593	−0.136	−2.92	−62.35	25.65	24.81
	−15.30	0.002	0.040	−0.031	−0.68	−14.85	8.06	11.11
4	−87.75	0.030	0.683	−0.167	−3.60	−77.20	33.71	35.92
	− 9.97	0.001	0.026	−0.020	−0.44	− 9.76	3.68	7.28
5	−97.72	0.031	0.659	−0.187	−4.04	−86.96	37.39	43.20
	27.20	0.042	0.925	0.065	1.44	31.70	0.92	2.24
6	−70.52	0.073	1.584	−0.122	−2.60	−55.26	38.31	45.44
	6.72	0.113	2.140	0.037	0.70	13.30	0.52	
7	−63.80	0.186	3.724	−0.085	−1.90	−41.96	38.83	
	7.63	0.234	3.642	0.063	0.98	15.33	0.71	
8	−56.17	0.420	7.366	−0.022	−0.92	−26.63	39.54	
	8.42	0.540	6.590	0.113	1.38	16.81	0.97	
9	−47.75	0.960	13.956	0.091	0.46	− 9.82	40.51	
	7.20	1.232	10.800	0.185	1.62	+14.21	1.10	
10	−40.55	2.192	24.756	0.276	2.08	4.39	41.61	
	3.90	2.730	14.890	0.244	1.29	6.77	0.74	
11	−36.65	4.922	39.146	0.520	3.37	11.16	42.35	
	0.14	4.060	7.220	0.131	0.23	0.42	0.27	
	−36.51	8.982	46.366	0.651	3.60	11.58	42.62	

designer in the work that follows. This table is then used in computating Table 7 which is obtained by substituting the crown values from the condensed table in Equations (2). Table 7 should be repeated for every point or section for which an influence line is required. Figs. 7 and 8 are plotted from Table 7 and similar tables not reproduced herein.

TABLE 2.—VALUES OF THE COEFFICIENTS, K (ALL LOADINGS).
($h = 20.5$ ft.; $L = 20.6$ ft.; $b = 66.9$ ft.; $\varepsilon = 1.192$; $\varepsilon^2 = 1.420$.)

$\Sigma y^2 \frac{b}{I}$	$= 553.6$	$= 554$
$- 2 h \Sigma y \frac{b}{I}$	$= - 2 \times 20.5 \times 42.6$	$= - 1\ 747$
$+ h^2 \Sigma \frac{b}{I}$	$= 20.5^2 \times 15$	$= 6\ 304$
$+ \epsilon^2 \Sigma x^2 \sin^2 \phi \frac{b \lambda}{F}$	$= 1.420 \times 1\ 000$	$= 1\ 420$
$\frac{b K_{11}}{s}$		$= 6\ 531$
$\frac{b K_9}{s} = - \epsilon \Sigma u x \sin \phi \frac{b \lambda}{F}$	$= - 1.192 \times 1\ 051$	$= - 1\ 253$
$\frac{b K_8}{s} = \epsilon \Sigma x \sin \phi \cos \phi \frac{b \lambda}{F}$	$= 1{:}192 \times 3.60$	$= 4.291$
$\frac{b K_5}{s} = \Sigma u^2 \frac{b \lambda}{F}$	$= 1\ 109$	$= 1\ 109$
$\frac{b K_6}{s} = - \Sigma u \cos \phi \frac{b \lambda}{F}$	$= (-0.083)$	$= + 0.083$
$\frac{b K_7}{s} = \Sigma \cos^2 \phi \frac{b \lambda}{F}$	$= 8.982$	$= + 8.982$
$\frac{b K_{10}}{s} = - \Sigma \sin^2 \phi \frac{b \lambda}{F}$	$= - 2.192$	$= - 2.192$
$- \Sigma y \frac{b}{I}$	$= - 42.6$	$= - 42.6$
$+ h \Sigma \frac{b}{I}$	$= 20.5 \times 15$	$= 307.5$
		$+ 264.9$

Where these curves should reach zero or unity at the point, $x = 20.6$, they do not do so for points along the abutment, because of a negligible approximation in the computation.

In a frame of the type shown in Fig. 6, the two critical sections are the crown and the springing line. It is customary to design these two sections first to test the adequacy of the assumed frame. If these two sections are about the right dimensions (small changes being unimportant), other sections are designed in order to determine the reinforcing steel patterns. Table 8 gives the total stresses for the two critical sections. The right half of the

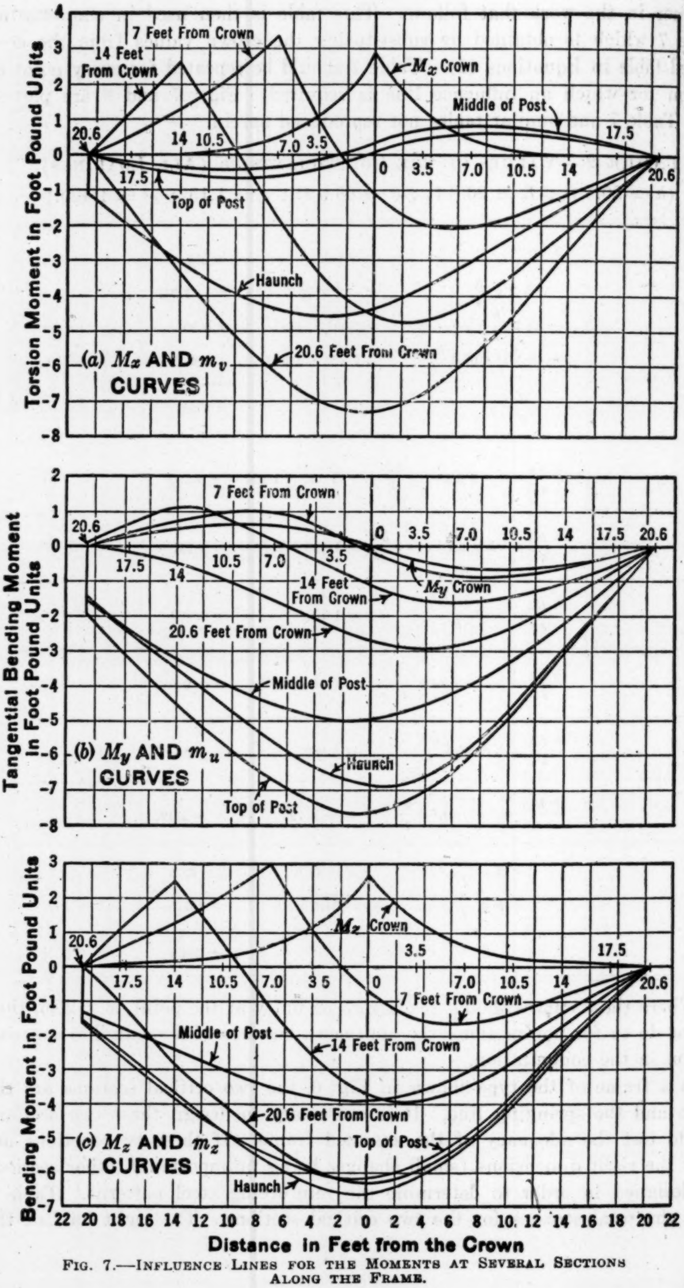


FIG. 7.—INFLUENCE LINES FOR THE MOMENTS AT SEVERAL SECTIONS ALONG THE FRAME.

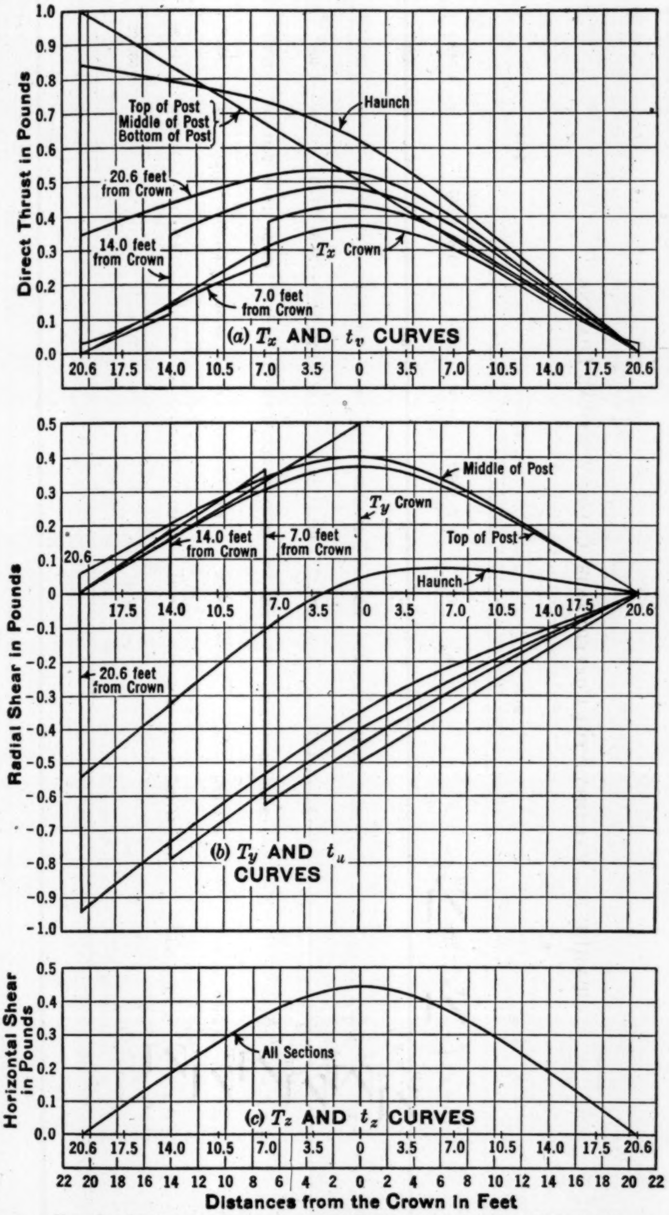


FIG. 8.—INFLUENCE LINES FOR THE THRUST AND SHEARS AT SEVERAL SECTIONS ALONG THE FRAME.

TABLE 3.—VALUES OF THE COEFFICIENTS, C, FOR VERTICAL LOADING.

Item. 1	$x' =$	20.6		17.5		14.0	
2	$(L-x')\left(-\sum y\frac{b}{I}+h\sum\frac{b}{I}\right) =$	0	= 0	(3.1) (265)	= 821.5	(6.6) (265)	= 1 749
3	$+\sum_A^P x\,y\,\frac{b}{I} =$	+831	= + 831	+841	= + 841	+852	= + 852
4	$-x'\sum_A^P y\,\frac{b}{I} =$	−(20.6) (38.3)	= − 789	−(17.5) (38.8)	= − 679	−(14.0) (39.5)	= − 553
5	$-h\sum_A^P x\,\frac{b}{I} =$	−(20.5) (63.2)	= −1 295	−(20.5) (66.4)	= −1 361	−(20.5) (71.6)	= −1 468
6	$+h\,x'\sum_A^P \frac{b}{I} =$	(20.5) (20.6) (2.90)	= 1 225	(20.5) (17.5) (3.07)	= 1 101	(20.5) (14.0) (3.41)	= 978.7
7	$+\epsilon^2\sum_A^P x^2\sin\phi\cos\phi\,\frac{b\,\lambda}{F} =$	+(1.420) (−55.3)	= − 78.53	(1.420) (−42.0)	= − 59.64	(1.420) (−26.6)	= − 37.77
8	$-\epsilon^2\,x'\sum_A^P x\sin\phi\cos\frac{b\,\lambda}{F} =$	−(1.420) (20.6) (−2.60) =	76.06	−(1.420) (17.5) (−1.9) =	47.22	−(1 420) (14.0) (−0.92) =	18.29
9	$2\,C_{11}\,\frac{b}{s} =$		− 30.47		711.08		1 539.22

TABLE 3.—(*Continued.*)

Item.	$x' =$	20.6	17.5	14.0
10	$- \epsilon \sum_A^P u\,x \cos\phi \frac{b\,\lambda}{F} =$	−(−70.5) = 70.5	−(−63.8) = 63 8	−(−56.2) = 56.2
11	$+ \epsilon x' \sum_A^P u \cos\phi \frac{b\,\lambda}{F} =$	(20.6) (−3.26) = −67.16	(17.5) (−2.91) = −50.93	(14.0) (−2.42) = 33.88
12	$2\,C_2 \frac{b}{s} =$	3.98 = ϵ (3.34)	15.35 = ϵ (12.87)	26.6 = ϵ (22.32)
13	$+ \epsilon \sum_A^P x \cos^2\phi \frac{b\,\lambda}{F} =$	1.584 = 1.584	3.72 = 3.72	= 7.37
14	$- \epsilon x' \sum_A^P \cos^2\phi \frac{b\,\lambda}{F} =$	−(20.6) (0.073) = −1.504	−(17.5) (0.186) = −3.255	−(14) (0.43) = −5.88
15	$2\,C_3 \frac{b}{s} =$	0.09534 = ϵ (0.080)	0.554 = ϵ (0.465)	1.776 = ϵ (1.49)
16	$2\,C_{14} = L - x' =$	0	3.1	6.6
17	$2\,C_{15} = 1 - \frac{x'}{L} =$	1 − 1 = 0	1 − 0.850 = 0.150	1 − 0.680 = 0.320
18	$\epsilon \sum_A^P x \sin\phi \cos\phi \frac{b\,\lambda}{F} =$	(−2.60) = −2.60	(−1.90) = −1.90	(−0.92) = −0.92
19	$- \epsilon x' \sum_A^P \sin\phi \cos\phi \frac{b\,\lambda}{F} =$	−(20.6) (−0.122) = 2.518	−(17.5) (−0.085) = 1.488	−(14) (−0.022) = 0.308
20	$2\,C_6 \frac{b}{s} =$	−0.1037 = ϵ (−0.087)	−0.4911 = ϵ (−0.412)	−0.728 = ϵ (−0.612)

TABLE 3.—(*Continued.*)

Item. 1	10.5	7.0	3.5	0.0
2	(10.1) (265) = 2 667	(13.6) (265) = 3 604	(17.1) (265) = 4 532	(20.6) (265) = 5 459
3	864 = 864	873 = 873	877 = 877	878 = 878
4	−(10.5) (40.5) = − 425.3	−(7) (41.61) = − 291.3	−(3.5) (42.4) = − 148.4	
5	−(20.5) (80.8) = −1 656	−(20.5) (95.7) = − 1 962	−(20.5) (115.2) = −2 362	−(20.5) (124.9) = −2 560
6	(20.5) (10.5) (4.16) = 895.4	(20.5) (7.0) (5.85) = 839.5	(20.5) (3.5) (9.55) = 685.2	
7	(1.420) (−9.82) = − 13.94	(1.420) (4.39) = 6.234	(1.420) (11.16) = 15.85	(1.420) (11.58) = 16.44
8	−(1.420) (10.5) (+0.46) = − 6.85	−(1.420) (7.0) (2.08) = − 20.68	−(1.420) (3.5) (3.37) = 16.75	
9	2 334.31	3 048.754	3 582.90	3 793.44
10	−(−47.8) = 47.8	−(−40.6) = 40.6	−(−36.7) = 36.7	−(−36.5) = 36.5
11	(10.5) (−1.727) = − 18.1	(7) (−0.904) = − 6.3	(3.5) (−0.164) = − 0.6	
12	35.4 = ϵ (29.7)	40.9 = ϵ (34.3)	43.1 = ϵ (36.1)	43.5 = ϵ (36.5)
13	13.96 = 13.96	24.76 = 24.76	39.1 = 39.1	46.4 = 46.4
14	−(10.5) (0.96) = − 10.08	−(7) (2.19) = − 15.33	−(3.5) (4.92) = − 17.22	
15	4.62 = ϵ (3.88)	11.24 = ϵ (9.43)	26.08 = ϵ (21.88)	55.3 = ϵ (46.4)
16	10.1	13.6	17.1	20.6
17	1−0.510 = 0.490	1−0.340 = 0.660	1−0.170 = 0.830	1 − 0 = 1.000
18	(+0.46) = + 0.46	2.08 = 2.08	3.37 = 3.37	3.60 = 3.60
19	−(10.5) (0.091) = − 0.9555	−(7) (0.276) = − 1.932	−(3.5) (0.520) = − 1.82	
20	−0.5906 = ϵ (−0.4955)	0.1764 = ϵ (+0.148)	1.847 = ϵ (1.55)	4.290 = ϵ (3.60)

TABLE 4.—VALUES OF COEFFICIENTS, C, FOR HORIZONTAL LOADING.

Item. 1	$x', y' =$	$x' = 22.6, y' = 20.5$	$x' = 22.7, y' = 17.0$	$x' = 23.0, y' = 13.55$
2	$(h - y')\left(-\sum y \frac{b}{I} + h \sum \frac{b}{I}\right) =$		+(3.5) (+265) = 927.5	(6.95) (265) = 1 841.8
3	$+\sum_A^P y^2 \frac{b}{I} =$		143.2 = 143.2	418.0 = 418.0
4	$-y' \sum_A^P y \frac{b}{I} =$		−(17) (7.65) = − 130.1	−(13.55) (25.7) = − 348.2
5	$-h \sum_A^P y \frac{b}{I} =$		−(20.5) (7.65) = − 156.8	−(20.5) (25.7) = − 526.8
6	$+h y' \sum_A^P \frac{b}{I} =$		(20.5) (17) (0.408) = 142.2	(20.5) (13.55) (1.583) = 440.0
7	$+\epsilon^2 \sum_A^P x^2 \sin^2 \phi \frac{b \lambda}{F} =$		(1.420) (124.8) = 177.2	(1.420) (534) = 758.3
8	$-\epsilon^2 x' \sum_A^P x \sin^2 \phi \frac{b \lambda}{F} =$		−(1.420) (5.91) (22.7) = − 190.5	−(1.420) (23.0) (24.8) = − 810.0
9	$2\, C_{11} \frac{b}{s} =$	0	912.7	1 773.1

TABLE 4.—(*Continued.*)

Item.	$x', y' =$		$x' = 22.6, y' = 20.5$	$x' = 22.7, y' = 17.0$	$x' = 23.0, y' = 13.55$
10	$-\sum_A^P u\,x \sin\phi \frac{b\,\lambda}{F}$	=		$-(157) = -\ 157$	$-(585)\quad -\ 585$
11	$+x' \sum_A^P u \sin\phi \frac{b\,\lambda}{F}$	=		$(22.7)\ (7.44) = 168.9$	$(23.0)\ (27.14) = 624$
12	$2\,C_2 \frac{b}{s}$	=	0	$14.2 = \epsilon\ (11.9)$	$46.5 = \epsilon\ (39)$
13	$\sum_A^P x \sin\phi \cos\phi \frac{b\,\lambda}{F}$	=		$(-1.77) = -\ 1.77$	$(-2.92) = -\ 2.92$
14	$-x' \sum_A^P \sin\phi \cos\phi \frac{b\,\lambda}{F}$	=		$-(-0.083)\ (22.7) = 1.88$	$-(-0.136)\ (23.0) = 3.13$
15	$2\,C_8 \frac{b}{s}$	=	0	$0.131 = \epsilon\ (0.11)$	$0.250 = \epsilon\ (0.21)$
16	$2\,C_{14} = h - y'$	=	M_s for total earth pressure can be computed directly.		
17	$2\,C_{15} = \frac{h - y'}{L}$	=	T_y for total earth pressure can be computed directly; also $T_y = 0$ for symmetrical loading.		
18	$\sum_A^P x \sin^2\phi \frac{b\,\lambda}{F}$	=	Need not be computed for symmetrical loading since $M_y = 0$.		
19	$-x' \sum_A^P \sin^2\phi \frac{b\,\lambda}{F}$	=			
20	$2\,C_6 \frac{b}{s}$	=			

TABLE 4.—(*Continued.*)

Item. 1	$x' = 23.4, y = 10.0$	$x' = 23.8, y' = 6.54$	$x' = 23.0, y' = -1.34$	
2	$(10.5)\ (265) = 2\,782.5$	$(13.96)\ (265) = 3\,699.4$	$(21.8)\ (265) = 5\,780$	
3	$(513.4) = 513.4$	$544 = 544.0$	$548 = 548$	
4	$-(10.0)\ (33.71) = -\ 337.1$	$-(6.54)\ (37.4) = -\ 244.6$	$-(-1.34)\ (38.3) = +\ 51$	
5	$-(20.5)\ (33.71) = -\ 691.0$	$-(20.5)\ (37.4) = -\ 766.7$	$-(20.5)\ (38.3) = -\ 782$	
6	$+(20.5)\ (10.0)\ (2.27$[illegible] $= 466.0$	$+(20.5)\ (6.54)\ (2.71) = 364.0$	$+(20.5)\ (-1.34)\ (2.90) = -\ 80$	
7	$(1.420)\ (776) = 1\,102.0$	$(1.420)\ (936) = 1\,329.0$	$(1.420)\ (985) = 1\,400$	
8	$-(1.420)\ (35.92)\ (23$[illegible] $= -1\,192.0$	$-(1.420)\ (43.2)\ (23.8) = -1\,460.0$	$-(1.420)\ (23.0)\ (45.4) = -1\,482$	
9	$2\,643.8$	$3\,465.1$	$5\,435$	
10	$-(837) = -\ 837$	$-(1\,001) = -1\,001$	$-(1\,043) = -1\,043$	
11	$(23.4)\ (38.6) = 903.2$	$(23.8)\ (46.0) = 1\,094.8$	$(23.0)\ (47.9) = 1\,102$	
12	$78.9 = \epsilon\ (66.2)$	$111.8 = \epsilon\ (93.8)$	$70.3 = \epsilon\ (59)$	
13	$(-3.60) = -\ 3.60$	$(-4.04) = -\ 4.04$	$(-2.60) = -\ 2.60$	
14	$-(-0.167)\ (23.4) = 3.91$	$-(-0.187)\ (23.8) = 4.45$	$-(-0.122)\ (23.0) = 2.81$	
15	$0.369 = \epsilon\ (0.31)$	$0.488 = \epsilon\ (0.41)$	$0.25 = \epsilon\ (0.21)$	
16				
17				
18 19 20				

table is carried out only far enough to determine the possibility of a reversal of stresses. In practice, Table 8 is continued to include every section for which an influence line is drawn.

TABLE 5.—DEAD LOAD, EARTH PRESSURE, AND TEMPERATURE COEFFICIENTS.

x' or y'.	Load, in kips.	$2\,C_{11}\frac{b}{s}$.		$2\,C_2\frac{b}{s}$.		$2\,C_8\frac{b}{s}$.	
		Unit load.	Dead load.	Unit load.	Dead load.	Unit load.	Dead load.
DEAD LOAD.							
20.6	119	—30.5	—3 630	3.98	473	0.0953	11.3
17.5	208	711	148 000	15.35	3 200	0.554	115.0
14.0	166	1 539	255 500	26.6	4 420	1.776	295
10.5	128	2 334	298 500	35.4	4 530	4.62	592
7.0	100	3 048	304 800	40.9	4 090	11.24	1 124
3.5	84	3 583	301 000	43.1	3 620	26.08	2 190
0	38	3 793	144 000	43.5	1 650	55.3	2 100
	843		1 448 170		21 983		6 427
EARTH PRESSURE.							
19.8	53	183*	9 720	3 *	159	0.026*	1.6
17.0	89	913	81 600	14.2	1 263	0.131	11.7
13.55	76	1 773	135 000	46.5	3 540	0.250	19.0
10.0	56	2 644	148 000	78.9	4 420	0.369	20.7
6.54	42	3 465	145 800	111.8	4 680	0.488	20.5
3.34	27	4 340*	107 200	86.3*	2 330	0.340*	9.2
	343		627 320		16 392		82.7
TEMPERATURE (FOR 100° RISE)							

$$C_{11}\frac{b}{s} = \frac{b}{s}\,E^{\dagger}\,c\,t\,L = \frac{66.9}{3.52} \times 432\ 000 \times 0.0000065 \times 100 \times 20.6 = 110\ 000$$

$$C_2\frac{b}{s} = \epsilon\,C_{11}\frac{b}{s} = \epsilon \times 110\ 000 = 131\ 500$$

$$C_8 = C_{14} = C_{15} = C_6 = 0$$

---

*These values are interpolated.

† For $n = 10$, $E_c = 30\ 000$ kips per sq. in. ÷ 10 = 432 000 kips per sq. ft.

It should be noted that the weight of the abutments has been omitted from the computations because the influence lines for the crown stresses are all zero over the reactions. In computing the direct thrust, t_v, for sections along the abutment, the weight of the wall should be included. Having the maximum stresses, the sections can now be proportional. This portion of the numerical work will be given in Part II, Example 2, where the "Theory of Proportioning Sections" is discussed.

TABLE 6.—SOLUTION OF EQUATIONS.

Item.			Values of x'.							Earth pressure.	Dead load.	Temperature, 100° rise.
			0	3.5	7	10.5	14	17.5	20.6			
1	$6\,531\ T_x - 1\,253\ T_z + 4.291\ M_x$	=	1 897	1 792	1 524	1 167	770	355	−15.24	627 000*	1 448 000*	110 000
2	$-1\,253\ T_x + 1\,109\ T_z + 0.083\ M_x$	=	21.7	21.6	20.5	17.7	13.3	7.67	1.99	16 390*	22 000*	131 500
3	$4.291\ T_x + 0.083\ T_z + 8.982\ M_x$	=	27.65	13.04	5.62	2.31	0.888	0.277	0.0477	82.7*	6 430*	0
4	$2.050\ T_x + 0.0398\ T_z + 4.291\ M_x$	=	13.21	6.229	2.684	1.103	0.424	0.132	0.0228	39.6	3 070	0
5	$0.0397\ T_x + 0.00077\ T_z + 0.083\ M_x$	=	0.257	0.12	0.052	0.0213	0.0082	0.0025	0.0004	0.763	57	0
6	$6\,529\ T_x - 1\,253\ T_z$	=	1 884	1 786	1 521	1 166	769.6	355.4	−15.22	627 000	1 445 000	110 000
7	$-1\,253\ T_x + 1\,109\ T_z$	=	21.4	21.5	20.5	17.7	13.3	7.67	1.99	16 390	22 000	131 500
8	$5.211\ T_x - T_z$	=	1.504	1.425	1.214	0.931	0.614	0.284	− 0.012	498	1 152	87.8
9	$-1.129\ T_x + T_z$	=	0.020	0.020	0.019	0.016	0.012	0.007	0.002	15	20	118.6
10	$4.082\ T_x$	=	1.524	1.445	1.233	0.947	0.626	0.291	−0.010	513	1 172	206.4
11	T_x	=	0.373	0.353	0.302	0.232	0.153	0.071	−0.0034	125	287	50.5
12	$5.883\ T_x - 1.129\ T_z$	=	1.699	1.609	1.370	1.051	0.707	0.329	−0.0137	563	1 302	99
13	$-5.883\ T_x + 5.211\ T_z$	=	0.104	0.104	0.094	0.083	0.063	0.036	0.0094	78	104	618
14	$4.082\ T_z$	=	1 803	1.713	1.464	1.134	0.770	0.365	−0.0043	641	1 406	717
15	T_z	=	0.442	0.420	0.359	0.277	0.189	0.089	−0.001	157	344	176
16	$4.291\ T_x$	=	1.601	1.519	1.296	0.996	0.657	0.305	−0.0146	537	1 232	217
17	$+ 0.083\ T_z$	=	0.037	0.035	0.030	0.023	0.016	0.007	0.000	13	28	15
18	$4.291\ T_x + 0.083\ T_z$	=	1.638	1.554	1.326	1.019	0.673	0.312	−0.0146	550	1 260	232

* These values are for the entire arch loaded, and equal $2\ C_{11}\ \frac{b}{s}$, $2\ C_2\ \frac{b}{s}$, etc.

TABLE 6—(*Continued*).

Item.			Values of x'.							Earth pressure.	Dead load.	Temperature, 100° rise.
			0	3.5	7	10.5	14	17.5	20.6			
19	$8.982\ M_s$	=	26.0	11.49	4.29	1.29	0.215	−0.035	0.062	−467	5 170	− 232
20	M_x	=	2.89	1.28	0.48	0.14	0.02	0	0	− 52	575	− 26
21												
22												
23	$M_s + 20.5\ T_x$	=	10.30	8.55	6.80	5.05	3.30	1.55	0			0
24	$20.5\ T_x$	=	7.65	7.26	6.19	4.76	3.14	1.46	−0.07			1 035
25	M_s	=	2.65	1.29	0.61	0.29	0.16	0.09	+0.07	−60 *	860*	− 1 035
26												0
27	T_y	=	0.500	0.415	0.330	0.245	0.160	0.075	0	0	0	
28												
29	$-2.192\ M_y + 4.291\ T_y$	=	2.15	0.924	0.088	−0.300	−0.364	−0.250	−0.052	0	0	0
30	$4.291\ T_y$	=	2.15	1.780	1.416	1.051	0.687	0.322	0	0	0	0
31	$-2.192\ M_y$	=	0	−0.856	−1.328	−1.351	−1.051	−0.572	−0.052	0	0	0
32	M_y	=	0	0.391	0.606	0.616	0.479	0.261	0.024	0	0	0

* For earth pressure and dead load, M_s is best computed directly (from considerations of static equilibrium) and is so computed in this table. The computation is not shown.

TABLE 7.—STRESSES 14 FEET 0 INCHES FROM CROWN.

$(x = 13.87\ ;\ y = 1.60\ ;\ \sin\phi = 0.233\ ;\ \cos\phi = 0.973\ ;\ \varepsilon = 1.192\ ;\ \varepsilon \sin\phi = 0.277\ ;\ \varepsilon\cos\phi = 1.159\ ;\ \varepsilon x \sin\phi = 3.86\ ;$
$\varepsilon x \cos\phi = 16.08\ ;\ u = 1.67\ ;\ v = 13.87\ ;\ w = 433K\ ;\ \bar{x}' = 8.00.)$

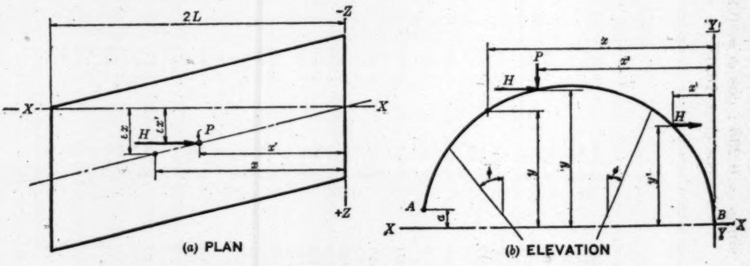
x'.	0	3.5	7.0	10.5	14.0	17.5	20.6	Dead load.	Earth pressure.	Temperature.
$T_x \cos\phi$	0.363	0.344	0.294	0.226	0.150	0.069	0	281	122	49
$\mp T_y \sin\phi$	∓ 0.116	∓ 0.096	∓0.077	∓0.057	∓0.035	∓0.017	0	0	0	0
$W \sin\phi$	+ 0.233	0.233	0.233	0.233	0.233	0	0	101	0	0
t_v (left)	0.480	0.481	0.450	0.402	{ 0.348 0.115 }	0.052	0	382	122	49
t_v (right)	0.479	0 440	0.371	0.283	0.185	0.086	0	382	122	49
$\pm T_x \sin\phi$	± 0.087	± 0.082	±0.070	±0.054	±0.036	±0.016	0	± 72	± 29	± 12
$+ T_y \cos\phi$	0.486	0.404	0.321	0.238	0.146	0.072	0	0	0	0
$- W \cos\phi$	− 0.973	− 0.973	−0.973	−0.973	−0.973	0	0	∓ 422	0	0
t_u (l)	− 0.400	− 0.487	−0.582	−0.681	{ −0.791 +0.182 }	+0.088	0	− 350	+ 29	+ 12
t_u (r)	0.399	0.322	0.251	0.184	+0.110	+0.056	0	+ 350	− 29	− 12
$t_z = T_z$	0.441	0.419	0.360	0.278	0.186	0.088	0	346	160	176
$T_z\, y$	0.597	0.566	0.483	0.371	0.246	0.117	0	462	200	80
$\pm T_y\, x$	± 6.935	± 5.756	±4.577	±3.398	±2.081	±1.026	0	0	0	0
M_x	2.640	1.280	0.590	0.290	0.150	0.080	0	820	− 51	−1 035
$- W (x - x')$	−13.870	−10.370	−6.870	−3.370	0	0	0	−2 542	0	0
m_x (l)	− 3.698	− 2.748	−1.220	0.689	2.477	1.223	0	−1 260	149	− 955
m_x (r)	− 3.698	− 3.910	−3.504	−2.737	−1.685	−0.829	0	−1 260	149	− 955
$\mp T_x\, \epsilon\, x \cos\phi$	∓ 5.996	∓ 5 691	∓4.855	∓3.730	∓2.476	∓1.130	0	∓4 641	∓2 010	∓ 812
$+ T_y\, \epsilon\, x \sin\phi$	1.918	1.592	1.266	0.940	0.575	0.284	0	0	0	0
$\pm T_z\, v$	± 6.117	± 5.812	±4.993	±3.856	±2.579	±1.221	±0.083	±4 799	±2 219	±2 434
$\pm M_x \sin\phi$	± 0.673	± 0.297	±0.111	±0.033	±0.005	±0	±0	± 133	∓ 12	∓ 7
$M_y \cos\phi$	0	0.380	0.585	0.622	0.447	0.250	0.023	0	0	0
$- W\, \epsilon\, (x - x') \sin\phi$	− 3.842	− 2.872	−1.903	−0.935	0	0	0	∓ 704	0	0
m_u (l)	− 1.130	− 0.482	−0.197	+0.786	+1.130	0.626	0.106	− 413	197	1 615
m_u (r)	1.130	1.554	1.572	1.403	0.914	0.443	−0.060	413	− 197	−1 615
$T_x\, \epsilon\, x \sin\phi$	1.431	1.358	1.158	0.890	0.591	0.274	0	1 107	479	193
$\pm T_y\, \epsilon\, x \cos\phi$	± 8.038	± 6.672	±5.305	±3.939	±2.411	±1.190	0	0	0	0
$- T_z\, u$	− 0.736	− 0.700	−0.601	−0.464	−0.310	−0.147	0	− 578	− 267	− 293
$M_x \cos\phi$	2.811	1.242	0.462	0.138	0.022	0	0	556	− 52	− 26
$\mp M_y \sin\phi$	0	∓ 0.091	∓0.140	∓0.149	∓0.107	∓0.060	0	0	0	0
$- W\, \epsilon\, (x - x') \cos\phi$	−16.076	−12.018	−7.962	−3.906	0	0	0	−2 940	0	0
m_v (l)	− 4.532	− 3.537	−1.798	0.448	+2.607	1.257	0	−1 860	160	− 126
m_v (r)	− 4.532	− 4.681	−4.146	−3.226	−2.101	−1.003	0	−1 860	160	− 126
$x - x'$	13.87	10.37	6.87	3.87	0			5.87		

TABLE 8.—TOTAL STRESSES.

	Loading.	COMBINATION FOR MAXIMUM TOTAL STRESSES.						COMBINATION FOR MINIMUM TOTAL STRESSES.					
		M_z.	M_y.	T_x.	M_x.	T_y.	T_z.	M_z.	M_y.	T_x.	M_x.	T_y.	T_z.
Crown	Dead loads.	+ 860	0	+ 289	+ 572	0	+ 346	+ 860					
	Earth pressures..	− 60	0	+ 125	− 53	0	+ 160	− 60					
	Live loads........	+ 192	± 7	+ 29	+ 209	± 38	+ 34	....					
	Total.............	+ 992	....	+ 443	+ 728	....	+ 540	+ 800					
	Temperature, 50°	+ 517	....	− 25	+ 13	....	− 88	− 517					
	Total.............	+1 509	....	+ 418	+ 741	....	+ 452	+ 283					
20.6 ft. from crown		m_z.	m_u.	t_v.	m_v.	t_u.	t_z.	m_z.	m_u.	t_v.	m_v.	t_u.	t_z
	Dead loads.......	−4 870	−2 071	+ 558	−5 848	− 695	+ 346	−4 870					
	Earth pressures..	+ 405	+ 393	+ 118	+ 422	+ 42	+ 160	+ 405					
	Live loads.......	−1 035	− 432	+ 103	−1 200	− 110	+ 73	....					
	Total.............	−5 500	−2 110	+ 779	−6 626	− 763	+ 579	−4 465					
	Temperature, 50°	− 425	+1 231	+ 24	− 115	+ 9	+ 88	+ 425					
	Total.............	−5 925	− 879	+ 803	−6 741	− 754	+ 667	−4 040					

UNSYMMETRICAL ARCHES

It is usually advantageous to take the origin either at the movable end of the transformed structure, or at the point of symmetry. The origin is, therefore, taken at the right abutment which is considered the movable end. (See Fig. 9.)



(a) PLAN (b) ELEVATION

FIG. 9.—DIAGRAMS SHOWING POSITION OF CO-ORDINATE AXES FOR UNSYMMETRICAL ARCHES.

The stresses at a section of an unsymmetrical arch in terms of the reactions may be found by substituting Equations (5) in Equations (2), and transferring the origin to the center of the right abutment:

$$t_v = -R_x \cos\phi - R_y \sin\phi + W \sin\phi - H \cos\phi \qquad (20a)$$

$$t_u = -R_x \sin\phi + R_y \cos\phi - W \cos\phi - H \sin\phi \qquad (20b)$$

$$t_z = R_z \qquad (20c)$$

$$\begin{aligned} m_u = {} & R_x\,\varepsilon\,x \cos\phi + R_y\,\varepsilon\,x \sin\phi + R_z\,v' - N_x \sin\phi + N_y \cos\phi \\ & - W\,\varepsilon\,(x - x') \sin\phi + H\,\varepsilon\,(x - x') \cos\phi \ldots\ldots\ldots\ldots (20d) \\ m_v = {} & - R_x\,\varepsilon\,x \sin\phi + R_y\,\varepsilon\,x \cos\phi - R_z\,u' - N_x \cos\phi - N_y \sin\phi \\ & - W\,\varepsilon\,(x - x') \cos\phi - H\,\varepsilon\,(x - x') \sin\phi \ldots\ldots\ldots\ldots (20e) \end{aligned}$$

and,

$$m_z = R_x\,y + R_y\,x + N_z - W\,(x - x') + H\,(y - y') \ldots\ldots\ldots\ldots (20f)$$

The equations of the co-ordinates of the center of the right abutment with respect to the point, P, of any section may be written as follows:

$$u' = x \sin\phi + y \cos\phi \ldots\ldots\ldots\ldots (21a)$$

and,

$$v' = x \cos\phi - y \sin\phi \ldots\ldots\ldots\ldots (21b)$$

The deformation formulas for this case are identical with Equation (7):

$$E\,d\,\gamma = m_z\,\frac{s}{I} \ldots\ldots\ldots\ldots (22a)$$

and,

$$E\,d\,\alpha = m_v\,\frac{\lambda\,s}{F} \ldots\ldots\ldots\ldots (22b)$$

Equations (20*e*) and (20*f*) have been rewritten by taking each reaction or load in turn and making the remaining terms zero. Equations (23*A*), therefore, are for the stresses at a section due to each reaction component at the right abutment.

EQUATIONS (23*A*).—STRESSES AT A SECTION DUE TO REACTION COMPONENTS.

Reaction component.	m_z.	m_v.
R_x	$R_x\,y$	$-R_x\,\epsilon\,x \sin\phi$
R_y	$R_y\,x$	$R_y\,\epsilon\,x \cos\phi$
R_z	....	$-R_z\,u'$
N_x	....	$-N_x \cos\phi$
N_y	....	$-N_y \sin\phi$
N_z	N_z	....

Stresses at a section due to a vertical load, W, at a point, $x = x'$, $z = \varepsilon\,x'$, are similarly given by:

$$\left.\begin{aligned} & m_z = - W\,(x - x') \\ \text{and,}\quad & m_v = - W\,\varepsilon\,(x - x') \cos\phi \end{aligned}\right\} \ldots\ldots\ldots\ldots (23B)$$

Stresses at a section due to the horizontal load, H, at a point, $y = y'$, $z = \varepsilon\,x'$, are:

$$\left.\begin{aligned} & m_z = + H\,(y - y') \\ \text{and,}\quad & m_v = - H\,\varepsilon\,(x - x') \sin\phi \end{aligned}\right\} \ldots\ldots\ldots\ldots (23C)$$

As in the case of the symmetrical arches, Equations (23*A*), when combined, are used, together with Equations (24*B*) and (24*C*).

Following the method used for the symmetrical arches, Equations (22) and (23) are substituted in Equations (24). For the fixed arch, all the deflections produced by the loads are equated to the deflections produced by the

reactions resulting in the final Equations (25), which are placed in the Appendix for the convenience of the designer. As in the case of Equations (16), the text numbering is used in the Appendix for convenience in referring back to the text when necessary.

EQUATIONS (24).—DIFFERENTIAL DEFLECTIONS OF THE RIGHT ABUTMENT.

Deflection.	Due to m_z.	Due to m_v.
$d\theta_x$	......	$d\gamma \cos\phi$
$d\theta_y$	......	$-d\alpha \sin\phi$
$d\theta_z$	$d\gamma$	....
$d\Delta_x$	$d\gamma\ y$	$-d\alpha\ \epsilon\ x \sin\phi$
$d\Delta_y$	$d\gamma\ x$	$d\alpha\ \epsilon\ x \cos\phi$
$d\Delta_z$	......	$d\alpha\ u'$

For the two-hinged arch, the fact that the left abutment is hinged prevents the assumption that the vertical deflections produced by the reactions and the loads are equal, and the fact that the right abutment is hinged prevents a similar assumption for the angular deflection of the right abutment about the *Z*-axis. Two additional equations are obtained from statics as in the case of the symmetrical arch, but in this case a correction must be made for the fact that the two abutments are not at the same elevation.

The structure has been transformed into a curved cantilever beam and the load has caused the right abutment to deflect vertically. Replacing the hinge in the left abutment, the entire arch is rotated until the right abutment returns to its original elevation. This rotation is about the *Z*-axis and corresponds to a γ-deflection. From Equation (24), it is seen that such an angular deflection produces a linear deflection along the *X*-axis, or a Δ_x. The Δ_x already derived for each load or reaction, therefore, must be corrected for this additional γ-deflection. These deflections will be denoted by the same symbols used previously, but primed.

In Fig. 10, the arch acting as a cantilever beam is deflected by the loads (not shown) so that *B* moves to *B'*. When the hinge at *A* is replaced, *B'* moves to *B''*, and the deflection, Δ_x, must be corrected by the amount, *B'' D*.

Since γ' is small, $\angle\ AB''\ B'$ may be considered a right angle, so that $\angle\ B''\ B'\ D = \angle\ E$. From similar triangles : $\dfrac{a}{2\ L} = \dfrac{B''\ D}{B'\ D} = \dfrac{B''\ D}{\Delta_y}$ or,

$$B''\ D = \left(\frac{a}{2\ L}\right) \Delta_y$$

The final deflection along the *X*-axis is $B\ B''$; and $B\ B'' = B\ D - B''\ D$. Therefore,

$$\Delta'_x = \Delta_x - \left(\frac{a}{2\ L}\right) \Delta_y \qquad (26)$$

For the two-hinged unsymmetrical arch, the Δ'_x produced by the loads must equal the Δ'_x produced by the reactions. For the symmetrical arch, $a = 0$, and $\Delta'_x = \Delta_x$.

Four equations are obtained by equating the deflections, and two more are obtained from statics, resulting in the six final Equations (27). (See Appendix I.)

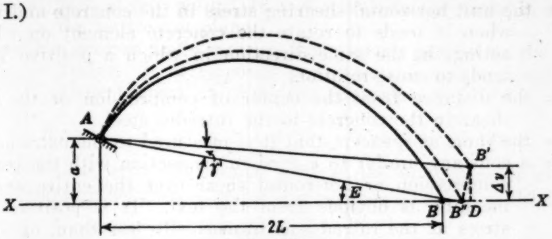


FIG. 10.

SUMMARY

This concludes Part I, which gives the derivation of the four sets of equations for obtaining the total stresses for the fixed and hinged arches, which may be either symmetrical or unsymmetrical. The unsymmetrical arch equations can be used for symmetrical arches, but would involve more numerical work. It should be remembered that the thrust, shears, and the bending deformations about the radial axes have been omitted. For large and important arches, the preceding theory should not be used without correction, or by an adjustment similar to that made for right arches.

PART II.—THEORY OF PROPORTIONING SECTIONS

The analysis of a structure may be divided into three parts:

(a) The computation of the forces and moments produced by the external loading at several sections of the structure.
(b) The distribution of the stresses over these sections.
(c) Design of the sections to resist the stresses acting on them.

The numerical work up to this point gives in detail all the steps necessary for plotting influence lines and the maximum moments, thrusts, and shears have been obtained from these curves in the usual way. The next step is to determine the distribution of the stresses over a section.

NOTATION

To the notation already given, the following symbols are added. In the design formulas, the standard reinforced concrete notation is used with the exceptions and additions noted. Symbols which are used only in the derivations, but which do not appear in the final formulas, are defined in the text and are not given here. Unless otherwise stated, symbols are always positive.

e = the distance from the intrados steel to the resultant normal thrust or horizontal shear acting on a section, and is positive in the direction away from the center of curvature of the arch.
e' = the same as e, except that it is measured from the extrados steel.
f_s = the unit stress in the steel in the intrados face; it is positive when the steel is in tension.
f_s' = the unit stress in the steel in the extrados face; it is positive when the steel is in tension.

f_c = the unit direct stress in the concrete; it is positive when the concrete is in compression.
h = the unit horizontal shearing stress in the concrete and is positive when it tends to rotate the concrete element on which it is acting, in the same direction in which a positive T_z (or t_z) tends to cause rotation.
j = the distance from the center of compression or the center of shear in the concrete to the intrados steel.
j' = the same as j, except that it is measured to the extrados steel.
q = a constant similar to k used in connection with the condition of compression or horizontal shear over the entire section. Its meaning is obvious from the text. It is positive when the stress at the intrados is numerically less than, or of opposite sign to, the stress at the extrados.
r = the distance between the intrados and extrados steel.
M = the direct moment per unit width acting on a design plane and is positive when it produces compression in the extrados fibers.
M_s = the horizontal shear moment per unit width acting on a design plane. Considering this moment as composed of two equal and opposite horizontal forces, it is positive when these forces tend to rotate the concrete element in the same direction in which the corresponding forces of the moment, m_v (or M_x), tend to cause rotation.
N = the direct thrust per unit width acting on a design plane and is positive when it produces compression.
S = the horizontal shear per unit width acting on a design plane and is positive in the same direction as h.
m = the direct moment per unit width acting on a plane perpendicular to the V-axis and is positive when it produces compression in the extrados fibers.
m' = the same as m, but acting on a plane perpendicular to the Z-axis.
t_n, t_n' = the direct thrust per unit width corresponding to m and m', respectively, and are positive when they produce compression.
m_s = the horizontal shear moment per unit width acting on a plane perpendicular to the V-axis and is positive in the same direction as m_v (or M_x).
m_s' = the horizontal shear moment per unit width acting on a plane perpendicular to the Z-axis and is positive in the same direction as M_s.
t_s = the horizontal shear per unit width corresponding to m_s and is positive in the same direction as t_z (or T_z).
t_s' = the horizontal shear per unit width corresponding to m_s' and is positive in the same direction as h.
V = the radial shear per unit width acting on a plane perpendicular to the V-axis and is positive in the same direction as t_y.
V' = the radial shear per unit width acting on a plane perpendicular to the Z-axis and is positive in the direction opposite to the positive direction of the shear, V, acting on the unbalanced triangle, ACD (Fig. 16(a)).
α = the angle between the direction of the steel and the direction of a design plane, and is positive in the same direction as m_y (or M_y).
ψ = the angle between a plane perpendicular to the V-axis and a design plane and is positive in the same direction as m_y (or M_y).

θ' = the skew angle measured in the plane tangent to the neutral surface at any point. It corresponds to the angle, θ, measured in the horizontal plane.

ϕ' = the central angle measured in a plane parallel to the face of the bridge. It corresponds to the central angle, ϕ, measured in a plane perpendicular to the abutments.

RELATION BETWEEN θ' AND θ AND BETWEEN ϕ' AND ϕ

Before entering upon the second step of the analysis—the stress distribution—the relation between θ' and θ and between ϕ' and ϕ will be found.

From Fig. 11: $\tan \theta' = \cos \phi \tan \theta$, or,

$$\varepsilon' = \varepsilon \cos \phi \quad \text{.........................} (28)$$

Also, $\tan \phi' = \dfrac{\sin \phi}{\cos \phi \sec \theta}$, or

$$\tan \phi' = \tan \phi \cos \theta \quad \text{.......................} (29)$$

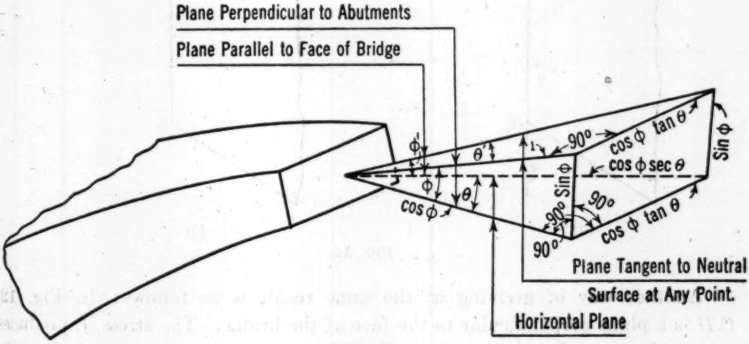

Plane Perpendicular to Abutments
Plane Parallel to Face of Bridge
φ'
θ'
φ
θ
1
90°
cos φ tan θ
Sin φ
cos φ sec θ
Sin φ
90°
90°
cos φ
cos φ tan θ
90°
Plane Tangent to Neutral Surface at Any Point.
Horizontal Plane

FIG. 11.

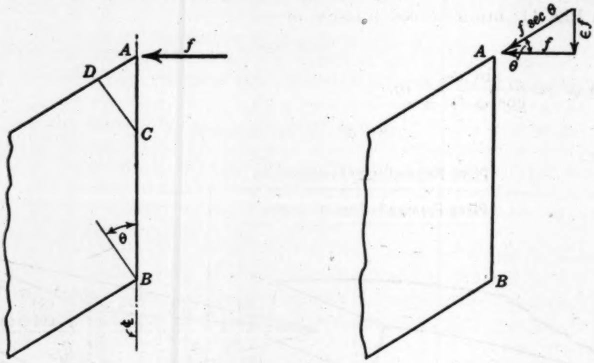
The relation between ϕ' and ϕ is convenient when laying out the arch for the purpose of scaling the properties of the sections. The intrados and extrados curves are usually determined in a plane parallel to the face of the bridge, while for the purposes of design, the arch must be laid out in a plane perpendicular to the abutments. This relation, therefore, gives a convenient method of finding ϕ which appears in the skew arch analysis. The relation between ϵ' and ϵ is important, because the former is used in the design of the steel at sections other than the crown.

SECTION 1.—STRESS DISTRIBUTION

Before the steel can be designed, the distribution of the stresses at any section produced by the various moments, shears, and thrusts must be determined. For simplicity, the crown section is analyzed in the introductory analysis, A, which follows. For other sections, the analysis is the same except that the sections are not vertical and the planes tangent to the neutral surface are not horizontal. The angle, θ', is then used instead of the angle, θ.

(A) Stresses at Face of Bridge

Resultant Thrust at Face of Bridge.—In Fig. 12(*a*), f is the unit stress produced at A by the direct thrust, T_x. Since there is no material to the left of the section to resist the stress, it is impossible for f to act by itself. There must be, therefore, a unit shear acting at the point, A, of such a magnitude that the resultant stress will be parallel to the face of bridge as shown in Fig. 12(*b*). The resultant stress is evidently equal to $f \sec \theta$, and the unit shear required to bring this stress in line with the face of bridge is εf.



(*a*) (*b*)

FIG. 12.

Another way of arriving at the same result is as follows: In Fig. 12, $C\,D$ is a plane perpendicular to the face of the bridge. The stress, f, produces a unit shear on the plane, $C\,D$, and an equal unit shear on the plane, $A\,D$, which is perpendicular to the plane, $C\,D$. However, the plane, $A\,D$, is an external face and can carry no stress. Hence, there can be no shear on Plane $C\,D$, and the resultant unit stress must be normal to it, that is, parallel to the face of bridge.

Thrust and Shears on Plane Perpendicular to Z-Axis.—In Fig. 13, a thrust, T_x, acts on the crown section, $A\,B$. At A, there is a direct unit stress, f, and a unit shear, εf, as previously shown. If the triangle, $A\,C\,D$, of dimensions, dx, dz, is cut from the arch, this triangle shown enlarged in Fig. 14, is a free body and must be in equilibrium. Face $A\,C$ is an external face in the uncut arch and carries no stress. For equilibrium there must be a unit shear, h, and unit thrust, t_n, on the face, $C\,D$.

Since $\Sigma X = 0$, $-h\,dx = f\,dz$; $-h = f\,\dfrac{dz}{dx} = \varepsilon f$; and,

$$t'_s = -t_s = -\varepsilon t_n \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (30)$$

This merely proves the familiar theorem that if there is a unit shear on one plane, there must be an equal unit shear on a plane at right angles to it.

Again, for equilibrium since $\Sigma Z = 0$, $t'_n\, dx = \varepsilon\, t_n\, dz$; $t'_n = \varepsilon\, t_n \dfrac{dz}{dx} = \varepsilon^2 t_n$; and,

$$t'_n = \varepsilon^2\, t_n = \varepsilon\, t_s \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (31)$$

Furthermore, since $\Sigma Y = 0$, $V'\, dx = \dfrac{T_y}{b}\, dz = V\, dz$, and therefore,

$$V' = \varepsilon\, V \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (32)$$

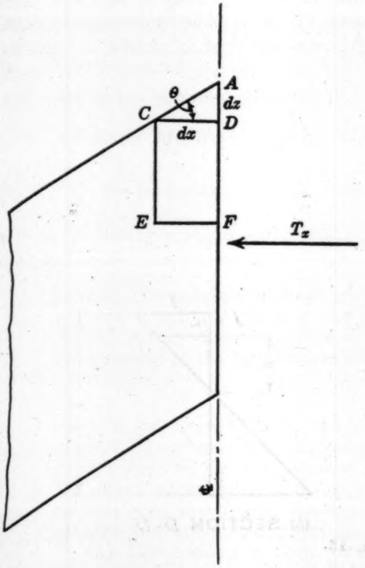


FIG. 13.—PLAN.

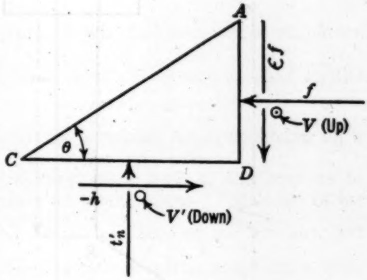


FIG. 14.

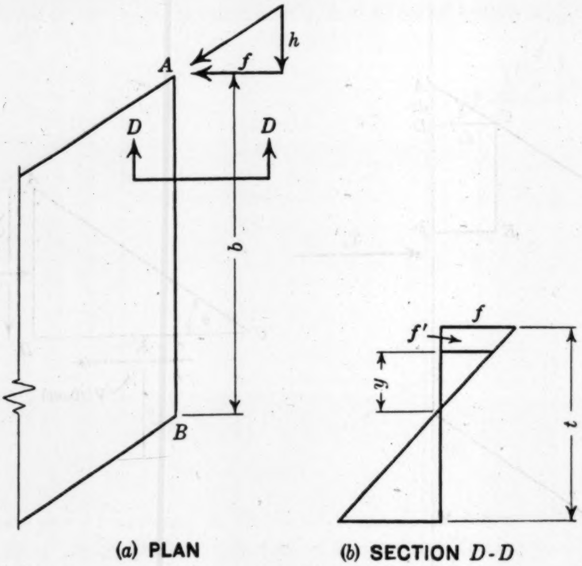
Direct Moment and Shear Moment on Plane Perpendicular to Z-Axis.—In the discussion of the direct thrust, it was shown that at the face of bridge a shear is present which makes the resultant thrust act parallel to that face. It was also shown that there is a direct thrust acting on the plane perpendicular to the section. Similar reasoning holds true for the stresses produced by M_x; but, whereas the stresses produced by the thrust, T_x, are uniformly distributed over the depth of the section, the distribution of M_x is variable.

In Fig. 15(a) a bending moment, M_x, acts on the crown section, $A\,B$. At A, f is the extreme fiber stress produced by the bending moment, and $h = \varepsilon\, f$ is the corresponding unit shear necessary to make the resultant stress act parallel to the face of bridge. The fiber stress at any other point on the radial line through A is f' (Fig. 15(b)). A corresponding unit shear, $h' = \varepsilon\, f'$, must accompany this fiber stress, f', in order that the resultant stress shall here also be parallel to the face of bridge; but f' varies along the depth, t, of the section. Therefore, h' must also vary along the depth of the section. The relation between these variables can be expressed in terms of moments; that is, $M_z = b \Sigma f' y \, dA$, or, $m = \frac{M_z}{b} = \Sigma f' y \, dA$; also, $\Sigma \varepsilon f' y \, dA = \Sigma h' y \, dA$. Therefore,

$$\varepsilon m = m_s \quad \text{(33)}$$

and, similarly,

$$\varepsilon^2 m = m' \quad \text{(34)}$$



(a) PLAN (b) SECTION D-D

FIG. 15.

The moment, m_s, is part of a torsion moment. It is the summation of the moments of the horizontal shears about the horizontal center of gravity of the section. It is also the shear moment required to make the resultant bending stresses at the face of the bridge act parallel to that face.

(B) Stresses at Any Point of a Section

Equations (30) to (34), inclusive, give the value of certain stresses at the face of bridge. The values of these stresses at any point of the section will now be found. As in the right arch, the effect of the curvature of the arch on the stress distribution will be neglected. The error involved is small for curves of large radii.

In the following discussion, the problem of a shaft subjected to torsion is encountered again. Before a correct design of a skew arch can be made, a correct torsion formula will have to be developed. Of course, the method presented herein is not exact, but it is developed with the idea of obtaining a logical design with the data available and of pointing out some of the problems involved.

There are various formulas for analyzing a shaft subjected to torsion. Most of them are very approximate except for the circular and the square sections. The usual assumptions are that an unstressed plane section remains plane after it is stressed and that only shearing stresses are produced. These assumptions are not true except in some special cases. St. Venant developed a more accurate theory which shows that a plane surface becomes warped when the shaft is twisted. This theory, like the more approximate theories, is developed for an isotropic and homogeneous shaft of constant cross-section normal to the axis. The derivation is also based on an assumption which leads to the conclusion that the only stresses produced are shears. However, since this formula agrees with such tests as are available, it is used in the present derivation.[4]

St. Venant's theory is rather complicated; but for large ratios of $\frac{b}{t}$, $\left(\frac{b}{t} > 3\right)$, the formulas reduce to simple terms.[5] The following facts which are proved by theory and, in some cases, substantiated by tests, are used in the design:

> (*a*) The maximum shear produced by a torsion moment acting on a rectangular shaft occurs at the middle of the long side.
>
> (*b*) Along the edges, the shears vary from zero at the corners to a maximum at the middle and are parallel to the edges. For large ratios of $\frac{b}{t}$, the shears parallel to the long edges are zero at the corners, but reach a value just inside the surface, which remains practically constant. The shears are, therefore, a maximum along the entire long edge, except at the corners where they are zero. The shears parallel to the short edges have a finite value only at the surface, but quickly become zero just inside the surface. These shears are also zero at the corners and are a maximum at the middle of the short edges.
>
> (*c*) The torsion moment can be divided into two parts, one-half of which produces shears parallel to the long edge and the other half, shears parallel to the short edge.

In the following derivation, it is assumed that the thrust, (T_x, t_v), the bending moment, (M_z, m_z), and the radial shears, (T_y, t_u), are uniformly distributed over the width, b, of any radial section parallel to the center line of the span, (Z-axis).

Distribution of Horizontal Shear.—Fig. 16(*a*) is a plan view of a skew frame showing the position of a strip of differential width. The strip is enlarged in Fig. 16(*b*) for convenience in illustrating the special notation that applies. In these diagrams, let,

t_3 = unit radial shear produced by the net torsion moment;
Q = static moment of the area, $A\ F$, about the axis, $U - U$ $= \frac{1}{2}\left(\frac{b^2}{4} - z^2\right) t$;

---

[4] A short discussion of torsion is given in *Bulletin No. 3* of the University of Toronto, previously mentioned.

[5] "Applied Elasticity," by John Prescott.

$A = b\,t;$

$a = A\,F \times t = \left(\frac{b}{2} - z\right) t;$ and,

$dv = dx$ (neglecting the curvature of the arch).

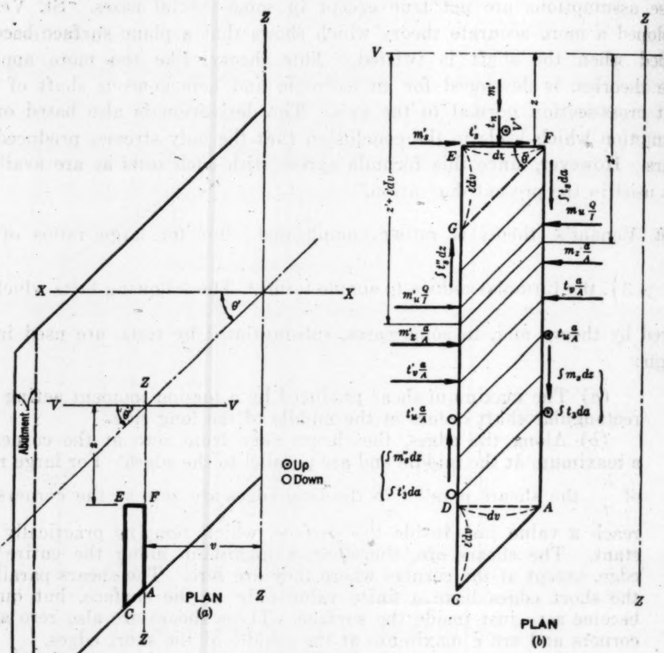

Z
V
X
θ'
Abutment
E
F
A
C
PLAN
(a)
⊙Up
○Down
G
D
m'_z
t'_s
dz
dv
z + ε dv
m_u Q/I
m_z a/A
t_v a/A
m'_u Q/I
m'_z a/A
t'_v a/A
t'_v a/A
∫ m'_z dz
∫ t'_s da
∫ m_z dz
∫ t_s da
∫ t'_s dz
PLAN
(b)

FIG. 16.

Moments are represented as two equal and opposite forces of which only the upper force is shown in the diagrams.

Since $\Sigma\, V = 0$ for equilibrium: $t'_s\, dv = - m'_u \frac{Q}{I} + m_u \frac{Q}{I} - m_u \frac{z}{I}\,\varepsilon'\, t dv - t'_v \frac{a}{A} + t_v \frac{a}{A} - \frac{t'_v}{b}\,\varepsilon'\, dv$, or, $t'_s = - \frac{dm_u}{dv} \frac{Q}{I} - \varepsilon'\, m_u \frac{tz}{I} - \frac{dt_v}{dv} \frac{a}{A} - \varepsilon' \frac{t_v}{b}$.

From Equations (2), in Part I, neglecting the effect of the curvature of the arch; that is, making $\frac{d\phi}{dv} = 0$; $\frac{dm_u}{dv} = (t_z - \varepsilon'\, t_v)$; $\frac{dt_v}{dv} = 0$. Furthermore, $t_s = - t'_s$, and therefore,

$$t_s = \frac{3}{2}\left(\frac{t_z - \varepsilon'\, t_v}{b}\right)\left(1 - 4\,\frac{z^2}{b^2}\right) + \varepsilon'\,\frac{t_v}{b} + 12\,\varepsilon'\,\frac{m_u\, z}{b^3} \quad\text{.........(35)}$$

The last term of Equation (35) is usually small and can be neglected.

This equation shows that the shear, t_s, is composed of two parts: (*a*), that produced by the computed shear, t_z; and (*b*), that produced by the effect of the skew on the thrust, t_v. When $\epsilon' = 0$, the shear, t_z, has the usual parabolic distribution for a rectangular section.

Equation (35) also agrees with the statement made by Professor Rathbun[6] that the portion of the shear, T_z, which is equal to $\epsilon\ T_x$, is distributed uniformly, the remainder being distributed parabolically. In other words, a certain portion of T_z is distributed uniformly across the section so that the resultant unit stress due to T_x and this portion of T_z is at every point of the section, parallel to the face of bridge. The remaining part of T_z is distributed in the same way as in a rectangular beam. If the moment, M_y (or M_u), should be large enough to require steel reinforcement to take the tension, this remaining portion would be distributed in the same way as the shear in a rectangular reinforced concrete beam.

In a right arch, it is generally assumed that the radial shear and the bending moment have the same distribution as in a straight beam. This assumption is the same as saying that $\frac{d\phi}{dv}$ is negligible or zero. **This derivative** is a function of the curvature, $\left(\frac{1}{r}\right)$, and for arches of large radii, the error involved is small.

Direct Thrust on Plane Perpendicular to Z-Axis.—From Fig. 16 (*b*), since $\Sigma\, Z = 0$ for equilibrium: $t'_n = \int_z^{\frac{b}{2}} \frac{\delta t_s}{\delta v}\, dz + \varepsilon'\, t_s$, and, from Equation (35): $\frac{\delta t_s}{\delta v} = 0$. Therefore,

$$t'_n = \varepsilon'\, t_s \qquad \text{(36)}$$

Distribution of the Torsion Moment.—From Fig. 16(*b*), since the summation of moments about the *Z*-axis must be zero for equilibrium:

$$m'_s\, dv = -\, dm_z \frac{a}{A} - \frac{m_z}{b}\, \varepsilon'\, dv + t_u \frac{a}{A}\, dv + t'_u\, dv\, dz$$

$$+ \left(\int_z^{\frac{b}{2}} \int_{-\frac{t}{2}}^{\frac{t}{2}} t_3\, dz\ du\right) dv + \left(\int_{-\frac{t}{2}}^{\frac{t}{2}} t_3\ du\right) dv\, dz.$$

Furthermore, since $m_s = -\, m'_s$ and $\frac{\delta m_z}{\delta v} = t_u$,

$$m_s = \frac{\varepsilon'\, m_z}{b} - \int_z^{\frac{b}{2}} \int_{-\frac{t}{2}}^{\frac{t}{2}} t_3\, dz\, du \qquad \text{(37)}$$

Equation (37) is similar to Equation (35) which shows that a certain portion of the horizontal shear, t_z, is distributed uniformly and the remainder is distributed the same as a shear in a similar rectangular (unskewed) beam. Similarly, Equation (37) shows that a portion of the torsion moment equal to $\varepsilon' m_z$ is distributed uniformly so that the resultant unit stresses produced by this moment and the direct moment, m_z, are at each point of the section parallel to the face of the bridge, and the remaining portion is distributed like a torsion moment. The two portions of the torsion moment are given by the first and second terms, respectively, of the right-hand member of Equation (37). The amount of the portion of this moment which is distributed like a torsion moment is still to be found. This unknown moment will be denoted by m'_v.

---

[6] *Transactions*, Am. Soc. C. E., Vol. LXXXVII (1924), p. 678.

In dealing with the torsion moment, m'_v, St. Venant's torsion formulas will be used.[7] A correct design is not possible until the problem of torsion in general is studied further and tests are made.

Integrating both sides of Equation (37) between the limits of $\frac{b}{2}$ and $-\frac{b}{2}$:

$$\int_{-\frac{b}{2}}^{\frac{b}{2}} m_s\, dz = \varepsilon'\, m_z - \int_{-\frac{b}{2}}^{\frac{b}{2}}\int_{z}^{\frac{b}{2}}\int_{-\frac{t}{2}}^{\frac{t}{2}} t_3\, dz\, dz\, du \ldots\ldots\ldots (38)$$

Substituting the value of t_3 in terms of m_v', as given by St. Venant's formulas, and performing the integrations indicated, the last term of Equation (38) can be shown to equal $-\frac{1}{2}\, m'_v$. Equation (38) can then be rewritten as follows:

$$\int_{-\frac{b}{2}}^{\frac{b}{2}} m_s\, dz - \frac{1}{2}\, m'_v = \varepsilon'\, m_z \ldots\ldots\ldots\ldots\ldots\ldots (39)$$

Since the sum of the moments produced by both the horizontal shears and the radial shears acting on the entire cross-section must equal the computed moment, m_v:

$$\int_{-\frac{b}{2}}^{\frac{b}{2}} m_s\, dz - \int_{-\frac{b}{2}}^{\frac{b}{2}}\int_{-\frac{t}{2}}^{\frac{t}{2}} t_3\, z\, dz\, du = m_v \ldots\ldots\ldots\ldots (40)$$

St. Venant's formulas show that the second term of Equation (40) also equals $-\frac{1}{2}\, m'_v$. Therefore,

$$\int_{-\frac{b}{2}}^{\frac{b}{2}} m_s\, dz + \frac{1}{2}\, m'_v = m_v \ldots\ldots\ldots\ldots\ldots\ldots (41)$$

Eliminating the term, $\int_{-\frac{b}{2}}^{\frac{b}{2}} m_s\, dz$, between Equations (39) and (41), and solving for m'_v:

$$m'_v = m_v - \varepsilon'\, m_z \ldots\ldots\ldots\ldots\ldots\ldots (42)$$

---

[7] These formulas are developed by Prescott in his "Applied Elasticity," Chapter VII, pp. 148 to 156, and are used because they are the best available.

The equation for m_s can now be found. At the face of the bridge, the second term of Equation (37) is given by St. Venant's formulas as zero. At any other point, this second term reduces to a value which for large ratios of $\frac{b}{t}$ $\left(\text{that is, } \frac{b}{t} > 3\right)$, is nearly constant and equals $-\frac{m'_v}{2\,b}$. Equation (37), therefore, reduces to the following expression:

At the face of the bridge:

$$m_s = \frac{\varepsilon'\, m_z}{b} \quad \text{.......................... (43)}$$

At any other point of the section:

$$m_s = \frac{m_v + \varepsilon'\, m_z}{2\, b} \quad \text{........................ (44)}$$

Two equations for the shear moment per unit width of section are necessary because, for large ratios of $\frac{b}{t}$, the torsion moment acting on a section which is normal to the sides of the shaft, produces stresses which are either zero at the surface of the short side and quickly reach a finite value just inside the surface; or, they have a finite value at the surface, and quickly reach zero just inside the surface. In a skew section, the surface stresses do not reach zero, but become a minimum which is a function of the skew angle and is produced by the effect of the skew on the bending moment, m_z.

Attention is called to the fact that the previous sentence is really the crux of the problem of the stress distribution on a skew section. The fact that the stresses on the section at the surface must be parallel to that surface is what causes the difference between the distribution on a right section and a skew section.

***Bending Moment on Plane Perpendicular to Z-Axis.*—From Fig. 16(*b*), since the summation of moments about the *V*-axis must be zero for equilibrium:**

$$m'\, dv = \int_z^{\frac{b}{2}} m''_s\, dz - \int_z^{\frac{b}{2}} m_s\, dz + m_s\, \varepsilon'\, dv - \int_z^{\frac{b}{2}} \int_{-\frac{t}{2}}^{\frac{t}{2}} t'_3\,(z' + \varepsilon'\, dv - z)\, dz\, du$$

$$+ \int_z^{\frac{b}{2}} \int_{-\frac{t}{2}}^{\frac{t}{2}} t_3\,(z' - z)\, dz\, du - \left(\int_{-\frac{t}{2}}^{\frac{t}{2}} t_3\, du\right) (\varepsilon'\, dv) \left(\varepsilon'\, \frac{dv}{2}\right)$$

$$- t'_u\, \frac{a}{A} \left[\frac{1}{2}\left(\frac{b}{2} - z\right) + \varepsilon'\, dv\right] + t_u\, \frac{a}{A} \left[\frac{1}{2}\left(\frac{b}{2} - z\right)\right] - t'_u\,(\varepsilon'\, dv) \left(\varepsilon'\, \frac{dv}{2}\right)$$

Combining and reducing,

$$m' = \int_z^{\frac{b}{2}} \frac{\delta\, m_s}{\delta\, v}\, dz + \varepsilon'\, m_s - \int_z^{\frac{b}{2}} \int_{-\frac{t}{2}}^{\frac{t}{2}} \frac{\delta\ t_3}{\delta\, v}\,(z' - z)\, dz'\, du$$

$$- \varepsilon' \int_z^{\frac{b}{2}} \int_{-\frac{t}{2}}^{\frac{t}{2}} t_3\, dz\, du - \frac{1}{2}\, \frac{\delta\, t_u}{\delta\, v}\, \frac{a}{A} \left(\frac{b}{2} - z\right) - \varepsilon'\, t_u\, \frac{a}{A} \quad \text{..... (45)}$$

From Fig. 17, since Angle $B\ A\ M$ is small and $A\ B = A\ M$, Angle $B\ M\ A$ may be considered a right angle. Let δ_1 be the unit elongation in the direction, $A\ F$; δ_2, the unit elongation in the direction, $B\ F$; and, f_d, the stress in the steel due to the direct stresses in the concrete. Then,

$$\delta_1 = \frac{B\ H}{B\ D} = \frac{B\ B' \cos\beta}{A\ B \cos\alpha} = \frac{B'\ M \sec(\alpha - \beta)\cos\beta}{A\ B \cos A}$$

or,

$$\delta_1 = \frac{f_d}{E_s}\,\frac{\cos\beta}{\cos\alpha\cos(\alpha - \beta)} \ldots\ldots\ldots\ldots\ldots\ldots\ldots (54)$$

Also,

$$\delta_2 = \frac{B\ G}{B\ F} = \frac{B\ B' \sin\beta}{A\ B \sin\alpha} = \frac{B'\ M \sin\beta}{A\ B \sin\alpha\cos(\alpha - \beta)}$$

or,

$$\delta_2 = \frac{f_d}{E_s}\,\frac{\sin\beta}{\sin\alpha\cos(\alpha - \beta)} \ldots\ldots\ldots\ldots\ldots\ldots\ldots (55)$$

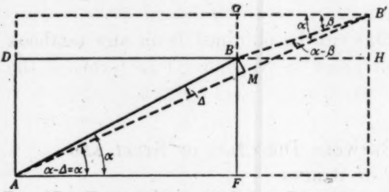


FIG. 17.—DEFORMATION IN STEEL PRODUCED BY DIRECT DEFORMATION.

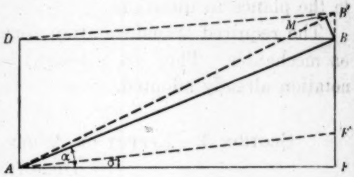


FIG. 18.—DEFORMATION IN STEEL PRODUCED BY SHEAR DEFORMATION IN CONCRETE.

Dividing Equation (54) by Equation (55), $\frac{\delta_1}{\delta_2} = \frac{\tan\alpha}{\tan\beta}$, or,

$$\tan\beta = \frac{\delta_2}{\delta_1}\tan\alpha \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (56)$$

From Equation (54)

$$\delta_1 = \frac{f_d}{E_s\cos\alpha\,(\cos\alpha + \sin\alpha\tan\beta)} \ldots\ldots\ldots\ldots\ldots (57)$$

Substituting Equation (56) in Equation (57), and transforming:

$$\frac{f_d}{E_s} = \delta_1\cos^2\alpha + \delta_2\sin^2\alpha \ldots\ldots\ldots\ldots\ldots\ldots\ldots (58)$$

(*B*) Relation of Stresses in Steel to Shear Deformation

From Fig. 18, neglecting the squares of small quantities: $(A\ M + B'\ M)^2 = (A\ F)^2 + (B\ F + B\ B')^2$, or $F\ F' = \frac{B'\ M \times A\ M}{B\ F}$. Writing for $F\ F'$ its value, $A\ F \times \sigma$, and assuming that f_v equals the stress in the steel due to the shear in the concrete; then, $\sigma = \frac{B'\ M}{A\ F} \times \frac{A\ M}{B\ F} = \frac{B'\ M}{A\ B\cos\alpha} \times \frac{1}{\sin\alpha}$. Therefore,

$$\frac{f_v}{E_s} = \sigma\sin\alpha\cos\alpha \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (59)$$

Combining Equations (58) and (59) to get the total stress in the steel,

$$\frac{f_s}{E_s} = \delta_1 \cos^2 \alpha + \delta_2 \sin^2 \alpha + \sigma \sin \alpha \cos \alpha \ldots\ldots\ldots\ldots(60)$$

Equation (60) gives the relation between the stress in the steel, the components of the elongation of the steel in the direction of the direct stresses on the faces of the concrete element, and the shearing deformation of that element.

In the usual simple case where the steel is parallel to the direction of the applied stress, the deformations in the steel and in the concrete are proportional to their distances from the neutral axis. In the present case, the components of the deformations in the steel and in the concrete in the same direction are proportional to their distances from their respective neutral axes.

SECTION 4.—DERIVATION OF DESIGN FORMULAS

The design formulas can be divided into twelve cases, each one of which has three sub-divisions. Each of these sets of formulas can be tabulated. The method of deriving the formulas is explained in detail for Case I in the discussion which follows.

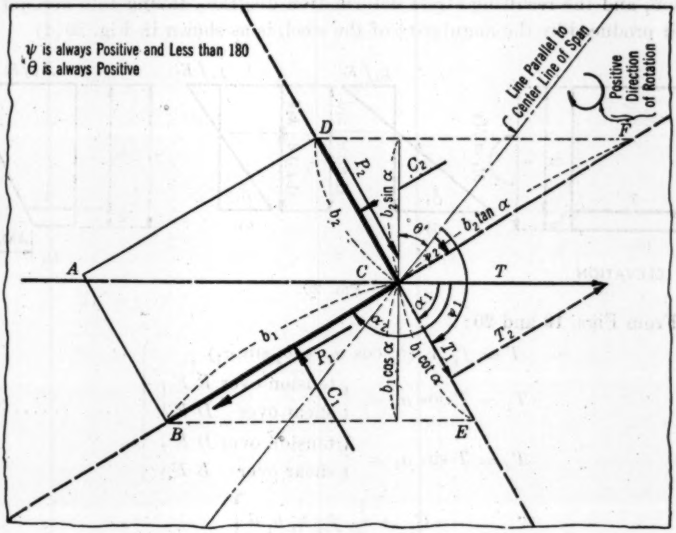

ψ is always Positive and Less than 180
θ is always Positive
Line Parallel to Center Line of Span
Positive Direction of Rotation
D
F
P₂
C₂
b₂ sin α
b₂
θ'
b₂ tan α
ψ₂
A
C
T
α₁
ψ₁
α₂
b₁
P₁
b₁ cos α
b₁ cot α
T₁
T₂
C₁
B
E

FIG. 19.—DIAGRAM SHOWING DESIGN PLANES.

Case I.—Simple Bending—Steel in One Direction and in One Face Only

In Fig. 19, *A B C D* is an element of concrete of small dimensions in which a strand of steel is embedded. The planes, *B C F* and *D C E*, are any two planes at right angles and will be referred to as "Design Planes". The subscript numeral, 1, is attached to all symbols on the plane, *B C F*, and the subscript numeral, 2, to all symbols on the plane, *D C E*.

From Fig. 17, since Angle $B\,A\,M$ is small and $A\,B = A\,M$, Angle $B\,M\,A$ may be considered a right angle. Let δ_1 be the unit elongation in the direction, $A\,F$; δ_2, the unit elongation in the direction, $B\,F$; and, f_d, the stress in the steel due to the direct stresses in the concrete. Then,

$$\delta_1 = \frac{B\,H}{B\,D} = \frac{B\,B'\cos\beta}{A\,B\cos\alpha} = \frac{B'\,M\sec(\alpha-\beta)\cos\beta}{A\,B\cos A}$$

or,

$$\delta_1 = \frac{f_d}{E_s}\,\frac{\cos\beta}{\cos\alpha\cos(\alpha-\beta)}\ldots\ldots\ldots\ldots(54)$$

Also,

$$\delta_2 = \frac{B\,G}{B\,F} = \frac{B\,B'\sin\beta}{A\,B\sin\alpha} = \frac{B'\,M\sin\beta}{A\,B\sin\alpha\cos(\alpha-\beta)}$$

or,

$$\delta_2 = \frac{f_d}{E_s}\,\frac{\sin\beta}{\sin\alpha\cos(\alpha-\beta)}\ldots\ldots\ldots\ldots(55)$$

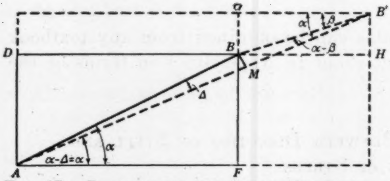


FIG. 17.—DEFORMATION IN STEEL PRODUCED BY DIRECT DEFORMATION.

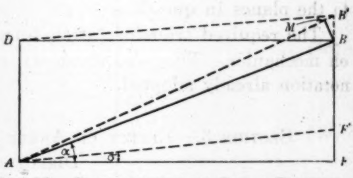


FIG. 18.—DEFORMATION IN STEEL PRODUCED BY SHEAR DEFORMATION IN CONCRETE.

Dividing Equation (54) by Equation (55), $\frac{\delta_1}{\delta_2} = \frac{\tan\alpha}{\tan\beta}$, or,

$$\tan\beta = \frac{\delta_2}{\delta_1}\tan\alpha\ldots\ldots\ldots\ldots(56)$$

From Equation (54)

$$\delta_1 = \frac{f_d}{E_s\cos\alpha(\cos\alpha + \sin\alpha\tan\beta)}\ldots\ldots\ldots\ldots(57)$$

Substituting Equation (56) in Equation (57), and transforming:

$$\frac{f_d}{E_s} = \delta_1\cos^2\alpha + \delta_2\sin^2\alpha\ldots\ldots\ldots\ldots(58)$$

(*B*) Relation of Stresses in Steel to Shear Deformation

From Fig. 18, neglecting the squares of small quantities: $(A\,M + B'\,M)^2 = (A\,F)^2 + (B\,F + B\,B')^2$, or $F\,F' = \frac{B'\,M \times A\,M}{B\,F}$. Writing for $F\,F'$ its value, $A\,F \times \sigma$, and assuming that f_v equals the stress in the steel due to the shear in the concrete; then, $\sigma = \frac{B'\,M \times A\,M}{A\,F \times B\,F} = \frac{B'\,M}{A\,B\cos\alpha} \times \frac{1}{\sin\alpha}$. Therefore,

$$\frac{f_v}{E_s} = \sigma\sin\alpha\cos\alpha\ldots\ldots\ldots\ldots(59)$$

Combining Equations (58) and (59) to get the total stress in the steel,

$$\frac{f_s}{E_s} = \delta_1 \cos^2 \alpha + \delta_2 \sin^2 \alpha + \sigma \sin \alpha \cos \alpha \ldots\ldots\ldots\ldots (60)$$

Equation (60) gives the relation between the stress in the steel, the components of the elongation of the steel in the direction of the direct stresses on the faces of the concrete element, and the shearing deformation of that element.

In the usual simple case where the steel is parallel to the direction of the applied stress, the deformations in the steel and in the concrete are proportional to their distances from the neutral axis. In the present case, the components of the deformations in the steel and in the concrete in the same direction are proportional to their distances from their respective neutral axes.

SECTION 4.—DERIVATION OF DESIGN FORMULAS

The design formulas can be divided into twelve cases, each one of which has three sub-divisions. Each of these sets of formulas can be tabulated. The method of deriving the formulas is explained in detail for Case I in the discussion which follows.

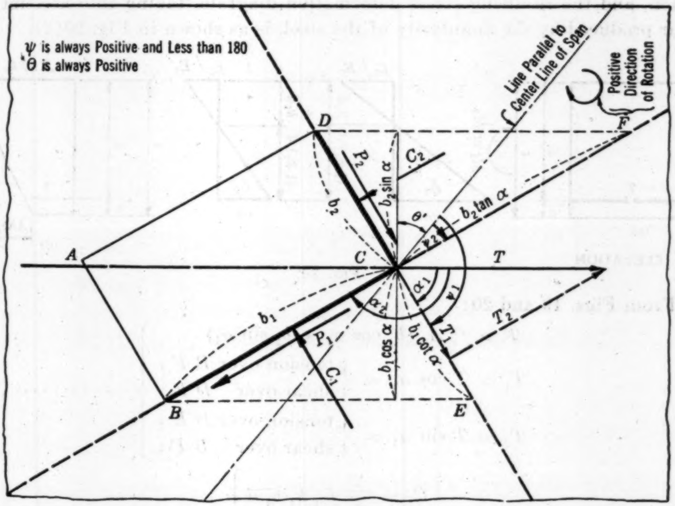

ψ is always Positive and Less than 180
θ is always Positive
Line Parallel to Center Line of Span
Positive Direction of Rotation
D
F
P_2
C_2
$b_2 \sin \alpha$
b_2
θ'
$b_2 \tan \alpha$
ψ_2
A
C
T
α_1
ψ_1
α_2
b_1
P_1
T_1
T_2
$b_1 \cos \alpha$
$b_1 \cot \alpha$
C_1
B
E

FIG. 19.—DIAGRAM SHOWING DESIGN PLANES.

Case I.—Simple Bending—Steel in One Direction and in One Face Only

In Fig. 19, *A B C D* is an element of concrete of small dimensions in which a strand of steel is embedded. The planes, *B C F* and *D C E,* are any two planes at right angles and will be referred to as "Design Planes". The subscript numeral, 1, is attached to all symbols on the plane, *B C F,* and the subscript numeral, 2, to all symbols on the plane, *D C E.*

The values of α for various conditions are:

Longitudinal steel, parallel to fascia	$(\psi + \theta') - 90°$
Longitudinal steel, perpendicular to abutment	$\psi - 90°$
Cross-steel, perpendicular to fascia	$\psi + \theta'$
Cross-steel, parallel to abutments	ψ

In the following derivations, Poisson's ratio is neglected because the value of this modulus is about the same for both concrete and steel. Poisson's ratio affects only the relative deformations and, since the value is about the same for both materials, there is no differential action on this account, and, therefore, no change in the relative stresses is produced. The stress deformation diagrams for each design plane are drawn as if there were no stress on the plane at right angles to it. The deformations shown are, therefore, only those produced by the stresses acting on the plane in question.

(A).—Tension Over Part of Section on Both Design Planes and on the Same Side of the Section.—For this case, assuming the usual straight-line distribution of flexural stress, the stress deformation diagrams on Planes $B\ C\ F$ and $D\ C\ E$, Fig. 19, are shown in Fig. 20(b) and Fig. 20(c). At the middle of the cross-section, the torsion moment stresses also have a straight-line distribution, and the resulting stress deformation diagram, taking into account the shear produced by the angularity of the steel, is as shown in Fig. 20(d).

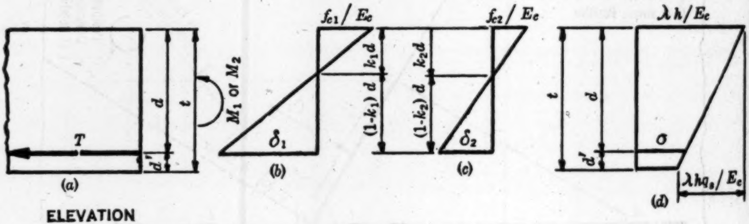


FIG. 20.

From Figs. 19 and 20:

$$\left.\begin{aligned} T &= f_s\, A_s\, (b_1 \cos \alpha_1 + b_2 \sin \alpha_1) \\ T_1 &= T \cos \alpha_1 = \begin{Bmatrix} \text{tension over } B\ F \\ \text{shear over } D\ E \end{Bmatrix} \\ T_2 &= T \sin \alpha_1 = \begin{Bmatrix} \text{tension over } D\ E \\ \text{shear over } B\ F \end{Bmatrix} \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots (61a)$$

$$\left.\begin{aligned} C_1 &= \frac{1}{2}\, f_{c1}\, k_1\, b_1\, d \\ C_2 &= \frac{1}{2}\, f_{c2}\, k_2\, b_2\, d \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots\ldots\ldots (61b)$$

$$\left.\begin{aligned} P_1 &= \frac{1}{2}\, h_1\, (2 - q)\, b_1\, t \\ P_2 &= \frac{1}{2}\, h_2\, (2 - q)\, b_2\, t \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots\ldots\ldots (61c)$$

On Plane $B\,C$, since the summation of all forces parallel and perpendicular to the plane must be zero for equilibrium:

$$\left.\begin{aligned}\frac{C_1}{b_1} &= \frac{T_1}{b_1 + b_2 \tan \alpha_1}\\ \frac{P_1}{b_1} &= \frac{T_2}{b_1 + b_2 \tan \alpha_1}\end{aligned}\right\}\dots\dots\dots\dots\dots\dots\dots\dots(61d)$$

Similarly, on Plane $C\,D$:

$$\left.\begin{aligned}\frac{C_2}{b_2} &= \frac{T_2}{b_1 \cot \alpha_1 + b_2}\\ \frac{P_2}{b_2} &= \frac{T_2}{b_1 \cot \alpha_1 + b_2}\end{aligned}\right\}\dots\dots\dots\dots\dots\dots\dots\dots(61e)$$

Substituting the values from Equations (61*a*), (61*b*), and (61*c*) in Equations (61*d*) and (61*e*), and writing $p\,d$ for A_s:

$$\left.\begin{aligned}f_s\,p \cos^2 \alpha_1 &= \frac{1}{2}\,f_{c1}\,k_1\\ f_s\,p \sin^2 \alpha_1 &= \frac{1}{2}\,f_{c2}\,k_2 = f_s\,p \cos^2 \alpha_2\\ f_s\,p \sin \alpha_1 \cos \alpha_1 &= \frac{1}{2}\,h_1\,(2 - q_s)\,\frac{t}{d} = \frac{1}{2}\,h_2\,(2 - q_s)\,\frac{t}{d}\\ f_s\,p \sin \alpha_2 \cos \alpha_2 &= \frac{1}{2}\,h_1\,(2 - q_s)\,\frac{t}{d} = \frac{1}{2}\,h_2\,(2 - q_s)\,\frac{t}{d}\end{aligned}\right\}\dots(61f)$$

$$\left.\begin{aligned}M_1 &= \frac{1}{2}\,f_{c1}\,k_1\,j_1\,d^2\\ M_2 &= \frac{1}{2}\,f_{c2}\,k_2\,j_2\,d^2\\ M_s &= \frac{1}{2}\,h_1\,(2 - q_s)\,j_s\,t^2 = \frac{1}{2}\,h_2\,(2 - q_s)\,j_s\,t^2\end{aligned}\right\}\dots\dots(61g)$$

$$\left.\begin{aligned}M_1 &= f_s\,p\,j_1\,d^2 \cos^2 \alpha_1\\ M_2 &= f_s\,p\,j_2\,d^2 \sin^2 \alpha_1 = f_s\,p\,j_2\,d^2 \cos^2 \alpha_2\\ M_s &= f_s\,p\,j_s\,dt \sin \alpha_1 \cos \alpha_1 = f_s\,p\,j_s\,d\,t \sin \alpha_2 \cos \alpha_2\end{aligned}\right\}\dots\dots(61h)$$

$$\left.\begin{aligned}j_1 &= 1 - \frac{k_1}{3}\\ j_2 &= 1 - \frac{k_2}{3}\\ j_s &= \frac{d}{t} - \frac{3 - 2\,q_s}{6 - 3\,q_s}\end{aligned}\right\}\dots\dots\dots\dots\dots\dots\dots\dots(61i)$$

From Fig. 20:

$$\left.\begin{aligned}\delta_1 &= \frac{f_{c1}}{E_c}\left(\frac{1 - k_1}{k_1}\right)\\ \delta_2 &= \frac{f_{c2}}{E_c}\left(\frac{1 - k_2}{k_2}\right)\\ \sigma &= -\frac{\lambda\,h_1}{E_c}\left(1 - q\,\frac{d}{t}\right)\end{aligned}\right\}\dots\dots\dots\dots\dots\dots(61j)$$

Substituting the values from Equations (61*j*) in Equation (60):

$$\left.\begin{aligned}\frac{f_s}{n} &= f_{c1}\left(\frac{1-k_1}{k_1}\right)\cos^2\alpha_1 + f_{c2}\left(\frac{1-k_2}{k_2}\right)\sin^2\alpha_1 - \lambda\, h_1\left(1-q_s\frac{d}{t}\right)\sin\alpha_1\cos\alpha_1\\ \frac{f_s}{n} &= f_{c1}\left(\frac{1-k_1}{k_1}\right)\sin^2\alpha_2 + f_{c2}\left(\frac{1-k_2}{k_2}\right)\cos^2\alpha_2 - \lambda\, h_2\left(1-q_s\frac{d}{t}\right)\sin\alpha_2\cos\alpha_2\end{aligned}\right\}\text{(61}k\text{)}$$

Eliminating f_s, f_{c1}, and f_{c2} between Equations (61*f*) and Equations (61*k*):

$$\left.\begin{aligned}\frac{1}{2\,p\,n} &= \left(\frac{1-k_1}{k_1^{\,2}}\right)\cos^4\alpha_1 + \left(\frac{1-k_2}{k_2^{\,2}}\right)\sin^4\alpha_1\\ &\quad - \lambda\left(\frac{d}{t}\right)\left(\frac{1-q_s\dfrac{d}{t}}{2-q_s}\right)\sin^2\alpha_1\cos^2\alpha_1\\ \frac{1}{2\,p\,n} &= \left(\frac{1-k_1}{k_1^{\,2}}\right)\cos^4\alpha_1 + \left(\frac{1-k_2}{k_2^{\,2}}\right)\cos^4\alpha_2\\ &\quad - \lambda\left(\frac{d}{t}\right)\left(\frac{1-q_s\dfrac{d}{t}}{2-q_s}\right)\sin^2\alpha_2\cos^2\alpha_2\end{aligned}\right\}\text{.........(61}l\text{)}$$

Also:

$$\begin{aligned}\frac{\cos^2\alpha_1\, f_s}{n\, f_{c1}\, k_1} &= \left(\frac{1-k_1}{k_1^{\,2}}\right)\cos^4\alpha_1 + \left(\frac{1-k_2}{k_2^{\,2}}\right)\sin^4\alpha_1\\ &\quad - \lambda\left(\frac{d}{t}\right)\left(\frac{1-q_s\dfrac{d}{t}}{2-q_s}\right)\sin^2\alpha_1\cos^2\alpha_1 \text{.............(61}m\text{)}\end{aligned}$$

$$\left.\begin{aligned}j_2 &= \left(\frac{M_2}{M_1}\right)\left(\frac{j_1}{\tan^2\alpha}\right)\\ j_s &= \left(\frac{M_s}{M_1}\right)\left(\frac{d}{t}\right)\left(\frac{j_1}{\tan\alpha}\right)\end{aligned}\right\}\text{................(61}n\text{)}$$

$$\left.\begin{aligned}k_2 &= 3\,(1-j_2)\\ q_s &= \frac{3+6\,j_s-6\,\dfrac{d}{t}}{2+3\,j_s-3\,\dfrac{d}{t}}\end{aligned}\right\}\text{................(61}p\text{)}$$

If the design planes are principal planes:

$$\left.\begin{aligned}M_s &= 0\\ j_s &= 0\\ q_s &= \frac{6\,\dfrac{d}{t}-3}{3\,\dfrac{d}{t}-2}\end{aligned}\right\}\text{..........................(61}q\text{)}$$

(B).—Tension Over Part of Section on One Design Plane, Compression Over Entire Section on the Other Design Plane.—In Fig. 21(*a*) is shown a section of a reinforced concrete beam on which a moment, M_2, is acting. The usual stress-strain diagram for this case is shown in Fig. 21(*b*). The stress in

the concrete acting with the stress in the steel produces a couple which balances the applied couple, M_2; but if there should be an initial tension in the steel, this stress (which is eccentric to the section), produces a moment which partly counteracts the applied couple, and at the same time produces a direct compressive stress on the section. Under certain conditions it may produce compression over the entire section as shown in Fig. 21(*c*). If the moment of the initial stress about the center of the section should happen to equal the applied moment, then the compression is uniformly distributed over the section, as shown in Fig. 21(*d*). If the initial stress moment is larger than the applied couple, the stress distribution may be as shown in Fig. 21(*e*).

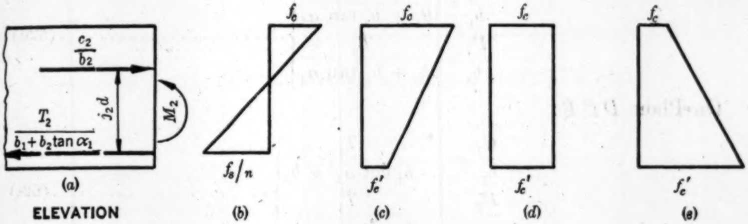

$\frac{c_2}{b_2}$
$j_2 d$
M_2
$\frac{T_2}{b_1+b_2 \tan \alpha_1}$
(a)
ELEVATION
f_c
f_s/n
(b)
f_c
f_c'
(c)
f_c
f_c'
(d)
f_c
f_c'
(e)

FIG. 21.

Since (as shown in Case I(*A*)), the stress in the steel is produced by moments on two planes, it may so happen that the moment of steel stress component on a plane may be greater than the applied couple and cause a kind of reversal of stress. The result will be similar to the case of initial stress just discussed. Case I(*A*) will fail when either value of k is greater than unity. For this contingency, Case I(*B*), or one of the following cases, must be used.

Referring to Fig. 19, the state of stress is assumed to be such that the stress-strain distribution on Plane $B\,C$ is as shown in Fig. 22(*b*), and that on Plane $C\,D$, as shown in Fig. 22(*d*).

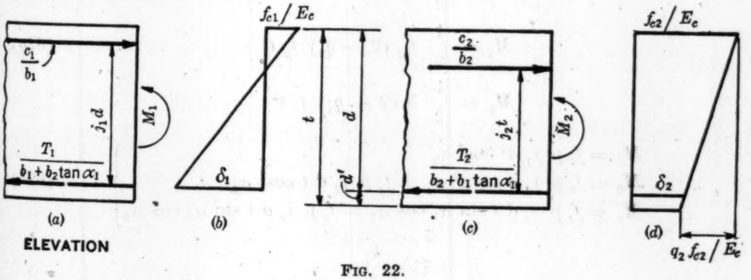

$\frac{c_1}{b_1}$
$j_1 d$
M_1
$\frac{T_1}{b_1+b_2 \tan \alpha_1}$
(a)
ELEVATION
f_{c1}/E_c
δ_1
(b)
t
d
d'
$\frac{c_2}{b_2}$
$j_2 t$
M_2
$\frac{T_2}{b_2+b_1 \tan \alpha_1}$
(c)
f_{c2}/E_c
δ_2
(d)
$q_2 f_{c2}/E_c$

FIG. 22.

From Fig. 19 and Fig. 22:

$$\left.\begin{aligned} T &= f_s\, A_s\, (b_1 \cos \alpha_1 + b_2 \sin \alpha_1) \\ T_1 &= T \cos \alpha_1 \\ T_2 &= T \sin \alpha_1 \end{aligned}\right\} \ldots\ldots\ldots\ldots\ldots (62a)$$

$$\left.\begin{aligned} C_1 &= \frac{1}{2} f_{c1}\, k_1\, b_1\, d \\ C_2 &= \frac{1}{2} f_{c2}\, (2 - q_2)\, b_2\, t \end{aligned}\right\} \cdots\cdots\cdots\cdots (62b)$$

$$\left.\begin{aligned} P_1 &= \frac{1}{2} h_1\, (2 - q_s)\, b_1\, t \\ P_2 &= \frac{1}{2} h_2\, (2 - q_s)\, b_2\, t \end{aligned}\right\} \cdots\cdots\cdots\cdots (62c)$$

On Plane BCF:

$$\left.\begin{aligned} \frac{C_1}{b_1} &= \frac{T_1}{b_1 + b_2 \tan \alpha_1} \\ \frac{P_1}{b_1} &= \frac{T_2}{b_1 + b_2 \tan \alpha_1} \end{aligned}\right\} \cdots\cdots\cdots\cdots (62d)$$

On Plane DCE:

$$\left.\begin{aligned} \frac{C_2}{b_2} &= \frac{T_2}{b_1 \cot \alpha_1 + b_2} \\ \frac{P_2}{b_2} &= -\frac{T_1}{b_1 \cot \alpha_1 + b_2} \end{aligned}\right\} \cdots\cdots\cdots\cdots (62e)$$

$$\left.\begin{aligned} f_s\, p \cos^2 \alpha_1 &= \frac{1}{2} f_{c1}\, k_1 \\ f_s\, p \sin^2 \alpha_1 &= \frac{1}{2} f_{c2}\, (2 - q_2)\, \frac{t}{d} = f_s\, p \cos^2 \alpha_2 \\ f_s\, p \sin \alpha_1 \cos \alpha_1 &= \frac{1}{2} h_1\, (2 - q_s)\, \frac{t}{d} = \frac{1}{2} h_2\, (2 - q_s)\, \frac{t}{d} \\ f_s\, p \sin \alpha_2 \cos \alpha_2 &= \frac{1}{2} h_1\, (2 - q_s)\, \frac{t}{d} = \frac{1}{2} h_2\, (2 - q_s)\, \frac{t}{d} \end{aligned}\right\} \cdots (62f)$$

$$\left.\begin{aligned} M_1 &= \frac{1}{2} f_{c1}\, k_1\, j_1\, d^2 \\ M_2 &= \frac{1}{2} f_{c2}\, (2 - q_2)\, j_2\, t^2 \\ M_s &= \frac{1}{2} h\, (2 - q_s)\, j_s\, t^2 \end{aligned}\right\} \cdots\cdots\cdots\cdots (62g)$$

$$\left.\begin{aligned} M_1 &= f_s\, p\, j_1\, d^2 \cos^2 \alpha_1 \\ M_2 &= f_s\, p\, j_2\, d\, t \sin^2 \alpha_1 = f_s\, p\, j_2\, d\, t \cos^2 \alpha_2 \\ M_s &= f_s\, p\, j_s\, d\, t \sin \alpha_1 \cos \alpha_1 = f_s\, p\, j_s\, d\, t \sin \alpha_2 \cos \alpha_2 \end{aligned}\right\} \cdots\cdots (62h)$$

$$\left.\begin{aligned} j_1 &= 1 - \frac{k_1}{3} \\ j_2 &= \frac{d}{t} - \frac{3 - 2\, q_2}{6 - 3\, q_2} \\ j_s &= \frac{d}{t} - \frac{3 - 2\, q_s}{6 - 3\, q_s} \end{aligned}\right\} \cdots\cdots\cdots\cdots (62i)$$

From Figs. 20 and 22:

$$\left.\begin{aligned}\delta_1 &= \frac{f_{c1}}{E_c}\left(\frac{1-k_1}{k_1}\right)\\ \delta_2 &= -\frac{f_{c2}}{E_c}\left(1-q_2\frac{d}{t}\right)\\ \sigma &= -\frac{\lambda\, h}{E_c}\left(1-q_s\frac{d}{t}\right)\end{aligned}\right\}\text{.....................}(62j)$$

Substituting Equations (62*j*) in Equation (60):

$$\left.\begin{aligned}\frac{f_s}{n} &= f_{c1}\left(\frac{1-k_1}{k_1}\right)\cos^2\alpha_1 - f_{c2}\left(1-q_2\frac{d}{t}\right)\sin^2\alpha_1 - \lambda\, h_1\left(1-q_s\frac{d}{t}\right)\sin\alpha_1\cos\alpha_1\\ \frac{f_s}{n} &= f_{c1}\left(\frac{1-k_1}{k_1}\right)\sin^2\alpha_2 - f_{c2}\left(1-q_2\frac{d}{t}\right)\cos^2\alpha_2 - \lambda\, h_2\left(1-q_s\frac{d}{t}\right)\sin\alpha_2\cos\alpha_2\end{aligned}\right\}(62k)$$

Eliminating f_s, f_{c1}, and f_{c2} between Equations (62*f*) and (62*k*):

$$\left.\begin{aligned}\frac{1}{2\,p\,n} &= \left(\frac{1-k_1}{{k_1}^2}\right)\cos^4\alpha_1 - \left(\frac{d}{t}\right)\left(\frac{1-q_2\frac{d}{t}}{2-q_2}\right)\sin^4\alpha_1\\ &\qquad -\lambda\frac{d}{t}\left(\frac{1-q_2\frac{d}{t}}{2-q_s}\right)\sin^2\alpha_1\cos^2\alpha_1\\ \frac{1}{2\,p\,n} &= \left(\frac{1-k_1}{{k_1}^2}\right)\sin^4\alpha_2 - \left(\frac{d}{t}\right)\left(\frac{1-q_2\frac{d}{t}}{2-q_2}\right)\cos^4\alpha_2\\ &\qquad -\lambda\left(\frac{d}{t}\right)\left(\frac{1-q_s\frac{d}{t}}{2-q_s}\right)\sin^2\alpha_2\cos^2\alpha_2\end{aligned}\right\}\text{......}(62l)$$

Also:

$$\begin{aligned}\frac{\cos^2\alpha_1\, f_s}{n\, f_{c1}\, k_1} &= \left(\frac{1-k_1}{{k_1}^2}\right)\cos^4\alpha_1 - \frac{d}{t}\left(\frac{1-q_2\frac{d}{t}}{2-q_2}\right)\sin^4\alpha_1\\ &\qquad -\lambda\left(\frac{d}{t}\right)\left(\frac{1-q_s\frac{d}{t}}{2-q_s}\right)\sin^2\alpha_1\cos^2\alpha_1\text{...............}(62m)\end{aligned}$$

$$\left.\begin{aligned}j_2 &= \left(\frac{d}{t}\right)\left(\frac{M_2}{M_1}\right)\left(\frac{j_1}{\tan^2\alpha}\right)\\ j_s &= \left(\frac{d}{t}\right)\frac{M_s}{M_1}\frac{j_1}{\tan\alpha_1}\end{aligned}\right\}\text{.....................}(62n)$$

$$\left.\begin{aligned}q_2 &= \frac{3+6\,j_2-6\frac{d}{t}}{2+3\,j_2-3\frac{d}{t}}\\ q_s &= \frac{3+6\,j_s-6\frac{d}{t}}{2+3\,j_s-3\frac{d}{t}}\end{aligned}\right\}\text{.....................}(62p)$$

If the design planes are principal planes:

$$\left.\begin{aligned} M_s &= 0 \\ j_s &= 0 \\ q_s &= \frac{6\,\dfrac{d}{t} - 3}{3\,\dfrac{d}{t} - 2} \end{aligned}\right\} \cdots\cdots\cdots\cdots (62q)$$

(C).—Tension Over Part of Section on Both Design Planes But on Opposite Sides of Section.—This case may arise in two ways: (1) recalling the discussion of Case I(*B*), if the initial tension moment is greater than the applied moment and produces a reversal of stress such that the resultant falls outside the middle third of the section; and (2) if the moments on both design planes are of opposite signs. If tension is allowed in the concrete, Case I (*B*) governs, q_2 will be greater than unity, and f_{c2} will be negative. If no tension is allowed in the concrete, or if it exceeds the allowable value, then Case I (*C*) must be used.

In Case I (*C*), the resultant of the compressive stresses falls outside the middle third and nominal steel should be used in the tension area. If f_{c2} is too high, Case II (*A*), or one of the following cases, will apply.

Fig. 23 shows the stress deformation diagram for the design plane on which there is tension over the extrados part of the section. The other diagrams are the same as in Figs. 21 and 22 and are not shown.

It will be noticed that Equations (62) are derived in the same way as Equations (61) and that the only differences between the corresponding equations are in the values of *k*, *q*, *j*, and in the expressions involving these symbols. The same thing is true for Equations (63) and the detailed derivation is not given:

$$\left.\begin{aligned} f_s\,p\cos^2\alpha_1 &= \frac{1}{2}\,f_{c1}\,k_1 \\ f_s\,p\sin^2\alpha_1 &= \frac{1}{2}\,f_{c2}\,k_2 = f_s\,p\cos^2\alpha_2 \\ f_s\,p\sin\alpha_1\cos\alpha_1 &= \frac{1}{2}\,h_1\,(2-q_s)\,\frac{t}{d} = \frac{1}{2}\,h_2\,(2-q_s)\,\frac{t}{d} \\ f_s\,p\sin\alpha_2\cos\alpha_2 &= +\frac{1}{2}\,h_1\,(2-q_s)\,\frac{t}{d} = \frac{1}{2}\,h_2\,(2-q_s)\,\frac{t}{d} \end{aligned}\right\} \cdots (63f)$$

$$\left.\begin{aligned} M_1 &= \frac{1}{2}\,f_{c1}\,k_1\,j_1\,d^2 \\ M_2 &= \frac{1}{2}\,f_{c2}\,k_2\,j_2\,d^2 \\ M_s &= \frac{1}{2}\,h_1\,(2-q_s)\,j_s\,t^2 = \frac{1}{2}\,h_2\,(2-q_s)\,j_s\,t^2 \end{aligned}\right\} \cdots\cdots (63g)$$

$$\left.\begin{aligned} M_1 &= f_s\,p\,j_1\,d^2\cos^2\alpha_1 \\ M_2 &= f_s\,p\,j_2\,d^2\sin^2\alpha_1 = f_s\,p\,j_2\,d^2\cos^2\alpha_2 \\ M_s &= f_s\,p\,j_s\,d\,t\sin\alpha_1\cos\alpha_1 = f_s\,p\,j_s\,d\,t\sin\alpha_2\cos\alpha_2 \end{aligned}\right\} \cdots\cdots (63h)$$

$$\left.\begin{aligned} j_1 &= 1 - \frac{k_1}{3} \\ j_2 &= \frac{k_2}{3} - \frac{d'}{d} \\ j_s &= \frac{d}{t} - \frac{3 - 2\,q_s}{6 - 3\,q_s} \end{aligned}\right\} \cdots\cdots\cdots\cdots\cdots\cdots (63i)$$

From Fig. 20:

$$\left.\begin{aligned} \delta_1 &= \frac{f_{c1}}{E_c}\left(\frac{1 - k_1}{k_1}\right) \\ \delta_2 &= \frac{f_{c2}}{E_c}\left(\frac{k_2 - \dfrac{d'}{d}}{k_2}\right) \\ \sigma &= -\frac{\lambda\, h}{E_c}\left(1 - q_s\,\frac{d}{t}\right) \end{aligned}\right\} \cdots\cdots\cdots\cdots\cdots\cdots (63j)$$

$$\left.\begin{aligned} \frac{f_s}{n} &= f_{c1}\left(\frac{1 - k_1}{k_1}\right)\cos^2\alpha_1 - f_{c2}\left(\frac{k_2 - \dfrac{d'}{d}}{k_2}\right)\sin^2\alpha_1 \\ &\quad - \lambda\, h_1\left(1 - q_s\,\frac{d}{t}\right)\sin\alpha_1\cos\alpha_1 \\ \frac{f_s}{n} &= f_{c1}\left(\frac{1 - k_1}{k_1}\right)\sin^2\alpha_2 - f_{c2}\left(\frac{k_2 - \dfrac{d'}{d}}{k_2}\right)\cos^2\alpha_2 \\ &\quad - \lambda\, h_2\left(1 - q_s\,\frac{d}{t}\right)\sin\alpha_2\cos\alpha_2 \end{aligned}\right\} \cdots\cdots (63k)$$

$$\left.\begin{aligned} \frac{1}{2\,p\,n} &= \left(\frac{1 - k_1}{k_1^{\,2}}\right)\cos^4\alpha_1 - \left(\frac{k_2 - \dfrac{d'}{d}}{k_2^{\,2}}\right)\sin^4\alpha_1 \\ &\quad - \lambda\left(\frac{d}{t}\right)\left(\frac{1 - q\,\dfrac{d}{t}}{2 - q_s}\right)\sin^2\alpha_1\cos^2\alpha_1 \\ \frac{1}{2\,p\,n} &= \left(\frac{1 - k_1}{k_1^{\,2}}\right)\sin^4\alpha_2 - \left(\frac{k_2 - \dfrac{d'}{d}}{k_2^{\,2}}\right)\cos^4\alpha_2 \\ &\quad - \lambda\left(\frac{d}{t}\right)\left(\frac{1 - q_s\,\dfrac{d}{t}}{2 - q_s}\right)\sin^2\alpha_2\cos^2\alpha_2 \end{aligned}\right\} \cdots\cdots (63l)$$

Also,

$$\left.\begin{aligned} \frac{\cos^2\alpha_1\, f_s}{n\, f_{c1}\, k_1} &= \left(\frac{1 - k_1}{k_1^{\,2}}\right)\cos^4\alpha_1 - \left(\frac{k_2 - \dfrac{d'}{d}}{k_2^{\,2}}\right)\sin^4\alpha_1 \\ &\quad - \lambda\left(\frac{d}{t}\right)\left(\frac{1 - q_s\,\dfrac{d}{t}}{2 - q_s}\right)\sin^2\alpha_1\cos^2\alpha_1 \end{aligned}\right\} \cdots\cdots (63m)$$

$$\left.\begin{aligned} j_2 &= \left(\frac{M_2}{M_1}\right) \frac{j_1}{\tan^2 \alpha_1} \\ j_s &= \left(\frac{d}{t}\right) \left(\frac{M_s}{M_1}\right) \frac{j_1}{\tan \alpha_1} \end{aligned}\right\} \quad \cdots\cdots\cdots (63n)$$

$$\left.\begin{aligned} k_2 &= 3 \left(j_2 + \frac{d'}{d}\right) \\ q_s &= \frac{3 + 6\, j_s - 6 \dfrac{d}{t}}{2 + 3\, j_s - 3 \dfrac{d}{t}} \end{aligned}\right\} \quad \cdots\cdots\cdots (63p)$$

If the design planes are principal planes:

$$\left.\begin{aligned} M_s &= 0 \\ j_s &= 0 \\ q_s &= \frac{6 \dfrac{d}{t} - 3}{3 \dfrac{d}{t} - 2} \end{aligned}\right\} \quad \cdots\cdots\cdots (63q)$$

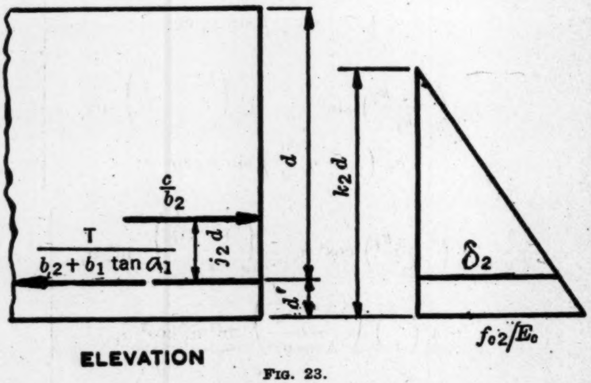


FIG. 23.

SECTION 5.—FINAL DESIGN FORMULAS

Section 4 shows the method of deriving the design formulas. In general, the stress in a strand of steel at an angle to a plane is produced by the forces acting on the plane in question and also by the forces acting on the plane at right angles to it. Furthermore, even the concrete stresses on a plane are partly dependent on the forces acting on the plane at right angles to it, and which are transmitted to it by the steel. Since two planes at right angles to each other must be analyzed together, many combinations of stress conditions on the two design planes are possible. If there are n stress conditions possible on a plane, there must be n^2 possible combinations when two planes are treated together. Hence, there would be a large number of cases to be analyzed if the design formulas were written according to the method used in the Section 4.

It is apparent from a study of Section 4, that the two design planes can be treated independently, except for the equations, (k), (l), and (m). There is, however, a certain symmetry about all the equations which makes it possible to treat each design plane independently and thus greatly reduce the number of cases to be analyzed. To analyze the stresses for any given set of conditions, the proper set of formulas for one design plane is combined with the proper set for the other plane. For brevity these formulas are tabulated and placed in Appendix I under the caption, "Equation (64)," in order to help the designer in referring back to the text when necessary.

EXAMPLE 2.—PROPORTIONING OF SECTIONS

Part I concluded with a numerical example which was carried up to the point at which the maximum moments, thrusts, and shears were obtained. Numerical Example 1 will now be resumed and several sections designed, thus illustrating the application of the design formulas which have been developed in Part II.

Design of Steel at Crown ($x = 0$)

$$\left.\begin{matrix} M_x = 741 \text{ kip-ft.} \\ T_s = 452 \text{ kips} \end{matrix}\right\} \quad \left.\begin{matrix} M_s = 1\,509 \text{ kip-ft.} \\ T_x = 418 \text{ kips} \end{matrix}\right\} \quad \left.\begin{matrix} t = 21 \text{ in.} \\ d = 18.5 \text{ in.} \end{matrix}\right\} \quad \left.\begin{matrix} \varepsilon = 1.192 \\ \varepsilon^2 = 1.420 \end{matrix}\right\}$$

$$\left.\begin{matrix} m = \dfrac{1\,509}{66.9} = 22.5 \text{ kip-ft.} \\ t_n = \dfrac{418}{66.9} = 6.25 \text{ kips} \end{matrix}\right\}$$

Fascia Steel.—

$$\left.\begin{matrix} M = m \sec^2 \theta' \text{ (See Equation (53}d\text{), Appendix I} \\ N = t_n \sec^2 \theta' \text{ (See Equation (53}f\text{), Appendix I} \end{matrix}\right\} \text{ and } \theta' = \theta$$

$$N = 6.25 \times 2.42 = 15.13 \text{ kips}$$

$$e = \frac{22.5}{6.25} \times 12 + 8.0 = 51.1 \text{ in.}$$

$$r = 16.0$$

$$e - r = 35.1 \text{ in.}$$

$$\frac{e}{d} = \frac{51.1}{18.5} = 2.76$$

$$K = \frac{N e}{b d^2} = \frac{15.13 \times 2.76}{12 \times 18.5} = 188$$

K (for tension steel only) $= 138$ (for $f_c = 800, f_s = 18\,000, n = 15$)

$$N' = \left(\frac{188 - 138}{188}\right) \times 15.13 = 4.03 \text{ kips}$$

$$f'_s = \frac{k d - d'}{k d} \times n f_c = \left(\frac{0.400 \times 18.5 - 2.5}{0.400 \times 18.5}\right) \times 12\,000 = 7.95 \text{ kips}$$

$$A'_s = \frac{N' e}{f_s r} = \frac{4.03 \times 51.1}{7.95 \times 16.0} = 1.62 \text{ sq. in.}$$

$$\left.\begin{matrix} A_{s1} = \dfrac{N'(e - r)}{f_s r} = \dfrac{4.03 \times 35.1}{18.0 \times 16.0} = 0.49 \\ A_{s2} = 0.0059 \times 12 \times 18.5 = \quad 1.31^{8} \end{matrix}\right\} 1.80 \text{ sq. in.}$$

---

[8] Hool and Johnson, p. 405.

The fascia section extends for a distance (parallel to the abutments) of about one and one-half times the depth of the section; and where cross-steel is required, it should extend for at least a bond length of the cross-steel.

Center Steel (Parallel and Perpendicular to Spandrel Walls).—

$$\frac{m_v}{b} = \frac{741}{66.9} = 11.1 \text{ kip-ft.}$$

$$\varepsilon' m = 22.5 \times 1.192 = \underline{26.8}$$

$$m_s = \frac{1}{2} \times 37.9 = 19.0 \text{ kip-ft. (See Equation (44) Appendix I)}$$

$$m' = \frac{\varepsilon' m_v}{b} = 1.192 \times 11.1 = 13.2 \text{ kip-ft.}$$

$$\frac{t_v}{b} = \frac{452}{66.9} = 6.75 \text{ kips}$$

$$\left.\begin{array}{r} \varepsilon' t_n = 1.192 \times 6.25 = 7.45 \text{ kips} \\ -\ \overline{0.70} \text{ kips} \\ -\ 0.35 \text{ kips} \\ -\ \overline{1.05} \text{ kips} \\ \varepsilon' t_n = 7.45 \text{ kips} \\ t_s = \overline{6.40} \text{ kips} \\ \text{(See Equation (35), Appendix I)} \end{array}\right\{ \begin{array}{ll} m = 22.5 \text{ kip-ft.} & t_n = 6.25 \text{ kips} \\ m' = 13.2 \text{ kips} & t'_n = 7.63 \text{ kips} \\ m_s = 19.0 & t_s = 6.40 \text{ kips} \end{array}$$

$$t'_n = \varepsilon' t_s = 1.192 \times 6.40 = 7.63 \text{ kips}$$

(*a*) Minimum Cross Steel.—

$$\begin{array}{rlrl} \psi = & 180 - \theta' & & \\ \sin\theta' = & 0.766 & \sin^2\theta' = & 0.587 \\ \cos\theta' = & 0.643 & \cos^2\theta' = & 0.413 \end{array}$$

$$\begin{array}{rrr} m \sin^2\theta' = & 22.5 \times 0.587 = & 13.2 \\ m' \cos^2\theta' = & 13.2 \times 0.413 = & 5.3 \\ -2\, m_s \sin\theta' \cos\theta' = & 37.9 \times 0.766 \times 0.643 = & -18.6 \\ & M = & \overline{0} \end{array}$$

$$\begin{array}{rrr} t_n \sin^2\theta' = & 6.25 \times 0.587 = & 3.7 \\ t'_n \cos^2\theta' = & 7.63 \times 0.413 = & 3.2 \\ -2\, t_s \sin\theta' \cos\theta' = & 2 \times 6.40 \times 0.492 = & -6.4 \\ & N = & \overline{0.5} \text{ kip (negligible)} \end{array}$$

The foregoing computations show that only nominal steel is required.

(*b*) Minimum Longitudinal Steel.—

$$\begin{array}{lrlr} m \cos^2\theta' = 22.5 \times 0.413 = & 9.2 & t_n \cos^2\theta' = 6.25 \times 0.413 = & 2.6 \\ m' \sin^2\theta' = 13.2 \times 0.587 = & 7.8 & t'_n \sin^2\theta' = 7.63 \times 0.587 = & 4.5 \\ 2\, m_s \sin\theta' \cos\theta' & 18.6 & 2\, t_s \sin\theta' \cos\theta' = & 6.4 \\ & \overline{35.6 \text{ kip-ft.}} & & \overline{13.5 \text{ kips}} \end{array}$$

$$e = \frac{35.6}{13.5} \times 12 + 8.0 = 39.7$$

$$\frac{e}{d} = \frac{39.7}{18.5} = 2.14$$

$$K = \frac{13.5 \times 2.14}{12 \times 18.5} = 130$$

$$f_c = 770 \text{ lb.}$$

$$f_s = 18\,000$$

$$p = 0.0050$$

$$A_s = 12 \times 18.5 \times 0.0050 = 1.11 \text{ sq. in.}$$

(*c*) **Principal Planes.**—

$$\tan 2\psi = \frac{2\,m_s}{m' - m} = -\,\frac{37.9}{9.2}$$

$$\tan(2\psi - 90) = \frac{9.2}{37.9} = 0.243$$

$$2\,\psi_1 - 90 = 13^\circ\, 40'\,;\ \psi_1 = 51^\circ\, 50'\,;\ \text{and } \alpha_1 = \psi_1 + \theta' - 90 = 11^\circ\, 50'$$

$$\sin\psi_1 = 0.786 \quad \sin^2\psi_1 = 0.618$$

$$\cos\psi_1 = 0.618 \quad \cos^2\psi_1 = 0.382$$

$$\sin\psi_1\cos\psi_1 = 0.485$$

$$\begin{aligned}
m\sin^2\psi_1 &= 22.5 \times 0.618 = 13.90 \text{ kip-ft.}\\
m'\cos^2\psi_1 &= 13.2 \times 0.382 = 5.05\\
2\,m_s\sin\psi_1\cos\psi_1 &= 37.9 \times 0.485 = \underline{18.40}\\
M_1 &= 37.35 \text{ kip-ft.}
\end{aligned}$$

$$\begin{aligned}
t_n\sin^2\psi_1 &= 6.25 \times 0.618 = 3.86 \text{ kips}\\
t'_n\cos^2\psi_1 &= 7.63 \times 0.382 = 2.91\\
2\,t_s\sin\psi_1\cos\psi_1 &= 12.8 \times 0.485 = \underline{6.21}\\
N_1 &= 12.98 \text{ kips}
\end{aligned}$$

$$\begin{aligned}
m\sin^2\psi_2 &= 22.5 \times 0.382 = 8.60 \text{ kip-ft.}\\
m'\cos^2\psi_2 &= 13.2 \times 0.618 = 8.15\\
2\,m_s\sin\psi_2\cos\psi_2 &= \underline{-\,18.40}\\
M_2 &= -\,1.65 \text{ kip-ft.}
\end{aligned}$$

$$\begin{aligned}
t_n\sin^2\psi_2 &= 6.25 \times 0.382 = 2.39 \text{ kips}\\
t'_n\cos^2\psi_2 &= 7.63 \times 0.618 = 4.71\\
2\,t_s\sin\psi_2\cos\psi_2 &= \underline{-\,6.21}\\
N_2 &= 0.89 \text{ kip}
\end{aligned}$$

The foregoing computations show that M_2 and N_2 are negligible.

Normal Steel Plane 1.

$$e = \frac{37.4}{13.0} \times 12 + 8.0 = 42.2 \text{ in.}$$

$$\frac{e}{d} = \frac{42.2}{18.5} = 2.28$$

$$K = \frac{13.0 \times 2.28}{12 \times 18.5} = 134$$

$$\left.\begin{aligned}
F_{cl} &= 770 \text{ lb.}\\
F_s &= 19\,200 \text{ lb. (trial value)}\\
P &= 0.00493
\end{aligned}\right\}$$

Normal Steel Plane 2.
(None required)

For Plane 1, use Case IV (*A*) (Equations (64)) and for Plane 2, use Case I (*C*):

$$\sin\alpha_1 = 0.205 \quad \sin^2\alpha_1 = 0.042$$
$$\cos\alpha_1 = 0.979 \quad \cos^2\alpha_1 = 0.958$$
$$\sin\alpha_1\cos\alpha_1 = 0.200$$

$$f_s\,p\cos^2\alpha_1 = \frac{1}{2}f_{c1}\,k_1 - \frac{N_1}{b\,d} = F_s\,P = 19\,200 \times 0.00493 = 94.5$$

$$f_s\,p = \frac{94.5}{0.958} = 98.7$$

On Plane 2:

$$M_2 = 0, \therefore j_2 = 0, k_2 = 3\left(j_2 + \frac{d'}{d}\right) = \frac{3 \times 2.50}{18.5} = 0.405$$

$$a_2 = -\left(\frac{k_2 - \frac{d'}{d}}{k_2}\right) = -\frac{2}{3}$$

$$f_s\,p\cos^2\alpha_2 = \frac{1}{2}f_{c2}\,k_2$$

$$f_{c2} = \frac{2 \times 98.7 \times 0.042}{0.405} = 20.4 \text{ lb. per sq. in.}$$

On Plane 1:

$$\frac{d}{t} = \frac{18.5}{21.0} = 0.882$$

$$q_s = \frac{3 - \frac{6\,d}{t}}{2 - \frac{3\,d}{t}} = 3.54$$

$$a_s = -\left(1 - q_s\frac{d}{t}\right) = -(1 - 0.882 \times 3.54) = 2.12$$

$$h_1 = \frac{2\,f_s\,p\,b\,d\sin\alpha_1\cos\alpha_1 + S}{b\,t\,(2 - q_s)}; \; S = 0 \text{ for principal planes}$$

$$h_1 = 2\,\frac{d}{t} \times \frac{f_s\,p\sin\alpha_1\cos\alpha_1}{(2 - q_s)} = \frac{2 \times 0.882 \times 98.7 \times 0.200}{-1.58} = -22 \text{ lb.}$$

$$\frac{f_s}{n} = \Sigma\,f_c\,a\cos^2\alpha + \lambda\,h\,a_s\sin\alpha\cos\alpha$$

$$n\,f_{c1}\left(\frac{1 - k_1}{k_1}\right) = F_s$$

$$f_s = 19\,200 \times 0.958 + 15 \times 20.4\left(-\frac{2}{3}\right)0.042$$
$$+\,37.5 \times (-22.0)\,(+2.12) \times 0.200$$

$$f_s = 18\,400 + (-8) + (-350) = 18\,050 \text{ lb.}$$

$$p = \frac{f_s\,p}{f_s} = \frac{98.7}{18\,000} = 0.00543$$

$$A_s = 0.00543 \times 222 = 1.20 \text{ sq. in.}$$

Maximum Horizontal Shear.—

$$\tan 2\,\psi = \frac{m - m'}{2\,m_s} = \frac{9.2}{37.9} = 0.243$$

$$\psi = 6^\circ\,50' \text{ and } \alpha_1 = (6^\circ\,50') + 50^\circ - 90^\circ = -\,(33^\circ\,10')$$

$$\left.\begin{aligned}\sin\psi_1 &= 0.119\\ \cos\psi_1 &= 0.993\end{aligned}\right\} \qquad \left.\begin{aligned}\sin^2\psi_1 &= 0.014\\ \cos^2\psi_1 &= 0.985\end{aligned}\right\} \qquad \left.\begin{aligned}\sin\alpha_1 &= -\,0.547\\ \cos\alpha_1 &= 0.837\end{aligned}\right\}$$

$$\sin^2\psi_1 - \cos^2\psi_1 = -\,0.971 \qquad \sin\alpha_1\cos\alpha_1 = -\,0.458$$

$$\sin\psi_1\cos\psi_1 = \quad 0.118$$

$$(m' - m)\sin\psi_1\cos\psi_1 = (-9.2)\,(0.118) = -\;\;1.08$$

$$m_s\,(\sin^2\psi_1 - \cos^2\psi_1) = 19.0 \times (-\,0.971) = \underline{-\,18.48}$$

$$M_s = -\,19.56 \text{ kip-ft.}$$

$$(t'_n - t_n)\sin\psi_1\cos\psi_1 = (-\,1.38)\,(0.118) = -\;\;0.16$$

$$t_s\,(\sin^2\psi_1 - \cos^2\psi_1) = 6.40 \times (-\,0.971) = \underline{-\;\;6.22}$$

$$S = -\;\;6.38 \text{ kips}$$

$$f_s\,p\sin\alpha_1\cos\alpha_1 = \frac{1}{2}\,h\,(2 - q_s)\,\frac{t}{d} - \frac{S}{b\,d}$$

$$f_s\,p\sin\alpha_1\cos\alpha_1 = 98.7 \times (-\,0.458) = -\,45.2$$

$$\frac{S}{b\,d} = \frac{-\,6380}{12 \times 18.5} = \frac{-\,29.7}{-\,74.9}$$

$$\frac{1}{2}\,h\,(2 - q_s) = \frac{d}{t} \times (-\,74.9) = -\,66.0$$

$$e_s = \frac{-\,19.56}{-\,6.38} \times 12 + 8.0 = 44.9 \text{ in.}$$

$$K_s = \frac{S\,e_s}{b\,t^2} = \frac{(-6.38) \times 44.9}{12 \times 21 \times 21} = -\,54.2$$

$$\left.\begin{aligned}\frac{K_s}{j_s} = \frac{1}{2}\,h\,(2 - q_s) &= -\,\frac{54.2}{j_s}\\ \frac{1}{2}\,h\,(2 - q_s) &= -\,66.0\end{aligned}\right\}\quad \begin{aligned}\frac{54.2}{j_s} &= 66.0\\ j_s &= 0.820\end{aligned}$$

$$q_s = \frac{3 + 6\,j_s - 6\,\dfrac{d}{t}}{2 + 3\,j_s - 3\,\dfrac{d}{t}} = \frac{3 + 6\,(0.820 - 0.882)}{2 + 3\,(0.820 - 0.882)} = 1.438$$

$$h = \frac{2 \times (-\,66.0)}{2 - q_s} = \frac{-132.0}{0.562} = -\,235 \text{ lb. ((O.K.), 240 lb. allowed)}$$

$$h' = (1 - q_s)\,h = (1 - 1.438)\,(-\,235) = 103 \text{ lb. (O.K.)}$$

Maximum Radial Shear.—

$$V = \frac{t_u}{b} - \frac{8}{5}\,\frac{m_v - \varepsilon'\,m_s}{b\,t}$$

$$V = \frac{38}{66.9} - \frac{8}{5}\,\frac{(-\,9.2)}{21 \times 66.9} = 0.58 \text{ kip.}$$

$$V_{\text{max.}} = 0.580 \times 2.42 = 1.4 \text{ kips}\,;\; v = \frac{1400}{0.8 \times 12 \times 18.5} = 7.9 \text{ lb. (O.K.)}$$

Design of Steel at the Section, $x = 20.6$

$$t = 54 \text{ in.}$$

$$\varepsilon' = \varepsilon = 1.192 \qquad \varepsilon'^2 = 1.420$$

$$\left.\begin{aligned} m_z &= -5\,930 \text{ kip-ft.} \\ t_v &= 803 \text{ kips} \end{aligned}\right\} \quad \left.\begin{aligned} m_v &= -6\,740 \text{ kip-ft.} \\ t_z &= 667 \text{ kips} \end{aligned}\right\}$$

Center Stresses.—

$$\begin{aligned} m &= -\ 88.6 \text{ kip-ft.} \\ m' &= -120.0 \quad \text{``} \\ m_s &= -103.3 \quad \text{``} \end{aligned} \qquad \left.\begin{aligned} t_n &= 12.0 \text{ kips} \\ t_n &= 6.56 \quad \text{``} \\ t_s &= 5.5 \quad \text{``} \end{aligned}\right\}$$

Fascia Steel.—

$$M = -88.6 \sec^2 \theta'$$

$$N = 12.0 \sec^2 \theta'$$

$$e' = \frac{-88.6}{12.0} \times 12 - 24.5 = -113.1 \text{ in.}$$

$$\frac{e'}{d} = -\frac{113.1}{51.5} = -2.20$$

$$K = \frac{12.0 \times (-2.20) \times 2.42}{12 \times 51.5} = -103.5$$

$$f_c = 650$$

$$f'_s = 18\,000$$

$$p' = 0.0039$$

$$A'_s = 0.0039 \times 51.5 \times 12 = 2.41 \text{ sq. in.}$$

Center Steel.—

(*a*) Minimum Cross-Steel.—

$$\psi = 180 - \theta'$$

$$\begin{array}{rlr} m \sin^2 \theta' = & -\ 88.6 \times 0.587 = & -\ 52.0 \\ m' \cos^2 \theta' = & -120.0 \times 0.413 = & -\ 49.5 \\ -2\, m_s \sin \theta' \cos \theta' = & 206.6 \times 0.492 = & 101.5 \\ \hline & & 0 \end{array}$$

$$\begin{array}{rlr} t_n \cos^2 \theta' = & 12.0 \times 0.587 = & 7.05 \\ t'_n \sin^2 \theta' = & 6.56 \times 0.413 = & 2.71 \\ -2\, t_s \sin \theta' \cos \theta' = & 11.0 \times 0.492 = & -\ 5.41 \\ \hline & N = & 4.35 \text{ kips} \end{array}$$

The foregoing computations show that no steel is required.

(*b*) Minimum Longitudinal Steel.—

$$\begin{array}{rlr} m \cos^2 \theta' = & -\ 88.6 \times 0.413 = & -\ 36.6 \\ m' \sin^2 \theta' = & -120.0 \times 0.587 = & -\ 70.4 \\ 2\, m_s \sin \theta' \cos \theta' = & & -101.5 \\ \hline & M = & -208.5 \text{ kip-ft.} \end{array}$$

$$\begin{array}{rlr} t_n \cos^2 \theta' = & 12.0 \times 0.413 = & 4.96 \\ t'_n \sin^2 \theta' = & 6.56 \times 0.587 = & 3.84 \\ 2\, t_s \sin \theta' \cos \theta' = & & 5.41 \\ \hline & N = & 14.21 \text{ kips} \end{array}$$

$$f_c = 600$$
$$f'_s = 18\,000$$
$$p' = 0.43\%$$
$$A'_s = 2.65 \text{ sq. in.}$$

(*c*) **Principal Planes (Case V).—**

$$\tan 2\psi = \frac{2\,m_s}{m' - m} = \frac{-\,206.6}{-\,31.4} = 6.57$$

$$\psi_1 = 40^\circ\ 40'$$

$$M_1 = -\,208.9 \qquad N_1 = 14.30$$
$$M_2 = 0 \qquad N_2 = \ \ 4.24 \text{ (negligible)}$$

$$\alpha_1 = \psi_1 + \theta - 90 = 0^\circ\ 40' \text{ (which is negligible)}$$

The design is for normal steel which gives the same condition as for minimum longitudinal steel.

Maximum Horizontal Shear.—

$$\tan 2\psi = \frac{m - m'}{2\,m_s} = 0.152$$

$$\psi_1 = 4^\circ\ 20' \qquad \alpha_1 = (4^\circ\ 20') + 50^\circ - 90^\circ = -\,(35^\circ\ 40')$$

$$\left.\begin{array}{r} M_s = 103.3 \text{ kip-ft.} \\ S = -\,5.5 \text{ kips} \end{array}\right\} \qquad \frac{d}{t} = 0.973 \qquad \frac{d'}{t} = 0.027$$

$$e'_s = \frac{103.3}{-\,5.5} \times 12 - 24 = -\,250$$

$$f'_s\,p' \sin\alpha_1 \cos\alpha_1 = \frac{1}{2}\,h\,(2 - q_s)\,\frac{t}{d} - \frac{S}{b\,d}$$

$$\begin{aligned} f'_s\,p' \sin\alpha_1 \cos\alpha_1 = 18\,000 \times 0.0043 \times (-\,0.473) &= -\,36.6 \text{ lb.} \\ \frac{S}{b\,d} = -\,\frac{5\,500}{54.5 \times 12} &= \underline{-\ \ \ 8.4\ ``} \\ &= -\,45.0 \text{ lb.} \end{aligned}$$

$$\frac{1}{2}\,h\,(2 - q_s) = -\,45.0 \times 0.973 = -\,43.8$$

$$K_s = \frac{S\,e'_s}{b\,t^2} = \frac{(-\,5.50)\,(-\,250)}{12 \times 54.0 \times 54.0} = +\,39.2$$

$$\frac{1}{2}\,h\,(2 - q_s) = -\,\frac{K_s}{j'_s} = \frac{-\,39.2}{j'_s} = -\,43.8$$

$$j'_s = \frac{-\,39.2}{-\,43.8} = +\,0.900$$

$$q_s = \frac{3 - 6\,j'_s - 6\,\dfrac{d'}{t}}{2 - 3\,j'_s - 3\,\dfrac{d'}{t}} = \frac{-\,2.562}{-\,0.781} = 3.28$$

$$h = \frac{-\,43.8 \times 2}{-\,1.28} = 68.5 \text{ lb.}$$

$$h' = (1 - q_s)\,h = -\,2.28 \times 68.5 \text{ lb.} = -\,156 \text{ lb. (O. K.)}$$

Section at Top of Post (Omitted from Table 8).

$$\left.\begin{array}{l} m_z = -\,6\,365 \text{ kip-ft.} \\ t_v = 1\,181 \text{ kips} \end{array}\right\} \left.\begin{array}{l} m_v = -\,1\,554 \text{ kip-ft.} \\ t_z = 630 \text{ kips} \end{array}\right\} \begin{array}{ll} t = 36 \text{ in.} & \epsilon' = \epsilon \cos 90^\circ = 0 \\ d = 33.5 \text{ in.} & \sec^2 \theta' = 1 \end{array}$$

$$\begin{array}{ll} m = -\,95.0 \text{ kip-ft.} & t_n = 17.8 \text{ kips} \\ m' = 0 & t'_n = 0 \\ m_s = -\,10.6 \text{ kip-ft.} & t_s = 14.1 \text{ kips} \end{array} \left.\vphantom{\begin{array}{l}a\\a\\a\end{array}}\right\} \text{at center}$$

Fascia Steel.—

$$\left.\begin{array}{l} M = m \\ N = t_n \end{array}\right\} \begin{array}{l} f_c = 660 \text{ lb.} \\ f'_s = 18\,000 \text{ lb.} \\ p' = 0.0042 \\ A_s = 0.0042 \times 12 \times 33.5 = 1.69 \text{ sq. in.} \end{array}$$

Center Steel.—

(*a*) Minimum Cross Steel.—

$$\psi = 180 - \theta' = 180^\circ$$

$$M = 0,\ N = 0 \text{ (no steel required).}$$

(*b*) Minimum Longitudinal Steel.—

$$M = m,\ N = t_n \text{ (same as fascia steel).}$$

(*c*) Principal Planes.—

$$\tan 2\,\psi = \frac{2\,m_s}{m' - m} = \frac{-\,21.2}{95.0} = -\,0.224$$

$$\tan (180 - 2\,\psi) = 0.224$$

$$\psi = 83^\circ\ 30'$$

$$\left.\begin{array}{l} M_1 = -\,96.7 \text{ kip-ft.} \\ N_1 = 21.0 \text{ kips} \end{array}\right\} \left.\begin{array}{l} M_2 = 1.22 \text{ kip-ft.} \\ N_2 = 3.07 \text{ kips} \end{array}\right\} \alpha_1 = (83^\circ\ 30') - 90^\circ = -\,(6^\circ\ 30')$$

This computation shows that the same steel as the fascia steel should be used.

Maximum Horizontal Shear.—Shear is "O. K." by inspection.

Example of Design for Steel in Two Directions

$$\text{Assume} \left\{\begin{array}{l} M_1 = 18.0 \text{ kip-ft.} \quad N_1 = 12.32 \text{ kips} \quad \alpha_1 = 20^\circ\ 25' \\ M_2 = 36.5 \text{ kip-ft.} \quad N_2 = 2.68 \text{ kips} \quad \alpha_2 = 110^\circ\ 25' \\ t = 21.0 \text{ in.},\ d = 18.5 \text{ in.} \\ \text{Minimum steel..}0.18\% \text{ longitudinal} \\ \text{Minimum steel..}0.63\% \text{ cross} \end{array}\right.$$

Normal Steel, Plane 1.	Normal Steel, Plane 2.
$K = 77,\ \dfrac{e}{d} = 1.38$	$K = 112,\ \dfrac{e}{d} = 9.28$
$F_{s1} = 19\,000$ lb. (trial value)	$F_{s2} = 16\,500$ lb. (trial value)
$F_{c2} = 575$ lb.	$F_{c2} = 710$ lb.
$P_1 = 0.18\%$	$P_2 = 0.70\%$

For the following computations use Equations (64), Case X (*A*).

$$\sum f_s\, p\, b\, d \cos^2 \alpha = \frac{1}{2} f_c\, k\, b\, d - N = F_s\, P\, b\, d$$

$$\left.\begin{aligned} 0.879 f_\alpha p_\alpha + 0.121 f_\beta p_\beta &= 19\,500 \times 0.0017 = 33.2 \\ 0.121 f_\alpha p_\alpha + 0.879 f_\beta p_\beta &= 16\,500 \times 0.007 \;\;= 115 \end{aligned}\right\}$$

Solving,

$$\begin{cases} f_\alpha p_\alpha = 20 \\ f_\beta p_\beta = 129 \end{cases}$$

from which, $q_s = 3.58$; $h = 40$ lb.; and $a_s = -2.16$

$$\lambda\, n\, h\, a_s \sin\alpha_1 \cos\alpha_1 = -1\,050 \text{ lb.}$$

$$f_s = \sum f_c\, a \cos^2\alpha + \lambda\, h\, a_s \sin a_1 \cos\alpha.$$

$$f_\alpha = 19\,500 \times 0.879 + 16\,500 \times 0.121 - 1\,050 = 18\,100 \text{ lb.}$$

$$f_\beta = 19\,500 \times 0.121 + 16\,500 \times 0.879 + 1\,050 = 17\,850 \text{ lb.}$$

$$p_\alpha = \frac{20}{18\,000} = 0.11\%$$

$$p_\beta = \frac{129}{18\,000} = 0.64\%$$

SUMMARY

In Part II, the theory for proportioning sections has been developed. Equations have been derived for (*a*) stress distribution; (*b*) transformation formulas; and (*c*) design formulas for skew reinforced concrete sections. The application of these formulas is illustrated by the preceding numerical examples. This part of the paper, together with Part I, gives the designer a complete working text for designing skew arches.

PART III.—APPROXIMATE ANALYSIS FOR CROWN STRESSES IN A SYMMETRICAL SKEW RIGID FRAME

The rigid frame is a special type of arch and is analyzed as an arch, but the shape of the frame is such that the angle, ϕ (as previously defined), is either nearly 0°, or nearly 90 degrees. From the analyses of two skew rigid frames, the writer found that a considerable variation in the value of the angle, ϕ, produced negligible changes in the crown stresses. These observations lead to the following approximations:

1.—(*a*) $\phi = 0°$ for the barrel of the frame.
(*b*) $\phi = 90°$ for the post of the frame.

2.—The effect of the curvature of the barrel of the frame can be neglected in places where other characteristics of the frame affect the magnitude of the stress.

These approximations have the effect of transforming the actual frame shown in Fig. 24 into the frame shown in Fig. 25. It should be noted that M_y is a direct result of the curvature of the neutral axis (that is, M_y would be zero if the neutral axis were truly straight), and the accurate analysis must be used to compute it. Fortunately, this is a simple matter. Furthermore, $M_y = 0$ for symmetrical loads and temperature variation. Making these approximations results in the following.

APPROXIMATE FINAL EQUATIONS FOR SYMMETRICAL TWO-HINGED RIGID FRAMES

Let T'_x equal the crown thrust of a right rigid frame produced by two parallel planes cutting the skew arch perpendicular to the abutments. All other symbols are as previously defined.

For Vertical Loading.—

$$\left.\begin{aligned}
T_x &= T'_x \\
T_z &= \varepsilon\, T_x \\
M_x &= \frac{\varepsilon\left[\sum_B^P x\,\frac{s}{I} - x'\sum_B^P \frac{s}{I}\right]}{2\sum_B^C \frac{s}{I}} \\
M_z\,,\ T_y &= \text{same as for right arch} \\
M_y &= \frac{C_6 - K_3\, T_y}{K_{10}}, \text{as given in accurate theory}
\end{aligned}\right\}\ldots\ldots\ldots(65A)$$

For Horizontal Loading.—

$$\left.\begin{aligned}
T_x &= T'_x \\
T_z &= \varepsilon\, T_x \\
M_x &= 0 \\
T_y,\ M_z &= \text{same as for right arch} \\
M_y &= \frac{C_6 - K_3\, T_y}{K_{10}}, \text{as given in accurate theory}
\end{aligned}\right\}\ldots\ldots\ldots\ldots(65B)$$

For Temperature Stresses.—

$$\left.\begin{aligned}
T_x &= T'_x \sec^2\theta \\
T_z &= \varepsilon\left(T_x + \frac{1.34\,E\,c\,t\,L}{\sum_A^B x^2\,\frac{s}{I}}\right) \\
M_x &= 0 \\
M_y &= 0 \\
M_z &= -\,h\,T_x \\
T_y &= 0
\end{aligned}\right\}\ldots\ldots\ldots\ldots\ldots\ldots(65C)$$

The origin for x in these equations is at the center.

DERIVATION OF APPROXIMATE COEFFICIENTS
(SEE EQUATIONS (19) AND FIG. 25)

For All Loading.—

From A to B ($\phi = 90;\ u = x$):

$$\left.\begin{aligned}
K_{11} &= \sum_A^B y^2\,\frac{s}{I} - 2\,h\sum_A^B y\,\frac{s}{I} + h^2\sum_A^B \frac{s}{I} + \varepsilon^2\sum_A^B x^2\,\frac{\lambda\,s}{F} \\
K_2 &= \varepsilon\sum_A^B x^2\,\frac{\lambda\,s}{F};\ K_3 = 0;\ K_5 = \sum_A^B x^2\,\frac{\lambda\,s}{F};\ K_6 = 0 \\
K_7 &= 0
\end{aligned}\right\}\ldots(66A)$$

From B to C ($\phi = 0$; $u = y = 0$):

$$\left.\begin{aligned} K_{11} &= h^2 \sum_B^C \frac{s}{I} \\ K_2 &= 0;\ K_3 = 0;\ K_5 = 0;\ K_6 = 0 \\ K_7 &= \sum_B^C \frac{\lambda\, s}{F} \end{aligned}\right\} \dots\dots\dots\dots\dots\dots (66B)$$

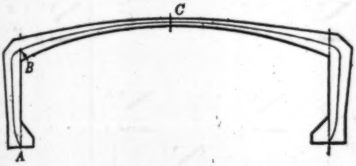


FIG. 24.—TYPICAL "RIGID FRAME."

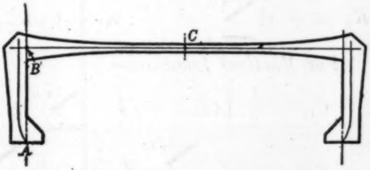


FIG. 25.—TRANSFORMED "RIGID FRAME."

For Vertical Loading.—

From A to B ($\phi = 90$; $u = x$):

$$\left.\begin{aligned} C_{11} &= \frac{1}{2}\Bigg[(L - x') \left(-\sum_A^B y\, \frac{s}{I} + h \sum_A^B \frac{s}{I} \right) + \sum_A^B x\, y\, \frac{s}{I} \\ &\qquad - x' \sum_A^B y\, \frac{s}{I} - h \sum_A^B x\, \frac{s}{I} + h\, x' \sum_A^B \frac{s}{I} \Bigg] \\ C_2 &= C_3 = C_6 = 0 \end{aligned}\right\} \dots (67A)$$

From B to C ($\phi = 0$; $u = y = 0$):

$$\left.\begin{aligned} C_{11} &= \frac{1}{2}\left[(L - x') \left(h \sum_B^C \frac{s}{I} \right) - h \sum_B^P x\, \frac{s}{I} + h\, x' \sum_B^P \frac{s}{I} \right] \\ C_2 &= 0 \\ C_3 &= \frac{1}{2}\left[\varepsilon \sum_B^P x\, \frac{\lambda\, s}{F} - \varepsilon\, x' \sum_B^P \frac{\lambda\, s}{F} \right] \\ C_6 &= 0 \end{aligned}\right\} . (67B)$$

For Horizontal Loading.—

From A to B ($\phi = 90$; $u = x$):

$$\left.\begin{aligned} C_{11} &= \frac{1}{2}\Bigg[(h - y') \left(-\sum_A^B y\, \frac{s}{I} + h \sum_A^B \frac{s}{I} \right) + \sum_A^P y^2\, \frac{s}{I} \\ &\qquad - y' \sum_A^P y\, \frac{s}{I} - h \sum_A^P y\, \frac{s}{I} + h\, y' \sum_A^P \frac{s}{I} \Bigg] \\ C_2 &= 0;\ C_3 = 0;\ C_6 = 0 \end{aligned}\right\} \dots (68A)$$

From B to C ($\phi = 0$; $u = y = 0$):

$$\left.\begin{aligned} C_{11} &= \frac{1}{2}\left[(h - y') \left(h \sum_B^C \frac{s}{I} \right) \right] \\ C_2 &= 0;\ C_3 = 0;\ C_6 = 0 \end{aligned}\right\} \dots\dots\dots\dots (68B)$$

For Temperature Stresses.—

$$C_{11} = E\, c\, t\, L;\ C_2 = \varepsilon\, E\, c\, t\, L;\ C_3 = C_4 = C_5 = C_6 = 0 \dots\dots (69)$$

APPROXIMATE COEFFICIENTS

Combining the two parts of Equations (66), (67), and (68), the approximate coefficients become:

For All Loading.—

$$K_{11} = \sum_A^B y^2 \frac{s}{I} - 2\,h \sum_A^B y\,\frac{s}{I} + h^2 \sum_A^C \frac{s}{I} + \varepsilon^2 \sum_A^B x^2 \frac{s}{I}$$

$$K_2 = -\,\varepsilon]\sum_A^B x^2\,\frac{\lambda\,s}{F};\ K_3 = 0;\ K_5 = \sum_A^B x^2\,\frac{\lambda\,s}{F};\ K_6 = 0;\ K_7 = \sum_B^C \frac{\lambda\,s}{F}$$

For Vertical Loading.—

$$C_{11} = \frac{1}{2}\left[(L - x')\left(-\sum_A^B y\,\frac{s}{I} + h \sum_A^C \frac{s}{I}\right) + \sum_A^B x\,y\,\frac{s}{I} \right.$$
$$\left. -\,x' \sum_A^B y\,\frac{s}{I} - h \sum_A^P x\,\frac{s}{I} + h\,x' \sum_A^P \frac{s}{I}\right]$$

$$C_2 = 0$$

$$C_3 = \frac{1}{2}\,\varepsilon\left[\sum_B^P x\,\frac{\lambda\,s}{F} - x' \sum_B^P \frac{\lambda\,s}{F}\right]$$

For Horizontal Loading.—

$$C_{11} = \frac{1}{2}\left[(h - y')\left(-\sum_A^B y\,\frac{s}{I} + h \sum_A^C \frac{s}{I}\right) + \sum_A^P y^2\,\frac{s}{I} \right.$$
$$\left. -\,y' \sum_A^P y\,\frac{s}{I} - h \sum_A^P y\,\frac{s}{I} + h\,y' \sum_A^P \frac{s}{I}\right]$$

$$C_2 = 0;\ C_3 = 0$$

For Temperature.—

$$C_1 = E\,c\,t\,L$$
$$C_2 = \varepsilon\,E\,c\,t\,L$$
$$C_3 = 0$$

DERIVATION OF FINAL EQUATIONS

For Vertical Loading.—Substituting the foregoing coefficients in Equation (19*b*) (Appendix I):

$$T_z = \frac{K_2\,T_x}{K_5} = \frac{\varepsilon \sum_A^B x^2\,\frac{\lambda\,s}{F}}{\sum_A^B x^2\,\frac{\lambda\,s}{F}}\,T_x$$

therefore,

$$T_z = \varepsilon\,T_x$$

Substituting this value of T_z and the foregoing coefficients in Equation (19*a*) (Appendix I):

$$T_x = \frac{C_{11}}{K_{11} + \varepsilon\,K_2} = \frac{C_{11}}{K'_{11}}$$

in which,

$$K'_{11} = \sum_A^B y^2\,\frac{s}{I} - 2\,h \sum_A^B y\,\frac{s}{I} + h^2 \sum_A^C \frac{s}{I}$$

therefore,

$$T_x = T'_x$$

Similarly, from Equation (19*c*) (Appendix I):

$$M_x = \frac{K_3}{K_7} = \frac{1}{2} \; \frac{\varepsilon \left[\sum_B^P x \, \frac{\lambda \, s}{F} - x' \sum_B^P \frac{\lambda \, s}{F} \right]}{\sum_B^C \frac{\lambda \, s}{F}}$$

but,

$$\frac{\lambda}{F} = \frac{3.58 \, \lambda}{b \, t^3} = \frac{3.58 \, \lambda}{12 \, I}$$

Substituting this value of $\frac{\lambda}{F}$ and cancelling out the constants:

$$M_x = \frac{\varepsilon \left[\sum_B^P x \, \frac{s}{I} - x' \sum_B^P \frac{s}{I} \right]}{2 \sum_B^C \frac{s}{I}}$$

For Horizontal Loading.—Similarly:

$$T_x = T'_x; \; T_z = \varepsilon \, T_x; \text{ and } M_x = 0$$

For Temperature Stresses.—Substituting the approximate coefficients in Equations (19*a*) and (19*b*):

$$\left. \begin{aligned} K_{11} \, T_x + K_2 \, T_z &= C_{11} \\ K_2 \, T_x + K_5 \, T_z &= C_2 = \varepsilon \, C_{11} \end{aligned} \right\} \quad \text{..................(70)}$$

Solving:

$$T_x = \frac{C_{11} \, K_5 - \varepsilon \, C_{11} \, K_2}{K_{11} \, K_5 - K_2^2}$$

but,

$$K_5 = -\frac{K_2}{\varepsilon}$$

Therefore,

$$T_x = \frac{C_{11} \, (1 + \varepsilon^2)}{K_{11} + \varepsilon \, K_2} = \frac{C_{11}}{K'_{11}} \sec^2 \theta$$

or,

$$T_x = T'_x \sec^2 \theta$$

From the second of Equations (70):

$$T_z = \frac{\varepsilon \, C_{11} - K_2 \, T_x}{K_5} = \frac{\varepsilon \, E \, c \, t \, L}{\sum_A^B x^2 \frac{\lambda \, s}{F}} + \varepsilon \, T_x$$

but,

$$\frac{\lambda}{F} = \frac{3.58 \times 2.5}{12 \, I} = \frac{1}{1.34 \, I}$$

Therefore,

$$T_z = \varepsilon \left[T_x + \frac{1.34 \, E \, c \, t \, L}{\sum_A^B x^2 \frac{s}{I}} \right]$$

Similarly, from Equation (19*c*), $M_x = 0$.

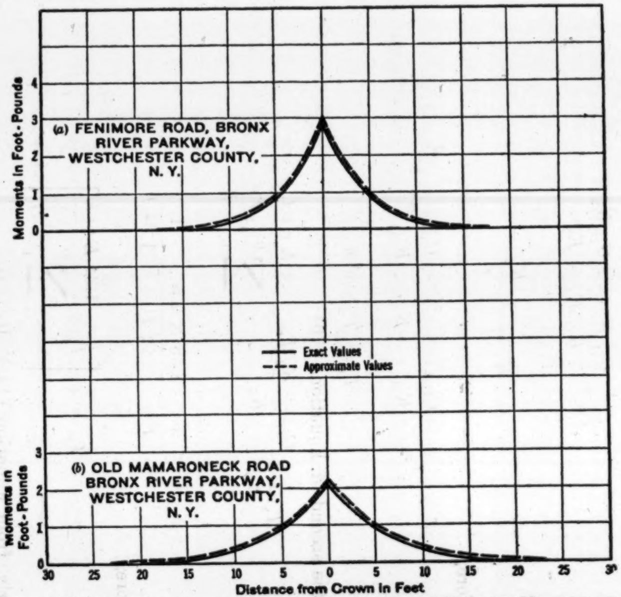


FIG. 26.—COMPARATIVE INFLUENCE LINES FOR TORSION MOMENT BY EXACT AND APPROXIMATE EQUATIONS.

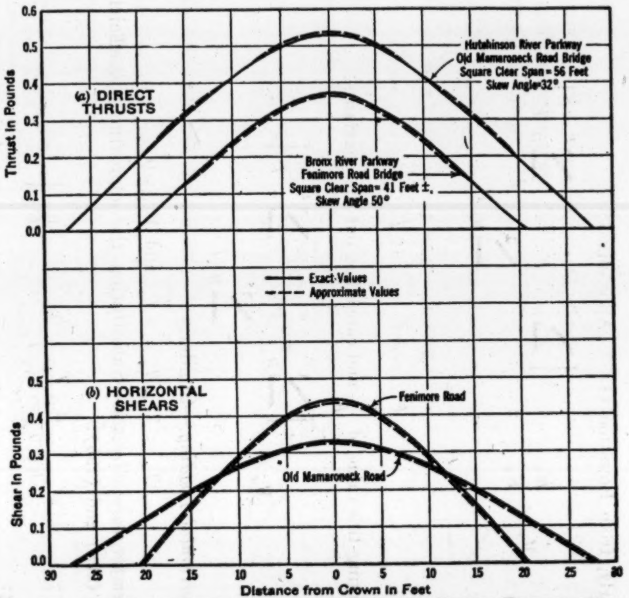


FIG. 27.—COMPARATIVE INFLUENCE LINES FOR DIRECT THRUST AND HORIZONTAL SHEAR BY EXACT AND APPROXIMATE EQUATIONS.

The relations just derived are true for the frame shown in Fig. 25. It is assumed that the same relations hold true for the frame in Fig. 24. The closeness of the approximation is shown by the influence lines of Fig. 26 and Fig. 27.

EXAMPLE 3.—APPLICATION OF APPROXIMATE THEORY

In a right symmetrical arch, it is somewhat more convenient to take the origin at the center of the span at the elevation of the hinges. When using Equations (2) and (3) of the more exact theory, $h-y$ must be written for y.

The right arch equations used herein can be derived from the skew arch equations by making $\epsilon = 0$ and writing $h-y$ for y, or they can be derived as follows.

For Vertical Loading.—From Fig. 28:

$$T'_x = \frac{\sum_A^B M y \frac{s}{I}}{\sum_A^B y^2 \frac{s}{I}} = \left[\frac{\sum_A^C \left(\frac{L-x'}{2L}\right)(L+x)\, y \frac{s}{I}}{\sum_A^C y^2 \frac{s}{I} + \sum_C^B y^2 \frac{s}{I}} + \frac{\sum_C^B \left(\frac{L-x'}{2L}\right)(L-x)\, y \frac{s}{I}}{\sum_A^C y^2 \frac{s}{I} + \sum_C^B y^2 \frac{s}{I}} - \frac{\sum_A^B (x-x')\, y \frac{s}{I}}{\sum_A^C y^2 \frac{s}{I} + \sum_C^B y^2 \frac{s}{I}} \right] P$$

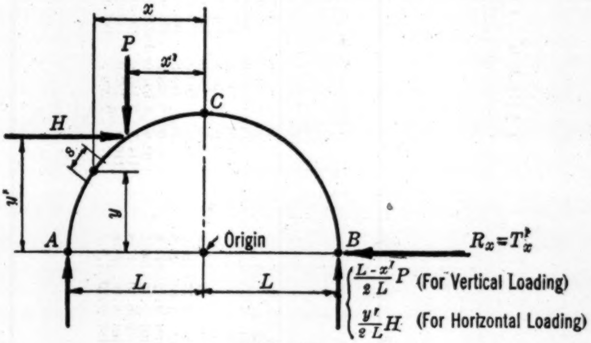

x
P
x'
C
H
s
y'
y
A
Origin
B
$R_x = T'_x$
L
L
$\frac{L-x'}{2L}P$ (For Vertical Loading)
$\frac{y'}{2L}H$ (For Horizontal Loading)

FIG. 28.

Since the arch is symmetrical, $\sum_A^C = \sum_B^C$:

$$T_x = \frac{(L-x')\sum_A^C y \frac{s}{I} - \sum_A^P x\, y \frac{s}{I} + x' \sum_A^P y \frac{s}{I}}{2\sum_A^C y^2 \frac{s}{I}} P$$

$$= \frac{a_v}{2\sum_A^C y^2 \frac{s}{I}} P \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (71)$$

TABLE 9.—CONSTANTS.

Point.	x.	y.	$\tan\phi$.	$\cos\phi$.	$\sin\phi$.	t.	I.	$\frac{1}{I}$.	$\frac{x}{I}$.	$\frac{x^2}{I}$.	$\frac{y}{I}$.	$\frac{y^2}{I}$.	$\frac{xy}{I}$.	$\frac{\sin\phi}{I}$.	$\frac{\sin^2\phi}{I}$.	$\frac{\sin\phi\cos\phi}{I}$.	$\frac{x\sin\phi\cos\phi}{I}$.	$\sum_A^P \frac{\sin\phi\cos\phi}{I}$.	$\sum_A^P x\frac{\sin\phi\cos\phi}{I}$.	$\sum_B^P \frac{1}{I}$.	$\sum_B^P \frac{x}{I}$.	$\sum_A^B \frac{x^2}{I}$.	$\sum_A^P \frac{y}{I}$.	$\sum_A^P \frac{y^2}{I}$.	$\sum_A^P \frac{xy}{I}$.
1	26.2	1.6	∞	0	1	4.8	9.2			75.0	0.17	0.3	4.6	0.1087	0.1087	0	0	0	0			75	0.17	0.3	4.6
2	26.3	4.9	∞	0	1	2.7	1.64			422.0	2.99	14.6	78.6	0.610	0.610	0	0	0	0			497	3.16	14.9	83.2
3	26.5	8.2	∞	0	1	3.0	2.25			312.0	3.64	29.9	96.5	0.445	0.445	0	0	0	0			809	6.80	44.8	179.7
4	26.7	11.5	∞	0	1	3.31	3.02			236.0	3.81	43.8	101.7	0.331	0.331	0	0	0	0			1 045	10.61	88 6	281.4
5	26.2	14.7	0.423	0.921	0.390	3.54	3.70			186.0	3.97	58.4	104.0	0.1055	0.041	0.0971	2.54	0.0971	2.54			1 231	14.58	147.0	385.4
6	23.3	15 8	0.357	0.941	0 337	3.20	2.73	0.37	8.6		5.81	91.8	135.4	0.1247	0.042	0.117	2.73	0.214	5.27	0.37	8.6		20.39	238.8	520.8
7	20.2	16.5	0.324	0.951	0.309	2.50	1.30	0.77	15.5		12.69	209.2	256.4	0 238	0.0735	0.226	4.56	0.440	9 83	1.14	24.1		33.08	448.0	777.2
8	17.2	17.2	0.278	0.963	0.268	1.94	0.61	1.64	28.3		28.24	485.5	485.8	0.439	0.118	0.423	7.29	0.863	17.12	2.78	52.4		61.32	933.5	1 263.0
9	14.1	17.7	0.250	0.970	0.243	1.57	0.322	3.12	43.8		55.0	973 5	775 0	0.759	0.184	0.736	10.35	1.599	27.47	5.90	96.2		116.32	1 907	2 038.0
10	11.0	18.2	0.206	0.979	0.202	1.33	0.196	5.10	56.1		92.9	1 691.0	1 022.0	1.030	0.208	1.010	11.10	2.609	38.57	11 00	152.3		209.22	3 598	3 060
11	7.8	18.7	0.129	0.992	0.129	1.25	0.163	6.10	47.8		114.7	2 145.0	895.0	0.786	0.101	0.780	6.08	3.389	44.65	17.10	200.1		323.92	5 743	3 955
12	4.7	18.9	0.107	0.994	0.107	1.25	0.163	6.10	28.8		116.0	2 192.0	545.0	0.652	0.069	0.648	3.04	4.037	47.69	23.20	228.9		439.92	7 935	4 500
13	1.6	19.1	0	1.000	0	1.25	0.163	6.10	9.9		117.2	2 239.0	187.0	0		0	0	4.037	47.69	29.30	238.8		557.12	10 174	4 687

TABLE 10.—SOLUTION FOR T_x, T_y, T_z, FOR UNIT VERTICAL LOADING.

Load point.	x'.	$(L-x')$.	$(L-x')\sum_A^C \frac{y}{I}$.	$x'\sum_A^P \frac{y}{I}$.	a_v.	Crown thrust = $T_x = T'_x$.	$T_y = \frac{L-x'}{2L}$.	$T_z = \epsilon T_y$.
5-6	24.8	1.4	780	362	757	0.037	0.027	0.0187
6-7	21.8	4.4	2 451	445	2 375	0.117	0.084	0.0596
7-8	18.7	7.5	4 178	620	4 021	0.198	0.143	0.1010
8-9	15.6	10.6	5 906	956	5 599	0.275	0.202	0.140
9-10	12.5	13.7	7 633	1 454	7 049	0.346	0.262	0.176
10-11	9.4	16.8	9 360	1 968	8 268	0.406	0.321	0.207
11-12	6 3	19.9	11 090	2 041	9 176	0.451	0 380	0.230
12-13	3.2	23.0	12 810	1 408	9 718	0.478	0.440	0.244
Crown	0	26.2	14 600	0	9 913	0.487	0.500	0.248

TABLE 11.—SOLUTION FOR T_x, T_y, T_z, FOR UNIT HORIZONTAL LOADING.

Load point.	y'.	$y'\sum_A^C \frac{y}{I}$.	$y'\sum_A^H \frac{y}{I}$.	a_h.	$T_x = T'_x$.	$T_y = \frac{y'}{2L}$.	$T_z = \epsilon T_y$.
1-2	3.3	1 838	0.6	1 838	0.090	0.063	0.046
2-3	6.6	3 677	21.0	3 671	0.180	0.126	0.092
3-4	9.9	5 515	67.0	5 493	0.270	0.189	0.138
4-5	13.2	7 354	140.0	7 303	0.360	0.252	0.184
5-6	15.4	8 580	225.0	8 502	0.418	0.294	0.213

TABLE 12.—SOLUTION FOR M_x FOR VERTICAL LOADING.

Load point.	x'.	$\sum_B^P \frac{x}{I}$.	$\sum_B^P \frac{1}{I}$.	$x'\sum_B^P \frac{1}{I}$.	$\sum_B^P \frac{x}{I} - x'\sum_B^P \frac{1}{I}$.	M_x.
6-7	21.8	8.6	0.37	8.07	+ 0.5	0.0
7-8	18.7	24.1	1.14	21.3	+ 2.8	0.024
8-9	15.6	52.4	2.78	43.3	+ 9.1	0.079
9-10	12.5	96.2	5.90	73.8	+ 22.4	0.195
10-11	9.4	152.3	11.00	103.4	+ 48.9	0.425
11-12	6.3	200.1	17.10	108.0	+ 92.1	0.800
12-13	3.2	228.9	23.20	74.2	+154.7	1.35
Crown	0	238.8	29.30	0	+238.8	2.08

TABLE 13.—SOLUTION FOR M_y FOR VERTICAL LOADING.

$\epsilon\sum_A^P \frac{x\sin\phi\cos\phi}{I}$.	$-\epsilon x'\sum_A^P \frac{\sin\phi\cos\phi}{I}$.	$-\epsilon\left(\sum_A^C \frac{x\sin\phi\cos\phi}{I}\right)T_y$.	$-\sum_A^C \frac{\sin^2\phi}{I}$.	M_y.
2.69	− 2.38	− 2.04	−2.329	0.814
5.02	− 4.20	− 3.48	−2.329	1.320
8.74	− 6.88	− 4.91	−2.329	1.71
14.00	−10.20	− 6.36	−2.329	1.91
19.65	−12.5	− 7.80	−2.329	1.82
22.8	−10.80	− 9.25	−2.329	1.40
24.3	− 6.6	−10.70	−2.329	0.795
24.3	0	−12.15	−2.329	0

TABLE 14.—TEMPERATURE STRESSES.

$$E\,c\,t\,L = 288\,000 \times 0.0000065 \times 100 \times 26.25 = 4920$$

$$T'_x = \frac{E\,c\,t\,L}{\sum y^2 \frac{s}{I}} = \frac{4920}{10170 \times 3.1} = 0.157 \text{ kip}$$

$$T_x = T'_x \sec.^2 \theta = 0.157 \times 1.26 = 0.198 \text{ kip}$$

$$T_z = \epsilon\left[T_x + \frac{1.34\,E\,c\,t\,L}{\sum_A^B x^2 \frac{s}{I}}\right] = 0.51\left(0.198 + \frac{1.34 \times 4920}{1231 \times 3.1}\right) = 0.983 \text{ kip}$$

For Horizontal Loading.—

$$T'_x = \frac{y' \sum_A^C y \dfrac{s}{I} + \sum_A^H y^2 \dfrac{s}{I} - y' \sum_A^H y \dfrac{s}{I}}{2 \sum_A^C y^2 \dfrac{s}{I}} H = \frac{a_h}{2 \sum_A^C y^2 \dfrac{s}{I}} H \ldots (72)$$

For Temperature Rise.—

$$T_x = \frac{E\,c\,t\,L}{\sum_A^C y^2 \dfrac{s}{I}} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (73)$$

CALCULATIONS FOR INFLUENCE LINES (SEE FIG. 29).

The following numerical example illustrates the use of the approximate theory and the saving of numerical work is apparent from a comparison of Tables 1 to 6, inclusive, and Tables 9 to 14, inclusive. In Fig. 29, Point *B* is on a vertical line through the reaction at *A*.

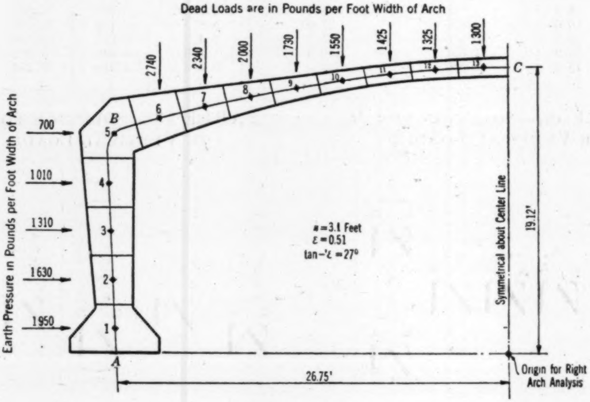

Dead Loads are in Pounds per Foot Width of Arch
2740
2340
2000
1730
1550
1425
1325
1300
C
Earth Pressure in Pounds per Foot Width of Arch
700
1010
1310
1630
1950
B
1
2
3
4
5
6
7
8
9
10
11
12
13
a = 3.1 Feet
ε = 0.51
tan⁻¹ε = 27°
Symmetrical about Center Line
19.12'
A
26.75'
Origin for Right Arch Analysis

FIG. 29.

ACKNOWLEDGMENTS

The writer wishes to express his gratitude to Mr. Hayden for the encouragement he has received in the preparation of this paper. He is also under obligation to two other members of the Design Division of the Westchester County Park Commission, Mr. Aaron Cohen, who checked Part I, and William R. Frederick, Jr., Jun. Am. Soc. C. E., who checked Parts II and III of the paper.

APPENDIX I

In this section of the paper all final equations necessary for the design of skew arches are grouped together. It will be noted that the equation numbers are discontinuous. The purpose has been to arrange the formulas in such a way that any one using them may turn back to the text for an explanation or derivation. For example, Equations (16) follow Equations (3) in Appendix I. Both these sets are essential in a design, whereas Equations (1), (2), and (4) to (15), inclusive, etc., in the main paper, are necessary only for the derivation of the final design equations. The number of the equations in Appendix I serve to identify their normal position in the paper, where the derivations are given.

EQUATIONS FOR COMPUTING TOTAL STRESSES

Equations (2), (3), (16), (19), (20), (21), (25), and (27), which follow, are explained in Part I.

Symmetrical Arches

For formulas to apply to the design of symmetrical arches use Equations (2), (3), (16), and (19), as follows:

Stresses at a Section.—

$$t_v = T_x \cos\phi \mp T_y \sin\phi + W \sin\phi - H \cos\phi \tag{2a}$$

$$t_u = \pm T_x \sin\phi + T_y \cos\phi \mp W \cos\phi \mp H \sin\phi \tag{2b}$$

$$t_z = T_z \tag{2c}$$

$$\begin{aligned} m_v = {} & T_x\, \varepsilon\, x \sin\phi \pm T_y\, \varepsilon\, x \cos\phi - T_z\, u + M_x \cos\phi \mp M_y \sin\phi \\ & - W\, \varepsilon\, (x - x') \cos\phi - H\, \varepsilon\, (x - x') \sin\phi \end{aligned} \tag{2d}$$

$$\begin{aligned} m_u = {} & \mp T_x\, \varepsilon\, x \cos\phi + T_y\, \varepsilon\, x \sin\phi \pm T_z\, v \pm M_x \sin\phi + M_y \cos\phi \\ & \mp W\, \varepsilon\, (x - x') \sin\phi \pm H\, \varepsilon\, (x - x') \cos\phi \end{aligned} \tag{2e}$$

$$m_z = T_x\, y \pm T_y\, x + M_z - W\,(x - x') - H\,(y - y') \tag{2f}$$

The upper signs in Equations (2) apply to the left half of the structure, and the lower signs apply to the right half. The loads, *W* and *H*, apply to either half of the arch.

Co-Ordinates of Crown Section with Respect to Any Point, P.—

$$u = x \sin\phi - y \cos\phi \tag{3a}$$

$$v = x \cos\phi + y \sin\phi \tag{3b}$$

Final Equations for the Fixed Symmetrical Arch (Origin of Co-Ordinates Is at Crown Center).—

$$K_4\, T_x + K_8\, M_z = C_4 \tag{16a}$$

$$K_1\, T_x + K_2\, T_z + K_3\, M_x + K_4\, M_z = C_1 \tag{16b}$$

$$K_2\, T_x + K_5\, T_z + K_6\, M_x = C_2 \tag{16c}$$

$$K_3\, T_x + K_6\, T_z + K_7\, M_x = C_3 \tag{16d}$$

$$K_9\, T_y + K_3\, M_y = C_5 \tag{16e}$$

and,

$$K_3\, T_y + K_{10}\, M_y = C_6 \tag{16f}$$

in which, the coefficients are, for all loading and temperature stresses:

$$K_1 = \sum y^2 \frac{s}{I} + \varepsilon^2 \sum x^2 \sin^2\phi\,\lambda \frac{s}{F}$$

$$K_2 = -\,\varepsilon \sum u\,x \sin\phi\,\lambda \frac{s}{F}$$

$$K_3 = +\,\varepsilon \sum x \sin\phi \cos\phi\,\lambda \frac{s}{F}$$

$$K_4 = + \sum y \frac{s}{I}$$

$$K_5 = + \sum u^2 \lambda \frac{s}{F}$$

$$K_6 = - \sum u \cos\phi\,\lambda \frac{s}{F}$$

$$K_7 = + \sum \cos^2\phi\,\lambda \frac{s}{F}$$

$$K_8 = + \sum \frac{s}{I}$$

$$K_9 = - \sum x^2 \frac{s}{I} - \varepsilon^2 \sum x^2 \cos^2\phi\,\lambda \frac{s}{F}$$

and,

$$K_{10} = - \sum \sin^2\phi\,\lambda \frac{s}{F}$$

For a vertical load, W, at the point, $x = x'$, $z = \varepsilon x'$:

$$C_1 = \frac{1}{2}\left[\sum_A^P x\,y \frac{s}{I} - x' \sum_A^P y \frac{s}{I} + \varepsilon^2 \sum_A^P x^2 \sin\phi\cos\phi\,\lambda \frac{s}{F} - \varepsilon^2 x' \sum_A^P x \sin\phi\cos\phi\,\lambda \frac{s}{F}\right] W$$

$$C_2 = \frac{1}{2}\left[-\,\varepsilon \sum_A^P u\,x \cos\phi\,\lambda \frac{s}{F} + \varepsilon\,x' \sum_A^P u \cos\phi\,\lambda \frac{s}{F}\right] W$$

$$C_3 = \frac{1}{2}\left[\varepsilon \sum_A^P x \cos^2\phi\,\lambda \frac{s}{F} - \varepsilon\,x' \sum_A^P \cos^2\phi\,\lambda \frac{s}{F}\right] W$$

$$C_4 = \frac{1}{2}\left[\sum_A^P x \frac{s}{I} - x' \sum_A^P \frac{s}{I}\right] W$$

$$C_5 = \frac{1}{2}\left[-\sum_A^P x^2 \frac{s}{I} + x' \sum_A^P x \frac{s}{I} - \varepsilon^2 \sum_A^P x^2 \cos^2\phi\,\lambda \frac{s}{F} + \varepsilon^2 x' \sum_A^P x \cos^2\phi\,\lambda \frac{s}{F}\right] W$$

and,

$$C_6 = \frac{1}{2}\left[\varepsilon \sum_A^P x \sin\phi\cos\phi\,\lambda \frac{s}{F} - \varepsilon\,x' \sum_A^P \sin\phi\cos\phi\,\lambda \frac{s}{F}\right] W$$

For a horizontal load, H, at the point, $y = y'$, $z = \varepsilon\,x'$:

$$C_1 = \frac{1}{2}\left[\sum_A^P y^2 \frac{s}{I} - y' \sum_A^P y \frac{s}{I} + \varepsilon^2 \sum_A^P x^2 \sin^2\phi\,\lambda \frac{s}{F} - \varepsilon^2 x' \sum_A^P x \sin^2\phi\,\lambda \frac{s}{F}\right] H$$

$$C_2 = \frac{1}{2}\left[-\varepsilon \sum\nolimits_A^P u\,x \sin\phi\,\lambda\,\frac{s}{F} + \varepsilon\,x' \sum\nolimits_A^P u \sin\phi\,\lambda\,\frac{s}{F}\right] H$$

$$C_3 = \frac{1}{2}\left[\varepsilon \sum\nolimits_A^P x \sin\phi \cos\phi\,\lambda\,\frac{s}{F} - \varepsilon\,x' \sum\nolimits_A^P \sin\phi \cos\phi\,\lambda\,\frac{s}{F}\right] H$$

$$C_4 = \frac{1}{2}\left[\sum\nolimits_A^P y\,\frac{s}{I} - y' \sum\nolimits_A^P \frac{s}{I}\right] H$$

$$C_5 = \frac{1}{2}\left[-\sum\nolimits_A^P x\,y\,\frac{s}{I} + y' \sum\nolimits_A^P x\,\frac{s}{I} - \varepsilon^2 \sum\nolimits_A^P x^2 \sin\phi \cos\phi\,\lambda\,\frac{s}{F} + \varepsilon^2\,x' \sum\nolimits_A^P x \sin\phi \cos\phi\,\lambda\,\frac{s}{F}\right] H$$

and,

$$C_6 = \frac{1}{2}\left[\varepsilon \sum\nolimits_A^P x \sin^2\phi\,\lambda\,\frac{s}{F} - \varepsilon\,x' \sum\nolimits_A^P \sin^2\phi\,\lambda\,\frac{s}{F}\right] H$$

For temperature stresses ($t°$ rise):

$$C_1 = E\,c\,t\,L;\quad C_2 = \varepsilon\,E\,c\,t\,L;\ \text{and,}\ C_3 = C_4 = C_5 = C_6 = 0$$

All the summations are from A to C (Fig. 29) unless otherwise noted.

Final Equations for the Two-Hinged Symmetrical Arch (Origin of Co-Ordinates at Crown Center).—As in the case of the fixed arch, the final equations are written in terms of the coefficients, K and C:

$$K_{11}\,T_x + K_2\,T_z + K_3\,M_x = C_{11} \dots\dots\dots\dots (19a)$$

$$K_2\,T_x + K_5\,T_z + K_6\,M_x = C_2 \dots\dots\dots\dots (19b)$$

$$K_3\,T_x + K_6\,T_z + K_7\,M_x = C_3 \dots\dots\dots\dots (19c)$$

$$M_z = C_{14} - h\,T_x \dots\dots\dots\dots (19d)$$

$$T_y = C_{15} \dots\dots\dots\dots (19e)$$

$$K_{10}\,M_y = C_6 - K_3\,T_y \dots\dots\dots\dots (19f)$$

All the coefficients given in Equations (19) have the same values as those in Equations (16), with the following additional values:

For all loading and temperature stresses:

$$K_{11} = \sum y^2\,\frac{s}{I} - 2\,h \sum y\,\frac{s}{I} + h^2 \sum \frac{s}{I} + \varepsilon^2 \sum x^2 \sin^2\phi\,\frac{\lambda\,s}{F}$$

For a vertical load, W, at the point $x = x'$, $z = \varepsilon\,x'$:

$$C_{11} = \frac{1}{2}\left[(L - x')\left(-\sum y\,\frac{s}{I} + h \sum \frac{s}{I}\right) + \sum\nolimits_A^P x\,y\,\frac{s}{I} - x' \sum\nolimits_A^P y\,\frac{s}{I} - h \sum\nolimits_A^P x\,\frac{s}{I} + h\,x' \sum\nolimits_A^P \frac{s}{I} + \varepsilon^2 \sum\nolimits_A^P x^2 \sin\phi \cos\phi\,\frac{\lambda\,s}{F} - \varepsilon^2\,x' \sum\nolimits_A^P x \sin\phi \cos\phi\,\frac{\lambda\,s}{F}\right] W$$

$$C_{14} = \frac{1}{2}\,(L - x')\,W$$

$$C_{15} = \frac{1}{2}\left(1 - \frac{x'}{L}\right) W$$

For a horizontal load, H, at the point, $y = y'$, $z = \varepsilon\, x'$:

$$C_{11} = \frac{1}{2}\left[(h - y')\left(-\sum y\,\frac{s}{I} + h\sum\frac{s}{I}\right) + \sum_A^P y^2\,\frac{s}{I} - y'\sum_A^P y\,\frac{s}{I}\right.$$
$$- h\sum_A^P y\,\frac{s}{I} + h\,y'\sum_A^P\frac{s}{I} + \varepsilon^2\sum_A^P x^2\sin^2\phi\,\frac{\lambda\, s}{F}$$
$$\left.- \varepsilon^2\,x'\sum_A^P x\sin^2\phi\,\frac{\lambda\, s}{F}\right] H$$

$$C_{14} = \frac{1}{2}\,(h - y')\,H$$

$$C_{15} = \frac{1}{2}\left(\frac{h - y'}{L}\right) H$$

For temperature stresses:

$C_{11} = C_1$

Unsymmetrical Arches

For formulas to apply to the design of unsymmetrical arches, use Equations (20), (21), (25), and (27), as follows:

Stresses at a Section.—

$$t_v = - R_x\cos\phi - R_y\sin\phi + W\sin\phi - H\cos\phi \ldots\ldots\ldots\ldots (20a)$$

$$t_u = - R_x\sin\phi + R_y\cos\phi - W\cos\phi - H\sin\phi \ldots\ldots\ldots\ldots (20b)$$

$$t_z = R_z \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (20c)$$

$$m_u = R_x\,\varepsilon\,x\cos\phi + R_y\,\varepsilon\,x\sin\phi + R_z\,v' - N_x\sin\phi + N_y\cos\phi$$
$$- W\,\varepsilon\,(x - x')\sin\phi + H\,\varepsilon\,(x - x')\cos\phi \ldots\ldots\ldots\ldots (20d)$$

$$m_v = - R_x\,\varepsilon\,x\sin\phi + R_y\,\varepsilon\,x\cos\phi - R_z\,u' - N_x\cos\phi - N_y\sin\phi$$
$$- W\,\varepsilon\,(x - x')\cos\phi - H\,\varepsilon\,(x - x')\sin\phi \ldots\ldots\ldots\ldots (20e)$$

$$m_z = R_x\,y + R_y\,x + N_z - W\,(x - x') + H\,(y - y') \ldots\ldots\ldots\ldots (20f)$$

Co-Ordinates of the Right Abutment with Respect to Any Point, P.—

$$u' = x\sin\phi + y\cos\phi \ldots\ldots\ldots\ldots\ldots\ldots (21a)$$

$$v' = x\cos\phi - y\sin\phi \ldots\ldots\ldots\ldots\ldots\ldots (21b)$$

***Final Equations for Unsymmetrical Fixed Arch (Origin of Co-Ordinates Is at Center of Right Abutment).*—Here, again, the final equations are written in terms of the coefficients, K and C, for brevity:**

$$- K_1\,R_x + K_2\,R_y + K_3\,R_z + K_4\,N_x - K_5\,N_y = C_1 . \quad (25a)$$

$$K_6\,R_x - K_7\,R_y + K_8\,R_z + K_5\,N_x + K_9\,N_y = C_2 . \quad (25b)$$

$$K_{10}\,R_x + K_{11}\,R_y + K_{12}\,N_z = C_3 . \quad (25c)$$

$$K_{13}\,R_x + K_{14}\,R_y + K_{15}\,R_z + K_7\,N_x + K_6\,N_y + K_{10}\,N_z = C_4 . \quad (25d)$$

$$K_{14}\,R_x + K_{17}\,R_y - K_{18}\,R_z - K_2\,N_x - K_7\,N_y + K_{11}\,N_z = C_5 . \quad (25e)$$

$$- K_{15}\,R_x + K_{18}\,R_y + K_{19}\,R_z + K_3\,N_x - K_8\,N_y = C_6 . \quad (25f)$$

in which, the coefficients for all loading and temperature stresses are:

$$K_1 = \varepsilon \sum x \sin\phi \cos\phi \frac{\lambda\, s}{F}$$

$$K_2 = \varepsilon \sum x \cos^2\phi \frac{\lambda\, s}{F}$$

$$K_3 = -\sum u' \cos\phi \frac{\lambda\, s}{F}$$

$$K_4 = -\sum \cos^2\phi \frac{\lambda\, s}{F}$$

$$K_5 = \sum \sin\phi \cos\phi \frac{\lambda\, s}{F}$$

$$K_6 = \varepsilon \sum x \sin^2\phi \frac{\lambda\, s}{F}$$

$$K_7 = \varepsilon \sum x \sin\phi \cos\phi \frac{\lambda\, s}{F}$$

$$K_8 = \sum u' \sin\phi \frac{\lambda\, s}{F}$$

$$K_9 = \sum \sin^2\phi \frac{\lambda\, s}{F}$$

$$K_{10} = \sum y \frac{s}{I}$$

$$K_{11} = \sum x \frac{s}{I}$$

$$K_{12} = \sum \frac{s}{I}$$

$$K_{13} = \sum y^2 \frac{s}{I} + \varepsilon^2 \sum x^2 \sin^2\phi \frac{\lambda\, s}{F}$$

$$K_{14} = \sum x\, y \frac{s}{I} - \varepsilon^2 \sum x^2 \sin\phi \cos\phi \frac{\lambda\, s}{F}$$

$$K_{15} = \varepsilon \sum u' x \sin\phi \frac{\lambda\, s}{F}$$

$$K_{16} = \varepsilon \sum x \sin^2\phi \frac{\lambda\, s}{F}$$

$$K_{17} = \sum x^2 \frac{s}{I} + \varepsilon^2 \sum x^2 \cos^2\phi \frac{\lambda\, s}{F}$$

$$K_{18} = \varepsilon \sum u' x \cos\phi \frac{\lambda\, s}{F}$$

$$K_{19} = -\sum u'^2 \frac{\lambda\, s}{F}$$

For vertical loading:

$$C_1 = \varepsilon \left[\sum_A^P x \cos^2\phi \frac{\lambda\, s}{F} - x' \sum_A^P \cos^2\phi \frac{\lambda\, s}{F}\right] W$$

$$C_2 = \varepsilon \left[-\sum_A^P x \sin\phi \cos\phi \frac{\lambda\, s}{F} + x' \sum_A^P \sin\phi \cos\phi \frac{\lambda\, s}{F}\right] W$$

$$C_3 = \left[\sum_A^P x \frac{s}{I} - x' \sum_A^P \frac{s}{I}\right] W$$

$$C_4 = \left[\sum_A^P x\, y \frac{s}{I} - x' \sum_A^P y \frac{s}{I} + \varepsilon^2 \left(-\sum_A^P x^2 \sin\phi \cos\phi \frac{\lambda\, s}{F} + x' \sum_A^P x \sin\phi \cos\phi \frac{\lambda\, s}{F}\right)\right] W$$

$$C_5 = \left[\sum_A^P x^2 \frac{s}{I} - x' \sum_A^P x \frac{s}{I} + \varepsilon^2 \left(\sum_A^P x^2 \cos^2\phi \frac{\lambda\, s}{F} - x' \sum_A^P x \cos^2\phi \frac{\lambda\, s}{F}\right)\right] W$$

$$C_6 = \varepsilon \left[\sum_A^P u' x \cos\phi \frac{\lambda\, s}{F} - x' \sum_A^P u' \cos\phi \frac{\lambda\, s}{F}\right] W$$

For horizontal loading:

$$C_1 = \varepsilon \left[\sum_A^H x \sin\phi \cos\phi \frac{\lambda\, s}{F} - x' \sum_A^H \sin\phi \cos\phi \frac{\lambda\, s}{F}\right] H$$

$$C_2 = \varepsilon \left[-\sum_A^H x \sin^2\phi \frac{\lambda\, s}{F} + x' \sum_A^H \sin^2\phi \frac{\lambda\, s}{F}\right] H$$

$$C_3 = \left[-\sum\nolimits_A^H y \frac{s}{I} + y' \sum\nolimits_A^H \frac{s}{I} \right] H$$

$$C_4 = \left[-\sum\nolimits_A^H y^2 \frac{s}{I} + y' \sum\nolimits_A^H y \frac{s}{I} + \varepsilon^2 \left(-\sum\nolimits_A^H x^2 \sin^2 \phi \frac{\lambda s}{F} + x' \sum\nolimits_A^H x \sin^2 \phi \frac{\lambda s}{F} \right) \right] H$$

$$C_5 = \left[-\sum\nolimits_A^H x y \frac{s}{I} + y' \sum\nolimits_A^H x \frac{s}{I} + \varepsilon^2 \left(\sum\nolimits_A^H x^2 \sin \phi \cos \phi \frac{\lambda s}{F} - x' \sum\nolimits_A^H x \sin \phi \cos \phi \frac{\lambda s}{F} \right) \right] H$$

$$C_6 = \varepsilon \left[\sum\nolimits_A^H u' x \sin \phi \frac{\lambda s}{F} - x' \sum\nolimits_A^H u' \sin \phi \frac{\lambda s}{F} \right] H$$

For temperature stresses (t ° rise):

$$C_1 = C_2 = C_3 = 0$$
$$C_4 = -2\, E\, c\, t\, L$$
$$C_5 = E\, c\, t\, a$$
$$C_6 = -2\, \varepsilon\, E\, c\, t\, L$$

All the summations are from A to B (Fig. 9) unless otherwise noted.

Final Equations for Unsymmetrical Two-Hinged Arch (Origin of Co-Ordinates Is at Center of Right Abutment).—

$$-K_1 R_x + K_2 R_y + K_3 R_z + K_4 N_x - K_5 N_y = C_1 \quad \text{(27a)}$$

$$K_6 R_x - K_7 R_y + K_8 R_z + K_5 N_x + K_9 N_y = C_2 \quad \text{(27b)}$$

$$K_{13} R_x + K_{14} R_y + K_{15} R_z + K_7 N_x + K_6 N_y - \left(\frac{a}{2\,L}\right)(K_{14} R_x + K_{17} R_y - K_{18} R_z - K_2 N_x - K_7 N_y) = C_4 - \left(\frac{a}{2\,L}\right) C_5 \quad \text{(27c)}$$

$$-K_{15} R_x + K_{18} R_y + K_{19} R_z + K_3 N_x - K_8 N_y = C_6 \quad \text{(27d)}$$

$$R_y + \left(\frac{a}{2\,L}\right) R_x = C_7 \quad \text{(27e)}$$

$$N_z = 0 \quad \text{(27f)}$$

All the coefficients are the same as in Equation (26) for the fixed arch except C_7, which varies under different conditions as follows:

For vertical loading, W,

$$C_7 = \left(1 - \frac{x'}{2\,L}\right) W$$

For horizontal loading, H,

$$C_7 = \left(\frac{y'}{2\,L}\right) H$$

For temperature stresses,

$$C_7 = 0$$

PROPORTIONING SECTIONS

All the following equations are fully explained in Part II.

Relations Between Angles

$$\varepsilon' = \varepsilon \cos \phi \quad \text{(28)}$$

$$\tan \phi' = \tan \phi \cos \theta \quad \text{(29)}$$

Equations for Stresses per Unit Width of Arch

$$t_s = \frac{3}{2}\left(\frac{t_z - \varepsilon' t_v}{b}\right)\left(1 - \frac{4 z^2}{b^2}\right) + \varepsilon' \frac{t_v}{b} + 12 \varepsilon' \frac{m_u z}{b^3} \quad \ldots\ldots (35)$$

$$t'_n = \varepsilon' t_s \quad \ldots\ldots (36)$$

$$m_s = \frac{\varepsilon' m_z}{b} \text{ (at the face of bridge)} \quad \ldots\ldots (43)$$

$$m_s = \frac{m_v + \varepsilon' m_z}{2 b} \text{ (at any other point of the section)} \quad \ldots\ldots (44)$$

$$m' = \frac{\varepsilon'^2 m_z}{b} \text{ (at the face of bridge)} \quad \ldots\ldots (48)$$

$$m' = \frac{\varepsilon' m_v}{b} \text{ (at any other point of the section)} \quad \ldots\ldots (49)$$

$$V' = \frac{\varepsilon' t_u}{b} - \frac{8}{5} \varepsilon' \frac{(m_v - \varepsilon' m_z)}{b t} \text{ (at the face of bridge)} \quad \ldots\ldots (51)$$

$$V' = \frac{\varepsilon' t_u}{b} \text{ (at any other point of the section)} \quad \ldots\ldots (52)$$

Transformation Formulas

Equations (53*a*) and (53*b*) are general formulas:

$$\left.\begin{aligned} M &= m \sin^2 \psi + m' \cos^2 \psi + 2 m_s \sin \psi \cos \psi \\ N &= t_n \sin^2 \psi + t'_n \cos^2 \psi + 2 t_s \sin \psi \cos \psi \end{aligned}\right\} \quad \ldots\ldots (53a)$$

$$\left.\begin{aligned} M_s &= (m' - m) \sin \psi \cos \psi + m_s (\sin^2 \psi - \cos^2 \psi) \\ S &= (t'_n - t_n) \sin \psi \cos \psi + t_s (\sin^2 \psi - \cos^2 \psi) \end{aligned}\right\} \quad \ldots\ldots (53b)$$

They may be adapted to special cases in the following ways:

For Principal Moment Planes.—

$$\left.\begin{aligned} \tan 2 \psi &= \frac{2 m_s}{m' - m} \\ M &= \frac{m' + m}{2} \pm \frac{1}{2} \sqrt{(m' - m)^2 + 4 m_s^2} \end{aligned}\right\} \quad \ldots\ldots (53c)$$

$$\left.\begin{aligned} \psi &= 90 - \theta' \\ M &= m \sec^2 \theta' \end{aligned}\right\} \text{ (at the face of bridge)} \quad \ldots\ldots (53d)$$

For Principal Thrust Planes.—

$$\left.\begin{aligned} \tan 2 \psi &= \frac{2 t_s}{t'_n - t_n} \\ N &= \frac{t'_n + t_n}{2} \pm \frac{1}{2} \sqrt{(t'_n - t_n)^2 + 4 t_s^2} \end{aligned}\right\} \quad \ldots\ldots (53e)$$

$$\left.\begin{aligned} \psi &= 90 - \theta' \\ N &= t_n \sec^2 \theta' \end{aligned}\right\} \text{ (at the face of bridge)} \quad \ldots\ldots (53f)$$

For Maximum and Minimum Torsion Moment.—

$$\left.\begin{aligned} \tan 2 \psi &= \frac{m - m'}{2 m_s} \\ M_s &= \pm \frac{1}{2} \sqrt{(m' - m)^2 + 4 m_s^2} \end{aligned}\right\} \quad \ldots\ldots (53g)$$

For Maximum and Minimum Shearing Thrust.—

$$\left.\begin{aligned} \tan 2 \psi &= \frac{t_n - t'_n}{2 t_s} \\ S &= \pm \frac{1}{2} \sqrt{(t'_n - t_n)^2 + 4 t_s^2} \end{aligned}\right\} \quad \ldots\ldots (53h)$$

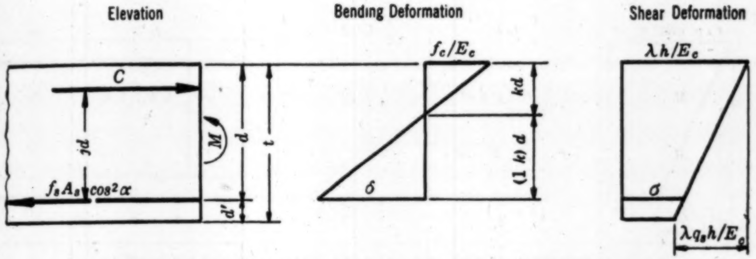
FINAL DESIGN FORMULAS (EQUATIONS (64))

CASE I.—SIMPLE BENDING; STEEL IN ONE DIRECTION, IN THE INTRADOS FACE ONLY (SEE FIG. 30).

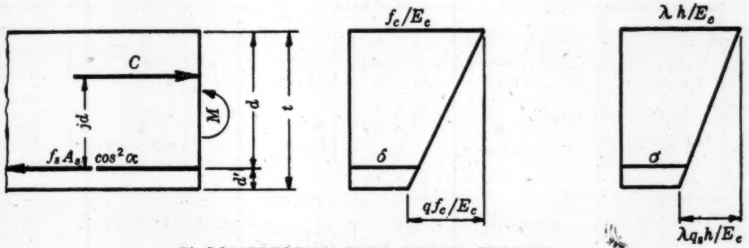
<table>
<tr><th></th><th></th><th>(A) Tension over part of section containing steel.</th><th>(B) Compression over entire section.</th><th>(C) Tension over part of section not containing steel.</th><th>Equations (64).</th></tr>
<tr><td>$f_s p \cos^2\alpha$</td><td>$=$</td><td>$\frac{1}{2} f_c k$</td><td>$\frac{1}{2} f_c (2-q) \frac{t}{d}$</td><td>$\frac{1}{2} f_c k$</td><td>(a)</td></tr>
<tr><td>M</td><td>$=$</td><td>$\frac{1}{2} f_c k j b d^2$</td><td>$\frac{1}{2} f_c (2-q) j b t^2$</td><td>$\frac{1}{2} f_c k j b d^2$</td><td>(b)</td></tr>
<tr><td>M</td><td>$=$</td><td>$f_s p j b d^2 \cos^2\alpha$</td><td>$f_s p j b d t \cos^2\alpha$</td><td>$f_s p j b d^2 \cos^2\alpha$</td><td>(c)</td></tr>
<tr><td>j</td><td>$=$</td><td>$1-\frac{k}{3}$</td><td>$\frac{d}{t}-\frac{3-2q}{6-3q}$</td><td>$\frac{k}{3}-\frac{d'}{d}$</td><td>(d)</td></tr>
<tr><td>a</td><td>$=$</td><td>$\frac{1-k}{k}$</td><td>$-\left(1-q\frac{d}{t}\right)$</td><td>$-\left(\frac{k-\frac{d'}{d}}{k}\right)$</td><td>(e)</td></tr>
<tr><td>c</td><td>$=$</td><td>$\frac{1-k}{k^2}$</td><td>$-\left(\frac{d}{t}\right)\left(\frac{1-q\frac{d}{t}}{2-q}\right)$</td><td>$-\left(\frac{k-\frac{d'}{d}}{k^2}\right)$</td><td>(f)</td></tr>
<tr><td>$\frac{1}{2pn}$</td><td>$=$</td><td>$\frac{f_s \cos^2\alpha}{n f_c k}$</td><td>$\left(\frac{d}{t}\right)\frac{f_s \cos^2\alpha}{n f_c (2-q)}$</td><td>$\frac{f_s \cos^2\alpha}{n f_c k}$</td><td>(g)</td></tr>
<tr><td>$f_s p b d$</td><td>$=$</td><td>$\frac{M}{j d \cos^2\alpha}$</td><td>$\frac{M}{j t \cos^2\alpha}$</td><td>$\frac{M}{j d \cos^2\alpha}$</td><td>(h)</td></tr>
<tr><td></td><td></td><td>$k=3(1-j)$</td><td>$q=\frac{3+6j-6\frac{d}{t}}{2+3j-3\frac{d}{t}}$</td><td>$k=3\left(j+\frac{d'}{d}\right)$</td><td>(i)</td></tr>
<tr><td>$f_s p \sin\alpha\cos\alpha$</td><td>$=$</td><td colspan="3">$\frac{1}{2} h (2-q_s) \frac{t}{d}$</td><td>(a')</td></tr>
<tr><td>M_s</td><td>$=$</td><td colspan="3">$\frac{1}{2} h (2-q_s) b j_s t^2$</td><td>(b')</td></tr>
<tr><td>M_s</td><td>$=$</td><td colspan="3">$f_s p j_s b d t \sin\alpha\cos\alpha$</td><td>(c')</td></tr>
<tr><td>j_s*</td><td>$=$</td><td colspan="3">$\frac{d}{t}-\frac{3-2q_s}{6-3q_s}$</td><td>(d')</td></tr>
<tr><td>a_s</td><td>$=$</td><td colspan="3">$-\left(1-q_s\frac{d}{t}\right)$</td><td>(e')</td></tr>
<tr><td>c_s</td><td>$=$</td><td colspan="3">$-\left(\frac{d}{t}\right)\left(\frac{1-q_s\frac{d}{t}}{2-q_s}\right)$</td><td>(f')</td></tr>
<tr><td>$\frac{1}{2pn}$</td><td>$=$</td><td colspan="3">$\left(\frac{d}{t}\right)\frac{f_s \sin\alpha\cos\alpha}{n h (2-q_s)}$</td><td>(g')</td></tr>
<tr><td>$f_s p b d$</td><td>$=$</td><td colspan="3">$\frac{M_s}{j_s t \sin\alpha\cos\alpha}$</td><td>(h')</td></tr>
<tr><td>q_s*</td><td>$=$</td><td colspan="3">$\frac{3+6j_s-6\frac{d}{t}}{2+3j_s-3\frac{d}{t}}$</td><td>(i')</td></tr>
<tr><td>$\frac{f_s}{n}$</td><td>$=$</td><td colspan="3">$\sum f_c a \cos^2\alpha + \lambda h a_s \sin\alpha\cos\alpha$</td><td>(j)</td></tr>
<tr><td>$\frac{1}{2pn}$</td><td>$=$</td><td colspan="3">$\sum c \cos^4\alpha + \lambda c_s \sin^2\alpha\cos^2\alpha$</td><td>(k)</td></tr>
</table>

---

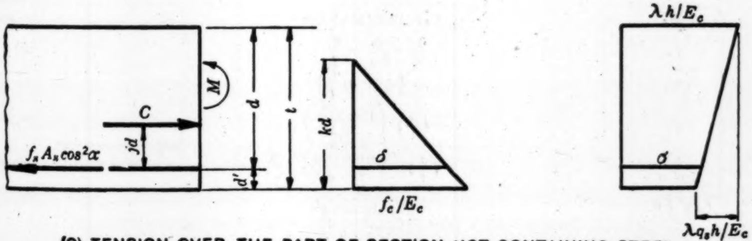
* If in any of the equations, $q_s = 2$, and $j_s = \infty$, that equation becomes indeterminate. By differentiation, it can be reduced to a determinate value: $\pm\frac{6 M_s}{b t^2} = h$, which means that the resultant is a couple.


Elevation
Bending Deformation
Shear Deformation
C
jd
M
d
t
$f_sA_s\cos^2\alpha$
d'
f_c/E_c
kd
(1 k) d
δ
$\lambda h/E_c$
σ
$\lambda q_s h/E_c$

(A) TENSION OVER THE PART OF SECTION CONTAINING STEEL


C
jd
M
d
t
$f_sA_s\cos^2\alpha$
d'
f_c/E_c
δ
qf_c/E_c
$\lambda h/E_c$
σ
$\lambda q_s h/E_c$

(B) COMPRESSION OVER ENTIRE SECTION

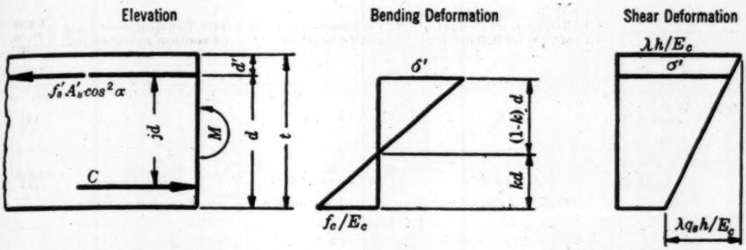

$\lambda h/E_c$
M
d
t
C
jd
kd
$f_sA_s\cos^2\alpha$
d'
δ
f_c/E_c
σ
$\lambda q_s h/E_c$

(C) TENSION OVER THE PART OF SECTION NOT CONTAINING STEEL

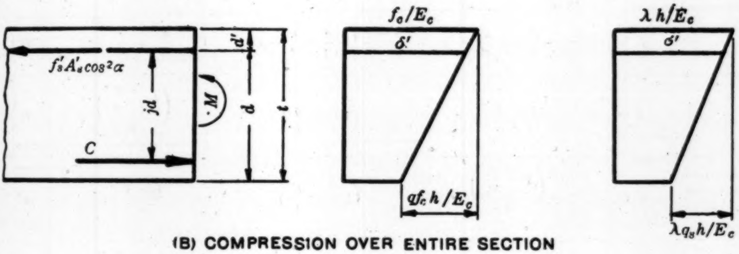
FIG. 30.—SIMPLE BENDING; STEEL IN INTRADOS FACE ONLY. (SEE CASE I FOR STEEL IN ONE DIRECTION AND CASE VII FOR STEEL IN TWO DIRECTIONS.)

CASE II.—Simple Bending; Steel in One Direction in Extrados Face Only (See Fig. 31).

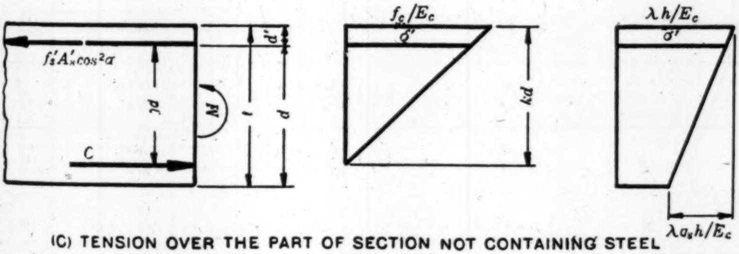
		(*A*) Tension over part of section containing steel.	(*B*) Compression over entire section.	(*C*) Tension over part of section not containing steel.	Equations (64).
$f'_s p' \cos^2 \alpha$	$=$	$\frac{1}{2} f_c k$	$\frac{1}{2} f_c (2-q) \frac{t}{d}$	$\frac{1}{2} f_c k$	(*a*)
M	$=$	$-\frac{1}{2} f_c k j' b d^2$	$-\frac{1}{2} f_c (2-q) j' b t^2$	$-\frac{1}{2} f_c k j' b d^2$	(*b*)
M	$=$	$-f'_s p' j' b d^2 \cos^2 \alpha$	$-f'_s p' j' b d t \cos^2 \alpha$	$-f'_s p' j' b d^2 \cos^2 \alpha$	(*c*)
j'	$=$	$\left(1-\frac{k}{3}\right)$	$\frac{3-2q}{6-3q}-\frac{d'}{t}$	$\frac{k}{3}-\frac{d'}{d}$	(*d*)
a'	$=$	$\frac{1-k}{k}$	$-\left(1-q\frac{d'}{t}\right)$	$-\left(\frac{k-\frac{d'}{d}}{k}\right)$	(*e*)
c'	$=$	$\frac{1-k}{k^2}$	$-\left(\frac{d}{t}\right)\left(\frac{1-q\frac{d'}{t}}{2-q}\right)$	$-\left(\frac{k-\frac{d'}{d}}{k^2}\right)$	(*f*)
$\frac{1}{2p'n}$	$=$	$\frac{f'_s \cos^2 \alpha}{n f_c k}$	$\left(\frac{d}{t}\right)\frac{f'_s \cos^2 \alpha}{n f_c (2-q)}$	$\frac{f'_s \cos^2 \alpha}{n f_c k}$	(*g*)
$f'_s p' b d$	$=$	$-\frac{M}{j' d \cos^2 \alpha}$	$-\frac{M}{j' t \cos^2 \alpha}$	$-\frac{M}{j' d \cos^2 \alpha}$	(*h*)
		$k=3(1-j')$	$q=\frac{3-6j'-6\frac{d'}{t}}{2-3j'-3\frac{d}{t}}$	$k=3\left(j'+\frac{d'}{d}\right)$	(*i*)
$f'_s p' \sin \alpha \cos \alpha$	$=$	$\frac{1}{2} h (2-q_s) \frac{t}{d}$			(*a'*)
M_s	$=$	$-\frac{1}{2} h (2-q_s) b j'_s t^2$			(*b'*)
M_s	$=$	$-f'_s p' j b d t \sin \alpha \cos \alpha$			(*c'*)
j'_s	$=$	$\frac{3-2q_s}{6-3q}-\frac{d'}{t}$			(*d'*)
a'_s	$=$	$-\left(1-q_s\frac{d'}{t}\right)$			(*e'*)
c'_s	$=$	$-\left(\frac{d}{t}\right)\left(\frac{1-q\frac{d'}{t}}{2-q_s}\right)$			(*f'*)
$\frac{1}{2p'n}$	$=$	$\left(\frac{d}{t}\right)\frac{f'_s \sin \alpha \cos \alpha}{h(2-q_s)}$			(*g'*)
$f'_s p' b d$	$=$	$-\frac{M_s}{j' t \sin \alpha \cos \alpha}$			(*h'*)
q_s	$=$	$\frac{3-6j'_s-6\frac{d'}{t}}{2-3j'_s-3\frac{d'}{t}}$			(*i'*)
$\frac{f'_s}{n}$	$=$	$\sum f_c a' \cos^2 \alpha + \lambda h a'_s \sin \alpha \cos \alpha$			(*j*)
$\frac{1}{2p'n}$	$=$	$\sum c' \cos^4 \alpha + \lambda c'_s \sin^2 \alpha \cos^2 \alpha$			(*k*)


Elevation
Bending Deformation
Shear Deformation
$f'_s A'_s \cos^2\alpha$
C
M
jd
d'
d
t
δ'
$(1-k)d$
kd
f_c/E_c
$\lambda h/E_c$
σ'
$\lambda q_s h/E_c$

(A) TENSION OVER THE PART OF SECTION CONTAINING STEEL


$f'_s A'_s \cos^2\alpha$
C
M
jd
d'
d
t
f_c/E_c
δ'
$qf_c h/E_c$
$\lambda h/E_c$
σ'
$\lambda q_s h/E_c$

(B) COMPRESSION OVER ENTIRE SECTION

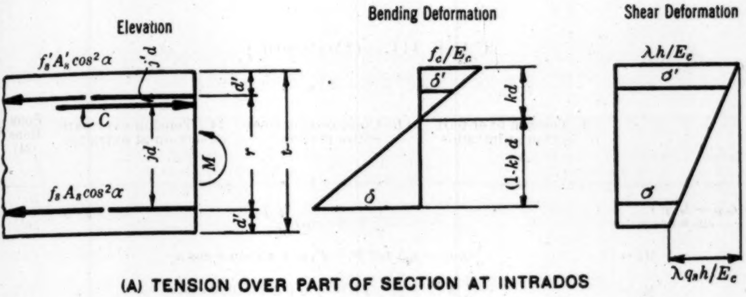

$f'_s A'_s \cos^2\alpha$
C
M
jd
d'
t
d
f_c/E_c
δ'
kd
$\lambda h/E_c$
σ'
$\lambda q_s h/E_c$

(C) TENSION OVER THE PART OF SECTION NOT CONTAINING STEEL

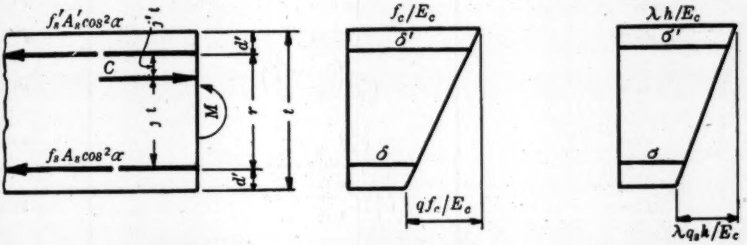
FIG. 31.—SIMPLE BENDING; STEEL IN EXTRADOS FACE ONLY. (SEE CASE II FOR STEEL IN ONE DIRECTION AND CASE VIII FOR STEEL IN TWO DIRECTIONS.)

CASE III.—SIMPLE BENDING; STEEL IN ONE DIRECTION ONLY BUT IN BOTH FACES (SEE FIG. 32).

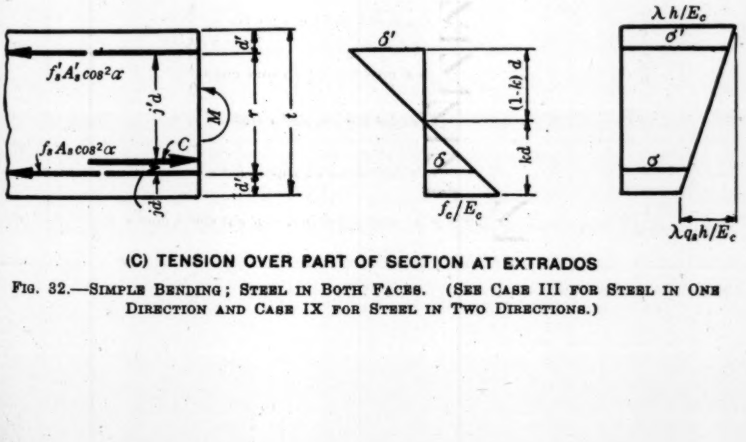
		(*A*) Tension over part of section at intrados	(*B*) Compression over entire section	(*C*) Tension over part of section at extrados	Equations (64)
	$(f_s p + f_s' p') \cos^2 a =$	$\frac{1}{2} f_c k$	$\frac{1}{2} f_c (2 - q) \frac{t}{d}$	$\frac{1}{2} f_c k$	(*a*)
	$M =$	$\frac{1}{2} f_c k j b d^2$ $- f'_s p' b d r \cos^2 a$	$\frac{1}{2} f_c (2 - q) j b t^2$ $- f'_s p' b d r \cos^2 a$	$\frac{1}{2} f_c k j b d^2$ $- f'_s p' b d r \cos^2 a$	(*b*)
	$M =$	$- \frac{1}{2} f_c k j' b d^2$ $+ f_s p b d r \cos^2 a$	$- \frac{1}{2} f_c (2 - q) j' b t^2$ $+ f_s p b d r \cos^2 a$	$- \frac{1}{2} f_c k j' b d^2$ $+ f_s p b d r \cos^2 a$	(*c*)
	$j =$	$1 - \frac{k}{3} = \frac{r}{d} - j'$	$\frac{d}{t} - \frac{3-2q}{6-3q} = \frac{r}{t} - j'$	$\frac{k}{3} - \frac{d'}{d} = \left(\frac{r}{d} - j'\right)$	(*d*)
	$j' =$	$\frac{k}{3} - \frac{d'}{d} = \frac{r}{d} - j$	$\frac{3-2q}{6-3q} - \frac{d'}{t} = \frac{r}{t} - j$	$1 - \frac{k}{3} = \frac{r}{d} - j$	(*e*)
	$a =$	$\frac{1-k}{k}$	$-\left(1 - q\frac{d}{t}\right)$	$-\left(\frac{k - \frac{d'}{d}}{k}\right)$	(*f*)
	$a' =$	$-\left(\frac{k - \frac{d'}{d}}{k}\right)$	$-\left(1 - q\frac{d'}{t}\right)$	$\frac{1-k}{k}$	(*g*)
	$c =$	$\frac{1-k}{k^2}$	$-\left(\frac{d}{t}\right)\left(\frac{1 - q\frac{d}{t}}{2-q}\right)$	$-\left(\frac{k - \frac{d'}{d}}{k^2}\right)$	(*h*)
	$c' =$	$-\left(\frac{k - \frac{d'}{d}}{k^2}\right)$	$-\left(\frac{d}{t}\right)\left(\frac{1 - q\frac{d'}{t}}{2-q}\right)$	$\frac{1-k}{k^2}$	(*i*)
	$g =$	$\frac{f_s \cos^2 a}{n f_c k}$	$\left(\frac{d}{t}\right) \frac{f_s \cos^2 a}{n f_c (2-q)}$	$\frac{f_s \cos^2 a}{n f_c k}$	(*j*)
	$g' =$	$\frac{f'_s \cos^2 a}{n f_c k}$	$\left(\frac{d}{t}\right) \frac{f'_s \cos^2 a}{n f_c (2-q)}$	$\frac{f'_s \cos^2 a}{n f_c k}$	(*k*)
For partial moments	$M_c =$	$\frac{1}{2} f_c k j b d^2$		$- \frac{1}{2} f_c k j' b d^2$	(*l*)
	$M - M_c =$	$f_s p_2 b d r \cos^2 a$		$- f'_s p'_2 b r \cos^2 a$	(*m*)
	$M - M_c =$	$- f'_s p' b d r \cos^2 a$		$f_s p b d r \cos^2 a$	(*n*)


Elevation
Bending Deformation
Shear Deformation
$f_s'A_s'\cos^2\alpha$
$j'd$
C
jd
M
$f_sA_s\cos^2\alpha$
d'
r
t
d'
f_c/E_c
δ'
kd
$(1-k)\,d$
δ
$\lambda h/E_c$
σ'
σ
$\lambda q_s h/E_c$

(A) TENSION OVER PART OF SECTION AT INTRADOS


$f_s'A_s'\cos^2\alpha$
j'
C
j
t
M
$f_sA_s\cos^2\alpha$
d'
r
t
d'
f_c/E_c
δ'
δ
qf_c/E_c
$\lambda h/E_c$
σ'
σ
$\lambda q_s h/E_c$

(B) COMPRESSION OVER ENTIRE SECTION


$f_s'A_s'\cos^2\alpha$
$j'd$
M
$f_sA_s\cos^2\alpha$
C
jd
d'
r
t
d'
δ'
$(1-k)\,d$
kd
δ
f_c/E_c
$\lambda h/E_c$
σ'
σ
$\lambda q_s h/E_c$

(C) TENSION OVER PART OF SECTION AT EXTRADOS

FIG. 32.—SIMPLE BENDING; STEEL IN BOTH FACES. (SEE CASE III FOR STEEL IN ONE DIRECTION AND CASE IX FOR STEEL IN TWO DIRECTIONS.)

CASE III.—(*Continued.*)

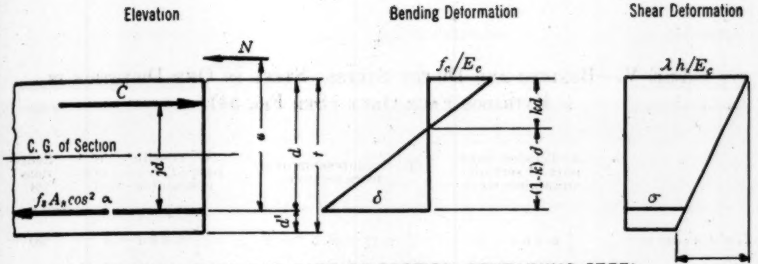
	(*A*) Tension over part of section at intrados	(*B*) Compression over entire section	(*C*) Tension over part of section at extrados	Equations (64)
$(f_s\, p + f_s'\, p') \sin a \cos a =$		$\frac{1}{2}\, h\, (2 - q_s)\, \frac{t}{d}$		(*a'*)
$M_s =$		$\frac{1}{2}\, h\, (2 - q_s)\, j_s\, b\, t^2 - f'_s\, p'\, b\, d\, r \sin a \cos a$		(*b'*)
$M_s =$		$-\frac{1}{2}\, h\, (2 - q_s)\, j'_s\, b\, t^2 + f_s\, p\, b\, d\, r \sin a \cos a$		(*c'*)
$j_s =$		$\frac{d}{t} - \frac{3 - 2\, q}{6 - 3\, q} = \frac{r}{t} - j'$		(*d'*)
$j'_s =$		$\frac{3 - 2\, q}{6 - 3\, q} - \frac{d'}{t} = \frac{r}{t} - j$		(*e'*)
$a_s =$		$-\left(1 - q_s \frac{d}{t}\right)$		(*f'*)
$a'_s =$		$-\left(1 - q_s \frac{d'}{t}\right)$		(*g'*)
$c_s =$		$-\left(\frac{d}{t}\right)\left(\frac{1 - q_s \frac{d}{t}}{2 - q_s}\right)$		(*h'*)
$c'_s =$		$-\left(\frac{d}{t}\right)\frac{1 - q_s \frac{d'}{t}}{2 - q_s}$		(*i'*)
$g_s =$		$\left(\frac{d}{t}\right)\frac{f_s \sin a \cos a}{n\, (2 - q_s)}$		(*j'*)
$g'_s =$		$\left(\frac{d}{t}\right)\frac{f'_s \sin a \cos a}{n\, (2 - q_s)}$		(*k'*)
For partial moments: $M_{sc} =$		$\frac{1}{2}\, h\, (2 - q_s)\, j_s\, t^2$ or $\frac{1}{2}\, h\, (2 - q_s)\, j'_s\, t^2$		(*l'*)
$M - M_{sc} =$		$f_s\, p_2\, b\, d\, r \sin a \cos a$ or $f'_s\, p'_2\, b\, d\, r \sin a \cos a$		(*m'*)
$M - M_{sc} =$		$f'_s\, p'\, b\, d\, r \sin a \cos a$ or $f_s\, p\, b\, d\, r \sin a \cos a$		(*n'*)
$\frac{f_s}{n} =$		$\sum f_c\, a \cos^2 a + \lambda\, h\, a_s \sin a \cos a$		(*p*)
$\frac{f'_s}{n} =$		$\sum f_c\, a' \cos^2 a + \lambda\, h\, a'_s \sin a \cos a$		
$g =$		$\sum c \cos^4 a + \lambda\, c_s \sin^2 a \cos^2 a$		(*q*)
$g' =$		$\sum c' \cos^4 a + \lambda\, c'_s \sin^2 a \cos^2 a$		
$\frac{1}{2\, n} =$		$\sum (p\, c + p'\, c') \cos^4 a + \lambda\, (p\, c_s + p'\, c'_s) \sin^2 a \cos^2 a$		(*r*)

CASE IV.—BENDING AND DIRECT STRESS; STEEL IN ONE DIRECTION IN INTRADOS FACE ONLY (SEE FIG. 33)

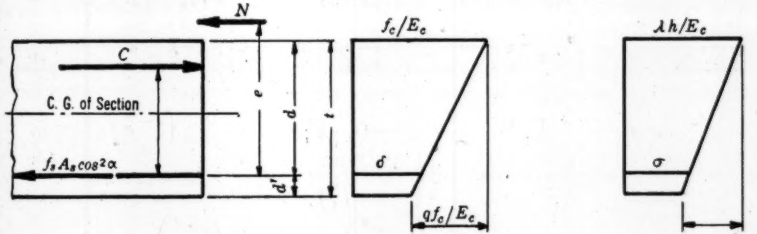
	(*A*) Tension over part of section containing steel	(*B*) Compression over entire section	(*C*) Tension over part of section not containing steel	Equations (64)
$f_s p b d \cos^2 a =$	$\frac{1}{2} f_c k b d - N$	$\frac{1}{2} f_c (2-q) b t - N$	$\frac{1}{2} f_c k b d - N$	(*a*)
$N e =$	$\frac{1}{2} f_c k j b d^2$	$\frac{1}{2} f_c (2-q) j b t^2$	$\frac{1}{2} f_c k j b d^2$	(*b*)
	$N(e - j d)$ $= f_s p j b d^2 \cos^2 a$	$N(e - j t)$ $= f_s p j b d^2 \cos^2 a$	$N(e - j d)$ $= f_s p j b d^2 \cos^2 a$	(*c*)
$j =$	$1 - \frac{k}{3}$	$\frac{d}{t} - \frac{3-2q}{6-3q}$	$\frac{k}{3} - \frac{d'}{d}$	(*d*)
$a =$	$\frac{1-k}{k}$	$-\left(1 - q\frac{d}{t}\right)$	$-\left(\frac{k - \frac{d'}{d}}{k}\right)$	(*e*)
$c =$	$\left(\frac{1-k}{k^2}\right)\left(\frac{1}{1 - j\frac{d}{e}}\right)$	$-\left(\frac{d}{t}\right)\left(\frac{1 - q\frac{d}{t}}{2-q}\right)$ $\left(\frac{1}{1 - j\frac{t}{e}}\right)$	$-\left(\frac{k - \frac{d'}{d}}{k^2}\right)\left(\frac{1}{1 - j\frac{d}{e}}\right)$	(*f*)
$\frac{1}{p n} =$	$\frac{f_s \cos^2 a}{n f_c k \left(1 - j\frac{d}{e}\right)}$	$\frac{f_s \cos^2 a \left(\frac{d}{t}\right)}{n f_c (2-q)\left(1 - j\frac{t}{e}\right)}$	$\frac{f_s \cos^2 a}{n f_c k \left(1 - j\frac{d}{e}\right)}$	(*g*)
$f_s p b d =$	$\frac{N\left(\frac{e}{jd} - 1\right)}{\cos^2 a}$	$\frac{N\left(\frac{e}{jt} - 1\right)}{\cos^2 a}$	$\frac{N\left(\frac{e}{jd} - 1\right)}{\cos^2 a}$	(*h*)
	$k = 3(1 - j)$	$q = \frac{3 + 6j - 6\frac{d}{t}}{2 + 3j - 3\frac{d}{t}}$	$k = 3\left(j + \frac{d'}{d}\right)$	(*i*)

CASE IV.—(*Continued.*)

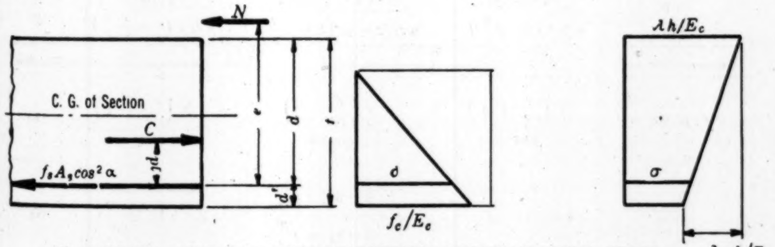
	(*A*) Tension over part of section containing steel	(*B*) Compression over entire section	(*C*) Tension over part of section not containing steel	Equations (64)
$f_s\, p\, b\, d \sin a \cos a =$		$\frac{1}{2} h\,(2-q_s)\, b\, t - S$		(*a′*)
$S\, e_s =$		$\frac{1}{2} h\,(2-q_s)\, j_s\, t^2$		(*b′*)
$S\,(e_s - j\,t) =$		$f_s\, p\, j_s\, b\, d\, t \sin a \cos a$		(*c′*)
$j_s =$		$\frac{d}{t} - \frac{3-2\,q_s}{6-3\,q_s}$		(*d′*)
$a_s =$		$-\left(1 - q_s \frac{d}{t}\right)$		(*e′*)
$c_s =$		$-\left(\frac{d}{t}\right)\left(\frac{1-q_s\frac{d}{t}}{2-q_s}\right)\left(\frac{1}{1-j_s\frac{t}{e_s}}\right)$		(*f′*)
$\frac{1}{2\,p\,n} =$		$\left(\frac{d}{t}\right)\frac{f_s \sin a \cos a}{n\,(2-q_s)\left(1-j_s\frac{t}{e_s}\right)}$		(*g′*)
$f_s\, p\, b\, d =$		$\frac{S\left(\frac{e_s}{j_s t}-1\right)}{\sin a \cos a}$		(*h′*)
$q_s =$		$\frac{3+6\,j_s-6\,\frac{d}{t}}{2+3\,j_s-3\,\frac{d}{t}}$		(*i′*)
$\frac{f_s}{n} =$		$\sum f_o\, a \cos^2 a + \lambda\, h\, a_s \sin a \cos a$		(*j*)
$\frac{1}{2\,p\,n} =$		$\sum c \cos^4 a + \lambda\, c_s \sin^2 a \cos^2 a$		(*k*)


Elevation
Bending Deformation
Shear Deformation
N
C
C. G. of Section
jd
$f_s A_s \cos^2 \alpha$
e
d
t
d'
f_c/E_c
kd
(1-k) d
δ
$\lambda h/E_c$
σ
$\lambda q_s h/E_c$

(A) TENSION OVER THE PART OF SECTION CONTAINING STEEL


N
C
C. G. of Section
$f_s A_s \cos^2 \alpha$
e
d
t
d'
f_c/E_c
δ
qf_c/E_c
$\lambda h/E_c$
σ
$\lambda q_s h/E_c$

(B) COMPRESSION OVER ENTIRE SECTION

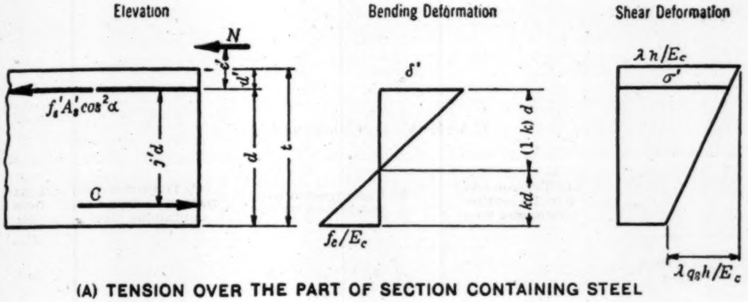

N
C. G. of Section
C
jd
$f_s A_s \cos^2 \alpha$
e
d
t
d'
δ
f_c/E_c
$\lambda h/E_c$
σ
$\lambda q_s h/E_c$

(C) TENSION OVER THE PART OF SECTION NOT CONTAINING STEEL

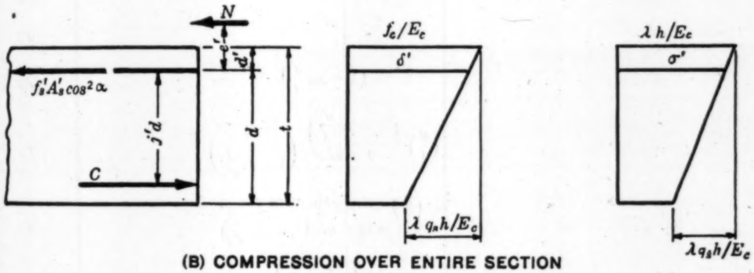
FIG. 33.—BENDING AND DIRECT STRESS; STEEL IN ONE DIRECTION IN INTRADOS FACE ONLY. (SEE CASE IV FOR STEEL IN ONE DIRECTION AND CASE X FOR STEEL IN TWO DIRECTIONS.)

CASE V.—BENDING AND DIRECT STRESS; STEEL IN ONE DIRECTION IN EXTRADOS FACE ONLY (SEE FIG. 34)

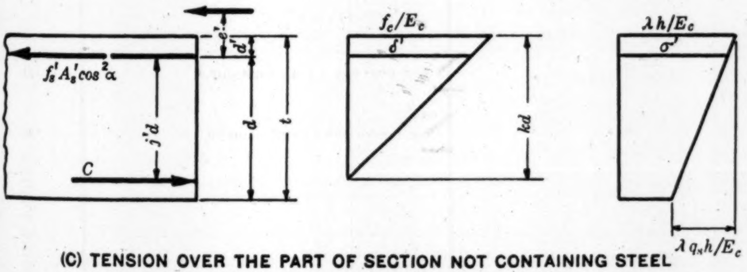
	(*A*) Tension over part of section containing steel	(*B*) Compression over entire section	(*C*) Tension over part of section not containing steel	Equations (64)
$f'_s p' b d \cos^2 a =$	$\frac{1}{2} f_c k b d - N$	$\frac{1}{2} f_c (2 - q) b t - N$	$\frac{1}{2} f_c k b d - N$	(*a*)
$N e' =$	$-\frac{1}{2} f_c k j' b d^2$	$-\frac{1}{2} f_c (2 - q) j' b t^2$	$-\frac{1}{2} f_c k j' b d^2$	(*b*)
	$N (e' + j' d) =$ $- f'_s p' b d^2 \cos^2 a$	$N (e' + j' t) =$ $- f'_s p' b d t \cos^2 a$	$N (e' + j d) =$ $- f'_s p' j' b d^2 \cos^2 a$	(*c*)
$j' =$	$1 - \frac{k}{3}$	$\frac{3 - 2q}{6 - 3q} - \frac{d'}{t}$	$\frac{k}{3} - \frac{d'}{d}$	(*d*)
$a' =$	$\frac{1 - k}{k}$	$-\left(1 - q \frac{d'}{t}\right)$	$-\left(\frac{k - \frac{d'}{d}}{k}\right)$	(*e*)
$c' =$	$\left(\frac{1 - k}{k^2}\right)\left(\frac{1}{1 + j' \frac{d}{e'}}\right)$	$-\left(\frac{d}{t}\right) \times$ $\left(\frac{1 - q \frac{d'}{t}}{2 - q}\right)\left(\frac{1}{1 + j' \frac{t}{e'}}\right)$	$-\left(\frac{k - \frac{d'}{d}}{k^2}\right)\left(\frac{1}{1 + j' \frac{d}{e'}}\right)$	(*f*)
$\frac{1}{2 p' n} =$	$\frac{f'_s \cos^2 a}{n f_c k \left(1 + j' \frac{d}{e'}\right)}$	$\frac{-\left(\frac{d}{t}\right) f'_s \cos^2 a}{n f_c (2 - q) \left(1 + j' \frac{t}{e'}\right)}$	$\frac{f'_s \cos^2 a}{n f_c k \left(1 + j' \frac{d}{e'}\right)}$	(*g*)
$f'_s p' b d =$	$-\frac{N \left(\frac{e'}{j'd} + 1\right)}{\cos^2 a}$	$-\frac{N \left(\frac{e'}{j't} + 1\right)}{\cos^2 a}$	$-\frac{N \left(\frac{e'}{j'd} + 1\right)}{\cos^2 a}$	(*h*)
	$k = 3 (1 - j')$	$q = \frac{3 - 6 j' - 6 \frac{d'}{t}}{2 - 3 j' - 3 \frac{d'}{t}}$	$k = 3 \left(j' + \frac{d'}{d}\right)$	(*i*)


Elevation
Bending Deformation
Shear Deformation
N
e'
d'
$f_s'A_s'\cos^2\alpha$
j'd
d
t
C
δ'
(1-k) d
kd
f_c/E_c
$\lambda h/E_c$
σ'
$\lambda q_s h/E_c$

(A) TENSION OVER THE PART OF SECTION CONTAINING STEEL


N
e'
d'
$f_s'A_s'\cos^2\alpha$
j'd
d
t
C
f_c/E_c
δ'
$\lambda q_s h/E_c$
$\lambda h/E_c$
σ'
$\lambda q_s h/E_c$

(B) COMPRESSION OVER ENTIRE SECTION


e'
d'
$f_s'A_s'\cos^2\alpha$
j'd
d
t
C
f_c/E_c
δ'
kd
$\lambda h/E_c$
σ'
$\lambda q_s h/E_c$

(C) TENSION OVER THE PART OF SECTION NOT CONTAINING STEEL

FIG. 34.—BENDING AND DIRECT STRESS; STEEL IN EXTRADOS FACE ONLY. (SEE CASE V FOR STEEL IN ONE DIRECTION AND CASE XI FOR STEEL IN TWO DIRECTIONS.)

CASE V.—*(Continued.)*

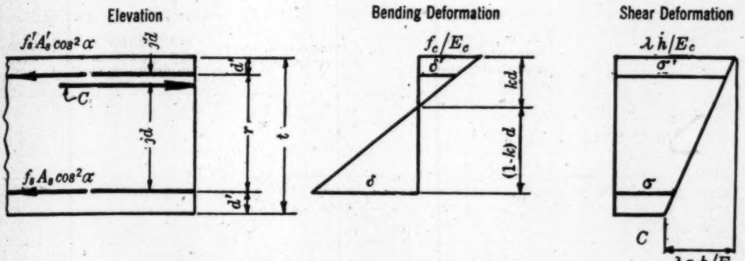
	(*A*) Tension over part of section containing steel	(*B*) Compression over entire section	(*C*) Tension over part of section not containing steel	Equations (64)
$\frac{f'_s p' d \sin\alpha}{\cos\alpha} =$	$\frac{1}{2} h(2-q_s)t - S$			(*a'*)
$S e'_s =$	$-\frac{1}{2} h(2-q_s) j'_s b t^2$			(*b'*)
$S(e'_s + j'_s t) =$	$-f'_s p' j'_s b d t \sin\alpha\cos\alpha$			(*c'*)
$j'_s =$	$\frac{3-2q_s}{6-3q_s} - \frac{d'}{t}$			(*d'*)
$a'_s =$	$-\left(1 - q_s\frac{d'}{t}\right)$			(*e'*)
$c'_s =$	$-\left(\frac{d}{t}\right)\left(\frac{1-q_s\frac{d'}{t}}{2-q_s}\right)\left(\frac{1}{1+j'\frac{t}{e'_s}}\right)$			(*f'*)
$\frac{1}{2p'n} =$	$-\left(\frac{d}{t}\right)\frac{f'_s \sin\alpha\cos\alpha}{h(2-q_s)\left(1+j'_s\frac{t}{e'_s}\right)}$			(*g'*)
$f'_s p' b d =$	$-\frac{S\left(\frac{e_s}{j'_s t}+1\right)}{\sin\alpha\cos\alpha}$			(*h'*)
$q_s =$	$\frac{3-6j'_s-6\frac{d'}{t}}{2-3j'_s-3\frac{d'}{t}}$			(*i'*)
$\frac{f'_s}{n} =$	$\sum f_c a' \cos^2\alpha + \lambda h a'_s \sin\alpha\cos\alpha$			(*j*)
$\frac{1}{2p'n} =$	$\sum c' \cos^2\alpha + \lambda c'_s \sin^2\alpha\cos^2\alpha$			(*k*)

CASE VI.—BENDING AND DIRECT STRESS; STEEL IN ONE DIRECTION ONLY BUT IN BOTH FACES (SEE FIG. 35)

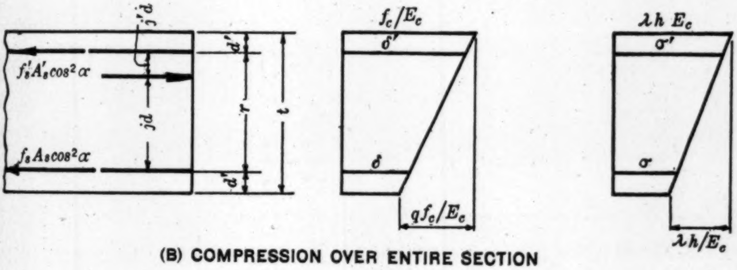
		(*A*) Tension over part of section at intrados	(*B*) Compression over entire section	(*C*) Tension over part of section at extrados	Equations (64)
	$(f_s p + f'_s p')\, b\, d \cos^2 a =$	$\frac{1}{2} f_c k b d - N$	$\frac{1}{2} f_c (2 - q) b t - N$	$\frac{1}{2} f_c k b d - N$	(*a*)
	$N e =$	$\frac{1}{2} f_c k j b d^2$ $- f'_s p' b d r \cos^2 a$	$\frac{1}{2} f_c (2 - q) j b t^2$ $- f'_s p' b d r \cos^2 a$	$\frac{1}{2} f_c k j b d^2$ $- f'_s p' b d r \cos^2 a$	(*b*)
	$N e' =$	$- \frac{1}{2} f_c k j' b d^2$ $+ f_s p b d r \cos^2 a$	$- \frac{1}{2} f_c (2 - q) j' b t^2$ $+ f_s p b d r \cos^2 a$	$- \frac{1}{2} f_c k j' b d^2$ $+ f_s p b d r \cos^2 a$	(*c*)
	$j =$	$1 - \frac{k}{3} = \frac{r}{d} - j'$	$\frac{d}{t} - \frac{3 - 2q}{6 - 3q} = \frac{r}{t} - j'$	$\frac{k}{3} - \frac{d'}{d} = \left(\frac{r}{d} - j'\right)$	(*d*)
	$j' =$	$\frac{k}{3} - \frac{d'}{d} = \frac{r}{d} - j$	$\frac{3 - 2q}{6 - 3q} - \frac{d'}{t} = \frac{r}{t} - j$	$1 - \frac{k}{3} = \frac{r}{d} - j$	(*e*)
	$a =$	$\frac{1 - k}{k.}$	$-\left(1 - q \frac{d}{t}\right)$	$-\left(\frac{k - \frac{d'}{d}}{k}\right)$	(*f*)
	$a' =$	$-\left(\frac{k - \frac{d'}{d}}{k}\right)$	$-\left(1 - q \frac{d'}{t}\right)$	$\frac{1 - k}{k}$	(*g*)
	$c =$	$\left(\frac{1 - k}{k^2}\right) \left(\frac{1}{1 - j \frac{d}{e}}\right)$	$-\left(\frac{1 - q \frac{d}{t}}{2 - q}\right) \left(\frac{1}{1 - j \frac{t}{e}}\right) \frac{d}{t}$	$-\left(\frac{k - \frac{d'}{d}}{k^2}\right) \left(\frac{1}{1 - j \frac{d}{e}}\right)$	(*h*)
	$c' =$	$-\left(\frac{k - \frac{d'}{d}}{k^2}\right) \left(\frac{1}{1 + j' \frac{d}{e'}}\right)$	$-\left(\frac{1 - q \frac{d'}{t}}{2 - q}\right) \left(\frac{1}{1 + j' \frac{t}{e'}}\right) \frac{d}{t}$	$\left(\frac{1 - k}{k^2}\right) \left(\frac{1}{1 + j' \frac{d}{e'}}\right)$	(*i*)
	$g =$	$\frac{f_s \cos^2 a}{n f_c k \left(1 - j \frac{d}{e}\right)}$	$\frac{d}{t} \frac{f_s \cos^2 a}{n f_c (2 - q) \left(1 - j \frac{t}{e}\right)}$	$\frac{f_s \cos^2 a}{n f_c k \left(1 - j \frac{d}{e}\right)}$	(*j*)
	$g' =$	$\frac{f'_s \cos^2 a}{n f_c k \left(1 + j' \frac{d}{e'}\right)}$	$\frac{d}{t} \frac{f'_s \cos^2 a}{n f_c (2 - q) \left(1 + j' \frac{t}{e'}\right)}$	$\frac{f'_s \cos^2 a}{n f_c k \left(1 + j' \frac{d}{e'}\right)}$	(*k*)
For partial moments	$N_c e =$ or	$\frac{1}{2} f_c k j b d^2$	$\frac{1}{2} f_c (2 - q) j b t^2$		(*l*)
	$N_c e' =$		$- \frac{1}{2} f_c (2 - q) j' b t^2$	$- \frac{1}{2} f_c k j' b d^2$	(*m*)
	$(N - N_c) e =$	$- f'_s p' b d r \cos^2 a$	$- f'_s p' b d r \cos^2 a$ or $- f'_s p'_2 b d r \cos^2 a$	$- f'_s p'_2 b d r \cos^2 a$	(*n*)
	$(N - N_c) e' =$	$f_s p_2 b d r \cos^2 a$	$f_s p_2 b d r \cos^2 a$ or $f_s p b d r \cos^2 a$	$f_s p b d r \cos^2 a$	(*p*)

CASE VI.—(*Continued.*)

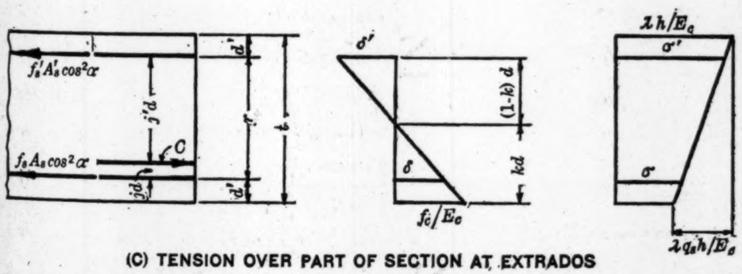
		(*A*) Tension over part of section at intrados	(*B*) Compression over entire section	(*C*) Tension over part of section at extrados	Equations (64)
	$(f_s\,p + f'_s\,p')\,b\,d\,\sin\alpha\cos\alpha =$		$\frac{1}{2}\,h\,(2 - q_s)\,b\,t - S$		(*a'*)
	$S\,e_s =$		$\frac{1}{2}\,h\,(2 - q_s)\,j_s\,b\,t^2 - f'_s\,p'\,b\,d\,r\sin\alpha\cos\alpha$		(*b'*)
	$S\,e'_s =$		$-\frac{1}{2}\,h\,(2 - q_s)\,j'_s\,b\,t^2 + f_s\,p\,b\,d\,r\sin\alpha\cos\alpha$		(*c'*)
	$j_s =$		$\frac{d}{t} - \frac{3 - 2\,q_s}{6 - 3\,q_s}$		(*d'*)
	$j'_s =$		$\frac{3 - 2\,q}{6 - 3\,q} - \frac{d'}{t} = \frac{r}{t} - j$		(*e'*)
	$a_s =$		$-\left(1 - q_s\frac{d}{t}\right)$		(*f'*)
	$a'_s =$		$-\left(1 - q_s\frac{d'}{t}\right)$		(*g'*)
	$c_s =$		$-\left(\frac{d}{t}\right)\left(\frac{1 - q_s\frac{d}{t}}{2 - q_s}\right)\left(\frac{1}{1 - j_s\frac{t}{e_s}}\right)$		(*h'*)
	$c'_s =$		$-\left(\frac{d}{t}\right)\left(\frac{1 - q_s\frac{d'}{t}}{2 - q_s}\right)\left(\frac{1}{1 + j'_s\frac{t}{e'_s}}\right)$		(*i'*)
	$g =$		$\left(\frac{d}{t}\right)\frac{f_s\sin\alpha\cos\alpha}{n\,h\,(2 - q_s)\left(1 - j_s\frac{t}{e_s}\right)}$		(*j'*)
	$g' =$		$\left(\frac{d}{t}\right)\frac{f'_s\sin\alpha\cos\alpha}{n\,h\,(2 - q_s)\left(1 + j_s\frac{t}{e'_s}\right)}$		(*k'*)
For partial moments	$S_o\,e_s =$		$\frac{1}{2}\,h\,(2 - q_s)\,j_s\,b\,t^2$		(*l'*)
	$S_o\,e'_s =$		$-\frac{1}{2}\,h\,(2 - q_s)\,j'_s\,b\,t^2$		(*m'*)
	$(S - S_o)\,e_s =$		$-f'_s\,p'\,b\,d\,r\cos^2\alpha$ or $-f'_s\,p'_2\,b\,d\,r\cos^2\alpha$		(*n'*)
	$(S - S_o)\,e'_s =$		$f_s\,p_2\,b\,d\,r\cos^2\alpha$ or $f_s\,p\,b\,d\,r\cos^2\alpha$		(*p'*)
	$\frac{f_s}{n} =$		$\sum f_c\,a\cos^2\alpha + \lambda\,h\,a_s\sin\alpha\cos\alpha$		(*q*)
	$\frac{f'_s}{n} =$		$\sum f_c\,a\cos^2\alpha + \lambda\,h\,a'_s\sin\alpha\cos\alpha$		
	$g =$		$\sum c\cos^4\alpha + \lambda\,c_s\sin^2\alpha\cos^2\alpha$		(*m*)
	$g' =$		$\sum c'\cos^4\alpha + \lambda\,c'_s\sin^2\alpha\cos^2\alpha$		
	$\frac{1}{2\,n} =$		$\sum (p\,c + p'\,c')\cos^4\alpha + \lambda\,(p\,c_s + p'\,c'_s)\sin^2\alpha\cos^2\alpha$		(*n*)


Elevation
Bending Deformation
Shear Deformation
$f'_s A'_s \cos^2 \alpha$
jd
C
jd
$f_s A_s \cos^2 \alpha$
d'
r
t
d'
f_c/E_c
δ'
kd
$(1-k)\,d$
δ
$\lambda h/E_c$
σ'
σ
C
$\lambda q_s h/E_c$

(A) TENSION OVER PART OF SECTION AT INTRADOS


$j'd$
$f'_s A'_s \cos^2 \alpha$
jd
$f_s A_s \cos^2 \alpha$
d'
r
t
d'
f_c/E_c
δ'
δ
$q f_c/E_c$
$\lambda h\ E_c$
σ'
σ
$\lambda h/E_c$

(B) COMPRESSION OVER ENTIRE SECTION


$f'_s A'_s \cos^2 \alpha$
$j'd$
C
$f_s A_s \cos^2 \alpha$
jd
d'
r
t
d'
δ'
$(1-k)\,d$
kd
δ
f_c/E_c
$\lambda h/E_c$
σ'
σ
$\lambda q_s h/E_c$

(C) TENSION OVER PART OF SECTION AT EXTRADOS

FIG. 35.—BENDING AND DIRECT STRESS; STEEL IN BOTH FACES. (SEE CASE VI FOR STEEL IN ONE DIRECTION AND CASE XII FOR STEEL IN TWO DIRECTIONS.)

CASE VII.—SIMPLE BENDING; STEEL IN TWO DIRECTIONS IN INTRADOS FACE ONLY (SEE FIG. 30)

	(A) Tension over part of section containing steel	(B) Compression over entire section	(C) Tension over part of section not containing steel	Equations (64)
$\sum f_s p \cos^2 \alpha =$	$\frac{1}{2} f_c k$	$\frac{1}{2} f_c (2-q) \frac{t}{d}$	$\frac{1}{2} f_c k$	(a)
$M =$	$\frac{1}{2} f_c k j b d^2$	$\frac{1}{2} f_c (2-q) j b t^2$	$\frac{1}{2} f_c k j b d^2$	(b)
$M =$	$\sum f_s p j b d^2 \cos^2 \alpha$	$\sum f_s p j b d t \cos^2 \alpha$	$\sum f_s p j b d^2 \cos^2 \alpha$	(c)
$j =$	$1 - \frac{k}{3}$	$\frac{d}{t} - \frac{3-2q}{6-3q}$	$\frac{k}{3} - \frac{d'}{d}$	(d)
$a =$	$\frac{1-k}{k}$	$-\left(1 - q\frac{d}{t}\right)$	$-\left(\frac{k - \frac{d'}{d}}{k}\right)$	(e)
	$k = 3(1-j)$	$q = \frac{3 + 6j - 6\frac{d}{t}}{2 + 3j - 3\frac{d}{t}}$	$k = 3\left(j + \frac{d'}{d}\right)$	(f)
$f_\alpha p_\alpha \cos^2 \alpha =$	$\frac{1}{2} f_c k - f_\beta p_\beta \cos^2 \beta$	$\frac{1}{2} f_c (2-q) \frac{t}{d} - f_\beta p_\beta \cos^2 \beta$	$\frac{1}{2} f_c k - f_\beta p_\beta \cos^2 \beta$	(g)
$\sum f_s p \sin \alpha \cos \alpha =$	$\frac{1}{2} h (2 - q_s) \frac{t}{d}$			(a′)
$M_s =$	$\frac{1}{2} h (2 - q_s) b j_s t^2$			(b′)
$M_s =$	$\sum f_s p j_s b d t \sin \alpha \cos \alpha$			(c′)
$j_s =$	$\frac{d}{t} - \frac{3 - 2q_s}{6 - 3q_s}$			(d′)
$a_s =$	$-\left(1 - q_s \frac{d}{t}\right)$			(e′)
$q_s =$	$\frac{3 + 6j_s - 6\frac{d}{t}}{2 + 3j_s - 3\frac{d}{t}}$			(f′)
$f_\alpha p_\alpha \sin \alpha \cos \alpha =$	$\frac{1}{2} h (2 - q_s) \frac{t}{d} - f_\beta p_\beta \sin \beta \cos \beta$			(g′)
$\frac{f_s}{n} =$	$\sum f_c a \cos^2 \alpha + \lambda h a_s \sin \alpha \cos \alpha$			(h)

CASE VIII.—SIMPLE BENDING; STEEL IN TWO DIRECTIONS IN EXTRADOS FACE ONLY (SEE FIG. 31)

<table>
<tr><th></th><th>(A) Tension over part of section containing steel</th><th>(B) Compression over entire section</th><th>(C) Tension over part of section not containing steel</th><th>Equations (54).</th></tr>
<tr><td>$\sum f'_s\, p' \cos^2 \alpha =$</td><td>$\frac{1}{2} f_c k$</td><td>$\frac{1}{2} f_c (2-q) \frac{t}{d}$</td><td>$\frac{1}{2} f_c k$</td><td>(a)</td></tr>
<tr><td>$M =$</td><td>$-\frac{1}{2} f_c k j' b d^2$</td><td>$-\frac{1}{2} f_c (2-q) j' b t^2$</td><td>$-\frac{1}{2} f_c k j' b d^2$</td><td>(b)</td></tr>
<tr><td>$M =$</td><td>$-\sum f'_s p' j' b d^2 \cos^2 \alpha$</td><td>$-\sum f'_s p' j' b d t \cos^2 \alpha$</td><td>$-\sum f'_s p' j' b d^2 \cos^2 \alpha$</td><td>(c)</td></tr>
<tr><td>$j' =$</td><td>$1 - \frac{k}{3}$</td><td>$\frac{3-2q}{6-3q} - \frac{d'}{t}$</td><td>$\frac{k}{3} - \frac{d'}{d}$</td><td>(d)</td></tr>
<tr><td>$a' =$</td><td>$\frac{1-k}{k}$</td><td>$-\left(1 - q \frac{d'}{t}\right)$</td><td>$-\left(\frac{k - \frac{d'}{d}}{k}\right)$</td><td>(e)</td></tr>
<tr><td></td><td>$k = 3(1-j')$</td><td>$q = \frac{3 - 6j' - 6\frac{d'}{t}}{2 - 3j' - 3\frac{d'}{t}}$</td><td>$k = 3\left(j' + \frac{d'}{d}\right)$</td><td>(f)</td></tr>
<tr><td>$f_\alpha p_\alpha \cos^2 \alpha =$</td><td>$\frac{1}{2} f_c k - f_\beta p_\beta \cos^2 \beta$</td><td>$\frac{1}{2} f_c (2-q) \frac{t}{d} - f_\beta p_\beta \cos^2 \beta$</td><td>$\frac{1}{2} f_c k - f_\beta p_\beta \cos^2 \beta$</td><td>(g)</td></tr>
<tr><td>$\sum f'_s p' \sin \alpha \cos \alpha =$</td><td colspan="3">$\frac{1}{2} h (2 - q_s) \frac{t}{d}$</td><td>(a')</td></tr>
<tr><td>$M_s =$</td><td colspan="3">$-\frac{1}{2} h (2 - q_s) b j'_s t^2$</td><td>(b')</td></tr>
<tr><td>$M_s =$</td><td colspan="3">$-\sum f'_s p' j'_s b d t \sin \alpha \cos \alpha$</td><td>(c')</td></tr>
<tr><td>$j_s =$</td><td colspan="3">$\frac{3 - 2q_s}{6 - 3q_s} - \frac{d'}{t}$</td><td>(d')</td></tr>
<tr><td>$a'_s =$</td><td colspan="3">$-\left(1 - q_s \frac{d'}{t}\right)$</td><td>(e')</td></tr>
<tr><td>$q_s =$</td><td colspan="3">$\frac{3 - 6j' - 6\frac{d'}{t}}{2 - 3j' - 3\frac{d'}{t}}$</td><td>(f')</td></tr>
<tr><td>$f_\alpha p_\alpha \sin \alpha \cos \alpha =$</td><td colspan="3">$\frac{1}{2} h (2 - q_s) \frac{t}{d} - f_\beta p_\beta \sin \beta \cos \beta$</td><td>(g')</td></tr>
<tr><td>$\frac{f_s}{n} =$</td><td colspan="3">$\sum f_c a' \cos^2 \alpha + \lambda h a_s \sin \alpha \cos \alpha$</td><td>(h)</td></tr>
</table>

CASE IX.—SIMPLE BENDING; STEEL IN TWO DIRECTIONS AND IN BOTH FACES (SEE FIG. 32)

	(*A*) Tension over part of section at intrados	(*B*) Compression over entire section	(*C*) Tension over the part of section at extrados	Equations (64)
$\sum (f_s p + f'_s p') \cos^2 \alpha =$	$\frac{1}{2} f_c k$	$\frac{1}{2} f_c (2 - q) \frac{t}{d}$	$\frac{1}{2} f_c k$	(*a*)
$M =$	$\frac{1}{2} f_c k j b d^2$ $- \sum f'_s p' b d r \cos^2 \alpha$	$\frac{1}{2} f_c (2 - q) j b t^2$ $- \sum f'_s p' b d r \cos^2 \alpha$	$\frac{1}{2} f_c k j b d^2$ $- \sum f'_s p' b d r \cos^2 \alpha$	(*b*)
$M =$	$- \frac{1}{2} f_c k j' b d^2$ $+ \sum f_s p b d r \cos^2 \alpha$	$- \frac{1}{2} f_c (2 - q) j' b t^2$ $+ \sum f_s p b d r \cos^2 \alpha$	$- \frac{1}{2} f_c k j' b d^2$ $+ \sum f_s p b d r \cos^2 \alpha$	(*c*)
$j =$	$1 - \frac{k}{3} = \frac{r}{d} - j'$	$\frac{d}{t} - \frac{3 - 2q}{6 - 3q} = \frac{r}{t} - j'$	$\frac{k}{3} - \frac{d'}{d} = \frac{r}{d} - j'$	(*d*)
$j' =$	$\frac{k}{3} - \frac{d'}{d} = \frac{r}{d} - j$	$\frac{3 - 2q}{6 - 3q} - \frac{d'}{t} = \frac{r}{t} - j$	$1 - \frac{k}{3} = \frac{r}{d} - j$	(*e*)
$a =$	$\frac{1 - k}{k}$	$- \left(1 - q \frac{d}{t}\right)$	$- \left(\frac{k - \frac{d'}{d}}{k}\right)$	(*f*)
$a' =$	$- \left(\frac{k - \frac{d'}{d}}{k}\right)$	$- \left(1 - q \frac{d'}{t}\right)$	$\frac{1 - k}{k}$	(*g*)
$(f_\alpha p_\alpha + f'_\alpha p'_\alpha) \cos^2 \alpha =$	$\frac{1}{2} f_c k - (f_\beta p_\beta$ $+ f'_\beta p'_\beta) \cos^2 \beta$	$\frac{1}{2} f_c (2 - q) \frac{t}{d} - (f_\beta p_\beta$ $+ f'_\beta p'_\beta) \cos^2 \beta$	$\frac{1}{2} f_c k - (f_\beta p_\beta$ $+ f'_\beta p'_\beta) \cos^2 \beta$	(*h*)

CASE IX.—(*Continued.*)

	(*A*) Tension over part of section at intrados	(*B*) Compression over entire section	(*C*) Tension over the part of section of extrados	Equations (64)
$\sum(f_s p + f'_s p') \sin\alpha\cos\alpha =$	$\frac{1}{2} h (2 - q_s) \frac{t}{d}$			(*a'*)
$M_s =$	$\frac{1}{2} h (2 - q_s) j_s b t^2 - \sum f'_s p' b d r \sin\alpha\cos\alpha$			(*b'*)
$M_s =$	$-\frac{1}{2} h (2 - q_s) j'_s b t^2 + \sum f_s p b d r \sin\alpha\cos\alpha$			(*c'*)
$j_s =$	$\frac{d}{t} - \frac{3 - 2q}{6 - 3q}$			(*d'*)
$j'_s =$	$\frac{3 - 2q}{6 - 3q} - \frac{d'}{t}$			(*e'*)
$a_s =$	$-\left(1 - q_s \frac{d}{t}\right)$			(*f'*)
$a'_s =$	$-\left(1 - q_s \frac{d}{t}\right)$			(*g'*)
$(f_\alpha p_\alpha + f'_\alpha p'_\alpha) \sin\alpha\cos\alpha =$	$\frac{1}{2} h (2 - q_s) \frac{t}{d} - (f_\beta p_\beta + f'_\beta p'_\beta) \sin\beta\cos\beta$			(*h'*)
$\frac{f_s}{n} =$	$\sum f_c a \cos^2\alpha + \lambda h a_s \sin\alpha\cos\alpha$			(*i*)
$\frac{f'_s}{n} =$	$\sum f_c a' \cos^2\alpha + \lambda h a'_s \sin\alpha\cos\alpha$			

CASE X.—BENDING AND DIRECT STRESS; STEEL IN TWO DIRECTIONS IN INTRADOS FACE ONLY (SEE FIG. 33)

	(*A*) Tension over part of section containing steel	(*B*) Compression over entire section	(*C*) Tension over part of section not containing steel	Equations (64)
$\sum f_s p\, b\, d \cos^2 \alpha =$	$\frac{1}{2} f_c\, k\, b\, d - N$	$\frac{1}{2} f_c (2-q)\, b\, t - N$	$\frac{1}{2} f_c\, k\, b\, d - N$	(*a*)
$N e =$	$\frac{1}{2} f_c\, k\, j\, b\, d^2$	$\frac{1}{2} f_c (2-q)\, j\, b\, t^2$	$\frac{1}{2} f_c\, k\, j\, b\, d^2$	(*b*)
	$N(e - j\, d) =$ $\sum f_s\, p\, j\, b\, d^2 \cos^2 \alpha$	$N(e - j\, t) =$ $\sum f_s\, p\, j\, b\, d^2 \cos^2 \alpha$	$N(e - j\, d) =$ $\sum f_s\, p\, j\, b\, d^2 \cos^2 \alpha$	(*c*)
$j =$	$1 - \frac{k}{3}$	$\frac{d}{t} - \frac{3 - 2q}{6 - 3q}$	$\frac{k}{3} - \frac{d'}{d}$	(*d*)
$a =$	$\frac{1-k}{k}$	$-\left(1 - q\frac{d}{t}\right)$	$-\frac{\left(k - \frac{d'}{d}\right)}{k}$	(*e*)
	$k = 3(1 - j)$	$q = \frac{3 + 6j - 6\frac{d}{t}}{2 + 3j - 3\frac{d}{t}}$	$k = 3\left(j + \frac{d'}{d}\right)$	(*f*)
$f_\alpha\ p_\alpha\, b\, d \cos^2 \alpha =$	$\frac{1}{2} f_c\, k\, b\, d - N$ $- f_\beta\ p_\beta\, b\, d \cos^2 \beta$	$\frac{1}{2} f_c (2-q)\, b\, t$ $- N - f_\beta\, p_\beta\, b\, d \cos^2 \beta$	$\frac{1}{2} f_c\, k\, b\, d - N$ $- f_\beta\ p_\beta\, b\, d \cos^2 \beta$	(*g*)
$\sum f_s p b d \sin \alpha \cos \alpha =$	$\frac{1}{2} h (2 - q_s)\, b\, t - S$			(*a'*)
$S e_s =$	$\frac{1}{2} h (2 - q_s)\, j_s\, b\, t^2$			(*b'*)
$S(e_s - j\, t) =$	$\sum f_s\, p\, j_s\, b\, d\, t \sin \alpha \cos \alpha$			(*c'*)
$j_s =$	$\frac{d}{t} - \frac{3 - 2q_s}{6 - 3q_s}$			(*d'*)
$a_s =$	$-\left(1 - q_s \frac{d}{t}\right)$			(*e'*)
$q_s =$	$\frac{3 + 6j_s - 6\frac{d}{t}}{2 + 3j_s - 3\frac{d}{t}}$			(*f'*)
$f_\alpha\, p_\alpha\, b\, d \sin \alpha \cos \alpha =$	$\frac{1}{2} h (2 - q_s)\, b\, t - S - f_\beta\ p_\beta\, b\, d \sin \beta \cos \beta$			(*g'*)
$\frac{f_s}{n} =$	$\sum f_c\, a \cos^2 \alpha + \lambda\, h\, a_s \sin \alpha \cos \alpha$			(*h*)

CASE XI.—BENDING AND DIRECT STRESS; STEEL IN TWO DIRECTIONS IN EXTRADOS FACE ONLY (SEE FIG. 34)

	(*A*) Tension over part of section containing steel	(*B*) Compression over entire section	(*C*) Tension over part of section not containing steel	Equations (64)
$\sum f'_s p' b d \cos^2 \alpha =$	$\frac{1}{2} f_c k b d - N$	$\frac{1}{2} f_c (2 - q) b t - N$	$\frac{1}{2} f_c k b d - N$	(*a*)
$N e' =$	$-\frac{1}{2} f_c k j' b d^2$	$-\frac{1}{2} f_c (2 - q) j' b t^2$	$-\frac{1}{2} f_c k j' b d^2$	(*b*)
	$N (e' - j' d)$ $= -\sum f_s p j' b d^2 \cos^2 \alpha$	$N (e' - j' t)$ $= -\sum f_s p j' b d t \cos^2 \alpha$	$N (e' - j' d)$ $= -\sum f_s p j' d^2 \cos^2 \alpha$	(*c*)
$j' =$	$1 - \frac{k}{3}$	$\frac{3 - 2q}{6 - 3q} - \frac{d'}{t}$	$\frac{k}{3} - \frac{d'}{d}$	(*d*)
$a' =$	$\frac{1 - k}{k}$	$-\left(1 - q \frac{d'}{t}\right)$	$-\left(\frac{k - \frac{d'}{d}}{k}\right)$	(*e*)
	$k = 3 (1 - j')$	$q = \frac{3 - 6 j' - 6 \frac{d'}{t}}{2 - 3 j' - 3 \frac{d'}{t}}$	$k = 3 \left(j' + \frac{d'}{d}\right)$	(*f*)
$f'_\alpha p'_\alpha b d \cos^2 \alpha =$	$\frac{1}{2} f_c k b d - N$ $- f_\beta p_\beta b d \cos^2 \beta$	$\frac{1}{2} f_c (2 - q) b t - N$ $- f_\beta p_\beta b d \cos^2 \beta$	$\frac{1}{2} f_c k b d - N$ $- f_\beta p_\beta b d \cos^2 \beta$	(*g*)
$\sum f'_s p' b d \sin \alpha \cos \alpha =$	$\frac{1}{2} h (2 - q_s) b t - S$			(*a'*)
$S e'_s =$	$-\frac{1}{2} h (2 - q_s) j'_s b t^2$			(*b'*)
$S (e'_s + j'_s t) =$	$-\sum f'_s p' j'_s b d t \sin \alpha \cos \alpha$			(*c'*)
$j'_s =$	$\frac{3 - 2 q_s}{6 - 3 q_s} - \frac{d'}{t}$			(*d'*)
$a'_s =$	$-\left(1 - q \frac{d'}{t}\right)$			(*e'*)
$q_s =$	$\frac{3 - 6 j'_s - 6 \frac{d'}{t}}{2 - 3 j'_s - 3 \frac{d'}{t}}$			(*f'*)
$f'_\alpha p'_\alpha b d \sin \alpha \cos \alpha =$	$\frac{1}{2} h (2 - q_s) b t - S - f'_\beta p'_\beta b d \sin \beta \cos \beta$			(*g'*)
$\frac{f'_s}{n} =$	$\sum f_c a' \cos^2 \alpha + \lambda h a'_s \sin \alpha \cos \alpha$			(*i*)

CASE XII.—BENDING AND DIRECT STRESS; STEEL IN TWO DIRECTIONS AND IN BOTH FACES (SEE FIG. 35)

<table>
<tr><th></th><th>(A) Tension over part of section at intrados</th><th>(B) Compression over entire section</th><th>(C) Tension over part of section at extrados</th><th>Equations (64)</th></tr>
<tr><td>$\dfrac{\sum (f_s p + f'_s p')}{b\, d \cos^2 \alpha} =$</td><td>$\frac{1}{2} f_c\, k\, b\, d - N$</td><td>$\frac{1}{2} f_c\, (2 - q)\, b\, t - N$</td><td>$\frac{1}{2} f_c\, k\, b\, d - N$</td><td>(a)</td></tr>
<tr><td>$N e =$</td><td>$\frac{1}{2} f_c\, k\, j\, b\, d^2 - \sum f'_s p'\, b\, d\, r \cos^2 \alpha$</td><td>$\frac{1}{2} f_c\, (2 - q)\, j\, b\, t^2 - \sum f'_s p'\, b\, d\, r \cos^2 \alpha$</td><td>$\frac{1}{2} f_c\, k\, j\, b\, d^2 - \sum f'_s p'\, b\, d\, r \cos^2 \alpha$</td><td>(b)</td></tr>
<tr><td>$N e' =$</td><td>$-\frac{1}{2} f_c\, k\, j'\, b\, d^2 + \sum f_s p\, b\, d\, r \cos^2 \alpha$</td><td>$-\frac{1}{2} f_c\, (2 - q)\, j'\, b\, t^2 + \sum f_s p\, b\, d\, r \cos^2 \alpha$</td><td>$-\frac{1}{2} f_c\, k\, j'\, b\, d^2 + \sum f_s p\, b\, d\, r \cos^2 \alpha$</td><td>(c)</td></tr>
<tr><td>$j =$</td><td>$1 - \frac{k}{3} = \frac{r}{d} - j'$</td><td>$\frac{d}{t} - \frac{3 - 2q}{6 - 3q} = \frac{r}{t} - j'$</td><td>$\frac{k}{3} - \frac{d'}{d} = \frac{r}{d} - j'$</td><td>(d)</td></tr>
<tr><td>$j' =$</td><td>$\frac{k}{3} - \frac{d'}{d} = \frac{r}{d} - j$</td><td>$\frac{3 - 2q}{6 - 3q} - \frac{d'}{t} = \frac{r}{t} - j$</td><td>$1 - \frac{k}{3} = \frac{r}{d} - j$</td><td>(e)</td></tr>
<tr><td>$a =$</td><td>$\frac{1 - k}{k}$</td><td>$-\left(1 - q \frac{d}{t}\right)$</td><td>$-\left(\frac{k - \frac{d'}{d}}{k}\right)$</td><td>(f)</td></tr>
<tr><td>$a' =$</td><td>$-\left(\frac{k - \frac{d'}{d}}{k}\right)$</td><td>$-\left(1 - q \frac{d'}{t}\right)$</td><td>$\frac{1 - k}{k}$</td><td>(g)</td></tr>
<tr><td>$\dfrac{(f_\alpha\, p_\alpha + f'_\alpha\, p'_\alpha)}{b\, d \cos^2 \alpha} =$</td><td>$\dfrac{\frac{1}{2} f_c\, k\, b\, d - N - (f_\beta\, p_\beta + f'_\beta\, p'_\beta)}{b\, d \cos^2 \beta}$</td><td>$\dfrac{\frac{1}{2} f_c\, (2 - q)\, b\, t - N - (f_\beta\, p_\beta + f'_\beta\, p'_\beta)}{b\, d \cos^2 \beta}$</td><td>$\dfrac{\frac{1}{2} f_c\, k\, b\, d - N - (f_\beta\, p_\beta + f'_\beta\, p'_\beta)}{b\, d \cos^2 \beta}$</td><td>(h)</td></tr>
<tr><td>$\dfrac{\sum (f_s p + f'_s p')}{b\, d \cos^2 \alpha} =$</td><td colspan="3">$\frac{1}{2} h\, (2 - q_s)\, b\, t - S$</td><td>(a')</td></tr>
<tr><td>$S e_s =$</td><td colspan="3">$\frac{1}{2} h\, (2 - q_s)\, j_s\, b\, t^2 - \sum f'_s p'\, b\, d\, r \cos^2 \alpha$</td><td>(b')</td></tr>
<tr><td>$S e'_s =$</td><td colspan="3">$-\frac{1}{2} h\, (2 - q_s)\, j'_s\, b\, t^2 + \sum f_s p\, b\, d\, r \cos^2 \alpha$</td><td>(c')</td></tr>
<tr><td>$j_s =$</td><td colspan="3">$\frac{d}{t} - \frac{3 - 2q_s}{6 - 3q_s}$</td><td>(d')</td></tr>
<tr><td>$j'_s =$</td><td colspan="3">$\frac{3 - 2q}{6 - 3q} - \frac{d'}{t} = \frac{r}{t} - j_s$</td><td>(e')</td></tr>
<tr><td>$a_s =$</td><td colspan="3">$-\left(1 - q_s \frac{d}{t}\right)$</td><td>(f')</td></tr>
<tr><td>$a'_s =$</td><td colspan="3">$-\left(1 - q_s \frac{d'}{t}\right)$</td><td>(g')</td></tr>
<tr><td>$\dfrac{(f_\alpha\, p_\alpha + f'_\alpha\, p'_\alpha)}{b\, d \sin \alpha \cos \alpha} =$</td><td colspan="3">$\frac{1}{2} h\, (2 - q_s)\, b\, t - S - (f_\beta\, p_\beta + f'_\beta\, p'_\beta) \sin \beta \cos \beta$</td><td>(h')</td></tr>
<tr><td>$\frac{f_s}{n} =$
$\frac{f'_s}{n} =$</td><td colspan="3">$\left.\begin{array}{l} \sum f_c\, a \cos^2 \alpha + \lambda\, h\, a_s \sin \alpha \cos \alpha \\ \sum f_c\, a' \cos^2 \alpha + \lambda\, h\, a'_s \sin \alpha \cos \alpha \end{array}\right\}$</td><td>(i)</td></tr>
</table>

DISCUSSION

G. D. Houtman,[*] Assoc. M. Am. Soc. C. E. (by letter).—This paper is without doubt a valuable and timely contribution to the science of skew arch design. It is founded, among other things, on the elastic theory.

This theory is known to apply almost perfectly (up to a certain point), to a steel beam. It applies with less accuracy to a concrete beam, and with still less precision to a narrow arch rib. Presumably, it is even less applicable to a wide skew arch, involving elastic deformations within cross-sections of considerable area. Granting that a concrete section, 1 ft. wide and 3 ft. deep, bends elastically, there is no positive assurance that another section 3 by 60 ft. will bend the same way. A low building can be erected safely perhaps on a shifty foundation; but one need not trust altogether the mathematical skyscraper built on a somewhat sandy base. At the same time, the art of design has always advanced by such steps as these, and the author is entitled to commendation for his paper.

It seems to the writer that skew arches can be explained and designed with much more simplicity by a method which is frankly approximate. Let the problem be, for instance, to design the skew arch, *A B C D,* in Fig. 36. The first step would be to design, by any method, a rectangular arch of the same longitudinal span and the same width, that is, the arch, *A B′ C′ D.* This rectangular arch is next conceived to be distorted into the skew arch, *A B C D.*

The next step consists in applying the principle of least work. It is well known that Nature is economical of its efforts, and that accordingly structures act and deflect in such a way that the work done is a minimum. Obviously, the least work is done when the compression paths are of least length, and as straight as possible. There surely would be more work done in carrying a load by a roundabout or longer path. Such being the case, the skew arch, *A B C D,* tends to divide itself into a central rectangular arch, *E B F D,* and two triangular arches, *A E D* and *C F B.* There is nothing mysterious about the central rectangular arch, *E B F D.* Regarding the triangular arches, any compression path emanating from the line, *F C,* converges upon the point, *B,* which is the

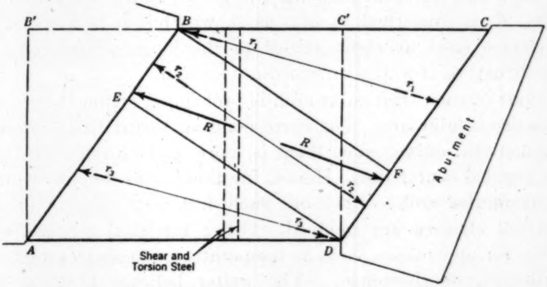


Fig. 36.—Approximate Analysis of a Shear Arch

nearest point to any other point on the line, *F C*. The same reasoning applies to the other triangular arch, *A E D*, which is similar in all respects.

---

[*] Designing Engr., County Engr.'s Office, Delaware County, Media, Pa.

Next, bisect each of the three arches with fine lines, at the extremities of which draw the three resultants, r_1, r_2, and r_3 ($= r_1$). The lengths of these three resultants are made proportional to the plan areas of their respective arches. Thus, r_1 is proportional in length or magnitude to the area, *B C F*, and r_2 is proportional to the area, *E B F D*. These three resultants are next combined to compose the final resultant, *R*. At present, this resultant is established as to direction and point of application, but not as to magnitude. It will be on the safe side to assume that its magnitude is equal to that of the total resultant acting on the center of line, *A B′*, in the original design of the rectangular arch, *A B′ C′ D*.

It is frankly acknowledged at this point that the procedure expounded in the preceding paragraph is not altogether correct, even if it possesses the virtue of simplicity. For instance, the magnitude of r_1 should be somewhat less than assumed herein, and, in reality, its point of application should be closer to the point, *C*, than to the point, *F*. There is also something to say regarding the different ratios of vertical to horizontal components.

The position and magnitude to the final resultant, *R*, determine the magnitude and distribution of stress along the abutment face, *D C*. Obviously, the compression stresses are greatest at *D* and least at *C*. In fact, tension may be indicated at *C*. If the compressive stresses at *D* are excessive, the arch can be thickened near that point, or, perhaps, a richer mix can be specified. The abutment is given a trapezoidal shape.

A triangular arch, such as *A E D*, obviously could not exist by itself, because the point, *D*, has no strength. It can exist only in conjunction with a neighboring arch, to which the thrust which the apex, *D*, cannot take, is transmitted by shear along the shear plane, *E D*. As far as the determination of the final resultant, *R*, is concerned, r_3 acts as shown; but it is necessary to place enough transverse steel at right angles to the longitudinal center line, to transmit the thrust, r_3, into the intermediate section.

The arch axis of each triangular arch is probably a symmetrical curve, such as a parabola or circular arc. The corresponding thrust line, however, would not be a symmetrical curve, since there is more load on one side than on the other, of the vertical center line. Hence, the thrust line departs from the arch axis in the triangular arch. Since one such arch is the reverse of the other, certain torsional stresses are induced. Other torsional stresses are caused when the arch, for any reason, such as temperature, rib-shortening, etc., is distorted, lengthened, or shortened. The writer believes however that these torsional stresses need give no concern, provided that conservative design stresses are used and that the arch contains somewhat more than the customary amount of longitudinal and transverse steel.

Some existing textbooks state that skew arches can be designed exactly as square arches. This assertion is known to be erroneous, but it must be admitted that a large number of skew arches have been built in complete ignorance of skew principles, and that only a few have failed. Usually, failure has resulted from excessive compression at the obtuse corner, and a cracking off of a section of abutment. There have been failures during construction caused by insufficient bracing of the centering. In general, the falsework of a skew arch should be braced three ways instead of the customary two. Of the existing skew arches which were designed as rectangular arches, some show peculiar cracks which indicate tension, or, at least, absence of compression, at the acute corner. Intermediate piers between skew arches are full of danger, since the skew thrusts tending to spin them about a vertical axis are doubled.

J. CHARLES RATHBUN,[10] M. AM. SOC. C. E. (by letter).—The writer has been much interested in Mr. Weiner's description of the methods of analyzing skew arches as practiced by the Westchester County Park Commission. The paper is valuable not only as a description of methods, but also because it brings the theory before the profession in a slightly different form from that used in its first presentation.[11] The numerous integral signs used in the writer's paper may have led to difficult reading for those who are more accustomed to the sigma sign for summation.

In the writer's opinion, the principal value of Mr. Weiner's paper is that it calls the attention of the profession to the fact that arches are being built successfully by this theory. When it was first published some question was raised as to the correctness of the mathematics. It was popularly believed that the stress distribution at the abutment was greatest at the obtuse corners, and when the equations failed to substantiate this idea they were not accepted by conservative members of the profession. Later, very careful and extensive experiments (unpublished) were conducted by George E. Beggs, M. Am. Soc. C. E., for the Society's Special Committee on Concrete and Reinforced Concrete Arches. The results of computations of all six abutment reactions for each of four different arches for a vertical load in a large number of positions showed substantial agreement with the results of these tests. As the report of this work for the Committee may not have been brought to the attention of all who have been interested in the subject, Mr. Weiner's paper brings before the public the fact that this method of analysis is a safe one to follow.

In discussing the changes in practice from the original, as introduced by Mr. Weiner, the writer does not feel that these changes seriously affect any results in the final design, but are simply differences in personal practice; or, as in the use of a sigma instead of an integral sign, the difference between a theoretical formula and its practical application. Some of the suggestions, such as the use of coefficients for Equations (25), K_1, K_2, etc., and the dropping of terms of low value will occur readily to any designer.

The practice of dropping terms of low value should be treated with great caution. Those terms that are dropped in analyzing one arch, may be important in another. For example, the arches designed by Arthur G. Hayden, M. Am. Soc. C. E., for Westchester County, will permit the omission of all terms containing the moment of inertia about the u-axis and these have been dropped by Mr. Weiner. These terms are indeed very small in barrel arches, but they have a great influence in arch ribs. The writer, therefore, considers it wise for an engineer who is designing a skew arch to study the question of omitting terms as it applies to his own problem. He may find an entirely different solution than the one given. The writer also was guilty of this practice in his paper,[12] although the equations as developed are found in full in the Appendix of that paper. The same idea may be extended to some of the other "short cuts".

---

[10] Associate Prof. of Civ. Eng., Coll. of the City of New York, New York, N. Y.

[11] *Transactions*, Am. Soc. C. E., Vol. LXXXVII (1924), p. 611.

A designer who has familiarized himself with the original theory may find some confusion in the notation in several places in Mr. Weiner's paper. It would be less confusing to have retained the writer's original notation as nearly as possible. For example R_x, R_y, and R_z, are used as deflections due to loads by the writer while Mr. Weiner uses them as abutment reactions. The introduction of θ for the writer's $\tan^{-1} \varepsilon$, in one place, and his Δ_α, Δ_β, Δ_γ, in another, might cause confusion.

The writer has found the change of signs suggested by Mr. Weiner very confusing and does not see what is to be gained by it. The sign convention introduced by the writer in his paper are self-consistent, and an engineer who follows these equations need fear no confusion. The sign of T_y was taken in the direction indicated because it is consistent with current practice in representing shear in beam analysis, and an arch approaches the beam in many cases. As it is customary to indicate positive shear in the manner shown, it is confusing to change it. If the arch tends toward the flat type the designer should not be confused by his familiarity with the beam theory. A change in the sign of the shear forces makes them inconsistent with the idea that the differential moment is a shear. The change of sign of T_y and sometimes of M_y and T_z is not wise. If familiarity with the figures of a skew as shown in Fig. 2(*a*) makes it easier for an engineer to keep his work more clearly in mind, there is no mathematical objection to computing an arch as there shown, and after the work is completed sketching the results as in Fig. 2(*b*). This will avoid using a term at times as plus and at other times as minus.

Another change that has been made, which has several disadvantages, is in the order of the equations. This appears, for example, in Mr. Weiner's equations for the solution of the unsymmetrical skew (Equations (25)). He is in error in his statement in the "Introduction" that "the theory as presented applied only to symmetrical skew frames fixed at the abutments". Much of the space in the paper devoted to the original theory, applied to the simplification of the equations due to the symmetry of the ring, but the theory as presented has been used by the writer without change for skews. If this symmetry does not exist the writer fails to see how the six equations (Equations (25)) given by Mr. Weiner, or those given previously by the writer,[13] can be simplified. Possibly the solution lies along the lines suggested by Carl B. Andrews, Assoc. M. Am. Soc. C. E.[14]

---

[12] *Transactions*, Am. Soc. C. E., Vol. LXXXVII (1924), p. 617.

[13] *Loc. cit.*, p. 640.

[14] *Loc. cit.*, p. 438.

Because Equations (29) of the writer's paper[13] supply the missing terms that may prove necessary in some designs they are here stated in Equations (74) in terms of the writer's notation.

The signs, order of equations, and form of integrals have been retained as in the original paper. Note that the coefficients, K, are symmetrical relative to rows and columns. Mr. Weiner assumed the position of the load such that $z' = \epsilon x'$.

$$\begin{array}{lllllll}
K_{13}\,T_x & + K_{14}\,T_y & + K_{15}\,T_z & + K_{10}\,M_z & + K_6\,M_y & + K_7\,M_x & = C_4 \ldots\ldots (74a) \\
K_{14}\,T_x & + K_{17}\,T_y & + K_{18}\,T_z & + K_{11}\,M_z & + K_7\,M_y & + K_2\,M_x & = C_5 \ldots\ldots (74b) \\
K_{15}\,T_x & + K_{18}\,T_y & + K_{19}\,T_z & & + K_8\,M_y & + K_3\,M_x & = C_6 \ldots\ldots (74c) \\
K_{10}\,T_x & + K_{11}\,T_y & & + K_{12}\,M_z & & & = C_3 \ldots\ldots (74d) \\
K_6\,T_x & + K_7\,T_y & + K_8\,T_z & & + K_9\,M_y & + K_5\,M_x & = C_2 \ldots\ldots (74e) \\
K_7\,T_x & + K_2\,T_y & + K_3\,T_z & & + K_5\,M_y & + K_4\,M_x & = C_1 \ldots\ldots (74f)
\end{array}$$

$$K_{13} = \int \cos^2\phi\,\frac{ds}{A} + \lambda\int \sin^2\phi\,\frac{ds}{A} + \int y^2\,\frac{ds}{I_z} + \varepsilon^2\int x^2\cos^2\phi\,\frac{ds}{I_y} + \varepsilon^2\lambda\int x^2\sin^2\phi\,\frac{ds}{F}$$

$$K_{14} = (1-\lambda)\int \sin\phi\cos\phi\,\frac{ds}{A} - \int x\,y\,\frac{ds}{I_z} + \varepsilon^2\int\left(\frac{1}{I_y} - \frac{\lambda}{F}\right)x^2\sin\phi\cos\phi\,ds$$

$$K_{15} = -\left\{\varepsilon\int v\,x\cos\phi\,\frac{ds}{I_y} + \varepsilon\lambda\int u\,x\sin\phi\,\frac{ds}{F}\right\}$$

$$K_{10} = \int y\,\frac{ds}{I_z}$$

$$K_6 = \varepsilon\int x\cos^2\phi\,\frac{ds}{I_y} + \varepsilon\lambda\int x\sin^2\phi\,\frac{ds}{F}$$

$$K_7 = \varepsilon\int\left(\frac{1}{I_y} - \frac{\lambda}{F}\right)x\sin\phi\cos\phi\,ds = K_1$$

$$K_{17} = \int \sin^2\phi\,\frac{ds}{A} + \lambda\int \cos^2\phi\,\frac{ds}{A} + \int x^2\,\frac{ds}{I_z} + \varepsilon^2\int x^2\sin^2\phi\,\frac{ds}{I_y} + \varepsilon^2\lambda\int x^2\cos^2\phi\,\frac{ds}{F}$$

$$K_{18} = -\left\{\varepsilon\int v\,x\sin\phi\,\frac{ds}{I_y} - \varepsilon\lambda\int u\,x\cos\phi\,\frac{ds}{F}\right\}$$

$$K_{11} = -\int x\,\frac{ds}{I_z}$$

$$K_2 = \varepsilon\int x\sin^2\phi\,\frac{ds}{I_y} + \varepsilon\lambda\int x\cos^2\phi\,\frac{ds}{F}$$

$$K_{19} = \lambda\int\frac{ds}{A} + \int v^2\,\frac{ds}{I_y} + \lambda\int u^2\,\frac{ds}{F}$$

$$K_8 = -\left\{ \int v \cos\phi \frac{ds}{I_y} + \lambda \int u \sin\phi \frac{ds}{F} \right\}$$

$$K_3 = -\left\{ \int v \sin\phi \frac{ds}{I_y} - \lambda \int u \cos\phi \frac{ds}{F} \right\}$$

$$K_{12} = \int \frac{ds}{I_z}$$

$$K_9 = \int \cos^2\phi \frac{ds}{I_y} + \lambda \int \sin^2\phi \frac{ds}{F}$$

$$K_5 = \int \left(\frac{1}{I_y} - \frac{\lambda}{F} \right) \sin\phi \cos\phi \, ds$$

$$K_4 = \int \sin^2\phi \frac{ds}{I_y} + \lambda \int \cos^2\phi \frac{ds}{F}$$

$$\begin{aligned} -C_4 = \Bigg\{ & (1-\lambda) \int_A^P \sin\phi \cos\phi \frac{ds}{A} - \int_A^P x\,y \frac{ds}{I_z} \\ & + \varepsilon^2 \int_A^P x^2 \sin\phi \cos\phi \left(\frac{1}{I_y} - \frac{\lambda}{F} \right) d\,s \Bigg\} W \\ & + \left\{ \int_A^P y \frac{ds}{I_z} \right\} W x' \\ & - \left\{ \varepsilon \int_A^P x \sin\phi \cos\phi \left(\frac{1}{I_y} - \frac{\lambda}{F} \right) d\,s \right\} W z' \end{aligned}$$

$$\begin{aligned} -C_5 = \Bigg\{ & \int_A^P \sin^2\phi \frac{ds}{A} + \lambda \int_A^P \cos^2\phi \frac{ds}{A} + \int_A^P x^2 \frac{ds}{I_z} + \varepsilon^2 \int_A^P x^2 \sin^2\phi \frac{ds}{I_y} \\ & + \varepsilon^2 \lambda \int^P x^2 \cos^2\phi \frac{ds}{F} \Bigg\} W - \left\{ \int_A^P x \frac{ds}{I_z} \right\} W x' \\ & - \left\{ \varepsilon \int_A^P x \sin^2\phi \frac{ds}{I_y} + \varepsilon \lambda \int_A^P x \cos^2\phi \frac{ds}{F} \right\} W z' \end{aligned}$$

$$\begin{aligned} -C_6 = & -\left\{ \varepsilon \int_A^P v\,x \sin\phi \frac{ds}{I_y} - \varepsilon \lambda \int u\,x \cos\phi \frac{ds}{F} \right\} W \\ & + \left\{ \int_A^P v \sin\phi \frac{ds}{I_y} - \lambda \int_A^P u \cos\phi \frac{ds}{F} \right\} W z' \end{aligned}$$

$$-C_3 = -\left\{ \int_A^P \times \frac{ds}{I_z} \right\} W + \left\{ \int_A^P \frac{ds}{I_z} \right\} W x'$$

$$\begin{aligned} -C_2 = & \left\{ \varepsilon \int_A^P \left(\frac{1}{I_y} - \frac{\lambda}{F} \right) x \sin\phi \cos\phi \, ds \right\} W \\ & - \left\{ \int_A^P \left(\frac{1}{I_y} - \frac{\lambda}{F} \right) \sin\phi \cos\phi \, ds \right\} W z' \end{aligned}$$

$$\begin{aligned} -C_1 = & \left\{ \varepsilon \int_A^P x \sin^2\phi \frac{ds}{I_y} + \varepsilon \lambda \int_A^P x \cos^2\phi \frac{ds}{F} \right\} W \\ & - \left\{ \int_A^P \sin^2\phi \frac{ds}{I_y} + \lambda \int_A^P \cos^2\phi \frac{ds}{F} \right\} W z' \end{aligned}$$

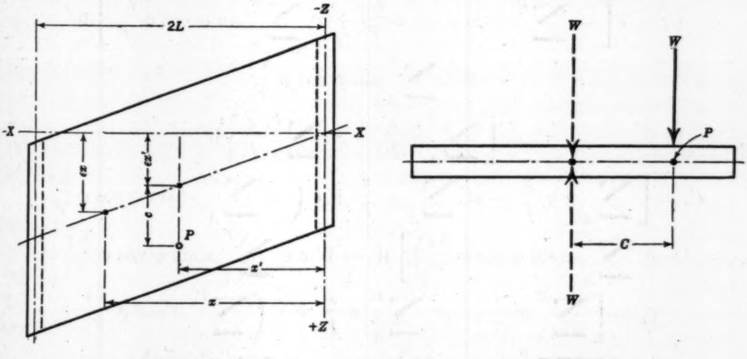
Equations (74) give the deflection of a skew cantilever under load and with end reactions. In the case of a skewed arch with fixed ends these deflections should be placed equal to zero and the end or abutment reactions computed.

Often the unsymmetrical skew arch has some degree of symmetry as will occur when the ring is symmetrical from the crown to the legs (in the case of an arch of the type used by Mr. Hayden); but when the legs have different lengths, or otherwise are not symmetrical, the theory as originally written can be applied, except that many of the terms that will cancel in the symmetrical arch must be retained, as there stated.

In the development of the unit stresses Mr. Weiner has used a different distribution for torsional stresses. Otherwise, it is fundamentally the same as that used by the writer for several years. It might be of interest to know the difference in results between these equations, especially as Mr. Weiner states in the "Introduction" that he has compared the several theories numerically. The matter of torsional stress distribution is a very vital part of this work in many cases.

Although the writer's discussion may appear to indicate that he differs from the author, in closing he wishes to state that his criticisms are all very minor and are given for the purpose of improving a very excellent paper. Mr. Weiner is especially to be commended in proving the correctness of the method of computing the unit stresses from the forces acting across a section.

A. A. Eremin,[15] Assoc. M. Am. Soc. C. E. (by letter).—The author has presented an extremely interesting paper on the design of a skew arch and "rigid frame." He has simplified the equations originally developed[16] by J. Charles Rathbun, M. Am. Soc. C. E., and has illustrated their application by a practical design problem.



(a) PLAN

(b) SECTION

Fig. 37.—Diagram Showing Position of Unsymmetrical Concentrated Load.

The equations for a skew arch with the loads symmetrical about the center line between spandrel walls, are discussed in this paper. For the stresses in a skew arch with unsymmetrical concentrated loads the equations will be slightly different.

---

[15] Asst. Designing Engr., Bridge Dept., State Highway Comm., Sacramento, Calif.

[16] *Transactions*, Am. Soc. C. E., Vol. LXXXVII (1924), p. 611.

Fig. 37(a) is the plan of a skew arch with a concentrated load, W, at the point, P. The eccentricity is designated by c. The elevation of the arch is the same as that shown in the author's Fig. 9(b). This unsymmetrical load may be replaced by a symmetrical load, W, and a moment, W_c. Then the equations for the moments will be:

$$m_u = R_x \epsilon x \cos\phi + R_y \epsilon x \sin\phi + R_z v' - N_x \sin\phi + N_y \cos\phi - W \epsilon (x - x') \sin\phi - W_c \sin\phi \quad \ldots\ldots (75d)$$

$$m_v = - R_x \epsilon x \sin\phi + R_y \epsilon x \cos\phi - R_z u' - N_x \cos\phi - N_y \sin\phi - W \epsilon (x - x') \cos\phi - W_c \cos\phi \quad \ldots\ldots (75e)$$

and,

$$m_z = R_x y + R_y x + N_z - W (x - x') \quad \ldots\ldots (75f)$$

The reactions at the arch may be obtained from the final equations:

$$- K_1 R_x + K_2 R_y + K_3 R_z + K_4 N_x - K_5 N_y = C_1 \quad \ldots\ldots (76a)$$

$$K_6 R_x - K_7 R_y + K_8 R_z + K_5 N_x + K_9 N_y = C_2 \quad \ldots\ldots (76b)$$

$$K_{10} R_x + K_{11} R_y + K_{12} N_z = C_3 \quad \ldots\ldots (76c)$$

$$K_{13} R_x + K_{14} R_y + K_{15} R_z + K_7 N_x + K_6 N_y + K_{10} N_z = C_4 \quad \ldots\ldots (76d)$$

$$K_{14} R_x + K_{17} R_y - K_{18} R_z - K_2 N_x - K_7 N_y + K_{11} N_z = C_5 \quad \ldots\ldots (76e)$$

$$- K_{15} R_x + K_{18} R_y + K_{19} R_z + K_3 N_x - K_8 N_y = C_6 \quad \ldots\ldots (76f)$$

in which, the coefficients, K, are as given by the author, in Equations (25), Appendix I, and the coefficients, C, are:

$$C_1 = \epsilon \left[\sum_A^P x \cos^2\phi \frac{\lambda s}{F} - x' \sum_A^P \cos^2\phi \frac{\lambda s}{F} \right] W + Wc \sum_A^P \cos^2\phi \frac{\lambda s}{F}$$

$$C_2 = \epsilon \left[- \sum_A^P x \sin\phi \cos\phi \frac{\lambda s}{F} + x' \sum_A^P \sin\phi \cos\phi \frac{\lambda s}{F} \right] W - Wc \sum_A^P \sin\phi \cos\phi \frac{\lambda s}{F}$$

$$C_3 = \left[\sum_A^P x \frac{s}{I} - x' \sum_A^P \frac{s}{I} \right] W$$

$$C_4 = \left[\sum_A^P x\, y \frac{s}{I} - x' \sum_A^P y \frac{s}{I} + \epsilon^2 \left(- \sum_A^P x^2 \sin\phi \cos\phi \frac{\lambda s}{F} + x' \sum_A^P x \sin\phi \cos\phi \frac{\lambda s}{F} \right) \right] W - W c\, \epsilon \sum_A^P x \sin\phi \cos\phi \frac{\lambda s}{F}$$

$$C_5 = \left[\sum_A^P x^2 \frac{s}{I} - x' \sum_A^P x \frac{s}{I} + \epsilon^2 \left(\sum_A^P x^2 \cos^2\phi \frac{\lambda s}{F} - x' \sum_A^P x \cos^2\phi \frac{\lambda s}{F} \right] W + W c\, \epsilon \sum_A^P x \cos^2 \frac{\lambda s}{F}$$

$$C_6 = \epsilon \left[\sum_A^P u' x \cos\phi \frac{\lambda s}{F} - x' \sum_A^P u' \cos\phi \frac{\lambda s}{F} \right] W + W c \sum_A^P u' \cos\phi \frac{\lambda s}{F}$$

The notation and initial assumptions are the same as those used by Mr. Weiner. If the eccentricity is zero, the coefficients, *C*, will be the same as given by the author in Equations (25), Appendix I, for the arch with symmetrical loads. For a more exact computation the neglected terms in Equations (75) and (76) may be obtained as shown by Professor Rathbun.[16]

If the abutments of a skew arch are likely to move due to the initial stresses, the error in computing the reactions depends upon the assumptions of the abutment displacements in six co-ordinates. The stress analysis may be simplified by introducing hydraulic jacks at the crown during construction, as shown in Fig. 38. Some of the unknown stresses at the crown may be eliminated by properly detailing the jack bearings. Moreover, the arch may be pre-stressed to almost any desirable condition by assuming a temporary load.

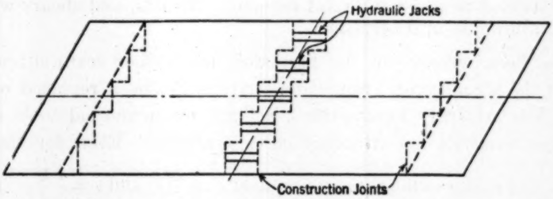


FIG. 38.—PLAN SHOWING HYDRAULIC JACKS METHOD OF POURING SKEW ARCH

It should be noted that a considerable departure from the fundamental requirements of the elastic theory of the statically indeterminate structures is made in applying the skew arch equations to the "rigid frame." This is due to the nearly right-angle corners in the neutral axis of the frame shown in Fig. 29.

For a rigid scientific analysis of stress distribution in a skew arch it has not yet been sufficiently proved that the effect of shear stresses at the sharp corner of a section, Fig. 12(*b*), will be carried uniformly over the entire width. Bearing in mind the elastic properties of concrete, it is more likely that the angle, θ (Fig. 14), decreases rapidly from the face toward the middle of the section.

The design of construction joints has tremendous influence on stress distribution in a skew arch. Experimental tests[17] with a hard rubber model of a skew arch has shown a symmetrical distribution of stresses due to the horizontal thrust parallel to spandrel walls and to the vertical reactions. This fact may be utilized in designing saw-toothed construction joints normal to the neutral axis in the arch and abutments, as shown in Fig. 38.

Numerous assumptions were made in these equations for stresses in a skew arch in order to simplify them and, therefore, experimental tests with models of a fairly large scale should be made to determine the magnitude of the errors introduced.

---

[17] *Engineering News-Record*, July 21, 1927, p. 106.

The author has performed a laborious task in presenting this contribution to the theory of skew arch design, and further tests are needed before conclusive statements can be made.

BERNARD L. WEINER,[18] ASSOC. M. AM. SOC. C. E. (by letter).—In engineering literature too little attention is given to the study of internal stress distribution. The design of any structure may be divided into two parts:

(1) The computation of the reactions, and from the reactions, the determination of the total moments, thrusts, and shears acting at the critical sections of the structure.

(2) The determination of the distribution of the unit stresses produced by the total stresses, in order that the sections may be proportioned to resist the total moments, thrusts, and shears within the allowable unit stresses.

Much has been written on the first step, but rather scant attention has been paid to the second step. Scientific analysis has been replaced repeatedly by more or less arbitrary assumptions, yet the mathematical tools necessary for the proper analysis are in many cases available. Even for the case of the straight beam for which the formulas, $f \doteq \frac{Mc}{I}$, and $v = \frac{VQ}{bI}$, apply, the unnecessary assumption is usually made that a plane section before bending remains a plane after bending. In this case, that assumption happens to be true, but it would be more scientific to prove it rather than merely to assume it. The proof also happens to be simple. What is true of structures, in general, is also true of the skew arch, in particular.

In 1924, J. Charles Rathbun, M. Am. Soc. C. E., published his masterly exposition of the analysis of the skew arch.[19] About two years later he revised and completed his theory as consultant for the Westchester County Park Commission. This theory formed the basis of Part I of the writer's paper; but this work concerned itself almost entirely with the first step; that is, it gave a method, or rather a mathematical theory, for computing the total moments, thrusts, and shears acting on any section of a skew frame. The second step was touched upon both in the report to the Commission and in correspondence with Professor Rathbun. The suggested treatment of the second step was entirely inadequate and scientifically unsound. The result of the writer's study of this part of the subject is given in Part II. Similar studies can profitably be made for many other types of structures, which are still designed more or less arbitrarily.

Such criticism as has been published deals only with Part I. The writer is rather disappointed on this account for he had hoped that criticism of Part II would bring to light valuable information on the theory of internal stresses, especially on the subject of torsion in reinforced concrete.

Mr. Houtman doubts the validity of Professor Rathbun's theory of skew arches when applied to a wide arch. This part of the theory can scarcely be considered open to reasonable doubt as it has been substantiated by tests on models. These tests were made by Professor Rathbun on plaster models,[20] and by George E. Beggs, M. Am. Soc. C. E., on hard rubber models,[21] and they check closely with the results obtained by computation. More recently, Professor Rathbun made similar tests on a plaster model of a multiple-span skew arch.[22] For design purposes, at least, the skew arch theory for obtaining the reactions, seems to be adequate beyond reasonable doubt.

---

[18] New York, N. Y.

[19] *Transactions*, Am. Soc. C. E., Vol. LXXXVII (1924), p. 611.

The writer fails to see the logic of Mr. Houtman's method of analyzing a skew arch. No proof whatever is given for the various statements, except some rather vague references to the theory of least work. Referring to Fig. 36, he says, "there is nothing mysterious about the central rectangular arch, *E B F D*." As a matter of fact, there is a great deal of mystery about this arch. For instance, it is not all clear how the vertical shears acting on the vertical surfaces, *D E* and *B F*, are taken into account. Reference to Part II of the writer's paper, will show that there are also a horizontal shear, a thrust, and a bending moment—the values of which per unit length are variable—acting on these surfaces. It is also not clear what the relation is between the analysis of the right arch, *A B′ C′ D*, and that of the skew structure, or why it is on the safe side to assume that the resultant, *R*, is equal to the resultant of this right arch. Furthermore, Fig. 36 may be redrawn so that the central right arch, *E B F D*, disappears entirely, as shown in Fig. 39. Following Mr. Houtman's method, the problem is changed from the design of one skew arch, to that of a smaller skew arch and two triangular arches. Fig. 39 may even be drawn so that the lines, *B F* and *D E*, will coincide. In this case two arches which cannot support themselves will have to support each other. This method of analysis will not bear close scrutiny.

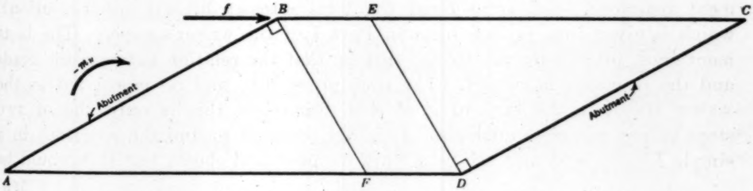


FIG. 39.

It is true that skew arches have been designed and built in entire ignorance of skew-arch principles and that only a few have failed. Of those that have not failed, nothing is known about their factors of safety, nor is anything known about their economy of design. Mr. Houtman stated that "usually failure resulted from excessive compression at the obtuse corner * * *." In view of the skew arch theory, the correctness of this diagnosis is doubtful. The prevailing opinions regarding the intensity of the compression at the obtuse corner seem to be very much exaggerated. In the writer's notation, the moment which causes this compression is m_u. Referring to Table 8 of the main paper, the dead load value of this moment is − 2 071 kip-ft. and acts as shown in Fig. 39.

---

[20] "Crown Stresses in a Skew Arch," *Transactions*, Am. Soc. C. E., Vol. 94 (1930), p. 135.

[21] *Loc. cit.*, Vol. 88 (1925), p. 1208.

[22] "An Analysis of Multiple-Skew Arches on Elastic Piers," Part IV, *Proceedings*, Am. Soc. C. E., April, 1931, Papers and Discussions, p. 539.

The stress, f, treating the arch as an unreinforced beam, is, in pounds per square inch:

$$f = \pm \frac{6\ m_u}{6^2\, t} \sec^2 \theta = \pm \frac{6 \times 2\,071\,000 \times 12}{(66.9)^2 \times 144 \times 54} \times 2.42 = \pm 10.35$$

The numerical value of this moment is large, but since the entire width of the arch acts as the depth of a beam to resist it, it follows that the resulting unit stress is small. Furthermore, the peculiar cracks at the acute corners of existing skew arches, to which Mr. Houtman refers, can scarcely be due to tension because of skew arch action, since the longitudinal reinforcement which is designed to resist the ordinary bending moment, M_z, will take up this tension in the action of the arch as a beam the depth of which is the width of the structure. Furthermore, it has just been shown that the estimates of the intensity of this tension are probably much exaggerated.

It is well to point out in this connection, that if the arch is poured in longitudinal strips, care must be taken that each section be well keyed in to the adjacent section by both horizontal and vertical keys, in order to provide adequate shear resistance. Lack of such shear resistance, of course, will destroy the ability of the arch to act as a single beam of depth, b (skew width of the arch).

In the second paragraph of his discussion, Professor Rathbun uses the expressions, "this theory," and "this method of analysis," rather loosely. What Professor Rathbun is referring to is his own theory for obtaining the abutment reactions (and from them the total stresses at any other section), which is given in a revised form in Part I of the writer's paper. The tests mentioned, prove only one thing; that is, that the relation between the loads and the reactions is correct. The tests prove this, and no more; but as the writer stated in the beginning of this discussion, this is only one of two steps in any complete analysis. It is easy enough to find the reactions in a simple I-beam, and also the maximum moment and shear; but this would be of little value in finding the unit stresses if one did not know that $f = \frac{My}{I}$, and that $v = \frac{VQ}{bI}$. Professor Rathbun made a slight and very unsuccessful attempt to solve the second and equally important part of the problem.

Toward the end of his discussion, Professor Rathbun states:

> "In the development of the unit stresses, Mr. Weiner has used a different distribution for torsional stresses. Otherwise, it is fundamentally the same as that used by the writer for several years."

The writer must take issue with Professor Rathbun on this point. He has been familiar with Professor Rathbun's method of stress distribution[23] for some years and disagrees entirely with that method. Obviously, since Professor Rathbun devotes only about four pages to his method, and the writer has thought it necessary to devote about sixty pages to this part of the subject (which forms Part II of his paper), there must be some fundamental difference between the two theories. The results obtained by the use of the two methods are radically different.

---

[23] "An Analysis of Multiple-Skew Arches on Elastic Piers," Pt. II, *Proceedings*, Am. Soc. C. E., April, 1931, Papers and Discussions, p. 524.

In this discussion it is not necessary to go into the differences between the theory of unit stress distribution as given in Part II and Professor Rathbun's method to which reference has been given. To do so would be mere repetition. Some of the more vital points have been discussed by the writer[24] in his criticism of the aforementioned paper by Professor Rathbun. Suffice it to say that the difference in the torsion formula used is only of minor importance. As the writer has remarked several times, the subject of torsion in reinforced concrete is still an open question; and much work remains to be done in this connection. Furthermore, in his development of the theory for proportioning the sections, the writer has given ample proof for each individual step and has not relied on mere assumption. The comparison, therefore, can be easily made by the interested reader.

As pointed out in the reply to Mr. Houtman's discussion, there is a difference of stress between the obtuse corner and the acute corner of the abutment, and the stresses are compression and tension, respectively. The general opinion as to the magnitude of this stress, however, is exaggerated, as shown by the numerical example which gives an indication of the range of the magnitude of these stresses. Professor Rathbun is in error in stating that his equations do not substantiate this belief. This belief is correct as to the presence of the stress, but not as to its magnitude.

In his discussion of Part I, Professor Rathbun is correct in stating that the practice of dropping terms of low values should be treated with caution. The writer expressed the same idea in his paper, and stated specifically that the question should be looked into carefully by the designer.

Professor Rathbun does not see what is to be gained by the changes in his part of the theory. The writer used Professor Rathbun's unrevised manuscript for a period of about two years. Blue-print copies were furnished by A. G. Hayden, M. Am. Soc. C. E., to many bridge designers all over the United States and abroad. It was found after this period of trial that, for one thing, the system of signs was very confusing and difficult to remember. Constant reference had to be made to the table of nomenclature in order to interpret the signs of the stresses in terms of actual directions. Professor Rathbun referenced his system of co-ordinate axes and his system of signs to directions in space, with the result that forces and moments changed signs with the direction of the skew for the same type of loading. The writer referenced his system to the actual structure, with the result that the stresses for the same type of loading always had the same sign. Furthermore, the sign system was organized in order to make it easier to remember. It was not a question of mathematical correctness, but one of practical convenience.

---

[24] *Proceedings*, Am. Soc. C. E., September, 1931, Papers and Discussions, p. 1107.

Professor Rathbun errs in saying that the direction of T_y makes it inconsistent with the idea that the derivative of the moment is the shear, and that it is inconsistent with the definition of shear in simple beams. The common definition of the vertical shear in a simple beam would make T_y act as shown in Figs. 4 and 5. Furthermore, from the equations it follows directly that $\frac{\partial m_z}{\partial v} = t_u$, and this expression is introduced several times in Part II. As a matter of fact, it would make no difference if the direction of the shear, T_y (t_u), were reversed. The most serious consequence would be that $\frac{\partial m_z}{\partial v}$ would equal $- t_u$. This might perhaps be inconvenient, but not incorrect. It might also be pointed out that Professor Rathbun, too, is guilty of using double signs for T_y and M_y. This is not caused by the direction of the stresses, but by the simplification made on account of symmetry, and applies only to symmetrical arches. He is in error, however, in including T_z in this category.

Mr. Eremin is correct in adding the terms containing Wc for eccentric loading. The reactions thus obtained for this type of loading would undoubtedly be correct; but whether this would be of any practical value at present is questionable, because the problem of concentrated load distribution would then have to be taken into consideration. This problem is not confined only to the skew arch; it is present also in the right arch. It is a question whether a single line of trucks near the curb of a bridge might not perhaps produce greater stresses at some points than are obtained for the bridge fully loaded.

In his discussion[24] of Professor Rathbun's paper on "Multiple-Skew Arches," the writer has pointed out that the tests made by E. F. Gifford, Assistant Engineer in the Engineering Department of the New York Central Railroad, showed in general that for eccentric loading there are two types of failure: (1) A local failure in the immediate vicinity of the load; and (2), a general failure at points far enough away from the load so as not to be affected by the local load distribution stresses.

For sections where the maximum moments occur with the load at the section, the local load distribution stresses become important since so little is known about them. For this reason knowing the correct reactions for eccentric loading does not help a great deal. Here, again, the all-important question of internal stress distribution enters, and even though the total stresses are available, the unit internal stresses still remain a mystery For the sake of completeness, it is well to include the terms representing the eccentricity of the load and the writer is grateful to Mr. Eremin for pointing this out and for supplying the missing terms.

The application of the skew arch equations to a rigid frame is not a departure from the fundamental requirements of the elastic theory, except that the stresses at the corners are unknown. For the right rigid frame the effect of the corner on the stresses in general has been found to be negligible by comparing the analytical results with those obtained by use of the Beggs' deformeter gauges. Before the first rigid frame was built, the corner was tested for strength, and it was found that failure occurred everywhere, except at the sharp corners where it was predicted.

Mr. Eremin is in error in his interpretation of Fig. 12(*b*) and Fig. 14. These two diagrams, as well as Equations (30), (31), and (32), apply only to the face of the bridge. Nothing is said at this point about the value of the unit shear toward the middle of the bridge. Mr. Eremin is correct in surmising that the shear at the sharp corner of the section is probably not carried uniformly across the section. This is proved by Equations (35) which give the value of the unit horizontal shear at any point of the section, and similar proofs are given for the vertical shear and shear moment per unit width. The angle, θ, does not vary since it is the skew angle. What Mr. Eremin has in mind is the angle, ψ, which gives the direction of the resultant stress at any point. At the face of the bridge where Fig. 12(*b*) and Fig. (14) apply, $\psi = 90° - \theta$, and the resultant stress is parallel to the face of bridge, as it should be.

The writer has found by experience that the radial shear is negligible. It can easily be obtained, however, from Equations (51) and (52), since $V' = s V$. The shear at any point in any radial plane is given by $V'' = V \sin \psi - V' \cos \psi$. The maximum or minimum radial shear is given by $V'' = \pm V \sec \theta'$, on a plane making an angle, $\psi = 90 + \theta'$. The writer has often been questioned in regard to these equations; they are given here for the sake of completeness.

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
TRANSACTIONS

Paper No. 1816

MANUFACTURING CONCRETE OF UNIFORM QUALITY[1]

BY WILLIAM M. HALL,[2] M. AM. SOC. C. E.

WITH DISCUSSION BY MESSRS. HENRY B. SEAMAN, THEODORE BELZNER, JOHN SANFORD PECK, J. W. KELLY, STANTON WALKER, EDWARD E. BAUER, O. BONNEY AND D. T. MITCHELL, C. E. ARNOLD, MORRIS MASON, JOHN H. GRIFFITH, W. B. WENDT, AND WILLIAM M. HALL.

SYNOPSIS

Most of the literature on monolithic concrete construction appears to be devoted to laboratory practice, the water-cement-ratio theory, and subjects relating thereto. Although the knowledge and technique thus obtained are essential for the construction engineer, they may not be as difficult to acquire, as it is to perfect the field methods of mixing and placing concrete of a desired uniform quality and strength.

In 1930, and a few years prior thereto, the United States Government completed, under direction of Lt. Col. George R. Spalding, U. S. A., nine locks and dams in the Ohio River between Louisville, Ky., and its mouth (nearly 400 miles). This task required more than 750 000 cu. yd. of concrete. In 1925, a systematic and uniform method of inspection and control of concrete manufacture was authorized, inaugurated for all the dams, and carried through to the end of the work. The purpose of this paper is to record the results obtained and to give a detailed description of the methods pursued with the hope of inspiring in others an ambition to adopt and perfect the practice of manufacturing concrete of a specified strength and grade with a reliability equal to that of the best of other structural materials.

With a realization of the importance of the perfect execution of all the inspection and of the co-operation of all the working forces in every detail of field work required in the manufacture and placing of concrete and

---

[1] Published in May, 1931, *Proceedings*.

[2] Civ. and Cons. Engr., Parkersburg, W. Va.

in the fair sampling and making, curing, and breaking of cylinders, the writer has included in Appendices a detailed description of the methods followed by two field inspectors, Messrs. R. W. McBeth and C. F. Little.

The nine locks and movable dams are the last of a series of fifty built for the improvement of the Ohio River so that with slack-water there will be a minimum depth of 9 ft. on the lock-gate sills and the navigable pass sills of the dams. All the structures are now (1931) in operation at a total cost of construction to the United States of about $115 000 000.

INTRODUCTION

The requirement of first importance for any well-designed concrete structure is that the concrete be satisfactory as to: (1) Soundness; (2) density; and (3) uniform strength, approximately as designed, throughout every part of the structure. Although due regard should be given to the element of cost, every concrete structure should have a probable life as long as its intended use, or if that is unknown, for a period of more than 100 years, except in the parts which may be worn out by severe abrasion.

Although it is now nearly forty years since, in the early Nineties, the design and construction of monolithic concrete structures became active in the United States and Europe, more than 50% of the concrete being made is of poor quality. Its probable life is short and only a small part will comply with conditions for quality as proposed for it. Therefore, the question of how to eliminate the causes of such failures has long been of great importance. Of the three major qualities sought in its manufacture, uniform strength is usually the one of greatest concern; or, expressed otherwise, the improvement most desired is to design a mix of minimum required strength and, then, with confidence, to be able to guarantee every batch of it.

Although the choice of materials is of paramount importance, the personal element is a large factor in the manufacturing and testing of the concrete. Variations in quality may occur due to: (1) The method of procuring, inspecting, and delivering the materials; (2) the choice and installation of the mixer; (3) the arrangement of all the features of transportation from mixer to the finished concrete in the form; (4) the measurement of all the materials entering the mixer; (5) the method of charging; and (6) the sampling, making, seasoning, and breaking of test cylinders. Not only do the inspection and cylinder records indicate the relative durability and value of concrete, but so, also, does its appearance when being placed and when finished. When inspection records are favorable with regard to these features the engineer, architect, and owner may feel confident of the safety and durability of the structure, provided its placement is known to be well and thoroughly done.

It is with this thought and for these reasons that the writer is submitting some records of work done, methods pursued, and experience gained in an effort to produce concrete of uniform quality. No one or two easy remedies can be formulated or found, but the way to success, like that which has been required for all other good construction, will include the systematic training of the workmen who are entrusted with the principal details of concrete manufacture, placing, and treatment. Uniform success in producing good concrete requires reasonably suitable materials; and it also requires contractors, superintendents, and foremen with the spirit to co-operate; competent inspectors; and workmen with training, skill, and reliability for measuring quantities of water, cement, and aggregates, day after day, with unfailing accuracy. It is not intended to discuss theories of concrete design, laboratory practice, or specifications, except as they affect the inspector's work in the field. This task involves the selecting and inspecting of materials and the teaching and directing of the workmen as to their duties and faithful performance. By excellency of control he may be able to make the contractor and other managers of the work realize that such methods are for their best interest in producing concrete with a favorable record for quality, and that the net result will be economy in the cost of the finished work.

The greatest reason for much defective work is due to the owner's ignorance of its quality (or of how badly he has been duped), immediately after completion. Many engineers in highest authority have accepted a poor quality of concrete only to realize it years later. The appearance of the mix or an immediate test of the concrete, when freshly mixed or placed, does not disclose its quality with sufficient accuracy to separate the poor from the good, except that it indicates when excess water is present, that aggregate has segregated, or that the mix was greatly deficient in cement, sand, or water. Furthermore, the need of either accepting the concrete immediately or of condemning and destroying it has tended to permit the construction of much poor work. The cost of repairs or early renewals of defective concrete constructed in the United States to date (1931) will probably be in the tens of millions of dollars.

Although in the Nineties monolithic concrete construction was strongly condemned by engineers of high repute as an unreliable structural material, the great urgency for speed in the construction of fortifications in 1898, at the twenty-six strategic points encircling the coast line of the United States from Maine to Oregon, so broadcast the knowledge of its adaptability for rapid masonry construction that its use in all classes of bridge and industrial masonry was thereby hastened. Since then its use has been extended rapidly to include almost all classes of masonry. It was possibly the great extent of that fortification construction program and the concurrent demand for speed that saved it from general condemnation as a fit structural material and substitute for stone and brick masonry. The great pity is that in the decade following 1895, some super-power could not have controverted entirely the arguments advanced by advocates of excessively wet concrete mixtures and thus have avoided the introduction of long chutes which were so injurious, or ruinous, to good concrete. These two wrong methods of construction and a few others, which were much in use from 1900 to 1920, were apparently in danger of eliminating almost entirely the use of the better methods of mixing and placing which were very well known and strongly advocated by a few.

PRACTICE ON THE OHIO RIVER

Most papers and published records show an almost complete lack of uniformity in terms of maintained strength for job concrete up to or above that designed, even with aggregates of fairly uniform quality of grading and of water content. For many years on the construction of the Ohio River locks and dams, sand and gravel (usually with poor natural grading), were dug at the work and delivered at the mixer for immediate use. This practice made the control of the water content much more difficult than with stored aggregates. Many efforts were made to secure better concrete under such difficult conditions, especially in the past five years (1926-1931) during which more than 750 000 cu. yd. of monolithic concrete were placed in the nine locks and movable dams constructed in the Louisville District of the Ohio River. A brief statement of the methods used and the results obtained may contribute to a better understanding of the requirements necessary in organizing and conducting all the most essential details of mixing, placing, and testing concrete.

The concrete construction of Auxiliary Lock 41, at Louisville, Dam 47, near Evansville, Ind., Lock 51 and Dam 51, at Golconda, Ill., was done by contract, and the remainder by hired labor forces under the direction of a resident "Assistant ,in Charge", reporting directly to the District Engineer. The location and general description of these structures have been discussed by the writer in another paper.[3]

The Concrete Inspectors.—The inspectors assigned to mixing and placing, and the cylinder makers, were part of a force organized in 1925 for the field work of laying out and inspecting the work of construction of the locks and dams. Many members of the assembled force and nearly all the men at the mixers had previously had very little experience, instruction, or laboratory and field training in manufacturing or placing concrete. This accounts for the large number of failing cylinders shown in the report for 1925.

At the beginning of the work, all the members of the concrete inspection force were required to study a digest of the literature on the subject, including the theory of Duff A. Abrams, M. Am. Soc. C. E., on the water-cement ratio, his proposed methods of determining fineness modulus for aggregates, and other methods of control. A system of making and testing concrete cylinders was also begun in August, 1925, A short time afterward the inspection forces, with some of the principal members of the construction force, were organized as a school. Its first meeting of a week, with two sessions daily, was held in January, 1926, at Dam 52, Brookport, Ill., the next two meetings were held at Owensboro, Ky., and the fourth was held in Louisville, in February, 1928. Under a leader, the inspectors were required to study at their respective stations, the specifications for the job, the specifications of the American Society for Testing Materials, and the several bulletins[4] and booklets[5] formulated by Professor Abrams. Then they attended classes, as stated previously, conducted by the writer, with J. W. Kelly, Assoc. M. Am. Soc. C. E. (through the courtesy of the Portland Cement Association) as principal teacher. Papers were also prepared, read, and discussed by members of the inspection force on questions relating to the plant, their own difficulties, and construction features at the various locks and dams. Copies of some of the complete papers, and carefully prepared minutes of all sessions, were later furnished to the entire force. Under the Abrams' standard for average strength of cement,[6] the specifications for the job should produce a concrete made with the average Ohio River gravel, with a compressive strength of about 1 700 lb. per sq. in. at 28 days. The job standard of minimum strength was set at 2 000 lb. per sq. in. for 28-day concrete, and 2 800 lb. per sq. in. at 6 months which is a high standard for the specified proportion of cement to aggregate.

---

[3] *Transactions*, Am. Soc. C. E., Vol. LXXXVI (1923), p. 94, Fig. 1.

[4] *Bulletins 1* to *12*, Lewis Inst., Structural Materials Research Laboratories, Chicago, Ill.

[5] Pub. by the American Portland Cement Assoc.

During 1925, 1926, and 1927, only one cylinder was made from each sample or batch of concrete. In 1928 and 1929, the cylinders were made in pairs, two cylinders from each sample or batch tested. In this manner one cylinder of the pair checked the fabrication of the other. If one cylinder failed in proposed strength, or if the tests showed a great difference in strengths, it was believed that the higher record could be assumed to be the true measure of quality of the entire batch from which the sample was taken. Throughout the seasons of 1925 to 1928, inclusive, the cylinders were packed in sand, two in a box, and shipped to a laboratory at Louisville. During 1929, the cylinders were broken under contract at the Testing Laboratory of the University of Louisville. The agreement included the care of the cylinders during the last days of seasoning, and re-capping them when the ends were not perfect. The record for 1929 included every cylinder made. For the other years only about half a dozen cylinders were rejected due to injury or other cause that clearly made the break recorded an unfair test. Table 1 indicates the strength of the concrete. Of the thirty-eight cylinders (nineteen pairs) made at Auxiliary Lock 41, Louisville, which were tested at the age of six months (see Column (12), Table 1), four (each cylinder of two of the pairs) were subjected to 200 000 lb. (7 100 lb. per sq. in.) and failed to break. This was the limit of compression for the testing machine available. One month later, one of these cylinders broke, in the same machine, under a total pressure of 194 000 lb., and its mate at 182 820 lb., or 6 450 lb. per sq. in.

SPECIFICATIONS

The specifications for the work provided that the concrete should be composed of cement, hydrated lime, sand, gravel, and water. The cement measure was by count of bags for each batch. The aggregates were measured by leveling up to exact marks set in the charging hoppers or by batchers and inundators. The water was measured by gauges in barrels or by water meters. The sand, gravel, and water were taken from the Ohio River, under specifications requiring the best that could be produced in the locality of the structure. The mix per cubic yard of concrete consisted of 5 bags of cement

---

[6] *Bulletin 1,* "Design of Concrete Mixtures," by Duff A. Abrams, Lewis Inst., Structural Materials Research Laboratory, Chicago, Ill.

(6 bags for all reinforced concrete and tops of walls), 25 lb. of hydrated lime, or other equally good admixture, and 25 cu. ft. of gravel. Sand was required sufficient to yield 15% or more of mortar than was needed to fill the voids in the gravel and sufficient water to produce a slump of 2 to 4 in. for mass concrete and 5 to 6 in. for reinforced concrete; that is, such that the feet would not sink in freshly spread concrete deeper than 10 in., nor less than 2 in.

TABLE 1.—CYLINDER TESTS OF CONCRETE IN DAMS ON THE OHIO RIVER IMPROVEMENT

Item	SEASONS:	1925		1926		1927		1928		1929	
	Age:	28 days	6 months	28 days	6 months	28 days	6 months	28 days	6 months	28 days	6 months
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
1	Number of cylinders..	81	44	172	90	236	134	206	109	64	40
2	Average strength per cylinder, in pounds per square inch.....	2 157	3 046	2 719	3 777	2 685	4 119	2 691	3 640	4 779	6 082
3	Average strength of all cylinders testing less than Item 2, in pounds per square inch......	1 590	2 321	2 186	3 047	2 128	3 394	2 215	3 194	4 308	5 529
4	Average strength of all cylinders testing more than Item 2, in pounds per square inch................	2 933	3 720	3 217	4 444	3 302	4 801	3 326	4 130	5 186	6 587
5	Percentage of cylinders testing less than Item 2..............	58.3	42.2	48.8	47.8	52.5	48.5	56.3	53.2	62.5	47.5
6	Percentage of cylinders testing less than 2 000 lb. per sq. in....	56.8	......	11.6	......	15.2	......	11.7	......	0	......
7	Percentage of cylinders testing less than 2 800 lb. per sq. in....	......	37.7	......	13.3	......	4.5	......	7.3	......	0
	Compressive strength, in pounds per square inch:										
8	Maximum.........	5 256	4 725	4 424	5 358	4 512	5 836	5 120	5 210	5 930	7 000+
9	Minimum..........	919	1 592	920	1 592	1 027	2 300	1 260	2 260	3 417	4 790
	Slump, in inches:										
10	Average.............	3.24	3.3	4.04	13.8	3.6	3.5	3.24	3.5	3.09	3.0
11	Maximum...........	7½	......	7	......	7	......	8½	......	6	4.5
12	Minimum............	½	......	1	......	1	......	½	......	2	1.5
13	Number of tests in which the slump was less than in Item 10..	49	19	91	53	130	63	111	59	40	14

It is believed that the workmen complied with these specifications in general, except for the batches that fell below the desired strength, or those below the proposed strength (see Items 6 and 7, Table 1). In these cases there was probably either some defect in complying with the specifications or in the making, handling, or breaking of cylinders that were defective. It was the rule to keep a complete record of every detail which might affect the strength of every cylinder from the time the sample was taken to the time the cylinder was broken. Even so, in the case of more than 50% of failures to exceed the proposed strength, it was impossible to find an adequate explanation of the cause. The observable cause of greatest frequency was the segregation of large-sized gravel which produced evident weakness because the mortar peeled off and left a clean, bright surface of stone. Another frequent condition observed was a lengthwise split in the cylinder, indicating an imperfect bearing on the end. In 1929, this defect was eliminated by greater care in detecting and re-capping slightly defective ends.

INSPECTION

The cost of maintaining any desired minimum strength will probably not be any more than that of concrete which actually fluctuated much above and below such standard of strength. Uniform maintenance certainly requires a knowledge of concrete materials and of manufacture on the part of all concerned and also a systematic and constant control, as may be better understood from Table 1 and a few comments based on the instructions given the inspection force from time to time.

Advance Study.—Before being placed on duty, inspectors were instructed and helped to become thoroughly familiar with the specifications, the tests relating thereto, and the methods of making them. Furthermore, they were taught the duties involved in the direction and instruction of workmen. They were also instructed in the technique of obtaining samples of materials and of analyzing the different grades of sand and coarse aggregates available.

Laboratory Experience.—For a better understanding of the literature studied, each inspector analyzed the sand and gravel being used. He mixed concrete by hand and made experimental cylinders from them, recording his results and reporting them in writing to his Senior Inspector. For each of the proposed slumps (approximately 2, 3, 4, 5, and 6 in. with the materials furnished and accepted for concrete on the particular job assigned), the inspector determined the water-cement ratio and kept a record of it. Aggregates were analyzed and, if the particular job received several distinct grades of aggregates, tests were made of each. As soon as possible after construction was in progress, the inspector plotted "typical job curves" of 28-day cylinders, indicating the water-cement ratio on each curve. Copies of such data were promptly forwarded to the office for review.

Practical Field Inspection.—The greatest possible care was required of inspectors in the field to obtain exact measurements of materials for each batch of concrete. The quantity of aggregates required by specification was their measure, "dry and rodded". Because the bulking of gravel is so slight and so nearly constant, its measure was considered exact or, at least, correct within 1 per cent. The measure of sand was variable because the gravel carried sand, and when an inundator was not used, the bulking was irregular. As nearly as possible, the measure of water was maintained so as to give a 3-in. slump, except that a 4 to 5-in. slump was required in case of concrete used with reinforcement and where it was difficult to place dry concrete. To maintain this slump uniformly every mixer inspector needed to understand the principles of the water-cement-ratio theory, the best and quickest methods of keeping informed as to measures of materials, quantity of water in aggregates, and method of correcting error in slump by changing the measure of water or sand, or cement in an emergency, and all things relating thereto at his plant. The slump and workability are functions of all the foregoing variables, and the importance of maintaining their uniformity was impressed on the inspectors. Before starting a monolith and at all times when the mixer was in operation the inspectors were expected to have perfect arrangements made for quick communication and co-operation, and for instant action.

As early as practicable, and before beginning the concrete mixing, the inspectors became thoroughly familiar with the operation of the mixing plant and with the personnel assigned to it; and they satisfied themselves that every one assigned to a particular task could perform his duties accurately and efficiently. Furthermore, they inspected the form to see that it was completely finished with a well prepared foundation bed properly unwatered. If it was a connection with old concrete, they looked for laitance, and directed its thorough removal. The old surface, thus prepared, was completely covered with a thick layer of mortar or grout.

The regulation of the quantities of materials and of the time of turning the mixer for each batch of concrete are the mixer inspector's most important duties. When the mixer is in operation he should remain on duty until relieved by another inspector. Dumping or chuting concrete into forms should be done so as to require spreading only, or a short movement, at most, not exceeding 10 ft. Great care should be taken to prevent any part of the concrete from striking the forms, the forming rods, or the bracing timbers. These items were emphasized in the instructions issued to inspectors.

The greatest care possible was taken to obtain fair samples for slump tests, and those for cylinders were preferably taken from a 5-bag mix. The inspector was instructed to indicate on the records whether lime or an extra half bag of cement was used, or whether any other variation or irregularity occurred. The age, make, and bin number of cement, and the seasoning temperature of cylinders and the maker's name, were also required.

Regarding slump tests, the instructions specified that: Samples of concrete for slump tests shall be taken within 10 min. of its discharge from the mixer. When starting concrete work, the slump test shall be made from the first batches mixed, and continued until a 3-in. slump is obtained and is running uniform; thereafter, while mixing is in progress, it shall be made at intervals of 1 hour, or oftener in case the consistency appears to change. Samples for cylinders shall be taken from forms immediately after being dumped and within 10 min. of discharge from mixer. A chart similar to progress drawings shall be kept, indicating (1) the location of each concrete monolith; (2) the cylinder batches in each monolith by serial number; and (3) its approximate location in the monolith.

Instructions as to Removal of Forms.—No forms were permitted to be removed in less than the time stated in the specifications without actual physical inspection by an assistant in charge to determine the set of the concrete and without concurrence by the senior inspector. During summer weather it was considered entirely safe to remove bulkhead forms after 24 hours from lifts of approximately 6 ft.

During the summer, the inspectors were permitted to allow a few men to walk on the concrete for the purpose of brushing after about 6 hours, but no succeeding lifts of concrete could be cast above one lift for a period of 48 hours.

Especial attention to the curing of concrete was required, and inspectors were instructed to follow in detail the specifications as to time of keeping forms on monoliths and of keeping surfaces wet.

Test Cylinders.—As every minute detail of the processes of sampling, transporting, making, curing, and breaking cylinders affects the record of breaking strength, the detailed account of the process, as described by Mr. Clifford F. Little, the Inspector who made all the cylinders at Auxiliary Lock 41 in 1929, is given in Appendix I. The standard cylinder, 6 in. in diameter and 12 in. high, was used for that and all other cylinders to which reference is made in this paper.

All the methods used were intended to be as prescribed by the Standard Specifications of the American Society for Testing Materials (Serial Designation, D 138-26, T), but, for obtaining uniformity, a detailed explanation or description of every movement was found desirable.

Due to defective bottoms, the entire first lot of moulds, purchased in 1925, was discarded. The second lot, purchased in 1926, from another maker, gave some trouble of the same kind, which is believed to account for some of the defective cylinder records from 1925 to 1928, inclusive.

A glimpse of the work and some further idea of its character may be obtained from Fig. 1. It shows the lower face of a part of the upper gate miter wall, a little of the finished face of the upper end of the right chamber wall, and the concrete forms erected for the head-house which includes the operating machinery of the upper filling valve. The masonry up stream from the gates is the wall of the old lock with its new concrete top which converted that lock into an upper guide-wall for the new lock. Fig. 2 shows the appearance of more of the face of the right chamber wall just after the forms and before the tie-rods were removed.

The specification for concrete in Auxiliary Lock 41 provided for about 24 cu. ft. of gravel instead of 25 cu. ft.; acting at his own suggestion and without change of specification or price, the contractor furnished double-ground cement instead of the standard required under the specifications. A comparison of the relative strengths of the 28-day cylinders and the cement with which they were mixed in 1929 (and some in 1928) and the relative strengths of the cements used in the corresponding years indicated clearly that the cement is usually the factor of first importance. Of course, it should always be realized that with good cement other errors may ruin the concrete.

In 1928, a particularly bad lot of cylinders was tested in a series made at one dam despite the constant vigilance maintained to prevent such errors. They were made and cured by a trained engineer, but due to other duties, he entrusted, to an experienced laborer, the task of transferring them from the seasoning bin to the place at which they were packed for shipment to the testing laboratory. Although the laborer had been around the laboratory and the

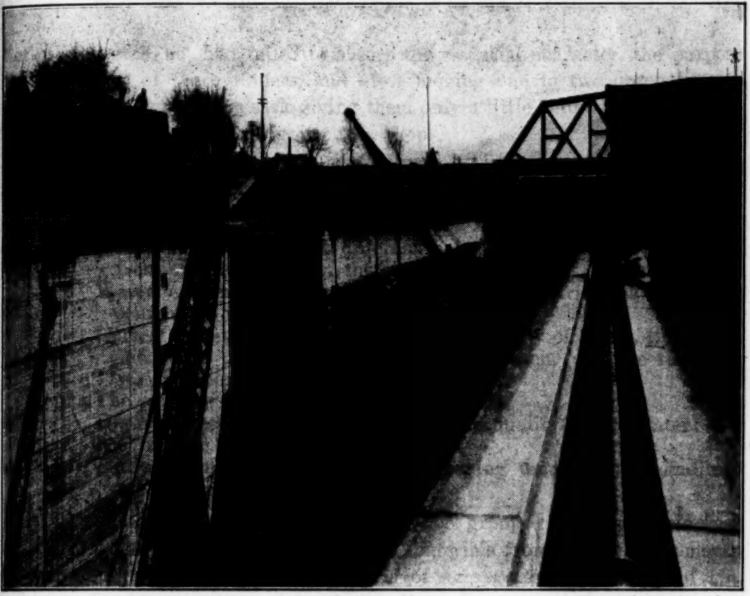


Fig. 1.—View of Chamber Wall Just After Removing Forms, Auxiliary Lock 41, Louisville, Ky.

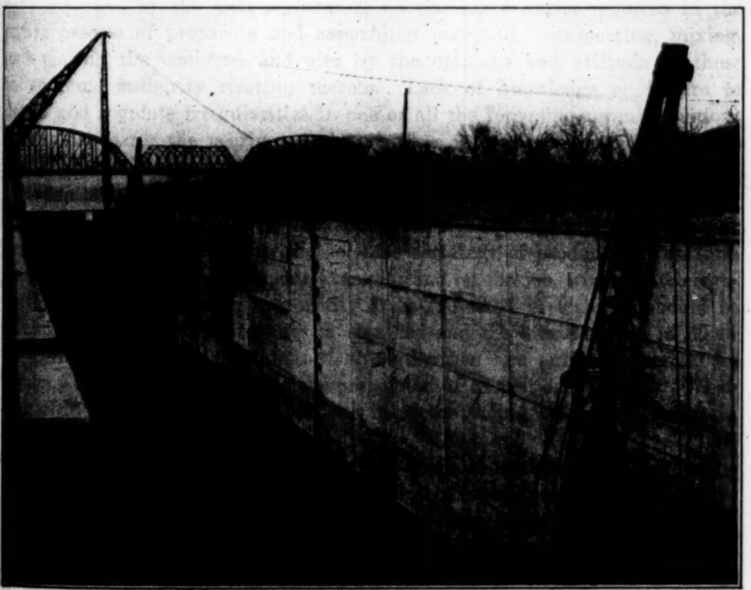


Fig. 2.—Construction View of Auxiliary Lock 41, Louisville, Ky.

work for months, he had failed to absorb the essential character, the purpose or importance of the cylinders, and after moving one to two dozen he was discovered with a pair in a sack giving them only a little better treatment than is customary with dull picks going to a shop.

Irregularities in the Strength of Concrete.—The principal causes of the great variations that occur in job concrete (as indicated by cylinder tests made on this work during four years) are the corresponding variations in:

(*a*) Strengths of cements of different brands, from different bins and from different parts of the same bin;
(*b*) Water content of aggregates, resulting in variations in water content of the batches;
(*c*) Correct measurement of aggregates (especially of sand), the water, and count of bags of cement for each batch (and the resulting water-cement ratio);
(*d*) The grading of aggregates;
(*e*) The cleanness of the aggregates and the quality of the water;
(*f*) The time of mixing the concrete; and
(*g*) The sampling, fabrication, curing, curing temperature, handling, or breaking of the cylinders.

It is evident that these variations may be grouped into two principal classes: (1) The variations in the natural materials from which the cements are manufactured, in the natural grading of aggregates and content of silt and other foreign matter, and in the quality of the water as found in the river or other source; and (2), the variation in men, the competency and care exercised by the men performing all the many duties required in the entire process of preparing and assembling materials, transporting, mixing, and placing the concrete, and also by the opinions and attitude of those in supreme authority relating thereto. Lack of knowledge or failure to detect and regulate irregularities in one or all the foregoing seven conditions is responsible for the nearly universal failure of engineers in their effort to specify, manufacture, and place concrete as good as any specified quality.

When materials of reasonably uniform quality are provided, the writer believes that men can now be so well trained as to manufacture concrete (using laboratory methods) with the same certainty of producing a uniform product as a bread-maker can, for example; and on the job, with great irregularities in materials, such uniformity as is indicated in Table 1 should be obtained. After such uniformity is obtained and the lowest record in 50 or 100 cylinders is above the designed strength, then and only then should an economy be effected by reducing the proportion of cement or otherwise. When such reductions are contemplated, the inspector should be sure to maintain the designed standard.

Cement.—An account in some detail of the field methods used and results obtained in the construction and management in assembling materials may be useful for reference and comparison. All cement for the Ohio River improvement was purchased under specifications and contract, and was sampled and tested by professional laboratories. Samples were taken from every bin of cement and reports were made as to chemical tests, physical tests for fineness, time of setting, and tensile strength of 7 and 28-day briquettes.

Table 2 is an abstract of the cement test reports for 1928 and 1929 at various dams as noted.

Curves were plotted showing fineness and strength tests for 7 and 28-day briquettes. Due to the difficulty of storing and tracing shipments the bin number of the cement in each cylinder was not kept, except for the year 1929. Variation in fineness and in strength between the different kinds of the same cement as well as differences in brands, are more readily and clearly understood by plotting them as curves than by the tabulations of figures from which the curves are made. The variations in fineness in 1927 and 1928 are probably greater than is realized by those not accustomed to make such plottings.

TABLE 2.—CEMENT TESTS, 1928 AND 1929, GIVING AVERAGE TENSILE STRENGTH FOR 7 AND 28-DAY BRIQUETTES AND FINENESS MODULUS FOR EACH BIN OF EACH CEMENT USED

(Mix: 1 part of cement and 3 parts of Ottawa sand.)

Brand of cement	Bin No.	Dam	Number of tests	7-day strength, in pounds per square inch	28-day strength, in pounds per square inch	Percentage of residue on a 200-mesh sieve	Quantity in bin, in barrels	7-DAY		28-DAY		Average fineness
								Minimum	Maximum	Minimum	Maximum	
CEMENT USED IN 1928												
A	11	51	20	315	424	10.3	4 000	267	339	385	447	10.3
B	4	51	18	277	384	16.6	3 500	241	317	353	413	
B	12	51	15	261	370	15.5	6 000	238	279	351	388	
B	13	51	15	269	378	15.7	6 000	242	282	356	390	15.9
C	2	51	80	316	421	13.4	16 000	270	405	381	459	
C	6	50	45	322	378	11.9	9 000	297	354	336	404	
C	12	46 51	60	314	421	12.6	12 000	280	339	388	448	
C	3	50	14	322	390	10.9	2 800	302	347	358	424	
C	4	50	14	291	339	12.0	2 800	272	320	326	367	
C	10	50	24	350	393	12.9	12 000	308	405	363	427	12.7
D	19	52	90	283	....	15.5	18 000	261	322	....	....	
D	20	52	75	278	389	16.8	15 000	244	320	309	424	16.1
E	18	53	12	287	395	13.1	2 400	251	305	366	412	
E	31	41	125	290	398	15.5	21 000	257	331	369	433	
E	34	41	125	279	384	19.0	21 000	250	399	360	423	
E	17	53	12	296	397	13.1	2 400	261	330	365	421	13.3
E	23	53	13	268	....	14.4	2 500	247	289	....	....	
CEMENT USED IN 1929												
F	4	41	16	369	465	3.20	9 000	340	403	420	500	
F	4 *B*	41	16	279	433	5.57	8 500	270	298	422	442	
F	4 *C*	41	16	266	399	9.55	8 500	253	275	388	408	
F	5 *C*	41	16	332	421	5.73	8 500	322	342	412	433	
F	6	41	34	326	461	5.84	17 500	303	360	443	483	
F	6 *B*	41	16	318	414	3.54	17 000	308	328	403	423	
F	10 *B*	41	16	323	413	3.77	8 500	313	333	430	447	
F	19	41	16	394	468	3.52	8 500	383	405	463	477	
F	6 *A*	41	20	299	420	6.25	20 000	292	312	407	432	5.46

In 1927 the 7-day briquettes of all brands used, varied in tensile strength from 241 to 345 lb. per sq. in.; and for 28-day briquettes of all brands used, the variation was from 326 to 455 lb. per sq. in. In 1928, the variation in tensile strength for 7-day briquettes was (see Table 2) from 238 to 405 lb. per sq. in. and for 28-day briquettes, from 309 to 459 lb. per sq. in. In 1929, for 7-day briquettes of Cement F only, the strengths varied from 253 to 405 lb. per sq. in., and for 28-day briquettes (see Table 2), the variation was from 388 to 500 lb. per sq. in.

In addition to the contract tests, complete field tests of cement that had been kept over winter were made according to the Ohio River General Specifications. Furthermore, two samples were taken from each car of cement as received at the work, one from each end of the car, and tested for soundness after the method specified[7] by the American Society for Testing Materials before being used. The use of cement stored over winter almost invariably resulted in low-strength concrete even when one or two additional bags of cement per cubic yard of concrete were used. This fact is painfully accentuated by the plottings in Fig. 3.

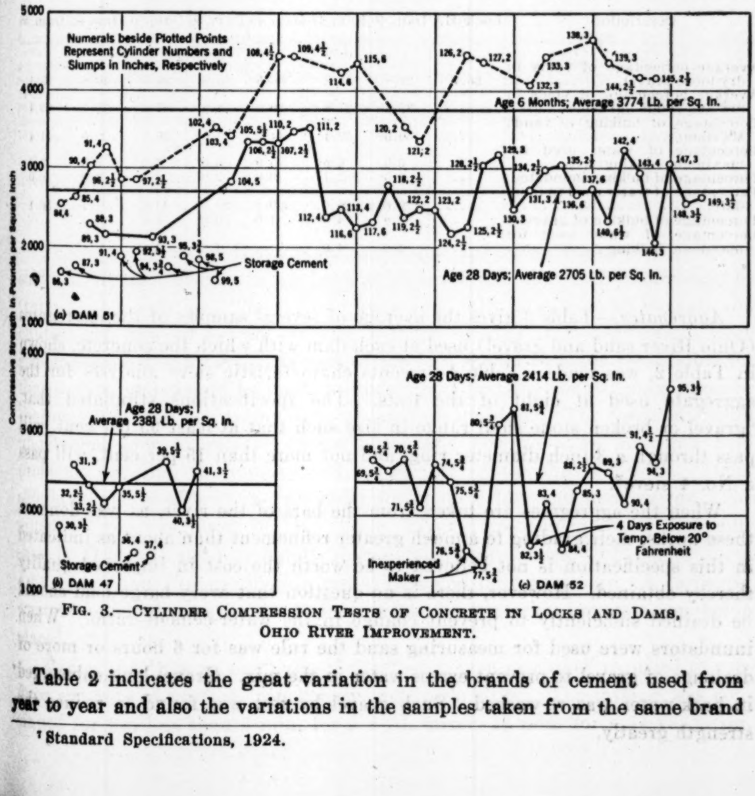


FIG. 3.—CYLINDER COMPRESSION TESTS OF CONCRETE IN LOCKS AND DAMS, OHIO RIVER IMPROVEMENT.

Table 2 indicates the great variation in the brands of cement used from year to year and also the variations in the samples taken from the same brand during the season of several months. All these tests were made at the highest class of professional laboratories and they show similar variations throughout all the reports.

---

[7] Standard Specifications, 1924.

In cases of carefully fabricated briquettes of cement and of cylinders of concrete made of cement from the same bin, although the two strengths do not uniformly correspond, it is not believed that all the variation is due to the cement. Frequently it is due to numerous causes, such as irregularities in water content, grading of aggregates, and other irregularities described elsewhere in this paper. However, the variation in cement probably causes the most frequent and the greatest variations in the strength of the concrete where there is reasonably competent supervision. With incompetent or careless supervision the variation in water content is, no doubt, the condition which causes the greatest variation in strength and which is probably causing more concrete to fall below the proposed strength than all other conditions combined.

TABLE 3.—QUALITY OF FINE AND COARSE AGGREGATES

Description	Lock 41A	Dam 46	Dam 47	Dam 49	Dam 50	Dam 51	Dam 52	Dam 53
Average percentage of voids in dry loose gravel	36.4	34.8	31.3	36.5	35.8	39.3	40.8	39.1
Average percentage of voids in dry and rodded gravel	....	33.3	27.8	30.3	34.8	37.1	37.7	33.4
Percentage of bulking of sand: Maximum	....	50.0	32.4	44.2	32.0	36.7	29.0	41.4
Percentage of water used for maximum bulking	....	6.0	6.0	5.0	5.0	4.0	6.0	3.0
Percentage of bulking, inundated.	....	0.0	−2.7	0.0	2.0	6.3	0.0	0.0
Percentage of water used to inundate	....	20.6	25.6	29.4	27.7	44.0	31.7	36.1
Percentage of bulking of gravel	....	2.9	4.0	< 1.0	6.8	6.4	....	....
Percentage of water used for maximum bulking	....	4.0	4.0	......	4.0	4.0	....	....

Aggregates.—Table 3 gives the average of several samples of the aggregates (Ohio River sand and gravel) used at each dam with which the concrete, shown in Table 2, was made. Table 4 presents characteristic sieve analyses for the aggregate used at eight of the tests. The specifications stipulated that, "gravel or broken stone shall range in size such that at least 95 per cent. will pass through a 3-inch diameter ring, and not more than 15 per cent. will pass a No. 4 sieve".

When the aggregates are taken from the bars of the river, as was done on these works, their grading to a much greater refinement than about as indicated in this specification is not believed to be worth the cost in improved quality thereby obtained. However, there is no question that every barge load should be drained sufficiently to prevent change in the water-cement ratio. When inundators were used for measuring sand the rule was for 6 hours or more of drainage of gravel to prevent excess water in the mix. Gravel long submerged in back-water was re-washed. Such gravel by test was found to reduce the strength greatly.

TABLE 4.—SIEVE ANALYSES (PERCENTAGE RETAINED) AND AVERAGE FINENESS MODULUS OF SAND AND GRAVEL

Sieve	DAM 46		DAM 47		DAM 49		DAM 50	
	Sand	Gravel	Sand	Gravel	Sand	Gravel	Sand	Gravel
1½-in.	00.00	00.00	00.00	00.00	00.00	09.75	00.00	......
¾-in.	00.00	8.70	0.40	10.11	00.00	19.55	00.00	19.73
⅜-in.	00.00	79.77	5.05	54.90	00.00	68.35	1.89	88.74
4-mesh	00.00	100.00	8.50	89.47	3.10	95.60	2.34	99.84
8-mesh	17.70	100.00	24.80	95.15	13.75	98.05	7.81	100.00
14-mesh	27.70	100.00	39.27	96.92	24.90	98.90	15.55	100.00
28-mesh	40.50	100.00	54.50	98.03	44.30	99.15	48.20	100.00
50-mesh	95.60	100.00	90.55	99.47	95.10	99.45	96.17	100.00
100-mesh	100.00	100.00	98.62	99.90	99.75	99.75	100.00	100.00
Average fineness modulus	2.82	6.88	3.22	6.44	2.80	6.88	2.71	7.08

Sieve	DAM 51		DAM 52		DAM 53		AUXILIARY LOCK 41	
	Sand	Gravel	Sand	Gravel	Sand	Gravel	Sand	Gravel
1½-in.	00.00	00.00	......	7.07	......	3.15	....	7.1
¾-in.	00.00	10.48	......	52.01	......	48.29	....	46.4
⅜-in.	00.00	69.68	1.84	90.61	......	88.05	....	82.6
4-mesh	00.00	100.00	4.59	98.01	6.48	96.98	1.6	98.3
8-mesh	7.42	100.00	21.36	99.91	11.56	99.23	16.0	99.9
14-mesh	22.60	100.00	34.86	99.97	15.43	99.80	35.8	100.0
28-mesh	48.09	100.00	58.37	99.98	24.90	99.87	69.7	100.0
50-mesh	95.21	100.00	98.29	99.99	95.07	100.00	93.7	100.0
100-mesh	99.75	100.00	99.79	100.00	99.76	100.00	98.5	100.0
Average fineness modulus	2.76	6.80	3.15	7.47	2.53	7.30	3.15	7.34

Tools for Placing.—When placing concrete it is most important to have a sufficient number of men in the form at all times, and that each man should be trained for his duties. They should know especially how to dump buckets and chutes, and how to spread concrete without causing it to segregate. Each spader assigned to a particular part of the face should be made responsible for that section and should be made to understand that the central mass must be thoroughly spaded and trampled to release entrapped air. The foreman should understand how to remove excess water and how to solve all other field problems affecting the quality and appearance of the finished work. There should be plenty of tools in good order for men on duty and some extras. The principal tools required on the Ohio River improvement were D-handle shovels, D-handle and long-handle spades, long wood spades, with blades about 4 by 10 in., and (for use in thin walls) bars and light hammers. A carpenter was stationed to work in the forms during the time that concrete was being placed.

Mixers.—All the mixers were of the "batch type machine" required to produce concrete of uniform consistency and workability. The mixing was continued for not less than 2 min. for a 2-yd. mixer, 1¾ min. for a 1½-yd. mixer, and 1½ min. for a 1-yd., or smaller, mixer, after completely charging and before beginning to dump. The batch was dumped into buckets on cars, lifted from the cars by derrick, and dumped (usually from bottom-door buckets), direct into place or into a hopper at the top of the form. From the hopper it was then conducted into place by a hose chute from 8 ft. to a maximum of about 25 ft. long, great precaution being required in all movements to reduce the segregation to the least possible. At the end of the specified mixing period for any batch, in case the dump bucket had not arrived and been made ready for receiving the charge, the mixing was continued until the bucket was ready, thereby improving the concrete, but introducing another factor for irregularity in strength.

The mixer for Lock 41 was of 2-yd. capacity, that for Lock and Dam 52 of 1¼-yd. capacity, and all the others were of 1 and 1½-yd. capacities.

Some instructions were given the foremen and men operating the mixers, but usually this was not begun early enough. The lack of trained and dependable men was a frequent cause of defect, which should be remedied by selecting the highest class of men procurable for experience and intelligence, and well known for integrity and reliability. Men who think the water-cement ratio useless, or that wet concrete is "good enough", should not be employed. They are especially objectionable in any position of authority. The crew on the concrete mixer and batch-placing forces in the forms should be selected with the greatest care as to experience and reliability some days in advance of commencing the work. Ordinarily, with competent inspectors, the direction of the principal men who are assigned to measure the materials, and the men in the forms, spreading, spading, and finishing the concrete, do most thorough work when under the direction of the inspector. The form inspector's proper position is close enough to the men to speak with ease to any one.

Throughout the season of 1928 Inspector R. W. McBeth had charge of the inspection of concrete materials, the mixing and placing of the concrete at Dam 51 and, during the season of 1929, he had similar duties at Auxiliary Lock 41. His recommendations for the proper inspection of concrete construction is described in Appendix II.

CONCLUSION

It is most fortunate for the artisan who can turn concrete mixing and the fabrication of cylinders into a game, or even more fortunate when he can, also, convert it into an art based upon highly scientific knowledge verified by laboratory experiments, as was done by Professor Abrams.

When the people who are paying for the concrete production of this country realize sufficiently the value of such skill, art, and knowledge as to demand uniform quality for soundness and a fixed minimum for strength, it is probable that the class of training indicated in this paper will be so perfected and extended as to make uniform products in concrete as usual as in other materials of construction.

The writer wishes to express here his appreciation of the great assistance of his associates with whom he has had much pleasure in accomplishing the tasks incidental to this class of work during many years.

APPENDIX I

MAKING AND SEASONING CONCRETE TEST CYLINDERS IN THE FIELD

BY CLIFFORD F. LITTLE,[8] ESQ.

The method given herein has been applied in moulding and seasoning test cylinders made from specimens of the concrete used in the construction of Auxiliary Lock 41. The process may be described in steps as follows:

(1).—Collecting the Sample.—After the first batch of concrete has been dumped the inspector hands two pails to one of the laborers in the form, giving him directions to fill them. The sample is promptly carried to the laboratory.

(2).—Making a Slump Test.—Upon arriving at the laboratory it is dumped from the pails to the dampened concrete floor and then immediately turned over by eight strokes of a large trowel. This operation will rectify any segregation which may have occurred in transportation and by dumping the sample from the bucket. Immediately thereafter, its slump is tested by filling the slump cone with three, 4-in. layers, puddling each layer with twenty-five strokes of a bullet-pointed $\frac{5}{8}$-in. rod of standard design. After the last layer has been puddled, the surplus concrete is struck from the top of the cone with the rod. Then, by use of the handles, the cone is slowly raised vertically until it clears the top of the concrete, and is placed beside the specimen. The rod is laid horizontally across the top of the cone, and, in that position, its distance above the top of the specimen is recorded, in inches, as the slump of the concrete.

(3).—Preparing the Cylinder Moulds.—The moulds are prepared for concrete before the sample is brought from the form. Every particle of old concrete is removed and then the inner surface is rubbed with a light oil. This treatment reduces the tendency of the new concrete to stick to the mould. Bottom plates of the brass and of the heavy cast-iron moulds were found unsatisfactory on the Ohio River work and they were replaced by steel plates 8 in. square by $\frac{1}{2}$ in. thick, machined to a true plane on one side. These plates produced very satisfactory results in securing true ends to the cylinders so that they rarely required re-capping.

(4).—Making the Test Cylinder.—Care should be exercised when placing the mould on the bottom plate to see that no small particles of sand have been caught between the bearing surfaces of the plate and the mould. After making these precautions, spread the concrete which was used to make the slump over the remaining concrete of the sample. Proceed immediately to make the cylinder by placing concrete from this sample in the mould in three, 4-in. layers, puddling each layer with twenty-five strokes of the $\frac{5}{8}$-in. rod. Care should be exercised to remove all pieces of aggregate that are more than 1 in. in diameter. After puddling the last layer, press down the top concrete in the mould with the blade of the large trowel thus removing all excess con-

---

[8] Insp., U. S. Engr. Office, Louisville, Ky.

crete and any air-pockets left by the rod. When all the excess concrete has been struck off with the trowel, give the top end a final finish by rubbing the rod over it.

(*5*).—*Capping and Storing the Cylinder.*—After the cylinder is made, prepare a stiff, neat paste for capping, with cement from the same bin as that used in making the cylinders. Soon after the cylinders and paste have shown the final set (by needle), the paste is re-worked until it is soft, and then it is spread on top of the cylinders in a thin layer, about $\frac{1}{16}$ in. higher at the center than on the edges. A sheet of thin oiled paper is spread over the top and one of the steel plates is placed over this paper. The plate is then pressed down and slightly turned from one side to the other until it has a firm bearing on the entire edge of the mould.

Thereafter, the cylinder is not disturbed for 24 hours. At the end of this period remove the moulds carefully, wipe all loose particles of sand from the cylinder, and place it on end on a perfectly plane surface. Place one hand on top of the cylinder and rock it slightly in all directions. Should the end be imperfect, a tilting motion of the cylinder will be felt, indicating that the end is not a perfect plane. It may be made perfect by rubbing it over the plane surface of the steel plate until no rocking motion can be detected by observation or by "feel".

The ends should also be tested to see that they are perfectly parallel. This may be done with a carpenter's square or by placing the cylinder in the testing machine. If they are not perfectly parallel, they should be made so by rubbing or by re-capping. After both ends of the cylinder have been treated in this manner, so that they will coincide perfectly with the plane surface of the testing-machine bearing-plates, they are marked and stored in damp sand which is kept within 15° of the average temperature of 75° Fahr. They are not again disturbed until the shipping date, when they are re-packed in shipping boxes filled with sand and shipped by truck to the testing laboratory. There, again, they are transferred into sand storage of about the same temperature as that in the field laboratory until they are broken in the testing machine.

APPENDIX II

Inspection of Concrete Construction

By R. W. McBeth,[*] Esq.

Hostility is one of the greatest obstacles to the control and inspection of concrete construction. Frequently, construction heads refuse to be informed, or to attend school. Indifference and hostility prevent the spreading of proper information and the co-ordination of operations between these workmen and the inspectors.

---

[*] Insp., U. S. Engr. Office, Paducah, Ky.

Plant Location.—The plant should be so installed as to get maximum output and this requires that every detail must be studied, and the installation made with care. Mixer frames must be secured firmly to solid foundations, and made plumb and level. Bearings and clutches, especially the tilting gear-clutches, need the closest attention. The charging hoppers should be set as close to the drums as clearance will allow. The throat of the hopper should be enlarged to the limits that the feed-opening of the drum will permit, and the tilting transfer scoop should be correspondingly enlarged, to prevent choking and spilling.

Mixing Water.—Water tanks should be cylindrical and set vertically, as nearly as possible over the drum feed, and the discharge pipe should be carried directly from the tank to the top of the drum feed-opening, without abrupt angles or bends. It should end in a flattened nozzle situated so as to deliver the mixing water on the rear third of the drum. This will give a quick, clean delivery of water. The tanks should be calibrated and fitted with gauges and floats automatically reading to gallons. The gauge marker should work accurately and smoothly.

Batch Mixing.—The inundator operator should be carefully trained by one familiar with its mechanism and "kinks" of operation. The use of a separate cement charging hopper is of much value because it eliminates hopper fouling. A common fault of concrete plants is a slow rate of feed to the drum. Many charging hoppers require 40 sec., whereas it should be done in less than 25 sec. A batchmeter should be installed on the mixer, so that the inspector will have a chance to watch the mix. Stairs and ladders should be placed so as to afford quick and easy access to all parts of the plant. The interior of the mixer drum should be brightly illuminated, day and night. When the hopper gate is open a spotlight may be thrown diagonally down the hopper into the drum. In non-tilt mixers, this is the only practical method. If bag cement is used the best method of charging is to empty bags into a skip that dumps at the top of the hopper, clearing the stream of mixing water. The skip should be as long as possible to minimize the splashing of cement. A locking and recording batchmeter should be a part of the plant equipment.

There are many things to be done beside checking tally and ringing bells, and the tallying is certain to be inaccurate when supervision and operation constantly interfere with timing and checking. The water tank should be covered and the valves and piping should be kept in good order. The platforms for observing changes in the mix and consistency and for workmen should be well lighted. When mixing is suspended for an hour or more, the mixer drum, and all the platforms, the hoppers, or chutes, at the mixer and in the form, should be thoroughly cleaned.

Many samples of sand and coarse aggregate should be carefully selected so as to represent the extremes of materials proposed for use. When separated into their respective sieve sizes they exhibit the grading. Weights of both materials per cubic foot in "dry rodded", "dry loose", and "damp loose" states, must be ascertained; and, also, the bulking of the sand with the probable moisture range. Piled in barges, stock piles, and plant bins, the aggregate naturally separates into coarse and fine particles and it is useful to know the effect of the different sizes of materials in a mix.

In general, a batch of coarse material, following a run of fine, is likely to produce a coarse, wet mix, which can be placed with comparative ease, but which results in poor construction. Fine material, on the other hand, will produce a wet mix with wet gravel, or a dry mix if the gravel is dry or low in moisture content. This is due to the much greater surface area, which takes up water from the mix if it is low in moisture, but adds water if it is wet. In either case, a run of fine gravel in a batch will cause a harsh mix, difficult to place, but one that produces good concrete if well compacted in place. However, it is usually difficult to get this tamping done properly.

To obtain good work, special care is required when the grading of sand or gravel is frequently changing. The specified measure of sand (equal to voids in gravel plus 15% of voids) will ordinarily give only a sufficient amount when a workable mass is well mixed; and, therefore, care must be used in making allowances for bulking. Sand thus proportioned should cover the maximum of voids likely to occur. Weighing batchers are preferable for obtaining accuracy in measuring coarse aggregate.

The mix should be watched and if the concrete is harsh, more sand is desirable. A slight reduction could be made in the gravel charge, but not quite enough to balance the increase of sand. Ordinary changes of ½ cu. ft. for fine or coarse aggregate will not unbalance the yield. Due care should be taken to see that the gravel "batcher" is set accurately. Variations in this unit will change the relative proportions twice as fast as the sand batcher, for a desired change of proportions in the charge.

A good method of determining proportions to be used at the beginning is the "trial method" devised by F. R. McMillan, M. Am. Soc. C. E., and recommended as standard by the Portland Cement Association. By this method, the mix is made in small batches to determine the correct point of workability with several water ratios. Coarse and fine gravel is analyzed in the various proportions and water-cement ratios. Various mixes are studied until the inspector is satisfied, and then the quantities required are determined for the batches so as to duplicate the best trial mix, in the actual structure.

Inspection During Operations.—When the plant has started, the slump test should be made of each batch until they are satisfactory. The batch should be observed carefully at the mixer, in the bucket, and when it is being placed in the form. The inspector should note how it acts, and should not hesitate to ask the foreman and form inspector, or the placing gang, how it behaves. He should spade and tramp it himself. In making the slump on trial a well-balanced mix should not slump off on the side and show gravel, provided the cone is lifted smoothly and steadily. Such a tendency indicates insufficient mixing or insufficient sand. By remembering that 4.7 lb. of cement is 0.01 of a 5-sack batch (1 cu. yd. as specified) and knowing the weight of all other materials per cubic foot in all conditions, computations required for changes are made easier. The material bins should have slopes of not less than 50° to the batcher gates. The grading of aggregates may be improved or injured by methods of loading and unloading.

The location of material supply piles and plant machinery is very important to efficient operation. For a river coffer-dam both should be situated so as to protect supply barges from the current, allowing easy handling for turning and cleaning off the barge. The relative position of barges, derrick, and mixer must be such that a minimum of movement is required to place material in the bins. By care in this operation the grading of gravel may be improved, or, by lack of care, it will be injured. To have a good signal man who understands this process of improving grading, is very advantageous. The bin should be in view of the derrick operator, and his operating house should be set so that he has a clear view of the plant and barge.

Concrete forms should be wire-brushed and oiled every time they are stripped. They should be kept wet so that cracks do not open up and the panels warp. This may cause complaint, but it is worth while, especially in close forms, and where finish is desired. Where reinforcing steel is used, the forms must be oiled before the steel is placed. Mortar should be placed in the bottoms of all lifts for bond; it is difficult to obtain neat cement, when so used, in sufficient quantity. Mortar will help correct any effect of dry batches when starting a monolith. Lack of control at the mixer may result in occasional dry or wet batches in the form, especially at the start, should the moisture content of the aggregates change quickly.

Plant Control Equipment.—The inspector or control man should have a laboratory on the charging floor, with a high-grade laboratory scale of large capacity. It should have a triple beam, graduated in decimals, a 5 000-gramme torsion balance measuring to ½ gramme, with special platform and a one-point suspension unit for specific gravity determinations. A 6-in. cylinder with a capacity of 0.2 cu. ft., fitted with double weirs and "strike off" cover, is useful. The following should also be on the list: A 1-cu. ft., bored steel measure; a 0.1-cu. ft. measure with scoops and rods; two slump cones; scale and balance scoop pans with counterweights; buckets and plenty of pans, large and small; an oven taking the largest pans, if practicable, with an electric heating unit of 1 800 watts and with three or four-point control; two 50-cu. cm., and three 150-cu. cm., cylindrical graduates; a pycnometer; a dozen mason jars; a dozen 8-oz. graduated prescription bottles, with corks; and a 1-gal. jug, with a cork. A full set of standard sieve analysis screens, with pan and cover, 12 in. in diameter; two steel shaker-frames, one for sand and one for coarse aggregate sieve nests, with electric vibrator for shaking samples of aggregates, are essential. Brooms and heavy dust brushes, a sink and large floor pan housed in a well-lighted and practically dust-proof room on the platform—all of these items comprise the essential equipment needed for plant control.

If thorough preliminary examinations have been made, the control man will have most of the necessary reference data and charts at hand; otherwise, he will have to develop them in the course of his control work. If the scales and balances are as specified herein, no conversion tables will be needed, and the work will be simplified and expedited. At any time the control man can catch a sample of either aggregate from the batch as it falls, and from it, in a few minutes, he can have any data desired.

All the apparatus mentioned is designed for rapid and accurate work, and, with some practice, two or more determinations can be made almost simultaneously, enabling the inspector to keep close check on all characteristics of the materials in use, and quickly to detect any changes.

With weight-batching, the proportions can be changed completely between batches if necessary, to meet any material changes in grading. The control man should be chosen not only for his knowledge, but for a deep interest in the work, and a known desire to produce the best possible concrete at the lowest possible cost. Such a man will have an enjoyable job. He should be in charge of the plant, including the batchmeter and the repair work. With such equipment, the inspector may be able to obtain satisfactory results unless by chance some one has omitted from the contract specifications one or two essential paragraphs.

The control of concrete is a great game and when one can look at thousands of yards of good concrete in a big structure, with few defects, he will feel that it was well worth the effort.

DISCUSSION

HENRY B. SEAMAN,[10] M. AM. SOC. C. E. (by letter).—With uniform materials and uniform methods of manufacture, such as those described by the author, one should obtain reasonably uniform results; a variation of 20 to 50%, as shown in Table 2, would seem to be as uniform as may be expected. In the construction of bridge masonry in large mass, however, more than uniformity is desired; solidity and durability of the structure are also necessary. Early strength may be misleading. It has not been considered a prelude to durability. Engineers have preferred a cement that is moderately slow-setting, with gradual and continuous increase of strength. Care must be taken not to "drown" the cement and produce laitance, but a moderately wet concrete has been found to give the most satisfactory results.

The importance of thorough mixing is as great as the quality and quantity of cement used, or of the water-cement ratio—whether it be 5½ or 6½ gal. of water per bag of cement—and usually much less than that will suffice. Then, too, a proper and complete distribution of aggregates in the forms is important.

Concrete of proper consistency for moulding will segregate in transportation, whether this is done by wheel-buggies, trucks, chutes, or blow-pipe. From wheel-buggies the fluent mortar will be deposited first and the coarse aggregate will follow, but from chutes or blow-pipe the coarse aggregate will be deposited first and the mortar will follow. This latter condition is the more desirable since, in spading, the mortar envelops the stone which rises, and the mortar flows down through the interstices. It is partly for this reason that chuted concrete, or blown concrete, has given such satisfaction in field work. Another, and an important, reason appears to be the fact that the chute, or blow-pipe, gives additional mixing to the mortar as it turns over while flowing.

Attention seems to be given, by some experimenters, to the preference of having the mixture so dry that it will not segregate during transportation, but such concrete will not fill the forms properly unless thoroughly rammed and compacted, and, even then, it has been found, upon removal of the forms, that voids have been left on the surface, and similar conditions may be expected in the interior of the mass.

For many years a 1:2½:5 mixture was generally used and was considered a theoretically perfect mix, but about 1900 the 1:2:4 mix was adopted and is now (1931) established as the general standard. Wastefully rich, it seemed at first, but experience has justified its adoption as giving assurance against imperfections that will creep into practical work of magnitude. The question that remains is how to obtain the best results with this mixture. Thorough mixing is the key to good concrete. If a single mixer will not produce concrete fast enough for an extensive job, tandem mixers, or even twin mixers delivering batches alternately, may be adopted. The extra cost will be insignificant on large work, and will secure a much more uniform and satisfactory product.

---

[10] Cons. Engr., Brooklyn, N. Y.

THEODORE BELZNER,[11] AFFILIATE, AM. SOC. C. E. (by letter).—The writer has read this timely paper and Appendices I and II with great interest. The excellent and painstaking work represented by the systematic and uniform method of inspection and control of concrete manufacture, authorized and carried out to the end, is highly commendable. The value of the results secured should serve as a strong incentive to other engineers to conduct similar investigations along these lines.

Furthermore, the absolute essentials necessary for securing uniform success in producing good concrete of uniform quality, resulting from the proper training, experience, and qualifications of those responsible for this class of construction in the field, cannot be too strongly emphasized. When structures of this class are so large and when the resources of construction in workmanship are taxed to a higher degree than usual, the results have far greater value than laboratory experiments could possibly give.

JOHN SANFORD PECK,[12] ASSOC. M. AM. SOC. C. E. (by letter).—This paper marks another milestone in the long struggle toward the general adoption of scientific and controlled methods of concrete manufacture in the field. Since the first promulgation of the water-ratio theory by Duff A. Abrams, M. Am. Soc. C. E., the battle to introduce his theories into practice has been a long and, at times, almost a losing one.

When Mr. Hall states, in his "Introduction," that more than 50% of the concrete made and placed to-day is of poor quality and will have a short life, he is erring strongly on the side of conservatism. The writer has yet to hear of any concrete being condemned or replaced because the test cylinders failed to meet the 28-day strength requirements; and the jobs on which sample cylinders are taken, constitute an almost infinitesimal proportion of the concrete work done in the United States. The use of "ready-mixed" concrete in large cities has gone a long step toward procuring concrete of reliable strength and durability, but even there, the room for improvement is vast.

As an example, the writer witnessed a mixer truck discharging ready-mixed concrete into the basement of a New York building. As the concrete left the mixer it seemed to be of good quality and consistency. At the top of the chute which led to the basement a workman stood with a hose, washing the concrete down. What became of the water-cement ratio in this case is easy to imagine. Such ignorance is a fair example of what still remains to be done in the line of education.

Of all the factors listed by Mr. Hall ("Irregularities in the Strength of Concrete"), which may affect the strength of concrete, the first, namely, the strengths of different brands of cement and the variation even in different parts of the same bin of a given brand, constitutes by far the most difficult variable to control. This wide variation has been long known in the laboratory. It is unfortunate that standard 2 by 4-in. compression cylinders were not included in the cement tests because it is the writer's opinion that these are a far better indication of the probable strength of concrete to be obtained from a given brand of cement than the tension briquettes. The compression tests as set forth in the American Society for Testing Materials Tentative Standards for 1930, Designation C 9-16T, should be adopted as a standard.

---

[11] Insp. of Steel and Bridge Insp. in Chg., Brooklyn Bridge, Dept. of Plant and Structures, City of New York, Brooklyn, N. Y.

[12] Instr. in Civ. Eng., College of the City of New York, New York, N. Y.

With the compressive strength of standard 1:3 cement mortars varying from 2 000 lb. per sq. in. (the American Society for Testing Materials minimum, which incidentally is too low) to 5 000 lb. per sq. in. at 28 days, the attempt to make concrete of a predetermined strength in the field is saddled with a severe handicap. The frequently expressed theory that all cements are equal seems absurd.

The account of the training and education of the field force of inspectors is worthy of more than passing comment. If the idea that special training and knowledge are necessary for the making of good concrete, could be impressed on the smaller contractors and job superintendents it might lead to the growth of a class of "concrete technologists" which in time would supplant the old-time concrete foreman or "mixer-man" who is supposedly endowed with a sixth sense whereby he knows all there is to know about concrete making.

The conditions under which the work described by Mr. Hall was done certainly were not of the easiest, and the fact that such satisfactory results were accomplished speaks volumes for the efficiency of his method of educating and training his inspection force. His experience in this regard should prove of great value in the planning of future work whether of the same magnitude or smaller, and it is to be hoped that his paper will establish a precedent for the construction industry as a whole to follow to the great benefit of the cause of good concrete.

J. W. KELLY,[13] ASSOC. M. AM. SOC. C. E. (by letter).—As stated by Mr. Hall, the writer was to some degree associated with the men of the inspection force in the Louisville District during 1926, 1927, and 1928. One could not fail to be impressed with the size and variety of the project, and with the organization's policy of thorough training for the men in order to produce a uniform concrete.

An apparent lack of uniformity in the strength tests, as exhibited by the maximum and minimum range of values in Table 1, has been pointed out to the writer. However, it must be remembered that the table averages nine jobs, spread out for nearly 400 miles, with concrete made by different organizations using different types of equipment and a variety of cements, admixtures, and river-bar aggregates; also, that the percentage of test specimens which were considered defective and which were omitted from the report is much smaller than is usual on concrete work. Unquestionably, the record of each job would show a smaller spread of strength values. With specification requirements which would normally produce 1 700-lb. concrete, as stated by the author, it is evident that the average quality of the work was greatly improved through the efforts of the inspection force.

---

[13] Eng. Materials Laboratory, Univ. of California, Berkeley, Calif.

Mr. Hall's analysis of the causes of variation in concrete or in specimen strength is a clear indication of the chief features to watch in order to obtain uniform results in any concrete construction. Elsewhere in the paper he emphasizes the training of individuals, even the spaders and mixer men. This reinforces the writer's observation that the human element enters largely into concrete making, and that while the process is simple it requires intelligent and conscientious workmanship to bring a "good enough" job into the category of a first-class piece of work. First-class concrete can be made without undue effort, but too many organizations seem to be satisfied with the line of least resistance.

Improved construction practices have been strongly urged by the Portland Cement Association, particularly in regard to placing and curing. It is gratifying to note that the care taken by the construction forces on the Ohio River bear out these recommendations by their example. Improvements in the strength of cements during the course of the project are noted, as well as the practice of re-washing coated aggregates, which process would tend also to reduce variations in grading through the attendant mixing of sizes.

Inspection has too often been regarded as a perfunctory matter, tending to retard construction and to increase cost, whereas the right kind of inspection (whether for concrete or for any other construction) is one of the most worth-while investments that can be made. It is the writer's belief that qualified inspectors, preferably but not necessarily with formal technical training, are worth far more than their present scale of salaries indicates, and that their services are as real and valuable to the project as those of the designer or the construction force. More and better inspection would eliminate many of the costly mistakes, early maintenance, and construction of inferior quality that are now encountered.

After a long experience with this subject, Mr. Hall considers that sound concrete of uniform strength is best obtained by good workmanship. Better realization of this by all who deal with concrete in its many angles, is sure to result in better structures.

STANTON WALKER,[14] ASSOC. M. AM. SOC. C. E. (by letter).—The factors to which emphasis should be given in the design and control of concrete on the job, have been clearly and concisely described in this paper. There are certain points, however, which, it is believed, will be clarified by further comment.

In the paragraph headed "Specifications," it is stated that "sand was required sufficient to yield 15% or more of mortar than was needed to fill the voids in the gravel * * *." In Appendix II, under "Batch Mixing," a considerably different rule for determining the quantity of sand is inferred, as follows: "The specified measure of sand (equal to voids in gravel plus

---

[14] Director, Eng. and Research Div., National Sand and Gravel Assoc., Washington, D. C.

15% of voids) will ordinarily give only a sufficient amount when a workable mass is well mixed * * *."

The design of proportions by the first rule would result in the use of very much less sand than is ordinarily considered necessary for workability. Taking the highest percentage of "dry-and-rodded" voids reported in Table 3 (37.7%) and assuming a percentage of voids of 35 for the sand and a net water ratio of 0.9, a mix of about 1:1.3:5.1 is obtained for concrete containing 5 sacks of cement per cu. yd. by the application of the first rule. The application of the second rule results in proportions more nearly those that are ordinarily used although, in the writer's opinion, such mixes are still considerably "under-sanded" for the usual conditions. Using the same assumptions, a 1:1.8:4.4 mix is obtained. The writer would be interested in knowing which of the two rules was actually applied in the design of the concrete. In his opinion neither one of them represents a satisfactory criterion on account of the resulting "under-sandedness" of the mix. A volume of mortar equivalent to, at least, 1.65 times the volume of voids in the coarse aggregate will be required to produce satisfactory workability for the usual conditions, and this factor will vary with the size and grading of the aggregate.

A procedure for determining whether or not the test cylinders have plane ends is described in Appendix I. This is one of the most important factors in securing uniform test results and one which is often overlooked. As much of the apparent lack of uniformity in tests of field specimens is due to poor testing technique as it is to actual variations in the quality of the concrete. The writer does not agree that it is sufficiently accurate to determine by the "feel," when a cylinder is rocked on a plane surface, if the end is in satisfactory condition; it may be concave and, if that is the case, it may "feel" perfectly solid. The ends should be tested with a straight-edge or by some equally satisfactory method.

The importance of having the ends of the test cylinders "perfectly parallel" is also stressed in Appendix I. This condition is practically impossible of attainment and reasonable deviation from it is not important if the cylinders are tested in a machine equipped with a suitable spherical bearing block.

Comprehensive information has been given by H. F. Gonnerman, M. Am. Soc. C. E., on these questions.[15] While Mr. Gonnerman showed convex ends to be considerably more detrimental than concave ones, the reductions in strength for the latter condition were important. Cylinders with the top surface inclined showed no reductions in strength for an inclination of ¼ in. in 6 in., and only about 5% reduction for ½ in. in 6 in.

Edward E. Bauer,[16] **Assoc. M. Am. Soc. C. E.** (by letter).—It is gratifying to note the care that was exercised in securing concrete of high quality on these particular projects. Too often the feeling exists that care and exactness have no place on the construction job; that it is only in the laboratory that these items need consideration. Mr. Hall and his assistants are to be commended for the excellence of their program for supervising the production of concrete.

---

[15] "Effect of End Condition of Cylinder on Compressive Strength of Concrete," *Bulletin 14*, Structural Materials Research Laboratory, Lewis Inst., Chicago, Ill.; reprinted from *Proceedings*, Am. Soc. for Testing Materials, April, 1925, Vol. 24, Part II, 1924.

[16] Associate, Civ. Eng., Univ. of Illinois, Urbana, Ill.

The writer wishes to emphasize the importance of attending to details in the making and testing of cylinders, in order to secure results that are uniform to any degree. Field cylinders are valuable, but unless certain rules are followed in their making and handling, the results may be erroneous and of no value. The cylinders must also be tested properly.

Mr. Hall has noted that many of his cylinders had to be capped just prior to testing, in order to secure an even bearing surface on both ends. The writer has noted in his testing work that whenever it is necessary to cap a cylinder with plaster of Paris, the crushing strength value is almost always lower than that of cylinders properly made. The ends should be made plane surfaces when the cylinders are cast in the field. It is difficult not to rock the plate or the cylinder in applying the plaster-of-Paris cap and thus produce a curved surface. The proper making of the cylinders should be given more attention by field men, in general. Cylinders that are patched up in the laboratory can not always be made to give correct results. Cylinders without plane ends should never be tested, yet one occasionally hears of this being done in some laboratory.

In Appendix I the statement is made that the ends should be "perfectly parallel." The writer sees no need for this, since the properly equipped testing machine has a spherical bearing block to be used on top of the specimen to allow for any lack of parallelism in the ends. It would seem that any attempt to secure perfectly parallel ends with an ordinary carpenter's square would be a hopeless task. While no statement is made as to whether or not a spherical bearing block was used, the precautions recommended lead one to believe that it was not. Without a spherical block there can be no positive assurance that the load is distributed over the entire cylinder. Concentration of pressure on a small area will cause the cylinder to fail at a lower load.

According to Appendix I the cylinders were kept in damp sand, the temperature of which may have varied between 60° and 90° Fahr. Tests at the University of Illinois[17] indicate that if the curing temperature after the first day is varied a uniform amount for the entire curing period, the strength is affected about 30 to 35 lb. per sq. in. per degree of change in curing temperature. The concrete in these tests showed a strength of 5 400 lb. per sq. in. at 28 days under standard curing conditions. The 1929 cylinders are fairly close to this figure, so that it may be safely assumed that the range of curing temperatures allowed on the Ohio River projects permitted a maximum variation in strength of 900 to 1 000 lb. per sq. in.

Cylinder strengths as reported vary over too wide a range. If the concrete placed in the structures was of uniform quality, then most certainly some of the cylinders did not receive the proper attention.

---

[17] See "High Early Strength Concrete," by the writer, *Proceedings*, Am. Concrete Inst., 1929.

O. Bonney,[18] M. Am. Soc. C. E., and D. T. Mitchell,[19] Assoc. M. Am. Soc. C. E. (by letter).—This paper presents and discusses a subject which is of interest and importance to all who have to do with the design and construction of concrete structures. As indicated by the author concrete of uniform quality has not always been produced in the past and experiences and information which may lead to more uniform concrete in the future are welcomed by engineers. When the art of making concrete reaches the state where uniformity is the regular occurrence it may then perhaps be possible to design structures of this material more economically.

The author emphasizes the fact that the production of concrete of uniform quality requires a thorough knowledge, on the part of all concerned, of the ingredients as well as a thorough knowledge of proportioning, mixing, transporting, placing, and curing. The engineer and the contractor are vitally cencerned and 100% co-operation is required of both.

The author sets forth under the caption, "Irregularities in the Strength of Concrete," the principal causes of the great variations that occur in "job" concrete. In this connection the writers feel that the method of mixing is a matter of considerable importance and, furthermore, that careful consideration should be given to the type of mixer used, the peripheral speed of its drum, the design of the mixing blades, the size of the drum, the mixing action, the time of mixing, and the ratio of the quantity of concrete mixed per batch to the volume of the drum.

Until recent years it has been a common practice to mix concrete in non-tilting, rotary, drum type mixers. Since about 1928, however, a type known as a truck mixer has been introduced and used on work of large magnitude, which permits the concrete to be mixed while in transit to the point of deposit, if that is desired.

Truck mixers which have come to the notice of the writers are designed to mix more concrete per batch in proportion to the total volume of the drum than the older types of mixers. Furthermore, the peripheral speed is less in the case of the truck mixer. Definite conclusions as to the effect of this on the quality of the concrete mixed in truck mixers have not been reached by the writers, but tests made on one type indicate that the mixing time for a truck mixer should be greater than that for the older type. Other tests indicated that there may be some variation in compressive strength between concrete taken from the first of the discharge of a batch and that taken from the last of the discharge. Other types of truck mixers might give different results.

During the three years since 1927 a part of what is known as the Olentangy-Scioto Intercepting Sewer has been constructed in Columbus, Ohio; this part is a rectangular, reinforced concrete structure, involving about 34 000 cu. yd. of concrete. Throughout the work attention has been directed toward making concrete of uniform quality, and the writers, therefore, submit the results of their compression tests of concrete as being of interest in connection with the results shown by the author. The results obtained on this work, together with weighted averages of those given by the author, are shown in Table 5.

---

[18] Sewerage Relief Engr., Div. of Eng., Dept. of Public Service, Columbus, Ohio.

[19] Field Engr., Sewer Dept., Div. of Eng. and Constr., Columbus, Ohio.

The work was divided into four sections, each section being a separate contract of the size and length shown in Table 5, Items 21, 22, and 23. With the exception of Section 4 and part of Section 1, the work was constructed in open trenches. Section 4 was constructed along the east bank of the Scioto River, the river side of the sewer being exposed. Approximately, 420 lin. ft. of Section 1 was constructed as a dam across the river.

Three brands of cement were used, one brand on Sections 1 and 4, another on Section 2, and still another on Section 3. The specifications for

TABLE 5.—COMPARISON OF CYLINDER TESTS OF CONCRETE IN THE OLENTANGY-SCIOTO INTERCEPTING SEWER, AT COLUMBUS, OHIO, WITH THOSE IN DAMS ON THE OHIO RIVER IMPROVEMENT. (28-DAY CYLINDERS ONLY.)

Item (1)	Description (2)	Section 1 (3)	Section 2 (4)	Section 3 (5)	Section 4 (6)	Weighted average (7)	Weighted average of cylinder tests on the Ohio River Improvement (8)
Results of Tests:							
1	Number of cylinders	217	108	407	319	1 051*	759*
2	Average strength per cylinder, in pounds per square inch	3 460	3 165	3 322	3 444	3 371	2 815
3	Average strength of all cylinders testing less than Item 2, in pounds per square inch	3 063	2 760	2 777	3 002	2 901	2 315
4	Average strength of all cylinders testing more than Item 2, in pounds per square inch	3 909	3 599	3 931	3 883	3 877	3 381
5	Percentage of cylinders testing less than Item 2	53	52	53	50	52	54
6	Percentage of cylinders testing less than 2 000 lb. per sq. in.	1	0	3	1	2	17
7	Percentage of cylinders testing less than 2 800 lb. per sq. in.	....	....	....	....	....	10†
	Compressive strength, in pounds per square inch:						
8	Maximum	4 990	4 210	5 940	5 060	5 940‡	5 930‡
9	Minimum	1 590	2 100	1 270	1 730	1 270§	919§
	Slump, in inches:						
10	Average	3.5	3.9	4.5	4.8	4.3	3.52
11	Maximum	7.0	7.5	8.75	7.75	8.75‡	8.5‡
12	Minimum	1.0	0.5	0.5	2.5	0.5§	0.5§
13	Number of tests in which the slump was less than in Item 10	108	52	199	159	518*	421*
14	Year constructed	1930	1930	1928–1930	1929–1930	1928–1930	1925–1929
15	Volume of concrete placed, in cubic yards	12 193	4 115	11 165	6 654	34 127*	750 000*
16	Mix, by volume	1: 6	1: 6	1: 6	1: 6	1: 6	(See text)
17	Water, in gallons per sack of cement	6	6	6.75	6	....	(See text)
18	Specified minimum strength, in pounds per square inch	2 500	2 500	2 500	2 500	2 500	2 000‖
19	Percentage of cylinders testing as much or more than Item 18	96	91	89	94	92	83
20	Percentage of cylinders testing less than Item 18	4	9	11	6	8	17
Dimensions of Sewer:							
21	Width, in feet	10.5	10.5	10.5	10.5	....	....
22	Height, in feet	5.5	12.0	12.0	12.0	....	....
23	Length, in linear feet	4 550	1 206	3 718	1 853	....	....

* Total. † Six-month cylinders. ‡ Maximum obtained. § Minimum obtained. ‖ Job standard.

aggregate were practically the same on all four sections. The fineness modulus of the fine aggregate was required to be between 2.75 and 3.25 and that of the coarse aggregate between 6.90 and 7.50. The fineness modulus of the fine aggregate was usually found to be near the upper limit and, in fact, on Sections 1 and 2 it was necessary to add fine sand in order to obtain a fineness modulus within the limits specified. Ordinarily, the coarse aggregate was found to be satisfactory with respect to the fineness modulus. City water, softened and filtered, was used.

All concrete was mixed for 3 min. in 1-cu. yd. paving mixers of the non-tilting, rotary, drum type. The water content was controlled for the most part by the slump test. The aggregates were measured by volume at the screening plant located from 1 to 3 miles from the mixer, and then hauled to the mixer in trucks which were provided with batch compartments. An inspector was stationed at the screening plant, another at the concrete mixer, and still another at the point of final deposit.

The test cylinders were made in accordance with the Standard Methods of Making and Storing Specimens of Concrete in the Field (Serial Designation: C 31-27), of the American Society for Testing Materials, with the exception that the concrete was obtained as near the point of final deposit as possible and the cylinders were stored in damp sand on the work for only 24 to 48 hours. At the end of this time they were removed to the testing laboratory where they were then stored in damp sand or in a damp room for the remainder of the time until broken. The cylinders were broken in accordance with the A. S. T. M. Standard Methods of Making Compression Tests of Concrete (Serial Designation: C 39-27).

C. E. ARNOLD,[20] ASSOC. M. AM. SOC. C. E. (by letter).—The author presents some interesting information which indicates that with ample, proficient, and well organized supervision, concrete of quite uniform quality can be manufactured even on detached or widely separated pieces of work.

During 1928 and 1929 the writer installed 41.33 miles of storm drains to intercept and remove the storm waters from an area of 22 319 acres lying adjacent to, and immediately east of, Los Angeles, Calif. The entire system is divided into four main outlet lines, lying several miles apart, the lower (larger) portions of which are of monolithic construction, rectangular in cross-section, ranging from 6 to 10 ft. in height and from 8 to 14 ft. in width; these portions have a combined length of 13.09 miles.

The structures were designed on the basis of 650 lb. per sq. in. for concrete in compression and 16 000 lb. per sq. in. for steel in tension. Naturally the cross-section area of concrete and the weight of steel (per foot of length) varied due to the superimposed load, which was from a few feet to as much as 40 ft. of cover (in which case an arch section was used).

Three of the main lines were built simultaneously, the fourth partly overlapping the construction period of the first three. Combined, they contain more than 141 000 cu. yd. of concrete, an average of a little more than 2 cu. yd. per ft. of length. In all cases the concrete was mixed on the ditch bank in large mixers and conveyed to the structure by a traveling bucket, belt conveyor, or chute. The fine and coarse aggregates were delivered to the mixers in 5-yd. batch trucks, the contents of each batch being weighed as loaded.

---

[20] Los Angeles, Calif.

In all, 510 pairs of standard 6 by 12-in. test cylinders were made, one-half of which were tested at 7 days and the remainder at 28 days. The strength developed by the 28-day cylinders led the writer to analyze the tests which resulted in the data plotted on Fig. 4. Fig. 4(*a*) is a curve of 510 tests made on 28-day cylinders. The cylinders were made of Cements *A* and *B* and Coarse Aggregates *A, B, C,* and *E;* Fig. 4(*b*) is an analysis of 495 tests segregated to show any difference in strength, due to the brand of cement used; Fig. 4(*c*) is an analysis of 507 tests segregated to show any difference in strength due to varying the brand of coarse aggregate; and Fig. 4(*d*) is the result of plotting curves of the 510 tests in classes according to the slump. It was thought that a higher unit concrete stress (about 800 lb. per sq. in.), might be used in future designs, which seems justified since only 4% of the tests showed strengths less than 2 500 lb. per sq. in. The writer would appreciate the author's comments relative to such an increase.

Inspection.—One chief storm-drain inspector, a graduate from a reputable engineering school, was employed to supervise and to assist, when necessary, the inspectors stationed on the job. The inspectors on the monolithic work were experienced men (some had been with the writer for more than three years), who knew the importance of the water-cement ratio, the fineness modulus, and the necessity for a workable mix. They were extremely loyal and more than faithful.

Tests.—More than 500 tests were made on the cement, by a reputable laboratory, before its delivery to the job. The average of these tests shows that 12.7% of the cement was retained on the No. 200 screen. The average tensile strength of 7-day standard briquettes was 330 lb. per sq. in. and 412 lb. per sq. in. at 28 days.

All test cylinders were made by the laboratory personnel; they were stored in wet sand for 24 hours and then removed to the laboratory. Concrete for the test cylinders was taken from within the forms immediately after it had been placed. Slump tests were made at the same time, the average of three being used.

Materials.—Cement was furnished from two mills, and the results obtained by their use were practically identical (see Fig. 4(*b*)). The tensile strengths of standard briquettes were quite uniform and nearly identical for both brands. Both fine and coarse aggregates were furnished by four firms. Aggregates *A, B,* and *C* (see Fig. 4(*c*)) came from the San Gabriel River, Washington; they were identical in character and quite uniform as regards size. Each of these coarse aggregates contained not more than 30% of crushed boulders. Aggregate *E* was furnished from a pit; it was slightly different in character, contained no crushed material, and was not as well graded as Aggregates *A, B,* and *C*. The results obtained by its use are quite plain.

Fig. 4(*a*) shows the results of all tests on 28-day cylinders. Fig. 4(*c*) shows the analysis according to the slumps. It will be noted that slumps of

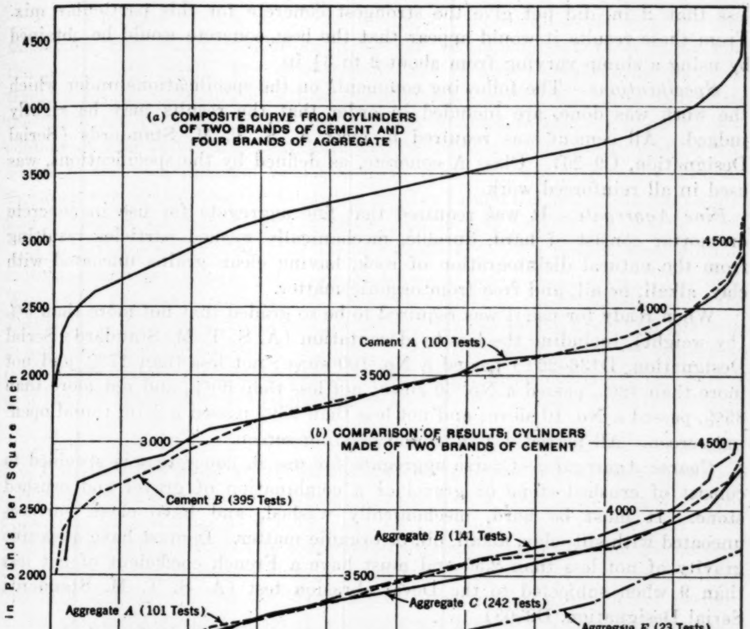


FIG. 4.—COMPARISON OF TESTS RESULTS ON CONCRETE CYLINDERS BY SEGREGATION OF VARIABLES.

less than 2 in. did not give the strongest concrete for this particular mix. From these results it would appear that the best concrete would be obtained by using a slump varying from about 2 to 3½ in.

Specifications.—The following comments on the specifications under which the work was done, are included in order that the results may be rightly judged. All cement was required to meet A. S. T. M. Standards (Serial Designation, C9-26). Class A concrete, as defined by the specifications, was used in all reinforced work.

Fine Aggregate.—It was required that fine aggregate for use in concrete or mortar consist of hard, durable, mechanically washed particles resulting from the natural disintegration of rock, having clean grains uncoated with clay, alkali, or oil, and free from organic matter.

When ready for use it was required to be so graded that not more than 6% (by weight), including the loss by decantation (A. S. T. M. Standard; Serial Designation, D136-22T) passed a No. 100 sieve; not less than 17%, and not more than 42%, passed a No. 30 sieve; not less than 60%, and not more than 85%, passed a No. 10 sieve; and not less than 93% passed a ¼-in. round opening screen. All passed a ½-in. round opening screen.

Coarse Aggregate.—Coarse aggregate for use in concrete was specified to consist of crushed stone or gravel or a combination of gravel and crushed stone. It must be hard, mechanically washed, and have clean surfaces uncoated with silt, clay, alkali, oil, or organic matter. It must have a specific gravity of not less than 2.50 and must have a French coefficient of not less than 9 when subjected to the Deval abrasion test (A. S. T. M. Standard; Serial Designation, D2-08).

When ready for use, it must contain not more than 30% of crushed stone; it must be graded so that not less than 95% (by weight) passed a 1½-in. round opening sieve, but not more than 7% passed a ¼-in. sieve, and not less than 30%, nor more than 55%, passed a ¾-in. round opening sieve.

Concrete.—The specification for Class A concrete required that it be composed approximately of the relative volumes of materials as follows: 1 cu. ft. of cement; 2 cu. ft. of sand; and 4 cu. ft. of gravel.

In all concrete, the volume of the sand and gravel could be varied as directed by the Engineer of Construction, but for Class A concrete, the sum of the volumes of sand and gravel (measured separately) must always be 6 cu. ft. The volume of sand specified for Class A concrete was on the basis of dry sand. The volume of any sand containing moisture could be increased as determined by the Engineer of Construction, in quantity sufficient only to allow for the bulking due to moisture content. No increase could be allowed in the volume of rock due to moisture content.

In Class A concrete, only sufficient water could be used to produce concrete having a slump of not to exceed 4 in. All concrete, whenever practicable, was mixed in batch-type mixers and Class A concrete was always so mixed.

It was required that the mixer drum have a speed of not less than 13 rev. per min., and each batch must be mixed in the drum of the mixer not less than 1½ min.

In the locality of this work, coarse and fine aggregates cost so nearly the same that there was no objection on the part of the contractors to changes in the proportions used. For this reason (and to avoid frightening the contractors by the inclusion of a fineness modulus specification, thus increasing the cost), the provision in the specification relative to the variation in percentage of fine and coarse aggregate was included and was freely used by the writer to secure a uniform workable mix. The proportions actually used on the work varied between the following limits: Cement, 1 part; fine aggregate, 2.25 to 2.35 parts; and coarse aggregate, 3.75 to 3.65 parts.

The writer feels that an analysis of the author's tests in a manner similar to that shown on Fig. 4 would be of value to the Engineering Profession. With the present knowledge of the manufacture of concrete and the methods and equipment available for testing the materials, a general increase in the unit working stress is in order. The presentation of such papers as that of Mr. Hall will enable engineers to judge, more correctly, the extent to which an increase seems advisable.

MORRIS MASON,[21] JUN. AM. SOC. C. E. (by letter).—It has been stated by Mr. Hall that the more scientific methods of concrete control have produced a more uniform concrete. Others have found that a designed strength of concrete may be produced at a lower cost by the application of these methods. If this is true, it is difficult to understand why the application has not become more general; but engineers show a great deal of stubbornness in maintaining formulas and rules which they have found to be usable, although not perfect.

In Appendix II, the statement is made that "hostility is one of the greatest obstacles to the control and inspection of concrete construction." This is the prevalent attitude on many concrete jobs. The engineer often encounters the following attitude among the men in charge of concrete work: "I was pouring concrete twenty years before your method was devised. I have poured good concrete and can continue to do so." Steel was made successfully for many years, by the same type of methods which are now in use in mixing concrete, but no steel company would consider using such methods under present conditions.

A number of methods of concrete control have been used successfully in the past. The Illinois State Highway Department has followed the "mortar-voids" method. The "surface-area" method has been used to some extent and the "water-cement-ratio" method has also been very generally adopted with success. A conclusive proof of the advisability of these methods is given in the fact that when any of the newer methods is used, the old "trial-batch" or "fixed-ratio" methods are not reinstated. It is left to the decision of the engineer as to which of the methods is more usable on the particular work with which he is connected. Even if the application of the method be imperfect, "a poor estimate is better than a good guess." The reports on these methods are unanimous in that more uniform concrete can be produced at a lower cost.

---

[21] Engr., Standard Oil Co. (Indiana), Wood River Refinery, Wood River, Ill.

When the practical concrete man has grasped the advantages of these methods he is generally in favor of their continued use, because he sees the improved quality and uniformity of the concrete. Excessively sloppy mixes are slowly being condemned. The presentation of conclusive proof that better concrete can be produced, even under adverse circumstances, as has been shown in this paper, will hasten the discarding of many of the older methods of concrete control.

JOHN H. GRIFFITH,[22] M. AM. SOC. C. E. (by letter).—The writer is interested in this paper because of the information afforded as to the range of variation to be expected in systematic analyses of concrete laid under first-class conditions of fabrication, control, and tests. He believes that a higher degree of efficiency could be obtained than is usual in the testing of concrete if the capping of cylinders could be replaced by grinding the ends of specimens to a true plane approximately normal to the axis of the cylinders. This is easy with a cast-iron disk mounted horizontally and rotating about 30 rev. per min. Ottawa sand is a good abrasive. A number of cylinders can be handled simultaneously with very little watching, the proper pressure being supplied by their own weights. Both brick and hollow building tile develop higher and more uniform strengths by this method than by the older methods of capping. As Mr. Little has remarked in Appendix I the operator cannot be sure of a plane surface in capping and by the use of an intermediate paper. As he rubs the cylinder on a steel plate in any case, he need only carry this a little further to get a perfect job once for all, and without rocking and other expedients.

The ideal theoretical method for mounting silicate specimens in compression is the hydraulic bearing originated by Mr. Mont Schuyler[23] formerly of the Municipal Laboratory at St. Louis, Mo. This method in principle consists in mounting the specimen on a hydrostatic base, eliminating entirely the uncertain and quite considerable radial friction developed under pressure by the concrete expanding radially under test. The common method of frictional reinforcement at the base produces a cone-shaped fracture, while the test under a pure compression gives vertical planes radiating from the axis of the specimen 120° apart. The method proposed by Mr. Schuyler needs mechanical development since there is the problem of a possible "blow-out" under the high pressures involved. While the surface grinding method has some radial friction, the writer believes that it will give efficient test results as high as or even higher than Mr. Schuyler's method as a result of the perfect square ends and because the vertical filaments are under a minimum of eccentricity, there being scarcely any incipient local failures by this method.

Rather frequently in testing practice use is made of hemi-spherical heads that are likely to "stick" as a result of gumminess and infrequent oiling. It is a good plan to give a hurried wiping and squirt of oil to these heads preliminary to every test so that they are sensitive to every eccentric load however slight. This plan avoids applying test loads at the peripheries of specimens, such that they lower the intrinsic strengths of concrete. Too often this danger is not realized in practical testing. One should seek to develop the full theoretical strength of a specimen even if it is difficult always to attain it in practice.

---

[22] Prof. of Experimental Eng., Iowa State Coll., Ames, Iowa.

[23] "Spherical Bearings," by Mont Schuyler, *Proceedings*, Am. Soc. for Testing Materials, Vol. XIII (1913), p. 1004.

The following method has been found of general utility in making specific gravity, displacement, and absorption tests of the silicate products, including sand, rock, refractories, and concrete. It has the advantage of being performed either on a small or a large scale, with practically any sort of container, a pycnometer, an Ehrlenmeyer flask, a Schuette cone, a can with a constant level spout, or even a pail. Three-figure accuracy is possible with reasonable care, and four figures by careful work with temperature correction determinations. It is a method of differential weighing; the equations are as follows: Let the tare of the receptacle be represented by $W_0 = w_0$. The weight of the container filled with water to a mark or to the top equals,

$$W_1 = w_0 + w_1 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

and the weight of the container, the water to the same mark, and the weight, S, of the specimen, equals,

$$W_2 = w_0 + w_2 + S \ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$$

Subtracting Equations (1) and (2),

$$W_2 - W_1 = S - (w_1 - w_2) = \Sigma\, w - \Sigma\, \frac{w}{t} = \Sigma\, w\left(\frac{t-1}{t}\right) \ldots\ldots (3)$$

in which, $\Sigma\, w = S$, is the weight of the specimen in terms of its separate components, if any, either intact or powdered in a mortar to suit, and sifted, and $\Sigma\, \frac{w}{t}$ is the summation of the displacements of components in terms of the specific gravities. The displacement, in grammes or cubic centimeters, of the specimen, $S = \Sigma\, w$, is $w_1 - w_2$.

In the tests of the silicate products, $\frac{t-1}{t}$ is approximately the fraction, $\frac{5}{8}$, and is to be chosen subject to the statistical experience of the operator or determined by sufficient check tests. Since the specific gravity of water is ordinarily taken as unity, $\frac{t-1}{t}$ for water vanishes. Hence, there results (to a close degree of approximation):

$$\tfrac{8}{5}\,(W_2 - W_1) = \Sigma\, w = S_d \ldots\ldots\ldots\ldots\ldots\ldots (4)$$

for the dry weight of the specimen, and for the absorption of water, $S_w - \frac{8}{5}\,(W_2 - W_1)$, the other elements of the specimen, if any, being found by sieve separation in the usual manner, dry or wet according to circumstances. The figures for a run of wet concrete will demonstrate its inherent advantages as an expeditious method, the process being still quicker in the cases of paving and building bricks, and refractories (see Table 6). A constant level container was used.

The difference between Items 3 and 2, Table 6, is 3.280 kg. Taking eight-fifths of this gives the dry weight of the mix as 5.248 kg. Subtracting this value from Item 8, Table 6, gives 0.377 kg. in contrast to 0.375 kg. for the original weighed water (which, of course, is frequently unknown in practice).

The specific gravity of Ottawa sand, using a 10 or 20-gramme sample in a pycnometer, can be found by the method mentioned herein so as to give the order, 2.65, as usually recorded with repetitions within the range, 0.01, sometimes recommended. For refractories and paving bricks it has been found that five samples can be tested in the space of a few minutes using a 10-gal. pail with a "fiduciary" pencil mark on the inner surface of the pail to give results of the orders published by the authorities. In the case of relatively non-absorbent materials like paving brick, scarcely any water can be absorbed in the short interval of the test. When absorption tests have been run, the true or apparent specific gravities are quickly found without pulverizing materials, which is sometimes a factor in hurried determinations.

TABLE 6.—WEIGHT OF A MIX, APPROXIMATELY SOAKED, BUT DRY ON THE SURFACE

Item	Symbol	Weight, in kilogrammes	Description
1	W_0	2.216	Tare
2	W_1	12.990	Container and water
3	W_2	16.270	Container, water, and specimen
4	w_g	3.000	Gravel
5	w_s	1.500	Sand
6	w_c	0.750	Cement
7	w_w	0.375	Water
8	S_w	5.625	Wet mix

In making routine tests it is advisable to obtain once for all, the values of W_0 and W_1 as averages from a number of determinations. Using an empty, dry receptacle the sample is then best weighed for the gross, and the water filled to the mark later to give W_2. In general, by the use of a strictly differential method with as few manipulative operations as possible high efficiencies are obtainable with few losses due to transfers and other handlings.

It is the purpose herein to give the writer's experience in testing rather than to criticize Mr. Hall's valuable paper on the difficult problem of handling a synthetic material.

W. B. WENDT,[24] M. AM. SOC. C. E. (by letter).—This discussion is confined to tests made at the laboratory of the University of Louisville, begun in May, 1928, as presented in Table 1 of the paper. The compressive strength of the 1929 concrete averaged the highest for the five years and the variations of Item 3 from Item 2 and of Item 4 from Item 2 (Table 1) are considerably less for 1929 than for any other year, as shown in Table 7.

The percentage differences in Table 7 show conclusively that the quality of the 1929 concrete was much more uniform than that of other years, and that the continued insistence on more uniform and better concrete on the part of inspectors on the job was bringing results. As the writer

---

[24] Prof., Civ Eng., Speed Scientific School, Univ. of Louisville, Louisville, Ky.

TABLE 7.—PERCENTAGE VARIATION IN STRENGTHS

Items corresponding to Table 1	1925	1926	1927	1928	1929
TWENTY-EIGHT DAY TESTS (VARIATION ABOVE THE AVERAGE)					
Item 2	2 157	2 719	2 685	2 691	4 779
Item 3	1 590	2 186	2 128	2 215	4 308
Difference	567	533	557	476	471
Percentage difference	26	20	21	18	10
TWENTY-EIGHT DAY TESTS (VARIATION BELOW THE AVERAGE)					
Item 4	2 933	3 217	3 302	3 326	5 186
Item 2	2 157	2 719	2 685	2 691	4 779
Difference	776	498	617	635	407
Percentage difference	36	18	23	24	9

understands it, one of the main objects of Table 1 was to point out that good concrete could be obtained and that it would be fairly uniform.

The nine jobs were spread out for nearly 400 miles, and concrete was made by different organizations using different types of equipment and a variety of cements, admixtures, and river-bar aggregates. It is because of these conditions that the small variation in test results for the year 1929 is remarkable.

Capping and Storing the Cylinders.—The method of rocking a cylinder by hand on a plane surface (or by the "feel") as reported in Appendix I, was not the only method used to determine whether the ends of the test cylinders were truly plane surfaces. At the laboratory, the rocking of the cylinder was only one of several tests to which the cylinder was subjected to determine whether the ends were satisfactory for testing. Each cylinder was given a thorough visual inspection and each was also subjected to a steel straight-edge, when considered necessary. The cylinders were tested with a spherical bearing-block in the testing machine, and, therefore, no error was caused by eccentric pressure due to any ends not being parallel.

Contrary to current belief it is quite possible to apply the plaster-of-Paris cap without rocking the plate or the cylinder in such a way as to produce a curved surface. The method of procedure is as follows: The plaster of Paris is mixed with water to produce a paste of very plastic consistency. The correct quantity (determined by experience) of this paste is applied to one end of the test cylinder. This end is immediately placed on the bottom machined plate, and the cylinder is rotated and pressed down on the plate by hand until the thickness of the plaster of Paris is about $\frac{1}{8}$ in., or less. The paste is then applied to the upper end of the cylinder. A machined plate is immediately placed on this upper end of the test specimen and the cylinder pushed into position in the testing machine with the upper and lower machined plates still in place. The moving head of the testing machine is brought into contact with the upper plate, a load of several hundred pounds is applied to the test specimen, and this load is kept on until the plaster of

Paris becomes hard, requiring at least 15 min. The cylinder is then tested to failure. The writer has examined several hundred of these plaster-of-Paris caps after removing the test specimen from the machine, and he has never found one with a crack that indicated any signs of failure.

Cylinders should not be discarded because the ends of the test cylinder are not parallel. They are made in the field in an endeavor to ascertain the strength of the concrete. When these cylinders reach a testing laboratory, the laboratory technician must test them, using every precaution to insure that the test results will have value. The writer used the plaster-of-Paris cap in an effort to secure pressure distribution over the entire surface of the test specimen. All the testing machines used are equipped with spherical bearing-blocks, because it is hopeless otherwise to expect results to have any value. The test results on pairs of cylinders made of the same concrete check remarkably well, and it might be well actually to compute the variation from the mean of each pair of cylinders in order to show just how close the check was.

In many pairs the breaks of the two cylinders were remarkably close, and, in fact, two pairs under loads of nearly 200 000 lb. were recorded as nearly equal as could be read on the pulling arm, 5 ft. long, the sensitivity of which was 10 lb.

All cylinders received from May, 1928, were tested either by W. R. McIntosh, Assoc. M. Am. Soc. C. E., or by the writer.

William M. Hall,[25] M. Am. Soc. C. E. (by letter).—It is gratifying to note that there is much unanimity of opinion in reference to the advisability of teaching and training inspectors (even after graduation from the best engineering schools) as to "job" methods, system, and technique, and the treatment and training of the men who do the real work of delivering, measuring, and mixing materials, and transporting, placing, and curing the concrete.

The writer agrees with Mr. Seaman in his statement as to quality requiring "more than uniformity," and that it should be of uniform good quality. It is desirable to secure such uniformity for every batch of concrete, and the record thereof, as indicated by every cylinder, should be made. All the details he names—thorough mixing, sufficient caution in "moulding" or placing to overcome the segregation resulting from transportation, thorough "spading" next to forms to give a mortar face on every stone or gravel, and thorough spading in the body to release the bubbles of air entrapped when dumping—are desirable or requisite. Such uniform care day after day, month after month, and year after year, can scarcely be attained except by selecting those inspectors and operatives in whom some pride and enthusiasm can be inspired for accomplishing good work, or better work than that which is just passable. When forms are removed, each man trained for his particular work, or part of the work of each monolith, should make an inspection of the finished work with a diagram in hand, showing the engineer in charge the part done by each. If systematically done, this tends to perfect such good accomplishment and interest in the finished work.

---

[25] Civ. and Cons. Engr., Parkersburg, W. Va.

Possibly, Mr. Peck's exception to the statement that 50% of the concrete made to-day has a short life, is well taken. It is to be hoped that such is the case, but that 50% is below the average quality is believed to be unquestionable, and much below the quality of most of the concrete tested and reported in Table 1, for the four years, 1926 to 1929, inclusive. A careful reading of Mr. Peck's comments following that statement tends to confirm the accuracy of the writer's estimate of poor concrete. Possibly 50% is a little high and possibly Mr. Peck has not seen so much concrete, not only poor, but failing, as it has been the writer's misfortune to see.

As Mr. Kelly, who had some years of association with Professor Abrams, was the principal and invaluable aid in conducting the training school for inspectors, he fully understands its importance, and, on large structures, the importance of perfecting and systematizing methods of directing the workmen and conduct of the work by any new assembly of inspectors, such service as he rendered is almost invaluable.

Mr. Walker's exception to the specification of "15% or more of mortar than was needed to fill the voids of the gravel," is well taken. The actual practice throughout all the work was (as stated in Appendix II under "Batch Mixing,") "the specified measure of sand (equal to voids in gravel plus 15% of voids)" and, in addition, due allowance was made for bulking where an inundator was not used in measuring the sand. In fact, emphasis was always placed on the importance of preventing any misunderstanding and of always having uniformly workable concrete. It is known that with the mortar specification in force, unless the sand carries an unusual quantity of fines, or unless an admixture is used, a harsh product may result. Therefore, the "plus 15%" of sand was the adopted practice, emphasis being placed on "workability."

Professor Wendt submits his comments on laboratory practice, in breaking test cylinders made on this work. The answer to the questions raised by Messrs. Kelly, Walker, and Bauer and by Professor Griffith will be found in this discussion. It is observed that the impression was given that the method of rocking a cylinder by hand, or by the "feel," was the only one used. As affirmed by Professor Wendt, this was not the case; however, it appears the simplest and one of the best for efficiency, and minimum time is required by the laboratory technician for that test.

In the paper, under the heading "The Concrete Inspectors," reference was made to nineteen pairs of cylinders at Auxiliary Lock 41, at Louisville, and tested at the age of six months. Fig. 5 is a view of some of these job cylinders after testing (from left to right, Cylinders Nos. 54, 53, and 71). Cylinders Nos. 53 and 54, tested at the age of six months, were subjected first to 200 000-lb. pressure without breaking. A month later Cylinder No. 53 broke at a pressure of 194 000 lb. Cylinder No. 71 broke, at six months, under a pressure of 182 820 lb., as reported in the paper. Note the numerous sheared pieces of gravel.

It is gratifying to have such an extensive exchange of experience on the important details of laboratory technique; however, the writer will add a few explanatory words. As an example of the practice, Appendix I describes only the making and finishing of cylinders in laboratories "on the job." Professor Wendt's remarks are all in reference to the treatment after the cylinders are transferred from the field to the University Laboratory—two entirely different parts of the process by different personnel. The writer has emphasized the importance of completing the cylinders entirely and curing them "on the job" to within about five days of breaking, thereby reducing the chance of fracture in transfer to a minimum. After arrival at the Laboratory, it was the exception for a cylinder to require more than an examination of the ends and its general condition. It was then immediately placed in storage for curing until the hour of breaking. Each cylinder requiring new plaster-of-Paris ends consumed from 15 to 30 min. additional. Hence, the detailed description of the two processes, and the effort to prevent re-capping with plaster of Paris.

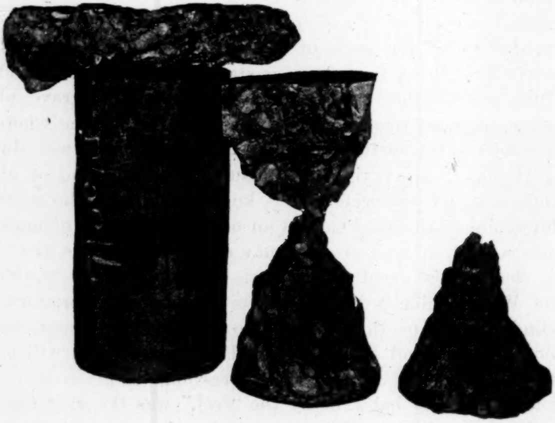


FIG. 5.—CYLINDERS BROKEN AT SIX MONTHS.

Preceding the engagement with Professor Wendt for breaking the cylinders, all the other cylinders included in Table 1, were broken by or under the personal direction of Herman H. Heck, Chemical Engineer, in a laboratory equipped and operated by the Byllesby Engineering and Management Corporation. Mr. Heck states that the methods of treatment, and the use of spherical head for applying and equalizing pressure conformed throughout with Professor Wendt's procedure. As regards finishing the ends of cylinders in field laboratories, it may be said in full justice to Mr. Little, that he has tried numerous methods of perfecting cylinders and, compared with the others, the "feel" appears to be the final test in his field laboratory for giving the best results, and the writer has observed no better method.

Messrs. Bonney and Mitchell who give such a good record of work accomplished for the City of Columbus, Ohio, are fortunate in the accomplishment recorded in Table 5. Mr. Arnold and the Los Angeles land owners are nearly as fortunate in obtaining concrete of about the same uniformity as that in Table 5, although with not quite so high an average strength. As the drains described are not subjected to freezing (which makes the question of waterproofness a minor factor of design), it appears that, should the uniformity in strength of the concrete be maintained it would be good practice to reduce the measure of cement to produce concrete slightly exceeding the designed strength. In fact, that is one of the most important reasons for such tests. It is regrettable that many other records of such uniform concrete were not given by other engineers or contractor managers. It adds to the evidence that such uniformity is only a question of training engineers, inspectors, and workmen, and that when owners become informed as to the increased durability of their structures, built of concrete of such uniform quality, it is probable that such methods and results will be demanded by them.

The writer wishes to express sincere appreciation and thanks to all who have contributed to this paper, including all those who took part in the construction described, and especially to all those with whom he was intimately associated in any part of the construction or testing work.

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
TRANSACTIONS

Paper No. 1817

SOME REFLECTIONS REGARDING ENGINEERING PRACTICE

ADDRESS AT THE ANNUAL CONVENTION, YELLOWSTONE NATIONAL PARK, WYOMING, JULY 6, 1932

BY HERBERT S. CROCKER,[1] PRESIDENT, AM. SOC. C. E.

It is safe, I think, to say that the minds of most engineers, at present are not overburdened with work. Such being the case it is only natural that they should be indulging in a great deal of solemn reflection concerning the internal problems of their profession; not only the problems of long standing but those which are bound up with the future.

My own reflections have had to do mostly with structural design and administration of construction work, since that is largely my field. There are two questions which I wish particularly to discuss. These questions may appear superficially to be unrelated, but since they affect every construction engineer profoundly, and every civil engineer to some extent, they may plausibly be discussed together. One is the sore question of the relations between those who design and supervise (the engineers) and those who undertake and execute (the contractors). The other is the question of the character which the work of the civil engineer may be expected to take in the future, of the opportunities which lie ahead not only for the experienced engineer but for the young engineering graduate. What I say must necessarily be taken merely as the speculation of an individual, based upon his personal experience and observation (which may be very different from that of other members of his profession). Also the matters discussed are controversial and are not susceptible of exact analysis. It may be profitable, nevertheless, to touch upon them.

In the earlier stages of engineering practice in the United States, there seems to have been a well defined line of demarcation between engineers and contractors. The contractor was a practical man in the most literal sense of the word; without especial pride in his important function; not overburdened with scruples as to the quality of his work so long as it yielded him a profit. If he found an opportunity to cut his costs, even though it meant a sacrifice of quality and workmanship, he seldom hesitated to take advantage of it. Indeed, this was sometimes a matter of pride with him.

---

[1] Cons. Engr., Denver, Colo.

The engineer on the other hand, both by education and technical training, held to a superior attitude. It was necessary and right that he should. As maker of the plans and specifications he was even more intimately acquainted with the detailed character of the work than was the owner himself. As the representative of the owner he also assumed the position of sole judge of the quality of materials and workmanship. His, in short, was the last word on any construction enterprise and he drew his specification accordingly.

This was, as I say, necessary and right at the time; but it led to a well-defined antagonism between the engineer and the contractor which has scarcely redounded to the benefit of either. The contractor frequently saw in the engineer, his enemy—the man whose principal aim, according to the contractor's point of view, was to keep a profit from being made. The engineer frequently saw the contractor, on the other hand, as an ever-present obstacle to good workmanship, a necessary evil who must be borne with, but only grudgingly. He usually approached the contractor in a spirit of suspicion, sometimes even with malice, and there were even engineers who held that they had not done their duty if the contractor did not wind up with a loss.

In recent years this situation has changed to a considerable extent. The traditional antagonism still hangs on here and there, but for a number of excellent reasons it has begun to die out. The experiences of the World War, when everything was subordinated to the one goal, had much to do with this. For another thing, there have been marked changes in the personnel of construction organizations. The increasing size of construction projects began to reveal the inadequacies and defects of the old-fashioned "practical" contractor. The more far-sighted contractors began to find that the technical man had much to offer them; and then began the infiltration of qualified engineers into the contracting service. To-day, many members of this Society are engaged in this work. The result of this introduction of engineers into contracting has been, naturally an improvement in the ethical standards of field management. Another important factor, of course, has been the laudable progress made by The Associated General Contractors of America in the promotion of higher standards of integrity in construction. As in many other lines of endeavor, it has become apparent that good workmanship is good business—that slipshod and "skimpy" construction is not, in the long run, really profitable. The "satisfied customer" is as great an asset to the contractor as to any other business man; perhaps even greater, for a construction contract is never entered into lightly and past reputation is frequently the deciding factor in awarding a contract. Still another influence in the improvement of the status of contracting is the contractor's realization that without close co-operation no work of construction may be carried on efficiently or with complete satisfaction to any of the parties concerned. Contractors taken as a body are to-day more reliable, more efficient, and more co-operative than ever before. Do not understand me to say that contractors have become perfect. Many "sharpshooters" are still to be found in the business and, on account of these, the unilateral contract is still a necessary club; but as a club it should be used sparingly and with discretion when a contractor is endeavoring to do good work.

During the upward progress of the contractor, how has the engineer occupied himself? He has made great advances in technique, in the knowledge of design, and in the art of construction; he has prepared himself by research, by careful technical training, and by experience, for the handling of works of hitherto undreamed magnitude. Not only that, the nature of his training has opened to him the field of executive management. Engineering training, with its emphasis on accurate analysis and close thinking, has proved itself of great value in the business world. The engineering-trained executive, placed in positions far removed from the actual technical problems of engineering, is now a commonplace.

But how about that difficult relationship between the engineer and the contractor? The contractor has made his advances toward a more satisfactory relationship. Has the engineer done his part? The answer, of course, is that in the minds of many engineers a great gulf still exists between the two. Contracts and specifications are still written upon the assumption that this gulf shall not—or cannot—be bridged. Many engineers, as individuals, still entertain the old hostility. There is a need for greater recognition of the unity of purpose by which the two are actually joined, and it is plainly up to the engineer to do his share in making this unity apparent.

The engineer should, in many cases, cast aside his "superiority complex," and recognize the qualified contractor as his co-equal, within the sphere of his special field; but the question of how this shall be done is not an easy one to answer.

One of the most difficult obstacles is the practice among engineers of placing upon their work as inspectors young engineers fresh from college, inexperienced, full of zeal, and with an abiding faith in the infallibility of plans and specifications. These become the rule and guide of their conduct, admitting no exceptions. Such faith in the infallibility of specifications is beautiful to behold, but hardly justified. If a perfect and all-inclusive set of specifications has ever been written, it has not been my pleasure to see it; and if so interesting a document exists anywhere it should be suitably enshrined where it may become the object of engineering pilgrimages.

Since such documents are so rare, it often happens that minor changes are either necessary to good workmanship or may be helpful to the contractor and at the same time be of benefit to the work. It is manifestly unfair, both to them and to the contractor, that young inspectors should be required to pass upon such points. Decisions of this kind require a background of experience and mature judgment which they cannot be expected to possess. The ideal inspector is capable of making fair decisions without the fear that the contractor may be deriving some hidden and diabolical advantage from them. A successful contractor usually has a fund of knowledge gained from practice—experience that stands him well in hand and that deserves the respect of any engineer. If relations between the contractor and the inspector are satisfactory, this fund of practical knowledge is available to the engineer at all times and he should not be afraid or unwilling to use it. The inspector most dreaded by the contractor is the young man who fears to depart from the letter of the specifications because of his inability to evaluate the advantages and disadvantages of such a departure.

Furthermore, in these days of highly competitive bidding and small margin of profit, a literal adherence to the plans and specifications in non-essential requirements may frequently change a profitable contract to a losing one without benefiting the finished product in any way. The old sport of "wearing out" a contractor is a vicious one, and we may frequently assume that the engineer who plays it is not an engineer of the best type. Rational practice requires that the field engineer in charge be a man of experience, endowed with sound engineering judgment and common sense. He should be capable of assuming responsibility and should be empowered to exercise it within reasonably broad limits; and he should be well paid for his work. Responsibility, also, should be on the job, not lodged in some distant office where the immediate requirements and peculiarities of the work are not known. In foundation work, for example, delays may be hazardous both physically and financially.

This is, in short, a plea for placing engineers of high calibre on the work and endowing them with greater powers of discretion than is now customary. It is a plea for recognition that the end of construction work is construction, and that the engineer charged with its immediate supervision is as important as the engineer engaged in design or remote supervision.

In Federal and State work this question of the relationship between engineers and contractors has peculiar difficulties not found in work of a private character. Both Federal and State highway departments are to be commended for the speed with which they have built up efficient organizations for the construction of a wonderful system of public highways during the past decade. However, owing to the handicaps of civil service and political preferment, they have been unable in many cases to place the field handling of the work in the hands of the experienced. This compels remote control, which, together with the necessity for dual supervision in the case of Federal Aid projects, has resulted in useless expense and unnecessary hardship to many capable and honest contractors. Many Government engineers are aware of these difficulties. How the difficulties may be overcome is a problem involving time, patience, and careful study by broad-minded engineers. I suggest that the qualifications of the men placed in charge in the field is the most important point to be considered.

So much for an old and difficult problem. I come now to another point, and one which is of even more concern to every engineer now engaged in construction work. I refer to the outlook for the future as it concerns these engineers and the young engineers who look forward to establishing themselves in this branch of the profession.

During the recent years of inflation, which were reflected among us by a tendency toward specialization and a feverish desire to outdo the magnitude of previous efforts of construction, engineers had difficulty at times in securing qualified assistants. (A qualified engineer I take to be one who possesses qualities of experience and ripened judgment, as well as of common sense and technical training.) Such well-rounded men were not to be found in sufficient quantity.

But that day is past—whether permanently or not no one can say. We have now entered into a period of smaller things, involving—so far as we are concerned—construction enterprises of less individual size. And although it may seem heresy to say so, I see in this new situation elements of possible benefit to the younger engineers. Huge organizations with centralized authority, as we have found out, do not solve all our problems. With the breaking down of many of them, and the dissolution of the mergers and combinations which brought them into being, construction is very likely to be resumed by smaller units. Each of these will contain all the elements that were contained in the larger. This means that instead of being swallowed up in a large enterprise and hence being forced into a one-sided development through over-specialization, the engineer may find himself in a more important and responsible position in a smaller concern. Engineering opportunities may be multiplied and diversified through local handling of construction. It may be that the field will open up once more for local consultants.

I think that whatever such a development might mean from the position of the economist, there is no doubt of its benefit not only to the individual engineer but to the profession. How much better for both that the young engineer should find opportunities and recognition in his own community than that he should be—as the bulk were rapidly coming to be—condemned to work in a narrow field.

It seems plausible that the answer to the question—"What is to be the future of our engineers in private practice?"—may be found in some such economic development. One need hardly fear that the collapse of the merging habit, which was carried to such extravagant lengths during the past decade, will dim the future of the engineer in private practice. On the contrary, it brightens it, and enlarges the opportunities for more well-rounded development of the younger engineer. Meanwhile, the opportunities for consultants, for pure technicians, and for those engaged in private practice, will still be open.

Another attractive field for the young engineer has been opened—as I have already suggested—in the contracting service. It will interest many to know that at present about 2 400 members of this Society, comprising about 16% of the total number, are engaged in this work. This proportion will unquestionably increase in the future, and by this leaven the improvement in the ethics and business standing of contractors will be carried still further—thus, incidentally, bringing the contractor-engineer problem a long step toward solution. That this entrance of more and more engineers into the contracting service will benefit the individual and the profession I have no doubt. I have still less doubt of the broad practical advantages, in terms of better and more economical construction, which will result from it. In entering this field the young engineer need have no fear that his professional standing will be jeopardized. He will have plenty of opportunity to demonstrate his skill and adaptability in the emergencies that will arise, and to broaden his experience.

Finally, in any consideration of the problems confronting engineers, the overwhelming importance of engineering education cannot be forgotten. Much progress has been made in this field of recent years, especially in the recognition that a purely technical training does not necessarily produce good engineers. Our engineering schools must impress upon their students—as many of them are doing already—the realization that when they leave college for the field they still have much to learn before they are really competent to take charge of work and to attain a true professional standing. No one ever learned the important qualities of diplomacy, common sense, and an understanding of men from purely technical courses—nor, indeed, from any other set course. But the wise educator can do much to further and encourage their development in the young engineer and hence broaden his usefulness to society.

I have sought in these remarks to touch upon some of the questions which deeply concern civil engineers at the moment. It must be confessed that the immediate outlook is not one to cheer the beholder. But that holds true not of engineering alone but of almost every field of activity. And unless we choose to share in the general discouragement, I see no reason for pessimism over the future of engineering. It is absurd to think of engineering as a decadent profession; in so vigorous a profession it is equally absurd to suppose that men of ability will not presently find congenial employment according to their talents. The high conception of professional responsibility which is held by engineers, combined with the indispensability of their services to our civilization, makes this inevitable. Although in the meantime we are beset by problems both personal and professional—problems which seem not easy of solution—we may assure ourselves that they will presently be solved.

MEMOIRS OF DECEASED MEMBERS

HOWARD ADAMS CARSON, Hon. M. Am. Soc. C. E.[1]

DIED OCTOBER 26, 1931

Howard Adams Carson, the son of Daniel B. and Mary (Pope) Carson, was born in Westfield, Mass., on November 28, 1842.

He first attended the public schools in Westfield, and from 1855 to 1861, his education was continued in the schools of North Oxford and Oxford, Mass. At the outbreak of the Civil War, he enlisted in the United States Corps of Engineers and served until 1864. At the end of the war he entered the Massachusetts Institute of Technology, from which he was graduated in the Class of 1869, with the degree of Bachelor of Science in Civil Engineering.

Immediately after his graduation, Mr. Carson began work with Shedd and Sawyer, Civil Engineers, in Boston, Mass. In 1870, he became an Assistant Engineer with the Brady's Bend Iron Company, at Weston, Pa. From 1871 to 1873 he was Assistant Engineer with the Providence (R. I.), Water-Works, and from 1873 to 1877 he served as Engineer in Charge of construction of sewers in Providence. He spent the winter of 1877–1878 in Europe studying sewerage systems.

From 1878 to 1884, Mr. Carson was Principal Superintendent of Construction of the Boston Main Drainage System. He invented the Carson trench machine which was used in the construction of this work, and he was afterward President of the Carson Trench Machine Company which built trench machines that were used extensively throughout the United States. From 1884 to 1887, he was engaged in general engineering practice. In 1887 and 1888 he designed for the State of Massachusetts the Metropolitan and the Charles River Valley systems of sewerage for about twenty cities and towns. In 1889, he was appointed Chief Engineer for the Metropolitan Sewerage Commission, which position he retained until 1894 when he was appointed Chief Engineer for the Boston Transit Commission.

In 1895, Mr. Carson toured Europe for the study of engineering works of interest in Italy, Austria, Hungary, Germany, France, and Great Britain. These are referred to by him in the First and the Second Annual Reports of the Boston Transit Commission. Mr. Carson designed and had charge of the construction of the Tremont Street Subway, which was the first subway for electric cars constructed in the United States; the East Boston Tunnel, which was the first under-water tunnel for car service in this country; and the Washington Street Tunnel. He designed and began the construction of a part of the tunnel under Beacon Hill for connection with the Cambridge Subway. In 1909, he resigned his position as Chief Engineer of the Boston Transit Commission.

Mr. Carson was Consulting Engineer on works in various parts of the United States, among which were the double-track railroad tunnel under the Detroit River and some of the early studies for the New York, N. Y., and the Philadelphia, Pa., subways. He contributed a long article on "Tunnels" for the Encyclopedia Britannica.

---

[1] Memoir prepared by Wilbur W. Davis and Thomas F. Sullivan, Members, Am. Soc. C. E., assisted by Edmund S. Davis, M. Boston Soc. of Civ. Engrs.

He was an Honorary Member and a Past-President of the Boston Society of Civil Engineers, and a member of the Institution of Civil Engineers, of London, England.

He was a life member of the Corporation of the Massachusetts Institute of Technology and the oldest in point of service, having served as a Trustee since 1880. He also was a Past-President of the Alumni Association. In 1906, he received the honorary degree of Master of Arts from Harvard University.

Mr. Carson was married to Nancy Wilmarth, of Oxford, Mass., on January 17, 1870. Mrs. Carson died in 1913.

No finer tribute could be paid to an engineer than to quote the following official statement entered on the records of the Boston Transit Commission when Mr. Carson submitted his resignation as Chief Engineer in 1909:

> "In accepting the resignation of Howard Adams Carson after a service of fifteen years as its Chief Engineer, the Boston Transit Commission places upon its records this expression of the great regret which it feels at his retirement from this position and of its high appreciation of the value of the work which he has performed for the public.
>
> "The Tremont street subway, the first for electric street car service constructed in this country, the East Boston tunnel, the first sub-marine tunnel built in this country for such service, and the Washington Street tunnel, stand as monuments to his engineering skill and judgment in the solving of difficult problems. The building of these great works, through narrow and crooked streets bordered by high buildings, through filled-in ground, or below the water level, has been a difficult achievement crowned with entire success. The record which he has made in the planning and execution of these novel and hazardous undertakings has placed him in the front rank of civil engineers.
>
> "Other qualities than those of technical skill have been shown by him in an equally notable degree. His tact in dealing with public and private interests, which in the progress of these works were of necessity seriously affected, his painstaking fairness in his relations with contractors, his skill in organizing and directing the engineering staff, his executive force, and his unflinching willingness to bear every proper responsibility of his office, have all been conspicuous. Scrupulous honesty and impartiality, moderation and modesty in all things, faithfulness to the point of extreme self-sacrifice have been characteristic of his service. At the termination of that service this Commission records its admiration of his professional skill and its high regard for his personal character."

He had been in ill health for several years before his death on October 26, 1931, and, consequently, had led a retired life during that time. The funeral services were held at his home on October 29, and burial was in the family lot at Woodlawn Cemetery in Malden, Mass. He is survived by a brother, Walter Scott Carson, of Greenfield, Mass.

Mr. Carson was elected a Member of the American Society of Civil Engineers on February 7, 1894, and an Honorary Member on October 10, 1921.

RAYMOND EDMOND ADAMS, M. Am. Soc. C. E.[1]

DIED DECEMBER 1, 1931

Raymond Edmond Adams, the son of Joseph Reed and Eliza Ann (Wescoat) Adams, was born at Hammonton, N. J., on May 26, 1874. He attended the public schools in Hammonton and in Philadelphia, Pa., and after graduation from the Philadelphia High School, and prior to 1898, gained considerable technical experience with various engineering concerns in Philadelphia and at Dauphin, Pa. During the Spanish-American War, he enlisted in the Corps of Engineers, United States Army, serving from May to December, 1898.

In 1899, Mr. Adams was employed in the Quartermaster General's Office, U. S. Army, at Washington, D. C., and served as Assistant Civil Engineer from 1902 to 1904 and as Civil Engineer from 1904 to 1917, his work pertaining principally to hydraulics and sanitation. While in this service, he enrolled in the Engineering Course at George Washington University, receiving the degree of Bachelor of Science in 1908 and that of Civil Engineer in 1909.

In May, 1917, Mr. Adams was commissioned a Captain in the Regular Army and served on the Quartermaster Corps Staff at the Headquarters of the Southeastern Department at Charleston, S. C., until injured in February, 1918. He was discharged from the Service in February, 1919, due to incapacitation as a result of his injuries. In 1919, he enrolled as a Vocational Training Student in the Wharton School of Finance, University of Pennsylvania, and was granted the degree of Bachelor of Science in Economics in 1921.

During 1921 and 1922, Captain Adams was a Resident Auditor for the Income Tax Unit, Washington, D. C., being advanced to Assistant Valuation Engineer in 1923, and Appraisal Engineer of the Engineering Division in 1924. From 1924 until his death he was Assistant to the Engineer in Charge of Civil Engineering, Hydraulics, and Sanitation in the Construction Division of the Quartermaster General's Office, in Washington.

As a young man, Captain Adams was intensely interested in athletics and was a bicycle rider of note during his residence in Philadelphia. He later became a devoteé of chess and checkers, and was President of the University of Pennsylvania Chess Club (undergraduate) during his studies there in 1919 and 1921. He was outstanding in his professional life as an advocate of high ideals in professional ethics, was painstaking in his work, and delighted in solving the abstruse and difficult problems encountered therein. Due to his long experience in engineering work relating to the Quartermaster Corps of the Army, he was a particularly valuable assistant.

Captain Adams was active in the United Spanish War Veterans, was a Charter Member of the Washington Society of Engineers, and also a member of the American Society of Political and Social Science.

---

[1] Memoir prepared by A. L. Anderson, M. Am. Soc. C. E.

He was a Mason, being affiliated with Olivet Lodge No. 607, Philadelphia, and Mount Horeb Chapter No. 9, Washington, D. C. He was a Protestant by faith and, although not affiliated with any church, was a splendid example of the true Christian, in that he inherently was possessed of a high standard of moral and social ethics, ordered his life in accordance therewith, and reflected this spirit by the kindly and generous manner he showed to all with whom he came in contact.

He is survived by his widow, Eleanore Hoffman Fletcher, whom he married in 1899, at Philadelphia, and by three children—two sons, Raymond Fletcher and Nellson Reed, and a daughter, Jessica (Adams) Lovewell.

Captain Adams was elected a Junior of the American Society of Civil Engineers on May 1, 1900; an Associate Member on December 6, 1905; and a Member on May 28, 1912.

WILLIAM GEORGE ADDIS, M. Am. Soc. C. E.[1]

DIED DECEMBER 17, 1931

William George Addis was born in Bristol, England, on May 6, 1862. His parents, Edwin J. and Elizabeth (Dimmer) Addis, were of English and Welsh descent. In 1870, the family, consisting of the parents, William, and a younger brother and sister, emigrated to the United States and settled first at Amherst, Mass., where the father began to practice his trade of cabinet maker, and where one more child was born to the family. The move to this country was probably influenced by the fact that two brothers of the elder Addis had previously come and were established in New Jersey and New York. Formal collegiate education was out of the question in this family, and as the oldest of four children, it was necessary for William George Addis to go to work in 1878, upon the completion of his public school education in Northampton, Mass. He was able, however, to follow a course of study and training in Civil and Mechanical Engineering privately under tutors, and for seven years found employment which afforded practical experience in these lines near home.

His active career in the practice of engineering began in 1885, and until his death, on December 17, 1931, Mr. Addis continued in engineering work. During this period he held many positions in his chosen profession which covered a wide field. The first eight years of practice were along mechanical lines. For five years he was Assistant Engineer on steam vessels on the Great Lakes and along the Atlantic Coast. From 1890 to 1893, he held the position of Construction Engineer with the Overman Wheel Company, of Chicopee Falls, Mass., and was in charge of the installation of power plants. From 1893 to 1901, he carried on a private consulting practice, and from 1901 to 1903, he was Chief Inspector of Reservoir Construction for the City of Northampton. In 1903, he was connected with the Metropolitan Water Board of Massachusetts as Inspector of Concrete Masonry in water tunnels.

---

[1] Memoir prepared by F. C. Pillsbury, M. Am. Soc. C. E.

For three years, from 1904 to 1907, Mr. Addis was Resident Engineer with the Massachusetts Highway Commission, in charge of construction. From 1907 to 1909, he held the positions of Designing Engineer and Superintendent of Construction with the R. B. Carter Company, of New York, N. Y., Water Supply Engineers and Contractors. From 1909 until his death, Mr. Addis was connected with the Highway Commission of Massachusetts (which, later, became incorporated in the Department of Public Works), except during 1918 when he was Inspector of Materials for the Construction Division, United States Army, at the Pittsburgh, Pa., District Office. During his connection with the State of Massachusetts, he served, successively, as Resident Engineer in charge of highway construction, as Survey Chief of Party, and as Assistant Engineer in charge of estimating.

Mr. Addis was deeply interested in the problems of the younger men in the profession. His cheerful philosophy, his keen sympathy and understanding, and his words of good advice, were ever a source of encouragement to those with whom he had contact, and his exemplification of living—of optimism—an inspiration. He was very fond of flowers; their cultivation was his hobby, and his brilliant garden annually was evidence of the success of his efforts in this direction. Throughout his life he had always been very active, but a serious illness in July, 1930, confined him to his home in Winthrop, Mass., for several months. In the spring of 1931 he was able to resume the duties of his employment, but early in December there was a recurrence of his illness, from which he never recovered.

Mr. Addis was a member of the American Association of Engineers, in which Association he had held the office of National Director for the New England District from 1927 to 1929, and in which, also, he was President of the Boston Chapter at the time of his death. He was affiliated with several Masonic bodies in the Connecticut Valley, including Jerusalem Lodge, A. F. and A. M., of Northampton, Northampton Royal Arch Chapter, Northampton Commandery, Knights Templars, and Melba Temple, A. A. O. N. M. S., of Springfield, Mass. He was also a member of the Canopy Club, of Boston, Mass.

He was married in September, 1893, to Anna H. Strong, of Glastonbury, Conn., who survives him. He leaves also a brother and other relatives both in this country and in England.

Mr. Addis was elected a Member of the American Society of Civil Engineers, on September 2, 1914. He was also a member of the Northeastern Section of the Society.

WILLIAM HENRY ADEY, M. Am. Soc. C. E.[1]

DIED MAY 17, 1930

William Henry Adey, the son of George and Katherine E. (MacNivin) Adey, was born in Cohoes, N. Y., on December 6, 1872, of English and Scotch ancestry. He was an alumnus of Rensselaer Polytechnic Institute, Troy, N. Y., from which he was graduated with the degree of Civil Engineer in 1895. He was a member of the Sigma Xi and Theta Nu Epsilon Fraternities.

---

[1] Memoir prepared by Dwight B. LaDu, M. Am. Soc. C. E.

After his graduation, Mr. Adey was employed on the construction of the Troy and New England Railroad and assigned as Inspector of pile-driving and pile trestles. He was also engaged for several years with Henry B. Seaman, M. Am. Soc. C. E., in land development work in Staten Island, New York, and, later, became identified with teaching his chosen profession at Swarthmore College, near Philadelphia, Pa. During several months of 1898, because of the disability of the Head of the Department, Mr. Adey was placed in complete charge of the Engineering Department. From May until November, 1898, he had charge, for the Dunsbach's Ferry Bridge Company, of the construction of a three-span pin-connected bridge, with a total length of 612 ft., across the Mohawk River, at Dunsbach's Ferry, N. Y.

In March, 1899, Mr. Adey accepted a position with the Engineering Forces of the Delaware and Hudson Railroad Company. He served in various grades, successively, from Rodman to that of Office Engineer, which position he held at the time of his death. During this period, he was engaged in preliminary surveys and surveys for improvements. Under the direction of the Chief Engineer, he designed coal-transfer trestles, a number of under and overcrossings, and was placed in charge of their erection.

As an Engineer, Mr. Adey was deeply interested in his profession and was esteemed by his associates. He was also interested in community affairs and was a member of the Masonic fraternity, the Young Men's Christian Association, the Albany Society of Civil Engineers, the National Geographic Society, and the Delaware and Hudson Veterans Association. He was a member of the Reformed Church of America.

He was married, in Cohoes, on May 1, 1902, to Myra Hermans Van Olinda. They had two children, Myra Harriet and Marion Elizabeth.

Mr. Adey was elected a Junior of the American Society of Civil Engineers on February 4, 1896; an Associate Member on December 3, 1902; and a Member on May 8, 1922.

ANDREWS ALLEN, M. Am. Soc. C. E.[1]

DIED MARCH 21, 1931

Andrews Allen was born at Madison, Wis., on January 11, 1870, the son of William Francis and Margaret (Andrews) Allen. His father was a direct descendant of James Allen, of Dedham, Mass. (1639), and his mother, of Thomas Andrews, of Hingham, Mass. (1642).

He attended the public and High Schools at Madison and entered the University of Wisconsin in 1887, from which he was graduated with the degree of Bachelor of Science in Civil Engineering, with honors, in 1891. In 1894, the University conferred upon him the degree of Civil Engineer, his thesis being "The Complete Design of a Railroad Swing Span, Including Steel Work, Machinery, and Power Plant." He was a member of Beta Theta Pi Fraternity and the Honorary Engineering Society, Tau Beta Pi.

---

[1] Memoir prepared by Joint Committee of the Society and of the Western Society of Engineers, consisting of C. W. Gennet, Jr., and A. J. Hammond, Members, Am. Soc. C. E., and C. W. Brooks, M. W. S. E.

Andrews Allen's early engineering training and experience was obtained with the Chicago and Northwestern Railway Company as Rodman, and as Assistant Engineer on city and county work in the vicinity of Madison, during vacation periods. After his graduation from college, he served with Professor Charles M. Van Hise, as Geologist for the United States Government Survey in Northern Michigan.

In the fall of 1891, he entered the employ of the Edgemoor Bridge Works, at Wilmington, Del., serving in the capacity of Draftsman, Assistant Engineer, and Designer. While he was at Wilmington he designed many railroad structures, including the Queen and Crescent Bridge over the Cumberland River, the Chicago, Lake Shore and Eastern Railway Company's swing span over the Calumet River, and a swing span for the City of Norfolk, Va.

In 1899, Mr. Allen became associated with the Wisconsin Bridge and Iron Company, of Milwaukee, Wis., as Contracting Engineer in charge of the Chicago Office. His work with this Company covered designing and contracting for steel structures of all kinds, including railroad and highway bridges and industrial and mining buildings. Some of the more notable structures designed under his supervision at this time included the Paoli Viaduct for the Monon Railroad Company, the Kankakee River Bridge for the Wabash Railroad Company, swing spans for the Chicago Junction Railway, and the Indiana, Illinois, and Iowa Railroad Companies, and the International Bridge, at Brownsville, Tex.

Mr. Allen was a specialist in the design of swing spans, and was frequently consulted by engineers where structures of this character were contemplated. The commercial side of his work, however, was not as interesting to him as the purely technical. Possessed of a keen scientific mind, and a thorough knowledge of mathematics and mechanics, he was always interested in developing a theoretically perfect structure. He developed the A-frame type of tower for coal mines, the three-column head-frame, and a special self-dumping cage for mining towers.

His desire for consulting work only, resulted in the formation, in 1911, of the Allen and Garcia Company, Consulting and Constructing Engineers. This Company, of which Mr. Allen was President, specialized in the design and construction of coal-mining plants throughout the United States. The Company was eminently successful, and, at the time of Mr. Allen's death, the members were Consulting Engineers for the Soviet Government in the development of its coal resources.

Mr. Allen was a non-tiring worker; on becoming absorbed in an engineering problem, he would labor from 16 to 18 hours a day until the task was completed. He frequently wore out his assistants, who were not able to bring the same energy and enthusiasm to their work that he always displayed.

He was Special Lecturer at Northwestern University on engineering subjects; Chairman of the Illinois State Board of Examiners for Structural Engineers; President (in 1909) of the Western Society of Engineers, and recipient of the Chanute Medal in 1916 for a paper on "The Engineering Opportunities of Our Coal Mining Fields".[2] He also contributed many articles to the technical magazines, as well as papers before the American Institute of Mining and Metallurgical Engineers and the Western Society of Engineers.

He was a member of the American Institute of Mining and Metallurgical Engineers, the American Railway Engineering Association, the Kentucky Mining Institute, the Chicago Engineers Club, the Western Society of Engineers, the Union League Club of Chicago, the Theosophical Society, the Wisconsin Society of Chicago, and the Masonic Orders, including Chapter, Commandery, and Shrine.

Andrews Allen was a rare character, loved by those who knew him for his great personal charm and willingness to help all those with whom he came in contact. He was universally admired for his engineering skill and attainments. He was a natural born teacher, and would work countless nights with the younger engineers, including many outside his employ, explaining mathematical and mechanical problems, expressing keen delight when the results of his teaching became apparent. He found his recreation in his great love of the out-of-doors, fishing, golf, and frequent camping trips to the Rocky Mountains. In the passing of Mr. Allen, the Engineering Profession has lost one of its most brilliant members, and the younger engineers a friend and counselor.

He was married, in 1916, to Elizabeth Emerson Cooke who, with four children, Francis, Henry, Andrews Jr., and Katherine, survives him.

Mr. Allen was elected a Member of the American Society of Civil Engineers on October 4, 1905.

LOUIS RUSSELL ASH, M. Am. Soc. C. E.[1]

DIED APRIL 7, 1930

Louis Russell Ash was born near Uniontown, Ky., on September 27, 1873. After completing his public school education, he attended the University of Arkansas and was graduated with the degrees of Bachelor of Civil Engineering in 1893 and Bachelor of Electrical Engineering in 1894. During his college years he had practical experience working in his father's saw-mills. With the early death of his father, he assumed with characteristic unselfishness the responsibilities of an eldest son in a large family—responsibilities which continued throughout his life. In 1895, he was called to the Chair of Mathematics in Coe College, Cedar Rapids, Iowa, where he taught for six years. During the summer vacations he was actively engaged in engineering work. He also attended the University of Chicago for special courses during one or two summers.

---

[2] *Journal*, Western Soc. of Engrs., Vol. XIX, May, 1914, p. 413.

[1] Memoir prepared by E. E. Howard and Henry C. Tammen, Members, Am. Soc. C. E.

Mr. Ash's active engineering work began in June, 1901, when he joined the staff of Waddell and Hedrick, Consulting Engineers, Kansas City, Mo., specialists in bridge design. His work was principally in the office in detailing and design work until he became Chief Draftsman, in which capacity he continued until the dissolution of that firm.

In January, 1907, Mr. Ash formed a partnership with Ira G. Hedrick, M. Am. Soc. C. E., as Consulting Engineers, and practiced for some years. Principal among the structures he designed during this period was the Oak Cliff Viaduct, at Dallas, Tex., a notable reinforced concrete viaduct of the arch type across the Valley of the Trinity River.

In 1910, Mr. Ash was made City Engineer of Kansas City, Mo., and began a notable service for that city which made him widely known and greatly respected, not only for his obvious ability as an engineer, but for his complete candor, open honesty, and unusual fairness. His character did not change; people merely came to know him until practically every one in the city knew him or knew of him and regarded him with both respect and affection which continued to the time of his death.

As City Engineer, Mr. Ash not only directed current work, but formulated extensive general plans for trafficways, and comprehensive sewerage and waterworks systems, many of which have since been carried out by others. He laid out and constructed the McGee Street Trafficway, one of the most heavily traveled thoroughfares of the city. He made a complete valuation of the Street Railways' properties. He had a principal part in the solution of the complex problems of trafficways into the low-lying Kaw Valley District which resulted in the construction of the Twelfth Street Viaduct and, later, the Twenty-third Street Viaduct. He evolved what came to be known as the Ash Plan for flood protection, and devised the Turkey Creek Tunnel diversion scheme. A change of administration after two years did not bring the customary and expected change of engineers, and Mr. Ash's services came to be so highly regarded that when he proposed to accept a more lucrative position, the Mayor increased his salary to an amount greater than that which he himself was receiving.

In 1913, however, Mr. Ash resigned from his municipal work and returned to private practice as a Principal Assistant Engineer of the consulting engineering firm of Waddell and Harrington. That firm was later dissolved, and Mr. Ash joined in forming the consulting engineering firm of Harrington, Howard, and Ash. He continued as a member of this concern until its dissolution in 1928, and then became a member of the firm of Ash-Howard-Needles and Tammen, carrying on a similar work. He continued with this firm until his death on April 7, 1930.

During this entire period Mr. Ash was very active in civic affairs in Kansas City. He served on two Charter Commissions and was active in promoting the present City Manager form of City Charter. He was a member of the first City Plan Commission and served thereon for several years.

His prominence in all efforts for civic betterment was so widely recognized not only in Kansas City, but in adjacent cities, that when, in 1917, Wichita, Kans., adopted the City Manager form of government, at that time the largest city that had yet turned to that plan, the municipal authorities impressed Mr. Ash into their service. Mr. Ash arranged a leave of absence with his firm in the summer of 1917 and served as City Manager of Wichita for about two and one-half years. This had exceeded his promised stay and he felt that he had demonstrated the merit and efficiency of the plan and had organized city affairs on a sound and business-like basis. Leaving a new host of friends and well wishers, and with testimonials of his successful administration, he returned to Kansas City late in 1919 and resumed practice with his firm.

He took a very active part in the work of all the firms with which he was connected, which covered the engineering of large bridges in many parts of the United States as well as abroad. Included were such structures as the Dumbarton Bridge across San Francisco Bay, the bridge across Mobile Bay, bridges across the Columbia, the Willamette, the Fraser, the Colorado, the Missouri, the Ohio, the St. Johns, the Hudson, the Arkansas, the White, the Mississippi, and other rivers, and many viaducts and similar structures. To this work Mr. Ash brought not only a sound technical ability and intense energy, but an unusual fair-mindedness in administration.

He had that rare sort of honesty that ignored entirely what might have been his personal interest or any thought of the effect of a decision on himself, and he had a fairness that could always see the viewpoint of the other man. These characteristics were so generally recognized and respected that he was frequently called upon for that trying task of acting as arbitrator in disputes between engineers, owners, and contractors.

The bridge at Vicksburg, Miss., which is the farthest down stream and probably the largest bridge yet constructed across the Mississippi River, was in a special way Mr. Ash's personal task and personal accomplishment. It was completed and opened to traffic only a few weeks after his death. He gave to it his closest attention—from the direction of the investigation of foundation conditions to substantial completion. He determined the location, projected the connecting railways and highways, directed the design, wrote the specifications, and gave personal supervision to the construction. His associates have felt that his intense fidelity to this work contributed to his early death. In the late fall of 1929 Mr. Ash visited the work to make a final decision on the foundation of one of the deep pneumatic piers. In spite of warnings from his doctor of a weak heart condition and high blood pressure, and contrary to other counsel, he felt so intensely his responsibility that he insisted on going himself into the deep working chamber under heavy air pressure, and there suffered a partial collapse. The owners are arranging for the erection of a memorial to Mr. Ash near the end of the bridge, comprising a stone with a memorial tablet and a bronze bas-relief portrait of him. From the sightly high hills of Vicksburg, this will overlook the bridge, his last great work.

His death came very unexpectedly and in the midst of activities. During March, 1930, he had a slight heart attack while on an inspection trip at Yankton, S. Dak., which incapacitated him for a day or two. On his return to Kansas City he pursued his usual work and was undergoing a complete clinical examination by his doctors and contemplating a considerable period for rest and recuperation. However, on April 6, 1930, during a Sunday of usual activity at church and at home, another heart attack occurred and Mr. Ash passed away on Monday morning, April 7, 1930.

Mr. Ash was a Member of the American Society of Municipal Improvements; Past-President of the Engineers Club of Kansas City; Past-President of the City Club of Kansas City; a member of Tau Beta Pi; a Mason; and was identified with other organizations. In addition to his professional work and civic and club activities, he was for many years active in the Linwood Presbyterian Church, serving as an officer in various capacities. He was also a Trustee of Park College.

His fellow engineers have expressed the opinion that there is no one left to occupy the unique place he held in Kansas City, and that the confidence of the people in him has reflected to the benefit of the entire Engineering Profession.

Mr. Ash was married in 1905 to Hadgie B. Davies, of Fayetteville, Ark., who, with one son, Samuel, survives him.

The *Kansas City Star* commented editorially as follows:

"Louis R. Ash was a type of citizen that every city needs. His profession of engineering may have had something to do with his high regard for system, efficiency, and order and with his intolerance toward things haphazard, including city government. He was assertively public-spirited. He recognized the obstructive effect of pernicious partisanship in city affairs. He helped to procure the adoption of the new charter after he had helped to frame it.

"Every genuine movement for municipal progress had his support. Some have been successful and some have failed, but Mr. Ash never lost his zeal for better standards of government. It was with better government as his objective that he led the recent non-partisan government movement before the city campaign. When, some years ago, he asked for leave of absence from his firm to accept the city managership of Wichita, the change must have involved some professional and business sacrifice, but he hoped to demonstrate the merits of the newer form of government.

"Mr. Ash was only 56. Actuated as he was by progressive, public-spirited ideas, he would have continued his efforts had he lived. In his death the city loses an individual factor in courageous civic effort. Even those who disagreed with him or lacked his faith in ultimate accomplishment, acknowledged his sincerity and respected his motives."

From *Engineering News-Record* is taken the following tribute:

"AN ENGINEER'S PUBLIC SERVICE"

"The quality of our municipal governments depends upon the collective efforts of the individual. It is good or poor depending upon whether the majority of those who direct its affairs are well trained or are uneducated in such matters.

"Civil engineers by their training are peculiarly fitted to advise cities upon many municipal matters and to take part in the direction of city affairs. But few of them do. Too many sit by and decry the fact that their qualifications in that respect are unappreciated. In this the fault lies with them rather than with the public; if they are to be appreciated they must take the initiative and so let the public know what they can do.

"The late Louis R. Ash was a splendid example of how effective engineers can become in municipal affairs if they make it their business to take more than a passive interest in civic problems. Of him, the *Kansas City Star* commented at the time of his death:

"'This country needs more such engineers. They may not be right at all times; they are sure to be misunderstood and abused frequently; but in the long run they will make an impression that will be felt and will be appreciated as Mr. Ash's service to Kansas City is appreciated.'"

Mr. Ash was elected an Associate Member of the American Society of Civil Engineers on December 5, 1906, and a Member on September 6, 1910.

PETER CHRISTIAN ASSERSON, M. Am. Soc. C. E.[1]

Died December 6, 1906

Peter Christian Asserson was born on the Island of Egersund, off the southwest coast of Norway, on January 5, 1839, the youngest of a family of twelve children. His forebears, as well as his own generation, were rugged Norse folk as evidenced by the family records which show that his paternal grandparents, his maternal grandmother, and his own mother reached the advanced ages of 100 years and more (the paternal grandmother, 104 years), and four of his own sisters and one brother were well over 90 years of age when they died.

His mother and father were staunch members of the Evangelical Lutheran Church and were deeply religious. He refers to them in writing as "my sainted mother and father to whose early teachings, precepts, and worthy example is due the little success that I may have been able to obtain in life." Although his father served as a Lieutenant in the Coast Guard from 1812 to 1814, at the time of the separation of Norway and Denmark, his real occupation was that of a farmer.

Peter Christian Asserson obtained his elementary education in the public schools of his district, and had his first engineering experience at the early age of 16 years, when he was employed as Second Assistant Engineer on the erection of a stone tower lighthouse built on an isolated and rocky promontory on Egersund's Island, about two miles from his father's farm. Toward the end of this year, however, an overpowering desire to follow the sea, so strong in the Scandinavian race, took possession of the lad and he shipped as cabin boy on a bark sailing from Stavanger, a near-by port, to the Mediterranean and Black Seas.

The following four years, from 1855 to 1859, were spent at sea, where he served as Second and First Officer and, finally, as Captain of merchant ships sailing from English, German, and Scandinavian ports. In 1859, at the age of 20 years, Mr. Asserson came to America with his mind made up to establish himself in this country. Soon after his arrival he obtained an appointment to the United States Coast Survey and Lighthouse Service. He first assignment was to the Gulf of Mexico where he was engaged on a hydrographic survey of the Gulf and where he also assisted in the erection of a large screw-pile lighthouse—"Shoal Light"—fifteen miles from shore, the first high-power lighthouse ever erected on a shoal in the ocean so far from the mainland.

---

[1] Memoir prepared from information on file at the Headquarters of the Society.

It was Mr. Asserson's intention to follow eventually the example of many of his Scandinavian compatriots, and as soon as circumstances and funds would permit, to make his way up the Mississippi River and to establish himself in the Middle West. Fate, however, dictated otherwise. When the storm, then hovering over the United States, broke on the morning of April 12, 1861, Mr. Asserson was faced with the alternative of making a hasty departure from the South or of acceding to the wishes and following the example of his friend and associate, Lieut. Joseph Fry, of Louisiana, by signing up for service with the Confederacy. As Mr. Asserson's sympathies were with the North, he was compelled to make a hasty departure from New Orleans, La., which he did by offering to take a merchant ship, which was loaded only in the afterhold, to her consignees in Spain. There was no time to shift the load so he ran the blockade which was already established in Southern waters, and, at great risk, due to the unequal ballast, set sail for Spain, where he safely delivered the ship and cargo.

As he had firmly made up his mind to become an American citizen, Mr. Asserson returned at once to this country and made preparations to fit himself for service in the United States Navy by taking special courses in engineering, navigation, and astronomy at Cooper Union, in New York City. He received instruction also under private tutors. It is of interest to note that he, with five other young men seeking Naval commissions, made up a class in navigation which was taught by the widow of a sea captain, Mrs. Thom, who had accompanied her husband for many years on his sea trips and who often assisted him in navigating his ship.

Mr. Asserson's industry and efforts were finally rewarded for he passed his examinations successfully and in May, 1862, received his appointment to the United States Navy with the rank of Master's Mate. He, thereafter, participated in many of the important naval campaigns of the Civil War. As Master's Mate he served throughout 1862 on the U. S. Ships *Relief, Vixen, Patroon,* and *Wamsutta,* on blockading duty. Toward the end of that year, November 24, 1862, he was promoted to the rank of Ensign and was given command of the *Zouave,* later, the *Cohasset, Shockokon,* and *Berberry.* In June, 1865, he was ordered to take the *Berberry* from Beaufort, S. C., to Hampton Roads, Va. On his departure, the Commanding Officer of the Station, Lt. Comdr. William C. West, wrote "your vessel has been ordered north—in parting with you I must express my appreciation of the ability and zeal you have shown in the performance of your duties while under my command. Wishing you every success in life, I am truly your friend." After this, Mr. Asserson served on the U. S. S. *Florida,* as Navigator and Division Officer. He participated in the attacks on Fort Fisher, Pocataligo Bridge, South Carolina; Castle Thunder, near Savannah, Ga.; Bermuda Hundred, Petersburg, Turkey Bend, and Drury's Bluff, in the James and Rappahannock Rivers. In these engagements he served under Rear-Admirals David D. Porter, S. F. Dupont, Hiram Paulding, and C. R. P. Rodgers. To the latter, Mr. Asserson was ever grateful for the encouragement and assistance he gave at a later date, toward helping him to get his appointment to the Civil Engineer Corps of the Navy.

From March, 1866, to 1868, Mr. Asserson was assigned to duty at the Norfolk (Va.) Navy Yard. From 1868 to 1869, he was on Coast Survey duty in command of the *Hasler* and, finally, in March, 1869, he was honorably discharged from the Service as a volunteer officer with the thanks of the Navy Department, which he had faithfully served from 1862 to 1869.

Mr. Asserson then became engaged in the important and difficult task of removing the obstructions and clearing the wrecks, the results of the war, from the Elizabeth, James, Rappahannock, and Potomac Rivers. He raised successfully four battleships and two frigates, besides a number of river craft, and in so doing performed some of the most difficult feats known to marine engineering. Among the wrecks removed were the famous gunboat *Merrimac,* sunk by the *Monitor* in Hampton Roads, near Crany Island; the U. S. S. *Cumberland,* sunk at Newport News, Va., the first of the Federal ships to be sunk by the *Merrimac;* the U. S. S. *Pennsylvania,* which was raised by means of a coffer-dam and towed to the dry dock at the Norfolk Navy Yard; the *Columbia;* the *Raritan;* and the U. S. S. *Delaware,* from which he secured, intact, the interesting figure-head of the famous Indian Chief, Tecumseh, and which, later (in the early Seventies), he presented to the U. S. Naval Academy where it was placed on a pedestal in the grounds of the Academy. Further honors were bestowed upon him (the old Chief) by the young midshipmen who dubbed him the "God of 2.5" (the passing mark in their studies), and who never fail to implore his favor before going up for their examinations. In recent years the picturesque old wooden figure which was painted to represent the regalia of an Indian chief, has been transferred to the Museum for preservation and has been replaced by a replica in bronze, which is still known as the "God of 2.5," and still receives the supplications of the midshipmen.

In June, 1871, Mr. Asserson received his final naturalization papers and thus became a citizen of the United States. In April, 1873, he was appointed Superintendent of Improvements at the Norfolk Navy Yard and Assistant in charge of the reconstruction work of that Yard, which, at the close of the conflict in 1865, was practically in ruins—the machine shops and office buildings had been burned to the ground, the dry dock blown up, and the gates and caissons destroyed.

In March, 1874, in President Grant's Administration, Mr. Asserson successfully passed his examinations and was commissioned a Civil Engineer in the U. S. Navy. He was at once placed in full charge of the reconstruction work already begun at the Norfolk Yard and within a period of ten years this station was practically transformed. Long stretches of substantially built wharves and quays replaced the old wooden ones; the dry dock was rebuilt; wet docks were built to receive timber needed for the ships of that date; and workshops and storehouses that covered many acres of ground were erected. The streets were well paved, and up-to-date sewerage and drainage systems were constructed. One of Mr. Asserson's outstanding accomplishments at this time was the installation of a salt-water fire system, the first in this country, and said at that time to be one of the most practical features introduced in any Navy Yard. By a unique automatic contrivance heavy pressure was carried on the salt-water mains only when needed.

In 1882, Mr. Asserson was promoted to the rank of Captain. This promotion, and similar promotions for other members of the Corps, came as a result of a long and vigorous campaign by him and his colleagues to have the same privileges of promotion and rank which the officers of the line and others of the staff were accorded. In 1885, Captain Asserson received due recognition for his outstanding accomplishments at the Norfolk Yard and was ordered by the Department to duty at the larger and more important naval station in New York, where he continued with indomitable industry to make that station one of the best equipped in the country. When he began his tour of duty the Yard was practically without a dry dock or wharf at which a ship could be tied up. There was only 15 ft. of water in the Wallabout Channel, the "cob dock" and ordnance docks were being eaten away by East River tides, many of the streets were unpaved, and the big granite Dry Dock No. 1 was leaking.

His first task was to save portions of the "cob dock" by building substantial sea walls in place of the old wooden ones. The new wall was 2 230 ft. long, built over solid cribwork of Georgia pine (2 by 12 in.), on which was laid a superstructure of concrete—the whole being capped with a coping of granite blocks. This piece of work was examined by experienced engineers who reported that there was not another like it in the United States. Here, again, he installed a salt-water fire system, as well as automatic sprinkler systems in the buildings; erected several new buildings, among them a mammoth machine shop; several miles of railroad; a new electric light and power plant; an immense coaling pier and coal-pockets to accommodate from 10 000 000 to 15 000 000 tons of coal. Dry Docks Nos. 1 and 2 were rebuilt of concrete. This was considered a triumph as they were designed, reconstructed, and enlarged without the help of outside contractors. For the former he built an underground electric pumping station, at that time the only one of its kind on record. This was constructed so as to save space and was located at the head of the dock, built entirely beneath the surface, encased within a caisson or wall of cement or stone, thus insuring the machinery against damage by cold, heat, or moisture. Centrifugal pumps were installed with a capacity of throwing 30 000 gal. of water per min. out of the dry dock, thus enabling the dock to be completely emptied of water in one-third the time and at one-half the cost of the former method. This also attracted much attention in the engineering world. Dry Dock No. 3 was built under his supervision and plans were made for Dry Dock No. 4, the latter, however, was not undertaken as his retirement from the service was then about due.

As early in his career as 1882 and 1884 Captain Asserson was called to the Bureau of Yards and Docks in Washington, D. C., as Senior Member of Boards to consider plans and specifications for the dry dock at Mare Island, California, and to examine and report on pumps in connection with dry docks.

In 1883, he received a note from the Hon. William E. Chandler, then Secretary of the Navy, in which he is commended for his "skill, energy and efficient service in successfully raising and docking the sea-going tug, the *Jean Sands.*"

In 1884, he was sent to St. John's, Newfoundland, to inspect a dry dock recently constructed for the Colonial Government and, a few months later, just prior to being sent to the New York Yard for permanent duty, he was ordered to examine the defective parts of the "cob dock" with a view to repairing it and to determine when and where the work of dredging near it should be done preparatory to the construction of certain timber cribs.

In 1889, Captain Asserson was appointed Senior Member of a Board for plans and specifications for a protective wall along the filled-in portion of the Delaware part of the Philadelphia, Pa., Navy Yard; in 1890, he was Senior Member of a Board for plans and specifications for the construction of a timber floating dock at Port Royal, S. C.; in 1894, Senior Member of a Board for plans and specifications for the construction of a dry dock in Puget Sound, Washington; in 1898, Member of a Board on "Location of Dry Docks"; in 1899 and 1900, Senior Member of a Board to inspect and consider certain changes in the sea wall and foundations of some of the buildings at the Naval Station in Annapolis, Md.; and, in 1900, on a board of inspection and survey of Port Royal and Charleston, S. C., for the purpose of examining the expediency of changing the location of the Naval Station from Port Royal to Charleston, or its vicinity.

His retirement from active duty was due January 5, 1901, but he served at the request of the Government, almost two years longer, and finally asked to be relieved of duty at the age of 64, as his health had become impaired. He was accordingly retired as a Rear-Admiral, the next higher grade to the one to which he had been promoted several years previously.

At the time of his retirement Rear-Admiral Asserson had made his reputation as an engineer of the highest ability. He was from the time of his appointment to his retirement senior member of the Corps of Civil Engineers, and was considered an authority on dry dock building, for which he was a Consultant at almost every Navy Yard in the country. He had a great fund of knowledge acquired largely in the school of experience and through constant reading and studying; he practically mastered the theory of his profession through his own efforts. He was an indefatigable and tireless worker and excelled as an administrator and executive in his ability to carry the important responsibilities of his department. One distinctive feature of his administration always was the moderate costs of his achievements, especially as compared with contract prices. He was always faithful to his trust and labored unceasingly for the best interests of the Government he was serving.

Loyalty to his Corps also led him and his colleagues, a small group of civil engineers, to try to break down the precedent which had been established for many years, of having a line officer in charge of the Bureau of Yards and Docks in Washington. In 1882, a petition signed by all members of the Corps was presented to Secretary Chandler, urging him in all justice and propriety to appoint a civil engineer to this important position. Finally, in 1898, success rewarded their efforts and Captain Asserson, as Senior Member and Dean of the Corps, was slated for this position, but he refused this honor as he had but two more years of service and the appointment was for four years. He believed that it would be unwise for the engineers to hold the hard-won position for so short a term, therefore, he insisted that the appointment should go to the late Rear-Admiral Moredecai T. Endicott, Past-President, Am. Soc. C. E., who received the appointment, much to the satisfaction of Captain Asserson and all the other civil engineers of the Corps.

Admiral Asserson was married in 1864, to Mary A. Wilson, a daughter of Henry and Rebekah Wilson, of Brooklyn, N. Y. Of their children, eight in number, seven were living in 1906 when he died of angina pectoris at the age of 68 years. Malena, the eldest, wife of Rear-Admiral W. B. Fletcher, U. S. N.; Henry R., a civil engineer in the Municipal Department of New York City, who served with the Engineers in France during the World War; Ane G., wife of Lt. Col. William F. Spicer, U. S. Marine Corps; M. Alice, a physician on the Medical Staff of Teachers College, Columbia University, who also spent a year in France during the World War organizing tuberculosis clinics under the auspices of the Rockefeller Commission for the Prevention of Tuberculosis in France; William C., a Captain in the U. S. Navy, who, as a young officer, took part in the Spanish- American War and who, later, in the World War, as Captain of the U. S. S. *Castine,* did patrol duty in the Mediterranean Sea; Agnes C., wife of Justin R. Swift, son of Rear-Admiral Swift, U. S. N.; and Frederick A., a surgeon in the U. S. Navy who, in the World War, was Executive Officer of the U. S. Naval Hospital in Newport, R. I., and, later, Medical Executive Officer of the U. S. S. *Leviathan* which was used to transport wounded soldiers from France to the United States. At present (1932), there are living four sons and daughters, fifteen grandchildren, six of whom were also commissioned officers in the U. S. Navy and served in the World War, and eighteen great-grandchildren.

Admiral Asserson's married life was a remarkably happy one—always a devoted, affectionate, and most unselfish husband and father. He had an inherent forcefulness and strength of character which was felt at once by all who came in contact with him, and although the practical side of his nature had been strongly developed his deeply religious and sympathetic nature was readily apparent to those who knew him best. He had a strong and rugged physique, inherited, no doubt, from his long-lived Norse ancestry, a cheerful, buoyant temperament, and a keen sense of fun and humor. He was a hard, conscientious worker, never satisfied with makeshifts or half-way measures; a favorite maxim and one which he quoted often to his children was "that which is worth doing at all, is worth doing well." He was a student and was blessed with an excellent memory. He found his greatest recreation in reading history and biographies of great men, and in his love for poetry and music. His mind was keen and alert, his vision far-seeing, his judgment good, and his ideals high. He was courageous to the point of aggressiveness if he felt it necessary to be so in upholding that which was right and just. A man, who with few opportunities and advantages in his young days, made a success of his life by cultivating the best that was in him and by giving generously of himself to the land of his adoption, his family, associates, and friends.

Admiral Asserson was elected a Member of the American Society of Civil Engineers on July 5, 1882.

FRANK HARRISON BAILEY, M. Am. Soc. C. E.[1]

Died April 29, 1931

Frank Harrison Bailey, the son of Harrison C. and Minerva L. (Stoddard) Bailey, was born in Wellsboro, Pa., on February 19, 1866. He was graduated from the Grammar School in the town of his birth, and from the High School and State Normal School of Mansfield, Pa.

Mr. Bailey was a student from the time he entered the Grammar School until his death. Graduating from High School at the age of 13, he passed the examinations to the State Normal School, but the State law would not allow his entrance until he was 16. He, therefore, returned to High School for special work until he reached the required age, when he entered the State Normal School, from which he was graduated in the spring of 1883 with the degree of Bachelor of Engineering at the age of 17.

His first work after graduation was with the Engineer Corps of the Fall Brook Railroad Company (now the Pennsylvania Division of the New York Central and Hudson River Railroad); after serving for a short time as Chainman, Rodman, and Levelman, Mr. Bailey went to Litchfield, Conn., as Transitman on a railroad locating party.

In 1885, he went to Boise, Idaho (then Boise City) where he was engaged as Transitman on location and construction work for the Union Pacific Railroad Company; in 1886, he went to Ouray, Colo., as Resident Engineer for the Denver and Rio Grande Railroad Company, and remained there until 1889 when he returned East. Shortly afterward he entered into partnership with Mr. William Rollins, then City Engineer of Elmira, N. Y. The firm carried on a general engineering and contracting business, but specialized in city improvement work, laying out and constructing additions to Elmira, as well as to many other towns in Southern New York and Northern Pennsylvania.

Early in 1895, Mr. Bailey was taken suddenly ill and for two months was not expected to survive, his partnership with Mr. Rollins in the meantime having been dissolved. It was not until the spring of 1896 that he was able to resume work, and he then took the position as Engineer for the New Orange Industrial Association, laid out and improved the town site of

---

[1] Memoir prepared by Ralph J. Bailey, Civ. Engr., Portland, Ore.

New Orange, N. J., and built a railroad connection from that place to the Central Railroad of New Jersey, at Roselle, N. J.

This work was completed in the fall of 1896, and Mr. Bailey returned to Elmira, where, for nearly two years, he was in the employ of the American Bridge Company's Engineering Department at its Elmira Plant. This engineering experience was invaluable to him, as it gave him a direct knowledge of design, layout, and construction of the various types of bridges, which was to serve him throughout the remainder of his engineering career.

In 1898, Mr. Bailey took the position of Resident Engineer at Caledonia, N. Y., with the Lehigh Valley Railroad Company, but only remained a short time, as in the early part of 1899, he accepted the position of Assistant Engineer, in the Construction Department, with the New York Central and Hudson River Railroad Company, with offices in Albany, N. Y. As such, he was in charge of reconstruction on the Middle Division, which included the elimination of grade crossings, track elevations, a new bridge over the Hudson River, at Albany, as well as a new passenger station, with subways to the various tracks. This work, including the new bridge over the Hudson River, on the site of the old bridge, was accomplished without the delay of a single train.

Completing this construction work in the summer of 1901, Mr. Bailey accepted a position as Construction Engineer with the Sonora Railroad of Mexico (Southern Pacific of Mexico), with headquarters at Guaymas, Sonora, Mexico, but as the climatic conditions were such that he could not withstand them in his physical condition, he soon returned to the United States. After recuperating for a short time, he accepted the position of Assistant Engineer in charge of construction with the St. Clair Bridge and Terminal Railroad Company (subsidiary of the Crucible Steel Company), at Pittsburgh, Pa. This work included the terminal facilities of the Clairton Steel Plant, at Clairton, Pa., as well as connections with the Pittsburgh, Virginia and Charleston Division of the Pennsylvania Railroad on the west side of the Monongahela River, and the Pittsburgh and Lake Erie Railroad on the east side of the same river. The bridge over the Monongahela River, with its approaches, was the principal engineering feature of this work. The structure has a clearance of 80 ft., with a main channel span of 500 ft., which is of steel arch design; it is also a combination railroad and highway bridge with upper and lower decks.

In January, 1902, after the completion of the Monongahela River Bridge, Mr. Bailey accepted a position with the Little Kanawha and the Buckhannon and Northern Railroad Companies, extensions of the Wabash Railroad in West Virginia, as Locating Engineer and, later, as Division Engineer, with headquarters first at Parkersburg and subsequently at Fairmont, W. Va. The construction of these railroads was abandoned in January, 1904, after all grading and culvert work was practically completed, which, again, made a change of position necessary.

Remaining in Fairmont, Mr. Bailey was soon employed on the location of the Fairmont and Mannington Railroad, a distance of 18 miles. After the completion of the survey, he accepted the position as Manager, and construction was immediately started, but because of the depression current at that time, the Company was unable to finance the completion of the work.

The Fairmont and Clarksburg Traction Company was contemplating the building of a connecting line between these two cities and early in 1905 Mr. Bailey was made Chief Engineer of this Company. Surveys were immediately started on this work, and by the end of 1906, the construction of a rapid transit line, 33 miles long, was completed between Fairmont and Clarksburg; this line passed through many mining towns and was a model of interurban construction.

In 1907, Mr. Bailey moved to Buffalo, N. Y., where he was made Assistant Chief Engineer of the Buffalo and Lake Erie Traction Company, in charge of all structural design; as this construction work closed down in the winter of 1907–1908, Mr. Bailey was sent by the holding syndicate of the Traction Company to report on an irrigation project in the Bitter Root Valley of Montana. His report was accepted, and he was made Chief Engineer of the Bitter Root Valley Irrigation Company. He returned to Hamilton, Mont., at once assuming active charge of the work, which included the construction of an immense earth dam at the head of the canal that impounded the waters for irrigation (now known as Lake Como); a main canal of 80 miles, with a cross-section of 18 ft. on the bottom, with 1 : 1½ slopes, and capable of carrying from 6 to 7-ft. depth of water. This necessitated the construction of thousands of feet of wood flume, two major inverted siphons—one crossing the Bitter Root River and the other the Sleeping Child Creek Canyon; also miles of lateral ditches for the distribution of the water. Mr. Bailey was Manager as well as Chief Engineer in 1910 and until the completion of the work in 1911.

From 1911 to 1914, he carried on a general engineering and contracting business in Hamilton, Mont. In 1914, he went to Alaska as Locating Engineer for the Alaska Engineering Commission, with headquarters at Fairbanks. While here Mr. Bailey located the Alaskan Railroad, from Seward to Fairbanks, a distance of 471 miles, and was Division Engineer in charge of construction until his return to his now favorite State—Montana—in 1920, where he remained until his death.

In 1921, he accepted the position of Bridge Engineer with the Montana Highway Commission. In this capacity, he designed and constructed numerous bridges throughout the entire State of Montana, the most important of which was one over the Missouri River in Eastern Montana. He remained with the Montana Highway Commission until his death which occurred on April 29, 1931, in Helena.

It was Mr. Bailey's interest in his work that brought on the sickness that caused his death; a few days before, he had gone from Helena to Cascade where some bridge work was in progress; while looking after this work, a sudden and extreme change in weather conditions (from mild spring weather to a blizzard) occurred; as accommodations were poor, the exposure was more than his constitution could withstand; returning to Helena on Saturday, he died the following Wednesday.

He was borne to his last resting place in Forestvale Cemetery, Helena, on Saturday, May 2, 1931, by members of the Engineering Staff of the State Highway Commission, with whom he had worked for years.

Although Frank Harrison Bailey was well and favorably known as an Engineer in a large part of the United States, it was in his adopted State of Montana that he was most intimately acquainted, not only with those of his profession, but with the people in general. He was loved by all who knew him for his pleasing disposition and his willingness to assist others in their work or in their daily life. He was greeted in all sections of the State with a glad hand and cheery "Hello Dad."

Shortly after Mr. Bailey was established in the engineering and contracting business in 1890, he entered into life partnership with Myrtle L. Welliver, then a school teacher and also a graduate of the Mansfield State Normal School, where they first met. Mrs. Bailey was his constant companion from this time on and was not only a faithful wife but a good adviser.

He is survived by his widow, Myrtle L. Bailey, of Helena; two sisters, Mrs. Eula I. Beach and Mrs. May H. Trowbridge, of Elmira, N. Y., and three brothers, Earle C. Bailey and J. O. Wright Bailey, of Fort Lauderdale, Fla., and Ralph J. Bailey, of Portland, Ore.

Mr. Bailey was elected a Member of the American Society of Civil Engineers on January 14, 1918.

SAMUEL POND BAIRD, M. Am. Soc. C. E.[1]

Died November 6, 1931

Samuel Pond Baird, the son of Samuel W. and Sarah (Pond) Baird, was born in Portsmouth, Ohio, on September 19, 1865.

Mr. Baird was always interested in things experimental and was a pioneer in the electrical field, being associated with the construction and operation of the first electric light and power plants in Portsmouth, Ohio, St. Louis, Mo., Sedalia, Mo., and Belleville, Ill. With the problems of power plant building, construction of track, etc., his interests gradually turned from the purely electrical to construction problems in general, and he built railroads, steel plants, brick plants, highways, sewerage systems, water-works, and laid out real estate developments. This work took him to many places where he always made many warm friends. In 1893, Mr. Baird built the first electric railway in Portsmouth, Ohio, and operated the System which is still running, as Superintendent and General Manager for twelve years.

After 1910, Mr. Baird maintained his home at Columbus, Ohio. He spent a number of years in Charleston, W. Va., where he rebuilt the plant of The Standard Brick Company and remained for a time as its Superintendent and General Manager. In Clearwater, Fla., and in Shelby, N. C., he was Resident Engineer in charge of the construction of large real estate developments.

---

[1] Memoir prepared from information on file at the Headquarters of the Society.

At the latter place, at Cleveland Springs Estates, the work included the building of a golf course, swimming pool, and Club House, as well as the paved roads, sewers and sewage disposal plant, water-works system, and electric lines needed to make a perfect residence community. His work in converting the hilltop section of his native town of Portsmouth into a residental section of beautiful homes, was also always a great satisfaction to him.

It had been his plan to retire at some future time to spend his last years in the congenial climate of Western North Carolina at a delightful spot, near Shelby. Little did he, or those nearest to him, dream that he would be cut off so suddenly from the work that he loved. Mr. Baird had always been a man of rugged health and great energy. When it was found that an operation was imperative, every one counted on his robust health to carry him through. He rallied splendidly until the fourth day when his heart suddenly failed and he passed away on November 6, 1931, in Grant Hospital, Columbus, Ohio.

Mr. Baird was married in Portsmouth, Ohio, on April 4, 1888, to May Johnson who survives him, with two children, Samuel P. Baird, Jr., of Akron, Ohio, and Madeline Baird Porter, of Youngstown, Ohio.

He was an active and interested member of the Protestant Episcopal Church, and wherever his work took him, he always made a church connection and became a valued member of a number of different parishes, serving on the vestries on several occasions.

In addition to his varied experience as an Engineer it is fitting that tribute be paid him as a man. Many a young engineer owes his success in no small way to Mr. Baird's kindly encouragement and timely advice. He was never too busy with personal affairs to lend a listening ear to another's need. A charming loyal friend, genial, even tempered, strong, capable, and willing, a lovable personality—his memory will be valued by many warm friends.

He had been a frequent contributor to such engineering magazines as *Engineering News-Record, Concrete,* and *Roads and Streets.* His ideas were always practical, and have been followed with profit by many.

Mr. Baird was elected an Associate Member of the American Society of Civil Engineers on March 1, 1899, and a Member, on September 4, 1906.

BERNARD ARTHUR BEHREND, M. Am. Soc. C. E.[1]

Died March 25, 1932

Bernard Arthur Behrend was born on May 9, 1875, at Villeneuve, Switzerland, the son of Moritz and Rebecca (Wolf) Behrend. His early education was by private tutor, and in England, followed by studies for the degrees of Civil Engineer and Mechanical Engineer at the Polytechnic Institute, at Charlottenburg, Germany. From 1890 to 1894, he continued his studies at the

---

[1] Memoir prepared by Carl J. Fechheimer, Cons. Engr., Milwaukee, Wis.

University of Berlin, and at Paris, France, and London, England. Subsequently, for about a year, he was Assistant to Gisbert Kapp, as a Designer of power plants.

From 1895 to 1898, Mr. Behrend was Assistant to the Chief Engineer of the Oerlikon Company, in Switzerland, in charge of the Experimental and Testing Departments, on the development and design of electrical machinery and the design of the Jungfrau Railway. It was during the early part of this period that he originated and developed the circle diagram for calculating the performance of induction motors in a simple and direct manner; this was an unusual accomplishment, particularly for one so young.

In 1899, Mr. Behrend came to the United States, and early in 1900 gave a course of lectures at the University of Wisconsin, Madison, Wis., on what was then a most obscure subject, "The Theory and Design of Induction Motors." The substance of those lectures he included in a book, "The Induction Motor, Its Theory and Design," which appeared in 1901. This book was also published in French in 1902 and in German in 1903. In that work the then little understood single-phase, as well as the polyphase, motor was treated.

Mr. Behrend became associated with the Bullock Electric Manufacturing Company, in Cincinnati, Ohio, in 1900, and was soon elevated to the position of Chief Engineer. Until the time of his connection with that Company, only direct-current machinery had been built there, and it was he who successfully designed alternating-current machinery, including alternating-current generators and induction motors, many of them of difficult ratings for the time at which they were built. His earliest salient pole alternators were of the revolving field type with the poles dovetailed to the spider, and the field windings were constructed of copper bent on edge, constructions which have been standard in most machines for many years. In 1903, he presented a paper entitled "The Experimental Basis for the Theory of the Regulation of Alternators,"[2] in which he clearly called attention to the errors of methods then in use for determining regulation, and suggested improved methods of computation, thereby paving the way to methods since adopted by the Standards Committee of the American Institute of Electrical Engineers.

Mr. Behrend recognized at an early date that it was more important that alternators be proportioned so that they maintain their voltages with loads at low lagging power factors than to possess close inherent voltage regulation, a practice that has since become universal with all builders. Whenever possible, his alternators were tested at nearly zero power factor, this being obtained by operating as load other synchronous machines under-excited and the machine in test over-excited. It was often difficult or impossible to secure an adequate load in that manner, so he devised the "split-field method," in which one part of the machine had lagging, and the other leading, current. He presented a paper[3] on that subject at the Electrical Congress at St. Louis, Mo., in 1904.

---

[2] *Transactions*, A. I. E. E., Vol. XXI (1903), pp. 497–517.

[3] The Testing of Alternating-Current Generators", *Transactions*, International Electrical Cong., St. Louis, 1904, Vol. I, pp. 528–541.

An interesting early achievement was the linkage, at Montreal, Que., Canada, in 1901, of the power from the Lachine Rapids, the Montreal Light, Heat, and Power Company, and Shawinigan Falls, eighty miles distant. It was the first successful paralleling of several large power stations connected through transmission systems and synchronous frequency changers, undertaken at a time when parallel operation of synchronous machinery was imperfectly understood. That work was done by Mr. Behrend in collaboration with Ralph D. Mershon, M. Am. Soc. C. E.

In 1903, the Bullock Company built its first turbo-alternator, which was exhibited at the Louisiana Purchase Exposition in 1904; it was driven by a Hamilton steam turbine. Mr. Behrend pointed the way in the mechanical construction of steam turbine-driven alternators at a time when salient poles and parallel slots were used. From the start he used radial slots, the coils were retained in position against centrifugal forces by non-magnetic wedges in the rotor body, and the coil ends were supported by steel rings placed over their outer periphery. That construction has since become standard with all manufacturers of turbine generators.

About 1904, the Bullock Company was taken over by the Allis-Chalmers Company, and a few years later Mr. Behrend and his staff were transferred to the main plant at West Allis, near Milwaukee, Wis., he retaining the position of Chief Electrical Engineer. He was also made Consulting Engineer of the Allis-Chalmers Bullock, Limited, at Montreal.

The Allis-Chalmers Company started to build steam turbines about the time it absorbed the Bullock Company, and Mr. Behrend designed and developed the generators which were driven by the turbines. For the time at which they were built, they were of difficult ratings, and many mechanical and electrical problems required his solution.

One of the problems that Mr. Behrend was called upon to solve was the successful parallel operation of gas-engine-driven alternators, about which little was known. Adequate damping to minimize the effects of fluctuation in applied torque was provided by means of solid steel poles and a low-resistance short-circuited winding. There was danger of having the natural and forced frequencies too near each other, with the possibilities of resonance. To avoid this, it was necessary to calculate the natural frequency from the torque in the generator per degree of displacement and the moment of inertia of the rotating parts. Mr. Behrend claimed that superior operation would be obtained by so choosing the fly-wheel that the natural frequency would be between two forced frequencies, rather than lower than the lowest frequency. The fly-wheels thus chosen were lightened, and the parallel operation was improved. Most of those large machines were built for the United States Steel Corporation, for use in its reconstructed power-houses at Rankin, Pa.; the Illinois Steel and the Indiana Steel Companies, at South Chicago, Ill., and Gary, Ind., respectively; and the American Steel and Wire Company, at Cleveland, Ohio. In all, the gas from the blast furnaces, which hitherto had been wasted, was used directly in the prime mover. In this work, his assistant, Mr. A. B. Field, collaborated.

Mr. Behrend early recognized that the distribution of stresses in rotating disks is a complicated phenomenon, and while with the Bullock Company he made tests on lead disks to study their permanent deformation when driven at high speed, thereby checking the theory on which he had also done considerable work. In his paper,[4] "A New Large Generator for Niagara Falls," presented to the American Institute of Electrical Engineers in 1908, he shows how the stresses are distributed, both radially and tangentially, in a high-speed disk. These were perhaps the earliest published curves of that character.

Mr. Behrend left the Allis-Chalmers Company in 1908, and assumed charge of the Power Division of the Engineering Department of the Westinghouse Electric and Manufacturing Company, at East Pittsburgh, Pa. He had realized the danger of failure occurring in high-speed turbine generator rotors from lack of homogeneity, from internal shrinkage stresses, or from pipes, cracks, etc., which are likely to appear in large steel castings or forgings. Recognizing that steel plates, when suitably forged or rolled, are practically homogeneous, he invented and developed the plate-rotor construction which the Westinghouse Company has used ever since in its large turbine generators of more than two poles. The construction is one that showed great ingenuity on the part of the inventor.

In 1911, Mr. Behrend opened an office as Consulting Engineer in Boston, Mass., and was retained by the Westinghouse Company as a Consultant, in which capacity he advised on many difficult engineering problems. In 1921, the second edition of his book, under the title of "The Induction Motor and Other Alternating Current Motors," appeared, much enlarged and revised, with treatments of certain types of motors omitted in the first edition. In 1926, he gave up the Boston Office, and continued his consulting practice at Wellesley Hills, Mass. He made an extensive study of machine tools and secured various patents on signal improvements that he affected.

Mr. Behrend was awarded a Gold Medal at the St. Louis Expostion in 1904, and the John Scott Medal by the City of Philadelphia, Pa., with the recommendation of the Franklin Institute, in 1912, for meritorious achievements in high-speed machinery. In 1931, the honorary degree of Doctor of Engineering was conferred upon him by the University of Darmstadt, Germany, "in appreciation of his meritorious development of electrical machinery, particularly of polyphase and single-phase induction motors."

Dr. Behrend, as a leader, inspired his associates and assistants and always offered words of encouragement. He was a man of vision, and one who knew well how to select men for his Engineering Staff. He was an able pioneer, who recognized while still very young, how machines should be designed, and always showed sound judgment. He was a great reader and student of the classics, in engineering and literature in general. He had a magnetic personality and a brilliant mind, but was of an exceedingly modest and retiring nature. Distinguished equally for scientific knowledge and constructive achievement, Dr. Behrend was an Electrical and Mechanical Engineer of more than National prominence. He had about seventy patents assigned to him, and was the author of numerous monographs and articles, among which may be mentioned: "The Debt of Electrical Engineering to C. E. L. Brown" (1901); "Engineering Education" (1907); "The Career of Oliver Heaviside" (1925); "The Work of Oliver Heaviside" (1928); and "Recent Developments in Precision Bench Tools" (1923). He was Non-Resident Lecturer at McGill University, the University of Wisconsin, and Leland Stanford, Jr., University.

---

[4] *Transactions*, A. I. E. E., Vol. XXVII (1908), pp. 1057–1068.

In 1926, he was married to Margaret P. Chase, of Boston, Mass., who survives him.

Dr. Behrend served on various committees of the American Institute of Electrical Engineers, such as the Edison Medal Committee, and was a Past-Chairman of the Electrical Machinery Committee. He was a Senior Vice-President of the Institute and was active on the Board of Directors.

He was a Fellow of the American Academy of Arts and Sciences, American Institute of Electrical Engineers, American Association for the Advancement of Science, and American Physical Society. He was a member of the British Institution of Electrical Engineers, American Society of Mechanical Engineers, Franklin Institute, and Society for the Preservation of New England Antiquities. He was also a member of the Engineers Clubs in New York, N. Y., and Boston, and the Athletic Club, in Pittsburgh, Pa.

Dr. Behrend was elected a Member of the American Society of Civil Engineers on October 5, 1909.

JAMES WARTELLE BILLINGSLEY, M. Am. Soc. C. E.[1]

Died August 17, 1931

James Wartelle Billingsley, the son of James J. and Georgine (Wartelle) Billingsley, was born at Arcadia, La., on January 29, 1880. He received his engineering education and training at Rensselaer Polytechnic Institute, Troy, N. Y., where as an outstanding student he was elected to Sigma Xi, an honorary fraternity for scientific research. He was graduated in 1902 with the degree of Civil Engineer.

Mr. Billingsley immediately began moulding an engineering experience which was to serve him well in his future work as a Consulting Engineer in extensive industrial and economic developments. His first position, in 1903, was with Sanderson and Porter, Engineers and Contractors, New York, N. Y., principally on electric interurban railway construction in Connecticut. In 1904 he was employed by the Westinghouse Electric Company at East Pittsburgh, Pa., in the Testing and Fabricating Departments. This engagement was in the nature of post-graduate work.

From 1905 to 1907, Mr. Billingsley was with Sanderson and Porter as Principal Assistant Engineer for the Local Manager in New Orleans, La., and was engaged in executing the work of reconstruction of the public utilities of New Orleans, including the installation of a new power plant for the New Orleans Railways and Light Company, with new electric and arc light system, the underground conduit system, and the laying of a submarine cable across the Mississippi River to furnish light and power to Algiers, New Orleans, La.

---

[1] Memoir prepared by a Committee of the Louisiana Section consisting of E. S. Lanphier, John T. Eastwood, and A. T. Dusenbury, Members, Am. Soc. C. E.

From 1908 to 1914, he was associated with Fred A. Jones, Consulting Engineers, of Houston and Dallas, Tex., first as Manager of the New Orleans Office and then as Assistant to the President. During this period Mr. Billingsley was in direct charge of all engineering work included in the construction of steam railways, power plants, water-works, hotels, and office buildings, and other types of construction.

In 1912, he was engaged with John W. Alvord, M. Am. Soc. C. E., Consulting Engineer, of Chicago, Ill., and the late Hiram Phillips, M. Am. Soc. C. E., to investigate and report upon a new water supply system for Oklahoma City, Okla. An exhaustive and complete report was made which was a most valuable contribution to the water-works art.

From 1914 to 1931, he was located in New Orleans, engaged in private practice as Consulting Engineer. Associated with him was his brother, Mr. Fred N. Billingsley, who, later, became his trusted and valued partner. It would be difficult, indeed, to cover in detail the many and varied extensive works of public and private character carried on by Mr. Billingsley during these seventeen years. Mention of the more important undertakings will suffice to convey the wide scope of his operations—water-works, sewerage and drainage, paving work, irrigation and reclamation, sea walls, highways, bridges, wharves and port development, electric and steam power plants, appraisals and reports on construction, and the supervision for bond and investment houses in connection with the financing of industrial plants and other construction work. He was engaged by public utility companies and cities, particularly upon matters pertaining to adjustment and arbitration, as an expert where conflict between the city authorities and the utility companies occurred in fixing rates for service. Notable among these cities are New Orleans, and Memphis, Tenn. For the last few years, he was retained by the City of New Orleans as Consultant in connection with regulating jurisdiction over the public utilities supplying electric, gas, and street railway service to the city.

Mr. Billingsley was also a pioneer in good roads in Louisiana, contributing valuable data and articles in his characteristic and sincere manner. He acted as Engineer for a number of Parishes in the State, namely, Richland, Claiborne, Catahoula, and Ouachita. His work involved many millions of dollars.

During the World War he served under the Chief of the Engineering Division of the United States Housing Corporation, in charge of all engineering work for housing development at New Orleans.

He was well known for his works of charity, both public and private, and gave generously to local charities.

Mr. Billingsley was a striking example of the economics engineer of to-day, with the ability to visualize in such manner as to prove practicable of execution. His judgment was sound and his wide knowledge of business methods, together with his insight of the financial operation of industrial, economic, and engineering problems, made him a most valuable asset to the community. To his social and personal charm he added the personality of his blithe and optimistic spirit, and, in his passing, the Engineering Profession and the City of New Orleans loses one of its ablest leaders and a genuinely constructive force for sound community upbuilding.

He is survived by his widow, the former Irene Rice, of New Orleans, whom he married in 1919.

Always closely in touch with technical activities and advancements, Mr. Billingsley was a member of the American Society of Municipal Engineers and the Louisiana Engineering Society. Prominent in the civic and social life of New Orleans, he was a member of the Boston, Pickwick, Louisiana, New Orleans Country, Metairie Golf, and Delta Duck Clubs.

Mr. Billingsley was elected an Associate Member of the American Society of Civil Engineers on May 6, 1908, and a Member on June 24, 1916.

FRED MILTON BISBEE, M. Am. Soc. C. E.[1]

DIED MAY 4, 1931

Fred Milton Bisbee was born on September 27, 1853, in Brunswick, Me., the son of John S. Bisbee and Eliza (Dustin) Bisbee. His mother was a descendant of Hannah Dustin of Colonial fame.

The early years of Mr. Bisbee's life were spent in Maine, and there he completed his education. He was graduated from Westbrook Seminary and the University of Maine, receiving the degree of Civil Engineer from the latter institution in 1876.

In 1878, Mr. Bisbee went West and for a few weeks was employed by the Kansas City Water-Works Company as Clerk. On May 1, 1878, he obtained employment in the Engineering Department of The Atchison, Topeka and Santa Fé Railway Company and was sent to "the front" of the main line then being built in New Mexico. When, in June, 1881, the line was completed into El Paso, Tex., he continued this pioneer railroad construction with the Mexican Central Railway Company, being placed in charge of rail-laying south of El Paso. There he constructed the first track connecting the United States with the Republic of Mexico. This event, attended with considerable ceremony, took place August 2, 1881, at El Paso. Governor Terrazas, of Chihuahua, Mexico, drove the golden spike.

Upon the completion of this work, Mr. Bisbee constructed another important stretch of track on the Mexican Central Railway at Chihuahua. This was a test of his untiring energy as it was necessary to expedite the work with all possible haste in order to obtain a Government bonus that was to be granted if Chihuahua was reached upon a stipulated date. This extension of the Mexican Central Railway was completed on September 15, 1882, and in celebrating the event, Governor Terrazas drove the same spike that was used at El Paso the previous year. Later, this spike was presented to Mr. Bisbee and was in his possession during his life time. It recently was added to the collection of the Texas State Historical Association, at Austin.

---

[1] Memoir prepared by William Pirschell, Office Engr., A. T. & S. F. Ry., Amarillo, Tex.

During 1883, Mr. Bisbee was Engineer in the Maintenance-of-Way Department of the National Railways of Mexico, which position he held until the latter part of 1884, when he returned to The Atchison, Topeka and Santa Fé Railway Company. During the remainder of that year and during 1885, he was in charge of construction of branch lines in Kansas, and, in 1886, was employed as Levelman on surveys. With the exception of a few months on the construction of the Colorado Midland Railroad, he continued with the Santa Fé, for the most part on construction work on the line between Kansas City, Mo., and Chicago, Ill., until February, 1888, when, for a time, he was employed as Superintendent of Construction for Lantry and Sons, Railroad Contractors. In December, 1890, Mr. Bisbee returned to the Santa Fé, and during 1891 and 1892 was General Roadmaster on the Gulf, Colorado, and Santa Fé Railroad. From 1893 to 1896 he was Superintendent of Bridges and Buildings for that Company, and then, for three succeeding years, he was with the St. Louis-San Francisco Railroad Company in a similar capacity. In 1899, he was employed by the Tennessee Central Railroad Company, as Chief Engineer, but again returned to the Santa Fé on July 1, 1904, as District Engineer of the Western Lines with headquarters at La Junta, Colo. On April 2, 1913, he was appointed Chief Engineer of the Western Lines with office at Amarillo, Tex., and continued in this position until his retirement on November 1, 1922. Since that time he had been living in Hollywood, Calif., where he passed away suddenly on May 4, 1931.

Mr. Bisbee's service with The Atchison, Topeka and Santa Fé Railway Company totaled thirty-three years of varied experience, such as Engineer of Track Construction on the pioneer extensions of the lines, General Superintendent of Bridges and Buildings, General Roadmaster on Maintenance of Way, and as District Engineer, and, during the closing years of this service, as Chief Engineer. In the latter capacities he had charge of many permanent improvements, such as construction of second track, new bridges, heavier rail laying, ballasting, changes of line, and reduction of grades; the westbound tunnel at Raton, N. Mex.; many station buildings and Harvey Houses, fuel and water stations, and mechanical facilities. A noteworthy example of the last mentioned class of work is the locomotive repair facilities and shops at Albuquerque, N. Mex.

He was a man of marked personality and untiring energy; essentially a man of action he vigorously prosecuted whatever work he had in hand. He was a loyal friend, constant and sympathetic. He was generous in giving advice and a helping hand to the unfortunate. His interest in civic matters in all communities in which he lived was very keen, and he had a wide acquaintance in all walks of life. He was approachable at all times, possessed sound judgment in making decisions, and was always fair in his business transactions.

Mr. Bisbee was a devoted husband. He took great interest in his home and those who knew him there, admired and loved him for the more sacred things of life for which he so firmly stood. Parity of age was not required by Mr. Bisbee in order to be listed among his friends, for he was particularly attracted to young people even as they were attracted to him.

He was a member of the Massachusetts Chapter of Sons of the American Revolution, and a Thirty-second Degree Mason.

He was married on November 4, 1884, to Addie Hamilton, who passed away June 11, 1911. On September 17, 1913, Mr. Bisbee was married to Mrs. Harriet C. Martin, who survives him.

Mr. Bisbee was elected a Member of the American Society of Civil Engineers on May 3, 1899.

HOWARD EDWARD BOARDMAN, M. Am. Soc. C. E.[1]

DIED FEBRUARY 28, 1932

Howard Edward Boardman was born in Meriden, Conn., on March 17, 1879, the son of Edward Allen and Nellie (Savage) Boardman. His ancestors had settled in Massachusetts Bay Colony in 1638. He prepared for college at the Meriden High School. In 1899, he was graduated with honors from the Sheffield Scientific School of Yale University with the degree of Bachelor of Philosophy. His engineering training as an undergraduate was in the Department of Civil Engineering and soon after graduation he served in that Department as an Instructor in a summer course in surveying. In 1931, the honorary degree of Master of Arts was conferred on him by Yale University.

Mr. Boardman began his professional career in the fall of 1899 as Draftsman in a consulting office in Philadelphia, Pa., but in the spring of 1900 he became Instrumentman on location with the Pennsylvania Railroad Company. During the next two years he was Assistant on the Engineering Corps and Acting Supervisor on the Pennsylvania Lines West, engaged in general maintenance on the Main Line Division and in charge of the construction of a large interchange yard.

For seven years, from 1902 to 1909, he was, successively, Instrumentman, Chief of Party, and Assistant Engineer on work pertaining to the construction of the North River Tunnels under the Hudson River to the Pennsylvania Station in New York, N. Y. His work involved the preliminary and final triangulation surveys, contract drawings, and estimates, and the construction of the cast-iron subaqueous and concrete-lined tunnels. He was in charge of all estimates, final detailed cost records, and reports, and of the alignment, until the junction of the tunnels from the opposite shores.

---

[1] Memoir prepared by J. C. Tracy, M. Am. Soc. C. E.

From 1909 to 1912, Mr. Boardman was Senior Assistant Engineer, and then Division Engineer, on the Missouri Pacific Railroad. He was in charge of the maintenance of a division 525 miles long and, later, of all construction work planned in the office of the Chief Engineer of Maintenance of Way.

From 1912 to 1916, he was Chief Assistant Engineer of the Underground Lines of the Buenos Aires Western Railway in South America. He was in direct charge of the construction of 3½ miles of single-track freight tunnel, ½ mile of double-track passenger tunnel, and the joint underground transfer station under the City of Buenos Aires.

Beginning in 1916, Mr. Boardman served for seven years as Engineering Assistant to the General Valuation Counsel of the New York Central Lines. As such, he was engaged in organizing the valuation work at the various headquarters of the System, and in making investigations, special reports, and cost analyses.

For a period of six years, beginning in 1923, he was not, strictly speaking, engaged in engineering. During this period he held three positions. As President and Treasurer of Foster Merriam and Company, of Meriden, hardware manufacturers and light gray iron and brass founders, he entirely and successfully re-organized the Company. Later, he became Manager of Engineering and Sales of the Divine Brothers Company, of Utica, N. Y.; and, still later, Manager of Research and Assistant to the President, National Industrial Conference Board, New York City.

In 1929 Mr. Boardman returned to engineering, as Construction Engineer for the Boston and Maine Railroad Company, in charge of important betterments, including nine miles of heavy, new-line construction in the Connecticut Valley.

In July, 1931, he began his work at Yale University as the first Dudley Professor of Railroad Engineering, a professorship established under the wills of the late Plimmon H. Dudley, Consulting Engineer of the New York Central Lines, and his wife, Lucy Bronson Dudley. This position offered an exceptional opportunity for research in the field of railroad engineering, and Professor Boardman entered upon the work with enthusiasm and energy. He began at once to make a thorough and comprehensive study to determine in what type of research he could render the greatest service. He sought and obtained the advice of many leading railroad officials and engineers and, at the time of his death, he was planning conferences with many others. It was his desire to secure the co-operation of the railroad companies in any research that he might undertake, and he had already received encouraging assurance of such co-operation. As a result of this preliminary study, he had begun to outline a program of investigations that gave promise of important results.

On September 24, 1903, Professor Boardman was married to Enna M. Jenkins, of Allegheny, Pa., who, with their two sons, Edward T. and Howard F. Boardman, survives him.

He was a member of a number of scientific and technical societies, including the American Society of Mechanical Engineers; the American Railway

Engineering Association; the Connecticut Society of Civil Engineers; and the Yale Engineering Association.

A hard and tireless worker, unusually efficient in everything he did, Professor Boardman, nevertheless, had broad interests outside his profession. He was active in promoting civic welfare, and was particularly interested in work for boys and young men. Beneath a quiet, modest demeanor and a winning personality, there were the qualities of a man of power and of high ideals. A forceful, successful engineer, he was also a thorough student, interested in research.

Professor Boardman was elected a Junior of the American Society of Civil Engineers on October 7, 1902; an Associate Member on January 4, 1905; and a Member on October 1, 1912.

WILLIAM JAMES BOUCHER, M. Am. Soc. C. E.[1]

DIED SEPTEMBER 14, 1929

William James Boucher was born in Jersey City, N. J., on January 19, 1875, the son of John and Lydia (Anderson) Boucher. He was of French-German and English descent; his ancestors on his father's side came from Holland in 1645, settled in Bowling Green, New York City, and, later, moved to Castleton, near Albany, N. Y., where Mr. Boucher's father was born. As a young man Mr. Boucher's father went to New York City, to enter business. He died when his son was twenty-two years of age and, after his death, his widow and son came from Albany to live in New York.

In his early years Mr. Boucher evidently had a strong liking for mechanical appliances, perhaps inspired by his occasional visits to the round-houses and shops of the railroads terminating on the west side of the Hudson River. A little later, electric apparatus, such as electric bells, induction coils, and frictional electric machines appeared to have engaged his attention. It was quite natural, therefore, that he should take up the study of Mechanical Engineering at the Stevens Institute of Technology in Hoboken, N. J., from which he was graduated as a Mechanical Engineer in 1896.

From 1896, following his graduation, to the time of his death in 1929, Mr. Boucher was professionally engaged in a considerable number of varied occupations which gave him a broad background of engineering knowledge. While his first engagements covered the field of mechanical engineering, his early association with contractors and contact with construction work, coupled with his adaptability to new problems, eventually led him to devote himself principally to civil engineering construction work. In this field he became connected, partly in responsible positions, with some of the most important projects.

Mr. Boucher's successive engagements and positions were, briefly, as follows: From 1896 to 1900, various positions which involved drafting, designing, shop work, inspection and installation of gasoline and oil engines, electric motors, elevators, controllers, steam engines, boilers, blowers, mechanical draft and heating apparatus, street cars, steam railroad cars, locomotives, and shop layouts; from 1900 to 1901, Chief Draftsman in the Mechanical Department of the Central Railroad Company of New Jersey; from 1901 to 1909, Draftsman and, later, Office Engineer with the Rapid Transit Subway Construction Company, Contractors for the first rapid transit subway in New York City; from 1909 to 1910, Office Engineer and Purchasing Agent for the Gore-Meenan Company, Contractors, in charge of purchase of equipment and supplies for the construction of 4 miles of the Catskill Aqueduct and a 1-mile rock tunnel in Connecticut; from 1910 to 1911, Engineer for the Chicago Subway Commission, specializing in designing and estimating of river tunnels, stations, drainage, and ventilation; from 1911 to 1912, Engineer for the Mineral Point Zinc Company and the New Jersey Zinc Company, Depue, Ill., and New York City, in charge of cost estimates for a $1 700 000 addition to the Company's zinc smelting and sulfur acid plants in Illinois and Pennsylvania; and from 1912 to 1919, Assistant Engineer with the Degnon Contracting Company, Contractors, in New York City, engaged in estimating costs, planning and supervising heavy construction work, comprising subways, tunnels, steel viaducts, foundations, sewers, water pipes, and electric-conduit lines.

---

[1] Memoir prepared by O. H. Ammann, M. Am. Soc. C. E.

During the World War (1918) Mr. Boucher was engaged as Engineer in the construction of the Port Terminal at Charleston, S. C., a $17 000 000 rail-and-water proposition, with buildings for the storage of explosives, barrack and camp buildings, roads, and dock.

From 1919 to 1920, he served as Engineer for the Philadelphia Subway Construction Company, Philadelphia, Pa., Contractors for $1 200 000 sewage treatment plant and for nine miles of State highway; and from 1920 to 1923, he was Assistant Supervisor, Department of Way and Structures, Eighth Avenue Railroad Company and Ninth Avenue Railroad Company, New York City, in charge of construction, maintenance, and repair of street railroad tracks, underground conduit systems, storage yard, and shop buildings.

From 1923 until the time of his death, Mr. Boucher was connected with the Port of New York Authority. In the capacities of Designing Engineer and Assistant Chief Engineer, he made investigations for additional tunnels under the Hudson River and for a belt-line railroad in New Jersey and in connection with inland freight stations. From 1926, he acted in the capacity of Engineer of Construction and, as such, had charge of the field operations in connection with the construction of the two bridges across the Arthur Kill, at Perth Amboy, N. J. (Outerbridge Crossing), and at Elizabeth, N. J. (Goethals Bridge), the bridge across the Kill van Kull, at Bayonne, N. J., and the preliminary field surveys for the bridge across the Hudson River, at Fort Lee, N. J. (George Washington Bridge).

Mr. Boucher manifested unusual loyalty to his employers and devotion to his work. Untiring in his efforts to accomplish results, he did not shrink from devoting long hours to his duties. Somewhat critical of those whom he did not know, he became their ardent collaborator when convinced of their sincerity and good will. Thus reserved in the selection of his friends, he was most devoted and loyal to those few with whom he was on intimate terms. All those with whom he came in daily contact benefited by his gentlemanly courtesy and cheerful, unassuming disposition.

Besides the heavy occupational duties of his last few years, Mr. Boucher found time and energy to devote himself wholeheartedly to various social and professional interests.

To his Alma Mater, the Stevens Institute, he remained a loyal friend. Its alumni, officers, and trustees find high praise for his devotion and they regret deeply his loss. Among his extra-curricular interests were the Stevens Engineering Society and the *Stevens Life,* which he edited in the last year of his college course. He was Secretary and later President of the Engineering Society. He accepted offices and responsibilities in the Alumni Association, passing through various duties to those of its Presidency in 1928, and, finally, in 1929, he was chosen a member of the Board of Trustees of the Institute.

Mr. Boucher also devoted much time and thought to the Broadway Presbyterian Church, of which he was an active Trustee.

He was not married, but his disposition reflected the happy home life which he shared with unusual filial devotion with his mother, whom he survived only a short time. He died suddenly of heart attack while bathing near Long Branch, N. J., on September 14, 1929. He lies buried at his ancestral home, Castleton, N. Y.

Mr. Boucher was elected an Associate Member of the American Society of Civil Engineers on September 6, 1905 and a Member on December 6, 1920. Throughout his membership he showed much interest in the activities of the Society and contributed to its publications and discussions. At the time of his death, he held the office of Assistant Treasurer of the Society.

WILLIAM ALGERNON BRACKENRIDGE, M. Am. Soc. C. E.[1]

DIED NOVEMBER 29, 1929

William Algernon Brackenridge was born at Richmond Hill, N. Y., on June 15, 1858. He was the son of Gavin Brackenridge and Anne (Hurry) Brackenridge. He attended the schools of New Brighton, Staten Island, N. Y., and Toms River, N. J.

Mr. Brackenridge commenced the practice of civil engineering on surveys and construction of elevated railroads in New York, N. Y., and was engaged on this work from 1877 to 1880.

From 1880 to 1882 he was a Transitman in the Survey Corps, and Resident Engineer on surveys and construction of the Western Extension of the main line of the Delaware, Lackawanna, and Western Railroad, from Binghamton to Buffalo, N. Y.

---

[1] Memoir prepared by Harry W. Dennis, M. Am. Soc. C. E.

During 1882 and 1883 he was engaged in making preliminary surveys for the proposed Hartford and Harlem Railway, from New York City to New Britain, Conn., a project which was not built. In 1883, he became Principal Assistant Engineer of the Brooklyn (N. Y.) Elevated Railway, and was in charge of all surveys and construction.

From 1885 to 1889, Mr. Brackenridge was Principal Assistant Engineer of the Brooklyn and Long Island Cable Railway and designed and had direct charge of all surveys for underground and elevated railway structures on Atlantic Avenue, Brooklyn. He also had charge of the construction of docks, ferry slips, and general improvements for the Long Island Railroad System. In 1889, he attended the International Convention of Civil, Mechanical, and Mining Engineers held in London, England, and spent eight months in investigation and study of the principal engineering works in England, and of hydraulic works on the Continent.

Upon his return in 1890, he was associated with the late Clemens Herschel, Past-President and Hon. M. Am. Soc. C. E., in making surveys, plans, and reports to the Railroad Commissioners of the State of Connecticut on the elimination of grade crossings in the City of Waterbury.

In 1891 Mr. Brackenridge began work along the special line of engineering to which he devoted practically all his subsequent professional career, namely, the development of hydro-electric power. He accepted an appointment with the Board of Consulting Engineers then engaged in preliminary plans for the development of water power at Niagara Falls, by the Niagara Falls Power Company, which engagement resulted later in his being appointed Engineer in Charge of all the construction operations of the Niagara Falls Power Company and its allied companies, at Niagara Falls, including the main tail-race tunnel and wheel-pits, and Power Houses Nos. 1 and 2 on the American side of the river, and the development of the Canadian-Niagara Power Company in Canada. He planned and directed the execution of work in connection with the abatement of floods on the lands of the Niagara Development Company, and superintended the construction of the industrial Village of Echota, including its sewerage system, sewage disposal works, land drainage, and street paving; built the Niagara Junction Railway; and was Engineer for, and superintended the works of, the Niagara Falls Water-Works Company, including the filter plant and hydraulic pumping plant. The power stations built under his direction at Niagara Falls, N. Y., are known collectively as the Edward Dean Adams Station. His active engagement with the Niagara Falls Power Company extended from 1891 to 1904.

On March 3, 1904, Mr. Brackenridge was appointed by Benjamin B. Odell, Jr., then Governor of New York, to be one of "five expert civil engineers to act as an Advisory Board of Consulting Engineers,"[2] in connection with the expenditure of $101 000 000 authorized by the Barge Canal Law for the improvement of the New York State Canals and the construction of the Barge Canal to connect the Hudson River with the Great Lakes. Nearly every question of moment concerning the new canal was submitted to the Advisory Board of which Mr. Brackenridge continued to be a member until he resigned in the autumn of 1909, when he changed his residence from Buffalo, N. Y., to Pasadena, Calif.

---

[2] Quoted from the Barge Canal Law.

Earlier in 1909 he had been called by the Southern California Edison Company to advise in reference to repairs and reconstruction of its Kern River No. 1 Hydro-Electric Plant, the operation of which had been suspended, due to the failure of a part of its pressure main and tunnel. The work was done after his design, under his direction, and within the estimates of cost and time. In October, 1909, upon the completion of this reconstruction work, Mr. Brackenridge became Vice-President and General Manager of the Company, continuing as such until March, 1918, when he was made President. This position he held for two years, retiring to that of Senior Vice-President in March 1920, and continuing in this position until he resigned in June, 1928.

Under his direction the Company made its initial steam installation of 47 000 kw., at Long Beach, Calif., with about 90 miles of steel tower transmission line, and built its 32 000-kw. hydro-electric plant known as Kern River No. 3. Two Francis turbines to operate under a head of 800 ft. (an extreme at that time), were installed.

In 1917, a consolidation with the Pacific Light and Power Corporation was effected, whereby the Southern California Edison Company became one of the largest central station companies in the United States. Subsequent to this consolidation, Mr. Brackenridge's duties with the enlarged company were mostly executive. Following the passage of the Federal Water Power Act, he prepared and filed with the Federal Power Commission, in behalf of the Southern California Edison Company, a comprehensive plan for the power development of the Colorado River from Lees Ferry, Ariz., to Needles, Calif.

He conducted a private practice as Consulting Engineer during the period of his engagement with the Niagara Falls Power Company and the Canal Board, and from time to time throughout his nearly twenty years of association with the Southern California Edison Company, he was called upon by other companies for consultation and advice.

Mr. Brackenridge designed the initial development for the Utica Gas and Electric Company, at Trenton Falls, N. Y., for the development of 8 000 h.p., under a 266-ft. head, requiring the building of a heavy masonry dam, pipe line, storage reservoir, and power house. He was Consultant for the construction of the Thomson Plant by the Great Northern Power Company, of Duluth, Minn.; for the second development of Snoqualmie Falls, for the Seattle (Wash.), Electric Company; and for the development on the White River in Washington, by the Pacific Coast Power Company, reporting later upon the entire property of the Puget Sound Traction, Light, and Power Company, including its system of electric railways.

He was also Consultant for the financiers in connection with the Buck's Creek development of the Feather River Power Company, in California. He made reports on hydro-electric properties throughout the United States, Canada, and Mexico.

Few engineers have possessed the background of broad experience and many years of practice in their special branch of the profession that belonged to Mr. Brackenridge. During more than forty-five years he had been identified with hydro-electric power development. For several years he worked with other leaders of the electrical power industry in co-operation with legislators in undertaking to prepare Federal water power legislation which would be acceptable, not only to Congress, but to the sources of capital that would invest in hydro-electric developments on Government land. The result was the Federal Water Power Act passed by Congress in 1920.

From 1920 to 1926 Mr. Brackenridge was a member of the Water Power Committee of the National Electric Light Association. In 1923, he proposed to the Arch Dam Committee of Engineering Foundation the construction of a large arch dam for experimental purposes, and, in behalf of the Southern California Edison Company, he offered a large contribution of funds. This was the beginning of the Stevenson Creek Experimental Arch Dam Investigation carried out by Engineering Foundation.[3] Mr. Brackenridge was Trustee of the funds which were used in connection with this experiment, and he maintained a keen interest in the work and a firm belief in its value to the Engineering Profession.

He was ever a lover of animals, deeply interested in their welfare, and was President of the Pasadena Humane Society. He was also most active in humanitarian work, and was a Director of the Boys and Girls Aid Society of Pasadena.

On June 17, 1902, Mr. Brackenridge was married at Buffalo, N. Y., to Margaret Lobdell, the daughter of the Rev. Dr. and Mrs. Francis Lobdell. There are four children—Marian, Margaret (Mrs. William Ellery Hale), Anne, and William Algernon, Jr. He was the brother of Charles D. Brackenridge, and of John C. Brackenridge, M. Am. Soc. C. E.

In April, 1929, he and his family moved for a few months to Santa Barbara, Calif., where Mr. Brackenridge died on November 29, 1929.

Mr. Brackenridge was elected a Member of the American Society of Civil Engineers on June 2, 1886.

GUSTAVE MAURICE BRAUNE, M. Am. Soc. C. E.[1]

DIED NOVEMBER 26, 1930

Gustave Maurice Braune was born at Eutaw, Ala., on March 3, 1872. His father, Gustave Braune, was a native of Freiburg, Saxony, who came to Athens, Ga., in 1850. Later, he moved to Eutaw, Ala., where he was engaged in several enterprises of a banking and financial nature. Mr. Braune's mother, Honoria Rooney, was born in Ireland. An only child, she was orphaned at an early age and came to Pensacola, Fla., with an uncle, John Gilmore, where she resided until she was married. His paternal grandfather was Charles Frederick Braune, a graduate of Heidelberg University, who later prepared students to enter that University; his grandmother was Emily Seibert.

---

[3] Report, Engineering Foundation Committee on Arch Dam Investigation, *Proceedings*, Am. Soc. C. E., May, 1928, Part III.

[1] Memoir prepared by Thorndike Saville, M. Am. Soc. C. E., and Robert Ridgway, Past-President, Am. Soc. C. E.

Gustave Maurice Braune received his preliminary education at Spring Hill College, Mobile, Ala. He then entered Washington and Lee University, Lexington, Va., from which he was graduated with honors in 1889, at the early age of seventeen.

From 1889 to 1890, he lived in Eutaw, Ala. In 1890, desiring to undertake special work in structural engineering, Mr. Braune became a student at the Royal Technical Institute at Dresden, Saxony. Here he studied under several noted German professors, and was a classmate of the late Allen Hazen, M. Am. Soc. C. E., under the famous Dr. Hubert Engels. Mr. Braune received his Diploma Ingenieure from the Dresden institution in 1895, after which he returned to Alabama. From 1896 to 1903 he was associated with the Union Bridge Company and Southern Bridge Company, first as Draftsman and, later, as Assistant Manager and Chief Bridge Designer. During this period, he designed and supervised the construction of numerous highway bridges, buildings, and bridge foundations, chiefly in Alabama and adjacent States.

From 1905 to 1907, Mr. Braune was associated with the late B. M. Hardaway, M. Am. Soc. C. E., as Chief Engineer of the B. M. Hardaway Contracting Company, in general charge of the construction of several large railway and highway bridges, as well as numerous small bridges and several concrete dams and water-power developments. Throughout his life, he was Mr. Hardaway's Principal Consultant on bridge projects, and, in his later years, developed unique methods of designing falsework and bridge erection practices which were utilized by the Hardaway Company.

From 1908 to 1911, Mr. Braune was Bridge Designer and, later, Assistant Chief Bridge Designer with the New York State Barge Canal. During this period he assisted in the design of numerous bridges and special steel structures constructed by the Barge Canal.

Late in 1911, he returned to Alabama, finding the northern winters somewhat severe for his family and himself. He was engaged for a year in Birmingham in general construction work under the name of the Braune Contracting Company, specializing in steel bridges and other structures.

Up to this time Mr. Braune had acquired a great fund of practical experience on a wide variety of problems in his special field of structural engineering. In 1912, he came into contact with Herman Schneider, Assoc. M. Am. Soc. C. E., who was just getting under way his notable development in co-operative engineering education as Dean of the Engineering School at the University of Cincinnati, at Cincinnati, Oho. Dean Schneider, with that rare vision which has been exemplified in his selection of associates, saw clearly that Mr. Braune was one of those unusual individuals who, after an extended period in the practice of engineering, are still able to adapt themselves to the rôle of engineering education.

Assistant Dean George Burns, of the University of Cincinnati Engineering School, states that Mr. Braune,

"* * * * came to this University with a wealth of practical experience, especially along the lines of bridge engineering, and this practical experience enabled him to vitalize and exemplify his instruction in Mechanics and Bridge Engineering. Shortly after coming here he was made Associate Professor, and a little later on he was given a full professorship and made Head of the Department. He was one of the outstanding men of the Engineering Faculty of the University of Cincinnati during the years he was associated here. He probably held as high a place in the affection of his students as any man who has ever been a member of this Faculty.

"Professor Braune took a very prominent part in the Engineers Club of Cincinnati and, at one time, he served as a Director of this organization. He was also very active in the Cincinnati Section of the American Society of Civil Engineers. At the University he organized the Student Chapter of the American Society of Civil Engineers, and the students, in return, as a mark of the high esteem in which they held him, called the organization the Braune Society of Civil Engineers, which still continues under that name.

"Professor Braune was one of Dean Schneider's ablest lieutenants in promoting the co-operative scheme of education. He had a very wide circle of friends among the engineering fraternities of Cincinnati."

He was a member of the City Planning Commission, of Cincinnati, and when he left, he was made one of the two Honorary Members of the Engineers Club of Cincinnati.

As Professor Braune had impressed Dean (now President) Schneider with his personality and experience, so he impressed President H. W. Chase, of the University of North Carolina, when they met while on a vacation in the North Carolina mountains. The Department of Civil Engineering was seeking a head at this time, and several candidates were under consideration. Professor Braune was not a candidate, but Dr. Chase was convinced that he was the man for the place, and in September, 1921, he became Professor of Civil Engineering and Head of the Department at the University of North Carolina.

In 1922 the several Departments of Engineering were co-ordinated to form the School of Engineering, and Professor Braune became the first Dean. He was responsible for introducing a modification of the Cincinnati plan of co-operative education and for greatly expanding all forms of engineering work at the University of North Carolina. When he came to the University, the Engineering Departments had a faculty consisting of five men of professional rank, which had been expanded to twelve when he died. Equal expansion had taken place in curricula, number of students, and equipment. Dean Braune had a most charming personality which endeared him to students and gave him marked opportunities to exercise his abilities in university affairs and in municipal and social organizations.

"As a teacher in the Engineering School, he was possessed of a quiet dignity that was always mellowed by a spirit of cheerfulness and a God-given sense of humor. He possessed that fine quality of being commanding without being dictatorial, and of being familiar without breeding contempt. His classes were always pleasant discourses upon subjects with which he was so familiar and those which he loved so well. He was academic, yet practical, with many a story to impress upon the student mind the application of theory to actual practice.

"As great as was Dean Braune's ability as a teacher, he stands forth more vividly as a true friend and trusted adviser. He considered no problem of the student too trivial to warrant his time and attention, and there was no problem for which he could not effect a solution. To him, there were no differences of class. All shared his confidence from Freshmen to Graduate, and all had for him the greatest love and admiration. He inspired in all, a desire to work for the pleasures of accomplishment, and not a fear of punishment for work poorly executed. His interest was fatherly, and a pleasant smile and 'well done' from him was more to be desired than the highest of grades."[2]

Dean Braune was not only most sympathetic in promoting engineering research, but contributed notably to experiments in earth pressures at both the Engineering Schools with which he was connected. He initiated the largest scale culvert experiment yet undertaken (in collaboration with Charles M. Upham, M. Am. Soc. C. E., Harold Janda, Assoc. M. Am. Soc. C. E., and the late William Cain, M. Am. Soc. C. E.), and the deflection tests made on the reinforced concrete bridge over the Yadkin River prior to its destruction.

He was active in the interests of the Society in North Carolina, as he had been in Cincinnati, and indeed all his life. He founded the Student Chapter at the University of Cincinnati (Braune Engineering Society) and that at the University of North Carolina, the William Cain Engineering Society, named after the late Professor Cain. Dean Braune was widely and favorably known in many parts of the South and his friends in North Carolina, Alabama, and Tennessee presented his name for Director of the Society in 1924. He was defeated for election by a very small number of votes by the late Paul H. Norcross, M. Am. Soc. C. E. Upon Mr. Norcross' death, soon after in the tragic *Norman* disaster, the Georgia Section of the Society most courteously urged the appointment of Dean Braune to succeed to the remainder of Mr. Norcross' term. This appointment was made, and Dean Braune served as Director from 1925 to 1927. It is of interest to note that the only other Director of the Society from North Carolina was Dean Braune's predecessor and collaborator, the late Professor Cain, of the University of North Carolina.

While a member of the Board of Direction, Dean Braune served as Chairman of the Student Chapter Committee in 1926-1927, and as a member of the Committees on Technical Procedure (1926) and on Research (1927). Later, in 1930, he served as a member of the Committee on Graded Schools. He was zealous in the performance of his duties as a member of the Board, and his associates were impressed by his earnest attention to matters under consideration. Jealous of the prestige of the Society, he was a supporter of its high standards, and a firm believer in its aims and activities. He earned the respect and esteem of his associates, and his delightful personal qualities endeared him to them all.

---

[2] Excerpt from address of T. M. Riddick, Representative of the Student Body at the presentation of a portrait of Dean Braune to the Engineering School of the University of North Carolina, on May 19, 1932.

Dean Braune was greatly admired and liked by the Faculty and students of the University, and the towns people of Chapel Hill, N. C. For many years he was a member of the Chapel Hill Board of Aldermen, a Vestryman of the Protestant Episcopal Church, and a member of many committees and boards of the University.

He had not been in good health for several years prior to his death, and when he suffered a violent attack of pneumonia, he was unable to throw it off, being ill only three days. He is survived by his widow, May Ida (Stoelker) Braune, to whom he was married on February 8, 1899, and his daughter, Anna.

He was a member of the Society for the Promotion of Engineering Education (Councillor, 1925-28), Phi Beta Kappa (honorary from Washington and Lee University), Sigma Xi, Tau Beta Pi, and (honorary) Engineers Club of Cincinnati.

Dean Braune was elected a Junior of the American Society of Civil Engineers on June 2, 1896, an Associate Member on September 4, 1901, and a Member on May 15, 1917.

HUGH BRODIE, M. Am. Soc. C. E.[1]

Died March 19, 1932

Hugh Brodie was born on July 12, 1871, near Mount Pleasant, Mich., of Scotch parentage. His father, a native of Scotland, came to Canada in 1854, where he taught school for three years. In 1857, he went to Michigan, and two years later entered a claim for a homestead. In 1866, he married Jennie McLeod, a pioneer school teacher of Central Michigan.

Hugh Brodie received his early mathematical training from his father, a man of keen and active mind. From his mother, who conducted the first art studio in Mount Pleasant, he inherited an appreciation of the beautiful.

Mr. Brodie grew to young manhood on the farm where he was born, enduring the inconveniences and hardships of a pioneer homestead. When he was about eighteen years old he suffered an illness involving a major operation. His health was so impaired following this experience, that he found it necessary to go to a milder climate. In 1889, accompanied by his mother and a sister, he went to Florida, where he spent several years. There he gradually regained his health, but was left with defective hearing and a permanently weakened heart. Against these physical handicaps he had to work throughout the remainder of his life.

While convalescing in Florida he spent as much time as possible in such schools as were available, and in other studies. He spent one year alone with his books in the solitude of the Florida pine forest, reviewing all his previous school work. He then entered the Florida State Teachers College, completing the course in 1897. The next few years were spent in teaching in the Florida public schools.

---

[1] Memoir prepared by C. O. Carey, M. Am. Soc. C. E.

From January, 1900, to January, 1902, Mr. Brodie was in the employ of the Seaboard Air Line Railroad Company as Tapeman, Rodman, Instrumentman, Inspector, and General Foreman of Construction. From January, 1902, to July, 1903, he was Assistant Superintendent of the Talantic Lumber Company. Through this experience he became interested in engineering and decided to obtain an engineering education. Although he was much older than the average student entering college, he enrolled at the University of Michigan, Ann Arbor, Mich., as a freshman in the fall of 1903, and was graduated in 1907 with the degree of Bachelor of Science in Civil Engineering. In 1914, he received the degree of Civil Engineer from his Alma Mater.

In the fall of 1908 Mr. Brodie became Instructor in the Department of Geodesy and Surveying in the University of Michigan. In 1914, he was made Assistant Professor, and, in 1925, Associate Professor, which rank he held at the time of his death.

A paper[3] entitled "Report on a Series of Tests on Concrete Columns Reinforced with a Spiral of Steel," by Professor Brodie, as co-author with C. G. Wrentmore, M. Am. Soc. C. E., and the writer, was presented to the Society at its meeting on March 18, 1914.

From August, 1918, to February, 1920, Professor Brodie was on leave of absence from the University, and, with a group of American engineers, with Joseph Ripley, M. Am. Soc. C. E., as Chief Engineer, was engaged in surveys and hydrographic investigations along the Grand Canal and Yellow River, in China, for the Grand Canal Improvement Board, at Tientsin. In this work Professor Brodie had charge of parties on topographic surveys along 120 miles of the Grand Canal, and the survey of dikes and flood conditions on about 250 miles of the Yellow River.

Professor Brodie was a lover of Nature, and enjoyed life in the open. During the years spent in Florida he traveled through all parts of the State. On these trips he saw many of the old Indian mounds, and became interested in prehistoric life. This interest he maintained throughout his life. He was a keen observer of wild animal life, and enjoyed the study of plants and trees.

He was a tireless worker, unsparing in his efforts to accomplish the task in hand. In his work as a teacher, he gave his time without reserve to his students, spending many hours outside the classroom discussing their problems with them.

He had a kindly, sympathetic nature, was always deeply concerned about those who were in trouble or distress, and willing to lend a helping hand to those in need. He maintained an uncompromising attitude when he thought an important principle was involved. He sometimes led those who did not know him well to feel that he was harsh and unyielding. His honesty of purpose and his keen desire to be fair were appreciated by those who knew him intimately.

Professor Brodie was not married, but his home in Ann Arbor during the latter years of his life provided living accommodations each year for one or more students who needed financial aid to continue their university work.

---

[3] *Transactions*, Am. Soc. C. E., Vol. LXXVII (1915), p. 97.

He was a member of the Michigan Engineering Society, the University Club, and the Ann Arbor Golf and Outing Club.

Professor Brodie was elected a Member of the American Society of Civil Engineers on October 14, 1929.

PAUL KING BUNN, M. Am. Soc. C. E.[1]

DIED JANUARY 23, 1932

Paul King Bunn was born at Buffalo Grove, Iowa, on August 6, 1891. His parents, Burris Harbis Bunn and Florence Adelia Bunn, were sturdy Americans, and Mr. Bunn received a sound heritage, both mentally and physically. His parents were residents of Rush Center, Kans., but, later, moved to La Crosse, Kans., and Mr. Bunn was educated in the grade schools of both places, but was graduated from the La Crosse High School. He then entered the University of Kansas, at Lawrence, Kans., from which he was graduated in 1914, with the degree of Bachelor of Science in Civil Engineering.

Mr. Bunn's first engineering work was with the Interstate Commerce Commission, Bureau of Valuation, Western District, where he served as Junior Civil Engineer from July, 1914, to October, 1917. He resigned this position to enlist in the Army as a Private. He was assigned to Company A, 29th Engineers, and was sent to France with his regiment after a short period of training. Mr. Bunn served nineteen months in France, most of which time was spent on the advanced line. During this period he was successively advanced in rank and was honorably discharged as a Second Lieutenant of Engineers in January, 1919.

After his discharge from the Service, Mr. Bunn again entered the employ of the Interstate Commerce Commission and served as Senior Civil Engineer until November, 1919. He then was engaged, successively, as follows: November, 1919, to August, 1920, Assistant Engineer, Leavenworth County, Kansas; August to November, 1920, Resident Engineer, Wyandotte County, Kansas; November, 1920, to January, 1922, Resident Engineer, Leavenworth County, Kansas, on concrete roads and bridges; and January to March, 1922, Resident Engineer, for V. V. Long and Company, on paving and sewer construction.

In March, 1922, Mr. Bunn entered the employ of The Missouri Valley Bridge and Iron Company, Contractors, of Leavenworth, Kans., with which Company he was engaged continuously until his death on January 23, 1932. During this time he was promoted, successively, from Timekeeper to Foreman to Field Engineer, and to Superintendent of Construction, which position he held at the time of his death.

It was in this position that he did his best work. He was well fitted for the job, being possessed of a fine constitution, and a calm, philosophical, well balanced mind. A hard worker himself, he won the good will and hearty co-operation of all his subordinates as well as the high esteem of his employers.

---

[1] Memoir prepared by E. H. Connor, and L. M. Bush, Members, Am. Soc. C. E.

As Superintendent for the Contractors, his work was quite varied and extensive, and Mr. Bunn became known as a builder of foundations and for caisson construction. He constructed the substructures of highway and railroad bridges across many of the large and difficult rivers of the Southwest, including the Cimarron at Guthrie, Okla.; the Brazos, in Texas; the Gila, at Yuma, Ariz.; the Canadian, at Canadian, Tex.; the Mississippi, at Louisiana, Mo., and at Cairo, Ill.; and the Missouri, at St. Charles, Mo.

While engaged on the Louisville and Nashville Railroad Bridge across the Ohio River, at Henderson, Ky., Mr. Bunn was stricken with appendicitis and died on January 23, 1932, after an illness of only one week. He passed away in the prime of life.

His was a valuable experience, with excellent prospects of greater usefulness in his chosen line of work. His many lovable, generous qualities endeared him to all his associates, and his upright, fair, and square treatment of his men and of all who came in contact with him, made them feel a deep loyalty and respect for him. They have lost a good friend. His love and thoughfulness in all his travels and labors for the comfort of his invalid mother, Mrs. Florence A. Bunn, who still lives at La Crosse, was characteristic of the man.

He was married on January 12, 1929, to Elizabeth Gray Dickison, at Louisiana, Mo., who survives him. Their only child, Paula Elizabeth, was born on April 5, 1932, nearly ten weeks after Mr. Bunn's death. He is also survived by a sister, Mrs. Z. L. Hall, of La Crosse, Kans.

Mr. Bunn was elected an Associate Member of the American Society of Civil Engineers on March 16, 1925, and a Member on May 25, 1931.

HECTOR ROBINS BURROUGHS, M. Am. Soc. C. E.[1]

DIED DECEMBER 21, 1929

Hector Robins Burroughs was born at Penns Manor, Pa., on August 1, 1878, the son of Henry Boswell, and Rebecca Wooley (Robins) Burroughs. In 1902, he received the degree of Civil Engineer from the National Institute of Technology and, later (1910), successfully completed a post-graduate course in Advanced Structures at the Brooklyn (N. Y.) Polytechnic Institute.

In 1901 and 1902, Mr. Burroughs was employed as Draftsman on structural details with the Post, McCord, and Jackson Iron Works of New York, N. Y. The following year he went to San Francisco, Calif., where he was engaged as Engineer for the Pacific Rolling Mills, in charge of drafting, estimating, and shop work for machines, bridges, and buildings. He also designed and supervised complete railroad turn-outs, cross-overs, crossings, yards, and special track work.

During a number of subsequent years he was engaged in various capacities among which are included the following: 1905 to 1906, Assistant Engineer for the Pacific Construction Company of San Francisco, designing and estimating steel and concrete-steel bridges and buildings (it was at this time that the great earthquake occurred in California and the reconstruction work arising therefrom gained for Mr. Burroughs a very valuable experience); 1906 to 1907, Checker and Draftsman with Frank B. Gilbreth, of New York, on drawings for the McGraw Building of that city; 1907 to 1908, Structural Engineer for the Southern Pacific Railroad Company; 1908 to 1909, Engineer and Superintendent of Buildings in charge of design and construction, for the Pacific Fruit Express Company, with headquarters at Las Vegas, Nev.; 1909 to 1910, Assistant Engineer in the San Francisco Office of Messrs. Sanderson and Porter, of New York, in charge of construction of a sub-station building, Bay Shore, Calif.; 1910 to 1911, Designer and Estimator, Office of Engineer of Structures, New York Central and Hudson River Railroad Company, engaged on bridges, viaducts, towers, retaining walls, etc.; and July, 1911, to January, 1912, Assistant Engineer, the Concrete Steel Engineering Company of New York, designing, estimating, and reviewing concrete-steel bridges.

---

[1] Memoir prepared from data on file at the Headquarters of the Society and from information supplied by the H. Robins Burroughs Co., Inc., New York, N. Y.

In January, 1912, Mr. Burroughs was engaged as Assistant to S. C. Weiskopf, M. Am. Soc. C. E., soon thereafter becoming a member of the firm known as Weiskopf and Burroughs, Consulting Engineers, of New York City. Many important buildings in and about New York were designed and built by this firm in succeeding years. Later, beginning in 1920, Mr. Burroughs conducted an independent practice of Consulting Engineering. It was during this period that he designed the steel structures and acted as Consulting Engineer on about seventy-five buildings in the vicinity of New York (among which was the Roxy Theater), and in other parts of the United States as well.

Also, during this period, he became interested in tennis court construction, this interest culminating in his development of the Bursco Fast Drying Tennis Court. At the time of his death he had perfected this surfacing and had a patent pending on it.

Mr. Burroughs was deeply interested in the Army and for a number of years belonged to the Engineers' Reserves. At the time of his death, he was a Major in the 368th Engineers. During the World War, he served with the Second Corps Engineers in Washington, D. C., and was employed on the construction of a torpedo plant.

He was a member of the American Institute of Consulting Engineers, the American Society of Military Engineers, the American Concrete Institute, and the American Society for Testing Materials. He was also a member of the Masonic Fraternity, and of the First Presbyterian Church, of Trenton, N. J. His recreations were tennis, squash, golf, and motoring, and he held membership in the Army and Navy, and the Crescent Athletic Clubs.

His high professional character and integrity made him a very valuable member of the Engineering Profession. He was killed in an automobile accident on December 21, 1929.

Mr. Burroughs was married in September, 1917, at Boston, Mass., to Mary Jane Owen, who survives him.

His death was a distinct shock to the large number of friends whose respect and genuine liking he had commanded because of his affability and genuine fineness of spirit. A credit to his profession, his outside interests were varied as witnessed by his Army and Masonic connections and his sporting activities. With regard to the latter, he was well known in both tennis and squash circles and, at the time of his death, was just beginning to get the fullest enjoyment out of a country place in Rhode Island, which gave him and his friends ample opportunities to satisfy all their sporting instincts.

Mr. Burroughs was elected a Junior of the American Society of Civil Engineers on March 3, 1908, an Associate Member on September 6, 1910, and a Member on July 7, 1915.

GEORGE RAYMOND CAMPBELL, M. Am. Soc. C. E.[1]

DIED APRIL 1, 1932

George Raymond Campbell was born on June 22, 1874, at The Dalles, Ore. He was the son of William Ellis Campbell and Sarah (Angell) Campbell, early pioneers of Oregon, his mother having come with her parents by covered wagon from Iowa to Oregon in 1851. On his father's side, he was a descendant of a Seventeenth Century settler from Scotland, and on his mother's, of the Thomas Angell who settled in Rhode Island, and who was one of the fifty-four associates who received land grants of property from Roger Williams, which later became the site of Providence.

After completing High School at The Dalles, Mr. Campbell worked his way through the University of Oregon, from which he was graduated in 1901, with the degrees of Bachelor of Science and Civil Engineer. He was the first man to receive the degree of Civil Engineer from that institution. While attending the University he was active in football and in debating societies.

For several years after his graduation, Mr. Campbell was in private practice at The Dalles, and made surveys and designs for irrigation, power, and storage projects along the Columbia River. In 1902, he was Engineer in Charge of three field parties on the survey of the Quinaielt Indian Reservation for the United States Department of the Interior. As such, he prepared the records and topographical maps and made the preliminary designs and report for drainage.

From 1904 to 1910, he was Senior Member of the firm of Campbell and Campbell, Civil Engineers, with offices in Spokane, Wash., and in this capacity served as Consulting, Designing, and Contracting Engineer on hydraulics, reinforced concrete, irrigation, power, and drainage problems. In 1906, he was engaged as Engineer in Charge of three field parties in surveying the Yakima Indian Reservation in the State of Washington for the Department of the Interior. This great area was surveyed in accordance with the old treaty stipulations on the long disputed boundary, and resulted in a $2 500 000 suit being decided in favor of the Government. Mr. Campbell prepared the reports and topographical maps, and make the preliminary report on the irrigation of that fertile but arid district.

---

[1] Memoir prepared by Carroll R. Campbell, U. S. Insp., Glasgow, Mo.

While in the Bridge Department of the City Engineer's Office, of Spokane, Mr. Campbell made the preliminary design for the Latah Creek (reinforced concrete) Bridge, checked the arches, and determined the unit stresses of the final design. He also made structural designs of truss and girder bridges, and was engaged on other municipal problems, including the Sprague Avenue Fill.

In 1912, deciding to enter the Government Service, Mr. Campbell passed the first two available Federal examinations, that for full grade United States Civil Engineer, and for United States Surveyor. For several years, he was U. S. Surveyor for the Department of the Interior on various surveys, being transferred in 1915 to the Federal Office at Olympia, Wash. Among his assignments were the hydrographic survey of the Nooksack River and Bellingham Bay, and the restoration surveys of Lumni and Swinomish Indian Reservations. In June, 1917, he was commissioned as Captain, in the Engineer Section, of the Officers Reserve Corps, and received training at the Presidio, in San Francisco, Calif. He was honorably discharged before being assigned to duty.

As United States Cadastral Engineer, Department of the Interior, in 1918 and 1919, Mr. Campbell made mine examinations and surveys, and hydrographic determinations of river channels and islands in Pend Oreille, Skagit, Snake, and Columbia Rivers; and located highways and Government lighthouses on the islands of San Juan Channel. He resigned in 1920, to accept a position with the Washington State Reclamation Service. He was in field charge of soil surveys and diking, drainage, and river and ocean control investigations for the State.

Since 1922, while engaged in private practice as a Consulting Engineer on irrigation problems, Mr. Campbell devoted much of his time to the running of his apple orchard in Yakima, Wash. One of his last engineering jobs was the supervision of steel erection on a four-story reinforced concrete cold-storage plant for the Yakima County Horticultural Union, of which he was a member.

In apparently sound health, his sudden passing in his sleep on April 1, 1932, was a great shock to his family and friends.

Few men knew the Pacific Northwest as did Mr. Campbell. His work as an engineer had taken him into the most inaccessible mountain areas. As a young man, earning his way through college by working in the summer months as a Surveyor in the mountains of Eastern Oregon, and, later, in private practice and as a Government Engineer, he saw and contributed to the development of the Pacific Northwest as his ancestors in Colonial times did to that of the Atlantic seaboard. It was natural that his principal hobby was the hunting of big game. His specimens of elk, deer, and bear, are outstanding trophies.

His interest in all phases of engineering was unlimited, as his excellent library of technical books and his bound volumes of engineering magazines and transactions of technical societies testify. In his spare time he could always be found inquiring into some engineering problem, often for its abstract interest alone. His associates in the profession and companions on the hunt respected him for his purposeful will and buoyant good spirits. Mr. Campbell was noted for his exemplary character and for his ability to get along with all kinds of men. He was a member of the Masonic Fraternity for thirty years, the Scottish Rite, and the Shrine.

On November 15, 1905, he was married to Lena Liebe, of The Dalles, Ore., who, with three children, Carroll R., Waldemar B., and Vera Lenoire, survives him. He is also survived by his 81-year old mother, Mrs. Sarah (Angell) Campbell, and by three brothers, Roy T., a U. S. Cadastral Engineer; Rex R., and Fred E. Campbell.

Mr. Campbell was elected an Associate Member of the American Society of Civil Engineers on August 31, 1915, and a Member, on April 21, 1920.

EDWARD CARLOS CARTER, M. Am. Soc. C. E.[1]

DIED DECEMBER 23, 1930

Edward Carlos Carter was born at Waverly, Ill., on January 11, 1854, the son of George and Louisa J. (Smith) Carter.

At the age of fifteen, he joined a surveying party of the Jacksonville, South Eastern, and North Western Railroad Company (later, a part of the Chicago, Burlington, and Quincy Railroad Company), on a line extending from Jacksonville to Virden, Ill. At intervals during the next three years, he worked as Levelman and Assistant Engineer on various railroads. In 1872, he entered Rensselaer Polytechnic Institute, at Troy, N. Y., from which he was graduated in June, 1876, with a degree of Civil Engineer.

Subsequent to his graduation, Mr. Carter's professional career was as follows: 1877 and 1878, Principal Assistant Engineer with the United States Engineers on the Mississippi River; 1878 and 1879, Assistant on Location, Chicago and Alton Railroad, Kansas City Division, and Assistant in Charge of Construction on the Blue Springs Division; 1879 and 1880, Assistant Engineer of Construction on the Indianapolis, Decatur, and Springfield Railroad; 1880 to 1882, Assistant Engineer of Maintenance, 1882 to 1884, Resident Engineer on the Middle Division, and 1884 and 1885, Principal Assistant Engineer on the Wabash, St. Louis, and Pacific Railroad; and 1885 to 1887, Assistant Engineer of the Detroit Bridge and Iron Works, at Detroit, Mich.

In 1887, Mr. Carter became Principal Assistant Engineer of the Chicago and North Western Railway Company, at which time, the late Mr. Horace G.

---

[1] Memoir prepared by a Joint Committee of the Society and the Western Society of Engineers, consisting of Charles F. Loweth, Past-President, Am. Soc. C. E., Bion J. Arnold, M. Am. Soc. C. E., George W. Hand, Assoc. M. Am. Soc. C. E., and Chester T. Dike, M. W. S. E.

Burt was Chief Engineer. He held this position during the remainder of the time that Mr. Burt was Chief Engineer and also during the tenure of Mr. Burt's successor, the late Mr. John E. Blunt. As Principal Assistant Engineer, Mr. Carter was in general charge of maintenance and construction. In 1899, he became Chief Engineer and retained this position until June, 1914, when, at the age of sixty, he decided to relinquish the active duties of his office and devote the remainder of his life to travel, study, and the practice of his profession as a Consulting Engineer.

In May, 1914, the City Council of Chicago, by ordinance, created the Chicago Railway Terminal Commission, consisting of municipal officials and others. In July, this Commission started abroad to study the railway terminals of other cities, and a few men, each expert in his field, were invited by the Commission to accompany it. Mr. Carter, among others, was selected to represent the steam railways.

Before sailing, however, the party visited Toronto, Ont., and Montreal, Que., Canada, Boston, Mass., and New York, N. Y. The Commission and its guests then went abroad and visited Liverpool, Manchester, and London, England, and Paris, France, where they were officially entertained. The itinerary called for an extensive trip through European countries, and the Commission was to leave Paris for Frankfort, Germany, on July 29, but the declaration of war by Austria against Servia on July 28, made a change in its plans necessary, and the party returned, *via* Brussels and Antwerp, Belgium, to London, and thence to the United States.

During Mr. Carter's service of more than twenty-seven years with the Chicago and North Western Railway Company, he manifested a broad and continuing interest in all phases of railroad construction, maintenance, and operation. He was a member of one of the earliest committees of the American Railway Association having to do with train rules. Early in his career he became interested in train signaling and was one of the first railroad engineers to give this important matter the study and attention it deserved. It was on his recommendation that the Chicago and North Western Railway Company, in preparation for handling the heavy traffic during the World's Columbian Exposition in 1893, installed extensive automatic block signals and interlocking plants in and near Chicago, including an extensive plant at the Chicago Terminal, which embodied the principles he had evolved, thereby contributing a marked advance in the art of signaling. Mr. Carter was a pioneer in promoting the use of semaphore signals operating in a quadrant above the horizontal, on a principle which had not been previously utilized, but which is now almost universal. He also brought about improvements in signal lighting, both in respect to the use of colors and their combinations and also in the details of lamps and lenses. In 1901, he invented a continuous-rail derail which obviated the troubles experienced with the common switch-point derail then in use. The principle he developed has been largely utilized in derails since that time, and a large number of derails of the type he invented are still in use.

While Chief Engineer of the Chicago and North Western Railway, Mr. Carter had responsible charge and direction of a large amount of new construction and improvements carried on by that Company, which included the location and construction of about 1 400 miles of new line; the major portion of the second track across the State of Iowa; the Boone Viaduct over the Des Moines River Valley, constructed in 1901—a bridge more than ½ mile long and 185 ft. high, which at the time it was built was a most notable structure; a bridge over the Missouri River at Pierre, S. Dak., completed in 1907; a double-track bridge over the Mississippi River at Clinton, Iowa, completed in 1909; and many other large and noteworthy structures. It was under his direction and supervision that the Company planned and constructed its modern passenger terminal in Chicago, which was completed and put in operation in 1911 at a cost of $25 000 000. During the years that he was Chief Engineer of the North Western Railway, that Company also carried out track-elevation projects in the City of Chicago alone, which cost more than $12 000 000.

Mr. Carter always manifested a broad and active interest in all engineering matters. He was one of the oldest members of the Western Society of Engineers, of which he was Vice-President in 1892 and President in 1905. In 1929, in recognition of his long membership and notable accomplishments in engineering, he was made an Honorary Member of that Society. He was also a member of the Special Committee on Standard Rail Sections of the American Society of Civil Engineers, a member of the Railway Guild of New York City, the Union League Club, of Chicago, the Glen View Golf Club, and the Camp Fire Club.

He was married, in 1880, to Fannie G. Fairbank, of Jacksonville, Ill. Mrs. Carter, with their three children, Mrs. C. M. Bell, Edward F. Carter, and Paul E. Carter, survives him.

Mr. Carter was elected a Member of the American Society of Civil Engineers on April 4, 1888. He served as a Director of the Society from 1901 to 1903.

WILLIS CHIPMAN, M. Am. Soc. C. E.[1]

DIED JANUARY 3, 1929

Willis Chipman was born in Elgin, Ont., Canada, on October 1, 1855, the son of Lewis Chipman of English lineage dating from the time of William the Conqueror. His mother was Abigail Jane (Delong) Chipman, the daughter of Jesse Delong of Elgin. It is recorded that the first member of the family to seek a home in America was John Chipman, who sailed from Barnstable, Devon, England, when he was 17 years of age and landed at Boston, Mass., on July 14, 1631.

Mr. Chipman attended the public school, at Athens, Ont., and the Weston, Ont., High School, later entering McGill University, Montreal, Que., Canada, from which he was graduated in 1876, with first rank honors in Civil and Mechanical Engineering. Following his graduation he became a teacher in the High School, at Napanee, Ont., Canada, serving as Mathematical Master on the Staff. He next entered the service of the Geological Survey of Canada, and afterward served as Assistant Engineer on the construction of the Toronto, Ont., Water-Works.

---

[1] Memoir prepared from information on file at the Headquarters of the Society.

Mr. Chipman then entered the profession of Land Surveying and became articled to Mr. G. B. Abrey, of Manitoulin, Ont., Canada, for one year, during which he was engaged on surveys in the then Canadian Northwest Territories. He obtained his commission as Dominion Land Surveyor on May 21, 1881, and was granted his commission as Provincial Land Surveyor for Ontario on October 4, 1881.

In November, 1881, Mr. Chipman began his practice, in Brockville, Ont., Canada, as a Land Surveyor and Civil Engineer, serving also as City Engineer for that city. For several years he carried on the dual practice, as well as, from January, 1889, to January, 1891, a partnership with the late Lieut.-Col. B. J. Saunders.

Convinced of the importance of old records in surveying work, Mr. Chipman acquired the field notes of many by-gone surveyors, and these are now in the hands of his surveying successors in Brockville. During his practice in Land Surveying, he served with the Department of the Interior from May to October, 1883, in surveys in the Northwest Territories. He also made the following sub-division surveys for the Department of Crown Lands (Ontario): 1884, Township of Mulock; 1886, Township of French; 1886, Township of Stewart; and, in 1890, the Township of Lockhart, all in the District of Nippissing. A plan of the Town of Brockville was prepared by him in 1883 and 1884.

Realizing the disadvantages both in regard to the members of the Profession as well as to the public—their clients—arising from the lack of cohesion among Land Surveyors, Mr. Chipman undertook the task of organizing the surveyors for Ontario. After continued correspondence, a meeting of representative surveyors was held in Toronto, Ont., Canada, on February 25, 1886, and the Association of Provincial Land Surveyors was formed, of which Mr. Chipman was Secretary-Treasurer; he continued in this office until 1890; in 1892, the Association became incorporated. Mr. Chipman held the office of Vice-President in 1895 and that of President in 1896, and, until his death, he continued his activities for the Association. To meet the requirements of his increasing practice, Mr. Chipman closed his Brockville Office in 1894, and removed to Toronto.

In his field of engineering, he won a Dominion-wide reputation, particularly in the branches relating to water-works and sewerage, his practice extending from Halifax, N. S., to Victoria, B. C., Canada. In 1893, he served as President of the Association of Health Officers of Ontario, and he also served as a member of the Commission appointed in 1911 to inquire into the situation of the collapse of the Toronto Water-Works. Among his published articles on

Sanitary Engineering are: "Ontario Waterworks System"[2]; "The Separate System of Sewerage in Ontario"[3]; "London Sewerage System"; and "Oakville Waterworks Auxiliary". He is regarded as the originator of the separate system for sanitary and storm-water sewers in Canada.

On April 1, 1910, Mr. George H. Power, who had acted as his Assistant from September, 1901, became Mr. Chipman's partner, and the firm continued in practice in all the Provinces of Canada as Chipman and Power until October 1, 1924.

In his quest for further information in his calling, Mr. Chipman visited the New England States, in 1885, to inspect and study water-works and sewerage problems; and in February and March, 1900, he investigated water-works, sewerage, and sewage disposal systems in England and Scotland. In 1891, he made a study of the sanitary systems of Denver, Colo., Seattle, Wash., and San Francisco, Calif.

Among the appointments and offices held by Mr. Chipman in connection with his Profession may be mentioned his service for three terms on the Council of the Canadian Society of Civil Engineers. During the transition of that body to the Engineering Institute of Canada, he was one of the prominent members of the Committee of Fourteen Engineers that was charged with the drafting of the Constitution and By-Laws. Later, he was one of the most active members of the Committee to which the drawing up of the bill for the organization of the Professional Engineers of Ontario was entrusted. The Engineers' Club of Toronto is another professional body which owes its inception and growth largely to the initiative and energy of Mr. Chipman. He was also a member of the American Waterworks Association throughout his active career.

Aside from his activities in professional matters, Mr. Chipman devoted much time to research in the history of the Province of Ontario, particularly that of Leeds County, and, as a member of the Ontario Historical Society, he made valuable contributions to its *Proceedings*. He also made a study of ornithology and was well versed in the subject. His Alma Mater meant much to him, and he was a member of the Graduates' Society of McGill University and a frequent contributor to its quarterly publication, *The McGill News*. In religion, Mr. Chipman was of the Anglican faith and, in politics, a Liberal.

Mr. Chipman was married on May 5, 1880, to Angeline Denison, who died in 1883 leaving a son, Harry D., now of Rhodesia, South Africa, and a daughter, Julia Angeline, of Toronto. He later was married to Elizabeth Bray Finlayson, the surviving children being Gertrude Elma, the wife of Charles W. Pennington of Dundas, Ont., Canada, and Lucy Kathleen, the wife of Arthus S. Runciman, of Montreal West. He was again married to Edith Venton Scott, who survives him.

Despite increasing weakness through illness Mr. Chipman continued to carry on as Chairman of the Biographical Committee of the Ontario Land

---

[2] *Canadian Engineer*, 1898.

[3] *Transactions*, Canadian Soc. of Civ. Engrs., January–December, 1897–98, Vol. 11–12, p. 163.

Surveyors' Association almost to the time of his death. He was interred in the old family plot at Brockville. Fittingly, his ring denoting "The Calling of an Engineer," was buried with him.

Mr. Chipman was elected a Member of the American Society of Civil Engineers on June 6, 1888.

JOHN CHRISTIAN VALDEMAR CHRISTENSEN, M. Am. Soc. C. E.[1]

Died June 16, 1931

John Christian Valdemar Christensen was born in Copenhagen, Denmark, on March 2, 1881, the son of Lars and Karen (Jorgensen) Christensen.

Mr. Christensen was graduated from the Polytechnic College of the Royal University at Copenhagen, with the degree of Civil Engineer, in January, 1903, and began his engineering career in May of that year as Draftsman with the Pittsburg, Shawmut, and Northern Railway Company, at Olean, N. Y. He remained in this position until August, 1903, when he went to Chicago, Ill., as Draftsman for the Chicago, Rock Island and Pacific Railroad Company. In June, 1904, he entered the employ of the Chicago and Eastern Illinois Railroad Company as Draftsman and, in December of the same year, he went in the same capacity to the Chicago Southern Railway Company, at Chicago.

In March, 1905, Mr. Christensen returned to the Chicago and Eastern Illinois Railroad Company, in Chicago, and remained there until September, when he joined the Engineering Organization of the Detroit River Tunnel Company, at Detroit, Mich., as Designer of tunnel sections, retaining walls, and a highway bridge, and as Inspector on the driving of the tunnel headings. On the completion of this work in March, 1907, Mr. Christensen went to New York, N. Y., as Draftsman for the New York Central and Hudson River Railroad Company, where he remained until March, 1908. From March, 1908, until May, 1909, he was engaged in private business in Copenhagen, Denmark.

He returned to New York in May, 1909, and became Designer and Squad Chief on the design of steel, concrete, and timber railroad and highway bridges, in connection with the Electric Zone Improvements on the New York Central and Hudson River Railroad. Mr. Christensen remained with the New York Central until April, 1913, when he was made Designing Engineer for the Gulf, Florida, and Alabama Railway Company, at Pensacola, Fla., where he was in charge of the structural design for bulkheads, piers, pier-sheds, substructures, and superstructures for bridges and coffer-dams.

In August, 1916, Mr. Christensen returned to the New York Central and Hudson River Railroad Company as Designer and remained in that capacity until November, 1917, when he went to the Calco Chemical Company, at Bound Brook, N. J., as Designing Engineer in charge of plant alterations and extensions. He remained with the Calco Chemical Company until October, 1918, when he returned to New York City, as Engineer for The Terry and Tench Company, for which he had charge of the engineering work on the Stapleton Piers, Staten Island, for the City of New York, underpinning the Lincoln Memorial, at Washington, D. C., and some of the work in connection with the Bear Mountain Bridge across the Hudson River.

---

[1] Memoir prepared by John M. Belknap, M. Am. Soc. C. E.

In March, 1926, Mr. Christensen was appointed Structural Engineer for the Foundation Company, of New York City and from that time until December, 1927, he was in charge of the structural design for the shops of the Wabash Railroad Company, at Decatur, Ill., a new plant for the Goodrich Rubber Company, at Los Angeles, Calif., and harbor improvements for ports in Venezuela and Peru.

In December, 1927, he went to Cincinnati, Ohio, as Engineer of Structures for the Cincinnati Union Terminal Company and was in charge of all structural designs for the new Union Passenger Terminal until July, 1929, when he was made Engineer of Design. In this capacity Mr. Christensen had charge of the design of all bridge structures, track work, and buildings connected with the new Union Terminal, with the exception of the Station Building.

The design of the Western Hills Viaduct, which was built in connection with the Terminal improvements, and the solution of the intricate engineering problems pertaining to this structure, was one of his most important achievements as a member of the Engineering Staff of the Cincinnati Union Terminal.

Mr. Christensen was a Structural Engineer of rare ability, which, with his innate frankness and pleasing personality, made for him a host of friends who unite in mourning his death.

On May 1, 1909, he was married to Dika Adelaide Martensen who, with a son, Paul Christensen, and a daughter, Lily Christensen, survives him.

Mr. Christensen was elected a Member of the American Society of Civil Engineers on October 1, 1928.

ST. JOHN CLARKE, M. Am. Soc. C. E.[1]

DIED MAY 22, 1931

St. John Clarke was born in Richmond, Va., on April 21, 1863, the son of Colonel John Graham Clarke and Lucy Selden (Macon) Clarke. His father, who was of English Quaker origin, was Chief of Engineers of Longstreet's Corps and, later, Chief of Engineers for General Robert E. Lee. Following the termination of the Civil War he was in charge of the reconstruction of railroads destroyed during Sherman's March to the Sea. On his mother's side, Mr. Clarke was of French Huguenot descent.

St. John Clarke began his professional career as a Rodman with the Richmond and Alleghany Railroad Company in June, 1881, being engaged on location until March, 1883, when he entered the employ of the Passaic Rolling Mills as Draftsman. He continued with that Company until the summer of 1888, during which period he gained a wide experience both in the designing room, shop, and erection fields of the then rapidly developing steel industry. As Resident Engineer for the Company, he had responsible charge of the steel work for the Washington Bridge over the Harlem River, New York, N. Y.

---

[1] Memoir prepared by George Hallett Clark, M. Am. Soc. C. E.

From June, 1888, to November, 1889, as Assistant Engineer for Hoffman and Bates of Portland, Ore., Mr. Clarke was engaged on the design and erection of combination bridges in the Northwest and also as a Transitman on a preliminary railroad survey from Portland to Tacoma, Wash. Returning east in November, 1889, he was employed as Draftsman in the office of the Engineer of Bridges of the Baltimore and Ohio Railroad Company until August, 1890, when he returned to the Passaic Rolling Mills as Assistant Engineer. As such he had charge in the office of most of the larger work and, at the same time, of the field work in erection of bridges, the most notable being the Macomb's Dam Bridge over the Harlem River, New York City.

From December, 1895, to June, 1896, he was with the Examining Department of the Department of Buildings, New York City, and from June, 1896, to May, 1900, in charge of the Estimating and Designing Department of Milliken Brothers, New York, preparing plans and estimates for high buildings and various other steel work.

While Mr. Clarke was with Milliken Brothers, his skill and thoroughness as a designing engineer brought him to the favorable notice of the late Theodore Cooper, M. Am. Soc. C. E., on whose recommendation he was selected for the responsible position of General Inspector of Designs in May, 1900, at the time of the organization of the Engineering Corps of the Board of Rapid Transit Railroad Commissioners, New York City, of which the late William Barclay Parsons, Hon. M. Am. Soc. C. E., was Chief Engineer.

The first subway constructed in the City of New York between May, 1900, and October, 1904, known as Contract No. 1 and, later, extended by Contract No. 2 into Brooklyn, N. Y., was built under a contract the drawings for which were typical and general in character. While the contract drawings were followed generally, the actual construction work required the preparation of a large number of "construction drawings" requiring the employment of a large force of designers and draftsmen, of which force St. John Clarke had personal direction and control. The expedition and economy with which the first subway was constructed in New York City, and its efficient operation and the low cost for maintenance of the structure, is a lasting monument to those responsible for its efficient planning.

Following the completion of construction and the beginning of operation of Contract No. 1 which extended through Manhattan from the City Hall to Van Cortlandt Park and to Bronx Park in The Bronx, Mr. Clarke resigned from the Board of Rapid Transit Railroad Commissioners, to become Chief Engineer of the Steinway Tunnel under the East River at 42d Street, which work he directed until its successful completion.

Following the completion of the Steinway Tunnel, Mr. Clarke engaged in general practice as a Consulting Engineer, his principal work being in the line of steel construction for buildings, tunnel methods, involving the use of compressed air, and details of design and analysis of stresses; for a time he was engaged in general contracting.

In addition to his professional activities, St. John Clarke was notable as a citizen in the community in which he spent the best years of his life. He became a resident of Bogota, N. J., in 1898, and at once took a special interest in the town in which he resided up to the time of his death. He became a member of the Board of Education in 1901 and was re-elected to the Board, serving until 1911. During his ten years of service, the latter eight were spent as President of the Board. Mr. Clarke was one of the founders and, for many years, an officer of the Bogart Memorial Church of Bogota, and as a Co-Trustee of the Bogart Estate had direct charge of the construction of the present edifice.

For several months prior to his death he took an active interest in grade elimination questions affecting the Town of Bogota, but did not live to see the results of his public service, which a month after his death resulted in the adoption by the State authorities of the plan which he had prepared in the interests of the community.

The following extracts from resolutions passed by official and other bodies, following his death, express the regard in which Mr. Clarke was held by his neighbors:

By the Mayor and Borough Council of the Borough of Bogota:

"*Whereas,* Mr. Clarke had given invaluable service during his thirty-three year residence in Bogota along religious and educational lines and to this Grade Crossing Elimination project, and in many other ways; therefore be it

"*Resolved,* That the Mayor and Council of the Borough of Bogota in meeting assembled on Thursday, May 28, 1931, express its deepest grief in the loss of such a staunch and stalwart citizen * * *."

By the Board of Education of the Borough of Bogota:

"*Whereas,* during his long residence in the Borough Mr. Clarke contributed generously of his time and his effort to the service of the public as a member of the Borough Council and as a member of the Board of Education, serving for eight years as the President of the Board; and

"*Whereas,* the wholesome influence which he exercised for so many years over public affairs in the Borough is sincerely recognized and will be long remembered,

"*Now Therefore Be It Resolved* that this Board of Education of Bogota hereby express its deep regret at his death, appreciating at the same time that the fine public service which he rendered is a far more fitting monument to his memory than any mere words which might be incorporated in a resolution of this nature and, in the hope that that service may act as an inspiration and a guiding light to the members of future boards, * * *."

By the Community Church of Bogota, N. J.:

"The Bogart Memorial Church learns with profound sorrow of the death on May 22, 1931, of Mr. St. John Clarke, one of its members and a former officer.

"Mr. Clarke moved to Bogota in 1898 and bore a prominent part in the early organization and administration of affairs of the Borough. He was a member of the Council, served eight years as President of the Board of Education, and was an officer of this Church from 1902 to 1906.

"He was one of the two Executors of the late Peter Bogart, whose benefaction made possible the construction of this church building. Through their fidelity and careful management they turned over funds to the officers of this Church for this building amounting to approximately twice the value of the properties before their stewardship; and this without fee to them."

Mr. Clarke was married in the fall of 1888 to Carrie S. Vanderlip, of Waverly, N. Y., a daughter of Stephen and Sylvinia Plummer Vanderlip, pioneer settlers of Tioga County, New York, who survives him. He is the father of John Graham Clarke (deceased), Charles Kennon Clarke, and Helen St. John Clarke.

An able, upright, and skillful Engineer, a genial and loyal friend, a useful and public-spirited citizen, who gave without compensation of his time and ability to the community in which he lived, St. John Clarke's career is both an example and an inspiration to his profession.

Mr. Clarke was elected a Junior of the American Society of Civil Engineers on September 5, 1888, and a Member on June 1, 1904.

JAMES RETZER COMLY, M. Am. Soc. C. E.[1]

DIED OCTOBER 6, 1931

James Retzer Comly was born to Harry R. and Beatrice (Seykora) Comly, on July 20, 1885, at Helena, Mont. He came from a family, of whom many were engineers, one, Henry Comly, having come with William Penn to the New World and having engaged in surveying and engineering in what is now the State of Pennsylvania. In 1893, Harry R. Comly moved with his family to San Diego, Calif., and from that time that city was the home of his son, James Retzer Comly.

Mr. Comly was graduated from the San Diego High School, in 1902, in the Scientific Course, and immediately entered the engineering field. His first work took him into Northern California and the Great Northwest where he held positions with the United States Reclamation Service, Bay Shore and Pacific Railroad Company, Peoples Water Company, and the Great Northern and the Southern Pacific Railroad Companies. During this period, covering nearly seven years of his early work, he spent his spare time studying and reading, and fitting himself for the Civil Engineering Profession.

In 1910, Mr. Comly became Locating and Constructing Engineer with the San Diego County Highway Commission, the first Highway Commission formed in the State of California.

In 1911, he became Assistant City Engineer of San Diego, and for the next four years was engaged in designing various municipal structures for water, harbor, and street and highway developments. Among the outstanding structures designed by Mr. Comly during this period were the Georgia Street Bridge across University Avenue, with its extensive retaining walls; the Seventeenth Street Bridge across E Street, and the Woolman Avenue Bridge. During this period he also made the first detailed survey of the water-front and San Diego Bay for a comprehensive study for future harbor development.

---

[1] Memoir prepared by Paul R. Watson, M. Am. Soc. C. E., and George Butler, Assoc. M. Am. Soc. C. E.

In 1915, he was engaged in private practice along engineering lines. In 1916, he was employed by the Sweetwater Company to design repairs of damage to the Sweetwater Dam and Distributing System occasioned by the heavy floods of that year.

From 1917 to 1920, except from January to November, 1918, during the World War, Mr. Comly was employed as Assistant Engineer by the San Diego and Arizona Railway Company, in charge of heavy mountain railroad construction. Later, he was put in charge of maintenance and valuation.

In January, 1918, Mr. Comly was commissioned a First Lieutenant in the Engineer Officers' Reserve Corps, and was assigned to the organization and training of Engineer Troops. He organized and commanded Company K, Seventh Engineers Training Regiment, and Company C of the Seventy-sixth Engineers. At the end of the World War he returned to San Diego.

In 1920, he again entered the employ of the City of San Diego as Senior Draftsman in the City Engineer's Office, devoting his time to study and investigation relative to water supply, flood control, harbor work, streets, and bridges. It was during this period that he suggested to the City Officials that some study be made along the lines of city planning, and he drew up a tentative plan in conformity with his ideas. Mr. Comly was one of the first to realize that it would be only a short time before through and heavy traffic would have to be by-passed around the business district on account of congestion and urged the City of San Diego to purchase rights of way while the land was still comparatively cheap and unoccupied, in order to provide for this contingency. This really inaugurated city planning in San Diego.

In 1922, he was engaged by the Spreckels Companies, of San Diego, to supervise the construction of the many buildings they were having built at this time, such as the San Diego Hotel and the Mission Beach Bath House.

From 1924 to the time of his death, Mr. Comly was engaged in private engineering practice. His chief activity during this period consisted of the design of municipal works connected with the laying out and development of high-class residential districts, such as La Jolla Country Club Estates, Ludington Heights, and Muirlands. He was also employed in numerous cases as Consulting Engineer in regard to water development, highways, and other work of a municipal character, and in this capacity rendered valuable service to the San Diego City Planning Commission.

Mr. Comly possessed marked ability to anticipate the engineering needs of the future, as shown by his endeavors to persuade the City of San Diego to adopt comprehensive plans for harbor development and city planning. In recent years, the wisdom of his ideas has been realized and many of his suggestions have been adopted. Mr. Comly was always painstaking in his work and was never satisfied until he had developed the problem to the utmost nicety.

The outstanding quality in his professional life was his high ideals of professional ethics. On account of these high ideals, he gained the respect of all those for whom he worked, as well as those with whom he was associated in his work.

Mr. Comly always took an active interest in the local affairs of the Society. He became a member of the Los Angeles Section shortly after its organization and, later, became active in the organization of the San Diego Section, of which he was a Charter Member and a Past-President.

His chief recreations were rowing and swimming and whenever he could spare the time from his professional work, he could be found at the San Diego Rowing Club. It was rarely that any water sports event took place without his presence, and especially outstanding was the frequency with which he officiated at aquatic meets.

Mr. Comly was a member of the San Diego Rowing Club and the San Diego Athletic Club. He was active in the Shrine Patrol and other Masonic organizations, and was a Past-President of the Local Chapter of the Sons of the American Revolution.

He was unmarried and is survived by a brother, Harry Comly, Assoc.M. Am. Soc. C. E., of Redding, Calif., and a sister, Cora Hornbuckle Comly, of San Diego.

Mr. Comly was elected an Associate Member of the American Society of Civil Engineers on April 2, 1913, and a Member on October 11, 1920.

HENRY ST. LEGER COPPÉE, M. Am. Soc. C. E.[1]

DIED MAY 8, 1901

Henry St. Leger Coppée, the son of Henry and Julia (DeWitt) Coppée, was born on March 30, 1853, at West Point, N. Y. His father, Henry Coppée, was a graduate of the United States Military Academy in 1845 and had also been a member of the Faculty, afterward becoming President of Lehigh University, at Bethlehem, Pa., which position he filled at the time of his death. After a preparatory education, the boy entered Lehigh University, from which institution he was graduated in 1872, with the degree of Civil Engineer.

After his graduation, Henry St. Leger Coppée entered the employ of the Southern Pacific Railroad Company and, in September, 1872, was engaged on a preliminary survey of that line extending from San Diego, Calif., to Maricopa and Puma Villages, in Arizona. He was afterward promoted to the position of Resident Engineer in charge of construction, with headquarters in San Diego.

During 1874 and 1875, he was Assistant Supervisor with the Pennsylvania Railroad Company, which position he held until 1876, at which time he entered the employ of the Bethlehem Iron Company, at Bethlehem, as Inspector of Materials and Civil Engineer. Mr. Coppée remained in this position until 1878, when he became associated with the Mississippi River Commission as an Assistant Field Engineer in charge of gauging operations near Memphis, Tenn. He remained in this District, under the late William H. H. Benyaurd, Major, Corps of Engineers, U. S. A., M. Am. Soc. C. E., until 1882, engaged on surveys and investigations.

---

[1] Memoir prepared by Charles M. Kerr, Assoc. M. Am. Soc. C. E.

In 1882, Mr. Coppée was transferred to the Vicksburg, Miss., Office of the Mississippi River Commission, and placed in local charge of improvements in the vicinity of that city. He continued in this position until 1892, at which time he was appointed Assistant Engineer of the Third Mississippi River District, which assignment he filled until his death in 1901. During this period he was actively engaged in the general supervision of levee building, bank protection, and other forms of river control and improvement along the Mississippi River and its tributaries in Arkansas, Mississippi, and Louisiana.

Mr. Coppée contributed, at various periods of his career, many technical writings based on his experiences and knowledge of river improvements and flood control. Principal among these was a paper entitled "Bank Revetment on the Lower Mississippi,"[2] for which he was awarded the Thomas Fitch Rowland Prize by the Society in 1896, and a medal at the Paris Exposition in 1900. He received recognition for this paper from all parts of the world from members of the Engineering Profession. Among his other contributions were a paper on "Standard Levee Sections,"[3] and discussions on "Dredges and Dredging"[4] and "The Discharge of the Mississippi River."[5]

Mr. Coppée enjoyed an enviable reputation among members of his profession, and was known as a capable engineer of the highest integrity; the public whom he served regarded him as a trustworthy official; his associates knew him as a constant and loyal friend. The last years of his life were spent in Greenville, Miss., where he served as a member of the Board of Education, and as a Vestryman of St. James Protestant Episcopal Church.

On July 15, 1884, he was married to Mary Bell Marshall, the daughter of Mr. and Mrs. Thomas A. Marshall, of Vicksburg. He is survived by Mrs. Coppée, and two daughters, who are now Mrs. Charles Macdonald Kerr, of New Orleans, La., and Mrs. William Barton Mason, of Orange, Va.

Mr. Coppée was elected a Member of the American Society of Civil Engineers on October 5, 1887.

CHARLES JAMES CRAIGMILE, M. Am. Soc. C. E.[1]

DIED SEPTEMBER 19, 1931

Charles James Craigmile, the youngest son of Alexander and Agnes (Calder) Craigmile, was born on December 27, 1889, at Gifford, Ill. His father and his grandfather came to the United States from Canada at the beginning of the Civil War, to fight with the Federal Armies. Mr. Craigmile was a graduate of the Rantoul, Ill., High School and of the College of Engineering of the University of Illinois, being President of the Class of 1913.

---

[2] *Transactions*, Am. Soc. C. E., Vol. XXXV (July, 1896), p. 141.
[3] *Loc. cit.*, Vol. XXXIX (June, 1898), p. 191.
[4] *Loc. cit.*, Vol. XL (December, 1898), p. 324.
[5] *Loc. cit.*, Vol. XXXV (July, 1896), p. 319.
[1] Memoir prepared by James H. Lynch, Jr., Assoc. M. Am. Soc. C. E.

Subsequent to his graduation, he was first employed in the Engineering Department of the Pennsylvania Railroad Company. His next position was with the Morgan Engineering Company in connection with a large drainage project in the Mississippi Bayou country; later, he was employed by the Illinois State Highway Department, in which position he superintended the construction of a section of the Great Highway, near Freeport, Ill. He was next engaged with the Engineering Force of the United States Interstate Commerce Commission in the valuation of railroad signal systems; while thus employed he was offered a position in the Engineering Department of the Burlington Railroad Company in a similar capacity, which he accepted.

Upon the entry of the United States into the World War, Mr. Craigmile immediately joined the Officers' Training Camp, at Fort Snelling, Minnesota, where he received his commission as First Lieutenant and was assigned to the 40th Infantry, at Fort Sheridan, Illinois; he was transferred, successively, to the Cavalry, at Camp Stanley, Leon Springs, Tex.; to the Artillery, at Fort Sill, Oklahoma; then to the School of Fire, at Fort Sill; and, later, to the Radio Department of the United States Army, Columbia University, New York, N. Y., where he was engaged at the time of the Armistice. At the conclusion of the war he was commissioned a Captain in the Reserve Officers Training Corps.

On December 23, 1918, Mr. Craigmile joined the Engineering Staff of the North Jersey District Water Supply Commission, being one of the first field engineers employed by the Commission. Work was just beginning on the construction of the Wanaque Reservoir at Wanaque, N. J., which reservoir furnishes the water supply for Newark, N. J., and seven neighboring municipalities. Mr. Craigmile held the position of Senior Assistant Engineer until September 1, 1931, the date of completion of construction on this project.

At the time of the illness which resulted in his death, he was on a vacation tour through the West, prior to joining the Engineering Department of the City of Newark.

His untimely demise has removed an Engineer who was devoted to his profession and whose genial nature had won him a host of friends, both professionally and socially.

Mr. Craigmile was a member of the American Association of Engineers, the American Legion, the Masonic Lodge, of Rantoul, Ill., the Benevolent and Protective Order of Elks, of Paterson, N. J., and the North Jersey Country Club.

He was married on December 12, 1927, to Irene Madeleine O'Crowley, daughter of Richard Joseph O'Crowley, of Newark, N. J., and is survived by his widow, his mother, three sisters, and two brothers.

Mr. Craigmile was elected an Associate Member of the American Society of Civil Engineers on October 14, 1919, and a Member on March 30, 1931.

WILLIAM WARREN CUMMINGS, M. Am. Soc. C. E.[1]

DIED APRIL 5, 1930

William Warren Cummings, the son of James Otis and Susan Celina (Bennett) Cummings, was born at Woburn, Mass., on September 13, 1864. He was a direct descendant of David Cummings, of Topsfield, Mass., who came to America in 1650. His mother was a member of the old Bennett family of Boston, Mass.

After receiving his preparatory training at the Woburn High School, Mr. Cummings entered the Rensselaer Polytechnic Institute, at Troy, N. Y., on February 4, 1881, one of the youngest members of his class. He became a member of the Pi Eta Society, later the Rensselaer Society of Engineers. He was graduated on June 18, 1884, with the degree of Civil Engineer, the only degree then given at the Institute.

The esteem in which he was held by his classmates was shown by his election to the office of Life President of the Class, and his excellent scholastic work was evidenced by his appointment, upon graduation, as Assistant in Rational and Technical Mechanics at the Institute.

He resigned this position in 1889, to engage in general engineering practice at Woburn, where he was City Engineer until 1892, when he was appointed to the Engineering Department of the City of Troy, N. Y. In 1893, he was the Engineer of the Mechanicville, N. Y., Water-Works, for which he built a reservoir and a dam. Later in the same year, he was an Instructor at the Colorado School of Mines, Golden, Colo., and, in 1894, he was Assistant Professor of Mathematics and Surveying at the same Institution.

In 1895, Mr. Cummings resumed the practice of his profession, as Engineer in Charge of the design of sewers at Mechanicville, in addition to his private practice. In 1897, he was connected with the Massachusetts Boundary Survey. In 1897-98 he was in charge of the erection of the works of the New England Gas and Coke Company, and, in 1898, he became Assistant Engineer of this Company and also Assistant Engineer of the Massachusetts Pipe Line Gas Company, and, later, the Chief Engineer of these Companies, until 1901.

He was Superintendent of the Isolated Plant Department of the Edison Electric Illuminating Company from 1901 to 1906, and Assistant Manager of the Commercial Department of the Boston Consolidated Gas Company from 1906 to 1910, resigning to resume his private practice, from 1910 to 1915. He then became Manager of the Industrial Fuel Department of the Boston Consolidated Gas Company and Engineer of Utilization for the same Company, from 1915 to 1924.

Mr. Cummings then returned to private practice as a Consulting Engineer, specializing in the industrial use of gas. He constructed coke ovens at

---

[1] Memoir prepared by Prof. C. W. Crockett, Rensselaer Polytechnic Inst., Troy, N. Y.

Everett, Mass.; three tunnels under the Charles and Mystic Rivers; a 42-in. double line of pipes in the congested streets of Boston in winter; in addition to railroad and mill construction, and a large amount of other work.

He was also President of the Industrial Appliance Company of New England from 1924 to January, 1930, at which time his resignation, tendered on account of poor health, was regretfully accepted. He died at Hanover, N. H., on April 5, 1930.

He was a member of the American Society of Mechanical Engineers; American Institute of Electrical Engineers; American Gas Association; New England Gas Association; American Society of Steel Testing; Boston Society of Civil Engineers; and other technical societies.

Mr. Cummings was married on June 28, 1893, to Mary F. McLane, of Troy, N. Y., who died on November 11, 1894, leaving no issue. He was married a second time on June 29, 1898, to Alice Jeeves, of Winchester, Mass., who, with four children (Carl Herrick, William Warren, Jr., Harry Blanchard, and Doris Elizabeth), survives him.

He was a man of great energy and activity, possessing rare judgment and thorough knowledge of his work. He was held in high esteem by all who knew him, not only because of his ability, but also because of his fine personality, high sense of duty, and lovable character.

Mr. Cummings was elected a Member of the American Society of Civil Engineers on June 7, 1899.

SVERRE DAHM, M. Am. Soc. C. E.[1]

DIED FEBRUARY 14, 1932

Sverre Dahm was born in Trondhjem, Norway, on January 2, 1858. His father, Dr. Jacob Christian Dahm, was a graduate of the University of Christiania, and for many years was a Government physician. Later, Dr. Dahm took up private practice in Drammen, Norway. He was greatly beloved as a physician, and continued active until he was more than 80 years of age. In his later life, the Order of St. Olaf was conferred upon him by King Oscar, of Sweden and Norway. Sverre Dahm's grandfather, von Dahm, was a Colonel in the Norwegian Army; the prefix, "von," was dropped when Norway adopted more democratic ideas and abolished all prefixes to proper names. Mr. Dahm's mother, Anneken Louise (Pehrsen) Dahm, was a woman of much originality of character, and was known for her interest in Norwegian antiquities. It was said that, at one time, she possessed the finest private collection of its kind in Norway.

After a preliminary education in the schools of his own country, Mr. Dahm studied Civil Engineering in the Polytechnicum, in Munich, Bavaria, from 1874 to 1878. For the following four years he was an Assistant Engineer on the Government Railways of Norway, engaged on the construction of the Western Railroad.

---

[1] Memoir prepared by Robert Ridgway, Past-President, Am. Soc. C. E., and A. I. Raisman, M. Am. Soc. C. E.

He came to the United States in 1883 and took up the design of bridges and other engineering structures. In the following year, he was a Draftsman for the Union Bridge Company, at Trenton, N. J., and in New York, N. Y. In the same year he was with Pedro Sosa, a contractor engaged in mapping the country about the proposed entrance to the Panama Canal, on the Pacific side, for the French interests then in control of the work. At that time, the control of yellow fever or the malarial diseases which were such scourges in the tropics, was not known, and practically all those who left the United States with Mr. Dahm to engage in this work died within a short time after their arrival on the Isthmus.

In 1885 he returned to the United States and was employed as a Draftsman with the late Theodore Cooper, M. Am. Soc. C. E. After six months' service with Mr. Cooper, he was appointed Assistant Engineer in charge of the Bridge Department of the Long Island Railroad, remaining in that position until 1889. Then, for six months, he was employed as a Draftsman and Computer for the Union Bridge Company, of New York. From 1890 to 1893, he was Chief Draftsman for Albert H. Wolf, Contracting Engineer, Chicago, Ill., and had charge of the steel design for various buildings, including the train-shed for the South Chicago Elevated Railroad.

Subsequently, Mr. Dahm was engaged, from 1894 to 1898, as a Draftsman in charge of structural work for the Jackson Architectural Iron Works, of New York City, and in 1899 and 1900, he was again with Mr. Cooper. All the assignments just described, served as a preparation for the important and responsible work which was to follow and with which he in his future life was to become identified.

On June 20, 1900, he reported as an Assistant Engineer to the Board of Rapid Transit Railroad Commissioners of New York City, then beginning the construction of the first rapid transit subway in that City under the direction of the Chief Engineer, the late William Barclay Parsons, Hon. M. Am. Soc. C. E. Mr. Dahm remained on that work, serving under the various Commissioners and Boards which succeeded each other, until his death.

On June 1, 1905, he was promoted to the position of General Inspector of Designs, succeeding the late St. John Clarke, M. Am. Soc. C. E., when the latter resigned. The position carried with it responsibility for the detail designing of all rapid transit structures, subway and elevated, and the work incidental thereto. He continued in that capacity with successive titles of Principal Assistant Engineer in 1909, Deputy Engineer of Subway Construction in 1919, and Engineer of Design from 1921 until June 16, 1925, when he was appointed Deputy Chief Engineer in Charge of Designs by the Board of Transportation. He was retired on his own application on December 25, 1929, and on the following January 1, 1930, was appointed Consulting Engineer by the Board of Transportation, which position he held until his death.

During the years of Mr. Dahm's service with the rapid transit bodies of New York City, work to the value of about $800 000 000 was designed and built, and for most of this time he was personally in charge of all the designing features. Mr. Dahm left a deep impression on the work and an enviable record for ability, efficiency, and loyalty. His thoroughness of training and the specialized knowledge he possessed of the principles of steel and reinforced concrete design were most valuable assets. His counsel was always sought on intricate matters of design, and his opinion commanded the high respect not only of the engineers of the Board, but of others with whom he came in contact. His sterling character and honesty of purpose impressed every one who knew him, and his kindly spirit and modesty endeared him to all his associates.

Following his death, the Board of Transportation of the City of New York, unanimously adopted the following resolution:

"*Resolved,* That the Board of Transportation of the City of New York does hereby express the sorrow of the members of the Board and of the entire staff of the Board at the loss of a beloved associate in the person of Sverre Dahm, Consulting Engineer of this Board, who died Sunday, February 14, 1932; and whose engineering ability and courteous manner, combined with the highest ideals of public service, attracted the admiration and affection of all of us who knew and worked with him;

"*Resolved,* That the Board of Transportation expresses its appreciation of the exceptional qualifications of Sverre Dahm as General Inspector of Designs, Principal Assistant Engineer, Deputy Engineer of Subway Construction, Engineer of Designs, Deputy Chief Engineer, and Consulting Engineer, and deplores his departure as a great loss to the City of New York."

Many of those who knew this man of slight physique, did not realize that he was just as active physically as he was mentally. One who was near to him writes:

"He was particularly fond of walking and in his earlier years he took long tramps in his native country. During his student years his vacations, all of them, were spent in long tramps in Germany, Italy, and Switzerland, when he averaged thirty or more miles a day for considerable periods of time. In Norway once, on a long tramp through the mountains, he walked over fifty miles in one day and used to tell with enjoyment that when they finally drew up at an inn his guide was so exhausted that he could not eat while he, himself, felt no discomfort.

"Mr. Dahm was also an experienced ski-runner and as long as he lived in Norway he kept up that sport. Running on skis began with him at a very early period of his life and while he was a small boy of five or six years he went to school each day on skis, a distance of six miles from his home in the northern part of Norway where his father was, at that time, government doctor.

"He was, when a young man, so proficient in gymnastics that his instructor urged him to become a professional in that field.

"Later in his life, while in New York, he took up boxing for a short time and for that exercise also he showed aptitude. His boxing instructor said of him, 'he is like lightning on his feet.'

"Mr. Dahm was a man of deep humane feeling and his abhorrence of cruelty extended not only to human beings, but also to dumb animals. For horses, dogs, and cats, he had a great liking. His attitude toward vivisection was characteristic. After a careful study of the vivisectors' published reports he became convinced of the cruelty and futility of vivisection. He was opposed to the practice because of the prolonged and often extreme suffering imposed upon the animals used, and because he considered the practice brutalizing in its effect upon the experimenters themselves. From this conviction he never wavered and until his death he remained affiliated with the Vivisection Investigation League.

"Mr. Dahm's interest in art, especially in his earlier life, was very pronounced and the closest friends of his student years were artists. Several of these young artists—Eilif Peterssen, Harriet Backer, Christian Skredsvig, and Hyerdahl—became, later on, numbered among Norway's most distinguished painters.

"He was keenly interested in literature and, while in Munich, as a young man, he became well acquainted with Henrik Ibsen and his wife who were then staying in Munich. On numerous occasions he was invited to their home and had amusing anecdotes to tell of Ibsen's very outspoken antagonism to 'herd opinion.'

"Mr. Dahm was always fond of reading and in his later years reading and walking were the recreations he enjoyed most."

He was married, in 1898, to Ellen L. Penrose, of Central Village, Conn., who survives him. It is interesting to note that although Mr. Dahm was a comparatively recent arrival in this country, he married Miss Penrose, who could trace her ancestry back to the earliest Colonial days. Roger Williams who settled Rhode Island was one of her ancestors.

Mr. Dahm was elected a Member of the American Society of Civil Engineers on February 7, 1906.

FRANK EUGENE DAVIDSON, M. Am. Soc. C. E.[1]

Died April 14, 1931

Frank Eugene Davidson was born near the Village of Hillsborough, Iowa, on June 7, 1867. He completed the course of Civil Engineering at Iowa State College at Ames, Iowa, in 1890 and immediately thereafter went to Chicago, Ill., where he resided for many years.

On his arrival in Chicago, Mr. Davidson secured employment as a Draftsman with Shailer and Schniglen and several other firms and, in 1895, he went into the general contracting business. In 1898, when Mr. Carter Harrison was elected Mayor of Chicago, he was appointed Superintendent of the Department of Sewers and, later, a member of the Board of Local Improvements. During the two years that he held this position he had direct charge of the redesigning of the intersecting sewer system of Chicago, and, during his administration, the Twelfth and Twenty-Second Street Systems were completed and the Thirty-Ninth Street and Lawrence Avenue Pumping Stations designed. Mr. Davidson recommended certain changes which were adopted by the City Council, that resulted in the saving of hundreds of thousands of dollars to the tax-payers of Chicago.

After two years of service with the City Government, Mr. Davidson resigned and entered the employ of the Illinois Steel Company as a Designer and Structural Engineer. He designed more than $50 000 000 worth of mill buildings for the Company and its allied interests. After four years, he entered the employ of the Western Electric Company as Architect, and as such had direct charge of building operations. In January, 1909, Mr. Davidson formed a co-partnership with the late William R. Patterson, M. Am. Soc. C. E., under the firm name of Patterson and Davidson, entering private practice as an Architect and Engineer, with offices in Chicago, where he remained until his death. In 1911, the co-partnership was changed to an Illinois corporation and Mr. Davidson shortly afterward purchased Mr. Patterson's interest in the firm.

---

[1] Memoir prepared by H. L. Palmer, Director, Illinois Soc. of Architects, Chicago, Ill.

Among the five hundred Chicago plants and buildings designed by Mr. Davidson may be noted the following: the Clearing Plant of the Continental Can Company; the plant of the Pyle National Company; the plants of the W. A. Jones Foundry and Machinery Company, and the Kaestner and Hecht Elevator Company; the Van Buren Street Plant of Hart, Schaffner and Marx; the plants of Metal Specialties Manufacturing Company, and the Standard Cork and Seal Company; and many other important industrial buildings.

Mr. Davidson always devoted a great amount of time to his professional societies and was a member of the American Institute of Architects, the Illinois Society of Architects, serving five times as its President, the National Fire Protection Association, the Chicago Chapter of the American Institute of Architects, the Western Society of Engineers, the Western Efficiency Society, the Architects Club of Chicago and other professional and technical organizations. At the time of his death he was a Director of the Illinois Society of Architects, and Treasurer of the Architects Club of Chicago, which he helped to organize. He was also Editor of the *Bulletin* of the Illinois Society of Architects, which position he had held since the paper was first published.

In politics, Mr. Davidson was a Democrat and was President of the Cook County Democracy, as well as Chairman of the Democratic Political League of Cook County. He devoted much of his time to civic affairs and was a member of the City Club and of several improvement societies. Governor Frank O. Lowden honored Mr. Davidson by appointing him as a member of the State Board of Parks and Buildings Advisers, and his counsel was of great value to the State officials.

He was an enthusiastic sportsman and a crack rifle shot, as well as an expert at trap shooting. He was also interested in billiards and played a good game of chess, but devoted his spare hours during the summer to cultivating a garden, as he was passionately fond of flowers and had one of the finest gardens on the east side of Lake Michigan.

Building contractors, in general, referred to Mr. Davidson as one from whom they always "got a square deal." His guiding policy as a professional and business man was the "Golden Rule."

At times, Mr. Davidson appeared to be rather abrupt with representatives of material houses, and when the writer remonstrated with him, he smiled and said,

"I never want to see salesmen unless I am in a position to do them some good and in the mean time I am doing them a real service by not taking up their time when they could be finding an architect in the market for their products."

His seemingly gruff exterior housed a heart of gold, and he was in reality very kindly, one who was ever ready to help the younger men who chose the Architectural Profession. His loyalty to his friends and business associates could not be exceeded, and those who knew him loved him. He will be missed by his friends and the organizations to which he belonged.

Mr. Davidson was elected a Member of the American Society of Civil Engineers on November 4, 1908.

JOSEPH PHINEAS DAVIS, M. Am. Soc. C. E.[1]

DIED MARCH 31, 1917

Joseph Phineas Davis was born in Northboro, Mass., on April 15, 1837, the son of William Eager and Almira L. (Sherman) Davis. All the lines of Mr. Davis' ancestry lead back to men and women who came early to New England, many of whom were active and efficient in the establishment of American institutions. On the paternal side, he was descended in the seventh generation, from Captain Dolor Davis and Margery Willard, who came to New England in 1634. On the maternal side, he was descended from Captain John Sherman, who came to America in 1634 and settled in Watertown, Mass. The home in Northboro was established in 1773 by Mr. Davis' great-grandfather, Deacon Isaac Davis, who went there from Rutland, Mass., and became a successful tanner.

Mr. Davis was graduated from Rensselaer Polytechnic Institute in Troy, N. Y., in 1856, at the age of 19. Immediately afterward, he was engaged as a Rodman at $1.25 per day on the Brooklyn, N. Y., Water-Works, later being promoted to the position of Transitman at $2.00 per day. The party had charge of the building of the foundations and the construction of Mt. Prospect Reservoir and Engine House. Mr. Davis directed the building of Engine House No. 2, in the Ridgewood Section of Brooklyn and his salary was increased to $60.00 per month.

In the summer of 1860, on hearing that the Peruvian Government wished the services of three American engineers, Mr. Davis made a proposition through the Peruvian Minister for a contract for five years at $4 000 per year, one-half payable in gold. While awaiting a decision, the Civil War broke out. Mr. Davis was drilling with a Brooklyn regiment when he saw the troops pass through New York, N. Y., on their way South in April, 1861. He then went to the Peruvian Minister to withdraw his proposition. To his surprise, the Minister drew out a bag containing $2 000 in gold and informed him that he was bound by his contract.

---

[1] Memoir compiled by Charles W. Sherman, M. Am. Soc. C. E., mainly from a memoir prepared by the late Frederic P. Stearns, Past-President, Am. Soc. C. E., and the late Edward W. Howe, M. Am. Soc. C. E., and published in the *Journal*, Boston Soc. of Civ. Engrs., Vol. 4, p. 37; also, from "The Story of Electricity" (1919), by T. C. Martin and S. L. Coles; and from "A Brief Sketch of the Telephonic Career of Joseph P. Davis," furnished in manuscript by the American Telephone & Telegraph Co. The compiler has made liberal transcripts from these sources, without using quotation marks.

In July, 1861, he went to Peru by way of Panama, and, during the next four years, made surveys, plans, and reports for water supplies, bridges, railroads, sewerage systems, and other public works. Probably, the most important piece of work he did while in Peru was to survey the great guano beds of Lobos and the coast north of Lima. This work lasted from October, 1862, to May, 1863.

Although the contract with the Peruvian Government was for five years, Mr. Davis returned to the United States in July, 1865, receiving a leave of absence, as a revolution was threatening. The revolution was successful, and the Engineer Corps was abolished. Mr. Davis was then appointed Assistant Engineer to lay a 48-in. main from the Ridgewood Reservoir to Brooklyn. In June, 1866, he was made Chief Engineer of Prospect Park, but in May, 1867, he was offered the position of Principal Assistant Engineer on the St. Louis (Mo.), Water-Works, which was about to be constructed. He accepted this offer, went to St. Louis, and, under the direction of the late T. J. Whitman, M. Am. Soc. C. E., at once began plans for putting the work under contract.

Early in 1870, he was appointed Chief Engineer of the Lowell, Mass., Water Commissioners, who had recently been chosen to build a new system of water supply for that city. Mr. Davis designed and placed under contract the new supply.

In November, 1871, he was appointed Chief Engineer of the Cochituate Water Board, to investigate and report on an additional water supply for Boston, Mass. After making a thorough investigation of all sources available, he recommended a supply from the Sudbury River. Works for this purpose were carried out under his direction and completed in 1880, within his estimate of cost, which was approximately $5 000 000.

In 1872, the office of City Engineer of Boston became vacant, and as there were differences of opinion in the City Government as to the engineer to be elected to this office, the matter was referred to a commission of prominent engineers, which reported, recommending the selection of Mr. Davis, and his election followed. The Commission stated in its report, after referring to the qualifications of other candidates:

> "Mr. Davis, in addition to a thorough special education as an engineer, has had a more varied and valuable professional experience than either of the other gentlemen. Especially in that branch of his profession which relates to the supply and distribution of water, and water work generally, does he appear to be eminently qualified. He has filled important positions upon several works of great magnitude, and of others he has had the full responsibility as chief engineer. The testimony of engineers of the highest professional reputation is fully given to his capacity and attainments, and to the excellence of the work he has done. As the final result of their examination and inquiry, the board are unanimous in the opinion that he best embraces the qualifications most important for the discharge of the responsible duties of city engineer."

It is to be noted that at the time of his election he was thirty-five years of age. He was City Engineer only from 1872 to 1880, when he resigned to accept a more important position, but during that comparatively brief time he completed an additional water supply for Boston, and, after visiting Europe and studying the sewerage systems of the larger cities, planned and began the construction of the comprehensive works for improved sewerage, now called the Main Drainage Works of Boston. This, however, was by no means the extent of his achievements in this position. He at once established the engineering of the city in all departments under his charge on a thoroughly sound basis.

At the time of Mr. Davis' election as City Engineer, the office had been organized only three years. His principal assistants were a group of young men who were extremely fortunate in having the benefit of the excellent engineering training which Mr. Davis could give, and in being subject to the remarkable influence which he had upon those with whom he was associated. He was thoroughly appreciative of the endeavors of his subordinates to do good work, and there are many of them who recall with pleasure and gratitude the time spent in service under his direction. Not only were these subordinates so greatly benefited by association with him, but they became so imbued with his views and methods that long after he resigned as City Engineer, his influence was a controlling factor in the conduct of the engineering work of the city.

In March, 1880, Mr. Davis joined the American Bell Telephone Company as Consulting Engineer; in 1891, he became its Chief Engineer, and continued in this position with the Company and its successor, the American Telephone and Telegraph Company, until 1905, when failing eyesight compelled him to relinquish the more active work; he remained, however, as Consulting Engineer of the Company until the end of 1908.

When he began his telephonic career, less than three years after the construction of the first telephone line, the streets and housetops of the larger cities were fast becoming unsightly from the mass of overhead wires. These, with the lofty poles and cumbersome roof structures which supported them, proved highly vulnerable to sleet storms, and were the frequent cause of serious disruption of the telephone service.

Mr. Davis was charged with the responsibility of devising a suitable method of placing the wires underground. Innumerable schemes had been suggested, practically all of which were valueless, but as there was no strictly underground telephone system in any part of the world, the Company had no guide except its own experiments.

His first important task was to design and direct the installation of the first "drawing-in" underground conduit system ever constructed. This work was carried out in Boston in the latter part of 1882, and served as a model for subsequent installations. Wrought-iron boiler tubes, encased in concrete, were used for the duct material, and manholes were placed at intervals for the purpose of drawing in and splicing the cables. Great care was taken to exclude water from the finished structure. So correct were the methods used in this first "drawing-in" system, and so workmanlike was its construction, that the conduits were reported as still in use nearly thirty years later.

During the years that followed, Mr. Davis placed the underground conduit construction of the Bell System on a sound engineering basis. He formulated the methods that should be used, not only in the actual building of the conduit systems but also in determining in which streets the conduits should be placed, and the most economical number of ducts to install, taking into account both present and future requirements. He and his assistants visited all the larger cities throughout the United States and gathered data to be used in the preparation of comprehensive conduit plans for those cities.

From these conduit plans of Mr. Davis developed the work on "Market and Development Studies and Fundamental Plans," which for many years has been one of the important engineering undertakings of the Bell System. These studies include a critical analysis of the existing market for telephone service in a community and pertinent facts as to the present sale of the various classes of service, upon which are based forecasts of the probable size, distribution, and character of the future market and its telephone requirements. These forecasts of future telephone requirements permit the preparation of a comprehensive plan to serve as a basis for the layout of the plant at a date fifteen or twenty years in the future.

During his long service with the Bell System, Mr. Davis was active as a Director and Officer of many of the local operating companies. He was one of the incorporators of the Metropolitan Telephone and Telegraph Company of New York City in 1880, and of its successor, the New York Telephone Company, in 1896. He was First Vice-President of these Companies during almost the entire period from 1880 to 1902, and a Director and member of the Executive Committee for even a longer period. He was appointed Consulting Engineer of the Metropolitan Telephone and Telegraph Company in July, 1887, and took an active part in the design and construction of the first underground conduit systems installed in New York City.

Mr. Davis was for several years President and a Director of the Hudson River Telephone Company and of the Westchester Telephone Company; he was also a Director of the following Companies: the Union Telephone Company; the Northern New York Telephone Company; the New York and New Jersey Telephone Company; the New York and Pennsylvania Telephone and Telegraph Company; the Empire City Subway Company; the Chesapeake and Potomac Telephone Company; the Southern Bell Telephone and Telegraph Company; the Bell Telephone Company of Buffalo; the Central District and Printing Telegraph Company; and the Chicago Telephone Company.

He acted as Consulting Engineer of boards and commissions on many important undertakings, including the Croton Aqueduct Commission from 1884 to 1886, the Massachusetts State Board of Health from 1886 to 1904, the Metropolitan Water Board and its successor, the Metropolitan Water and Sewerage Board, from 1895 to 1909; and the Metropolitan Sewerage Commission in 1898.

Mr. Davis was active in the re-organization of the Boston Society of Civil Engineers in 1874; he became its President in 1880, and an Honorary

Member in 1908. He was also a member of the American Society of Mechanical Engineers, the American Institute of Electrical Engineers, and the Society of Telegraphic Engineers and Electricians (of England).

Mr. Davis was elected a Member of the American Society of Civil Engineers on January 29, 1868, and served as Director in 1878 and again from 1881 to 1883, and as Vice-President in 1884.

JOHN STANLEY DAWSON, M. Am. Soc. C. E.[1]

DIED AUGUST 6, 1931

John Stanley Dawson, the son of Thomas and Hattie (Stanley) Dawson, was a native Kentuckian, having been born at Bloomfield, in Nelson County, on June 23, 1888.

Mr. Dawson received his early education at Bardstown, Ky., and then entered upon the study of Civil Engineering at the University of Kentucky, in Lexington, from which he was graduated on June 2, 1910, with the degree of Bachelor of Civil Engineering. During his college course he was rated as a splendid student and acquired a thorough knowledge of the fundamentals of his chosen profession.

After graduation he received his first position as a Rodman with the Louisville and Nashville Railroad Company, and continued in its service, with but little interruption, until August, 1916. During this period of employment he was promoted from Rodman to Assistant Resident Engineer and Resident Engineer, and received a broad, practical experience in railway location, relocation, staking out excavations, and bridge design and construction. As Resident Engineer he was in charge of heavy mountainous construction, including bridges, tunnels, and yard work, and discharged all the duties assigned to him with efficiency and dispatch.

In August, 1916, Mr. Dawson was appointed a Division Engineer with the Kentucky State Highway Department and served in that capacity until August, 1917. During this period he rendered meritorious service to his native State in the pioneer days of road and bridge building in Kentucky.

In August, 1917, he answered the call of his country and enlisted in the United States Army for service in the World War. He was assigned to the Second Officers Training Camp at Fort Benjamin Harrison, in Indiana. At the completion of this training he was commissioned a Second Lieutenant in the Field Artillery, which rank he held until the end of the war, serving as an Instructor in other training camps.

After Mr. Dawson's discharge from the Army he returned to the Kentucky State Highway Department and was reinstated in his former position of Division Engineer. In December, 1920, he was promoted to the position of Engineer of Maintenance of the Department and was placed in charge of the maintenance of all State highways in Kentucky, which position he held until his health broke down in March, 1924, compelling him to resign and seek relief in another climate.

---

[1] Memoir prepared by Joe S. Boggs, Esq., Frankfort, Ky.

During the four years that Mr. Dawson was Engineer of Maintenance the activities of this branch of the Department were greatly enlarged due to the rapid expansion of the road funds and the public demand for better maintained State highways. By careful selection of efficient employees, by close attention to details, and by the introduction of modern methods of highway maintenance and up-to-date machinery, Mr. Dawson built up in Kentucky a maintenance organization equal to that in any State.

After his resignation in the early part of 1924, he went to Tucson, Ariz., seeking the restoration of his health. He lived there until June, 1929, at which time he returned to Kentucky where he resided, until his death at the U. S. Veterans Hospital, at Lexington, on August 6, 1931. During all these years of illness, although unable to engage in any kind of employment, he displayed a remarkable fortitude and a cheerful patience with an incurable affliction, seldom seen in any one under like circumstances.

Mr. Dawson was married on November 10, 1915, to Bessie McDaniel, of Franklin, Tenn., a most worthy helpmate and companion during the years of his active professional life and his constant attendant after his health gave way. To her and his one child, a daughter, he was devoted, and longed to recover his health so that he could better safeguard their future welfare and happiness.

In the passing of John Stanley Dawson, Kentucky has lost a valuable citizen, his profession an outstanding member, his family a devoted husband and father, and his associates a staunch and loyal friend, whose place cannot be filled.

Mr. Dawson was elected a Member of the American Society of Civil Engineers on June 1, 1920.

FLOYD GOSSETT DESSERY, M. Am. Soc. C. E.[1]

DIED MAY 12, 1927

Floyd Gossett Dessery, the son of Alfred B. and Mary M. (Gossett) Dessery, was born in Leavenworth, Kans., on February 10, 1879. When the family moved to California he attended Grammar School and High School in Los Angeles. In June, 1898, he left High School and enlisted in Battery H, Third United States Heavy Artillery, and saw two years of active service in the Philippine Islands during the Spanish-American War and the Insular Insurrection.

On his return from Army service, Mr. Dessery began his engineering career as a member of field survey parties, working from the offices of the City Engineer of Los Angeles and the County Surveyor of Los Angeles County. Later, he was employed by the Los Angeles and Salt Lake Railway Company, and made a transmission line location survey in Kern County, California, for the Pacific Light and Power Company. During the construction of the Pacific Electric Railway Company's interurban system in Southern California, from 1902 to 1905, he was Transitman and Locating Engineer for that Company. In 1906 Mr. Dessery served the City of Los Angeles as Field Engineer and Hydrographer and, later, made hydrographic surveys for the Los Angeles Dock and Terminal Company's development of the inner harbor at Long Beach, Calif.

---

[1] Memoir prepared by Ralph J. Reed, M. Am. Soc. C. E.

From the fall of 1906 until the summer of 1908, he was associated with F. S. Noble, Consulting Engineer, on various water supply and highway projects; and at the conclusion of this engagement he opened his own office, continuing in private practice until his death. For several years he shared offices with the late James Dix Schuyler, M. Am. Soc. C. E., and was associated with Mr. Schuyler on a number of projects. From 1910 to 1912, he was in partnership with the late Mr. George W. West, under the firm name of Dessery and West.

Prior to the consolidation of Wilmington, Calif., and Los Angeles, Mr. Dessery was Harbor Engineer for the City of Wilmington in charge of the design and construction of harbor slips. As a result of his familiarity with conditions in the harbor district, he was employed as an Engineering Consultant in a number of lawsuits involving tide-land interests at San Pedro, Calif. For about twelve years he was Engineer for the Consolidated Lumber Company, designing and constructing the channel, turning basin, and wharves at its large Wilmington Plant.

Mr. Dessery was City Engineer of Covina, Calif., from 1908 to 1914, served for several years as City Engineer of Glendora, and of Upland, and from 1915 to 1918 was Consulting Engineer for the City of Coalinga, Calif., on the design and installation of its water supply system. In 1920 he was retained by the Governor of the Northern District of Baja California (Mexico) as Consultant on various water supply and sewerage systems, in particular the municipal water system for the City of Mexicali. About this time he was also engaged for several years in the improvement, for Mr. William R. Hearst, of the great San Simeon Ranch in San Luis Obispo County, California.

When the development of the extensive Palos Verdes Estates was undertaken in 1921 and 1922, Mr. Dessery was a member of the Engineering Commission directing this work and, about the same time, he served as Consulting Engineer to Los Angeles County Water-Works District No. 4. Throughout his practice, Mr. Dessery was frequently employed as a Consultant by land companies, water companies, and municipalities because of his wide experience in problems of hydraulics and water supply.

Ill health forced Mr. Dessery to give up active practice nearly a year before his death, which occurred at his home in Los Angeles, on May 12, 1927.

He was a member of the Sons of the American Revolution, The Society of American Military Engineers, the American Water Works Association, and the Jonathan Club of Los Angeles. As a Charter Member of the Los Angeles Section of the Society, he was an enthusiastic participant in its development and devoted much time to its interests, serving with great fidelity as its Secretary from 1919 to 1924.

He was married on September 6, 1905, to Julia M. Morrison who, with two sons, Floyd Gordon and Gerald Morrison, survives him.

Mr. Dessery was elected an Associate Member of the American Society of Civil Engineers on February 28, 1911, and a Member on July 11, 1921.

HARRY ESPENSHIP DETWILER, M. Am. Soc. C. E.[1]

Died February 19, 1931

Harry Espenship Detwiler was born at Oaks, Pa., on April 3, 1874. During his boyhood he attended the public schools of his native town, but owing to the early death of his parents, he was thrown upon his own resources when he was very young. Being of a resourceful and studious nature, he worked around the Pencoyd Plant of the American Bridge Company, and pursued his studies through correspondence courses in Structural Drafting.

His first regular work with the Pencoyd Iron Works was in 1896 and 1897 as Inspector in the Fabricating Shop. He later was appointed Shop Foreman.

In 1900, Mr. Detwiler became connected with the Structural Iron and Steel Company, of Baltimore, Md., as a Structural Draftman, and, from 1901 to 1906, he served the Lauer and Harper Company, of Baltimore, as Chief Draftsman and, later, as Designing Engineer on structural steel work for bridges, piers, etc.

He was employed by the City of Baltimore as Assistant Engineer in the City Park Department in 1906 and 1907. He then accepted a position as Assistant Engineer of Bridges and Buildings with the Seaboard Air Line Railway, with headquarters in Portsmouth, Va. In 1909, he associated himself with the York Bridge Company, of York, Pa., as Designer in the Engineering Department.

In 1914 Mr. Detwiler moved to Scranton, Pa., where he associated himself with Mr. Boyd A. Musser, in organizing the Anthracite Bridge Company, serving as Secretary-Treasurer, and also as Chief Engineer, until his death.

With his great capacity for detail, and studious application, Mr. Detwiler developed into a most valuable Designing and Consulting Engineer, particularly as applied to structural steel work. His training and experience were recognized, and his services appreciated, by the leaders in the anthracite coal industry, and he was continually being consulted on construction problems peculiar to this industry. He was also helpful to architects, county, city, and consulting engineers in their engineering problems. In all these contacts he was very zealous in maintaining a high code of ethics for the Engineering Profession.

He was not only a competent and efficient Engineer, but he was much interested in problems that were helpful in making communities more beautiful. He was recognized as an authority on wild flowers, and he took a great interest in legislation for their protection.

---

[1] Memoir prepared by Boyd A. Musser, Pres. and Mgr., Anthracite Bridge Co., Scranton, Pa.

Until his final illness which began on December 6, 1930, Mr. Detwiler was active in civic, engineering, and industrial organizations of the community in which he lived. He was especially active in Lions International, having served as President of his Local Club, and also as Governor for the State of Pennsylvania. He took great interest in the American Institute of Steel Construction, and served on committees of importance in that organization. He leaves a widow and one daughter, who reside in Philadelphia, Pa.

Mr. Detwiler was elected a Member of the American Society of Civil Engineers on October 14, 1929.

FREDERIC CLARK DUNLAP, M. Am. Soc. C. E.[1]

Died January 25, 1928

Frederic Clark Dunlap, the son of George and Jennie (Chattin) Dunlap, was born on February 8, 1869, at Mullica Hill, N. J. He received a common school education in Camden, N. J., and was a Special Student in Engineering at the Lauderbach Academy and Polytechnic School in Philadelphia, Pa., in 1885 and 1886.

In 1887, Mr. Dunlap entered the office of Howard Murphy, Civil Engineer, of Philadelphia, and in 1888 and 1889, was in charge of outside work, surveys, etc., for Mr. Murphy, including the construction of water supply works at Merchantville, N. J., and Roanoke, Radford, Richlands, and Luray Inn, Va., testing pumping machinery for Lynchburg, Va., and water power installations at New River, Va., and Fayetteville, N. C.

In private practice as Engineer and as Contractor, from 1890 to 1893, he built portions of water-works at Red Bank, N. J., and Hyndman, Pa. From 1894 to 1896, he was Assistant Superintendent of the Union Hydraulic Works, Philadelphia, engaged in the manufacture of supplies for water-works. In 1896 and 1897, he was Assistant Engineer for the contractor on the new water supply for Camden, N. J., Superintendent of Construction of water-works at Egg Harbor, N. J., and made surveys, plans, and estimates for water-power projects on the Delaware River at Trenton and Lambertville, N. J., and for a power project on the Susquehanna River.

In 1898, Mr. Dunlap was Superintendent of a building operation in Overbrook Heights, Philadelphia. For six months in 1899, he was an Assistant Engineer with the Bell Telephone Company, in Philadelphia, but resigned to accept an appointment with the City of Philadelphia then beginning an extensive water purification project. Ten years' experience in the construction of smaller water supply plants in many sections of the United States served as an invaluable training for the work which he now began and to which he gave the best of his life.

---

[1] Memoir prepared by Vernon R. Dunlap, M. Am. Soc. C. E.

In 1900 and 1901, he was Assistant Engineer on designs for the improvement of the Philadelphia Water Supply, under the late John W. Hill, M. Am. Soc. C. E., and from 1902 to 1905, he was in charge of the construction of the Torresdale Filter Plant, the largest filtration plant in the world at that time. In 1906, he was employed for a short time on designs for the subway in North Broad Street, Philadelphia, and then went to New York, N. Y., as an Assistant Engineer in the Department of Water, Gas, and Electricity in that city.

On May 1, 1907, Mr. Dunlap returned to Philadelphia as Chief of the Bureau of Filtration and remained in charge of the Water Bureau, with which the Bureau of Filtration was consolidated, until September 15, 1912. During this period the Queen Lane Filtration Plant was completed, extensive improvements were made in pumping and distribution mains, and filtered water for the first time was furnished to every section of the City of Philadelphia.

He resigned as Chief of the Water Bureau and spent the winter of 1912–13 in the West endeavoring to recover from a chronic rheumatic condition that he had endured for a number of years. He returned to Philadelphia and, in 1914 and 1915, was employed by the National Dredging Company in the development of its gravel and sand deposits. In 1916, he returned to the employ of the City in the Division of Sewage Disposal, and, later, as Assistant Chief Engineer, Department of City Transit.

With the outbreak of the World War Mr. Dunlap applied for a commission in the Corps of Civil Engineers, United States Navy, but was offered an appointment in the Bureau of Yards and Docks as a Civilian Engineer by the Navy Department. In the interim, he had accepted the position of Chief of the Bureau of Highways, Philadelphia and was requested to remain at his new post by the Mayor.

He served as Chief of the Bureau of Highways from June 1, 1917, until June 1, 1923, when he was transferred again to his former position as Chief of the Bureau of Water. The situation with regard to the water supply of the City of Philadelphia had reached a point at which it appeared that steps must be taken to secure an increased supply at an early date. A plan had been advocated and had received support looking for further development of the Schuylkill water-shed and that of the Lower Delaware. Mr. Dunlap's studies led him to believe that a better and more economical plan for an increased supply would be to adopt substantially the one advocated many years before to bring water from the Delaware River above Trenton, N. J., to Philadelphia. This plan developed and announced in detail aroused violent opposition from political sources, with the consequence that with a change in City Administration shortly thereafter, Mr. Dunlap's resignation was requested and accepted on February 1, 1924, although as yet no engineering investigation had been made into the question at issue.

The challenge presented to Mr. Dunlap was promptly met. Although he had never before engaged in public speaking, he began, on his own time and at his own expense, to interest civic bodies and societies in the questions involved. In plain language and with facts and figures presented in a manner that non-technical audiences could appreciate, he won the support of the newspapers as well as the public and prevented the adoption by the City of a plan which, in his experience and judgment, did not appear to be an economical and safe method of securing an increased water supply. He followed up this action by personally representing the City's interests before the Commission engaged in preparing a Tri-State agreement for the division of the water rights in the Upper Delaware River water-shed, avoiding the surrender of rights to which the City appeared to be entitled. His outstanding position in the water supply controversy was recognized by his appointment as Chief of the Bureau of Water for the third time in January, 1928, thus vindicating his action of four years previous.

Mr. Dunlap's restoration to the Water Bureau was followed almost immediately by the beginning of an illness which developed into pneumonia and resulted in his death on January 25, 1928. His passing came at his prime when his usefulness to his City which he had served for a score of years would have been most valuable. He had endeared himself to his subordinates and associates, many of whom he had taken from minor positions and developed as executives. His life exemplified that of many loyal municipal and Government employees who have and are giving their best to the service of their fellow men in a manner that frequently receives scant notice and recognition.

In 1891, Mr. Dunlap was married to Alice Palmer, the daughter of J. Albert and Mary Rea Palmer, of Camden. He is survived by his widow and by his two sons, Vernon R. Dunlap, M. Am. Soc. C. E., and George A. Dunlap.

He was a member of the Union League of Philadelphia, the Engineers Club of Philadelphia, the American Water Works Association, and the Municipal Engineers of New York.

Mr. Dunlap was elected a Member of the American Society of Civil Engineers on June 1, 1904.

JAMES HARVEY EDWARDS, M. Am. Soc. C. E.[1]

DIED AUGUST 14, 1930

James Harvey Edwards was born on June 19, 1864, in Oxford, N Y. His early education was obtained in the district school of Oxford and the Oxford Academy, where he prepared for Cornell University. His father owned and operated an iron foundry in Oxford, and the son spent his spare time from school and vacations in the foundry. He thus obtained a practical experience which was to prove of value in his later career as an engineer.

In the fall of 1884, he entered Cornell University, at Ithaca, N. Y., and was graduated in June, 1888, with the degree of Civil Engineer. During his college career, he was President of his Class in his Sophomore year, Captain of one of the cadet military companies, and Chief Engineer of his

---

[1] Memoir prepared by a Committee consisting of H. G. Balcom, Charles Stratton Davis, and J. E. Wadsworth, Members, Am. Soc. C. E., and R. Fleming, Esq., New York, N. Y.

Class in his Senior year. He was a member of the Delta Upsilon Fraternity and was elected a member of the honorary societies, Sigma Xi and Tau Beta Phi.

Immediately after his graduation, Mr. Edwards entered the employ of the Union Stock Yards in Chicago, Ill., where he remained until the fall of 1888, when he accepted a position with the Berlin Iron Bridge Company, of East Berlin, Conn., as a Structural Steel Draftsman. For the subject of his graduating thesis he had chosen a bridge built by the Berlin Iron Bridge Company, in Oxford. It was on the merits of this thesis that he obtained his position with the Company. He was promoted to be Chief Draftsman in 1890, made Assistant to the Vice-President in 1891, and Chief Engineer in 1897.

At the time that Mr. Edwards entered the employ of the Berlin Iron Bridge Company, steel was beginning to replace wood in the construction of factory and mill buildings and the Company seeing the possibilities in this type of construction, made it a specialty. The success of the Company was due largely to the ability of Mr. Edwards as a designer of this type of buildings.

In 1900, the Berlin Iron Bridge Company was merged into the American Bridge Company, and Mr. Edwards was made Structural Engineer, in charge of building work. In 1901, he was made Assistant Chief Engineer, and, in 1927, Chief Engineer, which position he held at the time of his death in New York City after an illness of only three days.

When the United States entered the World War, Mr. Edwards was assigned by the American Bridge Company as its representative on all war contracts with the United States Government.

Soon after the close of the war, Mr. Edwards took up the study of fusion welding. Although all his former training and experience had been along the line of riveted connections, he soon became convinced that there was a future in fusion welding. As he had been a pioneer in the design of steel mill buildings, so he now became a pioneer in the design of welded structures. At Mr. Edwards' suggestion, the American Bridge Company established a welding shop at Trenton, N. J. His combined engineering and business ability enabled the Company to secure contracts for a number of the principal welded structures of the United States.

He became an active member of the American Welding Society, of which he was elected a Vice-President and Chairman of the Committee on Structural Steel Welding. In the latter capacity, he was instrumental in having the Committee arrange for a series of tests which are now (1931) in progress. In 1929, the Society awarded to him the Miller Medal for conspicuous work in fusion welding.

In 1916, Mr. Edwards was placed in charge of the design of all steel structures for the plants of the United States Steel Corporation. In that capacity he had charge of the design of the new mills at Homestead, Pa., Gary, Ind., South Chicago, Ill., and Lorain, Ohio. He was also appointed a member of the Committee on the Design of I-Beam Sections, which Committee developed the new Carnegie Beam Sections of the Corporation.

Mr. Edwards was deeply interested in educational matters. He was elected Alumni Trustee of Cornell University for three successive 5-year terms (1906–1921). During his term of service he was instrumental in combining the Colleges of Civil, Mechanical, and Electrical Engineering into one College of Engineering. As Chairman of the Buildings and Grounds Committee, he had a large part in the building of the new Armory Hall and the new dormitories for men.

Mr. Edwards was a member of the American Railway Engineering Association, the American Society for Testing Materials, the American Welding Society, the American Iron and Steel Institute, the Society for the Promotion of Engineering Education, the Cornell Society of Engineers, the Engineers Club, and also of several social organizations. He was a member of the Merchants Association Commission, organized to prepare a new Building Code for New York City.

In 1892, Mr. Edwards was married to Mabel T. Davis, of Binghamton, N. Y., who, with four sons, Robert D., J. Leland (Assoc. M. Am. Soc. C. E.), John C., and David A., and one daughter, Elizabeth (Mrs. Richard W. Schabacher), survives him.

At the meeting of the Board of Direction of the Society held at St. Louis, Mo., on September 29, 1930, the following resolution in memory of Mr. Edwards was unanimously adopted:

"*Whereas:* In the providence of Almighty God: James Harvey Edwards, Member, has been removed from our midst, and

"*Whereas:* He has served with distinction as a Director of the Society during the years 1913 to 1915; as a member of its Committees on Steel Columns and Struts; on Stresses in Structural Steel; and as the Society's Representative on Building Code Committee of the U. S. Department of Commerce; Sectional Committee on Steel Shapes of the American Standards Committee; and on various other committees: and

"*Whereas:* He gave liberally of his time toward the advancement of engineering practice and methods of his energetic labors, tempered by good judgment and keen analysis of conditions, always keeping in mind the best interests of the Society, the Engineering Profession, and the Public:

"*Therefore Be It Resolved:* That the members of the Board of Direction of the American Society of Civil Engineers spread upon the records their profound sense of loss to the Society, the Engineering Profession, and the Public:

"That they express their deep sympathy for the family in their great bereavement: and,

"That a copy of this preamble and resolution be sent to Mrs. James Harvey Edwards, for the family."

The passing of Mr. Edwards leaves a large gap in the ranks of the Engineering Profession. He will be missed and mourned by a host of friends outside as well as within the Profession.

"He was a man, take him for all in all,
I shall not look upon his like again."

Mr. Edwards was elected a Junior of the American Society of Civil Engineers on May 31, 1892, an Associate Member on May 2, 1894, and a Member on May 4, 1898. He had served as a Director of the Society from 1913 to 1915.

GEORGE EZRA ELLIS, M. Am. Soc. C. E.[1]

Died December 17, 1931

George Ezra Ellis was born at Fairhaven, Mass., on January 13, 1870, the only child of Hiram B. and Sarah D. Ellis. His early education was secured in the public schools of Fairhaven, and he was graduated from Rensselaer Polytechnic Institute, at Troy, N. Y., in June, 1892, with the degree of Civil Engineer.

Upon graduation Mr. Ellis was engaged by the New York Central and Hudson River Railroad Company (now the New York Central Railroad Company) as Signal Inspector on various divisions until June, 1898. Among his duties in this connection was charge of the construction and operation of interlocking and block signals. From June, 1898, to June, 1900, he was Assistant Superintendent of Signals of the same road, during which time he designed and constructed an interlocking plant amounting to more than 200 levers. He was also Superintendent of the Hudson River Bridge Company, in charge of the maintenance and operation of two New York Central and Hudson River Railroad bridges (each about $\frac{1}{3}$ mile in length) over the Hudson River, at Albany, N. Y.

His subsequent career was as follows: From June 1900, to 1902, Signal Engineer, Standard Signal Company, in charge of estimating, designing, and constructing interlocking plants installed on various railroads, including 300 levers of mechanical and 180 levers of pneumatic interlocking on the New York Central and Hudson River Railroad, at Buffalo, N. Y. From 1902 to 1904 he was Western Agent for the Pneumatic Signal Company; and, during 1904, he was also Assistant Chief Engineer of the General Railway Signal Company.

In 1904, Mr. Ellis again entered active railroad work as Signal Engineer of the Chicago, Rock Island and Pacific Railroad Company, with headquarters at Chicago, Ill., and continued in this capacity until 1908. During the next two years, he was connected with the Federal Signal Company and the Hayes Track Appliance Company, re-entering the railroad field as Signal Engineer of the Kansas City Terminal Railway Company in 1910, which position he held until 1915. During this period he was active in the work incident to the rehabilitation of the Kansas City Terminal properties, involving elaborate construction work on the signal system of the new Terminal.

From 1915 to 1920, Mr. Ellis was with the Bureau of Safety of the Interstate Commerce Commission as Senior Signal Engineer, and during the last two years of that period he also served as Executive Secretary of the Automatic Train Control Committee of the United States Railroad Administration.

Upon the organization of the Committee on Automatic Train Control of the American Railway Association, in November, 1920, Mr. Ellis, as the individual best fitted by experience and qualification to handle this work, was appointed its Secretary, which position he held until his death. In April, 1930, when the Joint Committee on Grade Crossing Protection of the American Railway Association was organized, he was also chosen as its Secretary.

---

[1] Memoir prepared by E. A. Hadley, M. Am. Soc. C. E.

Practically the whole of Mr. Ellis' engineering career was in the field of railway signaling and automatic train control. His outstanding fitness for the position of Secretary of the Committee on Automatic Train Control had its foundation in his relationship as Executive Secretary to the Automatic Train Control Committee of the United States Railroad Administration. The investigations by this body were the basis of a knowledge that was increased with subsequent development of the art, so that, at the time of his death, Mr. Ellis probably had at his finger tips the details and characteristics of automatic train-control systems, to a greater extent than any one else in the United States. This is admirably indicated by the fact that he was selected to write the treatise on this subject for the World Engineering Congress held in Tokyo, Japan, in 1929, and a similar paper for presentation in the latest edition of the Encyclopedia Brittanica. He also wrote many articles for various technical magazines, and, upon request, delivered numerous addresses before railway and engineering societies upon the subject in which he was most interested.

Mr. Ellis always manifested an active interest in his Alma Mater, and was greatly interested in its Alumni activities. His realization of the value of an education, and his desire to help those not financially able to secure it, is indicated by the number of young men whom he assisted in procuring an education at Rensselaer Polytechnic Institute.

In his personal as well as in professional circles, Mr. Ellis made many warm friendships which, when once created, continued whole-heartedly. His friendship and loyalty were prized very highly among those who knew him best.

He was never married, and, at the time of his death, had no immediate relatives.

Mr. Ellis was elected a Junior of the American Society of Civil Engineers on March 2, 1897, an Associate Member on March 7, 1900, and a Member on December 4, 1901.

CHARLES CLEMENT ELWELL, M. Am. Soc. C. E.[1]

Died May 21, 1931

Charles Clement Elwell, the son of Benjamin T. and Martha (Wilson) Elwell, was born at Belfast, Me., on July 16, 1855. Before he was seven years old, the family moved to Aroostook County in the extreme northeastern corner of the United States. During the next five years his school attendance was confined to short terms of eight to twelve weeks in summer at crude schools taught by equally crude local schoolmasters or mistresses. Although his schooling, which had begun propitiously at Belfast, was interrupted by the move, the rough and independent life of a frontier farmer and woodsman may have given him as much benefit in sound health, a strong body, and ability to meet and overcome difficulties courageously, as he might otherwise have had from continuance in the city schools. However, from the age of twelve to sixteen years, he attended the Academy at Patten, Me., and came under the instruction of a master who inspired him with a desire for a better education than might be expected to come naturally to one in his situation. As the summer term of the country school near his home happened to be at just the time when Patten Academy had its vacation, he took advantage of the opportunity thus presented and these four years saw almost continuous attendance at school.

---

[1] Memoir prepared by Herbert C. Keith and Joseph P. Wadhams, Members, Am. Soc. C. E.

After graduation from the Academy, young Elwell became uneasy and "desired to strike out for himself and see the world." In April, 1872, his father consented, but "with the stipulation that the father should go along and start the son in the new venture." His study of history and geography had prepared him to enjoy a great thrill as they passed through Boston, Mass., and New York City, on their way "out West" to Elmira, N. Y. The travel also gave him a desire to become a railroad man, and he tried to get a position with the Erie Railroad Company. Although his youth prevented its accomplishment at that time, the desire never left him as long as he lived. The "hard times" of the following winter, and consequent unemployment, turned them homeward again, and they found a lighted lamp in the window as a beacon, although his mother had not received the letter advising of their intention to return (this light had been burning every night for them for several weeks). Although not profitable financially the trip had broadened Mr. Elwell's outlook and increased his desire for a better education, and he returned to Patten Academy for advance work. Another year also was spent there as a student, with the added position as Assistant to the Principal, and was followed by two years more of teaching in other Maine schools.

In 1876, Mr. Elwell entered the University of Maine, at Orono, in the Sophomore Class, and was graduated in 1878 with the degree of Civil Engineer. Fifty years after his graduation, the University gave him the additional degree of Doctor of Engineering.

In February, 1879, he entered the United States Lighthouse Engineer Corps, Sixth District, and continued therein until 1882, being stationed at various points on the South Carolina, Georgia, and Florida Coasts.

In January, 1882, Mr. Elwell entered the employ of the New York and New England Railroad Company as Assistant Engineer of Maintenance of Way, transferring a few months later (when the Maintenance-of-Way Department was abolished) to a similar position in the merged Engineering Department. Here, he had charge of building the docks and freight yards for the Hudson River Terminal at Fishkill-on-Hudson (later Beacon), N. Y.

In August, 1884, he was called by his old Chief in the Maintenance-of-Way Department of the New York and New England Railroad Company, Major F. L. Hills (who also had been his military instructor at the University of Maine and his superior in the Lighthouse Engineer Department), to the position of Assistant Chief Engineer of the Wilmington and Northern Railroad, with an office at Wilmington, Del.

In 1891, Mr. Elwell became Engineer of Maintenance of Way for the Philadelphia Division of the Baltimore and Ohio Railroad Company; and a few months later was transferred to the Pittsburgh Division with headquarters at Pittsburgh, Pa., where he replaced a large amount of timber and pile trestles by earth fills.

Two years later, he was called to the New York, New Haven and Hartford Railroad Company, beginning in April, 1893, as Roadmaster for the New York Division during its rebuilding, which consisted of four-tracking, making great changes of line and grade, and installing first-class construction and equipment throughout. On this work, under his direction, many alleged "impossibilities" were accomplished in the way of shifting track for long distances transversely and vertically without interference with traffic even to the extent of slowing down the expresses. These accomplishments were observed by the management with such appreciation that Mr. Elwell was appointed a Delegate to the International Railway Congress, at London, England, in 1895, and given a nine weeks' trip through Europe at the expense of the Railroad Company.

Soon after his return from the Railway Congress and the trip through Europe, in November, 1895, the New York and New England Railroad having been acquired by the New York, New Haven and Hartford Railroad Company and re-organized as the New England Railroad, Mr. Elwell was made Superintendent of its Norwich and Worcester Division, together with the steamboat line from Ally's Point, Conn., to New York City; this was his first position in charge of operating traffic other than "work trains." When the New England Railroad was absorbed by the New York, New Haven and Hartford Railroad Company in 1898, he continued to serve as Superintendent of these lines for the consolidated road. In 1901, he was transferred to the combined Air Line and Northampton Divisions, and, in 1904, to the Shore Line Division (which at that time combined what formerly had been three divisions).

In 1908, Mr. Elwell was made Chief Engineer of the Connecticut Company and several smaller separate lines, comprising all the trolley roads of the New York, New Haven and Hartford Railroad Company in Connecticut and New York State. During his administration there was considerable strengthening of the bridges on this 1 000-mile network of railways, and otherwise improving them.

When the Connecticut Public Commission was organized in 1911 to take over the authority and duties of the former Railroad Commission, together with similar and enlarged authority over all public utility companies of the State, Mr. Elwell was appointed its Chief Engineer, and served in that capacity until 1915, when he was appointed a Commissioner to fill the vacancy caused by the retirement of one of the original members. By two re-appointments he retained this position until his death, on May 21, 1931.

Mr. Elwell was a man of broad vision and a large capacity to create according to that vision; and he accomplished successfully, with economy, many tasks which observers had confidently declared to be impossible. He had a keen sense of justice, coupled with a kind and sympathetic nature, that enabled him in many times of stress and ill-feeling to bring order and harmony out of turmoil and hatred. Not only was he an able broad-minded engineer of large and varied experience, but he was also a man of such genial personality and human sympathy that he endeared himself to all with whom he came in contact whether a superior officer, a subordinate, or one with whom the association was only social or even casual; he inspired confidence in his judgment and ability and also in his fair-mindedness.

Among the publications prepared by Mr. Elwell are papers on: "Lighthouse Construction on the Southern Coast[2]"; "The Railroad Grade Crossing Problem"; "Development of Bus Transportation"; "Street Railway and Motor Bus Regulation"; "How Public Utilities Should Treat the Public"; and "Derailments."[3]

Mr. Elwell was married on March 25, 1885, to Nancy I. Bolton, who, with their sons, Francis Bolton Elwell and Charles Clement Elwell, Jr., survives him.

Although not very active in politics, and valuing a candidate according to his merit rather than according to his party, he was a Republican; in religion, he was a Congregationalist. In his club affiliations, he was a member of the Graduates Club of New Haven, Conn., and of the Hartford Club and the University Club of Hartford. He was an Honorary Member and Past-President of the Connecticut Society of Civil Engineers, and had been a member and President of the Association of New England Railroad Superintendents.

Mr. Elwell was elected a Member of the American Society of Civil Engineers on July 1, 1891, and served as a Director from 1919 to 1921, inclusive. As Director, he was active in organizing, and took great interest in, the Local Sections of the Society.

FRANK COLLINS EMERSON, M. Am. Soc. C. E.[1]

DIED FEBRUARY 18, 1931

Frank Collins Emerson, the son of Byron D. and Anne (Pendell) Emerson, was born on May 26, 1882, at Saginaw, Mich. He attended the public schools of his native town, graduating from the Saginaw High School in 1900. In the fall of that year he entered the University of Michigan, Ann

---

[2] *Proceedings*, Connecticut Soc. of Civ. Engrs., 1927, p. 22.

[3] *Loc cit.*, 1914, p. 80.

[1] Memoir prepared by H. D. Comstock, J. A. Elliott, and Z. E. Sevison, Members, Am. Soc. C. E.

Arbor, Mich., from which he was graduated in 1904, with a degree of Bachelor of Science in Civil Engineering.

Mr. Emerson's first professional employment was at his home town of Saginaw where he was Assistant City Engineer from June to September, 1904, when he felt the call of the West and moved to Wyoming which was thereafter his home. He first located in what is now Sublette County, near the Post Office of Cora. For the next few months he was engaged in private practice in that locality, largely on small irrigation projects, and it was during that time that he gave evidence of a characteristic which was to stamp his subsequent career, namely, that of making himself generally useful to his community.

From May to October, 1905, Mr. Emerson was employed by the La Prele Ditch and Reservoir Company, near Douglas, Wyo., in charge of canal location, and from October, 1905, to March, 1906, he served the Saratoga and Encampment Valley Railroad Company as Assistant Engineer in charge of the location of 45 miles of railroad through a mountainous country between Walcott and Encampment, Wyo. From March to June, 1906, he was employed by the State Engineer of Wyoming to locate two large canals on the Shoshone Indian Reservation. Fifteen years later the Wyoming Canal was built by the United States Bureau of Reclamation on substantially the location made by Mr. Emerson. From June, 1906, to April, 1907, he was Chief Engineer of the La Prele Ditch and Reservoir Company, in charge of canal construction.

In April, 1907, he was appointed Chief Engineer of the Wyoming Land and Irrigation Company in full charge of the location, design, construction, and operation of that Company's irrigation projects on Paint Rock and Shell Creeks in the Big Horn Basin in Wyoming. In 1909, the Shell Canal providing water for 11 300 acres was completed, and to provide against a possible shortage, the Adelaide Lake Reservoir was built in 1912 and 1913. He continued in charge of these projects until December, 1914. On account of financial difficulties the projects were taken over by the Court and from December, 1913, to March, 1914, Mr. Emerson acted in the capacity of Special Master Commissioner of the Fifth Judicial District of Wyoming, and, later, from March to December, 1914, he was Chief Engineer of the Wyoming Irrigation Company, successor to the Wyoming Land and Irrigation Company.

In December, 1914, he was appointed Superintendent of the Big Horn Canal Association at Worland, Wyo., and supervised the reconstruction of the Big Horn Canal, replacing temporary structures with permanent ones of concrete and steel. Later, he was also appointed Superintendent of the Lower Hanover Canal Association with similar duties. He continued in these positions until June, 1919. During much of this period Mr. Emerson was also Engineer for the Wyoming Sugar Company and the Worland, Worland-Bench, and Colter Drainage Districts. These positions involved the supervision of a large amount of construction work near Worland. At the same time, he was engaged in general engineering practice, mainly but not exclusively in irrigation and drainage.

In June, 1919, Mr. Emerson was appointed State Engineer of Wyoming and took office on July 1, 1919. Thereafter, until his untimely death, he played the leading part in the development of the water resources of Wyoming and, indeed, in all matters pertaining to irrigation and drainage. His work took him through all parts of the State, and he formed a wide acquaintance, especially in those communities where irrigation is extensively practiced.

In March, 1923, as the outcome of a controversy over his re-appointment as State Engineer, Governor William B. Ross summarily removed him from office. He contested his removal in the Courts, and the State Supreme Court held that the attempted removal was illegal, and that Mr. Emerson was in fact still State Engineer.

Irrigation and drainage bonds had lost favor with the investing public, and Mr. Emerson was largely influential in securing the passage by the Legislature of 1923 of the Act permitting the investment of the permanent funds of Wyoming in bonds of irrigation and drainage districts within the State. Under the terms of this Act a large amount of irrigation and drainage development was carried out, which otherwise could have been financed only on unfavorable terms, if at all.

In 1921, Mr. Emerson was appointed Wyoming Commissioner on the Joint Colorado River Commission, of which Herbert Hoover, Hon. M. Am. Soc. C. E., representing the Federal Government, was Chairman. After protracted negotiations covering the seemingly irreconcilable interests of the various States, the now famous Colorado River Compact was finally signed by all the Commissioners on November 24, 1922. This compact was then ratified by the Wyoming Legislature in January, 1923, the ratification being contingent on similar ratification by the Legislatures of the other six States in the Colorado River Basin, and by Congress. Two years later, in February, 1925, it became advisable to provide for ratification without waiting for Arizona, but a bill for that purpose met with indifference on the part of the Legislature and the State Administration, and it was only through the persistent and untiring efforts of Mr. Emerson that it was enacted, making possible the present development at Boulder Canyon on the Colorado River. A subsequent attempt by the 1929 Legislature to repeal this ratification was successfully resisted by him as Governor.

His abundant energy and concern for the development and welfare of Wyoming led to widespread demand that he serve the State as Governor and, after mature deliberation, in the spring of 1926 he "threw his hat in the ring." He won the Republican nomination by a good majority, and after a spirited contest in which unusual factors were involved, he was elected Governor by a plurality of about 1 200 votes over the Democratic candidate for re-election, Governor Nellie T. Ross, the widow of former Governor William B. Ross. With his record of an able active administration, Mr. Emerson was renominated for Governor on the Republican ticket in 1930, and after a hotly contested campaign he was re-elected by a plurality of about 1 000 over Mr. Leslie A. Miller, the Democratic candidate.

His work as Governor was a continuation and extension of his work as State Engineer. He considered himself not only the political head, but also the active manager of the Wyoming Administration and, as such, he took an active part in all departments of the State Government. He was intensely interested in all enterprises which seemed to effect the development, prosperity, and welfare of his State. This policy led him to be an ardent champion of States' rights as against Federal control in such matters as river regulation and the administration of the public lands. He worked constantly to safeguard for use in Wyoming the waters that arise in that State. He strongly opposed the Federal policy of limiting oil development on the public lands. Notwithstanding this attitude, no Governor enjoyed to a greater extent the respect and confidence of the Federal officials with whom he came in contact. He endeavored to promote the legitimate industries of the whole State and to that end he was a forceful and convincing speaker and was in great demand as such.

Governor Emerson's health was impaired by hard work during the campaign of 1930, and by equally intensive application to the duties of his office both before and during the session of the Legislature. He contracted a cold about February 10, but continued at the Executive Office until February 13, when pneumonia set in. His condition immediately became serious, and the end came on February 18, 1931.

Governor Emerson was a member and regular attendant of the First Baptist Church, of Cheyenne. He was a member of the Lions Club and of the Benevolent and Protective Order of Elks, a Thirty-second Degree Mason, and a Knight Templar. In 1918, he was one of the Founders and the first President of the Wyoming Engineering Society.

He was always active in community matters and took a leading part in many enterprises during his residence in Worland. Neither his character nor his integrity was ever questioned. Countless friends mourn his loss. He was essentially a builder, and belonged to that long line of pioneers who through unfaltering determination have developed, and are still developing, the resources of this Nation. His character has been well described in the following editorial from the *Wyoming State Tribune:*

> "The golden sunlight of a benign February day caresses Wyoming, but its brilliancy cannot dispel a pall of gloom that oppresses the State, nor its warmth cannot alleviate the benumbing chill of grief in hearts that mourn a beloved chief executive stricken by the hand of death in the full power of his high ability.
>
> "It is difficult for the people of the State he served so ably and with such self-immolating devotion to comprehend that virile, dynamic Frank C. Emerson is dead—that the brilliant mind that reasoned his shrewd and effective leadership is extinguished, that his body of seemingly inexhaustible energy is inert in death—we cannot think of him as insensate, immobile, he was so the embodiment of intellectual and physical vigor.
>
> "He died in time of peace, this devoted champion of the people's rights, but no embattled warrior that ever went down fighting in martial strife more truly laid down his life in patriotic service to the public weal than did Wyoming's heroic executive. Literally he sacrificed his life in his determination to give to his office the fullest measure of his abilities. He held service above self, he exhausted himself in service—he died for his ideal of service in the fullest need and so dying placed his State forever in memory under obligation to him.

"'Carry on' was the guiding rule of his devotion to his office; 'carry on' regardless of difficulties, regardless of obstacles, regardless of effects to self—'carry on' for Wyoming. He required the personnel of his administrative organization to adhere to that ideal, he adhered to it himself unswervingly—he 'carried on' to the very threshold of death and there gallantly faced the cost of his sacrifice, willing to pay it rather than to falter in devotion to official duty.

"What a glowing, an ennobling heritage that spirit has given his sons, his fellow-citizens, his successor in the governorship. Ever when we think of him we will have it in mind, visualizing his features as radiant with the light of a holy zeal to lead with the example and inspiration of service given by self as well as commanded of others. There could be no finer legacy.

"'Carry on'! That command, now a message from the dead, reverberates amid the gloom of grief that enshrouds a State to-day. The chosen leader is gone, but his precept is immortal—sorrowing, we 'carry on'."

On January 17, 1910, he was married to Zennia Jean Reynders who survives him, with three sons, Frank, David, and Eugene.

Mr. Emerson was elected a Member of the American Society of Civil Engineers on May 13, 1918. He was also a Member of the Irrigation Division of the Society, and, in 1926, he served on its Executive Committee.

LLOYD TILGHMAN EMORY, M. Am. Soc. C. E.[1]

Died January 5, 1931

Lloyd Tilghman Emory, the son of Edward B. and Henrietta (Tilghman) Emory, was born at "Poplar Grove", near Centerville, Md., on August 8, 1882. He was descended from a family which has been prominent on the Eastern Shore of Maryland since Colonial times.

It was necessary for Mr. Emory to start work when he was sixteen years old, and his first position was with the Washington Gas Light Company. It had always been his ambition to become an Engineer and in June, 1899, he was engaged as Instrument Bearer, Rodman, and Recorder on triangulation and shore-line work with the United States Coast and Geodetic Survey.

His next work was in the West as Assistant Draftsman on locating parties, running preliminary lines and locations in Oklahoma, for the Choctaw, Oklahoma, and Gulf Railroad Company. From November, 1902, to March, 1903, he was Topographer with locating parties of the Little Rock and Southern Railroad Company and, from March to September, 1903, he served as Topographer on location and Instrumentman on the construction of the Midland Valley Railroad Lines south of Fort Smith, Ark.

Throughout these years, Mr. Emory studied and saved and in October, 1903, he was able to enter the Senior Class at the Jacob Tome Institute, Port Deposit, Md. He was graduated in June, 1904, receiving the Lehigh University Scholarship, He entered the University of Pennsylvania in the fall of 1904, from which he was graduated in 1908, with the degree of Bachelor of Science in Engineering. In 1911, on submission of a post-graduate thesis, he received the degree of Civil Engineer from the University of Pennsylvania.

---

[1] Memoir prepared from information supplied by Mrs. Emory and on file at the Headquarters of the Society.

His first position after leaving college was that of Inspector and Chief of Party on the Hackensack Approach to the Pennsylvania Railroad Tunnels. In May, 1909, he was put in charge of construction of the McIntyre, Ga., Plant of the Kaolin Mining Company and, on its completion, he was made General Superintendent in charge of operation.

From 1911 to December, 1915, Mr. Emory was a member of the firm of Emory and Eisenbrey, Civil, Chemical, and Industrial Engineers, with offices in Philadelphia, Pa. His work consisted largely of making reports on various engineering projects, and plans and specifications for industrial plants and topographic surveys. It was during this period that he had an interesting trip to Sardis, in Asia Minor, with the Butler Archæological Expedition.

In December, 1915, Mr. Emory was engaged to go to South America on a short inspection trip which proved longer than he had anticipated. During 1916, he was Engineer in charge of field work in Dutch Guiana, South America, for the Republic Mining and Manufacturing Company, a subsidiary of the Aluminum Company of America. His work consisted of exploration of practically unknown territory in a tropical country, topographic mapping, test pitting, diamond drilling of bauxite deposits, and the preparation of estimates of ore bodies. In December, 1916, he was made Manager of the Demerara Bauxite Company, Ltd., of British Guiana, with headquarters in Georgetown, the capital of the Colony. He held this position until he joined the United States Army in 1918.

While in British Guiana, Mr. Emory was selected by the Governor of the Colony as one of three arbitrators to settle a dispute between the Government and the Case Coast Defense Company, a contracting firm then at work on the $2 500 000 sea wall. After a hearing, lasting eleven days, a compromise was effected.

After receiving his discharge from the Army at the close of the World War, Mr. Emory re-opened his office in Philadelphia as a Consulting Engineer, making reports and investigations for various clients, mostly in South America and Europe.

He was a man of great reserve, but those who knew him loved him, and men working with him admired and respected him for his depth of understanding and the sincerity with which he took hold of the job on hand. Whatever he achieved in life, he gained by his own determination to succeed. Engineering was his great hobby as well as his life work, although he had a wealth of information on many subjects and a great interest in outdoor recreations.

He had been engaged in work in Europe for some months previous to his death which came rather suddenly in Barcelona, Spain, on January 5, 1931, following a severe attack of influenza.

He was married on June 2, 1924, to Alice Martenis, of Wilmington, Del., who, with one child, Lloyd Tilghman Emory, Jr., survives him.

Mr. Emory was elected a Junior of the American Society of Civil Engineers on October 5, 1909, an Associate Member on April 2, 1912, and a Member on January 20, 1920.

HOWARD RICHARDS FARNSWORTH, M. Am. Soc. C. E.[1]

Died February 6, 1931

Howard Richards Farnsworth was born at Atchison, Kans., on December 21, 1886, the son of D. E. and Emma (Richards) Farnsworth.

Mr. Farnsworth attended the public schools of his native town and following graduation from Atchison City High School, he entered the University of Kansas, Lawrence, Kans. He completed the prescribed four-year course in the Civil Engineering School, attending as a resident student during the years 1904-1909, inclusive, with the exception of the fall and winter of 1907 and the spring of 1908, but it was not until 1921 that he submitted his thesis and was graduated from the University of Kansas, receiving the degree of Bachelor of Science in Civil Engineering on December 19 of that year. While a student at the University he was very prominent in musical activities, and was a member of Alpha Tau Omega, a social fraternity.

From June to August, 1906, Mr. Farnsworth was employed by the Kansas City-Southern and Louisiana Central Railroad Companies as Chainman in Louisiana. From August, 1907, to September, 1908, he was with a field party of the United States Examiner of Surveys in South Dakota and Nebraska, first as a Chainman and, later, as Principal Assistant.

From February, 1909, to April, 1924, Mr. Farnsworth was with the Cadastral Engineering Service of the United States General Land Office. During this period he was employed in the following capacities: From February, 1909, to April, 1917, as United States Surveyor, during which time the months from May to December of each year were spent as Chief of Party in the field in the execution of cadastral retracement resurveys in Nebraska, including stellar observations, investigations, and reports, and from January to April each year in the office of District No. 3 at Neligh, Nebr., computing proportions, balancing closures, checking astronomical data, and preparing records of field work with details and final plats; from May, 1917, to June, 1921, Reviewing Engineer in the Administrative Office of the Commissioner of the General Land Office, Washington, D. C., reviewing plans, specifications, and returns of cadastral engineering projects; from July, 1921, to April, 1924, as Cadastral Engineer in the Commissioner's Office and, during this period from February, 1922, to April, 1924, a member of the Manual Board in the preparation of the "Manual of Instructions for the Survey of the Public Lands of the United States."

---

[1] Memoir prepared by John M. Alden, Esq., Muskogee, Okla.

In April, 1924, Mr. Farnsworth decided to change his sphere of activity and enter the oil industry. In May, 1924, he was transferred to the Petroleum Division of the United States Bureau of Mines as an Assistant Petroleum Engineer and stationed at Taft, Calif., in connection with the supervision of oil and gas operations on the public domain and Naval petroleum reserves. In July, 1925, this activity was transferred from the Bureau of Mines to the United States Geological Survey. Mr. Farnsworth remained with the United States Geological Survey until his death in 1931.

In his new line of endeavor Mr. Farnsworth progressed rapidly. He interested himself in the geological and paleontological investigations in oil-field development in California and became well versed in these matters. His petroleum engineering activities also included sub-surface studies of the Elk Hills (Naval petroleum reserve), Kettleman Hills, and Maricopa Flats Oil Fields, on which he prepared extensive reports.

In January, 1931, he was transferred from Taft, Calif., to Tulsa, Okla., to continue his petroleum engineering work under the United States Geological Survey, and was made a District Engineer.

Mr. Farnsworth contributed materially to the various engineering activities in which he was engaged. During his association with the United States General Land Office, he published a paper[2] entitled, "A Review of Important Developments in the Science of Cadastral Resurveys as Executed by the United States Government, with Ethical Discussion Thereof."

Another paper[3] entitled "Elk Hills, California," of which Mr. Farnsworth was co-author with Messrs. W. P. Woodring and P. F. Roundy, is now (1932) in the course of publication. This paper deals with geological conditions and the accumulation and production of oil and gas in the Elk Hills Naval Petroleum Reserve, in California. Mr. Farnsworth was also the author of a "Report on the Sedimentation of the Sunset Oilfield and the Extension of the Thirty-Five Anticline, in California."[4]

He was a member of Trowell Lodge No. 71, A. F. and A. M., of Neligh, Nebr., Mt. Moriah Lodge, of Perfection, Nebr., Semper Fidelis Chapter, Knight Rose Croix, St. Andrew's Preceptory, Nebraska Consistory, and Tangier Temple, A. A. O. N. M. S., all of Omaha, Nebr.

The engaging personality of the man made lasting friends of his associates. His advancement in the several phase of the Engineering Profession was due to excellent ability and training. He not only took part in the practical phases of his calling, but also contributed to the literature of engineering and geology. Persistent and capable, likable and friendly, strong and efficient, Mr. Farnsworth left an impression that will not soon be forgotten by all who knew him. He is survived by his widow, Elizabeth Chestnut Farnsworth.

Mr. Farnsworth was elected an Associate Member of the American Society of Civil Engineers on April 17, 1917, and a Member on January 14, 1924.

---

[2] *Transactions*, Am. Soc. C. E., Vol. LXXXV (1922), p. 518.

[3] *Bulletin 835*, U. S. Geological Survey.

[4] *Oil Bulletin*, November, 1927.

FRANK FOSTER, M. Am. Soc. C. E.[1]

DIED APRIL 21, 1931

Frank Foster was born in Manchester, England, on May 1, 1865, the fourth son of T. Barham Foster, M. Inst. C. E. He was educated at Haileybury College, England, and in January, 1884, was articled for the usual period of three years to E. B. Thornhill, Assistant to Francis Stephenson, Chief Engineer of the London and North Western Railway.

The first half of this time was spent on the design of bridges and structures, principally for the Soho to Handsworth and Perry Bar Junction Railway, in the Birmingham District, and the other half upon the construction of that branch railway, which, although of no great length, included a tunnel through the most varying subsoils, a viaduct, large bridges, and heavy excavation and embankment.

On the termination of the three years, Mr. Foster joined his father's firm which then became Foster, Son, and Bardsley, and during the next four years was busily engaged with the construction of water service and drainage works at Motteram and Hollingworth and several other townships in the Midlands, in England, and attending as witness in Parliamentary and law cases referring to concessions for railways, water-works, floods, etc.

Wishing to increase his experience farther afield, Mr. Foster obtained the appointment as Junior Construction Engineer on the Buenos Aires Great Southern Railway, arriving in the Argentine Republic on November 16, 1890.

His first work was on the construction of a line from Las Flores to Tandil, and, on its completion, he took part in running a survey for a new line (to be constructed many years later), from Olavarria through Pringles and the Sierra Ventana Hills to Bahia Blanca, and on another line from General Alvear to Carhue.

As all construction work was being stopped owing to the severe financial straits of the country, known as the Baring crisis, Mr. Foster transferred to the maintenance side of railway engineering, becoming Assistant Engineer on the Buenos Aires Western Railway, in 1892, with which Company he remained during his working life. He was first stationed on the La Plata Lines, with headquarters at Tolosa, but in six months he was promoted to the Home Division radiating from the main terminal station in the City of Buenos Aires. He remained there for six years, during the last two of which he also acted as Engineer in Charge of the entire System. During the latter part of this period, the financial crisis began to pass off, and many minor but important works came under his care, apart from those of ordinary routine.

In September, 1898, Mr. Foster was definitely appointed Chief Engineer, and ten years later, in addition to this post, he was named Assistant Manager, passing to the General Managership in October, 1915. As General Manager, he remained until his retirement in 1928.

---

[1] Memoir prepared by A. W. Dannatyne, Esq., of Buenos Aires, Argentine Republic, and London, England.

During his tenure of office as Engineer-in-Chief, more than two-thirds of the 1 926 miles of line which now form the Buenos Aires Western Railway, were laid, and the present terminal station in Buenos Aires was built. This latter work involved the lowering and quadruplication of the main line to eliminate all grade crossings for a distance of 2¼ miles from the terminus, at a cost of $1 250 000. At the same time, new locomotive and carriage and wagon shops were built at a cost of $2 500 000, and sub-terminals with extensive marshaling yards and locomotive depots were laid down where necessary throughout the System.

In 1910, the preliminary works were begun for the construction of a tunnel 3½ miles long, under the city, to convey freight to the dockside for shipment, and, conjointly, the studies were made for the electrification of 23 miles of the main line, for suburban passenger traffic, electric traction being used likewise in the freight tunnel. A new underground passenger station was built in the city in conjunction with the Anglo-Argentine Tramway Company, for the quick transference of the passengers between train and tram. The electrification entailed heavy works and rebuilding in many of the suburban stations, the raising of the platforms in all stations to make them level with the floors of the cars, and the quadrupling of the tracks for 16 miles. The tunnel was completed on February 14, 1916, and was worked temporarily by steam locomotives, the World War delaying the inauguration of the electric service until April 30, 1923.

Mr. Foster devoted his energies during the war, to aid in every way the cause of the Allies and in spite of the difficulties due to the impossibility of obtaining materials, particularly coal, and the absence at the front of so many of the Staff, no interruption occurred in the transport of the countless head of cattle and tons of grain to the seaboard for shipment to the seat of war. For his patriotic services the distinction of the Order of the British Empire was conferred upon him.

Mr. Foster was elected a Member of the Institution of Civil Engineers (England), in December, 1904. He took a prominent part in the organization of the Association of British Engineers in the Argentine Republic in 1921, and was elected an Honorary Member for his services. He was also a member of the Institute of Transport, Argentine Center, and of the British Chamber of Commerce, taking an active interest in the British Engineering Standards Association.

He devoted his life to the interests of the Railway Company with which he was so long connected, and, at the same time, was always willing to give freely of his knowledge and experience to any who came to him for counsel or advice, not only as a skilled engineer, but also as a man of the world. He was greatly esteemed by all his associates and friends to whom his sudden death, so soon after his retirement, came as a severe blow.

In 1895, Mr. Foster was married to Caroline A. M. Adams, daughter of William Adams, M. Inst. C. E. (England), at that time Chief Engineer of the Buenos Aires Western Railway. He is survived by Mrs. Foster, two daughters, and a son.

Mr. Foster was elected a Member of the American Society of Civil Engineers, on June 30, 1911.

HERMANN FOUGNER, M. Am. Soc. C. E.[1]

DIED MARCH 9, 1932

Hermann Fougner was born at Hustad, Norway, on March 23, 1877, the son of Colonel Johan Herman Severin Fougner and Frederikke Margaretha (Jenssen) Fougner. He came from a prominent family of land owners, which for many generations has contributed to the Government services of Norway in high positions in the Army and Navy, as well as the Civil Service, and the Government itself. His uncle, Francis Hagerup, was twice Prime Minister of Norway under King Oscar II, and his father had received from King Christian IX, of Denmark, the rank of Commander of the First Class of the Order of Dannebroge. King Oscar II had himself honored Colonel Fougner on two occasions: First as Knight of St. Olaf, First Class; and, second, as Knight of the Order of the Sword, First Class.

As a boy, Hermann Fougner was permitted to accompany his father during school vacations on long surveying journeys through the mountains of Central Norway, where Colonel Fougner had charge of the Government surveys before taking command of the cavalry forces in Northern Norway. It was with this background that Mr. Fougner inherited his interest for civil engineering and love of outdoor life, which he also maintained in his adopted country.

After graduating from High School, Mr. Fougner studied engineering for four years at the Technical College, of Trondhjem, Norway, and was graduated, after taking the supplementary course of practical surveying, with the degree of Civil Engineer in 1896.

In the spring of 1897 he arrived in United States, and began his engineering career as Junior Engineer and Draftsman with the Pencoyd Iron Works, of Philadelphia, Pa. During the following three years, he made steady progress and gained a thorough experience in the design and erection of all kinds of structural work, as well as steel freight-car design, with the Koken Iron Works, of St. Louis, Mo., and the American Steel Foundry Company, of Granite City, Ill.

During the summer of 1900 Mr. Fougner visited Europe and, on his return to New York, became connected with Milliken Brothers, at that time, a leading concern of steel constructors in many parts of the world. The following year this Corporation placed him in charge of a large contract with the Russian Government for harbor works at Port Arthur, in Manchuria. This work comprised a naval basin, large cranes, storehouses, and magazines, for the Imperial Russian Navy. This construction was completed just before the Russo-Japanese War began. When the Japanese bombarded Port Arthur they destroyed almost all this development.

After more than a year in Port Arthur, Mr. Fougner proceeded to South Africa, as Chief Engineer for the Johannesburg Office of Milliken Brothers, traveling there *via* Shanghai, China, Singapore, Straits Settlements, and Colombo, Ceylon. For two and one-half years he had charge of the design and construction of some of the largest steel buildings in Johannesburg, including several leading banks, and office buildings for the gold and diamond mining companies. This marked the introduction of structural steel for buildings in South Africa.

---

[1] Memoir prepared by Charles T. Rundlet, Esq., Larchmont, N. Y., and Nicolay K. Fougner, M. Am. Soc. C. E.

On his return to New York in 1905, Mr. Fougner was engaged by the Trussed Concrete Steel Company (now the Truscon Steel Company) of Youngstown, Ohio, and assumed charge of its New York Office as Agent and Engineer. For more than twelve years he managed this office, and at the same time maintained a private consulting practice. Reinforced concrete was then still in its infancy, and few of the older generation of engineers and architects had any knowledge of the design or use of this material. Mr. Fougner saw the great possibilities of reinforced concrete and studied its development intensively. As a result he was recognized as one of the leading concrete engineers in the United States, and developed a large and profitable business for the products manufactured by the Corporation that he represented. During the years, 1909 to 1911, Mr. Fougner lectured on reinforced concrete at Pratt Institute, in Brooklyn, N. Y.

Many of the best known reinforced concrete structures of this pioneer period were designed in Mr. Fougner's office. As examples, the following may be mentioned: The reinforced concrete viaduct in Richmond, Va., which was built on a curve, and was the longest and most outstanding concrete viaduct in the world at that time; the Marlborough-Blenheim Hotel, in Atlantic City, N. J., with the largest concrete dome built to that date; the Traymore Hotel in the same city; and hundreds of other large buildings, bridges, reservoirs, and other structures, including the engineering design of the buildings at the United States Military Academy, at West Point, N. Y.

Early in 1918, during the World War, Mr. Fougner engaged in the construction of concrete ships, in partnership with his younger brother who, the previous year, had invented and successfully constructed the first sea-going concrete ship at Moss, Norway.

The Fougner Concrete Shipbuilding Company received a contract from the United States Shipping Board for several sea-going vessels, but was first required to construct a shipyard entirely at its own expense. Therefore, only the first of the Government ships was completed, when the war ended a few months later, and all Government contracts were cancelled. The enormous over-production of tonnage automatically stopped all further shipbuilding of steel, as well as of concrete.

After the war Mr. Fougner returned to his consulting practice, and of his work at this time a notable example is the Mergenthaler Linotype Company's large plant in Brooklyn, N. Y., and many other industrial buildings. For some years, from 1922 to 1927, he also took an active interest in contracting, as Vice-President of Thompson and Binger and, later, with the Industrial Engineering Company, both of New York City. He then formed a partnership, Fougner and Gautier, with Raoul C. Gautier, M. Am. Soc. C. E., a

French engineer and architect, and was engaged especially in the design of industrial buildings, swimming pools, breakwaters, and harbor improvements.

During his last years Mr. Fougner made a study of the engineering aspect of handling freight motor trucks in connection with terminals and warehouses, and was granted patents on improved methods of floor designs and layouts.

Hermann Fougner became an American citizen long before the war and joined heartily in the civic activities of Larchmont, N. Y., where he had resided for the last twenty years of his life. He was a member of the Ambulance Corps of the Larchmont Fire Department, and was elected a Trustee of the Village of Larchmont in April, 1931, which position he held until the time of his death.

He was a member of the Appawamis Club, the Larchmont Yacht Club, and the Engineers Club of New York, besides belonging to several fishing clubs and organizations for the preservation of game and fish.

Mr. Fougner's chief recreations were golf and fishing, but he was also fond of swimming and winter sports, in which he was naturally proficient from his early training in Norway.

He was a Licensed Professional Engineer in the State of New York, and was a member of the American Society for Testing Materials and the American Concrete Institute. He was also a Thirty-second Degree, Scottish Rite, Free Mason.

When Hermann Fougner, after fighting several weeks for his life, succumbed to an illness that developed from an accident, it was a great loss to his many friends in the engineering world. He was in the prime of life and had never been ill. His vigorous personality and alertness of mind invariably won him respect. His lovable character and unlimited capacity for friendship, endeared him to all with whom he came in contact. As an engineer and gentleman of lofty ideals and high principles his memory will long be cherished by his host of friends in all walks of life.

On February 13, 1903, at Colombo, Ceylon, Mr. Fougner was married to Stella Gray, of San Diego, Calif., who was touring the Orient. His one child, Frederikke, died at birth. He is survived by his widow, his sisters, Mrs. Elisabeth Hagen and Mrs. Annichen Aschehoug, of Oslo, Norway, and two brothers, Waldemar G. Fougner, a Consulting Engineer of Oslo; and Nicolay K. Fougner, M. Am. Soc. C. E., of New York City.

Mr. Fougner was elected an Associate Member of the American Society of Civil Engineers on February 1, 1905, and a Member on April 17, 1923.

JAMES HILLHOUSE FUERTES, M. Am. Soc. C. E.[1]

DIED JANUARY 30, 1932

James Hillhouse Fuertes, the son of the late Estevan Antonio Fuertes, M. Am. Soc. C. E., long Dean of the College of Civil Engineering, Cornell

---

[1] Memoir prepared by Frank W. Skinner, M. Am. Soc. C. E.

University, and Mary Stone (Perry) Fuertes, was born at Ponce, Porto Rico, on August 10, 1863. He spent his boyhood in Ithaca, N. Y., where he prepared for college, and, in 1883, was graduated from Cornell University with the degree of Bachelor of Civil Engineering, at that time the highest engineering degree bestowed by the University for undergraduate work. From 1883 until 1926, when failing health limited his more strenuous activities, Mr. Fuertes' professional engagements, of increasing magnitude and importance, although most prominently featuring Sanitary and Hydraulic Engineering—investigations, design, and construction—included notable examples of other civil engineering activities. In 1895, Mr. Fuertes made an extended European tour, visiting and studying important sanitary works, meeting distinguished engineers, and collecting data which enabled him subsequently to solve numerous important technical problems in a notably broad and thorough manner.

From 1883 to 1891, Mr. Fuertes served, successively, as Assistant, and as Engineer, for the Department of Parks, New York, N. Y.; the Missouri, Kansas and Texas; the St. Louis, Fort Scott and Wichita; the Yates Center and Winfield; the Dodge City and Southern; and (builders of) the Camden and Alexandria Railroads. From 1891 to 1893, he was engaged in private practice at Camden, Ark., designing and constructing mill railroads, electric light plants, water-works, sewers, etc. Thereafter, for a short time at Ithaca, N. Y., and subsequently permanently at New York City, he continued in private practice, associated at intervals with other eminent engineers on important investigations and constructions. He made investigations, reports, and designs for more than forty cities, towns, and private owners on the collection, storage, purification, protection, extension, and distribution of water supplies, and supervised much of the construction. Among his clients for this work were the Cities of Brooklyn, N. Y.; Philadelphia, Pa.; Baltimore, Md.; Dallas, Tex.; Charleston, S. C.; Lynchburg, Va.; Harrisburg, Pa.; Cumberland, Md.; Camden, Ark.; Gloucester, Mass.; Princeton, and Plainfield, N. J.; Winnipeg, Man., Canada (including a large concrete aqueduct 100 miles long); Caracas, Venezuela; and many other towns, corporations, and individuals.

Mr. Fuertes also made investigations and designs for drainage, sewerage, and sewage disposal for more than forty cities, towns, institutions, and private owners, including the Cities of Chicago, Ill., Philadelphia, Pittsburgh, Pa., Newark, Elizabeth, Hackensack, Lakewood, Plainfield, and Hoboken, N. J., Honolulu, Hawaii, Ottawa, Ont., Canada, Caracas, Venezuela, and for Santos and Para, Brazil, where his very efficient sanitary work eliminated a terrible scourge of yellow fever.

He designed and supervised the construction of important viaducts in Fitchburg, Mass., Lynchburg, Va., and Harrisburg, Pa., all of which included important developments of construction, the latter applying for the first time the cantilever method of erection for long-span reinforced concrete arches, which, by ingenious utilization of the permanent reinforcement steel to resist temporary erection tensile stresses, eliminated falsework centers, and proved of great economic value.

Mr. Fuertes designed and constructed works that, since 1907, have completely protected the City of Harrisburg from floods that had previously damaged it. He was a member of the Board of the Miami Conservancy Commission that approved the flood-regulation works executed at a cost of many millions of dollars to protect the Miami Valley from inundations that had ravaged it; and he was retained to report on flood prevention in Buffalo, N. Y. In 1899, he reported to New York State on important canal improvements; and, in 1900, he designed difficult foundations in the Jersey Meadows for the Snead Iron Works, and improvements for the State Hospital, at Warren, Pa.

In 1902, he was retained to investigate the disastrous dynamite explosion in the Fourth Avenue Subway Tunnel, in New York City. In 1905, he reported on the Conoduiguinet Creek water power, and investigated the typhoid fever outbreak at Tome Institute. In 1906, he prepared plans and specifications for fire protection for, and lighting of, a large estate near Harrisburg, Pa. In 1909 Mr. Fuertes reported on a dam at Harrisburg; on the improvement of the Susquehanna River; on the sanitary works of the Manhattan State Asylum; and was a member of the Water-Works Arbitration Board for Parkersburg, W. Va. In 1910, he was a member of the New York City Metropolitan Sewage Committee. In 1916, he reported on incinerators to the Mayor of Atlantic City, N. J. In 1924, he superintended the reclamation of coal from the Susquehanna River for shipment to Denver, Colo., where it was advantageously used instead of sand in the 60 000 000 gal. per day water supply filter which he had constructed. Mr. Fuertes was the first to substitute anthracite coal for sand in filters working under very low-head losses; he used it for the Dallas, Tex., Water-Works plant, and in other filters, and it has been adopted by other engineers.

In 1924, he was a member of the Board of Review of the Sanitary District of Chicago that investigated the relation of the Great Lakes levels to Chicago problems.

At different times, Mr. Fuertes was associated with the late Rudolph Hering, M. Am. Soc. C. E, and the late Allen Hazen, M. Am. Soc. C. E., on sanitary works of great importance, and he was frequently retained in a consulting capacity on important engineering projects by cities, towns, corporations, and individuals, and often served as expert witness in litigation and as arbitrator of engineering disputes. His services were so satisfactory that he was retained again and again by many clients, who insisted on receiving his opinions and counsel after his failing health no longer permitted him to undertake strenuous responsibilities. During the last few years of his life, when he was obliged to relinquish arduous activities, he kept fully abreast of developments in his own technical field, and was frequently consulted on important sanitary and hydraulic matters.

Mr. Fuertes was the author of "Water and Public Health" (1897); "Water Filtration Works" (1907); "European Sanitary Engineering Series," published in 1897–1898 in *Engineering Record;* numerous monographs on sanitary and engineering subjects; many technical articles in *Engineering Record* and *Engineering News,* and contributions to the *Proceedings* and

Transactions of the Society. He was a member of the American Institute of Consulting Engineers; the Canadian Institute of Civil Engineers; the Pennsylvania Society of Civil Engineers; the American Waterworks Association; the American Civic Association; and the Engineers Club, of New York City.

He was of an investigating and studious disposition, with keen perception, sound judgment, broad vision, and unusual ability to analyze conditions and solve problems on economic considerations of construction, operation, and efficiency advantages. He was an able mathematician, computer, analyst, and designer; well posted in precedents and standard practice, and brilliant in his adaptation of them for the requirements of special conditions, and in supplementing them with new features. He had great and practical ingenuity, which he frequently applied to advantage in major improvements, in designing important details, and in overcoming unusual difficulties. He had unusual architectural and artistic abilities that enabled him to create beauty and dignity, combined with strength and efficiency, in structures where they are too often absent. Mr. Fuertes' original development of thin parabolic arches for sewer construction was arrived at well in advance of arch computations that were standard at the time his designs were prepared. He designed a new type of reinforced concrete hollow dam of inexpensive construction, which has since been patented by others, and which has successfully resisted heavy ice pressures and severe flood conditions for twenty years in the Susquehanna River, at Harrisburg.

His drawings, often made by himself with great skill and rapidity, were unusually complete and clearly detailed, and his specifications were models of fairness and thoroughness, leaving nothing in doubt. He was devoted to the interest of his clients; considerate of his employees; helpful to his associates; just and generous in his dealings with all; loyal to his profession and faithful to his public duties as a citizen.

In his death the Engineering Profession loses an eminent member, whose achievements marked substantial progress in design and construction of varied important works that reflected honor and credit to it; benefited many cities, communities, and individuals; were an inspiration to those associated with him; and will be an enduring memorial to him. Mr. Fuertes' experience, abilities, and sympathetic interest inspired confidence in all with whom he had dealings, and his gracious personality and warm friendships endeared him to his fellows.

Although suffering from increasing disabilities that for several years had prevented laborious activities, Mr. Fuertes was able to enjoy his home and his friends, reading, writing, and driving until a few hours before his death, passing away peacefully and painlessly. He is survived by his two sisters, Katherine Fuertes, of Aurora, N. Y., and Mrs. Sarah F. Hitchcock, of New York City, and by three step-children, Cornelius and Lucian Cable, and Mrs. Samuel Hart.

Mr. Fuertes was elected a Junior of the American Society of Civil Engineers on May 2, 1888, and a Member on February 6, 1895.

EDWARD GAGEL, M. Am. Soc. C. E.[1]

DIED FEBRUARY 11, 1931

Although much of the fixed physical property of The New York, New Haven and Hartford Railroad Company stands as a monument to the stewardship of Edward Gagel as its Chief Engineer, his personal modesty and dislike of anything which, as he saw it, savored of self-advertising were such that he could not be persuaded to contribute papers or even discussions to the publications of the several engineering societies to which he belonged, and the character of his work and his ability are fully known only to those who were closely associated with him.

A Civil Engineer of fifty-three years' active practice, forty-seven of which were with The New York, New Haven and Hartford Railroad Company, of which for twenty-four years he was Chief Engineer, Edward Gagel had retired from professional activities on October 1, 1929, and was just beginning to enjoy a well earned rest. During the greater part of his life his health had been excellent, but a heart affection which until his retirement had caused little concern, became progressively worse thereafter, and eventually resulted in his death.

Edward Gagel was born on October 25, 1858, in that portion of the present Borough of The Bronx, New York, N. Y., which then was the Village of Mount Hope. His parents, Christian and Anna M. (Aulinger) Gagel, had limited means, but gave their son a public school education, and he subsequently attended a course in Civil Engineering at Cooper Institute, in New York City.

At the age of 18, on June 16, 1876, Mr. Gagel began his professional career as Draftsman and Field Engineer for Dennis and Mairs, of New York City, a firm engaged in general surveying, and, for the next six years, he had a varied experience. Leaving this service in December, 1877, he was employed, successively, by the Brooklyn, Flatbush, and Coney Island Railroad Company until March, 1879; by the Metropolitan Elevated Railway Company of New York City until January, 1880; by the West Side and Yonkers Railway Company until March of the same year; on a location survey for the New York and New England Railroad Company (later a part of The New York, New Haven, and Hartford Railroad Company), looking to an extension to the Hudson River, until January, 1882; on location work for the Erie and Wyoming Valley Railroad Company for the next seven months; then for two months as Contractor's Engineer on the construction of the Pittsburgh, McKeesport, and Youghiogheny Railroad; and, finally, for three months, as Draftsman with The New York Central Railroad Company. This ended his wanderings; on December 6, 1882, he entered the service of The New York, New Haven and Hartford Railroad Company, and remained with it thereafter until his retirement from active service.

---

[1] Memoir prepared by Charles Rufus Harte and Clarence Blakeslee, Members, Am. Soc. C. E.

Progressing steadily, Mr. Gagel, successively, held the positions of Assistant Engineer; District Engineer in charge of the New York, the Naugatuck, and the Highland Divisions; Assistant Engineer in charge of the Western District; and Principal Assistant Engineer; until, on October 1, 1905, he was made Chief Engineer, which position he held until his retirement on October 1, 1929. On March 1, 1913, he was also appointed Chief Engineer of the Central New England Railway Company, holding this position until December, 1926, when by merger with the New Haven Company this subsidiary lost its separate existence; and on September 3, 1918, he also was appointed Chief Engineer of The New York Connecting Railroad Company, owned jointly by The New York, New Haven and Hartford and The Pennsylvania Railroad Companies, between which it forms a direct physical connection. This last position, Mr. Gagel held until his retirement from the Chief Engineership of the New Haven Company.

Mr. Gagel served the New Haven under eight of its Presidents, coming to it under Mr. George H. Watrous, who had inaugurated the policy of improvement and expansion so well carried out by his successors. In 1882, the property was only one of the twenty independent small roads of Southern New England, and had only 192 miles of line, a small portion of which was double-tracked; at the end of 1929 it comprised substantially all the transportation agencies—railroad, trolley, bus, and steamship—of its territory, the railroad proper having 2 225 miles of road, of which 114 miles were four-tracked, while 169 miles, much of which was four-tracked, and a little six-tracked, were electrified. Moreover, under the masterly personal direction of the then President, the late Edward J. Pearson, M. Am. Soc. C. E., there had been a complete re-organization of the facilities of the property, bringing it to high operating efficiency.

Edward Gagel had personal charge of part of the first four-tracking and improvement work on the New York Division, and, later, as Chief Engineer, directed the completion of this work and that of six-tracking the Harlem River Branch. Under his Chief Engineership the line from Stormville, N. Y., through Danbury to Shelton, Conn., and that from Waterbury to Bristol, Conn., were both double-tracked, the work including much line improvement, particularly at Terryville, where a tunnel 3 680 ft. long effected a notable grade reduction, these improvements giving a double-track freight line from Maybrook, N. Y., *via* New Haven, Conn., and Providence, R. I., to Boston, Mass., and, except for a stretch of 20 miles of single-track between Vernon and Willimantic, Conn., another double-track line between the same terminals *via* Hartford, Conn., and Willimantic. Two other very important improvements during this period were the widening of the east and west cuts through New Haven, eliminating a bad bottle-neck in the main line, and the construction of a direct line from Providence to East Providence, R. I., the mile-long tunnel by which this was carried under a residential section of Providence calling for particularly careful treatment.

The change to heavier motive power under the Pearson program necessitated a large number of clearance changes and bridge renewals. In addition to the many minor spans, there were a number of large openings, the treatment of which involved major engineering problems. Notable among these were the rebuilding, under and with practically no disturbance to its heavy traffic, of the great Hudson River Bridge, at Poughkeepsie, N. Y.; the construction of a new bridge over the Thames River, at New London, Conn.; of the Connecticut River Bridge at Saybrook, Conn.; the Housatonic River bridges at Stratford and at Sandy Hook, Conn.; and the bridges at Cos Cob, Niantic, and Westport, Conn., and over the Neponset River at Boston.

Major grade-crossing elimination projects were carried out at Attleboro, Clinton, Harrison, New Bedford, and Worcester, in Massachusetts, and at Pawtucket and Westerly, in Rhode Island; a large number of individual crossings were also eliminated, while many of those crossings remaining were equipped with automatic warnings, making the property in this respect one of the best protected of the American railroads.

Besides these improvements there were the construction or reconstruction of great freight yards at Cedar Hill, New Haven, at Hartford, at Harlem River, and at Northrop Avenue, Providence; car-float bridges at Oak Point, N. Y., in connection with the transfer of freight cars around New York City by water; repair shops at Readville, Mass., and Van Nest, N. Y.; and many other projects, comparatively small in each case, but making up a very large total.

In addition to the steam road work, Mr. Gagel had under his charge the engineering in connection with the secondary transportation facilities of The New York, New Haven and Hartford Railroad Company, involving the construction of nearly 100 miles of trolley lines of interurban type, and the electrification for trolley operation of about the same mileage of steam road tracks, together with the construction of the necessary power-houses, transmission lines, sub-stations, car-houses, and repair shops. The electrification of the New York Division for railroad operation did not come directly under the jurisdiction of the Chief Engineer, but many of the physical features necessarily came before him for approval, and his advice and assistance were availed of to a large extent.

On October 14, 1885, at her home in Dunmore, Pa., Mr. Gagel was married to Jennie Field Smith who, with two children, Anna Smith and David Edward, survives him.

Although, as stated, Mr. Gagel took little active part in the technical proceedings of his engineering societies, he always was greatly interested in their welfare and progress. He was a member of the American Railway Engineering Association, of the New York Railroad Club, and of The Connecticut Society of Civil Engineers, which latter Society elected him an Honorary Member on January 13, 1931. The formal presentation of the attesting scroll at the Annual Meeting had to be made posthumously, but, fortunately, Mr. Gagel had been advised of, and was able to appreciate, the honor before his death.

While Mr. Gagel was strongly opposed to publicity regarding his work, this reticence did not extend to informal discussion; he was ever ready to talk over his own problems or those of others with his friends or with any one who came to him for advice, and he was particularly pleased when he was able to help some young engineer. The energy and hard common sense which were so characteristic of his professional work, were made equally manifest elsewhere, and his services were constantly sought in many directions; but while always willing to assist behind the scenes, he consistently avoided any but the most necessary publicity, and in spite of urgent pressure refused to be drawn into political activities. He served as Chairman of the Building Committee of the First Congregational Church, of West Haven, Conn., of which Church he was also a Deacon and a Trustee; he was a member for twenty-five years of the Board of Education of West Haven; he had been President of its Park Board; a member of its Paving Commission and of its Village Improvement Society; and, for four years, Chairman of its Fire Board. For many years, too, he was a member of the New Haven Harbor Commission, of which for six years he was President; he had long been a Director of the West Haven Bank and Trust Company, and he was active in several other business affairs.

A member of Trumbull Lodge, A. F. and A. M., Mr. Gagel was a Knight Templar, a Thirty-second Degree Mason, and a Shriner. He belonged to the Engineers Club of New York, and to the Union League and to the Racebrook Country Clubs of New Haven. An independent in politics, in religion he was a Congregationalist. Of fine presence and genial disposition, he easily made and held many friends who enjoyed his whimsical philosophy no less than his sympathetic attitude, while his kindly disposition made him an anchor to windward for many an unfortunate. His loss will be felt deeply in many directions.

Mr. Gagel was elected a Member of the American Society of Civil Engineers on April 3, 1907.

BENJAMIN HURT HARDAWAY, M. Am. Soc. C. E.[1]

DIED DECEMBER 13, 1928

Benjamin Hurt Hardaway was born at Valumbrosa, near Union Springs, in Bullock County, Alabama, on February 19, 1866. He was the son of Colonel Robert Archelaus and Rebecca Elizabeth (Hurt) Hardaway, parents of unusual strength of character, who through precept and example instilled in their children the highest principles of integrity. Benjamin Hurt Hardaway was one of three sons, his two brothers being Robert Early and George Stanfield Hardaway.

Mr. Hardaway's grandfather, Major Robert Stanfield Hardaway, had been President of the Mobile and Girard Railroad Company (in Alabama), and his father, Superintendent and Chief Engineer of the same railroad, before the Civil War. Colonel Hardaway had served as a Captain in the

---

[1] Memoir prepared by Frederick C. Ray, Assoc. M. Am. Soc. C. E.

Mexican War, and, in the Civil War, he was a Colonel of Artillery. About 1869, or 1870, he and his family moved to Opelika, Ala.; later, to LaFayette, Ala.; and then to Auburn, Ala. During this period, he was Chief Engineer and Superintendent of the East Alabama and Cincinnati Railroad, and, later, was Professor of Engineering, and Commandant, at the Alabama Polytechnic Institute. Some years later, Colonel Hardaway moved to Tuscaloosa, Ala., where he was Professor of Engineering at the University of Alabama.

During this period, Benjamin Hurt Hardaway was receiving his education. He was a student through his Sophomore year at the Alabama Polytechnic Institute, at Auburn, and at the University of Alabama, in 1883, from which he received the degree of Bachelor of Science; in 1884, the degree of Bachelor of Engineering; and, later, the degrees of Master of Arts and Civil Engineer. From 1884, to 1886, he was Assistant Professor of Mathematics at the University of Alabama. While teaching, he found time to devote to engineering pursuits, acting as City Engineer for the City of Tuscaloosa. In that capacity he designed and constructed works on the streets paralleling the Warrior River, for protection against erosion. In that early period, he wrote an Engineers' Handbook.

In 1886 and 1887, Mr. Hardaway was Assistant Chief Engineer of the Savannah, Americus and Montgomery Railroad, on construction work west of Lumpkin, Ga., working under his brother, Robert Early Hardaway, who was Chief Engineer. In 1887, the latter resigned, to enter other work, and Benjamin Hurt Hardaway, then less than 21 years of age, was promoted to the position of Chief Engineer. He continued the location and construction of the line between Americus and Abbeville, Ga., toward Savannah, Ga. Sixty miles of railroad and various bridges were built under his direction during this period.

In 1888, he left the service of the Savannah, Americus and Montgomery Railroad Company (afterward the Seaboard Air Line Railway Company) and became Division Engineer on the Chattanooga, Rome and Columbus Railroad, then under construction north of Rome, Ga. Later, he was Chief Engineer of Surveys for the East Alabama Railroad Company. Finishing this work in 1888, Mr. Hardaway went to Carrollton, Miss., where he was engaged as Resident Engineer, on construction, for the Southern Railroad Company, later, becoming Track Engineer. From Carrollton, he went to Gordon, Ala., as Resident Engineer for the Alabama Midland Railway Company (now the Atlantic Coast Line Railway Company) and, in this capacity, directed the construction of a bridge over the Chattahoochee River at Alaga, Ala., and fourteen miles of track on the west side of the river. This was the first of a number of bridges he built over the Chattahoochee. In 1890, he was promoted to the position of Assistant to the Chief Engineer, on this railroad.

From Gordon, Mr. Hardaway went to Ozark, Ala., and served as Assistant Chief Engineer on the extension of the line from Montgomery to Tuscaloosa, for the Montgomery, Tuscaloosa, and Memphis Railroad Company. In January, 1890, and until August of the same year, he was working near Maplesville, Ala., on this location.

About July, 1890, Mr. Hardaway returned to the service of the Savannah, Americus and Montgomery Railroad Company, as Assistant to the Chief Engineer and as Assistant Superintendent of the Georgia and Alabama Construction Company. He directed much field relocation from Louvale, Ga., to Montgomery, and was in special charge of the construction of the new drawbridge over the Chattahoochee River, at Jernigan, Ala. At this time, the severe financial panic of 1891 and 1892 was being felt and the Savannah, Americus and Montgomery Railroad Company went into the hands of a receiver; Mr. Hardaway, however, continued with the construction at Jernigan, and the bridge was completed in 1892. While with the Railroad Company, Mr. Hardaway and his brother, Mr. R. E. Hardaway, selected, bought, and laid out the Towns of Cordele and Helena, Ga.

The year 1891 marked a turning point in the career of Mr. Hardaway. Apparently, he had come to the conclusion that his efforts in the future would be along the lines of construction as a general contractor. He operated gravel pits at Americus, Ga., selling ballast to the Savannah, Americus, and Montgomery Railroad Company, and contracted for the construction of depots and section houses. He then contracted for the construction of all pile-driving and trestle work on the Savannah, Americus, and Montgomery Railroad from the Chattahoochee River to Fort Davis, Ala., and for a Y-trestle in the railroad yards in Montgomery.

Returning to Mississippi, in the spring of 1892, he entered into contract with the County of Carroll to erect numerous steel bridges throughout the County. This work was completed by fall and he then took contract for the construction of trestles between the Satilla River and the St. Mary's River, for the Florida-Central and Peninsular Railroad Company, near St. Mary's, Ga. During this year he also built a bridge over the St. Mary's River for this Company.

During 1892 and part of 1893, Mr. Hardaway was engaged in railroad construction work for the Atlantic Coast Line Railroad Company in South Carolina. In the latter part of 1893, he built the Belt Line Railroad through Petersburg, Va., the project including two treated wood-pile bridges with steel decking. He later constructed the Presbyterian Church in Rock Hill, S. C.

In 1894, he built eighteen miles of railroad between Seivern, and Batesburg, S. C., this line now being part of the Southern Railway. During the same year, he built the Louisville and Nashville Railroad Extension from Cusseta, to Prattville, Ala. While this work progressed, he contracted for a short railroad and bridge for the W. C. Alverson Saw-Mill, near Moselle, Miss. During this same year, he also built the pivot pier in the Warrior River at Tuscaloosa. This was for the highway bridge over the river and was built under a contract with Tuscaloosa County. At Montevallo, Ala., he removed an old wooden structure over the creek and replaced it with a steel bridge.

During 1895, Mr. Hardaway was interested in the construction of a dam and power house project for the Anderson Water, Light, and Power Company at Anderson, S. C., which was one of the first hydro-electric plants in the

South; a combination railroad and highway bridge over the Tallapoosa River, at Tallassee, Ala.; and various kinds of trestle work in Georgia. In 1896, his works were confined to railroad and bridge construction in the States of Georgia and Alabama. In 1897, continuing as an active contractor, and building with the South, Mr. Hardaway contracted for and built a dam and power house on the Yadkin River near Winston-Salem, N. C. This work was for the Winston-Salem Power Company; and, in 1898, he was interested in the construction of piers, head-gates, trestles, dams, power houses, etc. In 1899, Mr. Hardaway built a dam and power house at North Highlands, near Columbus, Ga., for the Columbus Power Company; a highway bridge at Fosters Ferry, on the Warrior River, between Tuscaloosa and Eutaw, Ala.; and a railroad from Searight, to Andalusia, Ala., for the Central of Georgia Railroad Company.

Beginning the Twentieth Century as a man 34 years of age, Mr. Hardaway had seen the completion of numerous projects with which he had been associated. In some of this work he was in partnership with Mr. B. O. Watkins, of Birmingham, Ala. His friends, clients, and associates recall with pleasure these earlier years in their dealings with him. "Honesty, Integrity, and Responsibility" was his motto from the very first contract. A job contracted for was a responsibility acquired, that had to be carried out, regardless of profit or loss.

In 1901, he executed contracts for the Central of Georgia Railroad Company, building a line from Columbia, to Coffee Springs, Ala. From 1901 to 1908, he was interested in and completed the construction of as many as thirty-two contracts throughout the States of Georgia, Florida, Alabama, Virginia, Mississippi, South Carolina, and Tennessee. In 1908, Mr. Hardaway signed a contract with the Southern Power Company for the construction of a dam and power house on the Broad River, at 99 Islands, near Blacksburg, S. C. In 1909, he entered into a contract with the Columbus Power Company for the construction of the dam and power house at Goat Rock on the Chattahoochee River above Columbus, Ga., and, in the same year, contracted with the United States Government for building Lock No. 16 on the Warrior River above Tuscaloosa. In the following year, he contracted with the Government for building Lock No. 17, just above Lock No. 16, and also entered into contract with the City of Columbus, for a bridge at Dillingham Street.

In 1911, Mr. Hardaway realized that his contracts were assuming large proportions for an individual, so, taking men who for years had been with him, he organized the Hardaway Contracting Company, a Georgia corporation. He was elected President of the Company and directed its activities until his death, on December 13, 1928. Mr. Hardaway was actively associated with the planning and carrying out of each and every one of the sixty-seven projects undertaken by his firm to that date and gave careful thought to all. Among these projects, and worthy of mention, were four hydro-electric developments for the Southern Power Company and subsidiaries in the Carolinas; three cantonments for the United States Government; two hydro-electric developments for the Georgia Power Company; two hydro-electric developments in North Carolina for the Aluminum Company of America; one hydro-electric development for the Carolina Power and Light Company, in North Carolina; besides seventeen concrete highway bridges for the States of North Carolina, South Carolina, Alabama, Georgia, Florida, and Tennessee. At the time of his death, his men were carrying out numerous works that had been planned by him.

With all his many activities, Mr. Hardaway never failed to discharge fully his civic and religious duties. He was a member of the Methodist Episcopal Church, serving as Steward for many years, and was ever a generous supporter not only of his church, but of all deserving causes brought to his attention. He held the respect, confidence, and affection of his fellows to a remarkable degree.

Concerning Mr. Hardaway a few days after his death, Mr. W. C. Woodall, a columnist of the Columbus *Enquirer-Sun,* wrote:

> "Mr. Hardaway was great as an engineer and builder. He was great as one of the outstanding figures in the development of his section of the country. * * * He was great in his high conceptions of business honor, and in the strict integrity which marked all big transactions. But he was greatest as a gentleman.
>
> "He was one of Nature's noblemen. There are some to whom the qualities of gentility are so inherently native that we accept them as such without wonder. Mr. Hardaway was instinctively and intuitively a gentleman. The memory of his courtesy, his modesty, his gentleness of manner and his perfect consideration of others will never fade from the hearts of his friends.
>
> * * * * * * * * * * *
>
> "He was distinguished in his field as an engineer, and builder; as a business man he was successful in unusual degree, and yet it is on his purely personal qualities that the memories of his friends linger most fondly. * * *."

Mr. Woodall has written ably and well of the feelings and thoughts of Mr. Hardaway's friends and associates toward him and little more can be said; however, it seems fitting to mention his attitude toward the men on his "jobs." On visits to his contracts and as he looked over the work, deep in thought, he was never too occupied, or too busy, to stop and speak with those with whom he had become personally acquainted; his hand was first to go out in welcome, whether to water boy, foreman, engineer, or superintendent. He liked to listen to the ideas and suggestions of those under him, no matter how radical or how impractical, and he would try to ferret out the reasonings behind the thought. His men were inspired by his thorough interest in every problem, large or small. He was the last to stop work; the first to begin. He would often say, "A contractor's profits depend not on how much he makes on any one contract, but on how well his clients are satisfied." This was the guiding advice to his organization.

On September 14, 1888, he was married to Rebecca Money, at Carrollton, Miss., who, with one son, Benjamin Hurt Hardaway, Jr., Assoc. M. Am. Soc. C. E., and a brother, Robert Early Hardaway, survives him.

Mr. Hardaway was elected a Member of the American Society of Civil Engineers on October 7, 1903.

JAMES CLARK HARDING, M. Am. Soc. C. E.[1]

Died November 16, 1927

James Clark Harding was born in Pittsfield, Mass., on September 28, 1873, the son of James Harding and Mary (Clark) Harding. His parents came from England, and his father was a well-known newspaper man who for many years published and edited a newspaper in Pittsfield.

Mr. Harding was educated in the public schools of his native city and after finishing his High School course became an Assistant to the late Mr. Ernest W. Bowditch on the construction of the Pittsfield Sewerage System. From 1896 to 1906, he was Assistant City Engineer of Pittsfield, in charge of the design and construction of sewer extensions and of highway and waterworks improvements. In 1902, under the direction of Mr. J. J. Van Valkenburgh, he was in immediate charge of the intermittent sand filter plant of Pittsfield and the associated sewage pumping station.

In 1906, Mr. Harding came to New York, N. Y., as a member of the staff of Hering and Fuller for five years, after which, for three years, he was a partner in the firm of George W. Fuller. From 1913 to 1917, he was engaged in private engineering practice and afterward was associated with the firm of Fuller and McClintock, until July, 1927, when he again established his own engineeering practice.

His work was principally in the fields of sewerage, sewage disposal works, water-works, and the valuation of public utilities. His best known work related to the Imhoff tank and sprinkling filters at Schenectady, N. Y.; the activated sludge plants at Mamaroneck, N. Y., and Tenafly, N. J.; the betterments to the sewerage system and the sewage disposal works for the General Electric Company, at Pittsfield and at Schenectady; the water filtration plant and extensions to the water-works system at Newburgh, N. Y.; the sewerage system and sewage treatment works at Glen Cove, Long Island, N. Y.; and the screen plants at New Rochelle, N. Y., and Bridgeport, Conn. He also collaborated in the preparation of numerous preliminary reports and designs in the fields of water supply, sewerage, and sewage disposal.

Mr. Harding was a diligent student of engineering practice and was widely read, not only on matters pertaining to sanitary engineering, but also to valuation of water-works properties. His work in this field, while connected with Fuller and McClintock, covered co-operation on the valuation of the New York Interurban Water Company, in Westchester County, New York; the Springfield Consolidated Water Company property, now the Philadelphia Suburban Water Company; the New Chester, Pa., Water-Works Company; and the Bluefield, W. Va., Water-Works Company.

In the sanitary field, his work covered surveys in connection with the investigation of typhoid fever epidemics at Cooperstown, N. Y., Des Moines, Iowa, Wilkes-Barre, Pa., etc.

---

[1] **Memoir prepared by George W. Fuller, M. Am. Soc. C. E.**

Mr. Harding's gifts as to clarity of expression, both orally and in writing, were of a high order and doubtless were influenced by his father. Combining this characteristic with his excellent executive ability, untiring efforts, and keen insight into human nature, he was more than ordinarily successful in carrying forward his undertakings in the field of public works. He was a member of the Masonic Order.

Mr. Harding was killed near South Canaan, Conn., in an automobile accident, in which his son, Joseph Buckley Harding, also lost his life. He is survived by his widow, Sarah Palmer Harding, and four sons, the eldest of whom, James Clark Harding, Jr., Assoc. M. Am. Soc. C. E., succeeded to his father's engineering practice with offices in New York City.

Mr. Harding was elected a Member of the American Society of Civil Engineers on January 2, 1912.

EDWARD WILLARD HOWE, M. Am. Soc. C. E.[1]

Died June 27, 1931

Edward Willard Howe, the son of William and Catherine (Willard) Howe, was born in Braintree, Mass., on August 27, 1846. He attended the public schools of Braintree and was graduated from Middlebury College, Middlebury, Vt., in 1869.

In 1874, Mr. Howe was appointed Assistant Civil Engineer for the City of Boston, Mass., which position he held until 1914. While serving in this capacity in the office of the City Engineer, he had a large and varied experience.

From 1883 to 1887, Mr. Howe had charge of the engineering work on Franklin Park, Bussey Park, Back Bay Park, Marine Park (Iron Pier and Castle Island Bridge) and Wood Island Park. As Engineer in Charge of Park Construction and Maintenance he continued on this work from 1887 to 1897, in addition to that on Franklin Field, Arborway, Jamaica Park, Leverett Park, Riverway, Dorchester Way (later, part of Columbia Road), Strandway (sea wall and filling), Charlestown Heights, Charlestown Playground, North End Park, Charlesbank (between Cambridge and Leverett Streets), Neponset Playground, and others. About 1897, a Superintendent of Parks was appointed by the Park Commissioners, but Mr. Howe continued to attend to the park engineering until early in 1898, when this work was taken out of the City Engineer's Office.

In 1898, he was placed in charge of building the Ipswich Street Bridge, in Boston, over the Fens Waterway, the Charlesgate West Bridge, over Ipswich Street, and the Columbia Road filling, in South Boston. About 1899, and later, he constructed the Columbia Road Bridge over the tracks of The New York, New Haven, and Hartford Railroad, and over Shoreham Street, and was in charge of work in connection with the abolition of the Dorchester Avenue grade crossing, including the bridges and streets at Dorchester Avenue, and Boston and Southampton Streets.

---

[1] Memoir prepared by Edgar S. Dorr, and Frank O. Whitney, Members, Am. Soc. C. E.

About 1901, Mr. Howe had charge of building the Broadway Bridge over Fort Point Channel, as well as the Atlantic Avenue Bridge over the railroad tracks near the South Station, Boston, and over Fort Point Channel. About 1905, he constructed the temporary bridge at Brookline Street (Cambridge), over the Charles River, and was in charge of work on the abolition of grade crossings, in East Boston, on the Boston and Albany Railroad. In 1907, the building of the boundary wall and the sea wall on Deer Island was entrusted to him.

From 1907 to 1914, Mr. Howe was engaged upon the sewerage system and surface drainage and road building for the Consumptives' Hospital, at Mattapan, Mass.; in 1909, he made many plans and estimates of schemes for the abolition of grade crossings on The New York, New Haven, and Hartford Railroad, in the Harrison Square District, of Boston; and, in 1910, he had charge of the addition to the Dorchester Avenue Bridge over the Shawmut Branch of The New York, New Haven and Hartford Railroad, at Ashmont, Mass.

In 1911, when the Public Works Department of the City of Boston was organized, Mr. Howe was appointed Engineer of Grade Crossings and Special Work. From 1911 to 1914, he was engaged on the abolition of grade crossings on the Boston, Revere Beach, and Lynn Railroad, in East Boston; and, in 1912, he designed and had charge of the construction of the Lawnsdale Terrace Subway under the Providence Division of The New York, New Haven, and Hartford Railroad.

In 1913 and 1914, he made surveys, plans, and specifications for rebuilding the Winthrop Bridge, between East Boston and Winthrop, Mass. His work included also many smaller jobs called for by the various Departments, which were done by the Engineering Department under his direction, until his retirement from active work about 1916.

In 1900, Mr. Howe became President of the Merchants' Co-Operative Bank of Boston (Director from 1884 to 1900, and President from 1900 to 1918). He was a member of the Boston Society of Civil Engineers, which he joined on January 8, 1874, and served as President in 1907. He joined the Appalachian Mountain Club in 1895, and was Vice-President in 1901 and President in 1902.

For more than fifty years he was a member of the Sons of the American Revolution, maintaining an active interest in the Boston Chapter. He was also a member of the New England Genealogical Society for many years. In religion, he was a member of the Unitarian Denomination and was especially interested in All Souls Church, of Roxbury, Mass.

Mr. Howe was throughout and invariably a mild, calm, and courteous gentleman under any and all circumstances.

On April 29, 1880, he was married to Addie E. Newell. He is survived by his widow and three nephews.

Mr. Howe was elected a Member of the American Society of Civil Engineers on September 7, 1887.

ANDREW MURRAY HUNT, M. Am. Soc. C. E.[1]

Died December 8, 1930

Andrew Murray Hunt, the son of Andrew Murray and Marion (Kent) Hunt, was born at Sioux City, Iowa, on August 12, 1859.

Mr. Hunt received his technical education at the United States Naval Academy, Annapolis, Md., from which he was graduated in 1879. While in the Navy he was Engineer Officer on several vessels; served as Instructor in Mechanical Engineering at the Orchard Lake Military Academy for a year; was a member of the Naval Steel Inspection Board, having charge of chemical inspection; organized the Chemical Laboratory at Mare Island Navy Yard; and was, for a time, Chief Engineer on the U. S. Fish Commission's Steamer, *Albatross*.

He remained in the Engineering Corps of the United States Navy until 1894, when he resigned to take up private practice. In 1893 he had been granted leave in order that he might serve the Midwinter Fair at San Francisco, Calif., as Chief of its Mechanical Department. His success in introducing novel effects of fountain and tower lighting attracted much attention at that time. Mr. Hunt's resignation from the Navy followed this engagement, and he then became active mainly on electrical projects as a member of the firm of Hasson and Hunt, of San Francisco. This firm began its activities at a time when electrical engineering was scarcely out of its infancy.

As a member of this firm, Mr. Hunt was Consulting Engineer to the Folsom Water Power Company, and to the Los Angeles Railway Company, on the problem of changing cable roads to electric roads. He also designed and installed an 800-kw. hydro-electric plant for the Nevada County Electric Power Company which for a time included the longest transmission line in the world; this line which, to-day, would be considered very short, was remarkable for its time, and, from the first, supplied the power necessary for operating the mines about Nevada City, Nev. This plant proved to be the beginning of the great system of the Pacific Gas and Electric Company. Mr. Hunt also designed and constructed an hydro-electric plant for the Victoria Electric Power Company.

In 1898, he was called to Alaska where he built four steamers for the Alaska Commercial Company. He was thereafter Consulting Engineer on various projects, such as the Northern California Power Company, the Truckee River General Electric Company, and the Vancouver Power Company, Limited, of Vancouver, B. C., Canada, a notable feature of which was a water tunnel 13 000 ft. long.

When Mr. Claus Spreckels decided that there should be competition with the established gas company at San Francisco, his selection of Manager and Chief Engineer fell upon Mr. Hunt who took personal charge, planned, and installed a successful competitive system at a cost of about $4 500 000 which, however, was taken over by the old company some years later.

---

[1] Memoir prepared by a Committee of the San Francisco Section, consisting of C. E. Grunsky, Past-President, Am. Soc. C. E., and H. L. Haehl and W. L. Huber, Members, Am. Soc. C. E.

Mr. Hunt removed from San Francisco in 1915, establishing himself in New York, N. Y., where he was a member and President of the Peyton Hunt Company, Incorporated, Consulting Engineers. The breadth of his activities and interests is indicated by the fact that he was also President of the Granton Chemical Company and a Director of the Western Pacific Railroad Corporation. During the World War he was a member of the Naval Consulting Board and also of the Ship Protection Committee of the United States Shipping Board. He had unusual skill in perfecting mechanical devices, one illustration of which is a mechanical locomotive stoker now in widespread use. His mental precision, combined with his absolute fairness, caused his services to be much sought in connection with litigation and also as an arbitrator.

When Mr. Hunt had reached a conclusion on any subject or problem there was a positiveness about his presentation thereof which was impressive and could not fail to carry conviction. He understood his subject and could be depended upon to present its aspects fairly. As a member of the Board of Direction of the Society he endeared himself to all his associates, generally taking but little part in the discussions, but at the proper time coming forward with a solution of some involved matter satisfactory to all.

Mr. Hunt was, as appears from this brief record of his activities, a highly trained, able, and conscientious member of the Engineering Profession, untiring in the interest of his clients, and a citizen of the highest type. In his death, which occurred at Berkeley, Calif., on December 8, 1930, the profession lost a most valuable member.

He was a member of the American Society of Naval Engineers; the American Society of Mechanical Engineers; the American Institute of Electrical Engineers; and the American Electrochemical Society. He was unmarried.

Mr. Hunt was elected a Member of the American Society of Civil Engineers on February 7, 1906. He served as a Director of the Society from 1918 to 1920, inclusive, and as Vice-President in 1921 and 1922.

EDGAR JADWIN, M. Am. Soc. C. E.[1]

Died March 2, 1931

Edgar Jadwin was born at Honesdale, Pa., on August 7, 1865, the son of the Hon. Cornelius C. Jadwin and Charlotte E. (Wood) Jadwin. His father served as a member of Congress from Pennsylvania, from March 4, 1881, to March 3, 1883. The son attended the public schools in Honesdale and entered Lafayette College, at Easton, Pa., in 1884. He completed the Freshman year and half the Sophomore year in the General Scientific Course, but left college on account of receiving an appointment to the United States Military Academy. He entered West Point on July 1, 1886, and was graduated therefrom on June 12, 1890, at the head of his Class.

---

[1] Memoir prepared by Herbert Deakyne, M. Am. Soc. C. E.

On his graduation, Cadet Jadwin was commissioned an Additional Second Lieutenant in the Corps of Engineers, and rose steadily through all the grades, to that of Colonel on September 10, 1919. On June 19, 1924, he was detailed as Assistant Chief of Engineers, with the rank of Brigadier General, and on June 27, 1926, as Chief of Engineers with the rank of Major General. He was placed on the retired list on August 7, 1929.

When war with Spain began, he was a First Lieutenant. He was commissioned as Major in the Third U. S. Volunteer Engineers, June 20, 1898, and as Lieutenant-Colonel in that regiment, September 15, 1898, serving as such until he was mustered out, on May 17, 1899. At the outbreak of the World War, he was advanced from his regular rank of Lieutenant-Colonel to the temporary rank of Colonel in the National Army, July 6, 1917, and to the temporary rank of Brigadier General, December 17, 1917, from which he was discharged November 1, 1919.

The first duty of Edgar Jadwin in his long and active career as an Engineer Officer was as Assistant to Capt. Thomas L. Casey, Corps of Engineers, U. S. A., in New York, N. Y., where he served as Senior Inspector on the enlargement of the site of the Immigration Station at Ellis Island, from August, 1890, to February, 1891. He then went to the Engineer School of Application at Willets Point, now Fort Totten, New York, to take the postgraduate course, and was graduated from this school in 1893. He was detailed as Quartermaster of the Battalion of Engineers in March, 1893, and continued on this duty until October, 1895. He was then engaged on river and harbor improvement and fortification work at Wilmington, N. C., as Assistant to the District Engineer, and remained on this duty for two years. In October, 1897, he was transferred to the Office of the Chief of Engineers in Washington, D. C., where he served as an Assistant in the Fortification Section of that Office.

After war with Spain was declared, as an officer of the Third U. S. Volunteer Engineers, Major Jadwin enlisted recruits for the regiment in Texas, Arkansas, and New Mexico, during June and July, 1898, and served with it in Missouri, Kentucky, and Georgia until December, 1898, when he went to Cuba, where he commanded a battalion of the regiment at Matanzas until the end of February, 1899. This Battalion raised the American flag at Matanzas on January 1, 1899, made a reconnoissance of the Province of Matanzas, and constructed camps, roads, water supply systems, a target range, and a dock. It also cleaned the streets and improved the sanitary conditions of the city. In March, 1899, Lt.-Col. Jadwin returned to the United States on account of illness, and was in the hospital and recuperating until the end of August. He then went on duty at New York as Recorder and Disbursing Officer of the Board of Engineers. At that time this Board was composed of three of the Senior Officers of the Corps of Engineers and was charged with the planning and development of the sea-coast fortifications of the United States. Its duties were of the greatest importance to the National defense and its work was of high interest from the technical and professional standpoint of the military engineer. He had charge of the designing force and the preparation of the reports of the Board.

Captain Jadwin remained on this duty until March, 1902, when he went to Los Angeles, Calif., in charge of river and harbor improvements and fortification work in Southern California. In June, 1903, he was transferred to Galveston, Tex., and placed in charge of this large and important district. In connection with the fortification work at Galveston, Captain Jadwin had charge of the building of the sea-wall at Fort Crockett, as a part of the comprehensive plan for the protection of Galveston against a recurrence of such a disaster as occurred in the hurricane of 1900.

His next duty was one of prime importance. He was selected as one of the Engineer Officers on the construction of the Panama Canal and took up this work in July, 1907. His first detail was as Division Engineer of the Chagres Division, engaged in dredging and straightening the Chagres River. On July 1, 1908, he became Resident Engineer of the Atlantic Division, Harbor and Channel Section, and took charge of the excavation of the sea-level section on the Atlantic side; the construction of the Colon breakwaters; the supply of broken stone and sand for lock and dam construction; and the operation of harbor dredges, dry docks, floating-plant repair shops, and sea-transportation plant. He continued on the Panama Canal work until August, 1911, when he returned to the United States, and for the short period from October to December, 1911, had charge of the Nashville Engineer District. He was then called to the Office of the Chief of Engineers, in Washington, where he organized a new section, the important Miscellaneous Civil Section, which passes on bridge plans and applications for authority to build structures in navigable waters, handles the removal of wrecks, and supervises various other minor activities.

Following this duty he was transferred, in June, 1916, to Pittsburgh, Pa., where he had charge of the improvement of the upper part of the Ohio River and of its tributaries in the vicinity of Pittsburgh. He was a member of the Ohio River Board which handled the questions pertaining to the great canalization project on the Ohio.

Lt.-Col. Jadwin was at Pittsburgh when the United States entered the World War. One of the first steps in the war was the organization of nine regiments of railway troops for early service in France, and he was selected to organize and command one of these regiments, known first as the 5th Reserve Engineers and, later, as the 15th Engineers (Railway). He proceeded to organize this regiment in May and June, 1917. The regiment was the first of the nine to sail for France, on July 9, 1917. It landed at Liverpool, England, on July 19, and at Le Havre, France, on July 26. This regiment had the distinction of being the first American regiment to pass through England under arms. In France, the regiment was stationed at Gievres, where it entered upon construction work and the selection of sites for depots and other facilities. Gievres became one of the greatest supply depots of the American Expeditionary Forces. The fall of 1917 was spent in the construction of row after row of storehouses and the necessary railway tracks to serve them, and in the receipt, storage, and issue of property and equipment.

On December 17, 1917, Colonel Jadwin was promoted to the rank of Brigadier-General in the National Army, and on February 17, 1918, was transferred to Langres as Chief Engineer of the Advance Lines of Communication. After a month at Langres he was ordered to Chaumont, the General Headquarters of the American Expeditionary Forces, where he became Director of Light Railways and Roads, on March 19, 1918. This duty lasted until May 23, 1918, when he went to the Headquarters of the Service of Supply, at Tours, as Director of Construction and Forestry. Here, he made his great contribution to the war. His work included the construction of docks, railroads, roads, barracks, hospitals, depots, bakeries, cold storage plants, machine shops, power plants, aviation fields, water supply, sewerage, oil and gasoline storage, and the management of forestry operations for the production of lumber. At the height of its activity, his command included 160 000 men.

After the conclusion of the war, these operations were gradually curtailed. General Jadwin was detailed in July, 1919, as a member of the American Mission to Poland and, later, as an observer in the Ukraine, to deal with post-war conditions in those sections of Europe. He returned to the United States in November, 1919, and resumed his regular rank of Colonel in the Corps of Engineers on November, 1919. He was assigned to duty as Engineer of the 8th Corps Area at Fort Sam Houston, San Antonio, Tex., and remained on that duty from February, 1920, to July, 1922. One of the principal activities of his office during that period was the mapping of sections of the Mexican Border. He was a member of the Board that prepared the project for flood control in the San Antonio River. It is noteworthy that during his tour in San Antonio one of the large floods of the Mississippi River occurred, and that, on his own initiative, Colonel Jadwin made a visit to the lower part of the Mississippi Valley to observe this flood. This was in 1922 and is significant in view of the events which occurred five years later.

In July, 1922, he was transferred to Charleston, S. C., where he took up the duties of District Engineer of the Charleston District and Division Engineer of the Southeast Division. His work covered the important sections of the Intracoastal Waterway along the South Atlantic Coast and the leading harbors in the States from North Carolina to Florida. This duty continued for only two years. On June 19, 1924, Colonel Jadwin became Assistant Chief of Engineers, with the rank of Brigadier General. During the period that he held this position, he was Senior Member of the Board of Engineers for Rivers and Harbors, handled the river and harbor activities in the office of the Chief of Engineers, and served on various boards, among them one of international importance, the Joint Board of Engineers on the St. Lawrence River.

General Jadwin became Chief of Engineers with the rank of Major General on June 27, 1926, and continued in that position until August 7, 1929.

He retained his position on the Board of Engineers on the St. Lawrence River, which submitted its report in November, 1926. During the first year of his incumbency as Chief of Engineers, occurred the great flood of 1927 in the Mississippi River. Secretary Hoover and General Jadwin visited the scene of the flood together and issued a joint statement regarding the flood and the principles upon which a plan should be based for preventing the recurrence of such a catastrophe. General Jadwin considered the time ripe for an attempt to lay down a project for the Mississippi River which would put an end to the menace of periodical floods. He spent a large part of his time during the summer and fall of 1927 on this problem. He visited the Valley repeatedly; he set in motion an intensive study of the records of past floods; he caused investigations to be made throughout the Mississippi water-shed to determine the feasibility of different methods of flood control; and he spent laborious days and nights in going over records and plans with the best talent at his command among the officers and civilian engineers in the Corps of Engineers and the Engineer Department. The result was his report submitted on December 1, 1927, embodying what has since been called the "Jadwin Plan" for the control of Mississippi River floods. Briefly, the plan proposed the strengthening of the existing levees, progressive revetment of caving banks, improvement of the channel by dredging and by contraction works, a controlled spillway just above New Orleans, La., and three floodways, one from Cairo, Ill., to New Madrid, Mo., one from the Arkansas River through the Tensas Basin to the Red River, and one from the Red River through the Atchafalaya River to the Gulf of Mexico. The cost of the work proposed was about $300 000 000. The plan aroused much controversy, but after protracted hearings and debates it was adopted by Congress in the Act approved May 15, 1928.

General Jadwin continued in service long enough to inaugurate the work under this plan and to see it in process of vigorous execution. He reached the age of 64 years on August 7, 1929, when, according to law, he passed to the retired list. By virtue of an Act of Congress passed in 1915, he was advanced one grade in rank upon retirement on account of his service in connection with the construction of the Panama Canal. This made him a Lieutenant General, one of the few men in American history who have reached that rank.

During his service as Chief of Engineers, General Jadwin was Chairman of the National Capital Park and Planning Commission, created in 1924 to deal with questions relating to streams, forests, and other natural scenery in and about Washington, and to develop the Park and Playground System. He was also a member of the Board of Commissioners of the United States Soldiers' Home.

After his retirement, General Jadwin went to Japan as a Delegate to the World Engineering Congress held in October and November, 1929. He then continued on around the world, visiting many lands that he had not had time to see during his busy career. He returned to the United States in June, 1930, in time for the Fortieth Reunion of his Class at West Point.

He was called into consultation on a number of engineering projects, and continued his membership on the Interoceanic Canal Board, which was appointed to study the matter of increasing the facilities at the Panama Canal and to bring up to date the investigations made many years ago with regard to the Nicaragua Canal. In February, 1931, he went to Panama on his way to Nicaragua to go over the ground and while in Panama, he was seized with a cerebral hemorrhage, from which he died on March 2, 1931, after an illness of only a few hours.

General Jadwin's crowning achievements were the execution of the great construction program in France during the World War and the development of the Mississippi flood-control plan. For his services in the war he received the Distinguished Service Medal, with citation as follows:

"For exceptionally meritorious and distinguished services. As commanding officer of the 15th Engineers, he inaugurated the important project at Gievres. Later, in charge of the Division of Construction and Forestry, he brought to this important task a splendidly trained mind and exceptionally high skill. His breadth of vision and sound judgment influenced greatly the successful completion of many vast construction projects undertaken by the American Expeditionary Forces."

He was also decorated by Great Britain with the order of Companion of the Bath, and by France with that of Commander of the Legion of Honor. In 1925, he received the degree of Doctor of Engineering from Lafayette College.

In his early life, General Jadwin was of rather slight physique. His four years at West Point did wonders for him, and the care that he always took of himself enabled him to meet the physical demands of his exacting duties. He suffered during much of his life from serious defects of eyesight, which required many severe operations and special glasses. At one time he had to depend on some one to read his papers to him and to look after his office work. Many a man would have given up the fight and gone on the retired list, and thereby would have missed the opportunities and achievements of the Panama Canal work, the vast construction program in France, and the Mississippi flood-control problem.

General Jadwin's equipment as an engineer was based on exceptional native ability, utilized to a high degree. He had a mind far above the average, and he was willing to trust his mental processes and to abide by the result to which they brought him. His tenacity would not allow him to give up a subject but partly finished. He liked to give the most thorough study to any subject in hand, but he had a pronounced aversion to anything savoring of delay in execution after the decision was reached. He had a faculty for looking far into the future and for probing deeply into the springs of human action.

He was married on October 6, 1891, to Jean Laubach, of Easton, Pa. She survives him, with their two children, Charlotte, the wife of Maj. Thomas G. Hearn, U. S. A., and Lieut. Cornelius C. Jadwin, also of the Regular Army.

General Jadwin was elected a Member of the American Society of Civil Engineers on April 20, 1925.

ANDREW MARTIN JENSEN, M. Am. Soc. C. E.[1]

DIED APRIL 26, 1932

Andrew Martin Jensen, the son of Martin and Adelheid (Jorgensen) Jensen, was born on January 10, 1889, in Fresno, Calif. After graduating from the public schools of Fresno City, he entered the University of California, from which he was graduated in 1917 with the degree of Bachelor of Science in Civil Engineering.

Municipal engineering, particularly sanitary engineering, seemed to be of especial interest to Mr. Jensen, and it was to this subject that he devoted most of his energy in college. He did not complete his university work in continuity, but chose rather to avail himself of a leave of absence in the middle of his course, in order to gain some practical experience before bringing his college education to a conclusion. His brother, Christian P. Jensen, M. Am. Soc. C. E., was engaged at that time in the design and supervision of a number of sewage disposal and water-works systems in the San Joaquin Valley, and he was given the opportunity to associate himself intimately with these projects. This, together with his subsequent graduation, formed an admirable background with which he established himself immediately as a Sanitary Engineer. During his college intermission he also held a Deputyship for eighteen months in the office of the City Engineer of Fresno.

From May, 1917, to January, 1919, Mr. Jensen was associated with his brother and Mr. J. A. Ross, under the firm name of Jensen, Jensen, and Ross, Civil Engineers, engaged in irrigation reclamation, structural design of buildings, sewerage, sewage disposal, and street paving, in a number of cities in the San Joaquin Valley. In January, 1919, he assumed complete charge of the business of the co-partnership, and continued to handle a large volume of professional work, under his own name, until April, 1925.

In April, 1925, Mr. Jensen was elected to the office of Commissioner of Public Works in the City of Fresno, and in April, 1929, was re-elected to the same office. His duties in this office were very extensive, including the design and maintenance of streets, sewers, sewage disposal, street-lighting systems, fire-alarm systems, public buildings, garbage collection and disposal, and a municipal airport. The airport design was particularly unique and effective and inspired a full measure of favorable comment in the scientific press as well as in newspapers of the State.

The one achievement for which the community of Fresno will always be grateful, was Mr. Jensen's initial proposal that the municipality purchase the water-works system, which, at the time, was owned and operated by a private corporation. Before resigning from his office, he was responsible for the accumulation of a large amount of statistical data, including a complete appraisal of the system, in conjunction with the State Railroad Commission, and also for an extensive publicity and educational program, resulting, eventually, in a favorable election and subsequent purchase of the system.

---

[1] Memoir prepared by Christian P. Jensen and C. G. Gillespie, Members, Am. Soc. C. E.

Just two years before his death, he resigned from his office as Commissioner of Public Works, to engage in private engineering practice in San Francisco, Calif. At the time of his tragic death he was working on an assignment from the Clay Products Institute of California. He was examining a sewer system at El Centro, Calif., and was overcome by sewer gas while in a deep manhole. Thus was brought to an end the life of one who had always been extremely careful to protect others from the very circumstances which led to his own death. He was always solicitous for the welfare of others, before his own, and never permitted his associates to undertake any hazardous investigation unless he himself was in the front line.

Mr. Jensen was favorably known among the members of the Engineering Profession in California, and his death came as a distinct shock to them, knowing that his work was his hobby, that to it he devoted all possible energy and earnestness, and that most of his life was given to public service.

Mr. Jensen was a member of the Sigma Zi and Tau Beta Pi (honor) Fraternities, and was a Thirty-third Degree Mason.

He is survived by his widow, Lillian M. Jensen, two sons, Philip and Martin, one daughter, Mildred Ann, and a brother, Christian P. Jensen, of Fresno.

Mr. Jensen was elected a Member of the American Society of Civil Engineers, on March 5, 1928.

WILLIAM HENRY JONES, M. Am. Soc. C. E.[1]

Died April 22, 1932

William Henry Jones, the son of John and Elizabeth Jones, was born on December 1, 1859, in London, England.

His education was obtained in London, and, at the age of twenty-one, he was employed as a Rodman on a Government survey in South Australia.

In 1882, Mr. Jones returned to England and came to the United States in 1883. From 1883 to 1887, he was in the employ of the Atchison, Topeka and Santa Fé Railroad Company, first (1883 to 1885) as Bridge Foreman and from 1885 to 1887, as Inspector of Bridges during construction.

In January, 1887, he entered the employ of The Missouri Valley Bridge and Iron Works, of Leavenworth, Kans., and was still actively engaged in work for the same Company at the time of his death. He had been in the employ of the Company for forty-five years, and had served as Principal Assistant Engineer for the past thirty-seven years.

Some of the larger projects on which Mr. Jones was engaged during this time, were: The bridge over Red River, Torras, La.; the bridge over Cañon de la Mauo, Mexico; Tampico wharf, Tampico, Tamps., Mexico; St. John's River Bridge, Jacksonville, Fla.; and the Broadway and Main Street Bridges, Little Rock, Ark.

---

[1] Memoir prepared by Clarence S. Jones, M. Am. Soc. C. E.

Mr. Jones was gifted with a marked ability in original design, and many projects completed by the Company bear evidence of his good judgment and engineering skill.

He took a personal interest in assisting young engineers under his charge, and his willingness and thoroughness in imparting practical knowledge will long be remembered by those who worked under him. He was of a genial disposition, and his many friends and business associates mourn his loss.

In 1886, Mr. Jones was married to Erminie J. Strain, of Lawrence, Kans., and they had lived in Leavenworth, Kans., for the past forty-six years. He is survived by his widow and a son, Clarence S. Jones, M. Am. Soc. C. E.

Mr. Jones was elected a Member of the American Society of Civil Engineers on May 4, 1904.

ALFRED LAWRENCE KEHOE, M. Am. Soc. C. E.[1]

DIED OCTOBER 17, 1931

Alfred Lawrence Kehoe, the son of Alfred L. and Christiana (Welde) Kehoe, was born on July 18, 1889, in New York, N. Y., where his father had practiced architecture for many years.

Mr. Kehoe attended the public schools of New York City, and at the age of 14 entered the employ of the American Bridge Company where he worked as a Draftsman on steel design and, later, on detail shop drawings. He remained with this Company for four years, during which time he took special work in Structural Engineering at Columbia University.

In 1908, he was employed by J. J. Smith, General Contractor, as Architectural Draftsman and Assistant to the Building Superintendent. He remained with Mr. Smith until the spring of 1909.

In the fall of 1909, Mr. Kehoe entered the service of Balcom and Darrow, Consulting Engineers, New York City, where he remained until the firm dissolved partnership on January 1, 1916. During this time, he was employed first as a Draftsman and, later, as Assistant Designing Engineer, working a large part of the time on buildings for the Grand Central Terminal Improvement.

When H. G. Balcom, M. Am. Soc. C. E., took over the business of Balcom and Darrow, Mr. Kehoe was made Engineer-in-Charge of the office, which position he held until 1919, when he became a member of the firm. During these years, he was connected with many important buildings, not only in New York City, but in many parts of the United States, including the Bowery and Greenwich Savings Banks, the Pershing Square, Johns-Manville, and Liggett-Winchester Office Buildings, and the Fifth Avenue Hospital, in New York City; the Allegheny General and Children's Hospitals, in Pittsburgh, Pa.; the Tennessee War Memorial and the Peabody Demonstration School, in Nashville, Tenn.; also, a large number of college buildings for

---

[1] Memoir prepared by H. G. Balcom, M. Am. Soc. C. E.

Columbia University, Barnard College, College of the City of New York, Hunter's College, Connecticut College for Women, University of Michigan, and Wellesley College. Mr. Kehoe also had charge of many more buildings too numerous to mention.

He was one of the founders and the first Secretary of the New York Chapter of the New York State Society of Professional Engineers and Land Surveyors. He was a member of the Leewood Country Club, and, in 1931, he was President of the Hundred-for-One (100 for 1) Club. He was also a member of Kane Lodge No. 454, F. and A. M.

Alfred Kehoe was of a genial disposition and a host of friends mourn his loss, not only as a loyal and pleasing friend, but as a man of exceptional ability.

In 1915, Mr. Kehoe was married to Grace Stearn, of New York City. They lived in Washington Heights until 1926, when they removed to Scarsdale, N. Y. It was here that he died suddenly of pneumonia, on October 17, 1931.

He is survived by Mrs. Kehoe, a daughter, Grace, a brother, Edward S. Kehoe, and two sisters.

Mr. Kehoe was elected an Associate Member of the American Society of Civil Engineers on September 10, 1918, and a Member on June 16, 1924.

SENTARO KONDO, M. Am. Soc. C. E.[1]

DIED JANUARY 22, 1931

Sentaro Kondo was born at Kanazawa, Japan, on April 24, 1859. He received his technical education at the Imperial University of Tokyo, Japan, and was graduated therefrom on July 10, 1883, with the degree of Bachelor of Science.

In September, 1883, following his graduation from college, Mr. Kondo entered the Government Service and was appointed Resident Engineer on the Mogami River Improvement. From 1885 until he retired in 1910 on account of ill health, he was connected with the Tokyo District, Home Department, where he served as Principal Assistant Engineer from 1885 to 1910 on the improvement of the Tonegawa River, as well as Resident Engineer on the Tone Canal. In 1887, he was also appointed Inspecting Engineer of Public Works, and as such was connected with all the public works, except railroads, in the Tokyo District.

In 1896 and 1897, Mr. Kondo gave a course of lectures at the Tokyo Imperial University, on River and Harbor Work, and in March, 1900, he was sent to Europe and the United States by the Japanese Government to study European and American methods in engineering.

In addition to his work as Director of Public Works in the Tokyo District of the Home Department, to which office he was appointed in 1906, Mr. Kondo was in charge of the Chair of Agricultural Engineering in Tokyo Imperial University from June, 1906 to March, 1922.

---

[1] Memoir prepared from information on file at the Headquarters of the Society.

In March, 1910, he was placed in charge of a land improvement project by the Department of Agriculture and Commerce, in which work he continued until March, 1925. He also served as Chief Engineer of the Toshin Water-Utilization, at Kintei, Korea, from 1920 to 1922, and from 1925 until ill-health compelled him to retire, he was again in charge of land improvement for the Department of Agriculture and Forestry.

In June, 1898, Mr. Kondo had conferred upon him the Sixth Order of the Sacred Treasure and in June, 1908, the Third Order. This decoration is given by the Emperor only to those who, for many years, have had full charge of important work under the Government. In 1912, he received the degree of Doctor of Engineering from the Department of Education.

Mr. Kondo devoted his life to the practice of hydraulics, irrigation, and drainage, in the service of the Japanese Government, his most important work having been the improvement of the Tone River

He was a member of the Engineering Society of Japan, the Civil Engineering Society of Japan, and an Associate Member of the Institution of Civil Engineers of Great Britain.

Mr. Kondo was elected a Member of the American Society of Civil Engineers on November 7, 1900.

WILLIAM HENRY LAWTON, M. Am. Soc. C. E.[1]

Died November 4, 1931

William Henry Lawton, a son of William H. and Charlotte Amelia (Tilley) Lawton, was born in Newport, R. I., on April 14, 1853. After his primary education, he attended the Newport High School for a time. In 1872, he received his first training in engineering when at the age of 19 he entered the office of a local engineer, remaining there about a year.

This work, although perhaps elementary in character, must have induced in him the desire for a real education, because he determined to continue his studies through high school and college. In 1873, the City of Newport constructed a fine new High School, the gift of Mr. William Sanford Rogers. The first class to be graduated from this new school was that of 1874, and Mr. Lawton, after his one year of further study, was of the eight then graduated. In September, 1874, he entered the Massachusetts Institute of Technology as a Sophomore and continued with his class at the Institute until May, 1876, completing the second and third years of the Civil Engineering Course.

Newport was then and is now an important military and naval center. For a year Mr. Lawton was in the employ of the City of Newport; but in April, 1878, he was appointed to a position in the United States Engineer

---

[1] Memoir prepared by George T. Seabury, M. Am. Soc. C. E.

Department and sent to superintend the construction of the Block Island Breakwater. For three years he was engaged in this branch of the Government service, at Block Island, on the Providence River improvement, and on surveys, until in April, 1881, he was made Assistant United States Engineer by Gen. G. K. Warren, U. S. A., and placed in charge of the construction of jetties at the entrance to Nantucket Harbor, Massachusetts. He continued there until February, 1884, after which, until the time of his death, he practiced civil engineering in Newport, specializing in sanitary engineering and in road work.

During this interval Mr. Lawton twice held a public engineering office; from 1885 to 1889 as Street Commissioner for the City of Newport, and from 1903 to 1914 as its City Engineer. He retired from active practice in 1922.

It was in the interval between these official connections and during the writer's first summer vacation of his college course that, wanting to do some practical work, he appealed to Mr. Lawton, who promised some day to give him a job. The promise was kept and one afternoon soon afterward his chance came to hold the zero end of the tape on a small survey. At the end of the afternoon Mr. Lawton generously gave him fifty cents. He demurred but Mr. Lawton insisted, saying the rate of pay for that position was $1.00 per day; the money was his and he must take it. Mr. Lawton was just as scrupulously punctilious in all business transactions to the day of his death.

Newport at the time of Mr. Lawton's greatest activity in private practice was developing as a "summer colony" for "the Four Hundred," and their magnificent estates on the rock-bound New England Coast presented many private sewage disposal problems the solution of which fell to Mr. Lawton. Access roads and grading, too, called for his advice, design, and supervision.

He loved his work for its own sake and for the sake of those with whom it brought him in contact. Sometimes it was only a farm to be surveyed, but the work was studied as meticulously as if it were in the heart of the metropolis.

He had a sense of humor that was delicious. On one farm survey it was reported to him that a boundary was of a certain length. He told his assistants that they must have made a mistake because the records of an earlier survey, made by a lame man, showed it to be 100 ft. shorter. His party re-measured the line, reporting their figure as correct and those of the earlier survey as incorrect. Mr. Lawton sighed and said "Well, maybe he paced it and lost that hundred feet with his short leg." However, he checked it again himself to be sure.

Mr. Lawton joined the Society when the total membership was only about 1 025. His Society number, 972, and his name were engraved on the back of an old-style Society badge which he wore as a necktie pin. On February 1, 1931, he suffered a paralytic stroke from which he never wholly recovered, dying on November 4, 1931, at the age of 78; but in that interval he sent the pin to the writer with the request that he wear it as his own after he had had the new name and number engraved on it. It has been an honor to write this memoir at Mr. Lawton's own request—to tell, with enthusiasm, of a generous, kindly, humor-loving man and withal a most scrupulous and capable engineer.

Mr. Lawton was married on December 4, 1889, to Mary J. Becar who, with a son, Charles S. Lawton, a daughter, Mrs. Evelyn Hubbard, and a sister, Mrs. B. B. H. Sherman, survives him. He was a member of the Protestant Episcopal Church, and Secretary of the Seamen's Church Institute.

Mr. Lawton was elected a Member of the American Society of Civil Engineers on June 2, 1886.

GEORGE HERBERT LELAND, M. Am. Soc. C. E.[1]

DIED FEBRUARY 11, 1931

George Herbert Leland was born in Cavendish, Vt., on May 16, 1849. He was the oldest son of Octavius A. and Adeline P. Leland (*née* Burnham), both of Lowell, Vt. His parents were direct descendants of Henry Leland, who settled in Grafton, Mass., in 1752. The boy's mother died in 1853, and for a time thereafter he was cared for by his Grandmother Leland.

In 1855, after his father had taken Martha J. Hayden as a second wife, the family moved from New England to Lambert, Wis., where they remained only a short time and then established their home on a farm a short distance west of Waukegan, Ill. In 1860, the family with goods and chattels in three "prairie schooners" went to a farm in Elgin, Minn., by way of the Overland Route, crossing the Mississippi River at Prairie La Crosse, and following the line of the present Winowa and St. Peters Railroad. Here, Mr. Leland remained for a time, taking advantage of such benefits as were offered by a pioneer country school. Meantime, the Civil War having broken out, his father had enlisted in the Union Army and was engaged in the military activities around Nashville, Tenn. His father died on his way home from the war; his stepmother had died in 1863. In November, 1865, as soon as business matters could be disposed of at Elgin, Mr. Leland went back to New England, to live with his grandfather at Athens, Vt. Here, he attended the district school and worked for his board.

In the spring of 1866, Mr. Leland entered the employ of the Lampson Goodnow Company, Machinists, at Windsor, Vt., with which Company he served a three-years' apprenticeship and then worked as a Journeyman covering a period of about fifteen years. In the autumn of 1881, he went to the Brown and Sharpe Manufacturing Company, at Providence, R. I., where he worked in the Tool Department until the following spring. Notwithstanding the fact that he was offered a responsible position with this Company, he decided, after careful consideration, to change from mechanical work to civil engineering.

---

[1] Memoir prepared by O. Perry Sarle, M. Am. Soc. C. E., from data assembled by Mr. Leland.

Early in 1882, Mr. Leland entered the office of the late J. Herbert Shedd, M. Am. Soc. C. E., as a student in Civil Engineering, and remained with him for about eight years. He then accepted a position as Engineer for the construction of a water-works plant at Troy, Ala. In 1891, he was in charge of construction of a system of sewers at Berkely, R. I. During the latter part of this year he accepted a position as Chief Assistant to the late Samuel M. Gray, M. Am. Soc. C. E., which he held for about thirty years. In this position Mr. Leland was engaged in engineering work on many important projects in the United States, Mexico, and Cuba.

Mr. Leland was a member of the New England Water Works Association, having been elected to that body in 1914, and of the Providence Engineering Society. He was a member of the First Baptist Church of Providence.

He was also a member of the New England and Rhode Island Botanical Clubs and was especially interested in botanical studies and in making collections of specimens of various sorts. His collection comprised about 4 000 specimens, including 400 ferns.

Mr. Leland was married, in 1870, to Amanda L. Harris, of Windsor, Vt. There were no children and, as his half-brother and sister had died a few years before, he was the last surviving member of his immediate family. He died in Providence on February 11, 1931.

Mr. Leland was elected a Junior of the American Society of Civil Engineers on March 2, 1887, and a Member on February 7, 1893.

MILO STUART MacDIARMID, M. Am. Soc. C. E.[1]

DIED JUNE 6, 1929

Milo Stuart MacDiarmid was born at Truxton, N. Y., on July 27, 1872. He was the son of Hugh Stuart and Lucy Louise MacDiarmid.

After graduation from Homer Academy, at Homer, N. Y., Mr. MacDiarmid entered Cornell University, from which he was graduated in 1895, with the degree of Bachelor of Science in Civil Engineering.

Nearly all Mr. MacDiarmid's engineering work was done in the service of the United States Government. This service began in 1900 as Inspector on Government projects at Chicago, Ill. He was employed by the United States Lake Survey, with headquarters at Detroit, Mich., in 1902, and continued in that service until his death, with the exception of several months when he was engaged on the survey of the Des Moines River, in Iowa.

In 1921, Mr. MacDiarmid was appointed Assistant Engineer in charge, at the headquarters of the Lake Survey, in Detroit, and, in 1926, he was made Associate Engineer. He was stricken with the illness which caused his death, on the U. S. Steamer *Margaret,* while actively engaged in deep-water soundings on Lake Superior, off the shore of Isle Royale.

---

[1] Memoir prepared by Charles Y. Dixon, M. Am. Soc. C. E.

Mr. MacDiarmid appreciated greatly the finer things of life and with him there was no compromise between right and wrong. His associates knew him as a remarkably splendid character, with a pleasing personality. Although others are carrying on his work, a void will long remain in the U. S. Lake Survey personnel who knew and respected him. His death is a distinct loss to the Engineering Profession.

He held the rank of Major in the Officers Reserve Corps, and was a member of the Presbyterian Church, the Masonic Order, Cornell University Club, and the Army and Navy Club.

He was married on May 5, 1900, to Katherine E. Cobb, who, with a son, Hugh Stuart, and a daughter, Louise, survives him.

Mr. MacDiarmid was elected an Associate Member of the American Society of Civil Engineers on September 4, 1907, and a Member on April 5, 1910.

ERNEST McCULLOUGH, M. Am. Soc. C. E.[1]

DIED OCTOBER 1, 1931

While newspapers were carrying complimentary reviews of his latest book, Colonel Ernest McCullough, engineer, architect, soldier, author, and journalist, died October 1, 1931, at the age of 64. His death marked the end of a versatile career, but his dynamic personality lives on in the memory of the many who prized his friendship and who respected his abilities as engineer, teacher, economist, and writer.

Throughout his life his friendships increased as his accomplishments extended to varied fields. Men admired his wealth of knowledge on many subjects gained through his intensive reading, study, and ability to retain what he read; they liked him for his kindly interest in and willingness to help young engineers plan their careers; and they acknowledged his professional ability as demonstrated on the many important projects with which he was associated.

Born on Staten Island, New York, on May 22, 1867, Ernest McCullough was the son of James and Caroline (McBlain) McCullough. After graduating from the Institute of Technology in Chicago, Ill., he went to the Pacific Coast where he received his engineering degree from the Van der Naillen School of Practical Engineering, in San Francisco, Calif. He also completed an Architectural Course at this school.

Originally, Mr. McCullough specialized in municipal work, acting as Consulting Engineer of the Merchants Association of San Francisco. There, he was Chief Engineer for the Midwinter Exposition and built the Sutro Baths. In 1898, he went to Lewiston, Idaho, where he held the office of City Engineer and was also engaged in private practice. From there, he moved to Chicago in 1903, remaining until 1917.

---

[1] Memoir prepared by B. F. Affleck, Esq., Chicago, Ill.

During his years in Chicago, Mr. McCullough made many new friendships and enjoyed a large private practice as a Reinforced Concrete Specialist. He was for a time, Associate Editor of *Engineering and Contracting* and, later, held the same position with *Railway Age Gazette* (now *Railway Age*). He also passed the State examinations for Architects and received a license to practice architecture.

He was one of the men who worked unceasingly for the passage of a Structural Engineers' License Law, and after much opposition finally succeeded in obtaining the passage of the State Law in 1916. In this campaign, he sacrificed much of his time and personal funds.

In addition to these activities, he taught night classes at the Young Mens' Christian Association College and at Lewis Institute, where he was well liked because of his ability to impart sound advice to students in a form easy to remember. His knowledge was more practical than theoretical. Students learned from him a number of "short cuts" and practical fundamentals which he had mastered while on construction and which, later, were valuable to them in their own work.

While in Chicago Mr. McCullough was employed by the City Building Department as a member of a Committee of Three to prepare regulations for concrete flat-slab design. He also served as Chief Engineer of the Municipal Engineering and Construction Company. He constructed more than 16 miles of sewers in St. Louis, Mo., and superintended the building of water purification plants for the Union Stock Yards, in Chicago. For a time he was employed as Engineer by the City of Cedar Rapids, Iowa. In 1916, he went to the Portland Cement Association, in charge of its Fireproof Construction Bureau; he left Chicago in 1917 to take a position with the Stone and Webster Corporation, in Boston, Mass.

However, the World War was raging in Europe, and Mr. McCullough was eager to do his share. Twenty-three members of the McCullough Clan in Ireland had been killed in action, and he felt that his place was on the battlefields where they had fought and fallen. In June, 1917, he was ordered to France as Acting Chief Engineer of the American Red Cross, later becoming a Student Officer in the Gas Schools of the French and British Armies. His wife, Elizabeth Townsend Seymour McCullough, died while he was in France.

Feeling that his fellow officers were making a mistake in not mastering the French language, Major McCullough mingled with the French and, after a time, was able to address French engineering groups in their own tongue. He was gassed at the Battle of Cambrai in 1917.

In 1919, he was married to Therese Claquin, at Tours, France. At that time, he was a Lieutenant-Colonel in the Chemical Warfare Service.

After the World War, his mastery of the French language made him valuable in handling damage claims in France. In August, 1919, he returned to America to become Assistant Commandant of the Lakehurst Proving Ground and Director of the Service School for Officers of the Chemical Warfare Service. In 1920, Oskaloosa College, Iowa, conferred upon him the degree of Doctor of Philosophy.

After resigning from the Army in July, 1920, Colonel McCullough was associated for a time with the Semet-Solvay Company as Structural Engineer and, later, was Associate Editor of the *American Architect,* in New York, N. Y. In 1923 and 1924, he was with the construction firm of Dwight P. Robinson and Company, Incorporated, in New York City, leaving this association to become Chief Structural Engineer for Vickers and International Combustion, Limited, London, England. During his connection with this organization in 1925, Colonel McCullough was awarded a medal for a paper presented to the British Institute of Structural Engineers. The paper dealt with American structural engineering practices, in which field he was an acknowledged authority.

Afterward, he became Editor-in-Chief of *Building Age* (now *American Builder and Building Age*), leaving, in 1928, to engage in writing, in addition to his engineering work. Two of his best-known books are "Practical Structural Design" and "Engineering as a Vocation." His last book, "How to Spend Your Money," published only a short time before his death, is described by reviewers as being full of practical advice on investments. He also wrote articles for magazines, being a frequent contributor to *Forbes* and other financial publications.

For twenty years, Colonel McCullough made a study of economics, with particular reference to the business cycle and possible means of eliminating the peaks and depressions. As a result of this research, he wrote during the last ten years a number of articles advocating Professor Irving Fisher's theory of currency control as a means of stabilizing business conditions.

At the time of his death, Colonel McCullough was making a study of the engineering aspect of package freight handling in connection with terminal warehousing. The result of these studies was to be embodied in a book dealing with terminal freight handling in all its aspects. His ability to write authoritatively and interestingly on many subjects made his books and articles widely read.

Colonel McCullough was a Member of the Société des Ingénieurs Civils de France and the Institute of Structural Engineers (Great Britain).

Surviving him are his widow and three children—Captain George Seymour McCullough, 16th Infantry, U. S. Army, Fort Jay, N. Y.; James David McCullough, of Staten Island; and Mrs. Paul C. Galleher, of Los Angeles, Calif.

Colonel McCullough was elected a Member of the American Society of Civil Engineers on May 31, 1910.

HORACE MILLER MARSHALL, M. Am. Soc. C. E.[1]

DIED OCTOBER 3, 1930

Horace Miller Marshall, the son of Thomas Alexander and Letitia (Miller) Marshall, was born in Vicksburg, Miss., on August 18, 1854.

---

[1] Memoir prepared by Charles M. Kerr, Assoc. M. Am. Soc. C. E.

His early education was obtained under private tutors, and at the age of fourteen he entered Lyons Academy, at Vicksburg. He then attended Bellevue High School, at Lynchburg, Va., later enrolling in the University of Virginia, from which he was graduated in June, 1877, with the degree of Civil Engineer.

Returning to Vicksburg after his graduation, Mr. Marshall entered into the private practice of his profession, his first engagement being a survey of the harbor and water-front. From 1879 to 1883, he managed his father's plantation, "Aubrey," and in the early part of the latter year entered the employ of the Louisville, New Orleans, and Texas Railroad Company (subsequently, the Yazoo and Mississippi Valley Railroad Company), in charge of location, and, later, construction, south of Clarksdale, Miss. In the summer of 1884, Mr. Marshall became associated with the United States Engineer Office at Vicksburg. He returned to private practice in the spring of 1885, and was engaged in research and levee work in Warren, Coahoma, and Tunica Counties, Mississippi. In the summer of 1887 he had charge of railroad work in Arkansas, with the Memphis, Arkansas, and Texas Railroad Company, between Memphis, Tenn., and Pine Bluff, and from Marianna to Clarendon. In the fall of that year he re-entered the employ of the United States Engineer Office at Vicksburg, where he remained until his retirement on August 18, 1924.

During this period, he was Assistant Engineer in charge of all survey work on the Red and Ouachita Rivers in Louisiana, and on the Yazoo and Big Sun Flower Rivers in Mississippi. During the term of his active Government service, Mr. Marshall had charge of the design, as well as the general supervision, of many important public improvements, principal among which was the work of diverting the waters of the Yazoo River into Lake Centennial, which restored the Harbor of Vicksburg; the plan for the improvement of Red River; the installation of the locks and dams on the Ouachita River; and the plans for a system of locks and dams on the Big Sun Flower River. To Mr. Marshall is credited the first successful attempt at tying the ends of crevasses. He did this first at the Grand Lake Crevasse in Arkansas, in 1888; and, later, in 1890, at the Duncansby Crevasse, in Mississippi, he prevented the natural widening of the break in the levee line by his personal efforts.

After his retirement in 1924 from the Government service, he resumed the private practice of his profession, in the capacity of Consulting Engineer. Mr. Marshall's efforts in the planning of the diversion of the Yazoo River into Lake Centennial, and in the subsequent successful restoration of Vicksburg Harbor, brought him widespread recognition in engineering circles, and he was accredited with the success of this scheme.

He was an acknowledged leader in his profession, and was considered an authority upon the subject of his special life work—flood control and the improvement of inland waterways. He conceived and developed a general plan for the establishment of the Industrial Canal and Inner Harbor at New Orleans, La., many years before a similar scheme was adopted and put into operation. He also planned for the development of Red River along ambitious lines, but lack of Government appropriations and funds prevented a realization of his well-conceived and thoroughly practical ideas.

Mr. Marshall was endowed by inheritance with a refinement of sentiment and manner, and was known for his devotion and loyalty to his friends and associates. He was possessed of a keen and analytical mind, and was well versed in the higher branches of mathematics. He was a deep and studious reader and, outside his books, his chief means of recreation lay in his flower garden, in which he spent his hours of leisure.

During 1895 and 1903, he served as Alderman, and, from 1903 to 1905, as Mayor, of the Town of Speeds, Miss. During the Spanish-American War, he served as a Colonel on the Staff of the Governor of Mississippi, and was appointed Chief State Engineer, and had charge of the protection of the mouth of the Mississippi River by the laying of bombs to prevent a possible invasion by the enemy fleet. Mr. Marshall was a member of Theta Delta Chi Fraternity, and was admitted into the Masonic Order in 1892.

On January 15, 1880, he was married to Elizabeth Taylor Henry, of Vicksburg. He is survived by Mrs. Marshall, and five daughters—Mrs. Kingsland Spencer, of Tarrytown, N. Y.; Mrs. Frederick Gray Hudson and Mrs. Harry Russell, of Monroe, La.; Miss Letitia Marshall, of Vicksburg, and Mrs. Henry Charles Yaeger, of Denver, Colo.

Mr. Marshall was elected a Member of the American Society of Civil Engineers on December 3, 1890.

MONTGOMERY MEIGS, M. Am. Soc. C. E.[1]

Died December 9, 1931

Montgomery Meigs, for forty years always called "The Major," was born at Rouses Point, N. Y., on February 28, 1847, the son of General Montgomery Meigs, Quartermaster of the Union Army during the War of the Rebellion, and Louisa (Rodgers) Meigs. Mrs. Meigs was the daughter of Commodore John Rodgers, of the United States Navy, and Commander of the famous old frigate, *Constitution*. After a distinguished career in the war, General Meigs was intensively engaged with public work in the National Capital, and, among other things, built the Washington Aqueduct.

Montgomery Meigs' early education was secured in private schools in Washington and Philadelphia, Pa., and he matriculated at Harvard University where he studied engineering for two years. After being graduated from Harvard, he studied in the Polytechnical School at Stuttgart, Germany.

Upon his return to the United States, he accepted a position with the Northern Pacific Railroad Company, then engaged in pioneer work from Lake Superior to Puget Sound. He located and built early sections of track between Jamestown, N. Dak., and Bismarck, and laid out the first squares of what is now the City of Bismarck, S. Dak. During his two years with the

---

[1] Memoir prepared by Hugh L. Cooper, M. Am. Soc. C. E.

Northern Pacific Railroad Company, he was with two surveying expeditions up the Yellowstone River and into the Little Big Horn country, where Custer's command was lost.

The panic of 1873 brought about the disbandment of the engineering corps of the railways, and Major Meigs then went to Rock Island, Ill., where he joined an expedition to the West and Southwest. He afterward returned to Washington, D. C., where he was located for some time in the Coast Survey Office of the Bureau of Weights and Standards.

He was next appointed by the Secretary of War as United States Civil Engineer, and was stationed at Rock Island. The position of United States Civil Engineer was created by Act of Congress and Major Meigs was one of four appointed under the provisions of this Act.

In 1876, he was married to Grace Lynde, of Rock Island, whose death occurred many years ago. There were six daughters born to this union, all surviving but one, Dr. Grace (Meigs) Crowder, whose death occurred about 1926. The daughters surviving are: Mrs. Louisa (Meigs) Green, of Winchester, Va.; Mrs. Mary (Meigs) Atwater, of Helena, Mont.; Mrs. Alice (Meigs) Orr, of St. Charles, Ill.; Mrs. Frances (Meigs) Fales, of Bryn Mawr, Pa.; and Miss Cornelia Meigs, of Keokuk, Iowa. Ten grandchilren of Major Meigs were living at the time of his death.

Major Meigs went to Keokuk, Iowa, on January 1, 1882, from Rock Island, and was given charge of that section of the Mississippi River from that date until he retired on September 20, 1926, thus completing a continuous service of forty-four years. He had supervision over all the Mississippi River work and its navigation facilities between Burlington, Iowa, and Hannibal, Mo. In addition to his regular duties in charge of this section of the river, he was given general supervision of the interests of the Federal Government during the time of the construction of the Mississippi River Power Company's power plant and its lock and dry dock at the foot of the Des Moines Rapids. This service was of great interest to Major Meigs, and his judgment on all matters coming under his review was of great value, not only to the Government, but to the engineers of the Power Company. His loyalty to the river interests and to the responsibilities imposed upon him by the Government was always of first importance to him, and his constant attention to every detail of the design and construction was a distinct contribution to the successes which have attended the operation of this entire installation.

Major Meigs was especially interested in the upper gate for this lock, built to take a submerged position when boats were entering from the upper pool. This submerged gate was operated by compressed air, and Major Meigs' interest in the working out of its details constituted a distinct engineering service.

Major Meigs' entire life was distinguished by devotion to every class of duty that is found in the ideal citizen. He was frequently confronted with offers of promotion, but they were always declined because of his devotion to his six daughters, and his belief that they could be given better foundations for their life work if they were freed from the setbacks that are always incident to moving the home from place to place. He took great pride in the achievements of his daughters in the various fields of endeavor in which they were engaged.

His retirement on September 20, 1926, came at a time when he was 79 years of age. This retirement was a distinct recognition of his splendid service to his Government, and to the unusual mental and physical capacity which had been in such full evidence during his extended public service.

Many interests occupied Major Meigs during the years after his retirement from service. He was a most delightful conversationalist and one of the best read and best informed men in his vicinity. He read widely, and his interest in books is shown in the unusually fine library which he possessed. With the late Howard Elliot, Affiliate, Am. Soc. C. E., Major Meigs organized the Keokuk Book Club forty-nine years ago, and it was one of his characteristics never to abandon an effort after he had once started it.

In his retirement he was greatly interested in bird life. He helped to band specimens of birds, and was identified with this work in other ways. Major Meigs was a photographer of considerable skill, and used his camera to make records of many happenings along the Mississippi River, both in an official and an unofficial capacity. He was an early devoteé of golf, and a member of the old Keokuk Bicycle Club.

In his religious life, Major Meigs attended St. John's Protestant Episcopal Church, of Keokuk. He was absolutely devoted to his family and his home life was ideal. He found a deep pleasure in what his numerous friends were doing, and they will always revere the memory of one who had a fine understanding of scientific principles and prime citizenship, and whose life made the world better.

Major Meigs was elected a Member of the American Society of Civil Engineers on March 5, 1879.

JOSEPH DAVIS METCALFE, M. Am. Soc. C. E.[1]

DIED FEBRUARY 2, 1931

Joseph Davis Metcalfe was born on January 21, 1886, at Georgetown, Tex., the son of J. A. Metcalfe. In 1906, he was graduated from the Agricultural and Mechanical College of Texas at which time he received the degree of Bachelor of Science in Civil Engineering.

In September, 1906, Mr. Metcalfe accepted a position with the Great Southern Lumber Company and the New Orleans Great Northern Railway Company as Masonry Inspector. He was also Instrumentman in charge of field work, and, later, in charge of detail work in the office of the Construction Engineer on the erection of saw-mill plant and railway shops at Bogalusa, La. He was thus employed until December, 1907.

---

[1] Memoir prepared by W. D. Dockery and G. R. Burtner, Assoc. Members, Am. Soc. C. E.

For the next three years, he was associated with the Altus, Roswell and El Paso Railway Company, and the Wichita Falls and North Western Railway Company, as Instrumentman and Assistant Engineer on surveys and in charge of construction. Mr. Metcalfe was a very capable engineer, and received a wide range of experience during the next six years. His dependability made him most acceptable as one to have charge of construction work.

In March, 1912, he was Instrumentman for the Quanah, Seymour, Dublin, and Rockport Railway Company. From April, 1912, to December, 1913, and from July to October, 1914, he was with the Houston and Texas Central Railway Company, as Instrumentman and Assistant Engineer in actual charge of construction, including roadbed, yards, bridges, and track. From January to July, 1914, he was Engineer in charge of construction on the Brazos River protection levee. In October, 1914, Mr. Metcalfe undertook a greater responsibility by accepting a position as Division Engineer in charge of highway construction in McLennan County, Texas. He remained there until July, 1915, when he became Assistant Highway Engineer in Bell County, Texas, for one year; during this time, he was also engaged in the preparation of plans.

From August, 1916, to April, 1917, Mr. Metcalfe increased his experience by doing valuation work with the Gulf, Colorado, and Santa Fé Railway Company. From May to July, 1917, during the planning of the road system, he was Highway Engineer on Road District No. 10 in Navarro County, Texas; and from August to November, 1917, he was Assistant Engineer and Superintendent on the construction of Camp MacArthur and Rich Field, at Waco, Tex. During the following year he filled the position of Assistant County Engineer in Dallas County, Texas, and prepared plans on highway projects.

In March, 1918, Mr. Metcalfe became a member of the firm of Isbell and Metcalfe, Civil Engineers, at Wichita Falls, Tex. The work of this firm included the location of pipe lines, the supervision of construction, refinery layouts, city and rural subdivisions, paving, the location and construction of roads and bridges, and surveys. The firm was in business about two years when in December, 1921, Mr. Metcalfe accepted a position as County Engineer in Frio County, Texas, with responsibility for the design and construction of the highway system. From 1923 to 1925, he served as Resident Engineer in the Texas Highway Department on highway construction. From 1925 to 1929, he was Consulting Engineer in Wichita Falls, and from March, 1929, until the time of his death, Mr. Metcalfe was Division Engineer in the Texas Highway Department, with headquarters at Paris, Tex.

As life is measured to-day, Mr. Metcalfe was a comparatively young man; in fact, his span was twenty-five years short of the three-score and ten allotted to men; but measured by events and achievements he had lived a life of much greater length. This untimely end cut short a career which promised great success in the engineering field. He had a high sense of responsibility and devotion to duty. He was energetic, resourceful, and all work which was entrusted to him, was fulfilled, with painstaking care.

In March, 1914, he was married to Mary Hudson, of Pearsall, Tex., and he is survived by his widow and two children, Joseph Davis and Robert Pryse Metcalfe.

Mr. Metcalfe was elected an Associate Member of the American Society of Civil Engineers on April 2, 1913, and a Member on October 22, 1924.

EUSEBIUS JOSEPH MOLERA, M. Am. Soc. C. E.[1]

DIED JANUARY 14, 1932

Eusebius Joseph Molera was born at Vich, Spain, on November 14, 1846. He received his technical education at the Royal Academy of Engineers at Madrid, from which he was graduated in 1867.

Two years later Mr. Molera came to America in the employ of the Spanish Government, collecting data relating to the manufacture of war materials. As early as 1870 he found employment with the United States Corps of Engineers on harbor work at San Francisco, Calif., his name being connected with the surveys of Blossom Rock preceding its removal as an obstruction to the navigation of San Francisco Bay. Thereafter, until 1876, he was Assistant Lighthouse Engineer of the 12th District, with headquarters at San Francisco.

Mr. Molera then established himself permanently in private practice as a Consultant Civil Engineer at San Francisco, making topographic surveys, planning irrigation works, etc. He was called to Mexico in 1881 by President Gonzales to outline a project for the drainage of the Valley of Mexico. A few years later, he was employed by a syndicate of capitalists as Consultant on the completion of the Tehuantepec Railroad. This work included studies for harbor terminals on both the Atlantic and Pacific Oceans. His professional practice covered not only a wide range in engineering, but extended, too, into the architectural field.

Mr. Molera was a student of early Mexican history and from time to time gave delightful accounts of the results of his studies to his favorite technical organization—the Technical Society of the Pacific Coast. He took great interest, too, in the advancement of science, as evidenced by his activities in the California Academy of Sciences of which he was long a member and which he had served as President from 1903 to 1909.

Mr. Molera was a polished gentleman, courteous to all and pleasant to meet, a sincere and dependable friend, an engineer of recognized standing, an esteemed citizen.

He was married on April 28, 1875, to Amelia Cooper, of Monterey, Calif., who died on August 19, 1918. Of his two children, Andrew J. and Frances M., the former passed away a few months before the father's death.

Mr. Molera was elected a Member of the American Society of Civil Engineers on October 5, 1904.

---

[1] Memoir prepared by C. E. Grunsky, Past-President, Am. Soc. C. E., and D. C Henny, M. Am. Soc. C. E.

RICHARD MONTFORT, M. Am. Soc. C. E.[1]

DIED FEBRUARY 7, 1931

Richard Montfort was born in County Carlow, Ireland, on March 4, 1854. He was the son of Henry Montfort, M. D., who was a nephew of the Right Hon. Abraham Brewster, Lord Chancellor of Ireland, in 1868, and of Elizabeth (Colles) Montfort, who was a niece of Dr. Abraham Colles, of Dublin, Ireland.

Mr. Montfort was educated in Kingstown School, County Dublin, and at the Royal College of Science, Dublin, in which he held a scholarship. He was graduated in Civil Engineering in 1876 and sailed for the United States in the same year.

His first professional engagement was with the Louisville Bridge and Iron Company, of Louisville, Ky., as Draftsman, Inspector, and Assistant Engineer until 1880, at which time he entered the service of the Louisville and Nashville Railroad Company as Bridge Engineer. He was employed in this capacity until 1883, when he became Resident Engineer of the System in charge of roadway, bridges, and buildings. In 1887, his title was changed to Chief Engineer, with increased duties.

On January 1, 1905, Mr. Montfort resigned his position as Chief Engineer of the Louisville and Nashville Railroad Company, on account of ill health, and the position of Consulting Engineer of the same Company was created for him, which position he held until his death.

Mr. Montfort was a Charter Member of the American Railway Engineering Association and served for many years on its Rail Committee. He was also a Charter Member of the Engineers and Architects Club of Louisville, and a member of the Pendennis Club of that City.

He was married to Henrietta Barret, of Louisville, who predeceased him many years. He left one son, Mr. Barret Montfort, who is Vice-President of the Chemical Bank and Trust Company of New York City.

Mr. Montfort was elected a Member of the American Society of Civil Engineers on June 6, 1888. He served as a Director of the Society from 1914 to 1916, and as Vice-President in 1916.

STEPHEN HENLEY NOYES, M. Am. Soc. C. E.[2]

DIED JANUARY 21, 1932

Stephen Henley Noyes was born on November 26, 1881, at Newport, R. I He was the son of Lieut. Boutelle Noyes, U. S. N., and Charlotte B. (Luce) Noyes. His grandfather was Rear Admiral Stephen B. Luce, U. S. N., Founder of the Naval War College and the Apprentice System of the United

---

[1] Memoir prepared by W. H. Courtenay, M. Am. Soc. C. E.

[2] Memoir prepared by Robert Farnham, M. Am. Soc. C. E.

States Navy. His great-grandfather was Commodore John Dandridge Henley, U. S. N. His paternal great-grandfather was the Hon. Timothy Boutelle, who was one of the organizers of the first railroad in Maine that, later, became the Maine Central Railroad. On both his father's and his mother's side, there were nine generations in America.

Mr. Noyes first attended school at the Chateau de Lancy, Geneva, Switzerland, then, later, he was for four years at St. Mark's School, Southboro, Mass. He was graduated from Harvard University with the degree of Bachelor of Arts in 1903, and that of Bachelor of Science "Cum laude" in 1905. While at Harvard, he was a member of the Zeta Psi, The Dickie, Institute of 1770, and the Hasty Pudding Club; he was also Editor-in-Chief of the *Harvard Engineering Journal.* He played on and captained his class football team in 1902, which won the Class Championship, and, later, played quarter-back on the University team. In 1904 and 1905, Mr. Noyes was a member of Battery A of the Massachusetts Light Artillery. During 1905, he traveled in the East and shot big game in India.

After his graduation from college, he was in the employ of L. J. Johnson, M. Am. Soc. C. E., engaged on making tests of reinforced concrete beams. He then went as a Draftsman with the Underwriters Engineering and Construction Company, of New York, N. Y., being employed on the design of reinforced concrete buildings. From October, 1906, to July, 1907, he was employed as a Civil Engineer by the Kittle Construction Company, of San Francisco, Calif., on the design and methods of constructing reinforced concrete mill buildings.

From September, 1907, to October, 1908, Mr. Noyes had experience with the American Bridge Company, at its Long Island City, N. Y., plant, detailing and checking shop drawings for structural steel buildings. In 1908, he was a member of the crew of the old Cup Defender, *Mayflower,* wrecked in a hurricane in the West Indies and abandoned at sea.

Mr. Noyes was with the Pennsylvania Steel Company from 1908 to 1913 detailing and checking shop drawings for steel bridges; he then served as Assistant to the Chief of Drafting-Room on methods of bridge erection, and, later, was in responsible charge of designing falsework and travelers. He was afterward placed in the field, in responsible charge of engineering, in connection with the erection of three large bascule bridges, and made a comprehensive study and estimates of costs of bridge erection of all kinds.

In 1913, he made an expedition into the little known interior of the Kamchatka Peninsula, in Siberia. In December, 1913, he entered the service of the Pennsylvania Railroad Company.

In 1916, believing that the entry of the United States into the World War was inevitable, Mr. Noyes took lessons in aviation, and was the first man to be graduated from the Philadelphia School of Aviation, at Essington, Pa. He was named a non-commissioned officer in the Air Service Reserve before the United States entered the war. On the declaration of war by the United States, he was sent to Kelly Field, Texas, and left for France in May, 1917, as First Lieutenant Flight Commander of the 1st Aero Squadron. He served, in turn, as Commanding Officer, 12th Aero Squadron, Commanding

Officer, 5th Corps Observation Group, and Commanding Officer, Corps Observation Group, 1st Army. He received the following promotions: Sergeant, February 5, 1917, Air Service Reserve Corps; First Lieutenant, May 10, 1917, Air Service Reserve Corps; Captain, August 1, 1918, Air Service; and Major, April 23, 1919, Air Service. He was honorably discharged on May 27, 1919. He took part in the following engagements: Marne-Aisne, St. Mihiel, and Argonne-Meuse.

In July, 1918, Captain Noyes was awarded the Croix de Guerre, with the following citation:

> "Pilote de premier ordre, calme et brave, modele de devoir pour son escadrille. Le 6 juillet 1918, attaque une premiere fois par une patrouille ennemie, la dispersa par sa manoeuvre hardie et permit a son observateur de prendre les photographies desirees. Attaque une deuxieme fois, a abattu un de ses adversaires au cours d'un severe combat; a termine son vol par une reconnaissance a 500 metres d'altitude dans les lignes allemandes."

In October, 1918, he was given the Distinguished Service Cross:

> "For extraordinary heroism in action near Chatel Chehery, France, October 16, 1918. Capt. Noyes volunteered under the most adverse weather conditions to stake the advance lines of the 82nd Division. Disregarding the fact that darkness would set in before he and his observer could complete their mission, and at the extremely low altitude of 150 feet, Capt. Noyes proceeded, amid heavy aircraft and ground machine-gun fire, until the necessary information was secured. On the return, due to darkness, he was forced to land on a shell-torn field, and proceeded on foot to Headquarters with valuable information."

After being discharged from the Service, Mr. Noyes again entered the employ of the Pennsylvania Railroad Company, where he remained until 1920. During his service with this Company, he made designs of all classes of railroad bridges, and had charge of the inspection and repair of existing highway bridges throughout the System.

From July, 1920, to February, 1927, he was head of the Bridge Department of the City of Philadelphia, in which position he had charge of the design and erection of all bridges built by the City. The outstanding bridges which he designed and erected while in the municipal employ are the South Street Bridge and the University Avenue Bridge over the Schuylkill River.

From 1927 to the day of his death, Mr. Noyes was Assistant to the Chief Engineer of the Philadelphia Improvements of the Pennsylvania Railroad, and, in this capacity, had complete charge of the design of all structural steel and reinforced concrete, as well as all other types of construction.

He was always a tireless worker, and was noted for his accuracy and sound judgment. He was recognized by his associates as an authority in his particular line of work, and his unfailing and absolute honesty of purpose was recognized and admired by all his co-workers. He had a wonderful sense of humor, was a delightful companion, and could play just as hard as he could work.

His principal recreation was golf, at which he excelled, and, during the last years of his life, he could generally be found on the golf links if he was not in the office.

Mr. Noyes was a member of the following clubs: Philadelphia Club, Philadelphia Cricket Club, Racquet and Tennis Club, Philadelphia; Metropolitan Club, of Washington, D. C.; University Club, and Harvard Club, of New York, N. Y.; the Military Order of the Loyal Legion; and the Mayflower Society.

His sudden death was a great shock to a host of friends in all parts of the country.

Mr. Noyes was elected a Junior of the American Society of Civil Engineers on October 1, 1907, an Associate Member on February 6, 1912, and a Member on January 20, 1922.

FREDERICK SYLVESTER ODELL, M. Am. Soc. C. E.[1]

DIED OCTOBER 20, 1931

Frederick Sylvester Odell, the son of Harry and Elizabeth Ward (Tompkins) Odell, was born at Mt. Kisco, N. Y., on June 8, 1850. He was educated at private schools and at Cooper Union, New York, N. Y.

Mr. Odell's first entry into the field of engineering in 1870 was as Assistant Engineer under Thomas C. Cornell, Civil Engineer, of Yonkers, N. Y., engaged in highway and dock construction and in general practice. In 1871, he was appointed Assistant Engineer in the Department of Public Parks, New York City. He remained in the employ of the City of New York until 1873, when he was placed in charge of highway and park development in the City of Cohoes, N. Y., under the direction of Robert Weir, Civil Engineer. In 1874, he returned to Yonkers and was in responsible charge of the preliminary surveys for a water supply for that city.

In 1875 Mr. Odell was re-employed by the New York City Park Department on the construction of Riverside Drive, remaining on this work until 1879, when he became Resident Engineer on the construction of the southerly end of the Putnam Division of the New York Central Railroad, completing this work. In 1880, he was placed in charge of the construction of sewers at Memphis, Tenn., under George E. Waring, Consulting Engineer. From 1881 to 1890, he was associated with the late J. James R. Croes, Past-President, Am. Soc. C. E., as Principal Assistant Engineer on surveys for the Suburban Rapid Transit Railways, and the Princeton, N. J., Water-Works, acting also as Engineer in Charge of Design and Construction of the sewerage systems for Lawrenceville, Seabright, and East Orange, N. J.

In 1890, Mr. Odell became associated with the office of the late Charles B. Brush, M. Am. Soc. C. E., and was actively engaged in the construction of water-works at New Rochelle, N. Y., and in North Hudson County, New Jersey. Also, during this period, he was employed on preliminary work for the proposed Hudson River Bridge. In 1892, he was appointed Commissioner of Public Works for the City of Mount Vernon, N. Y., and was instrumental in the design and construction of the many municipal improvements then being undertaken in that city. He remained in this position until 1898 when he resigned to become associated with the Engineer in charge of "Montreat," a large real estate development in the mountains of North Carolina. Completing this work, Mr. Odell returned north and entered general practice as a professional engineer, with an office at Port Chester, N. Y., where he remained actively engaged until 1916. During this period he designed and built the sewerage system and disposal works for that village.

---

[1] Memoir prepared by James R. Rundell, Esq., Port Chester, N. Y.

In 1916, Mr. Odell organized the F. S. Odell Engineering Corporation, and as its President specialized in large real estate developments, landscape engineering, and general surveying practice, being also one of several engineers employed in the real estate surveys and land acquisition for the Westchester County Park Commission.

In 1927, he retired to private life, spending his last remaining years at his home in Mount Vernon, and his country home at Wahackme Woodland, New Canaan, Conn. Able of mind and active of body until only a few months before his death, he suffered a general breakdown which led to a complication of diseases causing his death in the Mount Vernon Hospital, on October 20, 1931, at the age of 81 years. He was buried in Oakwood Cemetery, Mt. Kisco, N. Y.

He was married in June, 1888, to Carrie McLane Davis, of Mount Vernon, who died on November 21, 1914. He was married a second time to Mary Lyman, also of Mount Vernon. He is survived by a niece, Mrs. Grace Odell Burt, of Brooklyn, N. Y.

A deep student and of brilliant mind, Mr. Odell's retiring nature and modesty did not allow the world to know the fineness of the man. A gentleman of the old school, genial and friendly to his intimates, brave in adversity, the soul of honesty, and a most thorough and capable engineer, the profession and his associates will deeply feel his loss.

Mr. Odell was elected a Member of the American Society of Civil Engineers on March 5, 1884. He was also an Honorary Member of the Westchester Society of Civil Engineers.

ARTHUR ONDERDONK, M. Am. Soc. C. E.[1]

DIED JANUARY 22, 1932

Arthur Onderdonk, the son of Andrew Onderdonk and Sarah (Hillman) Onderdonk, was born at Yale, B. C., Canada, on January 29, 1881. His father was a noted engineer and contractor.

After attending McGill University, in Montreal, Que., Canada, and Columbia University, in New York, N. Y., each for one year, Mr. Onderdonk started his engineering career in 1900 under the late Ira A. Shaler, M. Am. Soc., C. E., Contractor. His work consisted principally in measuring materials and cross-sectioning in the tunnels of the New York City Rapid Transit Commission.

---

[1] Memoir prepared by John G. Claybourn, M. Am. Soc. C. E., and L. B. Moore, Assoc. M. Am. Soc. C. E.

From 1901 to 1903, as Assistant Engineer for his father, he was engaged on contract work and estimating. From 1903 to 1907 he was Assistant to his father's Chief Engineer on subaqueous tunnel works, mainly on the East River Tunnel and the Rapid Transit Loop, at Battery Park, New York City.

During parts of 1907 and 1908, Mr. Onderdonk was employed in different capacities on various projects as follows: Timber Superintendent, Philadelphia, Pa., Rapid Transit; Foreman and, later, Levelman, Panama Canal; Contractor's Assistant Engineer on railways in Costa Rica; and Civil Engineer on surveys and design of hydro-electric plants for the H. T. Purdy Engineering Company, at San José, Costa Rica.

For the next six years (1908–1914), Mr. Onderdonk was Engineer in charge of construction for the Abangarez Gold Fields of Costa Rica. He supervised the design and construction of roads, buildings, narrow-gauge railroads, aerial tramway, and a modern 60-stamp mill, with accessory and auxiliary plants.

During 1914 and 1915, he was engaged in the private practice of surveying for land improvements at Oyster Bay, N. Y.

From July, 1915, until January, 1916, he was Assistant Engineer for the Eden Mining Company, in Nicaragua, on the survey for and design of a new installation. From January to September, 1916, he was Assistant Engineer on surveys and location for the Tela Railroad, in Honduras. For a few months in 1916 and 1917, he assisted in installing plant on the East River Tunnel, New York City, for the Contractor, Patrick McGovern.

In March, 1917, Mr. Onderdonk returned to the Abangarez Gold Fields of Costa Rica as Superintendent and Assistant Manager, and, later, as Acting Manager. His work included the design and construction of structures for the San Rafael Mine. He remained in this position until November, 1919.

From December, 1919, to January, 1921, he was Chief Engineer and Manager of the Piermont Mines Company, in Nevada. His duties included the supervision of new installation and surveys, as well as examinations of various claims, properties, and structures.

From January, 1921, to April, 1924, he was Consulting and Chief Engineer for Mr. J. N. Popham in connection with the latter's development of manganese mines in Panama.

For a few months in 1924 Mr. Onderdonk assisted the Contractor on the Bear Mountain Bridge over the Hudson River. From December, 1924, to April, 1926, he was Estimator on the Canal Zone for the Constructing Quartermaster, U. S. Army. Since April, 1926, he had been engaged in structural design work for the Panama Canal. During this period he had also served as Engineer in charge of the Barraza Fill Development, in Panama City, and had done some work for the Panama Corporation, a gold mining company with extensive properties in the Republic of Panama.

During his long and varied career as an engineer over a period of thirty-one years, Arthur Onderdonk made an unusually large number of friends among whom his loss is deeply felt. He was a member of the Century Club, of Panama City.

He was married in 1911 to Gertrude Bruce, of Mount Vernon, N. Y., and is survived by his widow and two sons, Andrew and Arthur Bruce.

Mr. Onderdonk was elected an Associate Member of the American Society of Civil Engineers on January 6, 1915, and a Member on March 16, 1925.

WILLIAM COLLINS PHELPS, M. Am. Soc. C. E.[1]

DIED JUNE 19, 1931

William Collins Phelps was born on December 19, 1867, at Claremont, N. H. He was a descendant of Edward Phelps who came to America from Bolton, England, in 1634, and settled in Andover, Mass., and, on his mother's side, from the Denio family, connected with the early history of Deerfield, Mass., and the Allens, prominent in the history of Vermont in Revolutionary times.

Mr. Phelps attended Stevens High School, in Claremont, and was graduated from Dartmouth College in 1893, with the degree of Bachelor of Science, and from the Thayer School of Civil Engineering in 1895, with the degree of Civil Engineer.

During his vacation periods, while still attending college, Mr. Phelps worked as a Draftsman for the Boston Bridge Works, and as a Transitman for the City Engineer of Cambridge, Mass. Following his graduation, he went with the United States Engineer Corps on surveys in Northern Virginia. From September, 1895, to August, 1897, he was a Structural Steel Draftsman for C. L. Strobel, M. Am. Soc. C. E., in Chicago, Ill., and for the firms of Purdy and Henderson, and Post and McCord, in New York, N. Y.

From 1897 to 1923, Mr. Phelps was engaged in work on street railways, elevated lines, and subway lines in and around New York City. He started as a Draftsman with the Metropolitan Street Railway Company and one year later, 1898, became Assistant Engineer for the Edison Electric Illuminating Company of New York, preparing plans for central stations. In 1899, he was engaged by the Manhattan Railway Company as Assistant Engineer on the preparation of plans for the electrification of the elevated railroad lines. This work included the structural design of the main power station at 74th Street and the East River, and of seven sub-power station buildings.

In 1901, he became Assistant Engineer in the Mechanical Engineer's Department of the Rapid Transit Subway Construction Company and the Interborough Rapid Transit Company. In this capacity he was in charge of the structural design of the power station at 59th Street and Eleventh Avenue, nine sub-power stations, and repair shops, car sheds, and auxiliary buildings, at the Lenox Avenue and West Farms Terminals. He served with the Interborough Rapid Transit Company in the development, estimates, and construction of new subway lines until 1918. From 1918 to 1923, he was engaged on similar work with several independent lines in New Jersey. In 1923, he was with Post and McCord, in New York, on the Edison Power Station at 14th Street.

---

[1] Memoir prepared by A. J. Wilcox, M. Am. Soc. C. E.

From July, 1927, until his death, Mr. Phelps was employed by H. G. Balcom, M. Am. Soc. C. E., Consulting Engineer, New York City, as a Structural Engineer. In this position he was engaged on the design of various buildings and projects, which included work on some of the highest structures in New York City, and, notably, the framing for the elaborate stone work on the Department of Commerce and Post Office Department Buildings in Washington, D. C.

Mr. Phelps' natural inclination for painstaking attention to detail and thoroughness of analysis, made him especially suited for the most intricate design. He showed a conscientiousness and completeness in his work that was a satisfaction to all who had any occasion to deal with him or his efforts.

He was a great lover of Nature and of the out-of-doors. Camping was a favorite recreation with him. One of his camping trips had taken him to British Columbia a number of years ago.

Mr. Phelps was a member of the Thayer Society of Engineers of Dartmouth College, a life member of Prince of Orange Lodge, and a member of Constitution Chapter, Royal Arch Masons, New York City.

He died in Brooklyn, N. Y., after an illness of only a few weeks. The interment was at his boyhood home at Claremont, N. H. He never married, and the only surviving member of his immediate family is his sister, Elizabeth G. Phelps.

Mr. Phelps was elected a Member of the American Society of Civil Engineers on October 2, 1907.

HENRY TEGMEYER PORTER, M. Am. Soc. C. E.[1]

Died January 11, 1932

Henry Tegmeyer Porter was born in Piedmont, W. Va., on January 10, 1859, the son of William E. and Sarah (Paxson) Porter.

Mr. Porter was graduated from the Baltimore (Md.) City College in 1879, and began his engineering career in 1881 as Chainman with the West Virginia Central and Pittsburg Railway Company. In December of the same year, he became Transitman and Assistant Engineer with the New York, West Shore and Buffalo Railway Company, remaining in this position until February, 1884, during which time he was engaged largely on construction work.

In 1884 and 1885, he was Transitman with the West Virginia Central and Pittsburg and the South Pennsylvania Railway Companies and, during part of 1885 and 1886, was Contractor's Engineer for Beckwith, Quackenbush, and Company, on the construction of the Howard University Tunnel, Washington, D. C.

---

[1] Memoir prepared by W. S. McFetridge, M. Am. Soc. C. E., and F. R. S. Layng, Assoc. M. Am. Soc. C. E.

From 1886 to 1896, Mr. Porter was Resident Engineer and General Roadmaster of the Toledo, St. Louis and Kansas City Railroad Company, during which time the road was largely rebuilt and the track relaid with heavier rails. For a short time in 1897, he was Chief Engineer of the Chicago, Peoria and St. Louis Railroad Company, and then served as Engineer with the Continuous Rail Joint Company of America. In October, 1897, he entered the service of The Pittsburg, Bessemer and Lake Erie (now the Bessemer and Lake Erie) Railroad Company, as Engineer, Maintenance of Way, and, in 1898, he was appointed Chief Engineer of the same Company, which position he held until his retirement from active service on April 30, 1931.

From 1902 to 1906, Mr. Porter was also Chief Engineer of the Western Allegheny Railroad Company, extending from Kaylor to Newcastle, Pa. This road was located and constructed under his direction.

During Mr. Porter's long service of nearly thirty-four years with the Bessemer and Lake Erie Railroad Company, many major projects, changes of line, and other improvements were carried out under his direction. He was an early advocate of the use of heavier loadings in the design of bridges, and one of the first bridges, an 154-ft. through truss bridge, built shortly after he became Chief Engineer, was designed for Cooper's E-60 loading, and the Osgood Viaduct, built in 1901, was designed for the same loading.

Among the many projects carried out under his direction may be noted the following: The principal repair shops at Greenville, Pa., occupying 61 acres of land and constituting a modern up-to-date shop layout; the construction of the Bessemer and Lake Erie Railroad from Kremis to K O Junction, Pa., in 1901 and 1902 (this Company later took over the operation of The Pittsburg, Bessemer, and Lake Erie Railroad); the change of line from Culmerville to Cunningham, Pa., to improve alignment and grades, and which includes the Bull Creek Fill, 180 ft. high, containing 5 000 000 cu.·yd. of material; and the replacement of the single-track bridge over the Allegheny River, at River Valley, Pa., with a double-track bridge built for Cooper's E-75 loading. The single-track bridge had a viaduct approach at the north end and three 350-ft., one 520-ft., and one 207-ft. deck truss spans over the river; the total length, including the viaduct, was 3 538 ft. The new bridge had the same length of spans over the river, with one 273-ft. span, one 153-ft. span, and one 60-ft. span added at the north end; the remainder of the viaduct was filled. The new bridge was erected alongside the old one, on the extensions of piers required for double-track, and the main spans were built in two continuous sections erected as cantilevers, one section, 1 140 ft. long, and one section, 969 ft. long, each covering three spans; the other short spans at the north end were erected in the usual manner. Each section was moved laterally into place after the old bridge was torn down. The total weight in the 1 140-ft. section was 13 000 000 lb. and this section was moved into position in 27 min. In the 969-ft. section, the weight moved was 10 200 000 lb. The horizontal movement was 16 ft. 3 in. The embankment replacing the viaduct contains 1 382 000 cu. yd. This bridge was completed in 1918.

Another project was the enlargement of the Inner Basin at Conneaut Harbor, Ohio, where the ore docks of the Railroad Company are located. This work was completed in 1926 and included one of the heaviest and strongest reinforced concrete docks built on the Great Lakes. The dock front is constructed on piers and caissons, all of reinforced concrete, with their foundations carried to rock, the greatest depth being 41 ft. below mean lake level. This dock was built to carry ore-unloading machines, much larger and heavier than any used at the Harbor up to that time.

Other work on which Mr. Porter was engaged included: The new line from Filer to Coolspring, Pa., completed in 1929, which was named the Porter Cut-Off in his honor; in addition, many other smaller projects and changes were made, the road was double-tracked from North Bessemer to Conneaut Harbor and largely rebuilt, and the work done under his direction had much to do with enabling the Bessemer and Lake Erie Railroad to take its present position as a heavy tonnage line.

Mr. Porter was frequently consulted on engineering projects, especially those concerning his own community. He was Consulting Engineer on the Ohl Street Bridge, over the Shenango River, for the Borough of Greenville, and he was also consulted in regard to some of the highway bridges in county and other projects.

He was one of the original workers on the Pymatuming Conservation Project for building the Pymatuming Dam and Storage Reservoir, near Linesville, Pa., to provide flood control and water conservation for the Shenango Valley, and as a member of the Executive Committee of the Beaver and Shenango Water Conservation Association, he supported it from the start, about 1912, to its ultimate consummation and the beginning of construction work.

As a member of the Borough Council of Greenville for four years, he rendered valuable service to the community. He was a member of the Presbyterian Church of Greenville and always took a deep and sincere interest in its work.

Mr. Porter was a member of the International Railway Congress, American Railway Engineering Association, Engineers Society of Western Pennsylvania, and the Railway Club of Pittsburgh. Other organizations and clubs of which he was a member were: The Pennsylvania Society, Keystone Athletic Club of Pittsburgh, and American Iron and Steel Institute.

Locally, Mr. Porter belonged to the Greenville Country Club, Kiwanis Club, and Railway Club of Greenville. He was also a Director of the Greenville National Bank. Among Masonic Bodies he was affiliated with Eureka Lodge No. 290, F. and A. M., Mount Royal Chapter No. 212, Mount Calvary Commandery No. 67, Knights Templars, and the Acacia Club.

Mr. Porter exemplified and advocated high standards of honor, and diligent work for his profession. He was always active in, and gave of his time and experience to all, community affairs, and his public spirit led him into active participation in the activities of church, town, and business groups.

On September 23, 1886, he was married to Annie Hall, who died in Greenville on December 13, 1930. One son, William Hall Porter, of Youngstown, Ohio, and one daughter, Margaret E. Porter, at home, survive him.

Mr. Porter was elected a Member of the American Society of Civil Engineers on April 1, 1903.

HARRY LONGYEAR PRESTON, M. Am. Soc. C. E.[1]

DIED MARCH 25, 1930

Harry Longyear Preston, the son of Otis Marshall and Angeline (Lawrence) Preston, was born in Roxbury, N. Y., on November 29, 1877.

He was a descendant of Thomas Rogers who came over in the *Mayflower*, the same Thomas Rogers who is mentioned in Governor Bradford's Memoirs. The first Preston came to this country on April 8, 1635, from Cripplegate, London, on the ship *Elizabeth of London*, William Slagg, Master, An ancestress, named Sarah Lord, suffered a long imprisonment on the charge of witchcraft in 1692. Mr. Preston was a son of the American Revolution, with five ancestors who fought in that particular war. His great-grandfather was a Major General serving on the staff of Col. Jonathan Holman, when the regiment marched at the Rhode Island Alarm. The first of the family to settle in Roxbury, N. Y., was John Moore, who was a great friend of Joseph Brant, the Indian Chieftain, and credit is given to John Moore for saving Roxbury from destruction by Walter Butler and Brant at the time of the Cherry Valley Massacre. John Moore's daughter, Jean, was Mr. Preston's great-great-grandmother.

Mr. Preston was graduated from Sheffield Scientific School of Yale University in 1900 with the degree of Bachelor of Philosophy in Civil Engineering. In July 1900, he entered the service of the Pennsylvania Railroad, Lines West of Pittsburgh, beginning his engineering experience as a Rodman. In April, 1901, he left the Pennsylvania Railroad Company to accept a position as Assistant Engineer on construction for Jones and Laughlin, Limited, of Pittsburgh, Pa., in which position he had charge of a field party on steel mill and open-hearth furnace construction.

From March, 1902, until May, 1905, Mr. Preston was an Assistant Engineer for the Pittsburgh and Lake Erie Railroad Company, in charge of the construction of 10 miles of third and fourth-tracking, gravity yard, and shops, together with other general work.

In May, 1905, he accepted a position with the New York Central and Hudson River Railroad Company (later The New York Central System), acting as Transitman and Draftsman until November of that year, when he was made an Assistant Engineer in charge of construction work. He remained in the service of the New York Central and Hudson River Railroad Company until April, 1913, and during the period, May, 1905, to July, 1909, was in charge of the following construction work: Grade-crossing elimination, Tilly Foster Mines, Putnam Division; double-tracking, Goldens Bridge to Brewster, Harlem Division; Poughkeepsie improvement, Hudson Division; double-tracking, Salina to Liverpool, Rome, Watertown, and Ogdensburg Division; grade-crossing elimination, Grant Avenue, Sennett, Auburn Branch; and a 20-stall engine house, at Syracuse, N. Y. In July, 1909, Mr. Preston was made Assistant District Engineer, which title was changed to Resident Engineer on January 1, 1910, which position he held until he resigned on April 15, 1913. During the period from July, 1909, to April, 1913, he had charge of all the construction work in the vicinity of Syracuse, including third and fourth-tracking and grade revision of the Syracuse Junction Railroad, the construction of the Belle Isle Yard, the Barge Canal crossing of the West Shore Railroad, at Montezuma, N. Y., and a car repair shop, at Oswego, N. Y.

---

[1] Memoir prepared by E. B. Menuez, Valuation Engr., N. Y. Central Railroad, Lines Buffalo and East.

In April, 1913, Mr. Preston became Owner and Manager of the Jordan Wheelbarrow Company, engaged in the manufacture of agricultural implements, and continued in this work until the Company was discontinued in 1917.

On May 16, 1917, he returned to the New York Central Railroad Company as Assistant Engineer in the Valuation Department, taking charge of the field inventory of the New York District, which inventory was made by the Railroad Company at the request of the Bureau of Valuation, Interstate Commerce Commission. In September, 1917, Mr. Preston was promoted to the position of Field Engineer, Lines Buffalo and East, in charge of the Pilot Engineers and forces accompanying the field parties of the Bureau of Valuation, that were making an inventory of the railroad property.

On August 9, 1918, he was given a leave of absence for military duty and entered the military service of the United States as Captain of Engineers, U. S. Army, in command of Company K, 22d Engineers. In February, 1919, he was made Supply Officer, 4th Battalion, 22d Engineers, American Expeditionary Forces, France, Light Railway Division, Second Army, Toul Sector.

In July, 1919, Mr. Preston returned to the service of the New York Central Railroad Company, in the Valuation Department, Lines Buffalo and East, as Assistant Engineer in charge of grading distribution and reproduction program, and in January, 1925, was made one of the Assistant Valuation Engineers. During 1924, he was stationed in Washington, D. C., in charge of the forces co-operating with the Bureau of Valuation, in determining the quantities to be included in the proposed engineering report, and in 1926, during the conference with the Bureau of Valuation, in connection with the revision of the preliminary Engineering Report, he was in charge of certain negotiations and checking principally in regard to grading and allied items.

In 1928, he was in charge of the Railroad Company's forces engaged in the field inventory of the Hudson River Connecting Railroad which, having been constructed subsequent to the valuation date of the New York Central, was inventoried by the Bureau of Valuation as a separate corporate body. At the time of his death, Mr. Preston was engaged on certain details of this work.

Mr. Preston was a loyal friend and was highly esteemed and respected by all who came in contact with him. His personality was of a superior type and his integrity and sincerity without a blemish. At his death his family and many friends suffered a severe loss.

R. E. Dougherty, M. Am. Soc. C. E., Vice-President, New York Central Lines, who brought Mr. Preston into that service, writes as follows:

"It was my privilege to initially engage Mr. Preston for his New York Central service shortly after I had been appointed in charge of construction in the Eastern District of the New York Central. It was my own first appointment to responsibility and I was naturally much concerned about those who should collaborate with me.

"Mr. Preston quickly proved to be a man of keen intelligence, good judgment, and fine character. Outside of our business relations he and I established a personal friendship which continued until his death.

"His passing is a very distinct loss to the New York Central and to his friends who will not forget him."

Another of his many friends writes:

"My acquaintance with him dates back to the early days of Federal Railway Valuation work. My association with him was of a peculiarly intimate nature, and in my acquaintance with thousands of engineers engaged in valuation work, I have found no one endowed with a greater spirit of co-operation, fairness, and desire to intelligently arrive at facts in connection with the work at hand.

"Mr. Preston was not only a splendid engineer, thoroughly equipped with a knowledge of his road, but of fundamentals which go to make up a successful valuation engineer. All of his conferences were actuated with a spirit of fairness to the public as well as to the railroad that he served. He possessed to a high degree that engineering element known as diplomacy, and at the same time was earnest and firm in his convictions as to what was right and just in connection with his work. The engineering profession loses a splendid member in the passing of Mr. Preston."

From a letter written by another friend, the following is quoted:

"I will say that my acquaintance with Mr. Preston covered a term of perhaps eight years. Prior to that time I had heard of him as being an outstanding man among a group of men of the highest character. In my contact with the various valuation men from all over the country I have become acquainted with many men of the highest character and among them Mr. Preston was perhaps the outstanding man as a gentleman, a man who said nothing but good of everybody and who had the ability to play the game in this work."

He was married on September 8, 1903, to Ellen Marie Tracy, who died November 16, 1916. On September 15, 1920, he was married to Nona Nelson, who survives him.

Mr. Preston was elected an Associate Member of the American Society of Civil Engineers on April 1, 1908, and a Member on October 3, 1922.

ALBERT REESOR RAYMER, M. Am. Soc. C. E.[1]

DIED AUGUST 10, 1931

Albert Reesor Raymer, the son of John N. and Christina Raymer, was born on December 1, 1862, at Markham, Ont., Canada. He attended the Markham High School and was graduated from the University of Toronto, Toronto, Ont., Canada, in 1884, with the degree of Civil Engineer, entering at once into the practice of his profession.

From 1884 to 1887, Mr. Raymer was Rodman on Location and Resident Engineer on Construction of the Northern and Pacific Junction Railway (later, the Canadian National Railway); 1887 and 1888, Leveler on Location and Office Engineer in the office of the Chief Engineer on Construction, International Railway of Maine (later, the Canadian Pacific Railway); 1888, Office Engineer on Location, Toronto, Hamilton and Buffalo Railway, with headquarters at Hamilton, Ont., Canada; 1889, Locating Engineer, Grand Trunk Railway (later, the Canadian National Railway), from Waterloo to Elmira, Ont.; 1889 to 1891, Office Engineer on Construction, Cumberland Valley Extension of the Louisville and Nashville Railroad, from Cumberland Gap, Tenn., to Big Stone Gap, Va.; and 1891 to 1896, Assistant Engineer in charge of office and field work on maintenance, Lake Shore and Michigan Southern Railway (later, the New York Central Railroad), Toledo, Ohio. In 1896, he was naturalized as a citizen of the United States.

Mr. Raymer went to the Pittsburgh and Lake Erie Railroad Company (New York Central Lines) on August 19, 1896, as Assistant Engineer. On February 1, 1901, he was appointed Assistant Chief Engineer and held that position until March 1, 1920, at which time he was made Chief Engineer. He was appointed Assistant Vice-President and Chief Engineer on July 1, 1926, which position he held until July 1, 1931, when he was made Assistant Vice-President and continued as such until his death.

During his tenure of office as Assistant Chief Engineer Mr. Raymer was directly in charge of all engineering design which included the design and construction of large terminal, storage, and classification yards, with roundhouses, locomotive and car repair shops, etc., at various points along the road.

One of the major engineering structures built during that time was the pin-connected cantilever bridge over the Ohio River, at Beaver, Pa. This bridge, on account of the many novel features embodied in its design, attracted the attention of the Engineering Profession at the time of its construction. The substructure was designed under Mr. Raymer's personal supervision, and he prepared and presented a paper on the subject, entitled "The Substructure of the Pittsburgh and Lake Erie Railroad Bridge Over the Ohio River at Beaver, Pa.," before the Structural Section of the Engineers' Society of Western Pennsylvania.[2] He also prepared and presented a paper entitled "The Pittsburgh and Lake Erie Railroad Cantilever Bridge

---

[1] Memoir prepared by R. P. Forsberg and E. K. Morse, Members, Am. Soc. C. E.

[2] *Proceedings*, Engrs.' Soc. of Western Pennsylvania, Vol. 26, No. 1 (January 4, 1910).

Over the Ohio River at Beaver, Pa.," at the meeting of the American Society of Civil Engineers on March 1, 1911.[3]

Designs for other important engineering work made during his tenure of office included plans for a complete terminal layout in Pittsburgh, Pa., the principal items being a modern terminal passenger station and general office building, a power plant, and a large fireproof storage warehouse. Designs were also prepared by Mr. Raymer, covering the construction of several branch and tributary lines, the principal one of the latter being the Lake Erie and Eastern Railroad, a part of which is located through Youngstown, Ohio, crossing streets, railroads, several river crossings, etc., and involving many engineering problems. He designed, and under date of April 19, 1904, secured patents for a method of cleaning and restoring boilers to service condition.

Mr. Raymer was, in the strictest sense of the term, a Christian gentleman, always courteous, affable, and democratic. Inherently an Engineer, he derived genuine pleasure from the study and solution of all problems that came before him, usually treating them in an original manner. He possessed that trait which every engineer should covet, namely, the faculty of blending the ideal with the practical in his designs and in their execution.

He contributed his engineering knowledge and skill in a most generous manner, not only to the Company with which he was connected for a long term of years, but to the various organizations with which he was affiliated, and especially to any young engineer who came to him for counsel or advice.

He was President of the Beech Creek Railroad Company; the Beech Creek Extension Railroad Company; the Shenango Valley Railroad Company; and the Stewart Railroad Company; and a member of the Board of Directors of the Pittsburgh, McKeesport, and Youghiogheny Railroad Company; the Pittsburgh, Chartiers, and Youghiogheny Railway Company; the Chartiers Southern Railway Company; the Mahoning State Line Railroad Company; the Greene County Railroad Company; and the Pittsburgh and Clearfield Railroad Company.

Mr. Raymer was also a member of the Board of Directors and Past-President of the Standard Chemical Company of Pittsburgh, Pa.; Chairman of the Bridge Engineers' Committee of the New York Central Lines, which position he held continuously from the formation of that Committee in 1907 until his death; and a member of the Engineering Committee of the New York Central Lines.

He was a member and Past-President of the Engineers' Society of Western Pennsylvania; a member of the American Railway Engineering Association; the Duquesne Club, of Pittsburgh; and the Beaver Valley Country Club, of Beaver Falls, Pa.

He was a Thirty-second Degree Mason, a member of Pennsylvania Consistory, Valley of Pittsburgh, and A. A. O. N. M. S., Syria Temple, of Pittsburgh, and a member of the Methodist Episcopal Church, of Beaver, Pa.

Mr. Raymer was married to Frances Emmeline Hutcheson, who, with three children, Edith Alberta (Mrs. Frank T. Underhill), of Summit, N. J., Paul

---

[3] *Transactions*, Am. Soc. C. E., Vol. LXXIII (1911), p. 136.

Hutcheson Raymer, of New York, N. Y., and Dr. John George Raymer, of Norwich, Conn., survives him.

Mr. Raymer was elected a Member of the American Society of Civil Engineers on March 1, 1910, and served as a Director of the Society from 1925 to 1927.

ABRAHAM MOREAU REYNOLDS, M. Am. Soc. C. E.[1]

Died April 7, 1932

Abraham Moreau Reynolds was born in Newark, N. J., on December 1, 1869, the son of Abraham Moreau and Anna (Mason) Reynolds. He attended the Newark public schools and was graduated in a special mathematical course at Pingree Institute, Elizabeth, N. J., in 1888.

Mr. Reynolds began his engineering career, in February, 1888, with the firm of Earle and Harrison, Jersey City, N. J., which had an extensive business as Engineers and Surveyors in Northern New Jersey. While with this firm, he was engaged in a survey of the Raritan River Railway.

He left the employ of Earle and Harrison in the fall of 1892 to accept a position as Assistant Engineer with the Lehigh and Hudson River Railroad Company, at Warwick, N. Y., where he remained until March, 1895. At that time he was appointed Division Engineer on highway construction in Morris County, New Jersey, where he had charge of making surveys and plans for the improvement of about ten miles of county highways.

In August, 1895, Mr. Reynolds accepted a position with the Essex County Park Commission, at Newark, N. J., as Assistant Engineer. This Commission had just started its work of developing parks in Essex County and Mr. Reynolds continued with it until he retired in August, 1931, on account of ill health. From 1895 until December, 1901, he had charge of constructing three parks in the System and, at that time, was appointed Chief Engineer for the Commission in charge of all construction. In January, 1906, the Park Commission made him Superintendent of Maintenance, as well as continuing him as Chief Engineer, and he served in this capacity for more than twenty-five years, until his retirement.

The successful development, as well as the continued excellent state of maintenance of the parks of the Essex County Park Commission, was due to Mr. Reynolds more than any other individual connected with the System. His conscientious devotion to his work, his wonderful business ability, and his engineering skill were greatly appreciated by members of the Park Commission. This appreciation was expressed by the Commission in a handsomely embossed resolution presented to him at the time of his retirement.

Mr. Reynolds was a member of the American Society of Municipal Engineers, the New Jersey Society of Professional Engineers and Land Surveyors, and was on the Consulting Staff of The Regional Plan of New York and Its Environs, for several years prior to his retirement. He was also for many years a member of the Glen Ridge Country Club and the Down Town Club, of Newark.

---

[1] Memoir prepared by J. H. Philips, M. Am. Soc. C. E.

Besides his widow, Emily Sanford Reynolds, to whom he was married in 1899, Mr. Reynolds is survived by two sisters, Josephine and Clementine Reynolds, of Warwick, N. Y.

Mr. Reynolds was elected a Member of the American Society of Civil Engineers on November 30, 1909.

KNUD SOPHUS RISER, M. Am. Soc. C. E.[1]

Died November 1, 1931

Knud Sophus Riser was born at Horten, Norway, on August 15, 1858. His father was a lawyer, and for many years Chief of the Auditing Department in the Custom House at the Norwegian Capital. He had an older brother and one sister, who describe the home surroundings as unusually quiet and religious. He was graduated from the Polytechnic Institute at Christiania (now called Oslo), Norway, in 1880. He came to America soon afterward for, in 1881, he was employed on a railroad survey for the Pittsburgh, Alleghany, and McKeysport Coal Company, which he always spoke of as his "first job."

Subsequently, he served as a Draftsman with the Edgemoor Bridge Works, at Edgemoor, Del., and, later, with the Philadelphia (Pa.) Bridge Works. For three years Mr. Riser was employed as Assistant Engineer at the Milwaukee (Wis.) Bridge and Iron Works.

He became Chief Engineer of the Clinton Bridge and Iron Company, at Clinton, Iowa, in June, 1889, and among the larger of his works may be listed the design of a bridge across the Mississippi River, at La Crosse, Wis., consisting of three fixed and a single draw span of 450 ft.; also, the High Bridge, at Clinton, consisting of six spans of 150 ft. each, two spans of 210 ft. each, one span of 219 ft., and a cantilever span of 420 ft., resting on rocker towers, 55 ft. high, and which, together with an iron trestle approach, had a total length of 3 300 ft.

In 1893 he went to Detroit, Mich., and engaged in private practice, as a Consulting Bridge Engineer, until 1896 when he was appointed Engineer of Construction with the Penn Bridge Company, at Beaver Falls, Pa.

In 1901, Mr. Riser went to Grand Rapids, Mich., where he organized and became President and Chief Engineer of the Grand Rapids Bridge Company, which position he retained until 1907, when he again took up the private practice of engineering as a Civil and Architectural Engineer.

---

[1] Memoir prepared by his Fellow Members of the Society residing in Grand Rapids, Mich.

With his high regard for professional ethics Mr. Riser was never aggressive in seeking business. All work had to come to him, and he personally attended to every detail of it. In Grand Rapids, he earned the richly deserved reputation of "Court of Final Resort" in all problems involving steel design beyond the experience of local architects. For years, the City Building Inspector (who was not then a technically trained engineer) referred to Mr. Riser all difficult questions of structural design. Mr. Riser's reports defied political expediency. On one occasion he showed that with a few minor repairs a certain bridge was good for another fifty years' service, and local aldermen were compelled to employ outside talent to sanction the desired building of a new bridge on that site. The steel work for a new theater was delivered and in process of erection before he was called on to check the design, and the mistakes he compelled the steelwork sales engineer to rectify developed into a comedy when he found even the foundation reinforcement placed on the wrong side. He regarded it a mistake to print formulas in handbooks, for, in his opinion, unless each specific problem was sufficiently understood to derive the formula, the designer was not qualified to render judgment.

Mr. Riser achieved maximum economy of construction in designing the Street Railway Company's Bridge across the Grand River, using piers of twin steel cylinders sunk to firm bottom by interior mucking and then filled with concrete. The steel framework of many local high buildings, notably the Herpolsheimer Building, was his design, although his name was seldom mentioned with those of the architects. Another example is the steel work in the car-ferry dock of the State Highway Department, at Mackinaw City, Mich. The Ann Street Bridge, in Grand Rapids, was entirely his design.

In 1888, he was married to Sarah Evans, of Pottstown, Pa., who died on November 28, 1928. They had no children, and Mr. Riser felt her loss very keenly. At home, his greatest delight was in the perusal of his technical books.

His circle of intimate friends was small, but in that circle Mr. Riser was universally beloved. The social side of his nature was not developed sufficiently to overcome his innate modesty, and he avoided all publicity and seldom referred to his personal accomplishments. He could not be induced to attend Society meetings even when they were held in his part of the country, and he never attended the meetings of the Grand Rapids Engineers' Club, the requirements of which for membership failed to meet his high professional ideals.

Mr. Riser was killed on November 1, 1931, when the automobile he was driving was struck by a train at the Michigan Central crossing, at Dutton, Mich.

He was a member of the Masonic Fraternity, his Blue Lodge being York No. 410, in Grand Rapids, and his Chapter, Harmony No. 206, of Beaver Falls, Pa. He was a Knight Templar in De Molai Commandery, of Grand Rapids, which conducted his funeral services. In his passing, his professional associates mourn the loss of their ablest member.

Mr. Riser was elected a Member of the American Society of Civil Engineers on February 3, 1892.

EARL IVAN ROBERTS, M. Am. Soc. C. E.[1]

DIED MAY 4, 1932

Earl Ivan Roberts was born on September 5, 1890, at Bristol, Ohio. He was the son of Charles H. and Ella (Davis) Roberts.

Mr. Roberts attended the public schools and High School at McConnelsville, Ohio, from which he was graduated in 1910. Following a two-year period, during which he taught in a rural school near his birthplace, he entered Ohio State University, at Columbus, Ohio, and completed the course in the Sanitary Engineering Department, graduating in 1916 with the degree of Bachelor of Civil Engineering.

His first work after graduation was with W. J. Sherman, M. Am. Soc. C. E., Consulting Engineer, Toledo, Ohio, on street paving; later, Mr. Roberts became Topographer and Resident Engineer on sewer construction in the Sanitary Engineering Department of Lucas County, Ohio.

In March, 1917, he went with the Ohio State Department of Health as Assistant Engineer, where his duties required him to check designs and make inspections of water supplies, sewerage systems, and sewage treatment works.

In February, 1918, Mr. Roberts enlisted as a Private in the Medical Corps, U. S. Army, where his devotion to duty, cheerful disposition, and technical ability quickly won for him the attention of his superior officers, and, in March, he was promoted to Sergeant, First Class, in the Sanitary Corps, U. S. Army, in charge of the supervision of water-analysis laboratories. Soon after this promotion, Mr. Roberts was transferred to France. In October, 1918, he was commissioned Second Lieutenant in the Sanitary Corps and placed in charge of water laboratories and the supervision of operation of filtration plants, in which capacity he remained until May, 1919, when he was commissioned First Lieutenant. In June, 1919, his services no longer being required with the American Expeditionary Force, Lieutenant Roberts was returned to the United States, and was honorably discharged from the Army in July.

In August, 1919, Mr. Roberts again took up his work with the Ohio State Department of Health, and remained with the Department until March, 1924, in charge of the supervision of operation of water purification works.

From March, 1924, to March, 1927, he was associated with William G. Clark, Jun. Am. Soc. C. E., Consulting Engineer, of Toledo, during which period he was in responsible charge of the design and supervision of construction of water-works at Barberton, Piqua, and Fremont, Ohio, and Decatur, Ill.

In March, 1927, Mr. Roberts became an associate in the organization of H. P. Jones and Company, Engineers, of Toledo, and was with this firm until his death, as Engineer in charge of design and construction of sewerage, sewage treatment, and water purification works. During this period he played an important part in the design of works of large magnitude at Toledo, Findlay, and Lima, Ohio, all of which in no small degree bear silent witness to his skill as an Engineer, and his loyalty to the interests of his clients and business associates.

---

[1] Memoir prepared by Harvey P. Jones, M. Am. Soc. C. E.

Mr. Roberts met a tragic death in an heroic but futile attempt to rescue a plant operator at the sewage treatment works at Lima, Ohio. The plant operator had descended a shallow manhole alongside one of the digesters for the purpose of taking a sample of the sludge. After having taken the sample he came to the surface and passed it to one of the other employees who was assisting him, when he slipped back to the bottom, apparently having been partly overcome by gas which had accumulated in the manhole. Striking his head on the concrete floor, he suffered a fractured skull from which he later died. Mr. Roberts, who at the time was about a quarter of a mile away from the scene, hearing the alarm sent out by the operator's assistant, ran to the spot and after issuing orders that an ambulance and the rescue squad of the local gas company be sent for, descended into the manhole, hoisted the operator's body on his shoulders, and started out. He had almost accomplished his aim, when he himself was overome by the gas, slipped back into the chamber, and never regained consciousness despite all the efforts which were made to resuscitate him after his body was brought to the surface.

Mr. Roberts was admired and respected by all who knew him, and his friends loved him. His technical ability was rapidly winning high recognition in his profession to which he had always been an honor. He lived as unselfishly as he died, and his last act is by far the best eulogy to him that could be written.

He was married on June 15, 1922, to Marie S. Hoffman, at McConnelsville, Ohio. He is survived by his widow, and his daughter, Henrietta Jane, aged four, of Toledo; his parents, Mr. and Mrs. C. H. Roberts, of McConnelsville, and two sisters, Vera and Dorothy Roberts, of Cleveland, Ohio.

Mr. Roberts was elected a Member of the American Society of Civil Engineers on November 14, 1927.

WILLIAM LAWRENCE SAUNDERS, M. Am. Soc. C. E.[1]

Died June 25, 1931

William Lawrence Saunders, the son of William Trebell and Eliza (Morton) Saunders, was born on November 1, 1856, at Columbus, Ga. He was the grand-nephew of Robert Saunders, the fourteenth President of William and Mary College, at Williamsburg, Va., and a descendant of Sir Edward Saunders, one of the Knights of the Horseshoe, who discovered the Alleghany Mountains. His ancestors landed with the Jamestown Expedition, at Jamestown, Va.

---

[1] Memoir prepared from information supplied by the Ingersoll-Rand Co., New York, N. Y., and on file at the Headquarters of the Society.

Mr. Saunders was graduated from the University of Pennsylvania in 1876 with the degree of Bachelor of Science, and, in 1911, he received the degree of Doctor of Science, from the University. Previous to his graduation, he was Editor-in-Chief of *University Magazine* and the Class Poet. In 1925, he received the honor cup of the University of Pennsylvania, awarded annually to its most distinguished alumnus.

He was engaged in newspaper work in Philadelphia, Pa., from 1876 to 1878, but, in the latter year, he left this field for an engineering career which he began with the National Storage Company of Communipaw, N. J., as Engineer in Charge of Dock Construction at Black Tom Island, New York Harbor. He designed and patented apparatus for subaqueous drilling with a tube and water-jet system now in general use.

In 1882, he was appointed Engineer for the Ingersoll Rock Drill Company. Mr. Saunders subsequently served as Secretary, Vice-President, and Director of the Ingersoll-Sergeant Drill Company, and President of the Haesler Tool Company and the New York Imperial Tool Company. When the Ingersoll-Sergeant Drill Company was succeeded by the Ingersoll-Rand Company, he became President and, ultimately, Chairman of the Board of Directors of this firm, of which he was also Director.

In addition to his device for drilling rock under water, he was also the inventor of the Ingersoll track and bar channelers and gadders for quarrying dimension stone, marble, and limestone. He was interested particularly in the application of compressed air to engineering problems and relative to this pursuit made two balloon ascensions, reaching a height of 3½ miles, which necessitated his remaining above the earth over night. He also invented the radial axe system for coal mining and the method of pumping liquids by compressed air.

Mr. Saunders was for some time Editor of *The Compressed Air Magazine* and author of a number of books on compressed-air information and compressed-air production. He was also co-author of "The Subways and Tunnels of New York," "Rock Drilling," and numerous other pamphlets and articles in *Engineering* and other journals.

Mr. Saunders' later life comprises a record of distinguished public service. One of the most important public positions which he occupied was that of Chairman of the Naval Consulting Board to which he was appointed by President Wilson in 1915 and which he filled for many years. A prominent member of the Democratic party in New Jersey, he had been a member of the State Democratic Committee, and in the Presidential campaign of 1916, he was one of New Jersey's representatives on the Democratic National Committee. He had been twice Mayor of North Plainfield, N. J., and maintained his home in Plainfield.

During the World War, he was one of the organizers and a member of the Military Engineering Committee which at its own expense equipped a regiment and sent it to France. This was the first regiment of American engineers to pass through London, England, also the first in action at the Front. Mr. Saunders was an expert on submarine warfare and equipped the *Lucia,* an Austrian ship interned in an American port at the beginning of the war, as "an unsinkable ship." His theory was that 12 000 boxes fastened to the ship's framework below the water line would keep her afloat no matter how riddled she might be by torpedoes. This conviction, however, remained untested. Although the *Lucia* carried several contingents of troops to France, she was never struck by a torpedo. During the war, he also served on the District Committee on Capital Issues of the Federal Reserve Board and on the Advisory Board of the Fuel Administration Committee.

Mr. Saunders was a leader in the crusade against cancer and, in 1926, announced through the American Society for the Control of Cancer that he would present a prize of $50 000 for the discovery of the nature of human cancer and an equal amount for the discovery of a positive cure. After two years, he was convinced of the futility of the search and withdrew his offer.

He was a Past-President of the American Manufacturers' Export Association and had also served as President of the American Institute of Mining and Metallurgical Engineers. In 1926, he founded the Mining Medal of the Institute to recognize achievement in mining. He was a former Governor Director and Deputy Chairman of the Federal Reserve Bank of New York and had been a member of the New Jersey Harbor Commission, the New Jersey Board of Commerce and Navigation, and the Advisory Committee of the Federal Trade Commission. He was a former President, Trustee, and a member of the Finance Committee of United Engineering Society, and a member of the Chamber of Commerce of the United States, the National Foreign Trade Council, and the National Civic Federation. He was also a Fellow of the American Geographical Society, the Academy of Political Science, and the American Academy of Political and Social Science.

Mr. Saunders was a former President of the Machinery Club, and a member of the Engineers, India House, and University Clubs, and the University of Pennsylvania Club of New York, N. Y., and a Director of the Penn Alumni Realty Corporation. He was also a member of the Gogebic Country Club, of Ironwood, Mich.

He was married at Narragansett Pier, R. I., in 1886, to Bertha Louise Gaston, who died in December, 1906. He was embarked on a world cruise when he died in his seventy-fourth year in Teneriffe, Canary Islands. He is survived by two daughters, Mrs. Maxwell Evarts Perkins and Mrs. John E. Lancaster, Jr.

Mr. Saunders was elected a Member of the American Society of Civil Engineers on November 3, 1886.

ANTON SCHNEIDER, M. Am. Soc. C. E.[1]

DIED NOVEMBER 28, 1929

Anton Schneider was born on January 11, 1871, at Summit Hill, Pa. His grandfather was one of the patriotic citizens for whom the United States is indebted to the German Revolution of 1848. These citizens transferred their loyalty from the Mother Country, which rejected their ideas of freedom, to the new home of liberty and lavished on their adopted country the enthusiastic devotion to its principles which had been denied them at home. This patriotism was inherited by the grandson and was demonstrated in his efforts to do his part in the World War against those who were still contesting the right of individual freedom of thought and action. Mr. Schneider's mother, Sarah (Wintersteen) Schneider, was of Dutch extraction, the family having been early settlers of Pennsylvania; one of them was a victim of the Wyoming Massacre.

---

[1] Memoir prepared by Charles Carroll Brown, M. Am. Soc. C. E.

The record of Anton Schneider, the engineer, shows graduation from Lehigh University with the degree of Civil Engineer in 1892, and with vacation experiences as Chainman and Draftsman on the Central Railroad of New Jersey and in the Pencoyd Bridge Works. His employment after graduation began with the Utah and Idaho Division of the Union Pacific System on general maintenance of way, location of spurs to mines, etc. He also designed and supervised the construction of several bridges during the period from 1892 to 1898 for organizations which were affiliated with the Railroad System. These years also included work as Draftsman for the Pioneer Electric Power Company, Ogden, Utah, and as Assistant Engineer on the Oregon Short Line Railway.

Mr. Schneider enlisted for the Spanish-American War in the 2d United States Volunteer Engineers, stationed at Honolulu, Hawaii, and was rapidly promoted during the years 1898 and 1899 from Private to Sergeant, Second Lieutenant, and First Lieutenant.

The conclusion of the war gave him the opportunity of returning to transportation engineering under the former Chief Engineer of the Union Pacific System, George H. Pegram, Past-President, Am. Soc. C. E., who had become Chief Engineer of the Rapid Transit Lines, of New York, N. Y. Mr. Schneider served as Resident Engineer on elevated railroad construction on the Manhattan Elevated Railroad, including elevated tracks, yards, shop, terminal and passenger station buildings on the Third Avenue and Bronx Park Extensions, and in the 179th Street yards and shops.

He resigned late in 1902 to accept a position on the construction of the Cerro de Pasco Railway at the summit of the Andes Mountains in Peru—the highest railroad in the world. He served two months as Division Engineer and ten months as Acting Chief Engineer. Although the road was only 84 miles long it offered unusual opportunities for practice in surmounting unique difficulties. The construction period was short and the elevation too high for comfortable living—impossible for his family—so that he returned in 1904 to his position on the Manhattan Elevated Railroad in charge of station finish in New York subways. In 1905, he was transferred to the Rapid Transit Subway Construction Company, an associated company of what had then become the Interborough Rapid Transit Company. His work with this Company was as Division Engineer in charge of the East River Tunnels at the Battery, which work he followed from the bulkheads at the river bank to the completion of the tunnels in January, 1908.

In May, 1908, Mr. Schneider resigned to accept a position as Superintendent of the Pierce Phosphate Company, at Pierce, Fla. The phosphate mining industry in Florida, at this time, was more or less in an experimental stage, and Mr. Schneider demonstrated his abilities by improving the methods of operation of the plants and the handling of labor forces, including the beautification of the Company's property and the improvement of the living conditions of its operatives. This work was so notable that, in 1912, he was employed by the Amalgamated Phosphate Company, of which Mr. B. H. Brewster, Jr., was then President, and, in collaboration with Mr. Frank I. Wheeler, a Vice-President of the Company, he constructed an entirely new plant for mining phosphate on the Company's large properties at Brewster, Fla. This plant was considered for many years to be the largest and most efficient in the State and was designed so as to be the most economical in operation. Mr. Schneider continued in the operation of the plant under Mr. Brewster, and under the American Cyanamid Company, after the sale of the Brewster interests, until the climax of the labor troubles in the industry in 1919 following the World War. For a year or more, he aided the Tennessee Copper Company and its allied interests in plans for a new phosphate mining plant in Polk County, Florida, which plans, however, the Company finally decided to abandon.

In 1922, the Cities of Bartow and Lakeland, both in Polk County, received new charters and re-organized their City Governments. Bartow had long been the center of the phosphate industry in Florida and was the residence of Mr. Schneider at this time. Naturally, the municipal authorities of Bartow selected him for the new position of City Manager, but in less than a year the City of Lakeland offered him a larger field as its City Manager. The latter position he occupied for about four and one-half years during the period of the development and height of the rapid growth of so many Florida cities, from 1922 to 1928. His influence throughout these strenuous times was for the orderly advance of the city, with due attention to the economies of construction and the control of development, so that neither citizens nor city should be overloaded with debts. Complete control by any one was impossible, but the engineering and administrative skill exercised by Mr. Schneider and his associates has been evident to the many visitors, and the ability of the City Government to meet its indebtedness as it has become due, is unquestionably because of their intelligent efforts, supplemented by careful attention to the details of settlements by the Administrations following that of which Mr. Schneider was so valuable a member. During the last two or three years of his work for the City, his health was far from good, and his resignation was too long delayed, owing to his strong sense of duty. He lingered a few months and died at his home in Lakeland, on November 28, 1929.

The record of Anton Schneider, the man, is even more remarkable than that of the engineer. This may be illustrated by his treatment of difficulties connected with the performance of his technical duties. Perhaps the most outstanding of these were his acts and his attitude with reference to the labor troubles in the phosphate field immediately following the World War. A large share of the market for phosphate rock was in countries allied with Germany and, consequently, the business of the companies was very dull during and immediately after the war. When the cost of living increased, most of the phosphate companies were not in a position to increase wages without lowering dividends or suffering deficits. The Brewster Plant, as designed and operated by Mr. Schneider, was able to operate at a profit, even in those troublous times, and he could increase wages and operate in the face of strikes elsewhere, a result which was satisfactory to his own Company, but very unsatisfactory to others in the field.

Although he entered the contest subscribing to the views on the treatment of labor held by the other operators in the phosphate field, he studied the questions so thoroughly that he was converted to the view that economy of operation depended on the status of the employees. Unfortunately, the Eastern owners of most of the phosphate properties could not accept this view, partly because they were not able to operate their less ably designed properties in competition; consequently, pressure was brought to bear on the Eastern owners of the Brewster Plant which forced them to accept the common view and withdraw their support from Mr. Schneider. His sympathies, as well as the larger economic view which he had attained, supported him in his straightforward and strong adherence to what he considered to be right, even up to the point of causing his discharge. Expressions from several sources are to the effect that men familiar with the phosphate business, but not engaged in fighting the labor strikes, believed that he handled the situation for the best interests of his Company and was running his mines and works smoothly when others operated under great difficulties or shut down. His reputation as the best of plant designers and operators was well established by the results obtained until the severance of his connection with the field.

The handling of the difficult situations which arose during his term as City Manager of Lakeland also illustrated his tenacious adherence to what he considered just and right and his ability to devote himself to the solution of the larger problems, making his subordinates responsible for the development of the details. He had the ability to choose capable men, and he held the engineer's logical belief that they could and should do their best work if made responsible for the results. At times, the demands of those who were over-developing the city were too strong for the Commissioners to resist, but they accepted the responsibility for the consequent errors in policy and in technical matters which resulted.

The love of the arts—graphic, literary, and musical—was a dominant characteristic of Mr. Schneider's mind. His friends of early and middle life would feel that his story was incomplete if it did not emphasize, not only his appreciation of music, but the way in which he expressed it in performance, particularly of the works of Bach, Beethoven, and Brahms, to which he restricted himself as the years went on. His work in painting was also very creditable.

A few selections from many full expressions of appreciation of Mr. Schneider's qualities will show that his character and ability were recognized even by those who at times were opposed to his interests:

"Mr. Schneider was always cheerful, hopeful and enthusiastic in his work, of marked ability and thoroly reliable. He had a host of friends whom he was considerate enough to keep in touch with when in their vicinity."

* * * *

"He was a thoroughly competent engineer, resourceful, very energetic, and the quality of his integrity was such that he would not tolerate the slightest deviation from the truth as he saw it. * * * His personality was so strong and true that no separation can bring forgetfulness of the sympathetic understanding he gave in full measure to his friends. I knew and respected his ability as an engineer, but to me his capacity for friendship was so rare and fine a thing that I cannot write of him without trying to convey some idea of my appreciation of his personality."

* * * *

"The men who worked for him swore by him, loved him, worked over time for him, and would fight for him if necessary. Perhaps this is his best publicly known trait. He built for permanence. When Anton Schneider approached a piece of construction work he did not ask what is the cheapest way or what is the quickest way, or even what will please best. His question was what is the best known construction."

* * * *

"He searched out the strong points in individuals and stimulated them to their best efforts. He was happiest when helping in the improvement and the progress of his subordinates. I think his outstanding characteristic was his innate interest in and affection for people. * * * Mr. Schneider's frequent advice was to solve the main problem in a broad way in advance without cluttering the mind with too many details. This practice he followed. His scientific trend of mind enabled him to co-ordinate men, materials and equipment to obtain maximum results with minimum effort and waste. These qualities, together with his ability to obtain loyalty and co-operation from employes thru his sympathy and understanding, made him an exceptional executive."

* * * *

"He performed unusually difficult jobs and tackled them with a quiet concentration, carried them thru with highly original methods, and refused to talk about them or write about them. He abhorred the spectacular. When a job was finished he avoided the dedication functions and went fishing. His primary concerns in his professional tasks were a vigorously honest result and a vigorously humane treatment of the people who worked with him, down to the last laborer. While he trusted details to an astutely selected organization, he gave personal attention on his larger construction work to the living conditions, the medical care, and the recreational opportunities of his workers, skilled and unskilled, and he would not tolerate harsh treatment of any animals on the project. He showed equally to all men on his surface what he felt in his soul and he was frank and artlessly fearless. He experienced a lively satisfaction in solving the seemingly insoluble, and in meeting without a quibble his impregnable conscience concerning good workmanship. He would have revelled in the construction of medieval cathedrals, discussing stresses, form, balance, and line with the masters and apprentices; for he was blessed with an esthetic sense (oil painting was one of his hobbies) as well as an engineer's intuitive feeling of conformity with Nature and her laws and of the social service which his works rendered. Gentleness mixed with inflexible firmness, a feeling for solidity graced with line, a whimsical humor untinged with malice, an unaffected modesty modestly shown, and a sense of the reality of intangibles,—these were the attributes of soul which he brought to his work as an engineer."

At Ogden, Utah, Mr. Schneider met Mary Robinson and they were married on November 18, 1896. She survives him, together with two daughters.

Mr. Schneider was elected an Associate Member of the American Society of Civil Engineers on May 4, 1898, and a Member on July 2, 1913.

PHILIP SCHUYLER, M. Am. Soc. C. E.[1]

DIED DECEMBER 11, 1931

Philip Schuyler, the only son of the late Howard Schuyler, M. Am. Soc. C. E., and Frances (Brannon) Schuyler, was born in the City of Mexico, Mexico, on July 3, 1880. In him was the blood of pioneers on both the paternal and maternal sides. On his mother's side he was the grandson of Samuel Brannon, who came to California in 1846 and who was known as the "First Forty-Niner." His father, who traced descent from General Philip John Schuyler, of Revolutionary fame, was one of America's great railroad engineers; he located the Kansas Pacific and the Denver and Rio Grande Railroads, building the latter, as well as the Northwestern Pacific Railroad, out of San Francisco, Calif., and was Chief Engineer of the Mexican Central Railroad during its construction. Philip Schuyler also was the nephew of the late James Dix Schuyler, M. Am. Soc. C. E., one of the West's great hydraulic engineers.

His early childhood was spent in France and Switzerland, but after his father's death his mother returned with him to America, and Philip was placed in Trinity (Episcopal) School, in New York, N. Y., preparatory to entrance to Cornell University. It was necessary for him to leave Cornell in his Sophomore year (1899), at the age of 18, to assume the support of his mother.

Inherited instinct brought him to California, and, with the exception of two years in Western Nebraska and at Monterrey, N. L., Mexico, this State was the scene of his life work. His first jobs, from 1899 to 1902, were, successively, Rodman, Instrumentman, Draftsman, Engineer in Charge of Field Parties, and Inspector of Hydraulic Mines, for the California Débris Commission (United States Engineer Corps).

From 1902 to 1904, Mr. Schuyler was Engineer in Charge of Surveys for Mr. W. P. Hammon, at Oroville and Marysville, Calif., on gold-dredging operations. The years, 1904 and 1905, he spent partly in the employ of his uncle, Mr. James Dix Schuyler, and partly as City Engineer of Venice and Ocean Park, in Los Angeles County.

---

[1] Memoir prepared by Harold F. Gray and Clyde C. Kennedy, Members, Am. Soc. C. E.

Then followed his only professional excursions outside of his adopted State. In 1905 and 1906, he was Assistant Engineer with the Tri-State Land Company, at Scottsbluff, Nebr., on canal location and plane-table topography. In 1906 and 1907, he was Assistant Engineer in charge of construction of the water-works, and part of the time on supervision of the sewerage system, at Monterrey, N. L., Mexico, for the Monterrey Water-Works and Sewer Company, Limited.

He returned to California in 1907, and until 1912 was associated with Stephen E. Kieffer, M. Am. Soc. C. E., on subdivisions, sewers, water-works, and irrigation in various parts of the State.

In 1912 Mr. Schuyler entered the contracting field, constructing, among many other projects, the water supply system for the Panama-Pacific International Exposition, in San Francisco, and 8½ miles of heavy canal construction for the Orland (Calif.) Project, United States Reclamation Service.

In 1918 and 1919, he was Assistant Engineer, United States Housing Corporation, on the design and construction of paving, water and sewers, gas, street and house lighting, and pumping plant and reservoir, at the Marc Island Navy Yard, near Vallejo, Calif.

From 1919 to 1925, Mr. Schuyler was engaged in sales engineering work, in the promotion of sales of valves, water softeners, and vitrified pipe, being associated with the Coffin Valve Company, International Filter Company, Gladding McBean Company, and several others. In these years he made the contacts with engineers and contractors around which grew the rich friendships which were his most prized possessions during the remainder of his life.

This varied experience, supplemented by the extensive reading of an inquiring mind of broad interests, was preparing him for his real field of usefulness and achievement, and, in 1925, he became the first Managing Editor of *Western Construction News*, of San Francisco. He able editorship rapidly built up the new venture to a pre-eminent position in its field on the Pacific Coast, serving both engineer and contractor in the Far West.

Philip Schuyler was imbued with lofty, yet practical, idealism, and his magazine was dedicated to the cause of conservation of the West. He gave to *Western Construction News* its slogan "Keep the Far West Clean." Having always an especially keen interest in the public health relations of engineering, he labored and inspired others to labor toward the reduction of stream and bay pollution. He believed that civilization, through carelessness or neglect, should not destroy its natural resources, or defile its recreational areas.

He gave his time and strength unsparingly to the work of organizations which contributed to the engineering betterment of the West, holding membership in the American Water Works Association, the California Sewage Works Association, the Electrical Development League, of San Francisco, the Engineers' Club, of San Francisco, and the East Bay Engineers' Club, of Oakland, Calif. He was also a member of the Society of California Pioneers, the Demi-Tasse Club, of San Francisco, and the Oak Knoll Country Club, of Oakland.

There was no mean thing in the character of Philip Schuyler. Born a gentleman, he lived his life true to the instincts of a gentleman. Whatever he did, he did with enthusiasm, and he engendered enthusiasm in those associated with him. He dealt with men with courtesy and an instinctive consideration of their feelings, and he bred courtesy and high regard in return. He had a deep, friendly interest in every one he knew, and he numbered his friends by the hundreds.

In a memorial on the occasion of his death, the East Bay Engineers' Club characterized him, as follows:

"Exponent of high ideals of professional conduct; keenly interested in the progress of engineers and engineering; a strong contender for the right; acute in understanding of men and principles; warm-hearted, kindly and courteous; friendly to all and beloved by his friends; he was an ornament to the profession of engineering, and exemplified in the highest degree the attributes of a scholar and a gentleman."

In January, 1931, Mr. Schuyler was stricken with influenza, with finally fatal sequelæ, but true to his ideals of service, he refused to rest. Throughout his long illness, he never faltered nor lost courage.

He was married on April 16, 1912, to Cecily Ann McLaughlin, at San José, Calif., who, with his daughter, Barbara Anne, survives him.

Mr. Schuyler was elected an Affiliate of the American Society of Civil Engineers on December 15, 1924, and a Member on October 1, 1926.

MITSUGU SENGOKU, M. Am. Soc. C. E.[1]

DIED OCTOBER 30, 1931

Mitsugu Sengoku, the second son of Yajima Sengoku, a Samurai of the Kochi Clan, was born on June 2, 1857, at the Town of Kochi, Shikoku Island, Japan. He received his early education in his native town and, at the age of 16, he went to Tokyo, and under the guidance of the late Count Shojiro Goto, a prominent pioneer statesman and builder of the New Japan, who had come from the same clan, he entered the Science Department of the University of Tokyo, to study Civil Engineering, which course he finished in July, 1878.

Upon his graduation from the University Mr. Sengoku entered the service of the Local Government of Tokyo, where he was employed in the Civil Engineering Department, starting thus in his career as an engineer. After three years, however, he gave up this position and joined the North-Eastern Railway Company. In February, 1884, he entered the Government Railway Service, being appointed an Engineer of the Tokyo Bureau of Railways, then under the control of the Public Works Department. In December of the same year, when, at the request of the Nippon Railway Company, the Department undertook the construction work of that Company, Mr. Sengoku was ordered to take charge of the construction of the Nakata-Utsunomiya Section and, later, in September, 1885, that of the Shirakawa-Fukushima Section, both of which he accomplished successfully. In January, 1888, he was put in charge of the construction of the Shinjuku-Tachikawa Section, which now forms an important part of the Central Line, with its noted straight route extending more than 35 km. (about 22 miles). In June, 1888, he was ordered to go abroad to study railways, and was absent for 1½ years. During that trip he visited the Hartz Railway, in Germany, designed by Roman Abt, and, upon observing a mountain railway with a gradient of 16.2%, operated by a combined track of rack and ordinary rails, he submitted a report of what he had seen, and, as a result, it was decided to use the Abt System for the Yokokawa-Karuizawa Section, the present route of the Usui Pass (noted for its steep gradient) being selected.

---

[1] Memoir prepared by Mr. Shuzo Aoki, Tokyo, Japan.

In 1889, the street improvement of Tokyo was mapped out on a gigantic scale, the construction of an elevated urban line connecting Shimbashi Station with that of Ueno being included in the plan. Accordingly, in September, 1890, the Home Minister of the Imperial Government instructed the Director of the Railway Board to begin the construction work of this route, together with a large central station within the city limits. In this connection, Mr. Sengoku was appointed to direct operations and discharged his duty with his usual diligence.

In May, 1890, he was promoted to the rank of Railway Engineer of the Third Grade and in August, 1891, the title of Doctor of Science in Engineering was conferred upon him by the Japanese Government. In July, 1892, the Government Railways of Japan appointed a Committee of fifteen to make investigations and map out the best possible network of railways to be built in Japan. Dr. Sengoku was appointed a member of the Committee, and his advice proved invaluable. While acting as an Adviser, he was also made Supervisor of Surveying and Railway Construction, and the three Government Lines, the San-in, the Maizuru, and the Wakayama were surveyed under his supervision. At the time of the Sino-Japanese War (1894–1895), he was instructed to proceed to Korea on a mission entrusted him by the War Department and visited Fusan, Seoul, Chemulpo, etc., in the course of his trip. In November, 1895, after the close of the war, he was decorated by the Emperor with the Minor Cordon of the Rising Sun of the Fourth Grade, as well as a War Medal, in recognition of his services rendered during the war.

The next position, to which he was appointed in September, 1896, was that of Chief Engineer of Railways at that time under the control of the Minister of Communications. It was in this period of his life that the question of converting the gauge of the Government Railways from narrow to standard gave rise to a heated discussion in Japan. Accordingly, the Minister of Communications organized a committee to investigate the matter, instructing its members to inquire thoroughly into the question of gauge widths, expenditure involved, etc. Dr. Sengoku was included in the Committee, and, on his appointment, he made his opinion public in a pamphlet bearing the title "My Idea for Standardization of the Gauge", in which he vehemently advocated the necessity of reform. Notwithstanding his strong opinion, however, his attempt failed to obtain general support from other quarters, and the question was dropped, although it has since been repeatedly discussed both by the authorities and the politicians. In August, 1911, and again a few years later, another organization for the realization of the plan was created, and it was earnestly discussed; in each case Dr. Sengoku was appointed a member of the Committee. Especially in the latter case, when Dr. Sengoku was President of the Railway Board, a lengthy research was conducted extending over one and one-half years, and the standardization program became generally favored, but it was once again abruptly discarded, owing to his resignation and other financial difficulties, and the gauge has remained unchanged as it is to-day.

In October, 1896, Dr. Sengoku retired from the Government Service, in order to enter upon a business career, in which he achieved also a signal success. The first position he held was that of the Presidency of the Chikuho Railway Company to which he was elected in November, 1896. In August, 1896, while acting as President of the Company, he had been commissioned by the Government to organize the Imperial Iron Works, in Kyushu. In April, 1897, when the Chikuho Railway Company was amalgamated with the Kyushu Railway Company, he was named Vice-President and Managing Director of the latter Company until April, 1898, when he was elected President and Managing Director. Later, this Company amalgamated more railway companies in Kyushu, such as the Imari, the Hoshu, and, the Karatsu Railway Companies, under his Presidency. This position Dr. Sengoku held for ten years until July 1, 1907, when the railway was purchased and made over to the State Railways of Japan at a price of 118 850 000 yen. During his term as President the Russo-Japanese War broke out, and with the transportation of soldiers and provisions, the capacity of the railway was taxed to its fullest extent, but as President Dr. Sengoku was equal to the occasion. After the war, in April, 1906, in recognition of his service, the Third-Class Order of the Sacred Treasure was conferred upon him by the Emperor.

The first problem that presented itself in the wake of the war was how to turn to account the newly acquired rights in South Manchuria which had come under Japan's control by the Portsmouth Treaty concluded in 1905. Manchuria, in those early days, was nothing but a mere wilderness extending for hundreds of miles, uninhabited by any people, and, in order to develop its abundant natural resources, a better management of the railway ceded by Russia was judged indispensable. The Imperial Government thereupon appointed a committee to make the necessary investigations regarding its organization. Dr. Sengoku was summoned once again by the Government and was included in the Committee. His untiring efforts in conjunction with those of the Government resulted, toward the end of 1906, in the firm establishment of what is now known as the South Manchuria Railway Company.

With respect to his political career, Dr. Sengoku was elected to the Diet as a representative of his native town, Kochi, in May, 1908, having subsequently been returned three times in succession from the same constituency. He joined the ranks of the Kenseikai, or Constitutional Party, with Count

Kato at its head, and was reckoned as one of the most powerful members, feared and respected by his colleagues. About 1909, he became an Adviser of the Japan Nitrogen Fertilizer Company, a pioneer company that made a great success in the production of fertilizer and contributed much toward its improvement as used in Japan. In October, 1911, he became connected with the Inawashiro Hydro-Electric Company, a pioneer company that succeeded in effecting an epoch-making, long-distance transmission of a current of 100 000 volts.

Dr. Sengoku was not allowed to enjoy his calm private life for long, and, in April, 1914, he was recalled once again to the Government Service and appointed President of the Imperial Railway Board, which was later re-organized into the Department of Railways. While in this position, he established what was called the seven years' program of railway construction and improvement, practically fixing thereby the railway policy of Japan. On the occasion of the Imperial Enthronement that took place during his Presidency, in April, 1915, he was appointed a Councillor in connection with the Enthronement function and discharged his duties most successfully.

In January, 1921, he became the President of the Civil Engineering Society of Japan, for a term of one year.

In July, 1924, Dr. Sengoku was appointed Minister of Railways. Some of the principal innovations effected during his administration comprised a plan for establishing a hydro-electric power station on the River Shinano and a steam power station in the neighborhood of Tokyo. These plans were intended to materialize the long-cherished policy of the Government Railways, that is, to be self-supporting as regards power. The gigantic work of replacing the old couplers on the entire rolling stock on the Government Railway System with automatic couplers, that was effected in the course of a few days in July, 1925 (an incident unprecedented in railway history) was another service rendered by him during his administration.

In 1925, Dr. Sengoku was appointed a permanent member of the House of Peers by the Imperial Order, but in June, 1926, when he was relieved of his post as Minister of Railways, he resigned his membership in the House of Peers, at the same time withdrawing himself from the Minseito Party, thereby withdrawing from political life.

In January, 1926, after the World War, he was decorated with the First Class Order of the Sacred Treasure, on account of his services rendered during the war.

The position that eventually proved his final service for the State was that of the Presidency of the South Manchuria Railway Company, of which he had once been a promoter. This position he held for 1 year and 10 months, but during his Presidency, he encountered difficult management problems, owing to the financial depression throughout the world as well as the slump in the price of silver. Notwithstanding the general unfavorable situation, he was indefatigably striving to improve it when he was suddenly taken ill in February, 1931, on his way from Dairen, in Manchuria, to Tokyo, and, after a protracted illness, he died on October 30, 1931, at his villa in Katase, near Kamakura.

Dr. Sengoku possessed a somewhat unusual character in that he was extremely open-hearted, straightforward, and bold. In writing of him, Mr. Tsunego Baba, one of the most popular critics of present-day Japan, says:

> "While Mr. Sengoku was open-hearted, intrepid and even passionate in his rage, thus earning him his nickname 'Thunder', it must be remembered that he was also a man of genial, kindly nature, always ready to help others at the expense of his own interest. As such he was always loved and respected by all who knew him."

Dr. Sengoku's kindness might be illustrated by an incident that took place a few weeks before his death. For a long time he had cherished the hope of erecting a bronze statue in honor of the late lamented Cornelius Johannes Van Doorn, a Dutch engineer, who had done much in the cause of civil and hydraulic engineering work in Japan, but through some hindrance or other, he had long been unable to realize it. His untiring efforts, however, were crowned at last with success, and through the activity of the Tokyo Electric Company, and others, a statue in memory of the deceased engineer was at length erected on the shore of Lake Inawashiro and unveiled sixteen days prior to Dr. Sengoku's death.

As regards diversions, golf and the Japanese game of "go" were two pastimes in which he took much interest. Upon his death he was decorated by the Emperor with the First Class Grand Cordon of the Rising Sun, and his court rank was promoted to that of the Junior Third Class. Again, on the occasion of the funeral service, His Majesty, the Emperor, was pleased to grant a considerable sum of money toward the funeral expenses.

Dr. Sengoku was elected a Member of the American Society of Civil Engineers on June 4, 1890.

ROBERT AMES SHAILER, M. Am. Soc. C. E.[1]

Died July 7, 1930

Robert Ames Shailer was born in Roxbury, Mass., on July 27, 1852, the son of the Rev. Julius S. Shailer and Catherine Parmelee Read Shailer, of Haddam, Deep River, Conn. The Rev. Julius S. Shailer was Principal of the Suffield Literary Institution and Pastor of the Ruggles Street Baptist Church, of Roxbury.

Robert Ames Shailer was graduated from the Boston Latin High School and, subsequently, from the Massachusetts Institute of Technology in the Class of 1873.

Among the following are listed the various positions which he held after his graduation from college: 1873 to 1874, Draftsman, with the New England Iron Company; 1874 to 1875, Transitman for the Lawrence (Mass.) Water-Works; 1875 to 1878, Assistant Engineer for the American Bridge Company; 1876 to 1878, Resident Engineer on construction of foundations for the Poughkeepsie Cantilever Bridge over the Hudson River; 1878 to 1879,

---

[1] Memoir prepared by George H. Pegram, Past-President, Am. Soc. C. E., and George Perrine, M. Am. Soc. C. E.

Assistant Engineer, United States Engineers, in charge of an hydraulic survey of the Charles River, Massachusetts; 1879, Assistant Engineer with the late C. Shailer Smith, M. Am. Soc. C. E., Consulting Engineer, at St. Louis, Mo., and later in the same year, Assistant Engineer, the Edgemoor Iron Company. He was also employed from 1885 to 1890 by Rush and Coolidge, Chicago, Ill.

When Mr. Shailer was elected a Member of the Engineers Club in New York, N. Y., in 1906, the following additional professional record was supplied: Assistant Superintendent of the Edgemoor Iron Company; Engineer of Bridges and Buildings for the Chicago, Milwaukee and St. Paul Railroad Company; President of the firm of Shailer and Schniglau Corporation, Engineers and Contractors, of Chicago; President of the Boston, Mass., Tunnel Construction Company; and, later, Chief Engineer of the Degnon Contracting Corporation, in New York, during the construction of the Steinway Tunnel under the East River.

The Shailer and Schniglau Corporation constructed, during 1893 and 1896, one mile of the Chicago Drainage Canal and the Lake View Tunnel, two miles of which were under Lake Michigan, including the intake in the lake, and one mile of Hyde Park Tunnel; the two latter works were for the water supply of the City of Chicago. This firm also built the Scollay Square Section and the Haymarket Square Section of the Boston Subway. The Boston Tunnel Construction Company of which Mr. Shailer was President constructed a double-track concrete-lined tunnel, 5 000 ft. long, under Boston Harbor. Secretary of Commerce B. P. Lamont was Secretary and Engineer with the Shailer and Schniglau Corporation.

The contracting firm of Shailer and McCormick built the pneumatic foundations for the Newburyport, Mass., Bridge over the Merrimac River and, later, constructed a reinforced concrete viaduct at Jacksonville, Fla., over the tracks of the four railroads entering that city.

Mr. Shailer was also Managing Engineer for the T. A. Gillespie Company, General Contractors, on the construction of the Rondout Siphon of the Catskill Aqueduct, which supplies water to New York City.

Subsequently, he was Consulting Engineer for the General Contractors on the construction of the Cambridge Subway and the Beacon Hill Tunnel of the Boston Elevated Railway System. From 1915 to 1919, he served as Tunnel Engineer for the Rapid Transit Subway Construction Company, during the completion of the Queensboro Tunnel, originally the Steinway Tunnel, starting at 42d Street and Park Avenue, New York City, passing under the East River, and terminating in Long Island City. This tunnel was originally designed for the use of trolley cars, but, later, was changed to a third-rail system, and in 1913 was made part of the Interborough Rapid Transit System. This work was completed in conjunction with what was commonly called the "Diagonal Station" at 42d Street, and which has since become the Grand Central Station of the Interborough Lines.[2]

Mr. Shailer possessed an attractive personality, a keen mind, and a fund of energy, and was endowed with what might be termed a natural accuracy and directness. These qualities contributed largely to his unusual ability as an executive, and as an adviser. His statements even in ordinary conversation were clean-cut and definite, without loquacious or humorous "trimmings".

---

[2] *Transactions*, Am. Soc. C. E., Vol. LXXXII (1918), p. 327.

He was very fond of fishing and hunting—in fact, he might be called an expert in these sports, in which he was active until very near the end of his life, and, aside from his friendly manner, his stories of exploits made him a charming companion. He had a host of friends and could generally be found among a group of them in his leisure moments. He was a Thirty-second Degree Mason.

To his engineering brethren and the contractors with whom he was associated, he was frank and free in stating his experiences and helping them to solve doubtful problems. In short, he was a man whose demise is a notable loss to the Engineering Profession and a great bereavement to his many friends.

On May 31, 1877, Mr. Shailer was married to Grace Ellen Hooper, the daughter of Frederick T. Hooper and Louise (Stetson) Hooper, of Boston, Mass. Their family consisted of five daughters and one son, of whom three daughters, Mrs. Alfred C. Turner, Mrs. Wilson C. Handyside, and Mrs. Paul F. Avery, and the son, Harold Read Shailer, survive him.

Mr. Shailer was elected a Member of the American Society of Civil Engineers on March 3, 1880.

WILLIS APPLEFORD SLATER, M. Am. Soc. C. E.[1]

DIED OCTOBER 5, 1931

Willis Appleford Slater was born in the small town of Polo, Ill., on October 14, 1878, and received his early education in the country schools and High School of that community. Both his father, Belford Slater, and his mother, Ruth (Appleford) Slater, were of English descent, the first of his line having settled in Lynn, Mass., in 1684. The value of advanced knowledge and the discipline of work were thoroughly instilled in the boy, and formed the very apparent and sound foundation of his later achievements and reputation.

Willis Slater was graduated from High School at the normal age of sixteen, but from that time he alternated periods of gainful employment with those of further schooling. Three years after leaving High School he was able to attend Valparaiso University, at Valparaiso, Ind., for two years. After two more years, he entered the University of Illinois at Urbana, Ill., as an engineering freshman, and was graduated with his class in 1906, at the age of twenty-seven, with the degree of Bachelor of Science in Municipal and Sanitary Engineering..

Field experience as Axeman, Rodman, Leveler, and Chief of Party followed in 1906 and 1907, taking Mr. Slater eventually to Elgin, Ill., where he met and loved Clara Mary Knodle, whom he married on June 3, 1908.

---

[1] Memoir prepared by Arthur R. Lord and Franklin R. McMillan, Members, Am. Soc. C. E.

Meanwhile, he had moved on to Ely City, and McGill, Nev., and to Salt Lake City, Utah, working as Draftsman and Designer on municipal and structural projects. In the fall of 1908, he was appointed Research Fellow in the Engineering Experiment Station, and returned to the University of Illinois with his bride of three months. By this time he had determined to specialize in reinforced concrete, and here began, under the guidance of Arthur N. Talbot, Past-President and Hon. M. Am. Soc. C. E., the program of research in structural engineering that was to engage Professor Slater for the remainder of his life. Here was born his only child, Ruth Marjorie Slater, now a student in the University. At this time Willis Slater was a mature man of thirty, and his work attracted favorable attention at once. After receiving his degree of Master of Science in Theoretical and Applied Mechanics in 1910, he was retained at the University as First Assistant in the Engineering Experiment Station from 1910 to 1915, and as Research Assistant Professor from 1915 to 1917. His professional degree of Civil Engineer was awarded in 1912, in recognition of distinguished technical research already completed.

At this period began the extensive output of technical papers and bulletins on which his fame particularly rests. While still a Research Fellow, he became a member of the American Concrete Institute and of the American Society for Testing Materials and contributed to their proceedings and conventions. Research for Professor Slater meant the most careful preparation for the test program, with a full consideration of all available data in the same general field. His specimens were designed, his readings taken, so as to secure significant data only. Complicating variables were foreseen and controlled as necessary. With his data secured, there followed a long period of exhaustive study and analysis. Often additional tests were conducted to clear some doubtful point. Very commonly his analysis included the derivation of new mathematical formulas for the use of others in further investigation, or as a basis for design. Publication was always complete, with his data, analyses, and conclusions fully stated and most carefully and conservatively worded. The published papers of Professor Slater, as a result, commanded the respect and confidence of the Structural Engineering Profession by their uniformly high quality, and their practical usefulness.

The services of Professor Slater came greatly into demand for research undertakings away from the campus. He was employed as Consulting Engineer to conduct extensive investigations into the properties of gypsum and to establish standards for structural design in that material. Later standardization by the American Society for Testing Materials was largely founded on this work. With Professor Talbot he was closely connected with the early tests for actual stresses in reinforced concrete buildings, and they were joint authors of two *Bulletins*[2] on this subject. With H. M. Westergaard, M. Am. Soc. C. E., he prepared a paper, "Moments and Stresses in Slabs," which was first published by the American Concrete Institute,[3] and almost immediately reprinted by the National Research Council.

---

[2] "Tests of Reinforced Concrete Buildings Under Load," by Arthur N. Talbot and W. A. Slater, *Bulletin No. 64*, Univ. of Illinois Eng. Exper. Station; and "Tests of Reinforced Concrete Flat Slab Structures," by Arthur N. Talbot and W. A. Slater, *Bulletin No. 84*, Univ. of Illinois Eng. Exper. Station.

[3] *Proceedings*, Am. Concrete Inst., Vol. XVII (1921), p. 415.

With the entrance of the United States into the World War and the suggestion that ships be built of concrete in the emergency, Professor Slater was called to the Bureau of Standards, in Washington, D. C., and there made the preliminary study to determine the feasibility of such construction. A lack of adequate design data was discovered and a tremendous research program was instituted under Professor Slater's direction, on the results of which the design of concrete ships and barges was founded. While these tests were planned for this particular use, the data thus secured proved to be exceedingly valuable for the design of all types of reinforced concrete structures. Professor Slater's paper[4] before the American Concrete Institute in 1919, in which these data were summarized, was awarded the Wason Medal of the Institute.

These exhaustive data, still incompletely studied and analyzed, passed to the Bureau of Standards at the close of the war, where, through Professor Slater's further study, they formed the basis of several important *Bulletins*, applying the results to peace-time uses. Professor Slater remained at the Bureau from 1919 to 1928, becoming Chief of the Section of Masonry Structures. During these years he was frequently "borrowed" in order that he might direct some outside research project. Tests of clay-tile-and-concrete-joist construction in Ohio laid the solid foundation on which such construction is now universally designed. Most notable of all, however, was the Arch Dam Investigation of Engineering Foundation, which took Professor Slater to California for two years, from 1925 to 1927, and demanded the highest skill both in experimental technique and in mathematical analysis. During 1923 and 1924 he was happy to be able to serve his Alma Mater for sixteen months, supervising the concrete construction of the Illinois Memorial Stadium. During this period, also, he was honored by appointment of the American Society of Civil Engineers as Chairman of its representation on the Joint Committee for Specifications for Concrete and Reinforced Concrete.

In 1928, Professor Slater was appointed Research Professor of Engineering Materials and Director of the Fritz Engineering Laboratory at Lehigh University, Bethlehem, Pa. In this strategical position he was rapidly building up the research facilities and reputation of that laboratory. An outstanding investigation under way there was the fundamental column research of the American Concrete Institute (divided equally between Lehigh University and the University of Illinois), conducted by a committee of which Professor Slater was Chairman. Most of the testing had been completed and highly significant results already obtained at the time of his death. Two progress reports had been published, and the third and fourth reports were in preparation, when he was taken, in his fifty-third year, at the height of his power and usefulness.

Professor Slater had always so many things of overwhelming interest to himself, of so much importance to others, to attend to immediately, that he continually overtaxed his strength. He was not only a member, but was decidedly active in both administrative and technical committee work in many technical and scientific societies. He was a member of the American Society for Testing Materials, American Concrete Institute, Concrete Institute (British), American Association for the Advancement of Science, Washington Academy of Sciences, and the American Welding Society. He was Chairman of the present National Joint Committee on Specifications for Concrete and Reinforced Concrete. Only in recent years did he come to realize that he must give up some of the older obligations when he accepted the new and greater responsibilities that were continually seeking him out. While still at the University of Illinois he was subject to severe and prolonged headaches and these, later, became almost daily in occurrence. He worked steadily on. While in recent years this trouble had moderated somewhat, it was never completely overcome by medical care. The two years in California, on the Arch Dam Test at Stevenson Creek, built him up wonderfully, but the cumulative effect of long years of over-strain was apparent in the complications that followed suddenly upon an attack of tonsilitis and rendered it fatal.

---

[4] "Structural Laboratory Investigations in Reinforced Concrete, Made by Concrete Ship Section, Emergency Fleet Corporation," *Proceedings*, Am. Concrete Inst., Vol. XV (1919), p. 24.

Perhaps the best measure of Professor Slater's personality was the number and intimacy of his immediate friendships. His personal charm, his intellectual integrity, his unquestioned impartiality and fairness, and his rare patience won every one who became closely associated with him in professional work or engineering society activities. And these friendships were of the enduring kind; to know him better was to love him more. In the quality of his friendships, no less than in the thoroughness of his researches, he leaves a record in which any man could take pride. At conventions he was ever surrounded by a group of those who had come to know the pleasure it gave him to assist others, out of his great fund of knowledge and experience. He was fond of hearing and of telling stories—invariably clean stories—and was given to relieving the intensity of committee meetings by a keen humorous remark. He constantly encouraged younger research workers, and shared authorship with his assistants generously. Of him, as of the "Home on the Range," it could be said that:

"There seldom was heard
A discouraging word,
And the skies were not cloudy all day."

Professor Slater was elected an Associate Member of the American Society of Civil Engineers on June 24, 1914, and a Member on January 22, 1922.

JAY L. STANNARD, M. Am. Soc. C. E.[1]

Died March 13, 1931

Jay L. Stannard was born on September 15, 1866, at Chittenango, N. Y., the son of Gilbert and Esther (Heberd) Stannard. His father, a farmer, soon moved to Iowa where the boy attended the public school at Shell Rock, and received a two years' training at Cornell College, Mt. Vernon.

---

[1] Memoir prepared by R. K. Tiffany, M. Am. Soc. C. E.

At the age of 21 years, on being offered a position as Rodman on the Chicago, St. Paul, and Kansas City Railroad (now the Chicago Great Western Railroad), Mr. Stannard was attracted from the halls of learning to the active life of construction work, which he followed thereafter until his death. Many years of his young manhood were spent in railroad construction, which branch of the profession he followed in Iowa, Idaho, Washington, Pennsylvania, and Montana. For two years, beginning in 1890, he was employed on the Staff of the City Engineer of Whatcom (now Bellingham), Wash., and from that time his interest turned largely to water supply and power problems. For several years he was employed by the California Red Wood Pipe Company and the National Wood Pipe Company in charge of many large pipe installations.

From January, 1907, to January, 1910, Mr. Stannard served as Assistant City Engineer of Seattle, Wash., in charge of the construction of the second unit of the Cedar River Water Supply and the second unit of the Seattle Municipal Power Plant. Following 1910, for several years, he was engaged in private engineering practice, designing and building water supply and power systems for many cities in Oregon and Washington. In 1923, the City Commission of Tacoma, Wash., in anticipation of the building of a large hydro-electric project, employed Mr. Stannard as Chief Engineer. After a few years, impressed by the soundness of his judgment, not only in engineering, but in the business and economic phases of the city's development program, the Commission made him Chief Engineer of Public Utilities, which position he held at the time of his death.

The Cushman Power Project of the City of Tacoma will long stand as a material monument to Jay Stannard—"The Chief," as he was affectionately known by the "men on the job". To him is due the credit for the revision of the original project plans which resulted in the highest economical development of the project, and for the bringing together of a smooth working organization which, under his direction, fashioned those plans into reality.

To his friends, associates, and employees, Mr. Stannard's life was a constant inspiration and incentive to put forth their best efforts on the work at hand. His memory will live in their hearts not only because of his engineering genius, although he had the quick, keen mind that characterizes the great engineer, nor yet because of his sound judgment on business and financial matters, although in that field lay, perhaps his greatest value to those he served; but rather on account of his more personal qualities—the kindliness, the gentle dignity, the friendly humor, the sturdy honesty that flowed through his every action. He was one of those rare men of whom it may be truly said: "None knew thee but to love thee, nor named thee but to praise."

Mr. Stannard was married in 1902 at Waterloo, Iowa, to Carrie A. Nettleton, who survives him.

He was the leading spirit in the organization of the Tacoma Section of the Society and became its first President in January, 1930. He was unanimously re-elected for a second term, and his trip South, while undertaken chiefly for a rest and recuperation after the long and strenuous Cushman Project construction period, was planned with the intent of calling on the various California and Oregon Sections and extending to them in person a cordial invitation to attend the Annual Convention of the Society in Tacoma on July 8 to 11, 1931.

The Tacoma Section, on April 13, 1931, unanimously passed the following resolution and ordered that it be engrossed on parchment, signed by the members of the Section, and bound in leather for presentation to Mrs. Stannard:

> "*Whereas,* Jay L. Stannard, a beloved fellow engineer, having achieved works of outstanding merit, has been called to his reward by an All Wise and Loving Heavenly Father;
>
> "*Whereas,* the engineering profession has been uplifted by his faithful service and our community has been enriched by the ample fruits of his zeal, proficiency, and wisdom; and
>
> "*Whereas,* His professional associates mourn his loss and in so doing keenly realize that Mrs. Stannard has sustained a still greater loss; therefore be it
>
> "*Resolved,* That we bow to the Divine Will, with a spirit of thankfulness for the inspiration and enthusiasm gained by association with our lately departed brother;
>
> "*Resolved,* That we express our deep appreciation of his professional integrity, his loyalty to friends, his upright and noble ideals, and his earnest efforts in behalf of the Section; and
>
> "*Resolved,* That the heartfelt sympathy of this Section be extended to Mrs. Stannard and other members of the family."

Mr. Stannard was elected a Member of the American Society of Civil Engineers on August 31, 1915.

EUGENE WASHINGTON STERN, M. Am. Soc. C. E.[1]

Died November 9, 1931

Eugene Washington Stern was born at Toronto, Ont., Canada, on August 20, 1865, the son of Samuel and Alice (Block) Stern. His father fought under Kossuth in the Hungarian War of Liberalism in 1848, before coming to America.

Mr. Stern obtained his engineering education at the School of Practical Science of the University of Toronto, from which he was graduated in 1884. From August to November, 1884, he was Rodman on the Northern and Pacific Junction Railway, and from November, 1884, to November, 1885, Assistant Engineer on the same road, in charge of the construction of twelve miles of line. From November, 1885, to May, 1886, he was Fellow of Engineering in the School of Practical Science of the University of Toronto, in charge of drafting and the field work in surveying, also lecturing on descriptive geometry.

From August, 1886, to March, 1887, Mr. Stern was with the Passaic Rolling Mill Company, at Paterson, N. J., working on the shop plans of railway and highway bridges, including those of the Washington Arch Bridge over the Harlem River, in New York, N. Y. From March to September, 1887, he was an Assistant Engineer on the construction of the Grand Avenue Cable Railway, at Kansas City, Mo., and from September, 1887, to February, 1892, he was employed by the Chicago Bridge and Iron Company, first as Draftsman and, later, as Assistant Engineer on the design of highway and railway bridges, roofs, and substructures. From February, 1892, to 1897, he was with the Koken Iron Works, at St. Louis, Mo., as Engineer in responsible charge of the design and construction of structural iron work for buildings, roofs, bridges, water towers, etc.

---

[1] Memoir prepared by Henry Goldmark, M. Am. Soc. C. E.

In 1898, Mr. Stern came to New York City where he made his home during the remainder of his life. He was Chief Engineer of the Jackson Iron Works for about three years, and then established an office as Consulting Engineer, specializing in structural steel work for buildings, etc. In 1915, he received the appointment as Chief Engineer of the Bureau of Highways in the Borough of Manhattan, and continued in this position until 1917, when he entered the military service of the United States. He served in the World War as a Major of Engineers in the Officers' Reserve Corps of the American Expeditionary Forces. He was with the Eighth French Army, Toul Sector, from November, 1917, to December, 1918. Later, he became Chief Road Officer of Base Section No. 2, and was recommended for promotion to the rank of Lieutenant-Colonel. He received his discharge on January 16, 1919

After he left the Army, Mr. Stern returned to New York City and resumed his work as Consulting Engineer, specializing in steel construction He was associated in this work with his brother-in-law, Robert D. Kohn, Charles Butler, and Associates, Architects. As an Associate of this firm of architects, Mr. Stern was in charge of the design and construction of the engineering features of many large buildings in New York and other cities. Among the most important of these were the twenty-story additions to the Department Store of R. H. Macy and Company, at 34th Street and Broadway; the New York Evening Post Building, on Vesey Street; the Nurses Home of Mount Sinai Hospital; the new building for the Temple Emanu-El; the D. Auerbach and Sons Candy Factory, Eleventh Avenue, 46th and 47th Streets, New York City; the Namm Department Store, in Brooklyn, N. Y., etc. Among other buildings with which he was professionally connected should be mentioned, the Terminal Station of the Delaware, Lackawanna and Western Railroad, in Hoboken, N. J.; the B. Altman and Company Store, at Fifth Avenue and 34th Street, and the office building of the Guaranty Trust Company, in New York City; and the State Armory, and the Home Office Building of the Travelers Insurance Company, at Hartford, Conn.

Mr. Stern was a member of the Society of American Military Engineers, the American Legion, and the Military Order of the World War. He was also a member of the City Club and of the Society for Ethical Culture in New York, as well as the Engineering Institute of Canada, the Society of Professional Engineers of New York State, and the American Institute of Consulting Engineers. The American Institute of Consulting Engineers owed its success from its foundation largely to his unremitting efforts. He was specially interested in associations dealing with Structural Engineering, being a member and active worker in the American Concrete Institute and the Structural Engineering Society of New York State.

During an extended professional career, Mr. Stern accomplished a large amount of important engineering work. He was also much admired by all with whom he came in contact for his lovable and helpful personal character.

On February 2, 1899, Mr. Stern was married to Dorothea Kohn, of New York City, who, with two children, Helen (Mrs. Mark C. Meltzer), and Theodore, survives him.

Mr. Stern was elected a Member of the American Society of Civil Engineers on March 3, 1897.

FRANK STODDARD STEVENS, M. Am. Soc. C. E.[1]

Died May 26, 1931

Frank Stoddard Stevens, the son of Noah Miller and Martha Ann (Shute) Stevens, was born at Athens, Pa., on December 7, 1850. His maternal grandmother, Hannah Stoddard, a direct descendant of William the Conqueror, was born in Minisink, N. Y., on December 26, 1802. His maternal grandfather was Abel Shute, whose parents came from Holland and settled in Westchester County, New York. Both were Quakers. His paternal grandmother, Betsy Caton, and his paternal grandfather, John Stevens, were born in Orange County, New York. The forebears of his paternal grandmother were Irish Quakers, who came to America from the north of Ireland before the Revolution. Her grandfather was in the army of General Washington.

His mother, Martha Ann Shute, was born on October 2, 1826, at Horseheads, N. Y. His father, Noah Miller Stevens, was born on August 19, 1823, at Breeseport, N. Y. He was a graduate of Oberlin College, Oberlin, Ohio, and a practicing attorney of great promise. He passed away at Athens, Pa., in the early part of his career, on February 6, 1853. After his father's death, Frank Stoddard Stevens went to live with his grandmother at Horseheads, N. Y., where he received his early education.

In September, 1869, he entered Cornell University. On March 13, 1871, Ezra Cornell, President of the University, sent him with a corps of engineers to make a survey for and construct the Utica, Ithaca, and Elmira Railroad, in which President Cornell was interested. Mr. Stevens advanced rapidly until, in 1876, he became Chief Engineer of that railroad. About this time the Utica, Ithaca, and Elmira Railroad Company became interested in the Cazenovia, Canastota, and DeRuyter Railway. Mr. Stevens was made Chief Engineer of this road also, and had exclusive charge of building the new line. He held this dual position until March, 1881, when he went to the Long Island Railroad Company as Chief Engineer, and built the Long Beach Marine Railway.

---

[1] Memoir prepared from information on file at the Headquarters of the Society.

Mr. Stevens then moved to Wisconsin as Locating Engineer on the Chippewa Valley and Superior Railway; and in December, 1881, he went to Texas as Superintendent of Construction of the Sabine and East Texas Railway. He was soon made Assistant General Superintendent, which position he left in 1883 to become Assistant Division Engineer on the New York, West Shore, and Buffalo Railway. From 1884 to 1885 he was Principal Assistant to the Chief Engineer of the road, and in 1885, became its Roadmaster. From 1886 to 1887 he was, successively, Engineer of Maintenance of Way of the New York, Ontario, and Western Railway and the Elmira, Cortland, and Northern Railroad. On November 12, 1887, he went to the Reading Company as Division Engineer, where he remained until his retirement from railway service, serving as Division Engineer, Superintendent, and Engineer of Maintenance of Way, successively, until October 31, 1923.

He was married to Anne Clapp, of Athens, Pa., on December 21, 1882. His wife died on September 7, 1922. He passed away at the home of his daughter, Helen (Stevens) Morrison, May 26, 1931, at Springfield, Mass., and was buried in Reading, Pa., on May 29, 1931, at the age of 80 years. Two children survive him, Helen (Stevens) Morrison, of Springfield, Mass., and Harold Allen Stevens, of Allentown, Pa. He is also survived by a grand-daughter, Elisebeth Stevens, of Allentown, and by two sisters, Mrs. Elisebeth (Stevens) Stevens, of Kansas City, Mo., and Mrs. Bertha W. Myers, of Horseheads, N. Y.

Mr. Stevens was a Thirty-second Degree Mason, having been admitted into Horseheads Lodge No. 364 soon after his twenty-first birthday. He was a Charter Member of the American Railway Engineering Association; and a member of the American Academy of Political Science, and the Franklin Institute of Philadelphia. From the time of its inception until 1923, or over a period of about twenty years, he was an unusually active member of the American Railway Engineering Association, serving on its most important committees and contributing the value of his knowledge as a maintenance-of-way student to their work. Many of the standards adopted by the Association during the years, 1902 to 1920, were due to his suggestions and recommendations.

He was quiet and unassuming in his manner, lending freely of his time to helping others. As a committee member he was invaluable and tireless in his efforts. He was a man of great versatility, not confining himself to a knowledge of any one subject. An ardent student of every branch of engineering and architecture, he possessed not only a theoretical but a practical knowledge as well. Mr. Stevens was entirely devoid of ostentation, which in a large measure accounted for his versatility being known only among the circle of his friends and associates. His general and technical knowledge covered a wide variety of subjects. He was courageous in maintaining his opinion, but did not force it upon others. He had his own ideals, but did not insist upon others accepting them. His hobby was work. Throughout his entire life, he was a most prodigious student. He was a man of irreproachable character and integrity, and his friends and the Engineering Profession have suffered a loss by his demise.

Mr. Stevens was elected a Member of the American Society of Civil Engineers on October 3, 1883.

LEWIS THORNBURG, M. Am. Soc. C. E.[1]

DIED JUNE 3, 1931

Lewis Thornburg, the son of Charles L. and Eulalia (Green) Thornburg, was born at Nashville, Tenn., on November 23, 1894, his father being at that time Head of the Department of Mathematics and Astronomy at Lehigh University, Bethlehem, Pa. The boy's early education was obtained in several private schools, including the Moravian Preparatory School, at Bethlehem. Following in the footsteps of his father, he showed an inclination early in life for mathematical and technical subjects, and attended Moravian College, at Bethlehem, for one year. In 1910, Mr. Thornburg entered Lehigh University, from which he was graduated with first honors and the degree of Civil Engineer in June, 1914. He was very active and popular in the student life during his college days, in addition to being very industrious. He became a Member of Beta Theta Pi Fraternity, and, as a reward for his scholarship, was elected a member of the Honorary Societies, Tau Beta Pi and Phi Beta Kappa.

During the summer vacations of his college course, Mr. Thornburg was always on the lookout for practical experience, and secured employment on various survey work and with the Construction Division of the Pennsylvania Railroad Company. After his graduation, he was appointed as Assistant to the Bursar of Lehigh University, and did private tutoring in college mathematics. Teaching, however, did not appeal to him particularly, and he was anxious to get into the practical field of engineering.

From May to October, 1915, he was employed as a Draftsman in the Engineering Division of the Standard Oil Company of New Jersey. He left this work and obtained a position as Draftsman in the Maintenance-of-Way Division of the Pennsylvania Railroad, where he was employed until February, 1916. He then went with the Lupton Steel Sash Company as a Sales Engineer, where he was engaged until November, 1917.

At this time, when every one was looking forward as to how they could best serve their country, Mr. Thornburg took the professional examination for a commission in the Civil Engineer Corps of the United States Navy. He passed this examination and, in January, 1918, was commissioned Assistant Civil Engineer, United States Navy, with the rank of Lieutenant (Junior Grade) (Temporary). After a short course of instruction at the United States Naval Academy, he was ordered to the First Naval District, Boston, Mass., as Assistant to the Public Works Officer. At that time the construction program for the Navy, in this locality, was quite large, amounting to about $3 000 000 worth of work. Lieut. Thornburg was given considerable responsibility for a man of his age and experience, and his work proved that such assignment was fully justified.

---

[1] Memoir prepared by L. B. Combs, Lt.-Comdr. (C. E. C.), U. S. N., Assoc. M. Am. Soc. C. E.

In April, 1920, he was commissioned Assistant Civil Engineer, United States Navy, with the rank of Lieutenant (Junior Grade) (Permanent), and was advanced to the rank of Lieutenant (Permanent) on July 1, 1920. From December, 1922, to August, 1923, he served as Assistant to the Project Manager for Naval Hospitals at the Bureau of Yards and Docks, and in active charge of contracts at the Naval Hospital, at Washington, D. C.

From August, 1923, to March, 1925, he was Assistant to the Public Works Officer at the Navy Yard, at Portsmouth, N. H., handling the design, construction, maintenance, and operation of all public works and public utilities at this Station. While on this duty he began some experimental concrete work, comprising the construction, installation, observation, and report of concrete piles in sea water. This investigation which is still underway, is being carried on, under the jurisdiction of the Bureau of Yards and Docks, Navy Department, for making observations and drawing conclusions which may be of value in designing and constructing reinforced concrete piles and water-front structures for use in sea water.

In April, 1925, Lieut. Thornburg was ordered to report as Assistant to the Public Works Officer, Eleventh Naval District, San Diego, Calif., and supervised the design, construction, maintenance, and operation of public works and public utilities at the Naval Training Station and the Naval Radio Stations in this District. In July, 1926, he was commissioned a Civil Engineer in the Navy and advanced to the rank of Lieutenant Commander. He served at San Diego, until March, 1929, when he was ordered to the Navy Yard at Mare Island, California, as Assistant to the Public Works Officer.

During his career in the Navy, Lt. Comdr. Thornburg had suffered frequently from illness, usually brought on by nervous strain and overwork. He was of such an active nature it was difficult for him to take things easily. In the spring of 1930, he became seriously ill and was transferred to the U. S. Naval Hospital, at San Diego, for treatment, where he was retired from the Naval Service on account of physical disability. He was much disappointed in having to accept retirement as he was very fond of the Service and his work, and regretted intensely the idea of giving up his active naval career. He apparently recovered his health to a certain extent, but was not satisfied and was always reaching out for new treatment which he thought might accomplish a complete physical recovery. With this idea in view, he went to the Mayo Clinic, in Rochester, Minn., in May, 1931, where a serious operation was found to be necessary, the shock of which his system was not able to stand.

He was married to Julia W., the daughter of Rear-Admiral H. A. Field, U. S. N., in February, 1926, who, with two daughters, survives him. He is also survived by his father and mother, three sisters, and four brothers. He was buried with military honors at Arlington National Cemetery, on June 8, 1931.

Lt. Comdr. Thornburg was elected an Associate Member of the American Society of Civil Engineers on December 15, 1924, and a Member on July 8, 1929.

LESTER WALDO TUCKER, M. Am. Soc. C. E.[1]

DIED MARCH 24, 1931

Lester Waldo Tucker was born on February 12, 1865, in Arcade, N. Y., of staunch New England parentage. His early education was obtained in the district schools of Northern Rhode Island, and he qualified in 1880 for entrance to the Scientific Department of the English and Classical School at Providence, R. I., from which he was graduated in 1882. Later in that year, he entered the City Engineer's Office in Providence, as Student and Rodman, continuing until 1886, successively, as Leveler and Transitman, on land surveys, preliminaries for water-works, sewers, harbor improvements, and various kinds of construction work.

From 1886 to 1888, Mr. Tucker was with the National Water Works Syndicate of Boston, Mass., as Construction Engineer on the water-works at Grafton and Bridgewater, Mass., Bath and Skowhegan, Me., and on surveys for other similar projects throughout New England. Later in 1888 and in 1889, he was associated with the late John W. Ellis, M. Am. Soc. C. E., of Woonsocket, R. I., engaged on surveys for several branch-line railways, and also on general city engineering.

In August, 1889, Mr. Tucker received appointment as Division Engineer with the St. Johnsbury and Lake Champlain Railroad Company, and also as Assistant Engineer for the Boston and Maine Railroad Company, with headquarters at St. Johnsbury, Vt. He continued in this position until October, 1892, and in addition to the regular maintenance and repair work, had charge of filling Pumpkin Hill High Trestle, which entailed the construction of a masonry bridge and the placing of more than 300 000 cu. yd. of material without serious interference with traffic; also, he was engaged on other similar work, including Walden Trestle, 125 ft. high, and requiring 200 000 cu. yd. of filling. Several bridges were constructed, together with engine terminals and the Buffalo Branch of the St. Johnsbury Railroad.

He re-entered the service of Mr. Ellis in 1892 as Resident Engineer in charge of the construction of Pratt Dam and a seven-span railroad bridge over the Blackstone River, at Lonsdale, R. I. Later, he had charge of the design and construction of other masonry dams at Slatersville, R. I., and Wilksonville, Mass.

From 1895 to 1902, Mr. Tucker was with the Boston and Maine Railroad Company, first as Assistant Engineer, and, later, as Engineer of Construction in charge of abolishing grade crossings in Boston, Arlington, and Beverly, Mass., together with eighteen crossings in connection with the double-tracking of the Eastern Division. He also had charge of improvements at the Mystic and Portland docks and the reconstruction of many bridges, stations, and other structures, including the design of a large amount of work for future construction.

---

[1] Memoir prepared by Messrs. E. J. Beugler, J. C. Boyd, George W. Burpee, and J. P. Snow, Members, Am. Soc. C. E.

In 1902, he became Resident Engineer for the New York Central and Hudson River Railroad Company at Albany, N. Y., in charge of construction work on the Hudson, Mohawk, and West Shore Divisions, including shops at West Albany, Union Station at Troy, N. Y., Oneida Station, Hudson River freight-line bridge, and tunnels near Fort Montgomery and Anthony's Nose. In the latter part of 1903, he was engaged in general contracting, in Massachusetts and New Hampshire, with an office at Boston. During 1904 and 1905, he was Division Engineer on the Delaware and Hudson Railroad at Schenectady, N. Y., in charge of grade-crossing abolishment and maintenance on the Delanson Division.

From 1905 to 1907, Mr. Tucker was, successively, Designing, Division, and First Assistant Engineer for the Lake Shore and Michigan Southern Railway Company. His work included the design of the Toledo Freight Transfer, changes in the Collinwood Yard, and other miscellaneous work on yards and structures; also charge of the construction of the Franklin and Clearfield Railroad, from Franklin to Brookville, Pa., entailing heavy grading, bridging, and several tunnels.

For the next thirteen years, Mr. Tucker was connected with Westinghouse, Church, Kerr, and Company, first as Engineer-in-Charge, subsequently, as Special Engineer, and then for seven years as Managing Engineer in charge of many important engagements, including investigations and reports on a large number of operating properties and projects; the design and construction of steam and electric railway facilities, hydro-electric and steam power, industrial plants, coast protection, and other works throughout the United States, Canada, and Cuba. Outstanding among these are: A report on the physical condition and maintenance of the Southern Pacific Railroad Lines in Louisiana and Texas; a survey and valuation of the property of the American Agricultural Chemical Company (68 plants located in 20 States); reconstruction of the Arkansas and Louisiana Midland Railroad (50 miles); relining two single-track tunnels on the Pittsburgh and Shawmut Railroad, without interference with heavy traffic; building a fertilizer plant and deep-water quay at Matanzas, Cuba, for the Armour Company; and the construction of a sugar refinery at Savannah, Ga.

In 1920, Westinghouse, Church, Kerr, and Company was merged with Dwight P. Robinson and Company and Mr. Tucker continued with the combined organization on general engineering and construction until 1921, when he became Chief Engineer of Henry R. Kent and Company, Rutherford, N. J. The latter engagement covered a large amount of power-house work, including installations for Cornell University, Lock Haven (Pa.), Electric Light and Power Company, the Worcester (Mass.), Electric Light Company, and at various points on the New York, New Haven, and Hartford Railroad; also factory work, hotels, and other construction.

In 1925, he became associated with Haven and Hopkins, Engineers, at Boston, on general engineering and valuation. Later, until his death, he continued in business for himself as Consulting Engineer. As such, his work included considerable reconstruction in Vermont to make good the damage caused by the 1927 flood in New England, and general consulting practice.

After an unusually active career in connection with a wide range of engineering work, Mr. Tucker's sudden death at St. Petersburg, Fla., on March 24, 1931, came as a shock to his many friends and associates.

An outstanding trait was his resourcefulness in tackling difficult undertakings and reducing them to a common-sense basis. He solved most of his problems by the application of common sense rather than mathematics. He applied unlimited energy to any work in hand and followed a consistent schedule until the task was completed. He had good executive ability, largely due to the co-operative spirit toward and consideration shown to his assistants and associates. Men working under or with him had the satisfaction of knowing that their combined team work would give the desired result with little or no lost motion. He had an unusual amount of enthusiasm and the ability to transmit this enthusiasm to others. He never had any difficulty in surrounding himself with good assistants, because of the initiative which he allowed to his assistants and the consequent pride which they took in their work. He was extremely big-hearted and took a deep interest in the progress of his subordinates, going out of his way to teach them everything that he knew about the subject on which they might be engaged at the time.

Mr. Tucker was an Associate Member of the American Railway Engineering Association and a Member of the Boston Society of Civil Engineers. He was a Knight Templar and one of the oldest members of the Mystic Shrine of Montpelier, Vt.; he was also a member of the Engineers Club of Boston and the Boston City Club. He was a member of the Protestant Episcopal Church.

On September 26, 1892, Mr. Tucker was married at St. Johnsbury, Vt., to Bessie Thompson, who survives him.

Mr. Tucker was elected a Member of the American Society of Civil Engineers on December 4, 1901.

WILLIAM FRANCIS TYE, M. Am. Soc. C. E.[1]

Died January 9, 1932

William Francis Tye was born on March 5, 1861, in Haysville, Ont., Canada, the son of Francis Edward and Ann (Skelley) Tye. Mr. Tye received his higher education at the University of Ottawa, Ottawa, Ont., Canada, and the School of Practical Science of Toronto University. He began his professional career early in 1881 as Chainman on Dominion Land Surveys.

From the early spring of 1882 to December, 1885, he acted in various capacities from Rodman to Assistant Engineer on the construction of the main line of the Canadian Pacific Railway, between Brandon, Man., and Griffith Lake, B. C., Canada.

---

[1] Memoir prepared by John G. Sullivan and F. P. Gutelius, Members, Am. Soc. C. E.

From the spring of 1886 to the late fall of 1887 he was an Assistant Engineer on construction and Transitman on location for the St. Paul, Minneapolis, and Manitoba Railway (now the Great Northern Railroad).

Mr. Tye then went to Mexico and for a year was Track and Bridge Engineer on the Mexican Central Railway. During 1889, he was engaged as a Mining Engineer in Mexico.

From February, 1890, to the fall of the same year, Mr. Tye was Division Engineer in charge of the location and construction of the Great Falls and Canada Railway, in Montana, and from October, 1890, to May, 1893, he was with the Great Northern Railway Company as Locating Engineer and Division Engineer in charge of construction west of the summit of the Cascade Range, in Washington. He had charge of the construction of all that part of the Great Northern Railway west of the old Cascade Tunnel which was abandoned after the construction of the new 8-mile tunnel.

In 1893 and 1894, Mr. Tye was Chief Engineer of the Alberta Railway; in 1895, Chief Engineer of the Kaslo and Slocan Railway; and from 1896 to 1899, Chief Engineer of the Columbia and Western Railway. These three railways are now included in the Canadian Pacific System.

After the completion of the Columbia and Western Railway to Midway, B. C., Mr. Tye was appointed Chief Engineer of Construction of the Canadian Pacific System. In the spring of 1900 his title was changed to Assistant Chief Engineer, and, in 1904, he became Chief Engineer of the Canadian Pacific System, this position he retained until 1906, when he retired to enter private practice as a Consulting Engineer.

Mr. Tye's vast experience, and his well-earned reputation for fairness and clear thinking, made his professional opinion sought in connection with a number of important projects. His paper[2] on "Canada's Railway Problem and Its Solution" won for him the Gzowski Medal of the Canadian Society of Civil Engineers (now the Engineering Institute of Canada), in 1917.

In the spring of 1898, Mr. Tye was married to Mable Maloney, in London, Ont. He practically retired from active business about 1920, when he left Canada to live in Europe, and he and his wife traveled very extensively in Egypt, Spain, Italy, England, Scotland, France, and Switzerland. They were on their way from Switzerland, planning to return to Canada, when Mr. Tye was suddenly taken ill on the night train between Cannes and Paris, and died in Paris on January 9, 1932, less than 36 hours after arriving there. He was the last of his immediate family and is survived only by his widow.

Mr. Tye always took an actve part in the promotion of engineering ideas and, in addition to his membership in the Society, he was for more than thirty-five years a member of the Engineering Institute of Canada, in which he held various offices, being President in 1912. He was among the first to join the American Railway Engineering Association, and was an active member for more than twenty-five years. He joined the Institution of Civil Engineers (Great Britain) as a member in 1910, and was the Canadian member of the Council of that Institution during 1916 and 1917, and 1917 and 1918. He was also an active member of many social clubs, among which may be mentioned the St. James Club, of Montreal, the York Club, of Toronto, and the Manitoba Club, of Winnipeg. Mr. Tye was a member of the Roman Catholic Church.

---

[2] *Transactions*, Canadian Soc. of Civ. Engrs., January-June, 1917.

Of outstanding professional ability, truthfulness, and honesty, Mr. Tye was respected and loved by all who worked under his direction, and his death is deeply regretted by his wide circle of friends and by all members of the profession who knew him.

Mr. Tye was elected a Member of the American Society of Civil Engineers on September 1, 1897.

GEORGE WILLIAM VOLCKMAN, M. Am. Soc. C. E.[1]

Died July 15, 1931

George William Volckman was born at Stratford, Essex, England, on March 27, 1873, the second son of Charles and Elizabeth Volckman. His father, grandfather, and great-grandfather were all Freemen of the City of London. He was educated at Dulwich College and in the Engineering Department of King's College, in London.

Mr. Volckman gained his first practical experience on the construction of the Manchester Ship Canal, first as a pupil, and, later, as an Assistant Engineer on the Contractor's Staff. The work upon which he was engaged comprised two very heavy sections, 7 miles in length, and included large quantities of rock and earth excavation, canal and river embankment and walls, canal and river locks, sluices, swing bridges, and the diversion of sewers and water supply mains.

From 1893 to 1896, he was Contractors' Engineer on the construction of two dry docks for the British Admiralty at Portsmouth, England. After the first year, the work on these docks was carried on continuously day and night and was completed in three years, or two years before the contract time. At the same time that the two docks were under construction, Mr. Volckman was acting as Contractors' Engineer on the dredging of 1 000 000 cu. yd. of gravel from the entrance channel to Portsmouth Harbor, to a depth of 36 ft. at low water.

From 1896 to 1898, Mr. Volckman was Contractors' Engineer on the construction of a $3\frac{1}{4}$-mile length of the Central London Tube Railway, including double-shield driven tube tunnels and station tunnels. For the next two years he was engaged as Contractors' Engineer on the construction of the Crowhurst, Sidley, and Bexhill Railway, in Sussex, England, a branch line $4\frac{1}{2}$ miles in length, embracing heavy main-line permanent way, a brickwork viaduct with sixteen arches of 60-ft. span, the pier foundation of which had to be carried down in timbered trenches through peat and earth bog, in many cases, to 50 ft. below ground level.

---

[1] Memoir prepared by Gerald Fitz-Gibbon, Esq., Hampstead, England.

The next year (1899), he was employed as Contractors' Sub-Agent and Engineer on the construction of the Whitechapel and Bow Railway, an extension of the London Underground Railway to connect with the London, Tilbury, and Southend Railway. This was a heavy piece of engineering work done by the "cut and cover" method. From 1900 to 1902, Mr. Volckman carried out several important engineering works, mostly in connection with railway or docks in England, as Contractors' Agent and Engineer.

From 1902 to 1907, he was engaged for the contracting firm of C. H. Walker and Company, Limited, of Westminister, as Agent and Engineer in charge of the construction of the extension of H. M. Dockyard, in Bermuda, for the British Admiralty. The works included the removal of a rock cliff ½ mile in length, on the coast line of Ireland Island, and the reclamation of the foreshore to form a quay. The quay wall, 2 200 ft. long, in 35 ft. of water, was built of 10-ton concrete facing blocks with mass concrete backing. A breakwater, 3 000 ft. in length, extended from one end of the quay wall, being constructed of 10 to 13-ton concrete blocks laid sloping on a rubble stone foundation in a depth of water running up to a maximum of 60 ft. At the same time, Mr. Volckman was engaged in carrying out, for his Company, the reconstruction of all the batteries surrounding the dockyard, and inspecting, reporting on, and making designs and estimates for large engineering works in South America. He also visited Canada and inspected and reported on the route of the proposed Montreal, Ottawa, and Georgian Bay Canal.

From 1908 to 1927, he resided in Ottawa, Ont., Canada, as Engineer for the proposed canal and surveyed the route, designed the necessary locks, dams, etc., and made detailed estimates of the cost of the work. He was also Vice-President, General Manager, and Chief Engineer of the Canadian Engineering-Contracting Company of MacArthur Perks and Company, Limited, from 1909 to 1927. During this time he examined and reported on many large propositions in Canada, the United States, and in other countries, including work executed by his Company for the Southern Pacific Railway, in Oregon, and the construction of docks at Havana, Cuba.

In 1928, Mr. Volckman was engaged by the firm of Sir Alexander Gibb and Partners, the eminent Consulting Engineers of Westminster, as its Representative resident in Bombay of all the firm's Indian work. He was on leave when he died in a London Nursing Home after a serious operation, on July 15, 1931, at the age of 58 years. In 1923, he was married to Anna Maude West, who survives him.

From the outset of his career, Mr. Volckman had exceptional opportunities for gaining experience in the construction of large engineering works of a widely varied character, and he took the fullest advantage of such opportunities. He was a most capable and energetic Engineer, and it would not be possible to speak too highly of his industry, character, or professional ability.

Mr. Volckman was elected a Member of the American Society of Civil Engineers on March 5, 1912. He was also a Member of the Institution of Civil Engineers, of London, and of the Engineering Institute of Canada.

FRED J. WAGNER, M. Am. Soc. C. E.[1]

Died May 24, 1930

Fred J. Wagner was born on June 30, 1870, in Georgetown, N. Y., the son of Charles C. and Marcelia (Jones) Wagner.

Mr. Wagner attended the local schools and after his graduation from Cazenovia Seminary in 1890, he entered Union College at Schenectady, N. Y., with the class of 1894, in the Civil Engineering Course. He was a member of the Phi Delta Theta fraternity. During the college vacations, Mr. Wagner was employed in the New York State Engineer's Department as an Assistant to the Engineer in Charge of the construction of a spillway for the Eaton Brook Reservoir, the Genesee Street lift bridge over the Erie Canal, at Utica, N. Y., and other State construction projects.

From 1894 to 1910, Mr. Wagner was continuously in the State Engineer's Department, advancing through all the then established civil service grades to and including that of First Resident Engineer. During the first part of that period, he had field charge of the construction of many important contracts, among them being a difficult repair job on the apron of a State dam at Minetto, N. Y., on the Oswego River; a stone dam at Baldwinsville, on the Seneca River; four important lift and bascule bridges over the Erie Canal at Syracuse, Utica, and Canastota; a concrete dam across the Black River at Forestport, N. Y.; and a sea-wall at Auburn, N. Y., on Owasco Lake.

In 1905 and 1906, Mr. Wagner was in charge of Barge Canal Contract No. 4 at Sylvan Beach, N. Y., which involved the construction of a lock, bridges, breakwater, and several miles of channel dredging. This contract presented some unusual difficulties, due to the character of the sand encountered. While not a quicksand, this material would assume an exceedingly flat slope when submerged and its control was a serious problem. Following 1906, Mr. Wagner was in charge of a section of more than 25 miles of Barge Canal Route, completing the surveys and making preliminary plans and estimates for future contracts. The surveys and plans of this section were complicated by several railroad and highway intersections which necessitated relocation and adjustment of grades.

In 1909, he was again promoted, this time to the position of First Resident Engineer with headquarters at Syracuse, in general charge and supervision of all Barge Canal contracts then in force on the Middle Division, which covered approximately 140 miles, comprising the Cayuga and Seneca, the Oswego, and about one-third of the Erie, Canals. During this time, a large engineering force was required to man the contract and survey parties, the chiefs of all the different parties reporting to Mr. Wagner and acting under his orders.

In 1910, Mr. Wagner resigned from the State service and became associated with the Scott Brothers Construction Company, as Chief Engineer. From that time until 1916, he had engineering charge for the contractors of important Barge Canal work, comprising heavy excavation, concrete structures of many kinds, bridges, etc.

---

[1] Memoir prepared by George D. Williams, M. Am. Soc. C. E.

In 1917, he accepted a position as Advisory Engineer for the New York State Department of Public Works. Just before the outbreak, and during the first few months, of the World War, many State contracts had been let and were under way. The attendant economic upheaval and general dislocation of peace-time industries had made further progress impossible, in many cases, necessary materials being no longer available and all the construction forces being rapidly demanned by reason of the men being drawn into industries engaged in manufacturing munitions. To make arrangements to close, suspend, or complete those contracts on a basis equitable to the State and the contractors, required a considerable knowledge of general conditions, of engineering, contracting possibilities, fairness, and justice. Mr. Wagner was largely instrumental in making the necessary adjustments to co-ordinate such contracts with the conditions then existing.

Having completed this work successfully, he opened an office as Consulting Engineer in Oneida, N. Y., and also accepted the position of County Superintendent of Highways for Madison County, New York, which position he occupied until his death. During this period of his life, he was Consultant on several important projects both as adviser to other engineers and for contractors on heavy construction, his experience having prepared him thoroughly for both these classes of work.

For several years, the City of Oneida had been hampered in its business growth and as a residential city by an unsatisfactory and inadequate water supply, which was impounded a short distance south of the city among hills of limestone formation. The collectible quantity was small and the quality poor, being hard and unsuitable either for industrial or domestic purposes. Agitation for a supply, better both in quantity and quality, had long been carried on by many far-sighted citizens. In 1927, this agitation crystallized into action, and Mr. Wagner was engaged as Consulting Engineer for the City. The source chosen was on Florence Creek, near its head-waters. This stream flows southerly from the hilly plateau in the northern part of Oneida County and, eventually, its waters find their way into Oneida Lake. The reservoir site selected is in a sparsely settled sandy country which provides natural filterage in large part and in which danger of pollution is practically negligible. The system included a dam and reservoir of considerable size, about twenty miles of conduit, a distributing reservoir, and connection with the existing city mains. Under Mr. Wagner's supervision the work was carried to a successful conclusion.

In New York State, in addition to the through routes and other important roads which are built by the State, the individual counties annually construct a large mileage of permanent surfaced roads, many of which are of importance. The method under which Madison County operated was to build by its own forces. Mr. Wagner organized a construction force for the County with all the equipment necessary to build roads and, during this period, was actively engaged in county road construction, together with the building of such bridges as came under his jurisdiction. A large mileage of well-planned and executed concrete and bituminuous roads as well as numerous bridges will bear witness for years to his ability. During this period and on this work, the results of his broad experience in engineering and contracting were most evident. His success there was due to his engineering knowledge and his experience as an organizer and practical constructor.

Mr. Wagner was a member of Georgetown Lodge, F. and A. M., and after locating in Oneida he joined and maintained an active interest in Oneida Lodge No. 767 Benevolent and Protective Order of Elks, and the Rotary Club of that City.

He was the descendant of pioneering ancestors and like them, looked on difficulties and obstacles as things not to be avoided if they were in his path, but to be overcome. Due partly to his early environment, family training, and inherited trend of mind, and partly to his experience as a consultant and as an engineer in contracting, he had developed to a high degree the faculty of quickly considering all accessible viewpoints of a question before coming to a decision. This faculty made it easy for him to understand another's opinion and possible for him to differ on questions of county construction policy and still maintain harmonious relations.

Mr. Wagner possessed a genial and winning nature. He was kindly and courteous both in his official and in his social relations. He was held in high esteem not only for his ability as an engineer and constructor, but also for his uprightness and usefulness as a citizen.

Mr. Wagner was married, in 1893, to Cora Brown, of Cortland, N. Y., who survives him, with two daughters, Marcelia Wagner and Hazel Wagner Lavender, and a son, Charles C. Wagner.

Mr. Wagner was elected an Associate Member of the American Society of Civil Engineers on April 4, 1906, and a Member on September 6, 1910.

SAMUEL TOBIAS WAGNER, M. Am. Soc. C. E.[1]

DIED AUGUST 7, 1931

Samuel Tobias Wagner, the son of John and Sara A. (Wood) Wagner, was born in Philadelphia, Pa., on August 30, 1861. He attended the University of Pennsylvania, receiving the degree of Bachelor of Science in 1881, and, three years later, that of Civil Engineer from the same institution.

On leaving college, Mr. Wagner was employed by the Phœnix Bridge Company, at Phœnixville, Pa., and, in the period from 1881 to 1890, he held, successively, the positions of Draftsman, Assistant to the Master Mechanic, and Superintendent of Shops. From 1890 to 1893, he was a Resident Engineer for the Company, in Louisville, Ky.

---

[1] Memoir prepared by a Committee of the Philadelphia Section consisting of S. M. Swaab, *Chairman*, R. I. D. Ashbridge, Clark Dillenbeck, and E. B. Temple, Members, Am. Soc. C. E.

He entered the employ of the Bureau of Surveys of the City of Philadelphia in 1894, as Assistant Engineer in charge of the Reading Railroad Subway and Tunnel on Pennsylvania Avenue. This work was described by Mr. Wagner and the late George S. Webster, Past-President, Am. Soc. C. E., in a paper[2] presented before the Society entitled "The Pennsylvania Avenue Subway and Tunnel, Philadelphia, Pa."

Upon the completion of this work, Mr. Wagner was appointed Assistant Engineer in Charge of the Improvement and Filtration of the Water Supply of Philadelphia, which position he held from 1900 to 1902, inclusive, when he entered the service of the Philadelphia and Reading Railroad Company as Assistant Engineer on the elevation of tracks on Ninth Street, in Philadelphia. This work occupied his time from 1902 to 1915, when he was appointed to the position of Chief Engineer of the Philadelphia and Reading Company (now the Reading Company). At the time of his death, he held the position of Consulting Engineer to that Company.

He wrote a number of papers and prepared numerous discussions for the Society, one of which, in 1914, received the James Laurie Prize.[3]

Mr. Wagner was a Fellow of the American Association for the Advancement of Science; a member of the American Railway Engineering Association; the American Society for Testing Materials, of which he was Vice-President; the Franklin Institute, of which he was a Director; the Philadelphia Academy of Natural Sciences; and President of the Board of Trustees of the Wagner Free Institute of Science, a free co-educational institution founded by one of his forebears. Mr. Wagner lectured for many years on Civil Engineering at this institution, and was succeeded by one of his sons. He was also a member of the General Alumni Association and a Past-President of the Engineering Alumni Society of the University of Pennsylvania; a member of the Engineers Club of Philadelphia; and a Vestryman of the Protestant Episcopal Church of St. James the Less, Philadelphia, and was active in this capacity. Until a short time before his death, he was a member of the State Registration Board for Professional Engineers in Pennsylvania.

Mr. Wagner was married on November 13, 1888, to Mary C. R. Scull who, with four children, Mary, John, William Worrell, and Thomas Rowan, survives him.

He was a man of great energy and activity, deeply interested in his profession. He possessed ripe judgment and a thorough knowledge of engineering, gained in many years of experience and contact with important engineering works, and was held in high esteem by all who knew him. Any commentary on Mr. Wagner's life would be incomplete, that failed to emphasize his fine personality, lovable character, and high sense of duty, which endeared him to his many friends and associates. With his passing, the Engineering Profession has lost one who, during his whole career, exemplified and upheld its highest traditions.

---

[2] *Transactions*, Am. Soc. C. E., Vol. XLVIII (1902), p. 470.

[3] "The Elevation of the Tracks of the Philadelphia, Germantown, and Norristown Railroad, Philadelphia, Pa.", *Transactions*, Am. Soc. C. E., Vol. LXXVI (1913), p. 1819.

He had recently celebrated the Fiftieth Anniversary of his graduation from college, and although, at the time of his death, he was seventy years of age, he was more than usually active; as a matter of fact, his untimely death resulted from his physical activities.

Mr. Wagner was elected a Member of the American Society of Civil Engineers on February 2, 1887. He served as Director of the Society from 1918 to 1920, and as President of the Philadelphia Section in 1917 and 1918.

JOHN SIMPSON WALKER, M. Am. Soc. C. E.[1]

DIED MAY 12, 1931

John Simpson Walker was born at Florence, Ala., on July 2, 1849. His father was Richard Wilde Walker, a Confederate States Senator and, later, Judge of the Superior Court of Alabama. His mother was Mary Ann (Simpson) Walker, of Florence, Ala.

Mr. Walker was educated at private schools and at the University of Virginia. His first work in engineering was as Assistant Engineer with the State Board of Improvement, Mobile Bay, during 1871 and 1872. He served as Assistant Engineer with the Corps of Engineers, United States Army, on the survey of the Tombigbee River, in November and December, 1872.

In 1873, he became Resident Engineer on the Savannah and Memphis Railroad and in June of that year, he was appointed Private Secretary to the late A. L. Rives, M. Am. Soc. C. E., Chief Engineer and General Superintendent for the Mobile and Ohio Railroad Company. In 1874, Mr. Walker was placed in charge of improvements in the Mobile Yard and Round-House, at Whistler, Ala. He remained with this Company until September, 1876, as Assistant Division Superintendent and Division Superintendent, respectively, in charge of transportation and maintenance. He then became Assistant Engineer in charge of location for the Mexican National Railroad Company; Engineer for the Vicksburg, Shreveport, and Pacific Railroad Company; and Engineer of the Birmingham, Ala., Water-Works, in which position he had charge of the construction of the extension at Cahaba.

On June 5, 1881, he was appointed U. S. Assistant Engineer, on the improvement of Elk River Shoals, on the Tennessee River. Mr. Walker was engaged in private practice at Huntsville, Ala., from December, 1883, until February 1, 1891, when he became Assistant Engineer in the office of the U. S. District Engineer, at Nashville, Tenn. He remained in this position until his retirement, on an annuity, on August 20, 1922. Practically all the locks and dams in the Cumberland River, above Nashville, and some of those below the city, were constructed under his supervision.

Mr. Walker was the ninth Charter Member of the Engineering Association of the South, organized at Nashville in 1889 and chartered in 1892. He was made an Honorary Member of the Engineering Association of Nashville on May 9, 1921. Upon his retirement from active service, the City of Nashville honored him by the presentation of an inscribed cane in recognition of his great service to the community while he was a resident of Nashville.

---

[1] Memoir prepared by Hunter McDonald, Past-President, Am. Soc. C. E.

Mr. Walker was a great student, modest and retiring in disposition, genial and friendly among those with whom he became intimate. His reading was wide, and, in addition to his outstanding engineering knowledge, he was well versed in history and literature.

After his retirement, except for a year spent with his son, Colonel Walker, at Fort Leavenworth, Kansas, and at Fort Meade, South Dakota, Mr. Walker made his home at the Tulane Hotel, in Nashville, until his death on May 12, 1931. A prayer service was read at Nashville by the Rev. J. F. McCloud, Assistant Rector of Christ Protestant Episcopal Church, and the funeral took place at Huntsville, Ala., on May 15, 1931.

He was married in 1875 to Nani Herndon Rice, at Mobile, Ala. Mrs. Walker died several years before him. In addition to his son and only child, Col. Richard W. Walker, U. S. A., and four grand-children, he is survived by two brothers, Judge Richard W. Walker, of Huntsville, and Percy H. Walker, Chemist, Bureau of Standards, at Washington, D. C., and a sister, Mrs. John M. Bolling, of Huntsville.

Mr. Walker was elected a Member of the American Society of Civil Engineers on January 5, 1881.

ANDREW JACKSON WILEY, M. Am. Soc. C. E.[1]

Died October 8, 1931

The sudden and unexpected death of Andrew Jackson Wiley ended an engineering career of notable achievements. For nearly half a century Mr. Wiley was identified with the engineering development of the West, especially in irrigation work and in the construction of many of its outstanding dams. His professional connections covered most of the Western States, in addition to India, the West Indies, and the Panama Canal Zone. When death overtook him, his consulting engagements included such outstanding projects as the $165 000 000 Boulder Canyon Project of the United States Bureau of Reclamation, the $220 000 000 aqueduct of the Metropolitan Water District of Southern California, and the $400 000 000 Columbia Basin Project under investigation by the Government in the State of Washington.

Andrew Jackson Wiley was born in New Castle County, Delaware, on July 15, 1862, the son of John and Mary Wiley. He attended the Newark Academy in Newark, Del., graduating at the head of his Class and winning a scholarship to Delaware College. He was graduated from Delaware College as Valedictorian of his Class in 1882, with the degree of Bachelor of Philosophy in Engineering.

---

[1] Memoir prepared by J. L. Savage, M. Am. Soc. C. E.

After his graduation Mr. Wiley spent a year on surveys and construction for the Baltimore and Ohio Railroad Company in Delaware and Maryland. His first assignment on irrigation work, and the connection which established him in the West, extended from 1883 to 1886, when he served as Rodman and Assistant Engineer with the Idaho Mining and Irrigation Company, of Boise, Idaho. From 1886 to 1888 he was Assistant Engineer on construction for the Union Pacific Railroad Company, at Butte, Mont., but returned to irrigation work in Idaho in 1888. From this time until 1892 he was Chief Assistant Engineer for the Idaho Mining and Irrigation Company, on the construction of an irrigation project in Southern Idaho. From 1892 to 1898 he was Chief Engineer and Manager of the Owyhee Land and Irrigation Company, constructing a large irrigation project near Grandview, Idaho.

This period of his long experience was perhaps the most uncertain and trying time in his life, due to the financial pitfalls which constantly faced these developments. The mining operations connected with these early developments, although promising at first, were fraught with many disappointments and often ended in financial failure. The land development and irrigation work was most difficult and discouraging during that period of agricultural and financial depression when prosperity was always "just around the corner." These were years of adventure and of great effort and personal sacrifice on Mr. Wiley's part.

Some of his friends refer to this period in Mr. Wiley's life as his "lean years." Measured by dollars received, the results of his labor were almost nil, but he made many friends among the mining and irrigation fraternities. He was known to many leading business men of Idaho, and to many officials in Washington, D. C., as a man of integrity, whose word alone was a guaranty of performance. His opinion in relation to the development of the water and land resources of the desert regions was accepted and valued by those who knew him. His knowledge of the gold value of Snake River gravels was usually considered final and, from time to time, he was employed to examine tracts that had been presented to financiers for promotion. During this period he had many days when work was not pressing, but they were happy days, when the tendrils of his friendship were finding their way deep into the hearts of all with whom he came in contact, from teamsters to investment bankers; from prospectors to enterprising promoters.

It was about 1900 when the "corner" was finally turned and prosperity overtaken. The population of the arid inter-mountain States had greatly increased and plans for the further development of irrigation were being seriously considered. Hydraulic engineers were also scouting in the interest of the great utility companies for favorable sites for the development of hydroelectric power. Southern Idaho received its full share of attention, and this period marks the beginning of Mr. Wiley's intensive activities in both these fields. Probably one of the brightest spots in Mr. Wiley's early experience was his association with the famed engineer, Arthur D. Foote, M. Am. Soc. C. E., in the first construction work of the New York Canal, one of the early irrigation developments in Boise Valley.

Following these early beginnings Mr. Wiley worked through the next thirty years on a continuous procession of great irrigation and power projects in Idaho, Oregon, California, and other States in the West. His contacts were with the large Carey Act companies, irrigation districts, power development companies, and municipalities.

A partial list of the large non-Federal projects on which Mr. Wiley acted in a consulting capacity includes the following: 1900–02, Swan Falls, Idaho, power plant and later extensions; Trade Dollar Mining Company, Idaho; 1902–04, Chief Engineer, Boise-Payette River Electric Power Company; Horseshoe Bend power plant, Idaho; Granite Springs Dam, City of Cheyenne, Wyo.; 1904–05, Barber Lumber Company dam and power plant, Idaho; 1906–14, repairs and extensions, Milner Dam, Idaho; Twin Falls North Side Irrigation Project, Idaho; Twin Falls-Oakley Project, including Oakley Dam, Idaho; Shoshone Falls power plant, Idaho; Lower Salmon Falls power plant, Idaho; Salmon River Dam and Irrigation Project, Idaho; Belleview Dam and power plant, Idaho; Payette River Dam and power plant, Idaho; American Falls power plant, Idaho; Southern Alberta Land and Irrigation District, Canada; 1914–20, Ochoco Dam and Irrigation District, Oregon; Warm Springs Dam and Irrigation District, Oregon; Grants Pass Irrigation District, Oregon; Sweetwater Dam, San Diego, Calif.; 1920–26, Don Pedro Dam and power plant for Turlock and Modesto Irrigation Districts, California; Murphy Dam, Big Wood River Dam, and Mackay Dam, Idaho; projects for Kern River Water Storage District, California; Crow Creek Irrigation District, Montana; projects for San Joaquin River Water Storage District, California; Exchequer Dam and power plant for Merced Irrigation District, California; Melones Dam for South San Joaquin and Oakdale Irrigation District, California; reconstruction of irrigation system, Bitter Root Valley Irrigation District, Montana; reconstruction of irrigation system, West Okanogan Irrigation District, Washington; Logan City, Utah, power plant; Washington Water Power Company, Spokane, Wash.; Blitzen River Reclamation District, Malheur Land and Cattle Company, Oregon; Bull River Dam, Oregon; 1927–31, Sutlej River Project, in India, for the British Government; Reeder Gulch Dam, Ashland, Ore.; reviewing design and stability, Lower Otay, Barrett, Morena, Chollas Heights, and Hodges Dams for San Diego, Calif.; investigation of St. Francis Dam disaster for Los Angeles, Calif.; Los Angeles County Flood Control District, California; reconstruction of the Lafayette Dam, California; flood control project for Orange County, California; Don Pedro power plant, California; and San Gabriel Dam, California.

During much of the period when Mr. Wiley was engrossed with his work for private companies, he was also engaged in a consulting capacity by the United States Bureau of Reclamation. He served as Consulting Engineer for the Bureau from the inception of its reclamation work in 1902, and from 1925 held an appointment as Consulting Engineer for the Interior Department at large.

He was consulted in connection with the design and construction of such major Government dams as the Belle Fourche, Shoshone, Lower Yellowstone, Roosevelt, Pathfinder, Minidoka, Jackson Lake, Arrowrock, Cold Springs, Sun River Diversion, Tieton, Black Canyon (Idaho), McKay, American Falls, Guernsey, Gibson, Stoney Gorge, Deadwood, Echo, Owyhee, Cle Elum, Boise Diversion, Deer Flat (Upper and Lower), Dodson, Conconnully, Vandalia, Sherburne Lakes, Hubbart, Gerber, Coolidge, and Hoover. His consideration of these projects included intimate studies of many of the design features, in addition to periodical field inspections during construction.

In the case of most of these dams the foundation conditions were carefully inspected and approved by Mr. Wiley before the construction was permitted to be started. Likewise, the concrete materials, mixtures, and control, the character of the embankment materials, methods of compacting, and many other fundamental construction details were submitted to him for review and approval.

The officials of the Bureau of Reclamation, and his associates among the engineers of the Bureau, had implicit confidence in his judgment, and felt a genuine sense of security when his approval of a project had been given.

While the foregoing list mentions dams only, Mr. Wiley's assignments for the Bureau of Reclamation were by no means limited to that field. He was consulted in connection with the design and construction of many other features of Government irrigation and power projects, particularly in regard to the main canals and important canal structures, including diversion works, tunnels, flumes, siphons, bridges, and wasteways.

It is not surprising that with his background of twenty-eight years' service in Government reclamation work, and an even longer record on Carey Act and district irrigation and power development, Mr. Wiley was the first engineer selected for the Hoover Dam Consulting Board. His broad experience and sound judgment were invaluable in connection with the preliminary studies of this stupendous project. This was especially so, when the recent unemployment situation created an emergency which suddenly shortened the allotted time for the completion of the designs and specifications a matter of six months.

Mr. Wiley's extensive experience in the design of dams and power plants was also recognized and utilized by other departments of the Federal Government. He was Consulting Engineer for the United States Indian Service on the design and construction of the Coolidge Dam in Arizona; he also prepared reports on the feasibility and method of constructing the Kickinghorse Storage Dam, the raising of the Tabor and Pablo Dams, and plans for the Lower Crowe and Twin Reservoir Dams, on the Flathead Project, in Montana. The Panama Canal Zone selected him as one of the Consultants on the design of the Madden Dam and power plant, the contract for the construction of which has only recently been awarded. He was consulted during the summer of 1931 by the United States War Department in connection with the major structures on the great Columbia Basin Power and Irrigation Project in the State of Washington.

Any engineer with such an impressive list of successful projects to his credit could not fail to receive international notice. Mr. Wiley acted as Consulting Engineer for the British Government from October, 1927, to February, 1928, in the preliminary investigation of twenty-six dam sites in the

Himalayan Mountains in India. One of these sites proved suitable for a large storage project, and careful design studies were made, under Mr. Wiley's direction, for a dam about 500 ft. high, known as the Bhakra Dam, which it is proposed to construct at this site.

In 1927, Mr. Wiley visited Porto Rico in connection with the Guajataca Dam of the Isabella Irrigation Service. He again visited the Island in 1929 in connection with the Guineo Dam, and a third time in 1931 in connection with the Guineo and Metrullas Dams, all of these assignments being for the Porto Rico Government Irrigation Service.

When stricken, Mr. Wiley was engaged on work in San Gabriel Canyon, California, where the Los Angeles Flood Control District is planning two flood-control dams. He was also in California for conferences in connection with the great aqueduct which will carry Colorado River water to Los Angeles and the surrounding Metropolitan District. He had also been requested to reserve time during the latter part of November for an important meeting of the Hoover Dam Consulting Board.

It is significant of Mr. Wiley's career as a designer and builder of important dams that he was Consulting Engineer on the succession of the ranking high masonry dams of the world, as follows: Shoshone (1905–10), height, 328 ft.; Arrowrock (1911–15), height, 349 ft.; Owyhee (1928–32), height, 405 ft. (530 ft. in fault zone); Bhakra, India (1928), height, 500 ft.; and Hoover (1930), height, 727 ft.

Following the great St. Francis Dam disaster in Southern California on March 13, 1928, great official and public concern was felt as to the safety of all storage dams whose failure might endanger life. On this account the Secretary of the Interior, on March 15, 1928, ordered that all such dams of the Bureau of Reclamation be promptly examined by a competent engineer and reported on as to their safety. It is significant of the confidence had in Mr. Wiley's judgment and ability that he was chosen in this connection to report on twenty of the more important storage dams of the Bureau. In a similar manner, Mr. Wiley was also retained by the City of Los Angeles to investigate the safety of thirty-two dams.

Mr. Wiley's death ends an exceptionally brilliant engineering career. It deprives the United States Government and many large organizations throughout the world of a Consultant who was highly valued. He held the highest respect of all who knew him. During his fifty years in the West, he made a host of intimate friendships, especially among his engineering associates and others who were connected with his many successful and monumental engineering projects.

His professional counsel will be keenly missed by many organizations. His qualities of mind, his sterling character, and his lovable and winsome nature endeared him to a wide circle of friends and associates who sincerely mourn his passing and in whose memory he will always live.

He was suddenly stricken with the illness that caused his death while engaged on a Consulting Board considering the San Gabriel Canyon development of the Los Angeles County Flood Control District in California. The remaining members of the Board, L. C. Hill, M. Am. Soc. C. E., and Messrs. Robert T. Hill, and C. F. Lohman, in their report, stated:

"We fellow members of the Board are sincerely grateful that we had the benefit of his [Mr. Wiley's] association on the work and deeply feel his unexpected loss."

The reports from those who knew Mr. Wiley well in the years of his youth and early manhood show that he then possessed the same admirable traits of character and lovable nature that distinguished him in later life. As a young boy entering Newark Academy, and while attending Delaware College, he made his home with Dr. Henry, of Newark, Del., who previously was one of his school teachers. Mr. Wiley worked in Dr. Henry's drug store in return for his living and school expenses. His relation to the Henry family then and throughout his life was that of a devoted and generous son and brother. Dr. Henry's daughter, now Mrs. Edna Henry Cooper, of Wilmington, Del., tells of many delightfully characteristic incidents of young Wiley's life and of his great devotion to her widowed and invalid mother.

Mr. Wiley never married, but he loved children—girls more than boys—and especially during the earlier years of his life in Boise there were established many beautiful friendships which continued to his death. To be a member of "Mr. Wiley's Set" was assurance of seeing all the circuses and best shows, not to mention birthday and Christmas parties, all personally conducted. Initiation into this happy circle generally occurred during the early doll days and the years of beautiful friendships were marked by birthday and Christmas gifts and, in many cases, wedding gifts as the years passed. All these children cherished his friendship to mature years and some of them owed their college education to his generosity. The principal benefactors of his estate were these young friends who had known him from their childhood days.

Mr. Wiley was averse to publicity and seldom spoke before the public, even during meetings of his own Engineering Profession. He enjoyed the companionship of friends and was always a genial and entertaining host. His kindness and consideration of others engendered the loyalty and diligence of his associates. He was always ready to assist his friends in time of trouble.

He loved his work and devoted practically all his time to engineering matters. His intense and untiring studies probably accounted for his unusual capacity for handling details. He never wished to undertake more work than he could personally supervise and often declined to accept professional engagements because of his inability to give the work his personal attention.

There follow a few characteristic quotations from letters that have been received from some of Mr. Wiley's friends and associates:

Mrs. Edna Henry Cooper, of Wilmington, Del.:

"We do not understand why such good useful men are taken in the midst of their precious, busy lives. His work was his great love—so often he has written mother that work was recreation to him and it was just so in his college days—it was his keen delight to help his classmates who had not the ability he possessed. The years did not change his modest, charming personality."

Mr. J. H. Lowell, of the Gem Irrigation District, Homedale, Idaho:

"It was as a friend that I knew him, retiring, thoughtfully considerate, a most delightful companion. His estimate of people might be whimsical or humorous, never unkindly. A man who always looked for the best in others and who was intensely loyal to friends and associates. * * * Whether as a business associate or as a friend Mr. Wiley inspired confidence, loyalty, and love."

Charles H. Paul, M. Am. Soc. C. E.:

"His kindly disposition which was so apparent to all who knew him; his consideration for the comfort of others; his open-minded regard for others' points of view; his absolute integrity, including his abhorrence of sham or pretence and his hatred of deception or misrepresentation; his sound judgment and common sense; his fine sense of humor; and the high regard accorded him without exception by his clients and associates."

Louis C. Hill, M. Am. Soc. C. E.:

"There was never another man with whom I have served on boards whose opinion merited and received such consideration as did the opinions so modestly and considerately expressed by Mr. Wiley. * * * During the long years of frequent contact with Mr. Wiley, I was always impressed with his modesty, unselfishness, and consideration for others. A great man and a most lovable friend has passed on."

Major John S. Butler, U. S. A., M. Am. Soc. C. E.:

"I was very much impressed with Mr. Wiley's earnest desire to get at the truth of a question; with his wide fund of engineering knowledge along broadest lines, which was always so readily available; with his sound judgment and, notwithstanding a certain firmness of manner, with his gentleness and courtesy to all."

Mr. Frederick Irwin, of San Francisco, Calif.:

"I was never associated with another man who was so uniformly kind and courteous, in all his contacts, and one of whom I was so proud to call my friend."

R. E. Shepherd, Manager of Twin Falls North Side Land and Water Company:

"Death found him at the head of his profession, honored and beloved by all who knew him."

Mr. Otto P. Cherdron, of Salt Lake City, Utah:

"What a pity it seems, that a mind such as this great engineer had, cannot be endowed to some one else. What an accumulation of experiences has been lost which has to be laboriously gathered by other kindred minds."

D. C. Henny, M. Am. Soc. C. E.:

"Mr. Wiley's associates loved him for his unselfish nature, his open-mindedness, his almost excessive modesty, and his constant geniality. The description of a great predecessor (Alfred Noble) may be truthfully applied to Andrew Jackson Wiley: 'An upright man, a good citizen, an eminent engineer, whose life and labors contributed greatly to the high honor of his profession and to the success of many important public works.'"

D. W. Ross, M. Am. Soc. C. E.:

"Mr. Wiley belonged to no religious organization nor did he openly subscribe to any creed, but he gave liberally to every good cause, and like coin of the realm, it seemed always to be taken for granted that he was just as sound as he appeared to be. He truly loved his neighbor as himself. But his 'neighbor' was not the one who had *given* assistance, but the unfortunate one who was in need of help. He had a remarkably keen sense of humor and was the life of every small company of friends, but shunned the limelight, and never found it difficult to put the time in happily and profitably when alone. The greatest engineering works, the dams which he helped to plan and construct, if arranged end to end would extend for many miles and their continuous profile would make a most imposing skyline—fitting monuments to his great engineering skill. But greater than these works was their designer, this quiet, unassuming, smiling gentleman, who cared neither for riches nor fame and who had such a big place in his heart for children and dogs. It can be truly said of him that 'his reward was with him and his work before him.'"

Mr. Wiley was elected a Member of the American Society of Civil Engineers on December 5, 1894.

HENRY DICKINSON WOODS, M. Am. Soc. C. E.[1]

Died April 2, 1931

Henry Dickinson Woods was born in Paris, France, on October 3, 1852, the son of Henry Woods and Annie L. (Dickinson) Woods. His father was long a member of the old-time firm of C. F. Hovey and Company, representing that house in Europe and making his home in Paris for thirty years. It was his custom to return to the United States at least once every four years in order, as an American citizen, to vote in the National elections. It was in Paris, therefore, that his son, Henry Dickinson Woods spent his boyhood and received most of his education, with an interval of two years in 1866 and 1867 when he came to this country and attended a boarding school at Newport, R. I. In Paris, he attended the College Chaptal and, later, the Ecole Centrale des Arts et Manufactures, from which he was graduated in 1875.

After his graduation Mr. Woods came to the United States and spent his first two years in this country as a Draftsman and Rodman on the Dam Division of the Boston, Mass., Water-Works. He then returned to Paris and spent a part of the next year in association with Eduard Badoit, Civil Engineer, on plans for the St. Louis, Senégal, Water-Works. He spent the year, 1879, working with Mr. H. T. Bartlett, in Paris, on board-cutting machinery, and, in 1880, he again returned to the United States to accept a position as Assistant in the office of the City Engineer of Newton, Mass., which position he retained until 1884. During his four years as an Assistant Engineer, Mr. Woods was engaged in general municipal work, highways, heating and ventilating school-houses, and alterations to the City Hall and other municipal buildings. He spent the year, 1884, in traveling between France, Belgium, and England, in the interest of the Carson Trench Machine Company.

---

[1] Memoir prepared by William P. Morse, M. Am. Soc. C. E.

In the latter part of 1884 Mr. Woods engaged in the private practice of surveying and building construction, remaining in this business until the latter part of 1887. At that time he gave up his practice to accept a position in the office of the Chief Engineer of the Massachusetts State Board of Health, to work on investigations relative to the various water supplies of the State and the question of sewerage purification abroad. After leaving the Massachusetts State Board of Health, he was employed for a year as Inspector of Masonry Foundations for bridge work for the City of Newton and the Town of Wellesley, Mass.

In 1889, Mr. Woods again returned to Europe as "Ingénieur Expert" with the United States Commission, Paris Exposition, coming back in 1890 to the office of the City Engineer of Newton, as Principal Assistant. In 1893, he was appointed City Engineer, in which capacity he remained until January 13, 1900. At that time he retired from the Engineering Profession and began the work which he so loved among his greenhouses, on the hill close to his home.

In his six years as City Engineer, the position he filled so well because of his education and wide experience in the various lines of engineering, both in the United States and abroad, Mr. Woods accomplished a great deal for the fast-growing City of Newton. The Commonwealth Avenue Boulevard, constructed to a width of 120 ft. through the center of the city, was completed during his administration.

Because of many fatal accidents it became necessary to abolish the grade crossings along the main line of the Boston and Albany Railroad, and to widen Washington Street to 75 ft. It was during the years that Mr. Woods was City Engineer that this great engineering feat was accomplished, which included the widening of Tremont and Park Streets and the reconstruction of many main drains along the line of the railroad. During his administration, Bulloughs Pond, which had formerly been a municipal eyesore, was also changed, and is to-day a beauty spot, in the center of the city.

In 1899, studies and negotiations were made with the Boston and Albany Railroad Company and plans were drawn for the abolition of the grade crossings on the south side of the city. Before this work was definitely started, however, Mr. Woods retired from his position as City Engineer, leaving behind him many treasured friends and lasting monuments of work which he had created during the time of his association with the City of Newton.

He was greatly interested in charities all over the world and contributed liberally to a large number. During and after the World War he was especially interested in the Phare de France, for the re-education of men blinded in the war, and also in the Preventorium, a charitable health center for children established by the Lafayette Association at Chavagnac, France, to which he gave a refectory building. His generosity was well known and in his will he made bequests to many charities which he had supported during his life, among which were: The New England Home for Deaf Mutes; Massachusetts Association for Promoting Interests of Adult Blind; New England Peabody Home for Crippled Children; Industrial School for Crippled and Deformed Children; Infants' Hospital; Bethesda Society; Boston Floating Hospital; Boston Society of Natural History; New England Moral Reform Society; Massachusetts Charitable Eye and Ear Infirmary; Stone Institute and Newton Home for Aged People; and Prentiss Normal and Industrial Institute, Prentiss, Miss.

Mr. Woods was a member of the Boston Society of Civil Engineers, the New England Water Works Association, and the Massachusetts Genealogical Society.

He was married, in 1894, to Jennie A. Pond, of Brookfield, Mass. He is survived by one daughter, Mrs. Ethel P. Coty, and two grandchildren.

Mr. Woods was elected an Associate Member of the American Society of Civil Engineers on March 1, 1893, and a Member on June 5, 1895.

CHARLES WORTHINGTON, M. Am. Soc. C. E.[1]

Died August 29, 1931

Charles Worthington, the son of Edward and Carolina C. (McLean) Worthington, was born in Worthington Valley, Baltimore County, Maryland, on May 12, 1871.

His early education was received entirely at home until the fall of 1888 when he entered Rensselaer Polytechnic Institute, at Troy, N. Y., from which he was graduated in June, 1892, with the degree of Civil Engineer.

Mr. Worthington's first position was that of Draftsman for the Maryland Steel Company, of Sparrows Point, Md., which he held during July, 1892. From August to September of the same year, he served as Draftsman for the Link Belt Engineering Company, of Philadelphia, Pa., and from October, 1892, until June, 1894, he was Instructor in Civil Engineering at the University of Pennsylvania, at Philadelphia.

Subsequently, Mr. Worthington held the following positions: From July, 1894, to January, 1895, as Draftsman, making shop drawings, with the Keystone Bridge Works, at Pittsburgh, Pa.; from the latter date until December, 1895, as Draftsman and Estimator with the Phoenix Bridge Company, at Phoenixville, Pa.; from December, 1895, to July, 1896, as Bridge Engineer in the Maintenance-of-Way Department, reporting on the strength of old structures, etc., of the Pennsylvania Railroad Company, at Philadelphia; from July, 1896, to January, 1897, in the Estimating Department of the Pennsylvania Steel Company at Steelton, Pa., making designs and estimates of various kinds of structures, principally railroad and highway bridges; from January to July, 1897, as Resident Engineer and Inspector on the erection of the superstructure of the Duluth-Superior Bridge (one 500-ft. draw and two 300-ft. approach spans) under the late Alfred P. Boller, M. Am. Soc. C. E., Chief Engineer; from July, 1897, to July, 1899, in the Estimating Department of the Keystone Bridge Works, making designs and estimates of steel structures of all kinds, including the following: One 450-ft. and two 300-ft. deck spans of the Allegheny and Western Railroad Bridge over the Allegheny River at Mosgrove, Pa.; two viaducts, 100 ft. high and about 400 ft. long, for the same Railroad Company; design for a cantilever bridge, 1 600 ft. long, center to center, river piers, and two 600-ft., center to center, anchor piers over the St. Lawrence River, at Quebec, Que., Canada, including estimate of material, as well as many other railroad and highway bridges, mill buildings, etc.; during August, 1899, with Brown Hoisting and Conveying Machine Company, at Cleveland, Ohio; from September to October, 1899, with Wilkins and Davison, Engineers, of Pittsburgh, on the design of several steel viaducts for a street railway and a cantilever bridge for a city street; and from November, 1899, to May, 1900, with the Keystone Bridge Works, in charge of designing and detailing several mill buildings and plate-girder bridges.

---

[1] Memoir prepared from information on file at the Headquarters of the Society.

For the next ten years, Mr. Worthington was in private practice as Consulting Engineer in Pittsburgh and New York, N. Y. During this time, he designed and superintended the erection of a coal plant for the Virginia and Pittsburgh Coal and Coke Company, near Fairmont, W. Va., which cost about $40 000. Other work included the design of bridges, mill buildings, steel frames for office buildings, etc.

Some of the later engagements of Mr. Worthington were: The design of all steel work for the Carnegie Library at Pittsburgh, costing about $5 000 000; the design and supervision of construction of the steel work for the Municipal Ferry Terminals at the foot of Whitehall Street, costing about $1 100 000, and also for the new 22d Regiment Armory at West 168th Street, both in New York City; the design and construction of a composite building (of reinforced concrete and structural steel) to serve as a covered tennis court on a private estate near New York, for Warren and Wetmore, Architects, costing about $150 000; the design of a number of buildings of the mill type for the Aluminum Company of America; and the design of all bridges for the Little Falls and Johnstown Railroad Company, and many other structures.

From September, 1914, until April, 1915, Mr. Worthington was engaged as Contracting and General Engineer for the Columbus Mill and Mine Company, of Columbus, Ohio, and from April, 1915, until February, 1917, he was Designer and Estimator for Milliken Brothers, of New York City.

In February, 1917, Mr. Worthington received the appointment of Civil Engineer in the United States Navy, at Washington, D. C., where he remained until April, 1920, leaving to accept a position with Stone and Webster, Incorporated, in Boston, Mass.

On account of ill health, in November, 1929, Mr. Worthington severed his connection with Stone and Webster, Incorporated, and went to Florida where he resided until his death.

He was married in 1906 to Fannie A. Crumpton, of Pittsburgh, who, with a son, Charles C. Worthington, survives him. A daughter, Elizabeth C. Worthington, died in July, 1930.

Mr. Worthington was elected a Member of the American Society of Civil Engineers on November 6, 1901.

ADOLPH YAPPEN, M. Am. Soc. C. E.[1]

Died April 4, 1931

Adolph Yappen was born on the Island of Fehmern, Denmark, on November 5, 1860, the son of Richard N. and Caroline Yappen. In 1867, the family emigrated to the United States and settled at Sabula, Iowa, and it was there that Mr. Yappen's boyhood was spent and there that he received his public school education. Possessed of a roving disposition, he went to the Pacific Coast when he was only fifteen years old and, later, entered the University of California, working his way through college, from which he was graduated in June, 1882.

On leaving college, Mr. Yappen secured a position with the Northern Pacific Railroad Company as Rodman on new construction, subsequently being promoted to Instrumentman and Assistant Engineer, working under Mr. Bogan who, at that time, was Principal Assistant Engineer.

Mr. Yappen's experience after leaving the employ of the Northern Pacific Railroad Company can best be told in his own words:

"I was what might well be called a tramp engineer—worked as Instrumentman on the jetty work at the mouth of the Columbia River. Later, I find I was on a survey between Antioch and Bakersfield, Calif., now a part of the Southern Pacific. At that time it was the San Joaquin and Kern Valley Railway. Later, on a location of a proposed road in Mexico. Later, with a Government survey through Ecuador over the Andes through part of Brazil and from south to north through Venezuela. At one time went to Panama as Assistant Engineer when the French were trying to dig the ditch, but got disgusted and went back to Oregon, and also visited the Alaska gold field at the junction of the Yukon and Stewart Rivers."

Among other work during this period, he assisted in laying out the City of Tacoma, Wash.

In 1893, Mr. Yappen entered the Bridge and Building Department of the Chicago, Milwaukee and St. Paul Railway Company and remained in its service and that of its successor, the Chicago, Milwaukee, St. Paul and Pacific Railroad Company, until his death.

In 1898, he was on the staff of W. A. Rogers, M. Am. Soc. C. E., at that time Engineer of Subways on the first Chicago Track Elevation work undertaken by the Railway Company and, later, he served as Assistant Engineer on masonry construction and on bridge work in Illinois, Iowa, and Missouri. In 1901, he became District Carpenter, with headquarters, successively, at Green Bay, and Milwaukee, Wis., and Chicago, Ill. In 1919, he was made Assistant

---

[1] Memoir prepared by T. L. D. Hadwen and Walter A. Rogers, Members, Am. Soc. C. E.

Engineer of Bridge Maintenance on the System, which position he occupied until his death.

Mr. Yappen showed in his massive physique, which stood him in such good stead in his early wanderings, the traces of his Viking ancestry and, in his character, the persistent determination to achieve correct results, of his Teuton forebears. Quiet and unassuming in manner and highly conscientious, he possessed a sense of humor and a tolerance which made him very successful in handling men. His wide experience and sound judgment were always at the service of his friends. He had a fondness for mathematical problems and found recreation in their solution. His early experience in pioneer construction had given him a fund of knowledge on timber construction, which few possess.

He was a member of the Western Society of Engineers, and of the American Railway Bridge and Building Association. He was also a member of various Masonic bodies.

In 1909, he was married to Belle Paul, of Viroqua, Wis., who, with his younger brother, Reinhart Yappen, of Rock Rapids, Iowa, and a number of nieces and nephews, survives him.

Mr. Yappen was elected an Associate Member of the American Society of Civil Engineers on June 1, 1909, and a Member on July 7, 1915.

ELMER ZARBELL, M. Am. Soc. C. E.[1]

DIED SEPTEMBER 10, 1931

Elmer Zarbell, the son of Iver C. and Pauline (Hane) Zarbell, was born in Chicago, Ill., on May 25, 1874. His education was obtained in the public schools of Chicago and at Cornell University, Ithaca, N. Y. He was graduated from the latter institution in the Class of 1895, with the degree of Civil Engineer.

Immediately on the completion of his college course, Mr. Zarbell went to Harrisonburg, Va., where, until January, 1897, he held a position as Draftsman and Assistant Engineer with the Chesapeake and Western Railroad Company. In this position, his duties were general, in connection with the construction of twenty-seven miles of railroad and several hundred miles of surveys.

In January, 1897, he became associated with the Mobile and Ohio Railroad Company as Instrumentman, on preliminary and location surveys in Bibb County, Alabama; somewhat later he was made Assistant to the Resident Engineer in charge of construction of the Mobile and Ohio Railroad Bridge across the Alabama River, at Montgomery, Ala., and, during the early part of 1898, acted as Resident Engineer in charge of the construction of yards and terminals for the Mobile and Ohio Railroad Company at Montgomery.

Mr. Zarbell served during the Spanish-American War from August, 1898, to March, 1899, as Second Lieutenant with the Third Regiment, U. S. Volunteer Engineers, at Jefferson Barracks, Missouri; as First Lieutenant, at Lexington, Ky. and Macon, Ga.; and as Adjutant of the Third Battalion, at Matanzas, Cuba, and Atlanta, Ga. He was in charge of various harbor improvements, made in Cuba, and commanded an expedition, including a squad of the Second U. S. Cavalry, making a reconnoissance of half of Matanzas Province.

---

[1] Memoir prepared by G. R. Smiley, Assoc. M. Am. Soc. C. E.

From March, 1899, until March, 1900, Mr. Zarbell was with the Special Commission on the Chicago Drainage Canal, at Chicago, in charge of re-measurements and re-calculations for capacity, velocity, etc.

He became associated with the Engineering Department of the Louisville and Nashville Railroad Company in March, 1900, and, thereafter, until the time of his death, he was engaged on the maintenance, construction, and design of important lines and projects for that Company. For a year subsequent to 1900, he served as Assistant Engineer in charge of the construction of the Camden Branch. From March, 1901, to January, 1903, he was Resident Engineer in charge of the construction of the Alabama River Bridge and eight miles of roadbed at Selma, Ala. From January to March, 1903, he was Assistant Engineer at Pennsacola, Fla. During the remainder of that year, he was in charge of the field surveys then being made for reduction of grades and elimination of curvature on the Henderson Division.

In January, 1904, Mr. Zarbell was appointed Engineer of Maintenance of Way of the Alabama Mineral Division, in which position he remained until August, 1905. At this time he was transferred to Louisville, Ky., as a General Assistant to the Chief Engineer, where he had charge of the design and general supervision of the Company's concrete structures of all types as well as assisting generally in the administrative work of the office.

Impeccable in character, Mr. Zarbell was a man of positive characteristics. He was intolerant of anything that was wrong, and drove direct to the conclusion that in his judgment was correct. Absolutely loyal, he was indefatigable in his attention to all work he had in charge. He mastered the details of each problem and overcame its difficulties. Because of his qualities his friendship will be remembered long and appreciated deeply.

He was married on October 12, 1904, to Fay Smith, of Denver, Colo., who, with a son, Iver, and a daughter, Dorathea, survives him.

Mr. Zarbell was elected a Junior of the American Society of Civil Engineers on January 3, 1899; an Associate Member on February 7, 1900; and a Member on May 5, 1908.

JULIAN WILMOT ALGER, Assoc. M. Am. Soc. C. E.[1]

DIED JANUARY 27, 1932

Julian Wilmot Alger, the only son of Wilmot R. and Carrie (Tomlinson) Alger, was born in Stowe, Vt., on October 27, 1890. He was graduated from Norwich University, at Northfield, Vt., in 1910, with the degree of Bachelor of Science in Civil Engineering.

---

[1] Memoir prepared by Henry C. Porter, M. Am. Soc. C. E.

Mr. Alger received his first engineering experience on water-power and municipal investigation work at Wells River, Vt. However, most of his active professional life was spent in Texas to which State he removed in 1911. At that time he accepted a position with the San Benito Land and Water Company, San Benito, Tex., and during the next six years he was engaged upon the following engineering projects in Texas: The construction of the Medina Lake Dam, near San Antonio; with the United States Engineering Department, on surveys of the Guadalupe River; with the San Benito and Rio Grande Valley Railway Company; the American Rio Grande Land and Irrigation Company, of Mercedes; and the United States Engineering Department, at Galveston.

In 1917, Mr. Alger entered the First Officers' Training School for the United States National Army, at Leon Springs, Tex.; he finished the Engineer Officers' Training School at Fort Leavenworth, Kansas, on August 15, 1917, received a commission as First Lieutenant, and was assigned to the 315th Engineer Regiment, 90th Division, of the United States Army. Upon his arrival at Camp Travis, Texas, he was assigned to C Company, and, in France, he commanded C Company, with the rank of First Lieutenant, during its entire seventy-four days of service at the front in the St. Mihiel and the Meuse-Argonne Offensives. He received a citation for seventy-three hours of duty without rest or sleep during that time.

On April 28, 1919, he was commissioned Captain of Engineers. In the latter part of May of that year, he was appointed Personnel Adjutant of the 315th Engineer Regiment, and, a few days later, was made Regimental Adjutant, serving in that capacity until his organization was mustered out of service on June 26, 1919. He was honorably discharged on July 12, 1919.

In 1920, Mr. Alger was Field Construction Engineer with American Dreseler Tunnel Kilns, Incorporated, of Cleveland, Ohio, and, in 1921, he was with the Valuation Department of the St. Louis and San Francisco Railway Company, engaged in computing and compiling data. Later, he served as Computer and Estimator in the City Engineer's Office of Dallas, Tex., on street paving. In 1922, Mr. Alger was employed by Sutton County, Texas, to locate, design, and construct State highways in that County, with State and Federal Aid. In October, 1923, he was made County Highway Engineer of Dickens County, Texas, on the location, preparation of plans, and construction of State highways in that County until the completion thereof in December, 1926. From January to April, 1927, he was employed by the State Highway Department as Resident Engineer for Lubbock County, Texas, on plans and construction of State highways, and from April to October, 1927, he brought to completion, as Resident Engineer for the State Highway Department at Breckenridge, Tex., the State highway program in Stephens County. In November, 1927, he was appointed Resident Engineer of Navarro County, for the State Highway Department, at Corsicana, Tex., on the location, preparation of plans, and construction of State highways in that County, which work was not completed at his death on January 27, 1932.

Mr. Alger's illness was of short duration, and his death came as a great shock to his loved ones and to a wide circle of friends in Texas and elsewhere.

A man of simple and temperate habits in all things, it was expected that a long and useful future stretched before him. Well-trained, capable, thorough, and conscientious in his work, his career as a Civil Engineer will always reflect honor upon his name and upon the profession to which he belonged; widely read and well informed on all subjects, he was a most delightful and interesting conversationalist; cheerful and optimistic by nature, daily association with him in his work and elsewhere was most congenial and pleasant; possessed of a high sense of honor, he expected and demanded the same honesty from those with whom he dealt; holding himself aloof from the petty, inconsequential things about him, he seemed to dwell serenely and thoughtfully in an atmosphere all his own, to which he welcomed his friends with all the kindliness, consideration, and cordiality of the loyal friend he was. Honest citizen, painstaking workman, fearless soldier, firm but just executive, and cultured gentleman, Texas has lost much in the passing of Julian Wilmot Alger.

Immediately after his return from France, Mr. Alger was married, on July 15, 1919, to Margaret Bushwall, who survives him, and with his many friends mourns his passing.

Mr. Alger was elected an Associate Member of the American Society of Civil Engineers on March 16, 1925.

ALBERT ROSS BAILEY, Assoc. M. Am. Soc. C. E.[1]

Died June 29, 1931

Albert Ross Bailey was born in Grand Rapids, Mich., on February 28, 1877. He was the son of Charles and (Susan) Bailey, who had moved to Michigan from the State of New Hampshire.

Mr. Bailey was educated in the public schools and at the University of Michigan where from 1899 to 1903 he was a student in the Engineering College. He returned to the University in 1909 as an Instructor in Surveying and Drawing and, in 1913, received the degree of Bachelor of Arts.

From 1903 to 1909, he was employed by the Engineering Departments of the New York Central and Baltimore and Ohio Railway Companies, on construction work and also in the office as Draftsman and Chief Draftsman

From 1909 to 1914, he served on the Faculty of the University of Michigan as Instructor in Surveying and Drawing, and, from 1914 to 1917, as Assistant Professor in Civil Engineering, teaching Railway Engineering.

During his University service, he was employed by Mortimer E. Cooley, M. Am. Soc. C. E., in connection with valuations of the Pere Marquette Railroad and the Detroit United Railroad. He received a leave of absence for the last year of his connection with the University and was engaged on the valuation of the Public Service Railways of New Jersey, associated with Dean Cooley and Mr. H. C. Anderson.

---

[1] Memoir prepared by a Committee consisting of Henry E. Riggs and W. C. Hoad, Members, Am. Soc. C. E.

From 1917 to 1920, Mr. Bailey was Engineering Assistant to the General Valuation Counsel of the New York Central Lines, with headquarters in New York, N. Y., devoting his time to the study of questions of valuation theory and making investigations and reports on railway construction throughout the eastern half of the United States.

In 1920, he returned to Ann Arbor, Mich., and associated himself with Mr. H. H. Atwell in engineering and construction work, under the firm name of Bailey and Atwell. This partnership continued until 1922, during which time the firm was engaged in the construction of bridges and roads in Southeastern Michigan.

In 1922, Mr. Bailey was appointed Engineer-Manager of the Washtenaw County Road Commission and was placed in charge of all highway activities in the County, which position he held at the time of his death. He had the responsibility for all construction and maintenance of roads, county parks, and parkways. He established the accounting and statistical records of the Highway Department on the highest plane. He designed and built the larger part of the present permanent roads of the County. He conceived and carried to completion the construction of the Huron River Drive, a parkway extending for nine miles along the Valley of the Huron River, from Ann Arbor to Dexter, Mich. This was one of the pioneer high-class parkway developments in the State. During his administration, approximately 300 miles of paved highways and gravel roads were built in the County, and considerable construction work was done in landscaping and planting.

Throughout Mr. Bailey's connection with the work of the Washtenaw County Highway Commission, he gave special attention to the scenic development of highways and parkways and had a very keen sense of the value of this class of improvement to the people. His untimely death just at the commencement of the finer and more artistic highway development in Eastern Michigan is a real loss to the community.

Mr. Bailey's annual reports have been among the most complete and comprehensive of any of the County reports and have set a high standard for other counties in Michigan.

He was a member and Director of the Ann Arbor Chamber of Commerce, and a Director of the Ann Arbor Flying Club, in which connection he was responsible to a large degree for the airport development in the County.

Mr. Bailey was a competent and skillful engineer, a man who was thoroughly engrossed in his work, a good citizen, and a good public servant. His outstanding qualities were his gentleness of nature and his loyalty to his friends. He made a large place for himself in Ann Arbor and in Michigan.

Mr. Bailey's death resulted from injuries received in an automobile accident while going to his work, and was directly due to his attempt to avoid injury to a runaway team.

On April 6, 1905, he was married to Effie Reid, of Battle Creek, Mich. He is survived by his widow, a daughter, Virginia, and a son, Reid.

Mr. Bailey was elected an Associate Member of the American Society of Civil Engineers on July 2, 1913.

GEORGE CASTLEMAN BRECKINRIDGE, Assoc. M. Am. Soc. C. E.[1]

Died April 26, 1932

George Castleman Breckinridge was born in Huntsville, Mo., on January 16, 1885, the son of John and Nellie Breckinridge. His father was a Civil and Mining Engineer and his grandfather was Judge S. M. Breckinridge, of St. Louis, Mo. He was a nephew of Onward Bates, Past-President and Hon. M. Am. Soc. C. E., of Chicago, Ill., noted Civil Engineer.

In 1903 he entered the Missouri School of Mines, but transferred to the University of Missouri the second semester of 1904-05, finishing that term and the following summer session. In 1915, he re-entered the University of Missouri and continued his studies with interruptions until 1917.

Mr. Breckinridge began his professional career as Assistant Surveyor and Mine Foreman for the Illinois Jalisco Copper Mining Company, at Ayutla, Mexico, in July, 1902, continuing in this capacity until he entered the Missouri School of Mines in the fall of 1903.

Upon leaving school in 1905, he entered the City Engineer's Office at Joplin, Mo., as Rodman and Chainman. In February, 1906, he engaged in private practice in Pittsburg, Kans., his work consisting largely of surveying and mapping for coal mines in the district.

In November, 1906, Mr. Breckinridge returned to Mexico as Mine Foreman for the Mexican Coal and Coke Company, at Las Esperanzas, and in June, 1907, became Engineer in charge of three mines for the Compania Carbonifera de Monterey, at El Menor. He returned to Huntsville, Mo., in August, 1908, in the capacity of Engineer for the Northern Central Coal Company, taking care of three mines at Huntsville and Higbee, Mo. He also managed his mother's farm during this period.

In September, 1911, Mr. Breckinridge entered the employ of the Manufacturers Railway at St. Louis, Mo., and was engaged in the reconstruction of switching yards, roundhouses, shops, and track work. In August, 1912, he joined the staff of the St. Louis Engineer's Office of the Chicago, Burlington and Quincy Railroad Company and continued with that organization until June, 1914, principally on double-track construction work. Following this, he was with the Southern Railway Company and the Wabash Railway Company, both at St. Louis, engaged in construction, valuation, and maintenance-of-way work.

In August, 1915, Mr. Breckinridge became Resident Engineer on bridge construction for the Illinois State Highway Commission. Returning to the University of Missouri in the fall of 1915, he continued his studies and also served as Assistant City Engineer of Columbia, Mo., while attending the University. In May, 1917, he returned to railroad work with the Missouri Pacific Railroad Company, at Omaha, Nebr., on track elevation.

In March, 1918, he became Chief Engineer of the Bell and Zoller Coal and Mining Company and, later, was appointed Superintendent of that

---

[1] Memoir prepared by William von Meding, Esq., Chicago, Ill.

Company's No. 2 Mine at Zeigler, Ill., continuing in this position until July, 1919, when he resigned to become Superintendent of Mine 27 of the Peabody Coal Company, at Shelburn, Ind. He served in this capacity until he was transferred to Bellwood, W. Va., in May, 1928, as Superintendent of Mine 71, owned by the Bellwood Coal Company, a subsidiary of the Peabody Coal Company.

In February, 1929, Mr. Breckinridge again returned to railroad work with Walter A. Shaw, M. Am. Soc. C. E., of Chicago, Ill., in connection with grade separation in the North Shore suburbs. While engaged in this work he became interested in developments in the Soviet Union and accepted a position as Senior Mining Engineer with the Commission sent to Siberia by Allen and Garcia Company, of Chicago. He left Chicago in the latter part of March, 1930, arriving in Tomsk, Siberia, on April 18. His work was mostly of a consulting nature, in connection with the development of the major Siberian fields. He traveled extensively through the various mining districts and planned several large operations, chiefly in the Cheremhovo Basin. In October, 1931, this work was terminated, and the Commission, as a whole, returned to America.

Mr. Breckinridge, however, in company with two others, chose to remain and continue as individuals, concluding a separate contract with the Soviet Coal Trust known as Vostugol. He became District Engineer of the Prokopievsk Field, and continued as such until March 1, 1932. In returning, he contracted a severe cold at Tiaga, Siberia, a junction point on the Trans-Siberian Railway, and arrived in Berlin, Germany, suffering from pleurisy. In spite of the best care of Berlin physicians, Mr. Breckinridge contracted pneumonia and died on April 26, 1932.

He was married in Joplin, Mo., on September 11, 1905, to Pearl Coles, who survives him. He was the father of Albert C., John, George C (deceased), and Virginia.

Mr. Breckinridge was a member of Shelburn, Ind., Lodge No. 369, A. F. and A. M., Jerusalem Chapter No. 81, Royal Arch Masons, Sullivan, Ind., and Zorah Shrine Temple, of Terre Haute, Ind.

He was a Registered Civil and Mining Engineer in the State of Indiana, and a member of the American Institute of Mining and Metallurgical Engineers.

Mr. Breckinridge was elected an Associate Member of the American Society of Civil Engineers on February 24, 1924.

MORTON BURDEN, Assoc. M. Am. Soc. C. E.[1]

DIED APRIL 15, 1931

Morton Burden, the son of John and Jenet (Duncanson) Burden, of Stirling, Scotland, was born at Troy, N. Y., on April 13, 1874. His parents were married in Stirling and came to America a few years later. For a short period his father was connected with the Burden Iron Works in Troy.

---

[1] Memoir prepared by Richard Khuen, Jr., M. Am. Soc. C. E.

Mr. Burden studied civil engineering at the Rensselaer Polytechnic Institute, in Troy, and was graduated in 1895. From September of that year to September, 1897, he was in the Bridge and Construction Office of the Pencoyd Iron Works, and from September, 1897, to November, 1898, he was in the Drafting Office of the Keystone Bridge Works, making and checking shop drawings of bridges and buildings.

In November, 1898, Mr. Burden entered the Engineering Office of the Ohio Steel Company (now a part of the United States Steel Corporation), where he served as Chief Draftsman of the Structural Department, designing the buildings and other structural work required for the various plants, including the ore, coke, and limestone bins and trestles for the four large blast furnaces which were built at the Ohio, New Castle, and Mingo Works.

In 1901, Mr. Burden became connected with the American Bridge Company in the Designing Department, at Pittsburgh, Pa., making designs and estimates of various classes of structural steel work, including the design of such structures as pipe mills and steel plant buildings for the National Tube Company, at Lorain, Ohio; pipe mills at McKeesport, Pa.; a steel plant for the Brier Hill Steel Company, at Youngstown, Ohio; buildings for the American Rolling Mill Company, at Middletown, Ohio; plate and jobbing mills for the Otis Steel Company, at Cleveland, Ohio; factory buildings for the Pittsburgh Plate Glass Company, at Ford City, Pa.; mill buildings for the Pittsburgh Crucible Steel Company, at Midland, Pa.; a 16-story office building for the Eastman Kodak Company, at Rochester, N. Y., and numerous other structures. From 1914 to 1926, he was in charge of the Designing and Estimating Office of the Pittsburgh Division.

From 1926, until the time of his death, Mr. Burden was on special assignments of designing and supervising large corporation work, such as new mill buildings for the National Tube Company, at McKeesport; a new seamless mill, at Lorain; a new roof for pipe mills, at McKeesport; a new beam shop for the American Bridge Company, at Ambridge, Pa.; a new warehouse for the Carnegie Steel Company, at Pittsburgh; and a new coal washery, at Clairton, Pa.

To his many friends and business associates Mr. Burden was a pleasant companion, a man of sterling character, and a steady, hard worker. He derived a great deal of pleasure out of working in his garden, but did not indulge in the more strenuous outdoor pastimes.

He was married to Jane Nevin Knox, of Sewickley, Pa., on October 29, 1906. He is survived by his widow and one son, Morton Burden, Jr., a Junior at Dartmouth College, Hanover, N. H.

Mr. Burden died of pneumonia on April 15, 1931, in the Sewickley Valley Hospital. His passing leaves a void in the hearts of many who knew him and who had learned to love him for his personal qualities.

Mr. Burden was elected a Junior of the American Society of Civil Engineers on February 4, 1896, and an Associate Member, on January 2, 1901.

ROBERT MORRIS DRAKE, Assoc. M. Am. Soc. C. E.[1]

Died October 25, 1931

Robert Morris Drake, the son of Colonel J. M. Drake and Angelina (Robb) Drake, was born in Portland, Ore., on April 2, 1869. He entered Leland Stanford University with its pioneer class in 1891, and received the degree of Bachelor of Arts in Civil Engineering in June, 1894.

Previous to entering the University, Mr. Drake had been employed from March to December, 1890, as Leveler on a location survey for the Oregonian Railway Company; from February to July, 1891, on surveys for the Portland Water-Works; and from July to October, 1891, as Leveler on a location party for the Union Pacific Railroad Company. During the summer of 1892, he was engaged as Assistant Engineer with the East Side Railway Company, on the construction of an electric railway connecting Portland and Oregon City, Ore.

Mr. Drake entered the service of the Southern Pacific Company in July, 1895, as an Assistant Engineer in the Maintenance-of-Way Department, and for seven years was engaged in the work of that Department, including new bridges, terminals, changes of alignment, and revision of grades.

In 1902, he was given charge of bridge and building work on the Coast Division, and, in 1906, he was promoted to the position of Division Engineer. In 1907, he became District Engineer, Maintenance of Way, for the Northern District of the Southern Pacific Company, continuing in such office until the time of his retirement in 1918.

Owing to failing sight, Mr. Drake lived very quietly at his home, in San Francisco, Calif., after leaving the railroad service. He died suddenly on October 25, 1931, and his funeral services were conducted by California Lodge No. 1, F. and A. M., of which he was a member.

He was married in San José, Calif., on November 11, 1900, to Caroline Mosher, who survives him.

Mr. Drake was elected an Associate Member of the American Society of Civil Engineers on March 7, 1900.

PRESTON BURT EASTERBROOKS, Assoc. M. Am. Soc. C. E.[2]

Died February 1, 1931

Preston Burt Easterbrooks, the son of Ezra Otis and Nora Barton Easterbrooks, was born in Swansea, Mass., on February 26, 1876. He could be classed among the self-educated since he assumed the duties of his first position before completing his High School course. Soon realizing his limited future without further technical knowledge, he availed himself of the evening engineering courses at The Rhode Island School of Design, and thereafter continued by himself in the higher mathematics, ever increasing and widening his knowledge of his profession.

---

[1] Memoir prepared by W. H. Kirkbride and George W. Rear, Members, Am. Soc. C. E.

[2] Memoir prepared by C. E. Hunt, Affiliate, Am. Soc. C. E.

In 1892, Mr. Easterbrooks entered the service of the New York, New Haven, and Hartford Railroad Company, being employed, successively, as Chainman, Rodman, Draftsman, and Transitman, until January, 1898. Subsequently, he was employed by the Chihuahua and Pacific Railroad Company, as Chief Draftsman and, as such, had charge of the drawing of all plans (except steel), including all masonry plans, used in the construction of 125 miles of road.

He was next engaged by the Chief Engineer of the Mexican Central Railroad Company, City of Mexico, Mexico, and made surveys of betterments in the Corcovada Hills, on the Tampico Branch, and was on reconnaissance with the late J. T. Norton, M. Am. Soc. C. E., from Guadalajara to Aguas Calientes (250 km.). In July, 1900, he was placed in responsible charge of construction on the division covering the Parral Extension, Parral, Mexico. He resigned his position with the Railroad Company in October, 1900, and was associated with the late E. S. Safford, M. Am. Soc. C. E., on surveys for a dam site and increased water supply for the City of Chihuahua, Mexico.

In January, 1901, Mr. Easterbrooks entered the service of the Cuba Company as First Assistant Engineer, and, later, as Superintendent of Construction, Eastern Division, remaining with this Company until June, 1902. During this time he had responsible charge of masonry construction, erection of steel bridges, wooden trestles, track-laying, and material yards.

In July, 1902, he entered the employ of Westinghouse, Church, Kerr, and Company, Engineers, New York, N. Y., and, for eighteen months, served as Chief of Party on the location of the Rochester, Auburn, and Syracuse (N. Y.) Electric Railway. He was next placed in charge of construction of the Scranton Tunnel (4 700 ft. long), as Resident Engineer, for the Lackawanna and Wyoming Valley Railroad Company, at Scranton, Pa. On the completion of this work, he returned to the New York Office of the Company and was engaged on various engineering works, including plans and surveys for electric transmission lines in Northern New York, proposed underground railways for New York City, railway passenger terminals, power developments, etc.

From October, 1911, to May, 1912, he was Chief Assistant Engineer of the Caribbean Construction Company, Port au Prince, Haiti, for which F. W. Andros, M. Am. Soc. C. E., was Chief Engineer. In this capacity, Mr. Easterbrooks was in charge of location and construction work on 100 miles of road under construction and 350 miles contemplated.

In June, 1912, he returned to Westinghouse, Church, Kerr, and Company and was actively engaged on many important engineering projects, including studies for railroad terminals, investigation, and report on an irrigation project in Montana; building hotel and terminal facilities for the Canadian Pacific Railroad Company, at Winnipeg, Man., Canada; constructing difficult foundations for a mole in connection with a large plant development for the Armour Fertilizer Company, at Matanzas, Cuba; and building a sugar refinery having a capacity of 1 000 000 lb. per day, for the Savannah Sugar Refining Company, at Savannah, Ga.

In the spring of 1918, Mr. Easterbrooks was made Construction Engineer of a $25 000 000 contract for the Air Nitrates Corporation, in connection with the Muscle Shoals Development, known as United States Nitrate Plant No. 2. It was the expectation at the time of his assignment to this work that a contract for Plants Nos. 3 and 4 also would be awarded the Company, in which event it was the intention and understanding that Mr. Easterbrooks would be made Resident Engineer in full responsible charge of that project. This opportunity did not materialize, however, and he stayed through the operation of the work on Plant No. 2 and was one of the responsible officials in charge of its construction. It has been written of him that "he did the work to which he was assigned with an enthusiasm and loyalty that was his chief characteristic, putting his shoulder to the wheel where the going was hard and filling in where the ranks were thinnest."

On the completion of this work he returned to New York and was placed in charge of the construction of a plant for the manufacture of complete locomotive superheaters for the Locomotive Superheater Company, at East Chicago, Ill.

During 1919 and 1920, he was Superintendent of Construction on an extensive addition to the plant of the American Rolling Mill Company, at Middletown, Ohio, including warehouse and storage building, galvanizing, finishing, cold-roll distributing furnace, and continuous annealing furnace buildings, sheet mill, fire-bed mill, sewers, roads, track construction, water and fire protection, electrical distribution, boiler-house extension, power-plant addition, spray pond, cooling system, and a large amount of miscellaneous work. The buildings alone covered about 10 acres of ground area.

In 1920, Westinghouse, Church, Kerr, and Company was consolidated with Dwight P. Robinson and Company, Incorporated, and, subsequently, this latter Company was merged with three other prominent engineering organizations, forming the United Engineers and Constructors, Incorporated, and Mr. Easterbrooks was retained by the latter as one of its Superintendents of Construction and, subsequently, was made Construction Manager.

During the next ten-year period, until the time of his death, in addition to his administrative responsibilities, he was in direct charge of the construction of a great variety of projects among which were the Islin Shops at Jackson, Tenn., for the Mobile and Ohio Railroad Company; extensions to the New York, Westchester, and Boston Railway, from Harrison to Rye, and from Rye to Port Chester, N. Y., a two-track electrically operated system, including grading, culverts, fencing, tracks, car lay-up yards, stations and platforms, bridges, tunnels, and stairways, etc. This work was contiguous to the high-tension electrified lines of the New York, New Haven, and Hartford Railroad Company, and was accomplished without interruption or delays to its train service. Mr. Easterbrooks was in charge of the work of extension to the South Street Station of the Narragansett Electric Lighting Company, at Providence, R. I., including switch-house and installation of switching equipment; also, changes and extensions in sub-stations and underground transmission work. He also built the Queensboro Apartments, at Jackson Heights, Long Island, consisting of two groups of seven 5-story elevator apartment houses, containing a total of 140 apartments; the American Embassy at Rio de Janeiro, Brazil; a theatre and office building for the Statler Hotels, in Buffalo, N. Y., and "The Barclay," a high-class fireproof apartment hotel, in Philadelphia, Pa., overlooking Rittenhouse Square.

Mr. Easterbrooks' experience covered a wide range of subjects, and his recognized ability and intimate knowledge of the details which go to make up good construction, on heavy construction as well as on high-class building work, were outstanding characteristics.

He was interested in any young men who came under his direction, who showed promise of future development, and spent a great deal of time in his own way in trying to develop their latent technical ability and to fit them for administrative responsibility.

Personally and professionally, he maintained a high standard of ethics and was unswerving from a course which he believed to be right. Tenacious of purpose, with unflagging energy, he gave himself wholly to his profession and took a justifiable pride in the accomplishment of work well done. Mr. Easterbrooks was a member of Clinton Lodge No. 54, F. and A. M., at Savannah.

He was married in Providence, R. I., on August 9, 1904, to Janet M. Auty, who, with a son, Preston B., Jr., a brother, Frank D. Easterbrooks, and three sisters, Clara B. Easterbrooks, Mrs. Henry J. Peck, and Mrs. Leon H. Read, survives him.

Mr. Easterbrooks was elected an Associate Member of the American Society of Civil Engineers on May 1, 1907.

JAMES HENRY GALLIVAN, Assoc. M. Am. Soc. C. E.[1]

Died February 12, 1931

James Henry Gallivan was born at Northampton, Mass., on September 21, 1876, the son of Patrick and Bridget (Kiley) Gallivan.

After graduating from the Northampton High School, and taking courses in Civil Engineering with the International Correspondence Schools, Scranton, Pa., and the Alexander Hamilton Institute, Poughkeepsie, N. Y., he entered the employ, in 1897, of E. C. and E. E. Davis, Civil Engineers, of Northampton, as a Rodman, continuing with this firm until July, 1902.

---

[1] Memoir prepared by Paul Sterling, M. Am. Soc. C. E.

From April until November, 1898, he served with the 2d Massachusetts Regiment, United States Volunteers, in the Spanish-American War.

In July, 1902, Mr. Gallivan was employed as Transitman with the Metropolitan Water and Sewerage Board of Massachusetts, and in March, 1903, started his railroad career as a Transitman with the Chesapeake and Ohio Railroad Company. In May, 1904, he entered the employ of The New York, New Haven, and Hartford Railroad Company, continuing with that Company, with uninterrupted service, until his death.

Starting as a Transitman, Mr. Gallivan worked his way through the various engineering grades in both the Maintenance and Construction Departments of the Railroad Company. He was appointed Division Engineer of the Central New England Division, at Poughkeepsie, N. Y., in July, 1912. In November, 1917, he was transferred to Boston, Mass., as Division Engineer of the Midland Division, which position he held at the time of his death.

He is survived by his widow and two daughters, Mrs. C. W. Maloney and Mrs. T. B. Gray.

Mr. Gallivan's engineering work was confined almost entirely to railroad new construction and maintenance. The former consisted of double-tracking forty miles of railroad, which involved changing line, modifying grades, and numerous over and under-grade railroad bridges; also, the construction of a large engine terminal yard at Maybrook, N. Y., which included one of the first so-called "hump" yards, with its large supporting receiving, departure, and classification tracks. He had an unusual faculty for handling men, winning their loyalty and support through his always genial personality.

He will be sorely missed by his many personal friends. Full of Irish wit, his stories and personal magnetism always brought him to the leadership of any social gathering. No party was complete without "Jim" Gallivan's presence.

Mr. Gallivan was elected an Associate Member of the American Society of Civil Engineers on January 2, 1912.

JAMES HENDRICKS HALLETT, Assoc. M. Am. Soc. C. E.[1]

DIED JANUARY 11, 1931

James Hendricks Hallett was born in Cambridge, Mass., on September 14, 1874. His father, Nelson Arnold Hallett, was a descendant of distinguished loyalist families that fled from New York in 1783, at the close of the War of the Revolution, and settled in St. John, N. B., Canada. His mother was Jennie Burns (Cunningham) Hallett, whose father, John Cunningham, was a prominent Architect and Civil Engineer for more than half a century in Scotland and America. The boy's early education was completed in the local schools of Cambridge.

---

[1] Memoir prepared from information on file at the Headquarters of the Society.

Mr. Hallett was associated with Hodges and Harriman, Civil and Consulting Engineers, of Boston, Mass., as Rodman, Levelman, and Transitman from 1896 to 1899. There was a year's interlude in this period, during the Spanish-American War, at which time he volunteered and served in the United States Army, in Cuba.

From 1899 to 1903, Mr. Hallett was with Ephraim Harrington and Company, Civil Engineers, of Boston, as Inspector and Superintendent of Construction. At this time the measurements and physical examinations were made for the consolidation of seventy-one street railway companies into the Massachusetts Electric Company. This System included 900 miles of track, with 200 bridges as well as highway construction. He was also engaged on the construction of street railways, pavements, and bridges for the Norwich and New London, Conn., Street Railway Company, the Milford, Attleboro, and Woonsocket, Mass., Street Railway Company, and with companies of many other New England towns in the immediate vicinity.

In 1904, Mr. Hallett entered the employ of the Stone and Webster Engineering Corporation, as Superintendent of Construction. He remained with this Company throughout a period of four years, during which time he was engaged on the development of bridges, dikes, street railways, power-house and sub-station construction in various parts of the United States. He later became General Manager for the Columbia Improvement Company, of Paducah, Ky., and Construction Agent and Resident Engineer for the Baton Rouge, La., Gas and Electric Light Company. He also served as Superintendent of Construction for the Jacksonville Traction Company, of Jacksonville, Fla.

In 1908, Mr. Hallett was employed by the J. G. White Company as General Manager for the L. N. Farnham Construction Company, on the Little River Water Supply of Springfield and Westfield, Mass. In 1909, he again went South and served as Assistant City Engineer, Assistant Building Commissioner, and Chief Clerk for the City of Jacksonville.

In 1918, at the expiration of ten years of service in Jacksonville, Mr. Hallett went North to accept a position with the Massachusetts State Highway Department where he remained for about three years. His subsequent positions were with the American Woolen Company, when Shawsheen Village was planned and developed, with the Boston and Maine Railroad Company, and, at the time of his death, he was Engineer in Charge of the construction of the Manger Hotel, in Boston.

He was a Mason, a member of Friendship Lodge, Taylors Falls, Minn. He was also a member of the Florida Engineering Society and of Adelbert Ames Camp No. 19, Spanish-American War Veterans, Lowell, Mass.

He had a winning personality and made many friends in the various places where he had lived, to all of whom his untimely death was a great shock.

Mr. Hallett was married on June 28, 1905, to Ethel Foster, of Tewksbury, Mass., who survives him. They had one son, James, who was born in Florida on December 9, 1907.

Mr. Hallett was elected an Associate Member of the American Society of Civil Engineers on June 23, 1916.

LEWIS STIRLING HEREFORD, Assoc. M. Am. Soc. C. E.[1]

DIED APRIL 13, 1931

Lewis Stirling Hereford was born in Baton Rouge, La., on April 15, 1875. His father was a Captain in the Confederate Army and a Louisiana sugar planter. His mother was Rowena (Percy) Hereford, also a native of Louisiana.

Mr. Hereford was educated at the Louisiana State University and his first engineering work was on the Mississippi River levees; afterward, he went to Helena, Ark., where he was engaged in municipal engineering. From there he went to Atchison, Kans., where he served as Assistant City Engineer under the late Frederick Giddings, M. Am. Soc. C. E. After several years, he was appointed County Engineer and, in that office, built many bridges and roads. In 1912, he went to Lake Charles, La., where he was engaged in road and bridge building.

In 1920, Mr. Hereford went to Charlotte, N. C., and started the first Piggly Wiggly Store, gradually increasing them in number until he had seven, when he finally sold them. They were very successful and were outstanding in their upkeep and cleanliness, thereby setting a standard that competitors had to try to meet or suffer by comparison.

After Mr. Hereford sold his stores in 1927, he did not engage actively in any business. He was Treasurer of the Carolina Cork Roll Company of Charlotte. He was also active in the affairs of the Rabbit Breeders Association, and was the pioneer in the introduction of Rex rabbits in this section of the United States. He had many fine specimens which he had imported from England and Canada. The same scrupulous care was given to his rabbits that went with everything he undertook, and his hutches were taken as models for many others that were started later.

Mr. Hereford had a host of friends who regret his passing and who sincerely realize that in him the community has lost a fine type of citizen.

In 1903, Mr. Hereford was married to Christina Giddings, the daughter of the late Mr. Giddings, who survives him.

Mr. Hereford was elected an Associate Member of the American Society of Civil Engineers on January 17, 1916.

WILLIAM CHAFFIN HOWE, Assoc. M. Am. Soc. C. E.[2]

DIED SEPTEMBER 16, 1930

William Chaffin Howe was born in Worcester, Mass., on April 19, 1876, the son of William B. and Harriet (Chaffin) Howe. His early education was received in the Worcester Public Schools, from which he entered the Worcester Polytechnic Institute with the Class of 1893.

---

[1] Memoir prepared by L. W. Cuddy, Pres., Carolina Cork Roll Company, Charlotte, N. C.

[2] Memoir prepared by Philip L. Wilder, Esq., Worcester, Mass.

Mr. Howe's first position after leaving the Institute was with the Washburn and Moen Company, in Worcester, with which he served as Designer of automatic machinery until 1894, when he went to Norcross Brothers, of Worcester and Boston, Mass., as Structural Draftsman and Checker. In 1897, he severed his connections with Norcross Brothers and entered the employ of the A. F. Hawkins Iron Works, in Springfield, Mass., as Office Assistant in charge of structural work, and he continued in the same capacity with the Mace Moulton Construction Engineering Company, when it took over the Hawkins Company.

In 1901, Mr. Howe went to the Eastern Bridge and Structural Company, at Worcester, Mass., as Checker on structural details. In 1905, he was made Contracting Engineer, in charge of the New England Territory, a position he held until shortly before he passed away.

Mr. Howe was a member of the Central Congregational Church, the Unitarian Men's Club, the Worcester Society of Civil Engineers, the Fay Club of Fitchburg, Mass., Past Master of Athelstan Lodge, A. F. and A. M., a member of the Scottish Rite Bodies, the Worcester County Commandery, and Phi Gamma Delta Fraternity.

He was married on November 4, 1898, to Ella Harned, of Philadelphia, Pa., and their son, Wilfred Harned, of Winchester, Mass., survives his parents.

Mr. Howe's unfailing cheerfulness won for him a wide circle of friends throughout New England, and to those whose privilege it was to know him intimately, his thoughtfulness, his love of doing for others, and the enjoyment it gave him, are constant sources of inspiration. William Chaffin Howe was a success in the broadest sense of the word. He is greatly missed.

Mr. Howe was elected an Associate Member of the American Society of Civil Engineers on September 2, 1914.

FRANK BURDETTE KIEL, Assoc. M. Am. Soc. C. E.[1]

DIED DECEMBER 31, 1930

Frank Burdette Kiel, was born in Lowell, Mich., on March 16, 1889, the son of Edward E. and Rose (Kinyon) Kiel. His grandfather, Simon Everett Kiel, served three years with the Third Michigan Regiment of Engineers and Mechanics during the Civil War.

Mr. Kiel attended school at Lowell and at South Bend, Ind., where he was graduated from the High School in 1907, winning special distinction in mathematics. He attended the Engineering College of the University of Michigan and after a complete four-year course in Civil Engineering received the degree of Bachelor of Science in Engineering in June, 1912.

---

[1] Memoir prepared by a Committee consisting of Henry L. Fruend, William G. Fargo, Horace S. Hunt, the late W. W. Tefft, and A. Streiff, Members, Am. Soc. C. E., Herman H. Block, Assoc. M. Am. Soc. C. E., and C. H. Wolf, Jun. Am. Soc. C. E.

Following his graduation from college, he entered the Engineering Department of the City of Flint, Mich., where, for three months, he was engaged on location for a proposed extensive sewerage system. He then entered the employ of the Grand Rapids, Holland, and Chicago Railroad Company, and was placed in charge of the surveys and mapping for extension of trackage.

His next appointment was with the Kent County, Michigan, Good Roads Commission, for which he took charge of surveys and preliminary work on a proposed highway system.

In January, 1913, Mr. Kiel entered the employ of the Fargo Engineering Company of Jackson, Mich., conducting field surveys and investigations for proposed water power and reservoir sites in Michigan, Indiana, and Wisconsin. He was in charge of the field work for reports on the physical condition of properties in the appraisal of the Pere Marquette Railroad System and of its proposed electrification. He was also in charge of field investigations for storage reservoirs and hydro-electric plants on the Red Cedar and Chippewa Rivers in Wisconsin, conducting surveys, river gaugings, railroad and highway relocations, and diamond drill test borings for five dam sites.

From November, 1915, until February, 1917, he was Principal Assistant Engineer under Joel D. Justin, M. Am. Soc. C. E., Resident Engineer, on the construction of the Wissota Plant, a 50 000-h.p. hydro-electric development on the Chippewa River, for the Wisconsin and Minnesota Light and Power Company.

Mr. Kiel was then appointed Resident Engineer on the construction of the Alcona Hydro-Electric Development on the Au Sable River, in Michigan, for the Consumers Power Company, at the same time having supervision of construction at both the Mio and Foote Developments on the same river.

In December, 1917, he was made Assistant Construction Superintendent on the Junction Dam on the Manistee River, in Michigan, in special charge of sluicing operations for the placing of embankment material. The Junction Development is a 16 500-kw. hydro-electric plant of the Consumers Power Company of Michigan.

In April, 1918, Mr. Kiel enlisted for World War Service in the United States Army, Ordnance Department, Nitrate Division, and was stationed at Sheffield, Ala., on the construction and operation of United States Nitrate Plant No. 2. He was specially assigned to the construction of a 90 000-kw. steam power plant. He was honorably discharged from service in April, 1919, but remained by special request, under civil service, as Civil Engineer in the Ordnance Department at Large, in charge of various tests and reports on the manufacture of ammonium nitrate. In June, 1919, at the completion of this assignment, he was granted an indefinite leave of absence. For his war service he was commended by his commanding officer, Lt. Col. J. K. Clement, Ordnance Dept., U. S. A., and recommended for a commission.

On leaving the Government Service Mr. Kiel organized the firm of Bird and Kiel, Consulting Engineers, engaged in general engineering practice at Little Rock, Ark., making investigations and reports on hydro-electric developments, design and construction of highways and bridges, and investigations for drainage projects, levees, and revetments.

He returned to Michigan in July, 1920, and again entered the employ of the Fargo Engineering Company. He was placed in charge of construction for the Elm Street Steam Plant, at Battle Creek, Mich., for the Consumers Power Company of Michigan. After this assignment, during 1921 and 1922, he was Resident Architect, in charge of the construction of the First Methodist Episcopal Church of Jackson, a $350 000 edifice.

In July, 1922, Mr. Kiel returned to Wisconsin as Resident Engineer, for the Fargo Engineering Company, on a 20 000-h.p., hydro-electric development at Jim Falls, on the Chippewa River, for the Chippewa Power Company. On the completion of this project, he was engaged in field investigations and compilations of reports for proposed power-plant developments in North Carolina and Michigan. In June, 1924, work was resumed on the Alcona Development, on the Au Sable River of Michigan, and Mr. Kiel was again placed in charge of construction, with special supervision of difficult deep quicksand foundations. In the fall of 1924, he made surveys of the grounds and layout for a $1 250 000 High School Building, in Jackson.

In November, 1924, Mr. Kiel entered the organization of the Commonwealth Power Corporation, of Jackson, for which he worked upon special investigations and reports for various power plants and water supply systems. During this assignment, he was engaged on the design and supervision of construction of the Hales Bar Steam Plant for the Tennessee Electric Power Company, of Chattanooga, Tenn., on the Saginaw River Steam Plant, and on the Wealthy Street, Grand Rapids, Steam Plant for the Consumers Power Company of Michigan. He was then transferred to the Commonwealth Power Corporation and was engaged on the completion of the Wealthy Street Steam Plant. He also collaborated on an extensive power survey and reports for the Republic Iron and Steel Company, at Youngstown, Ohio, making estimates of cost and preliminary layouts for a project involving the proposed expenditure of $15 000 000. He was then engaged on several proposed steam power projects, with special attention to condenser cooling water problems and hydraulic requirements.

On the organization of the Allied Engineers, Incorporated, as successors to Stevens and Wood, Mr. Kiel was assigned to special problems in connection with a large coal dock at Saginaw, Mich., for the Saginaw River Steam Generating Plant, a 140 000-kw. installation for the Consumers Power Company, and also on extensions to the Saginaw Heating Plant.

In May, 1930, he rejoined the organization of the Fargo Engineering Company and was engaged on specifications and preliminary plans for a series of hydro-electric developments.

Mr. Kiel passed away on December 31, 1930, after a short illness. At the time of his death, he was working on construction problems in connection with a 20 000-kw. hydro-electric development, storage reservoir, and long multiple-arch dam across the Colorado River, in Texas, for the Central

Texas Hydro-Electric Company, to which he had been assigned as Division Engineer with headquarters at Austin, Tex.

He belonged to the American Legion. He was a Mason, affiliated with Jackson Lodge No. 17, and an active member in the Masonic Country Club. He was a member of the Official Board of the First Methodist Episcopal Church of Jackson, and took a deep interest in the religious life of the community. He was an active member of the Civic Music Association. He was a life member of the Michigan Union, a registered Civil Engineer in the State of Michigan, and a Past-President and one of the founders of the Jackson Engineers Club. The engineers of Jackson feel that by the death of Frank Kiel, they and the profession in general have lost a true friend and valued associate. Resolutions and a testimonial to this effect were passed by the Engineers Club.

In December, 1924, Mr. Kiel was married to Helen Peters, of Battle Creek, Mich. He is survived by his widow and a son, Donald Franklin Kiel.

Mr. Kiel's character and personality are best described in the funeral tribute paid him by his friend, pastor, and associate, the Rev. Frederick Spence, D.D., as follows:

> "Everybody liked Frank. He had a rare personality. He made friends and held them. He was honest, kind to the point of personal sacrifice, giving himself to others in a way that is very rare among men in this highly individualistic age. He radiated friendliness and good will, and those who knew him trusted and loved him. Efficient in his chosen field, he yet found time to be interested in everything that was for the good of Society."

Mr. Kiel was elected an Associate Member of the American Society of Civil Engineers on November 25, 1919.

JOHN BRUCE MAILEY, Assoc. M. Am. Soc. C. E.[1]

DIED APRIL 29, 1932

John Bruce Mailey was born in Lynn, Mass., on August 22, 1886, the son of John Augustus and Rebecca (Twisden) Mailey, who were also born in Lynn.

Mr. Mailey's early life was spent in Lynn, where he received his preliminary education. He continued his studies at Tufts College, Medford, Mass., from which he was graduated with the degree of Bachelor of Science in Structural Engineering in 1910. In 1911-12, he took a course in Bridge Design at the Harvard Graduate School of Applied Science.

From 1910, following his graduation from Tufts, until his death in 1932, Mr. Mailey specialized in structural engineering. His successive engagements and positions were, briefly, as follows: From 1910 to 1912, various positions which involved structural detail drafting and design; and from 1912 to 1917, with the Boston and Maine Railroad Company as Structural Designer and Squad Boss in the Bridge Department and Assistant Engineer in the Valuations Department. During the World War (1917-18) he served in the Air Service.

---

[1] Memoir prepared by Dana N. Peaslee, Esq., Boston, Mass.

From 1919 to 1921 he was connected with the Bridge Department of the Boston and Maine Railroad Company; from 1921 to 1923 with the United States Railroad Administration, as Bridge and Building Engineer; and from 1923 to 1924 with Charles T. Main Company, Boston, Mass. From 1924 until the time of his death, Mr. Mailey was connected with Stone and Webster Engineering Corporation. In the capacities of Structural Designer and Engineer, he assisted in the design and construction of central power stations and industrial plant developments.

He manifested loyalty to his employers and devotion to his work. Untiring in his efforts to accomplish results, he did not shrink from devoting long hours to his duties. He was a cheerful and likable person, making true friendships with a large number of men to whom he was devoted and loyal.

Mr. Mailey was interested in the Masonic Bodies, and was a Thirty-second Degree Mason.

He was not married, but his disposition reflected the happy home life which he shared with unusual filial devotion with his aged mother. He died rather suddenly of spinal trouble, in Andover, Mass., on April 29, 1932, and is buried in Pine Grove Cemetery, in Lynn.

Mr. Mailey was elected a Junior of the American Society of Civil Engineers on April 30, 1912, and an Associate Member on May 15, 1917.

ALFRED STOWE MIRICK, Assoc. M. Am. Soc. C. E.[1]

Died December 6, 1931

Alfred Stowe Mirick, son of Ira and Sarah K. (Croul) Mirick, was born at Lyons, N. Y., on February 16, 1877. His maternal grandfather was Columbus Croul, a pioneer settler of Lyons. Mr. Mirick attended the Lyons Union School, from which he was graduated in 1896. He then entered Cornell University, and received his degree in Civil Engineering in 1901.

Upon graduation, he went with the Chicago and Northwestern Railroad Company as Rodman and Instrumentman. In July, 1902, he became Draftsman for the American Bridge Company, at Canton, Ohio, and Pittsburgh, Pa. In November of that year, he joined the Columbia Bridge Company, at Carnegie, Pa., as Squad Master in charge of details. In the autumn of 1903, he took a position with the Altman Company, of Canton, in charge of the structural work of the Elevating and Conveying Department. When this Department was sold to the Interstate Engineering Company, of Bedford, Ohio, in 1904, Mr. Mirick became Chief Draftsman of the Interstate Engineering Company, organizing and overseeing the drafting-room, laying out and designing new work, and submitting preliminary sketches and specifications.

---

[1] Memoir prepared by William W. C. Perkins, M. Am. Soc. C. E.

In March, 1908, Mr. Mirick went with the New York State Engineer and Surveyor as Assistant Engineer, first in charge of highway improvements and, in December, on the Barge Canal, in the Bureau of Locks, designing concrete structures. Later, he went to Albany, N. Y., with the State Highway Department, where he remained as Resident Engineer until 1917. Mr. Mirick was then appointed Chief Road Engineer and Chief of Construction of the Nebraska State Highway Department at Lincoln, Nebr., where he remained until 1923, when he became District Engineer for the Eastern Paving Brick Manufacturers Association, at Pittsburgh, Pa. He continued with the Association until August 1, 1930, with the exception of one year when he was General Sales Manager for the Clydesdale Brick and Stone Company, at Pittsburgh.

In August, 1930, he went to California and remained there until August, 1931, when he accepted a position with the Nebraska State Highway Department as Project Engineer, with headquarters at Lincoln, which position he held at the time of his death.

Mr. Mirick died on December 6, 1931, after suffering a stroke of apoplexy in his room at the Young Men's Christian Association Building, at Lincoln. He was buried in the Rural Cemetery at Lyons, N. Y., where a number of his ancestors lie. His funeral was conducted by Lodge No. 19, F. and A. M., of Lyons, to which he belonged.

Mr. Mirick leaves a widow and two daughters, Mrs. Jeanette (Mirick) Button and Florence Mirick, all of Hollywood, Calif. Other survivors are an aunt, Anastasia Mirick, a brother, Nelson C. Mirick, both of Lyons, and a half-brother, Clarence Beckwith, of Rochester, N. Y.

He was of a quiet and retiring disposition, and a man who made and kept a great many friends. He was a high type of Engineer and especially good in promotion work.

A friend and business associate of Mr. Mirick, at Lincoln, writes:

"I knew him well from 1919 to 1923 and wrote to him occasionally until last summer when he returned to Lincoln to work. It was common comment around the office about Mr. Mirick's courtesy and gentlemanly behavior. He was a good engineer and worked well with us. The men in the office held him in very high esteem."

Mr. Mirick was elected an Associate Member of the American Society of Civil Engineers on October 5, 1909.

JOSEPH LINCOLN MURPHY, Assoc. M. Am. Soc. C. E.[1]

Died February 20, 1932

Joseph Lincoln Murphy, the son of Joseph D. and Isabelle (Evans) Murphy, was born at Creola, Ohio, on January 29, 1877.

His preliminary education was obtained in the schools of Vinton County, Ohio. All his boyhood was spent there with the exception of one year in Nebraska when his family moved to that State to take up a homestead claim.

---

[1] Memoir prepared by G. F. Schlesinger, M. Am. Soc. C. E.

The most vivid memories of his boyhood were the trip to Nebraska in a covered wagon, the sod house his father built, and the hardships endured before it was finally decided to return to their old home in Vinton County.

Circumstances would not permit the boy's regular attendance at school, but possessed of tireless ambition, the early age of fifteen found him working in a saw-mill, performing the tasks of a grown man, so that he might have enough money to go to school.

On entering the High School at McArthur, Ohio, Mr. Murphy lived in the home of Dr. Rannells, and his time outside school hours was employed in driving the doctor through the country on his round of calls. Here, he came to know Miss Eliza Rannells, a teacher, who took a great interest in him, encouraged him in his efforts to continue with his education, and it was through her direction and influence that he acquired his taste for fine literature. Reading was always the source of his greatest enjoyment, and his ready fund of knowledge on a multitude of subjects made him an interesting conversationalist.

Extremely interested in military affairs, on May 11, 1897, at the age of twenty, and while still a student in High School, he enlisted in the Ohio National Guard, becoming a member of the Hospital Corps of the 17th Ohio Infantry.

On April 26, 1898, Mr. Murphy enrolled in Company B, 17th Regiment of Infantry, Ohio Volunteers, to serve two years, or for the duration of the Spanish-American War. He served seven months, receiving his discharge on November 6, 1898; during this period his Company was encamped at Columbus, Ohio, and at Washington, D. C. His services in the Army left his health in such condition that he could honorably have asked for a pension, but, characteristically, refused to do so.

During the World War, he twice attempted to enter the service of the Government, but each time, greatly to his disappointment, was unsuccessful, owing to defective eyesight caused from an injury sustained when a boy.

Upon receiving his discharge from the Army, in 1898, Mr. Murphy returned to McArthur, Ohio, and determining at all costs to continue his education, re-entered High School, graduating in April, 1899, at the age of twenty-two.

He was anxious to take up the study of Law, and encouraged and aided by Dr. Rannells and Miss Rannells, entered Ohio State University, at Columbus, Ohio, in the fall of 1899. Unable to qualify for the Law School because of a lack of proper credits, he enrolled in the Civil Engineering School. Four years of unceasing work and study, with little or no recreation followed, and, in 1903, he received his degree in Civil Engineering.

In 1905, Mr. Murphy moved to the Hocking Valley, in the southeastern part of the State, where the remainder of his life, with the exception of the two years preceding his death, when he resided at Columbus, Ohio, was spent.

During his last year in college, Mr. Murphy met Mr. E. M. Poston, President of the New York Coal Company, who was prominent in the coal and power industries in Ohio, and particularly in the Hocking Valley, where most of his large holdings were located. Following his graduation, Mr. Murphy was employed by Mr. Poston as Engineer for his many enterprises.

Assuming the position of Chief Engineer, he organized and built up the Engineering Department of the New York Coal Company, and, in the ensuing years, his opinion on tax and mine engineering problems was widely sought and highly valued.

Possessed of rare judgment in the selection of his men, Mr. Poston readily perceived that here was a man in whom he could place unbounded confidence and responsibility, and the story of Mr. Murphy's career is the story of the coal, clay, oil, and power developments of Mr. Poston in the last twenty-eight years—a story written deeply in the annals of the Hocking Valley. In the years of his association with the New York Coal Company, Mr. Murphy was actively engaged in all the various branches of the organization, and was an Officer and Director in many of its associated companies.

When the Manhattan Oil Company, a subsidiary of the New York Coal Company, was organized, Mr. Murphy was made its managing head, and, for many years, was a familiar figure in the oil and gas developments of the New Straitsville, Ohio, Lima, Ohio, and Hocking Valley fields.

In 1912, the paving-brick plant of the Hocking Valley Brick Company, at Logan, Ohio, was added to the already large holdings of the New York Coal Company, Mr. Murphy becoming President of the new Company. Thus, began the work to which the remaining years of his life were dedicated. From that time on, his activities were principally directed in the Brick and Tile Department of the organization. Subsequently, the plants of The Nelsonville Brick Company and The Ohio Fire Proofing Company at Nelsonville, Ohio, and The Zanesville Clay Products Company, at Zanesville, Ohio, were acquired, with Mr. Murphy holding the position of Chief Executive. It was chiefly through his association with these Companies that he was so widely known, and, in the course of years, became a recognized leader in the clay industry throughout the State.

He was intensely interested in the civic welfare of the Hocking Valley and it was to a great extent due to his efforts that a Young Men's Christian Association Building was erected at Nelsonville. He also designed and helped in the construction of the Protestant Episcopal Church, at Nelsonville, and contributed greatly toward its maintenance. He gave to any cause for the advancement of public welfare a full measure of time and devotion. Increased recognition for the Hocking Valley in the industrial affairs of the State was his greatest desire, and the goal toward which he worked. In his going, the Hocking Valley District of Ohio lost one of the most devoted of her adopted sons.

His keen Irish wit, unlimited capacity for friendship, and unquestioned ability, made him a dominant figure in any group. Mr. Murphy was a leader in a number of clay products associations, having served the Ohio Paving Brick Manufacturers Association as President for thirteen years, and at the time of his death, he was Chairman of the Executive Committee. A resolution passed by this Association, states in part:

"* * * a fair fighter and an honest competitor. He won every advancement entirely through his own efforts. His was a dynamic force that knew no such word as 'fail' ".

For a number of years he served on the Board of Directors of the National Paving Brick Association, and at all times was actively engaged in its affairs. He was also Treasurer of the Structural Clay Tile Association for a number of years, subsequently serving as a Director and a member of the Executive Committee.

Recognizing the need for scientific research in the ceramic industry and realizing that few, if any, of the ceramic corporations of Ohio were large enough to support this much needed research in their own organization, Mr. Murphy, with several other forward-looking, prominent ceramic men, set about to devise means whereby a ceramic research bureau might be established to serve all branches of the industry in Ohio. It soon became evident that some organization, or group of ceramic manufacturers, would have to stand sponsor for the research. Accordingly, these men, inspired largely by the enthusiasm of Mr. Murphy, set about to organize what has since become the Ohio Ceramic Industries Association, now comprising nearly one hundred and twenty corporations, and fully three-quarters of the production capacity of the State.

Under the direction of this organization, when Mr. Murphy was President, the Ohio State University was persuaded to make ceramic research a major project in its Engineering Experiment Station. Contracts were entered into between the Ohio State University and the State Welfare Department which made possible the establishment of an experimental research station in connection with the State Brick Plant, at Roseville, Ohio. Thus, Mr. Murphy's dream became a reality. This Station has developed until it is now the most outstanding ceramic research organization in the United States and in the world. During the years since the formation of this Association, Mr. Murphy was continually one of its active and enthusiastic members. In recent years, he served as Chairman of the Public Relations Committee, in which capacity he was notably successful in representing the ceramic industry of Ohio, and presenting its cause and viewpoint to the State Legislature and to other individuals, boards, and bureaus associated with the State Government. The part he played in the organization of the Ohio Ceramic Industries Association ranks as one of the major achievements of his life. The active head of this Association states:

> "The entire ceramic industry of Ohio owes a debt of gratitude to J. L. Murphy who gave so unstintingly of his time and efforts and accomplished so much for the industry as a whole."

Early in 1931 Mr. Murphy, through his activities in the American Ceramic Society, was named Chairman of the Executive Committee in charge of the ceramic "Century of Progress" exhibit at the Chicago World's Fair Exposition to be held in 1933.

In the fall of 1931, death claimed Mr. Poston, and Mr. Murphy, with countless others, was deeply grieved at the loss of friend and employer. Under Mr. Poston's will, Mr. Murphy became one of the Trustees of the Poston

Estate, an added honor and responsibility which he keenly felt. From the time of Mr. Poston's death until his own, Mr. Murphy was noticeably changed. Always, he had welcomed with eager enthusiasm every problem of his work-crowded life, and was alert and active until the very end. He died suddenly from a heart attack on February 20, 1932, after returning home from a short walk with his dog.

He was a staunch Republican, and consistently took a keen and active interest in politics. He was a member of the Masonic Lodge, including the Scottish Rite and the Shrine; a member of the Benevolent and Protective Order of Elks; and, because of his interest in the Mooseheart School for children, held a life membership in the Order of Moose. He was a member of Kiwanis, the Engineers Club of Columbus, The Ohio Engineering Society, Columbus Athletic Club, and the Ohio Society of New York, N. Y.

Mr. Murphy was individualistic by nature; yet an editorial appearing in a Columbus newspaper following his death, said in part:

> "Among those prominent in the development of the Hocking Valley, few, if any, were better known or better liked than 'Pat' Murphy. He had a rare faculty for making and keeping friends, and he had the gift of retaining the respect even of those who disagreed with him."

Deeply interwoven in the threads of his life was his love for children. It was chiefly through his concern for the welfare of their families that his employees came to know him more as friend than employer.

Because his own early struggles were ever fresh in his mind, many young men and women acknowledge their debt of gratitude to him for the educational advantages he so willingly offered, asking only that they, in turn, whenever possible, help another deserving student.

Generous to a fault, giving freely of himself and his means to those who called on him, "he saw keenly enough where he was imposed upon, and yet for fear of dropping below the abnormally high standard of generosity and honor he held himself to, went on doing for the impostors. This was the combination of characteristics which made him what he was."

In one of the last books Mr. Murphy read, this passage was found marked: "* * * note that a little height made a sight of difference the way a body saw things," and those nearest this gracious, kindly man knew that he reached heights from which harsh judgment was rarely pronounced; that he had touched greatness. Easily approached, his fellow-workers came to him with their sorrows, nor were turned away unheeded; in their hearts his memory will long endure; for them a light has gone out.

On June 14, 1905, he was married to Margaret Duvall, of Columbus, Ohio, who, with two brothers, Daniel Murphy of Columbus, and Hulbert Murphy, of Chicago, Ill., and two sisters, Mrs. A. V. Allen and Mrs. Homer Timms, of Columbus, survives him.

Mr. Murphy was elected an Associate Member of the American Society of Civil Engineers on May 6, 1914. He was also a Member of the Central Ohio Section of the Society.

ROBERT McKNIGHT PARDEE, Assoc. M. Am. Soc. C. E.[1]

Died April 10, 1930

Robert McKnight Pardee was born in Fort Wayne, Ind., on March 27, 1885, the son of the late William McKnight Pardee, Division Engineer of the New York, Chicago, and St. Louis Railroad Company (The Nickel Plate).

Mr. Pardee received his early education in Fort Wayne and New Harmony, Ind. He entered New York University in the fall of 1902, and was graduated with the degree of Bachelor of Science in Civil Engineering from the School of Applied Science, with the Class of 1906. During his Junior and Senior years at the University, he was Business Manager of *The Triangle,* a weekly publication of the students.

After his graduation, from August, 1906, to October, 1911, Mr. Pardee was a member of the Engineering Department of the Grand Trunk and Pacific Railway Company in Western Canada, and worked on every part of the route from Winnipeg, Man., to Prince Rupert, B. C., on preliminary location and construction. Beginning as Tapeman and Rodman, he rose to the grade of Instrumentman, then Assistant to the Resident Engineer, in which position he handled numerous difficult pieces of high side-hill and canyon work in the mountains of British Columbia, involving tunneling and heavy embankment cribbing. During the period of his employment with the Grand Trunk and Pacific Railway Company, there was, in 1907, a suspension of railroad construction for the winter due to depressed business conditions. During this time he acted as a Reporter for *The Telegram,* a Winnipeg newspaper, covering night work. In 1912, he was in the employ of Isaac S. Rossell, General Contractor, of New York, N. Y., and, later, with the Turner Construction Company, on various buildings in and around Buffalo, N. Y., including the Bell Telephone Company Building. He rose in the building field to the position of Assistant Superintendent of Construction for the Turner Construction Company.

From the latter part of 1913, until July, 1915, Mr. Pardee was a partner in the firm of Pardee and Backus, Builders, of Edmonton, Alberta, Canada, during which time numerous buildings were constructed. When the World War put a stop to the building boom in that section of Alberta, Mr. Pardee went East and again found employment with the Turner Construction Company, from July, 1915, to September, 1921. During this time he served in various capacities on construction work, in the office, as Assistant Superintendent, and as Purchasing Agent on numerous reinforced concrete buildings in the cities of the South.

In October, 1921, he left the Turner Construction Company, and established the firm of Robert McKnight Pardee, in Atlanta, Ga., representing a number of manufacturers in the sale of their building materials and specialties throughout the Southeast.

---

[1] Memoir prepared by Robert S. Fiske, M. Am. Soc. C. E.

Mr. Pardee built a home in Atlanta, and made a large circle of friends in that city. He was active in the Georgia Section of the Society for a number of years, and was a respected and honored citizen. He possessed a very pleasing personality and a genial disposition, which won for him many close friends, and he was ever ready to assist a fellow-man in any way and give aid to any worthy cause. He was punctilious in his habitis, and painstaking in the performance of his duties.

He was married in 1912, and is survived by his widow, Mrs. Emily Purvis Pardee, and four children—Jack, Constance, William, and Robert, Jr.—all of whom reside in Atlanta.

Mr. Pardee was elected an Associate Member of the American Society of Civil Engineers on May 19, 1924.

WILLIAM COUTS RANDOLPH, Jr., Assoc. M. Am. Soc. C. E.[1]

Died January 17, 1931

William Couts Randolph, Jr., was born in Atmore, Ala., on June 5, 1897, the son of William Couts, and Allie (Hill) Randolph. Soon after his birth the family moved to Russellville, Ky., about two years later moving to Springfield, Tenn., where they resided until William was four years of age. In 1901, the family made their home in Andalusia, Ala., where his youth and boyhood were spent.

Mr. Randolph attended the public schools of Andalusia and, following his graduation from the High School in 1915, he entered the University of Alabama at Tuscaloosa, Ala., where he remained for two years. While at college, he was very prominent in campus activities and was a member of Pi Kappa Phi Fraternity.

From June, 1917, to October, 1918, he was employed by J. B. McCrary Company, of Atlanta, Ga., as Rodman, Instrumentman, and Tracer in connection with the construction of Southern Field at Americus, Ga. Following this engagement, he spent about four months in the employ of the United States Corps of Engineers at Charleston, S. C., as Piling Engineer in connection with dock construction at that place.

From January until June, 1919, Mr. Randolph was employed by the Lockwood-Greene Company as Chief of Topographic Party in connection with maps and surveys at Fort Benning, Georgia; and from June, 1919, until February, 1920, he was Resident Engineer on general highway work in the southern part of Georgia.

Beginning in February, 1920, he served for ten years with the State Highway Department of Georgia, where his progress was rapid and where he built for himself a remarkable reputation in the construction of paved highways. For two years he was Division Draftsman and, for another two-year period, he was Resident Engineer in charge of the construction of highways in the Americus Division of the Department. From July, 1924, until May, 1928, he was Senior Engineer in technical charge of all highway construction in nineteen counties for the State Highway Department. His ability was fully recognized by this time, and he was designated as Supervising Engineer and placed in charge of the paving on the Atlanta-Chattanooga Road. This work included the paving of about ninety-two miles of road and involved five different contracts.

---

[1] Memoir prepared by a Committee of the Georgia Section consisting of Thomas M. Neibling, Assoc. M. Am. Soc. C. E., *Chairman*, and W. R. Neel, and S. B. Slack, Members, Am. Soc. C. E.

In January, 1930, Mr. Randolph left the Georgia Highway Department to become associated with The Dixie Culvert and Metal Company in charge of engineering, research, and development in the Southeast. He carried on this work enthusiastically until he was seized with an acute attack of appendicitis which caused his death in Atlanta, on January 17, 1931.

Although a comparatively young man, Mr. Randolph was an ardent and enthusiastic Engineer, and at all times exemplified and advocated the high standards of honor, diligent work, and efficiency for which his profession stands. In every community in which he lived he was regarded as a useful and highly respected citizen, and he was held in high esteem by his professional associates everywhere. His fine character, enthusiasm, and interesting personality made for him a wide circle of friends.

On June 1, 1918, he was married, at Americus, to Genevieve Prather, who survives him, with two daughters, Ruth and Elizabeth, and one son, James.

Mr. Randolph was elected an Associate Member of the American Society of Civil Engineers on June 9, 1930.

HOMER AUSTIN REID, Assoc. M. Am. Soc. C. E.[1]

DIED JUNE 9, 1931

Homer Austin Reid, the son of Elias H. and Maria U. (Ulrich) Reid, was born in Bristol, Ohio, on September 30, 1869. He attended the public schools in Warren, Ohio, and secured his engineering education at Lehigh University, Bethlehem, Pa., which he attended from September, 1891, until his graduation in June, 1896, with the exception of one school year (1893–1894), when he managed the business of the firm of E. H. Reid and Son, lumber dealers, in Warren. During other summer vacations, he worked on survey parties for the City Engineer of Warren.

After his graduation, Mr. Reid's first employment was from June, 1896, to April, 1897, with the Norfolk and Western Railroad Company, Lynchburg Division, in charge of maintenance-of-way work. Later, until August, 1897, he worked for the Warren Gas Light Company, compiling a map of the City of Warren. From August, 1897, to August, 1899, he was employed as Draftsman on structural steel detailing by the Youngstown Bridge Company, of Youngstown, Ohio, and by the Penn Bridge Company, of Beaver Falls, Pa. This work had a particular appeal to Mr. Reid and led him to specialize in the structural engineering branch of the profession. His subsequent engagements for the Greenbrier Railway Company (a branch of the Chesapeake and Ohio System), for the State Engineer and Surveyor, at Albany, N. Y., and for Milliken Brothers, in New York, N. Y., were on bridge and structural design.

---

[1] Memoir prepared by John W. Cunningham, M. Am. Soc. C. E.

In March, 1901, Mr. Reid entered the employ of the New York Rapid Transit Commission. Starting as Inspector, he rose to the position of Division Engineer. On the completion of his Section, he became Chief Engineer of the Bureau of Buildings, Borough of Manhattan. Using this wide acquaintance and experience with municipal work, in 1900, Mr. Reid opened a consulting office in his own name. He was immediately successful and handled a number of large buildings. One feature of his work was the design of the framework for many of the large electrical signs on Broadway.

During the period of the World War, Mr. Reid was Engineer and Manager for the Foundation Company in New York and Canada. Afterward, in 1919, he was Resident Engineer on the repair of the Grand Trunk Bridge, at Niagara Falls, N. Y., and Ont., Canada. From 1919 to 1922, he was Office Engineer for Lockwood Greene and Company, Architects and Engineers, in the New York Office, Field Engineer for Hardy S. Ferguson, M. Am. Soc. C. E., and in charge of extensive port improvements at Puerto Tarafa, Cuba, the largest sugar port in the world. For the following years, until 1927, he was with the organization of Albert Kahn, Architects, in Detroit, Mich., as Assistant Chief Engineer, having to do principally with large industrial buildings.

In 1927, in order to secure a climate more favorable to his wife's health, Mr. Reid moved to Los Angeles, Calif. He handled several structural jobs in that city and in Long Beach, Calif. In the fall of 1930 he went to Portland, Ore., where he intended to locate as a Consulting Engineer. His plans were interrupted, however, by his sudden death on June 9, 1931.

Mr. Reid was by nature quiet and was devoted to his studies along professional lines. In 1907, when the theory and practice of concrete design was being most rapidly developed, he wrote an extended treatise entitled "Concrete and Reinforced Concrete Construction," which met an immediate acceptance by the profession. The preface of this work contains a plea for engineering judgment and conservatism that deserves quotation:

> "Although the increase in the use of concrete, both with and without reinforcing metal, has become so great in the past few years as to almost warrant it being said that we are going concrete mad, it should be remembered that reinforced concrete does not possess wonderful and mysterious properties such that it may be unscientifically or recklessly used. To the contrary, it should be used with the same care and judgment that has made other and older kinds of construction both safe and satisfactory. It should also be remembered that there are other and tried kinds of construction, which are much more suitable for use in many situations. Under such conditions enthusiasm for a given form of construction should be tempered with good judgment, and the most suitable building material chosen * * *."

Mr. Reid was a Mason, affiliated with the Youngstown, Ohio, Lodge, and a member of the Protestant Episcopal Church.

He is survived by his widow, Ednor (Smith) Reid, and a daughter, Mrs. John O. Davis, of Portland; also, a brother, Halbert Reid, of Liberal, Kans., and a sister, Mabel Reid, of Warren.

Mr. Reid was elected an Associate Member of the American Society of Civil Engineers on December 4, 1901.

HERBERT JONES SOWDEN, Assoc. M. Am. Soc. C. E.[1]

DIED NOVEMBER 14, 1931

Herbert Jones Sowden, the son of George and Anna Sowden, was born in Philadelphia, Pa., on November 8, 1883. He was educated in the public schools of Philadelphia, and was graduated from the Northeast High School in 1901. His technical training was received through attendance in the evening classes at Franklin Institute from 1901 to 1903.

Mr. Sowden was ambitious to become a Civil Engineer and, after his graduation from High School in 1901, he entered the employ of the Philadelphia and Reading Railway Company as a Chainman, and remained in its service until January, 1914. While with this Company he received several promotions, and eventually attained the grade of Assistant Engineer of Construction, which position he held until the termination of this connection. During this period he was engaged on the elimination of grade crossings on the 9th Street Section of the Philadelphia, Germantown, and Norristown Railroad, and on the Port Richmond Branches of the Philadelphia and Reading Railway, in Philadelphia. He was also employed in the laying out of the New York Short Line Division of this Railway Company. In 1914, he assumed charge of the building of the Wayne Junction Station, on the main line of the Philadelphia and Reading Railway, as Superintendent of Construction for the Contractor, Charles P. Bower, M. Am. Soc. C. E.

Upon the completion of this assignment, in 1915, Mr. Sowden accepted an appointment with the City of Philadelphia, as Sewer Inspector in the Bureau of Surveys, Department of Public Works. He held this position until 1923, when he was promoted to the rank of Assistant Engineer in the Division of Grade Crossing Elimination, Bureau of Surveys, Department of Public Works, and was assigned to the South Philadelphia Terminal Improvements. This work included the abolishment of numerous grade crossings upon the lines of The Pennsylvania Railroad, The Baltimore and Ohio Railroad, and The Schuylkill River East Side Railroad Companies, in the southern section of the city, together with several extensive viaduct approaches and the construction of railroad terminal and river-front improvements.

---

[1] Memoir prepared by Thomas Buckley, M. Am. Soc. C. E.

He was promoted in 1924 to the position of Assistant Engineer of Sewer Construction, Bureau of Engineering and Surveys, Department of Public Works, and was placed in responsible charge of all sewer construction work in the City of Philadelphia, which position he occupied until his death. As Assistant Engineer of Sewer Construction Mr. Sowden had direct charge and supervision of the present amplified sewerage system of the City of Philadelphia, during a period wherein were built many miles of branch and main sewers and appurtenant work.

Mr. Sowden was a member of Mitchell Lodge, No. 296, F. and A. M., of Pennsylvania, and the Germantown Royal Arch Chapter. He was also a member of the American Society of Municipal Engineers.

He is survived by his widow, Alice (Hobson) Sowden, a daughter, Gloria Sowden, and two sons, Herbert J. Sowden, Jr., and Richard Sowden, all of Philadelphia.

Mr. Sowden was elected an Associate Member of the American Society of Civil Engineers on October 10, 1927.

PAUL BERTRAM SPENCER, Assoc. M. Am. Soc. C. E.[1]

DIED MARCH 26, 1932

Paul Bertram Spencer was born in Boston, Mass., on October 6, 1881, the son of Lafayette and Mary (Batchelder) Spencer. He spent his boyhood on a farm at Tatnic Hill, Brooklyn, Conn., attending the Brooklyn Center School, where he prepared for college.

Mr. Spencer received his technical and engineering training at the Worcester Polytechnic Institute, Worcester, Mass., from which he was graduated in 1902, with the degree of Bachelor of Science in Civil Engineering, receiving one of the six "Graduate Aid Fund" prizes for scholarship. In 1914, his Alma Mater granted him his Civil Engineer degree.

In June, 1902, Mr. Spencer began his professional career in the Engineering Department of the New York, Ontario, and Western Railway Company, where he progressed from the position of Draftsman through the grade of Inspector to that of Assistant Engineer in November, 1902, which position he held until November, 1905.

From November, 1905, until September, 1906, Mr. Spencer served as Junior Assistant Engineer with the New York, New Haven and Hartford Railroad Company, and was engaged on the six-tracking of the Harlem River and Portchester Railroad (Harlem River Branch), between the Harlem River and New Rochelle, N. Y., particularly the river crossings at Bronx River and Pelham Bay, and grading for the Oak Point Classification and Float-Bridge Yards, in New York, N. Y.

From September, 1906, until May, 1907, he was Resident Engineer with the Charlotte Harbor and Northern Railway Company (now part of the Seaboard Air Line Railway Company) engaged on the construction of the new line from Hull to Boco Grande, Fla., which included two long trestle bridges, one involving a drawbridge span over Gasparilla Bay.

---

[1] Memoir prepared by A. C. E. Mullen, Constr. Engr., N. Y., N. H. & H. R. R., New York, N. Y.

From June, 1907, until January, 1908, as Instrumentman with the Cambria Steel Company, at Johnstown, Pa., Mr. Spencer was employed on the construction of new buildings and the erection of mine-shaft machinery.

In January, 1908, Mr. Spencer returned to the New York, New Haven and Hartford Railroad Company as Second Assistant Engineer of Construction at New York City and was again assigned to the six-tracking of the Harlem River Branch.

From June until November, 1912, he was Assistant Engineer in charge of the completion of the Harlem River Branch project, the Oak Point Yard and Float-Bridge, Van Nest and Westchester Yard improvements, and the construction of the electric repair shop at Van Nest, New York City.

From November, 1912, until March, 1916, as Division Engineer of Construction at New Haven, Conn., Mr. Spencer was in general charge of all construction work west of New London and Willimantic, Conn., the consolidation of the East River Piers and the Woodlawn Junction improvements in New York City, the double-tracking of the line, and the elimination of grade crossings between Berkshire Junction and New Milford, Conn., constituting the major projects. During this period Mr. Spencer made the studies for the location of the new Thames River Bridge, at New London, Conn.

From March, 1916, until March, 1920, Mr. Spencer, with headquarters at New London, devoted practically all his attention to work in that vicinity. This work consisted of the construction of the railroad bridge (a four-track substructure, with two-track superstructure) spanning the Thames River; the deep foundations of the river piers presented an interesting problem. This work involved the re-alignment of the railroad at Winthrop Cove on the westerly approach, and the four-tracking of the road from Groton to Midway, Conn., on the easterly approach, the enlargement of the Midway Yard, and the improvement of the engine facilities.

In March, 1920, Mr. Spencer was appointed Assistant to the late W. H. Moore, M. Am. Soc. C. E., then Engineer of Structures at New Haven, and, at Mr. Moore's death, succeeded him on October 1, 1920, as Engineer of Structures, which position he held until his sudden death on March 26, 1932.

Although no new projects of major engineering importance were carried out by the Company during this period, many varied and difficult reconstruction and maintenance problems were presented to Mr. Spencer. These he solved in a manner which indicated that his professional ability in design had been enriched by a thorough and intimate knowledge of general construction methods and railroad operating problems.

Mr. Spencer was a member of the Connecticut Society of Civil Engineers; American Railway Engineering Association, of which he was a member of Committee No. 15, "Iron and Steel Structures"; American Railway Bridge and Building Association; a member of the Presidents' Conference Committee on the Federal Valuation of Railroads; and, since 1928, a highly valued member of the "Contact Committee" for the Department of Civil

Engineering of the Worcester Polytechnic Institute. He was also a member of the New Haven Railroad Club, and Annawon Lodge No. 15, A. F. and A. M., of West Haven, Conn.

On September 21, 1904, Mr. Spencer was married to Anna A. Basler, of Newark, N. J., who, with an only daughter, Doris Marie Spencer, survives him.

To the end, an earnest student, an indefatigable worker, Mr. Spencer possessed, to a marked degree, those fundamental attributes of the successful engineer—perseverance and a logical creative mind. His was the nature that inspired loyalty, commanded respect, won admiration from all. His kindly generous spirit prompted him to assist all who sought his counsel.

His death was a veritable loss to the Engineering Profession, to his colleagues, and to his many friends in whose thoughts his memory will ever linger as a true gentleman and a lovable character.

Mr. Spencer was elected an Associate Member of the American Society of Civil Engineers on August 31, 1915.

SAMUEL HOUSTON WARE, Assoc. M. Am. Soc. C. E.[1]

DIED APRIL 14, 1931

Samuel Houston Ware, the son of the late James Ware, was born at Fortress Monroe, Va., on October 26, 1879, where for many years his father was Chief Civilian Engineer.

Mr. Ware received the customary High School education and attended night school for three years, taking special courses in Engineering and Mechanical Drafting. Thereafter, he maintained a continuous study of engineering throughout his life.

He began work in 1899, and, as a civilian with the United States Engineer Department, in Norfolk, Va., supervised the fortification and river and harbor work in that vicinity until 1907. At this time, he was sent to Hawaii with Col. E. E. Winslow, U. S. Corps of Engineers, to construct emplacements for the 6-in. and 14-in. guns at Fort DeRussy. He also installed the fire-control system at Diamond Head, constructed the concrete wharf at Fort Armstrong, emplacements for guns at Fort Kamehameha, and many other projects. From 1915 to 1917, he was Superintendent on a land defense project, building land fortifications, redoubts, trenches, and anti-aircraft emplacements which required skillful engineering, due to location.

Mr. Ware was in command of Company I, Hawaiian National Guard, for three years, and, on June 1, 1918, when the Guard was called into active service, he was commissioned a Captain in the National Army of Engineers, and appointed Assistant to the District Engineer for the District of Hawaii. One year later, he was ordered to Camp A. A. Humphreys, Virginia, and, in 1920, accepted a commission in the Regular Army, transferring to the Quartermaster Corps.

---

[1] Memoir prepared by Maj. Henry Hockwald, Q. M. C., U. S. A., Fort Bragg, N. C.

Captain Ware served at the Army Base in Brooklyn, N. Y., for three years. During that time he was sent to Tobyhanna, Pa., as Constructing Quartermaster, where he remained for about three months. On his return to Brooklyn, he was sent to Newark, N. J., to close the Base Camp there.

From there Captain Ware went to California, serving as Transport Quartermaster on the United States Army Transport, *Cambrai,* for about six months. He was then ordered to the Presidio of Monterey, where he was stationed for three years, part of which time he was Post Quartermaster. He also served as Quartermaster for the Citizens Military Training Camp and the California National Guard Camp, at Del Monte, Calif., every summer, and was highly commended for his efficiency.

In 1927, he was ordered to Hawaii, where he served for three years at Fort Kamehameha. On completing this tour of duty, Captain Ware was ordered to Fort Bragg, North Carolina, where he remained for about three months. In March, 1931, he was ordered for duty as Constructing Quartermaster, at Fort Benjamin Harrison, Indiana. While driving to this station, with his family, they spent the night of March 14, 1931, at a hotel in Asheville, N. C., where he suffered a paralytic stroke. He was taken to the Mission Hospital, in Asheville, where he remained until his death on April 14, 1931. He was buried in Arlington Cemetery, Washington, D. C., with military honors.

While on duty at Fort Bragg, Captain Ware was Fire Marshal and in charge of the Post Utilities. In addition to these duties, he was Acting Quartermaster much of the time, which duty he performed most efficiently and satisfactorily to his superior officers. He was well liked by the employees working under his supervision, who regretted to see him leave this station, because of his exceptional fair treatment in dealing with them. Captain Ware was of an excellent character, and a very hard and efficient worker. He was a man of high moral character; delightful to meet, and well liked by every one.

He was married in 1910, to Mable McAndrews of Honolulu, who, with two sons, Everett Houston and Samuel Houston, Jr., survives him. He is also survived by two brothers, James E. Ware, Chief Clerk of the U. S. Engineering Department of Norfolk, Va., and Lieut.-Col. Joseph I. Ware, U. S. A. (*Retired*), of Dierdorf, Germany, and two sisters, Mrs. J. H. Cook, of Westbrook, Conn., and Mrs. J. B. Dennis, the wife of Capt. John B. Dennis, U. S. N., Assistant to the Chief of the Bureau of Medicine and Surgery, Navy Department, Washington.

Captain Ware was elected an Associate Member of the American Society of Civil Engineers on May 12, 1919.

CHARLES EDWARD WILLACKER, Assoc. M. Am. Soc. C. E.[1]

Died November 4, 1930

Charles Edward Willacker, the son of Sylvia (Scisinger), and George D. Willacker, was born at Willard, Ohio, on May 25, 1888. His parents were native born; his mother's people came from Holland, and his father was of German descent. The Willacker family dated back to the Fifteenth Century, the coat of arms having been granted in 1493, by Kaiser Maximilian I, in the City of Worms, Germany.

---

[1] Memoir prepared by George A. Taylor, M. Am. Soc. C. E.

Mr. Willacker's boyhood was spent on a farm, and his early education was obtained in the common schools of Huron and Norwalk, Ohio. He attended a private night school, studying mathematics and engineering, and received a diploma from the International Correspondence Schools on the completion of a course in Surveying and Mapping.

In 1910, Mr. Willacker went to Toledo, Ohio, where he was employed by William H. Gould, Surveyor, as Rodman on city surveys, subdivisions, and construction work. The following year he acted as Instrumentman in Charge of Party on a similar detail. In 1912, he became associated with the late Joseph Hofman, M. Am. Soc. C. E., of Toledo, specializing in land surveys and real estate subdivision, and being licensed to practice in the State of Michigan.

His association with the Joseph Hofman Company continued throughout his life. At brief intervals, he was engaged as a Draftsman on airplane structures and on school heating plans. Subsequently, however, his successive positions with the Joseph Hofman Company were, as follows: 1912, Instrumentman and Inspector in charge of construction (paving and sewers); 1912-1916, Instrumentman in charge of party for city surveys; 1916-1924, Engineer in charge of city surveys, subdivision development, and construction (paving, sewers, walks, and culverts); 1924, in partnership with Mr. Hofman, as Engineer in charge of office and all other work coincident with his previous experience.

On the death of Mr. Hofman, in 1924, Mr. A. L. Hofman took over the interests of the business and, as a partner of the latter, Mr. Willacker continued as Engineer in charge of office and field work. He was later made Manager of the Company and was appointed Engineer for the Village of Sylvania, Ohio, which positions he held at the time of his death on November 4, 1930.

Mr. Willacker was a member of the Pilgrim Congregational Church, a Rotarian, an Odd Fellow, and a member of the Masonic Order, having been Eminent Commander of Lafayette Lytle Commandery. For seven years he was Scout Master of Boy Scout Troop No. 24. He was an enthusiastic supporter of all the organizations with which he was connected.

He was a loyal citizen and a careful and conscientious Engineer, held in high esteem by all his associates. He had many friends in every walk in life and was dearly loved by his family.

On November 6, 1912, he was married to Edna E. Clark, of New London, Ohio. He is survived by his widow, a three-year old son, Joseph Hofman Willacker, and by a brother, Elmer Willacker, of Cleveland, Ohio.

Mr. Willacker was elected an Associate Member of the American Society of Civil Engineers on November 15, 1926.

FERD CLARK WINTERMUTE, Assoc. M. Am. Soc. C. E.[1]

DIED JANUARY 26, 1931

Ferd Clark Wintermute was born in the Charles Miner farmhouse on old River Road at Wilkes-Barre, Pa., on July 1, 1883, the son of Lemuel C. and Ida Ethel Belle (Mott) Wintermute. His father was a farmer, and a native, and a member of one of the oldest families, of the State of New Jersey.

Ferd Clark Wintermute grew up in his home town where he attended the public schools, but being filled with a desire to acquire more knowledge, he studied surveying and mapping by correspondence, and was tutored in higher mathematics and engineering. During his school days, he was employed as Time-Keeper for the Barber Asphalt Company. Later, he gained valuable experience in surveying and engineering with Mr. I. E. Hartwell, and the late N. B. Rutter, former County Surveyor for Luzerne County.

From 1902 to 1905, Mr. Wintermute served as Transitman and Assistant Engineer with the firm of Stevenson and Knight, of Scranton, Pa. After this experience, he was connected with the New York, New Haven, and Hartford Railroad Company as Assistant to the Engineer of Location, and served in that capacity until March, 1907. At this time he returned to Wilkes-Barre and opened an office in association with Mr. Fred A. Young, under the firm name of Young and Wintermute, Consulting and Civil Engineers. This partnership continued until Mr. Young's death in 1924. For a time, Mr. Wintermute remained in practice alone, but on January 1, 1927, Mr. Thomas J. Halsey entered the business with him under the name of Wintermute and Halsey. The members of the firm devoted their time largely to municipal and mining engineering, serving as Engineers for the Boroughs of Swoyerville and Kingston, Pa., and for the Central Coal Company and the Lehigh and Wyoming Valley Coal Mining Company; they were also employed as Civil Engineers for a number of realty companies, development concerns, and architects.

From 1909 to 1911, Mr. Wintermute served as Engineer in Hanover Township, Pennsylvania, on road and bridge construction; and in 1911 and 1912, he was Supervisor of field work on the construction of the Plymouth, Pa., Bridge; in 1913, he designed the steel and reinforced concrete for, and supervised the construction of, the Post Bailey Store and Office Building, in San Diego, Calif.; in 1914, he supervised the construction of the Crane Brothers Silk Mills, in Kingston, Pa.; and from 1917 to 1919, he served as Inspector of Municipal Improvements for the Wyoming Valley Taxpayers Association. He was employed as Resident Engineer for water supply improvements costing more than $2 500 000 for the City of Bayonne, N. J., in 1920 and 1921, and also designed and supervised the construction of the water supply filtration plant and sewers and sewage disposal plant for new buildings, for the Ranson District Poor Board.

---

[1] Memoir prepared by T. J. Halsey, Esq., Wilkes-Barre, Pa.

In 1921 and 1922, the firm designed and supervised the construction of the Lock Haven Steel Bridge, at Lock Haven, Pa., and from 1923 to 1925 was engaged on the design of original and revised plans for the proposed steel and reinforced concrete North Street Bridge, in Wilkes-Barre. In 1926, Mr. Wintermute made comprehensive plans for a sanitary sewer and storm drainage system for Kingston Borough. In 1927, he completed the building of the dam and the 60-acre Wyoming Lake Reservoir, in Franklin Township, Pennsylvania. At the time of his death, he was engaged as Engineer on the construction of a dam, roads, bridges, and other improvements on the Indian Lake Development of Stout-Roup, Incorporated, in the Pocono Mountains, in Pennsylvania, and a sanitary sewer system for the Borough of Swoyerville.

He was a Registered Professional Engineer and Land Surveyor of Pennsylvania and New Jersey, and from 1911 until his death in 1931, had been one of the Engineer Members of the County Board of Viewers of Luzerne County.

Mr. Wintermute was a member of Fidelity Lodge, No. 655, F. and A. M., of Wilkes-Barre; of Shekinah Chapter, No. 182, Royal Arch Masons; Dieu le Veut Commandery, No. 45, Knights Templars; Keystone Consistory, of Scranton, Ancient Accepted Scottish Rite Masons of the Thirty-Second Degree; and Irem Temple, Ancient Arabic Order, Nobles of the Mystic Shrine. He was a member of the Irem Temple Country Club, the Craftsmens Club, the Wyoming Valley Motor Club, and the Franklin Club, of Wilkes-Barre, and the American Water Works Association. He was also a member of the Junior Order of United American Mechanics, Patriotic Order of Sons of America, the Kiwanis Club, the Chamber of Commerce, and a Director and Vice-President of the Liberty State Bank and Trust Company, of Wilkes-Barre, and Director and Secretary of the Wilkes-Barre Mortgage Company. He was a member of the Westminster Presbyterian Church, of Wilkes-Barre.

On March 31, 1908, Mr. Wintermute was married to Jessie Archer Zundt, the daughter of Alexander and Elizabeth Zundt, of Brooklyn, N. Y. They had four children, Donald Stewart, Jack Lee, Lois Jean, and Janice Louise.

Mr. Wintermute was elected a Junior of the American Society of Civil Engineers on July 1, 1909, and an Associate Member on November 4, 1914.

THOMAS ROGERS HUGHES, Jun. Am. Soc. C. E.[1]

DIED AUGUST 2, 1931

Thomas Rogers Hughes, was born at Oxford, Ohio, on September 30, 1902. He was graduated from Miami University, Miami, Ohio, with the degree of Bachelor of Arts in 1924, and continued his studies for the next two years at the Massachusetts Institute of Technology, from which he received the degree of Bachelor of Science in Civil Engineering.

On July 1, 1926, Mr. Hughes became a member of the Engineering Staff of the American Water Works and Electric Company and continued in this capacity until his death. For the past few years, he had supervised the selection and installation of a major portion of the electrical equipment in the pumping stations of plants controlled by the Company. His work in revamping the pumping stations of the Old Dominion Water Company, at Hopewell, Va., and The Wichita Water Company, at Wichita, Kans., was exceptionally well carried out. In addition, he superintended extensive improvements in the pumping stations of the East St. Louis and Interurban Water Company, at East St. Louis, Ill.

---

[1] Memoir prepared from information supplied by American Water Works & Elec. Co., Inc., and on file at the Headquarters of the Society.

Mr. Hughes was considered by his associates to be one of the most able and promising young men in the organization. His pleasing manner and marked degree of engineering ability made him a valuable asset to the Company's Engineering Staff, and his death was a great loss as he appeared to be just upon the threshold of a brilliant career in his chosen profession.

Mr. Hughes was a Junior of the American Society of Mechanical Engineers, and an active member of the American Water Works Association. His activities were not limited to engineering fields, as he was Chairman of the Scholarship Committee of the National Inter-Fraternity Group, the purpose of which was to raise the scholastic standing of college fraternities. Mr. Hughes was a member of Delta Kappa Epsilon Fraternity.

In October, 1930, he was married to Juliet Garrett, of Dongan Hills, Staten Island, N. Y. Mr. Hughes died suddenly of infantile paralysis at St. Joseph, Mo., on August 2, 1931. In addition to his widow, he is survived by his father, Dr. Raymond M. Hughes, President of Iowa State College, by his mother, and by a sister, Emily Hughes.

Mr. Hughes was elected a Junior of the American Society of Civil Engineers on November 15, 1926.

ELMER BARTHOLD LAWSON, Jun. Am. Soc. C. E.[1]

DIED OCTOBER 8, 1931

Elmer Barthold Lawson, the son of Frank H. and Jennie A. Lawson, was born in Minneapolis, Minn., on December 18, 1904. He obtained his early education there, including two years in the College of Engineering at the University of Minnesota. In 1925, he transferred to the Massachusetts Institute of Technology, at Cambridge, Mass., where he specialized in hydraulic and water-power engineering. He received the degree of Bachelor of Science from the Institute in June, 1927.

Following his graduation, Mr. Lawson was connected with the Randolph-Perkins Company, of Chicago, Ill., first as Resident Engineer in charge of the construction of a sewerage system in Hinsdale, Ill., and, later, he studied the phenomena of flood waves in channels. He was also engaged in the research work for a report on the 9-ft. channel for the Upper Mississippi

---

[1] Memoir prepared by F. I. Lawson, Esq., San Francisco, Calif.

River. As a result of his ability and resourcefulness in analyzing these problems through original thought, Theodore Brent, Consulting Engineer on waterways, requested his services, and Mr. Lawson became associated with Mr. Brent in New Orleans, La., in February, 1929.

This work, which covered the period until his health failed in March, 1931, had to do with economic and engineering surveys connected with the development of water transportation on the Mississippi River and its tributaries, as well as the development of interchange terminals, and work in preparation of important cases before the Interstate Commerce Commission.

Both his employers have remarked about Mr. Lawson's ability to couple a highly trained engineering mind with the practical requirements of the problems put before him. In addition, he developed a personality which made friends even with those whose assertions it became necessary to modify, unobstrusively ingratiating himself with others so that his untimely illness and passing are deeply felt by all those who came in contact with him.

Mr. Lawson was elected a Junior of the American Society of Civil Engineers on November 14, 1927.

BERNARD JOSEPH MERICKEL, Jun. Am. Soc. C. E.[1]

DIED DECEMBER 21, 1931

Bernard Joseph Merickel was born in Toledo, Ohio, on February 14, 1902, the son of the late George Joseph and Theresa Merickel.

Mr. Merickel attended the grade school and the Jessup W. Scott High School, in Toledo. He matriculated at the Ohio State University, at Columbus, in 1922, from which he was graduated with the degree of Bachelor in Civil Engineering on June 15, 1926. During the vacation periods of 1924 and 1925, he was employed in the office of Mr. Fred L. Kuebler, in Toledo, on miscellaneous surveys.

Following his graduation, Mr. Merickel entered the service of the Building Products Company, in Toledo, as Designer and Detailer on structural iron work, serving until the fall of 1926, when he entered the City Engineer's office, as Assistant Engineer in charge of concrete analysis, which position he held until the fall of 1927.

He then joined the Engineering Department of the New York Central Railroad Company, serving as Transitman until June, 1930. At this time, he was appointed an Assistant Engineer, with the Ohio State Highway Department. In July, 1931, he was given a leave of absence on account of ill health, and he died five months later.

Mr. Merickel was married, on June 18, 1927, to Helen Miller, whom he had met when they were students at the Ohio State University, and who, with a young daughter, Marylin, survives him.

---

[1] Memoir prepared by George Champe, M. Am. Soc. C. E.

He was a member of the various Masonic organizations, including the Scottish Rite and Zenobia Shrine. He was also a member of the Chi Phi Fraternity.

It was the privilege of the writer to have known Mr. Merickel from his early boyhood and to have watched, with much interest, his uniform advancement through the schools and University and his increasing interest and advancement in engineering experience, his activities in civic affairs, and his devotion to his home and family.

Mr. Merickel was elected a Junior of the American Society of Civil Engineers on October 1, 1926.

JOHN HARRY MILLER, Jun. Am. Soc. C. E.[1]

DIED OCTOBER 14, 1931

John Harry Miller, the son of John Harry and Catharine (Kearney) Miller, was born in Philadelphia, Pa., on October 24, 1902. His mother was a native of County Donegal, Province of Ulster, Ireland, where she spent her childhood days and her girlhood. His maternal grandparents were of Northern Ireland stock and lie buried beside Doe Castle, a famous landmark in County Donegal. His paternal grandparents were early settlers of Oxford County, Pennsylvania, and his father was born in the Town of Oxford.

Mr. Miller attended the public schools of Philadelphia, and was graduated with honors from the West Philadelphia High School in June, 1921, despite having left school to work for a year in the shipyards at Hog Island, Philadelphia. He entered Drexel Institute, Philadelphia, in September, 1921, and had completed one year's work at the time of his father's death, in 1922. He then left school and became a Machinist at the Baldwin Locomotive Works, in Philadelphia, which position he held until September, 1923.

Through the interest of his good friend, Mr. Thomas B. Donaldson, he obtained a scholarship at the University of Pennsylvania, and in September, 1923, he enrolled in the Civil Engineering Course in the Towne Scientific School. In addition to making a creditable record in his scholastic work, Mr. Miller also engaged in athletics to the fullest extent of his allowable time. He played football all four years of his college career, receiving the coveted Varsity award, the "P." He also rowed on the crew for the University and received the Junior Varsity award. He was elected to the Honorary Engineering Society, Tau Beta Pi, in the fall of 1926, in recognition of his high scholastic standing and well-rounded undergraduate life. He was also a member of the Delta Sigma Phi Fraternity and of the Varsity Club. He was graduated in June, 1927, with the degree of Bachelor of Science in Civil Engineering

Immediately after his graduation, Mr. Miller entered the employ of the Austin Company, of Cleveland, Ohio, as Engineer of lines and grades on the construction of an industrial building in Baltimore, Md. After the completion of this building and another in Newark, N. J., on which he had been engaged, the shortage of current work caused him to leave this Company and enter the employ of Engstrom and Wynne, in the same capacity, on the construction of a new unit for the Bryn Mawr Hospital, in Bryn Mawr, Pa. When this project was completed, he became Field Engineer on the construction of the new Maternity Building of the Pennsylvania Hospital, in Philadelphia.

---

[1] Memoir prepared by Charles S. Doerr, Jun. Am. Soc. C. E.

Following the completion of this latter program, he entered the employ of Fuller and McClintock, Consulting Engineers, of Philadelphia, as an Office Engineer and Designer, principally on hydraulic and sanitary installations. However, Mr. Miller was by nature and training a rugged, outside construction man, and was not happy in this connection due to the close confinement of office work. Consequently, he went to the Stone and Webster Corporation in the capacity of Field Engineer on the construction of buildings for the Westinghouse Lamp Company, at Nutley, N. J. This project was about finished when the Austin Company sent for him to return as Resident Engineer and Superintendent on the construction of the Frederick Brant Rentschaler Airport, affiliated with the Pratt and Whitney Aircraft Company and the Chance-Vought Corporation, in East Hartford, Conn. This airport was one of the largest in the Eastern United States, and included several features unique in their types, at the time of their installation. Following this project, Mr. Miller was engaged for a time on a municipal airport for Scranton, Pa., being built by the Austin Company. He was erecting an industrial building for this Company at Hawthorne, N. J., and was in the performance of his duties when he met his untimely death.

Mr. Miller's experience in airport construction had developed him into a valuable engineer and superintendent and, no doubt, he would have become an expert in this field in latter years. He inherited from his greatly admired mother a keen wit, an alert mind, and a strong sense of humor that made him a most congenial companion. He learned from his early association with his father to develop a wholesome interest in all topics and in people in general, and he often referred to his father's wise counsel. His father had made a practice of taking his son on long walks and had thereby developed a real friendship between them, which Mr. Miller always cherished.

He was a composite type of American manhood, clear-thinking, clean-minded, athletically active, strong-bodied, fearless, and possessed of the highest ideals from which he never swerved. He held the esteem and respect of his fellow men and was genuinely liked by all with whom he came in contact. He knew how to be and was a true friend; he also had true friends, and he was loyal to them always. The world has lost a real man and the profession has lost a capable engineer, while those who knew him best, have only an immortal, wistful memory.

On October 17, 1930, he was married to Margaret Dill, of Philadelphia, and is survived by his widow, his mother, and two sisters, Irene (Miller) Wilson and Sara (Miller) Wanner.

Mr. Miller was elected a Junior of the American Society of Civil Engineers on December 5, 1927.

WILLIAM JOSIAH KARNER, Affiliate, Am. Soc. C. E.[1]

Died September 21, 1931

William Josiah Karner, son of Pliny and Eliza (Bacon) Karner, was born at South Egremont, Mass., on January 10, 1840. He had two sisters, Lucy and Sarah, and one brother, Andrew. He was educated in the "Little Red Schoolhouse" at South Egremont, and at Hudson River Institute, Claverack, N. Y. A report from the latter school to Mr. Karner's parents shows a grade of 100 in all his studies, and remarks that "William should become one of the best school teachers in the State." However, Mr. Karner, after a short trial of country school teaching, turned to a commercial life.

In 1862 and 1863, during the Civil War, Mr. Karner was a Corporal in Company E of the 49th Regiment, Massachusetts Volunteer Militia, and was honorably discharged on account of illness. In January, 1865, he was appointed to a first-class clerkship in the Government Post Office at Washington, D. C., but ten months of routine desk work was too much for an active man and he resigned to accept a clerical position in New York, N. Y. In 1869, he moved to Hannibal, Mo., where he was engaged for four years in the lumber business.

In February, 1873, Mr. Karner formed an association with the late E. L. Corthell, Past-President, Am. Soc. C. E., which lasted twenty-three years, on engineering and construction work throughout the United States, in Mexico, Central America, and South America. One of their early works in 1873, 1874, and 1875, was the construction of the Sny Levee on the east or Illinois bank of the Mississippi River and of a bridge across that river at Louisiana, Mo., with a draw span 444 ft. long, at that time the longest in the world. From July, 1875, to May, 1880, Mr. Corthell, as Chief Engineer of Construction, and Mr. Karner, as Purchasing Agent and Cashier, were associated with the late James B. Eads, M. Am. Soc. C. E., in the construction of the South Pass Jetties at the mouth of the Mississippi River which, by the opening of a 30-ft. channel into the Gulf of Mexico, made the Mississippi accessible to ocean-going vessels.

From September, 1880, to August, 1881, Mr. Karner was an active partner on a construction contract from the United States Government for the protection of a section of the Mississippi River bank between Baton Rouge and New Orleans, La., and was in charge of this work for six months. From October, 1881, to July, 1884, he was Office Assistant and Secretary of the North River Construction Company which held the contract for financing and constructing the West Shore Railroad from Weehawken, N. J., to Buffalo, N. Y., and the Weehawken Tunnel and Terminal. The late Walter Katté, M. Am. Soc. C. E., was Chief Engineer of the Construction Company and Mr. Corthell, Chief Engineer of the Railroad Company in charge of the work. In October, 1884, Mr. Karner joined the Eads Concession Company, of which

---

[1] Memoir prepared by Lincoln Bush, Past-President, Am. Soc. C. E., and S. Dewey Brown, Assoc. M. Am. Soc. C. E.

Mr. Eads was President and Mr. Corthell, Chief Engineer, and served until May, 1887, as Secretary of the Company and Private Secretary to Mr. Eads. This Company was formed to make the surveys and studies for a projected ship railway across the Isthmus of Tehauntepec and obtained a concession for it from the Mexican Government, but on Mr. Eads' death, in 1887, the project was dropped.

From May, 1887, to May, 1889, Mr. Karner was Chief Clerk in charge of accounts and estimates in the Chicago Office of the late George S. Morison, Past-President, Am. Soc. C. E., and Mr. Corthell, engaged in designing and constructing railroad bridges over the Ohio, Missouri, and Mississippi Rivers, including the Ohio River Bridge at Cairo, Ill., for the Illinois Central Railroad, then the longest steel bridge in the world. Among the engineers in the employ of Messrs. Morison and Corthell at this time was Ralph Modjeski, M. Am. Soc. C. E. From May, 1889, until Mr. Corthell's retirement from active practice in the summer of 1895, Mr. Karner was the former's Assistant on many important engineering and construction projects, including the improvement, by means of jetties, of Tampico Harbor, at the mouth of the Panuco River, Mexico, the Merchants Bridge at St. Louis, Mo., improvements at the mouth of the Brazos River in Texas, and a joint entrance into the City of Chicago for the Atchison, Topeka, and Santa Fé Railroad and a subsidiary of the Illinois Central Railroad.

In 1895, Mr. Karner went as Assistant and Chief Clerk to the late John F. Wallace, Past-President, Am. Soc. C. E., Chief Engineer of the Illinois Central Railroad, and continued to hold the same position under David Sloan, M. Am. Soc. C. E., when the latter succeeded Mr. Wallace. In 1903, Mr. Karner resigned his position with the Illinois Central Railroad Company and joined Mr. Modjeski, at that time Engineer in charge of the construction of a railroad bridge across the Mississippi River, at Thebes, Ill. In 1904, when Mr. Wallace was appointed by President Roosevelt as the first Chief Engineer of the Isthmian Canal Commission, Mr. Karner went with him to Panama as Resident Engineer and, in June, 1905, was appointed Special Disbursing Agent. From this time, until his resignation in July, 1909, Mr. Karner had charge of recruiting and shipping to the Canal Zone laborers from Barbados and certain other West Indian Islands, and, later, from Spain and other European countries. From 1909 to 1921, Mr. Karner was Director, Secretary, and Treasurer of Bowne and Company, Incorporated, New York, printers and manufacturers of commercial stationery, retiring on October 31, 1921, from which time until his death he made his home at Danville, Ill., with Federal Judge Walter C. Lindley whose wife is a niece of Mrs. Karner.

It is a remarkable fact that Mr. Karner was associated in service with three Presidents of the Society, namely, George S. Morison, John F. Wallace, and Elmer L. Corthell, and with one Vice-President, James B. Eads.

Mr. Karner was widely known for his excellent reputation and integrity and for his capable executive ability, gracious personality, and his loyalty to his friends and associates. He always had a great interest in young men, and while with the Illinois Central Railroad Company he instituted a "Track

Apprentice" system which many present-day engineers will remember as being their start in the Civil Engineering Profession. He was a member of the Western Society of Engineers, the New England Society of New York, the Visiting Women's Press Club of New York, the Thursday Morning Club of Great Barrington, Mass., and was a Mason, an Elk, and a Kiwanian, a Congregationalist in religion, and a Republican in politics.

On January 25, 1871, he was married to Maria Louise Dewey, of North Egremont, Mass., who died on September 11, 1917. They had no children.

Mr. Karner was elected an Affiliate of the American Society of Civil Engineers on July 2, 1890.

EDWARD ALFRED SIMMONS, Affiliate, Am. Soc. C. E.[1]

DIED SEPTEMBER 30, 1931

Edward Alfred Simmons, the son of John Wesley and Agnes Anne (Owers) Simmons, was born on March 20, 1875, in Brooklyn, N. Y.

He attended the public schools of that city only as far as the Grammar Grades, for at the age of fourteen it became necessary for him to help support his widowed mother. Through the recommendation of his Sunday School teacher he secured employment as a counter clerk in the department store of A. D. Mathews and Son, in Brooklyn, which employment paid him $1.50 per week. He did not remain long at this post, however, as one of the members of the choir of St. Mary's Protestant Episcopal Church, in Brooklyn, in which he sang, informed him that he could get $5.00 per week with The Railroad Gazette.

His first job with the Company which he later controlled, was as a clipper of news items. He was rapidly promoted to Assistant to the News Editor, Advertising Make-up Clerk, Advertising Solicitor, Western Manager, with headquarters in Chicago, Ill., Secretary, Treasurer, Director, Vice-President, and (from 1911) President. In 1912, the corporate name of this Company was changed to Simmons-Boardman Publishing Company. It numbers among its publications the *Railway Age, Railway Signaling, Railway Mechanical Engineer, Railway Electrical Engineer, Railway Engineering and Maintenance, Marine Engineering and Shipping Age,* and *The Boiler Maker;* as well as three technical encyclopedias and numerous books. Its widely known slogan name, "The House of Transportation," was originated by Colonel Simmons. In 1928, the American Builder Publishing Corporation was purchased and Colonel Simmons was elected Chairman of the Board. Its principal publication is now known as *American Builder and Building Age.* Both these companies were taken over later in the same year by a new holding company, called the Simmons-Boardman Publishing Corporation, of which Colonel Simmons was President.

---

[1] Memoir prepared by Jessica Husk, New York, N. Y., Executive Secretary to the late Colonel Simmons.

An authority of international reputation on rail and water transportation, Colonel Simmons was appointed by two Presidents of the United States as a Government Delegate to International Railway Congresses, the first at London, England, in 1925, and the second at Madrid, Spain, in 1930. He was also designated a Delegate on the part of the United States to the Fifth International Congress of Building and Public Works, held in London, in May, 1930.

As President of the American Marine Association he was an outspoken advocate of the development of a great merchant marine for the United States, and the achievements of the American Marine Standards Committee, which organization he founded, are largely due to his capable guidance as Chairman of its Executive Board.

Industrial interests were also his. In 1904, he organized the American Saw Mill Machinery Company with a plant at Hackettstown, N. J., and served thereafter as its executive head. The Company has two subsidiaries, the American Saw Works and the American Machine Tool Company.

He heard the call of his country at the time of the World War and was commissioned, in February, 1918, a Major in the National Army. He served as Regional Constructing Quartermaster until February, 1919, in charge of all new Army construction work in and around New York Harbor, except the Brooklyn Army Base, having at one time twenty-two projects under his supervision. Among them were the $2 500 000 General Hospital, at Fox Hills, Staten Island, four different sets of barracks, a coal trestle, a dock, an engine house, an aviation field, a repair pit, a garage, an incinerator, and a fresh-water supply line.

His work in this connection was highly praised by Brig.-Gen. R. C. Marshall, Jr., U. S. A., M. Am. Soc. C. E., Chief of the Construction Division, in a citation which read:

"A spirit of loyalty and competent effort has characterized your service in the Construction Division and was particularly helpful in the successful prosecution of the largest war construction program ever undertaken."

His transfer to the Engineer Corps, with detail for service with the American Military Railways in France, was in process of consummation when the Armistice was signed.

In 1918, he was commissioned a Lieutenant Colonel in the Quartermaster Section of the Officers Reserve Corps; in 1922, he was promoted to the rank of Colonel; and recommissioned in 1927, which commission ran to November 8, 1932.

Colonel Simmons' activities in the American Legion began in 1919 when he helped to organize several posts. He was elected Commander of Kings County Post No. 500 (now Brooklyn Post No. 500) and commanded the Kings County Organization of the Legion for two terms. He had charge of the funeral services at the Brooklyn and Hoboken piers for the soldiers brought from France for reburial in this country. For several years he served on the National Transportation Committee of the Legion. He was Treasurer of Construction for Division Post No. 10 at the time of his death.

He took an active part in organizing the Citizens' Military Training Camp Clubs and was untiring in his efforts in behalf of Reserve Officers, having twice served as President of the Brooklyn Chapter of the Reserve Officers' Association of the United States, and was discharging the duties of that office for the Department of New York at the time of his death. Other military organizations of which he was an active member were: The Construction Division Association; Military Order of the World War; New York Society of Military and Naval Officers of the World War; Society of American Military Engineers; and Quartermasters' Association.

The career of Colonel Simmons was conspicuous in civil as well as industrial and military service. He was a member of the Board of Regents and Treasurer of the Long Island College Hospital from 1914 to 1922. Here, he had studied medicine in 1908, principally before working hours in the morning, with no idea of becoming a physician, but with the thought of devoting his knowledge to charity. He had accepted the Chairmanship of a commission appointed to provide an adequate water supply for the Town of Inlet, N. Y., adjacent to his country home, "Albedor," situated at Eagle Bay on the fourth of the Fulton Chain of Lakes, where he spent as much time as business permitted. Always a zealous American, he offered gold, silver, and bronze medals to school children for the three best essays selected in competition on such subjects as "The American Flag" and "Why I Love America"; and frequently presented schools with large American flags. His philanthropies were many.

Colonel Simmons was an Associate Member of both the American Society of Mechanical Engineers and the Society of Naval Architects and Marine Engineers, and Chairman of the Endowment Committee of Engineering Foundation, Inc.

While a registered Republican, and a member of the Union League Clubs of both New York and Brooklyn, Colonel Simmons always supported a candidate rather than his party, and numbered many Democrats among his friends. As one of the foremost leaders of the business press, he was selected to head the Business Paper Publishers' Committee which successfully campaigned for President Hoover in 1928. He was once mentioned as a possible candidate for Borough President of Brooklyn, but gave his supporters no encouragement.

Colonel Simmons took an active interest in winter sports, donating a silver loving cup to the American Legion of Old Forge, N. Y. An ardent fisherman himself, he served as Honorary President of the New York State Division of the Izaak Walton League of America. He was also an enthusiastic golfer and was prominently identified with the activities of the Inlet Golf Club, Inlet, N. Y. The Adirondack League, Old Forge, N. Y., the Thendara Golf Club, Thendara, N. Y., the Engineers' Club, Roslyn, Long Island, and the Congressional, Washington, D. C., were among his country clubs. His name was also on the membership roll of a number of New York City clubs, including the Army and Navy, Automobile, Bankers', and Engineers'; and, for several successive terms, he was a Governor of the Railroad Club.

A Thirty-second Degree Mason, and a member of the Benevolent and Protective Order of Elks, his fraternal orders were: Aurora Grata Consistory; Clinton Commandery; Kismet Temple; Long I Grotto; Orient Chapter; Mistletoe Lodge (F. and A. M.), and Brooklyn Lodge 22 (B. P. O. E.).

Scheduled to attend the Washington Conference of the President's Committee on Home Building and Home Ownership the following day, Colonel Simmons was in the Pennsylvania Railroad Station, New York City, on the night of September 29, 1931, when he was taken ill. The following evening he died at his home in Brooklyn, of cerebral hemorrhage. Having come into the world simply, he passed out of it with simple, but impressive, services at his residence. However, full military honors were accorded him en route to his final resting place in Greenwood Cemetery, Brooklyn, with Reserve Officers acting as Honorary Pallbearers and a detachment from Fort Hamilton firing a farewell salute as "Taps" was sounded.

Among the numerous floral tributes was a wreath sent by special messenger from President Hoover, with a letter in which he said:

> "I am greatly shocked to learn of the death of Colonel Simmons, who had long been my friend. He had a distinguished career of public service and his passing is indeed a great loss."

Professor Arthur T. Hadley, before he became President of Yale University, said of Colonel Simmons:

> "It has been my fortune to know many men to whom journalistic responsibility attached, but never one who more seduously was devoted to the belief that out of ill there would come good. He was not impulsive, he was not voluble; but as an optimist, in the higher sense of living optimism, he was action perfect."

Following twenty years of association, Samuel O. Dunn, Editor-in-Chief of the publications of the Simmons-Boardman Publishing Corporation, wrote:

> "Colonel Simmons never issued a single order that any editorial, article or item should be published in or omitted from one of these papers, or that any editorial policy should be adopted, changed or abandoned. His course in this respect was not dictated by expediency, but by principle. Like every publisher, he desired that the company of which he was the head should be financially successful. His fundamental principle as a publisher, however, was that a business paper should be published primarily to render the greatest possible service to the industries in the field that it covered. He believed that such service could be rendered only by a paper that published the most complete and accurate information that it could get regarding all the developments in the field, and that boldly criticized unsound policies and advocated sound policies, regardless of any fears that might be felt as to the probable effects upon its circulation and advertising. He never once failed unflinchingly to support his editorial department when convinced that the editors themselves were satisfied as a result of adequate investigation and study that the policies they were following were sound from the standpoint of the interests of the industries the papers were undertaking to serve. If the various Simmons-Boardman papers have been outstanding in the services they have rendered to the business fields that they have undertaken to cover, the credit has been mainly due to Colonel Simmons' courage and public spirit as a publisher."

And so, this able publisher, manufacturer, soldier, civic worker, and sportsman, actuated by the highest motives, and with the utmost consideration for his associates, despite a lack of technical training, achieved unusual prominence. Colonel Simmons will long be remembered in the words of Ward Gedney, as "frank, cordial, but unfailingly serious; smiling with the warm smile of a man big physically, mentally and morally, but slow to smile."

He was survived by his widow, Ida Rines Simmons, to whom he was married on October 12, 1914, and their three daughters, Aline, Doris, and Elizabeth.

Colonel Simmons was elected an Affiliate of the American Society of Civil Engineers on October 14, 1919.

EDWARD DEAN ADAMS, F. Am. Soc. C. E.[1]

Died May 20, 1931

Edward Dean Adams, the eldest son of Adoniram Judson Adams and Harriet Lincoln Norton Adams, was born in Boston, Mass., on April 9, 1846. He was a lineal descendant of Henry Adams of Braintree, Mass., who came to America from Somersetshire, England, in 1638, and whose descendants included two early Presidents of the United States, John Adams and John Quincy Adams.

Graduating from Norwich University at Northfield, Vt., in 1864, as a Bachelor of Science, Mr. Adams spent fifteen months traveling in France, Italy, Greece, Turkey, Palestine, Egypt, and Spain. Returning to America, he studied at the Massachusetts Institute of Technology during the college year, 1865-1866, as a member of the Class of 1869 in Engineering and Architecture. The year following was devoted to practical experience in all departments of a paper mill in Berkshire, Mass., with some thought of engaging in this industry. Study was continued after leaving the Institute, with special attention to accounting and business management, developing aptitude for analysis of accounts.

Mr. Adams began his business career in 1866 in Boston as Bookkeeper and Cashier of Thomas J. Lee and Hill, Stock Brokers, soon representing the firm as member of the Stock Exchange. In 1870, the Middle West attracted him. He selected Appleton, Wis., for its water-power possibilities and other natural resources and organized a bank there; but a Boston friend dissuaded him from settling in the West. Instead, he joined this friend and another in establishing in Boston the banking house of Richardson, Hill and Company.

In 1878, Mr. Adams removed to New York, N. Y., and became a partner in the banking house of Winslow, Lanier and Company. This connection was maintained for fifteen years. From 1893 to 1914, he was American representative of the Deutsche Bank of Berlin. In his seventy-fifth year, he curtailed his business interests, in order to devote energies to the fine arts and sciences in which he had long been particularly interested.

---

[1] Memoir prepared by Lewis B. Stillwell, M. Am. Soc. C. E.

During fifty-four years in business, Mr. Adams was an officer or director in more than forty corporations engaged in transportation, manufacture, power development, or other enterprises. His constructional and financial aptitudes manifested themselves in combination throughout his business activities, and in numerous voluntary public services. His business undertakings embraced organization, re-organization, or development of the New York, West Shore and Buffalo Railroad, Central Railroad of New Jersey, Northern Pacific Railroad, two railroads in Mexico, Cataract Construction Company, The Niagara Falls Power Company, East Jersey Water Company, All America Cables, Incorporated, Allis-Chalmers Company, American Cotton Oil Company, Edison Electric Illuminating Company, Hammond Typewriter Company, Davis Coal and Coke Company, Union Petroleum Company, Bullock Electric Manufacturing Company, and many more. These transactions involved hundreds of millions of dollars and tens of thousands of bondholders, stockholders, and employees, and necessitated thorough familiarity with the industries of the entire United States. Under a concession from the Mexican Government, Mr. Adams and a few associates for several years minted much of the coinage of that country.

Greatest of the services by Mr. Adams to his country was his leading part in preventing suspension of specie payment in President Cleveland's second term, threatening as an outcome of the "free silver" fallacy. On January 8, 1896, the Deutsche Bank conferred,

> "Special power of attorney unto Edward D. Adams to enter into an agreement on our behalf with the Secretary of the Treasury of the United States of America for the sale and delivery to said United States of a certain quantity of gold payable in United States 4 per cent 30-year coupon or registered bonds, part of a total not exceeding two hundred millions of dollars nominal capital, and we authorize the said Edward D. Adams to stipulate on our behalf and in his discretion all conditions of said agreement."

With this backing Mr. Adams invited Messrs. J. Pierpont Morgan and James Stillman to join him in a syndicate with Mr. Morgan as Manager, each of the three subscribing \$25 000 000. A contract was made with the Government. An issue of 4% bonds, due in 1925, was fivefold over-subscribed by the public. A shipment of several million dollars in gold from Germany was arranged to meet immediate needs and restore confidence. On February 21, 1896, the gold reserve was re-established in excess of \$100 000 000; it had fallen to \$31 862 000 on January 31. Suspension would have staggered industry and commerce.

Mr. Adams' interest in science and in engineering development led him to take keen interest in Edison's invention of the electric lamp and the establishment of the first electric lighting central station.

In 1885, the late William B. Rankine, brought to the attention of a group of New York capitalists and bankers a charter for the development of power at Niagara Falls. Among the early members of this group were D. O. Mills, John Jacob Astor, Edward A. Wicks, Francis Lynd Stetson, and Mr. Adams. These gentlemen decided to form a corporation to undertake the enterprise, and selected Mr. Adams as executive leader. The Cataract Construction Company was organized with Mr. Adams as President, and the comprehensive and exhaustive investigation of possible methods of developing and utilizing the power of Niagara which he directed during the years immediately following, is a classic among researches in the field of engineering.

The development of power on a large scale at Niagara Falls by hydro-electric equipment and the transmission of this power in the form of alternating current by electrical conductors to distant points of use, attracted universal attention and established the confidence of investors and of the public in this far-reaching development of applied science. So pervasive of all engineering, industrial, communal, and household operations have been the results of this pioneering large-scale work at Niagara that the generation since born little realizes the boldness of that forward step, the immaturity of the sciences and arts upon which success depended at the time when it was taken, or the courage it demanded amid diametrical divergences of opinion among acknowledged scientific and engineering leaders. To-day, 96% of the electricity utilized in the United States is generated as alternating current and all long-distance transmissions of power are by alternating current at high potential.

When attacking the Niagara problem, Mr. Adams retained the late Coleman Sellers, M. Am. Soc. C. E., of Philadelphia, Pa., a mechanical engineer of great ability, sound judgment, and long experience, and accompanied by him traveled extensively seeking the advice of the foremost engineers and scientists in both America and Europe. The problem was presented to those whom he consulted in its broadest aspects. Here was a vast amount of undeveloped hydraulic power. Its utilization, of course, involved the use of some type of hydraulic turbine. How should the mechanical power at the shafts of the turbines be utilized—by direct connection or by some form of transmission—by belts or cables, by compressed air, by hydraulic transmission, or by electrical transmission?

To make sure that the great enterprise should have the benefit of the latest advances, five distinguished scientists and engineers were invited to be the members of the International Niagara Commission, to determine the forms of applications of the latest knowledge to the "harnessing of Niagara." Sir William Thomson (later, Lord Kelvin) was Chairman; his associates were Dr. Coleman Sellers, of Philadelphia, Lieut.-Col. Theodore Turrettini, of Geneva, Switzerland, Professor E. Mascart, of the College of France, and Professor William Cawthorne Unwin (now Hon. M. Am. Soc. C. E.), of London, Secretary. This was the first known inter-continental council of leaders in scientific fields to consider practical applications of the physical sciences to an industrial project of international interest.

It was inevitable and of course expected that among the many engineers consulted who were not members of the Commission there should be wide divergence of opinion and advice. Even when the Commission reached the conclusion that electricity should be used to transmit and distribute power from a central station, some of its members advocated direct current while others believed that alternating current should be adopted. Throughout this difficult period of investigation, the great interest of Mr. Adams in science and engineering, his vision, sound judgment, wonderful tact, and systematic methods guided his Company to a conclusion which has abundantly stood the test of time and experience.

The 5 000-h. p. turbine and generator on the same shaft built for Niagara Falls Power House No. 1, constituted a unit of unprecedented magnitude. The ten units there installed became the progenitors of the 32 000-h. p. units placed in operation at Niagara Falls in 1914. The 22-mile transmission line of 1896 to Buffalo, N. Y., at 11 000 volts tension, with transformers arranged so that 22 000 volts could be used when adequate line insulators might become available, pointed the way to the numerous high-tension lines since erected in many parts of this country, scores, and even hundreds, of miles long, at voltages up to 220 000, being increasingly linked together to form super-power networks, each dependably serving industries and communities in several States.

The installation thus constructed under Mr. Adams' executive leadership was the first central station in America built and equipped for the purpose of developing and distributing power in the form of electric energy—a conception which has contributed greatly to economy, convenience, and dependability in the production and delivery of power whether generated from falling water or from fuel. The story of the development of power at Niagara Falls has been told by Mr. Adams in a work of two volumes, entitled "Niagara Power."

In April, 1926, the John Fritz Medal, esteemed the highest recognition in the gift of the Engineering Profession in the United States was conferred upon Mr. Adams "for great achievements as Engineer, Financier, Scientist, whose vision, courage and industry made possible the birth at Niagara Falls of hydro-electric power."

A service of peculiar and enduring interest to the Engineering Profession was rendered by Mr. Adams only a few years before his death. In 1927, while representing Engineering Foundation at the celebration of the Five Hundredth Anniversary of the founding of the University of Louvain, in Louvain, Belgium, he conceived the idea that a clock and carillon for the tower of the University Building, the reconstruction of which by contributions from America was then approaching completion, would constitute a fitting memorial to the American Engineers who lost their lives in the World War. His idea was adopted by the American Society of Civil Engineers, the American Society of Mechanical Engineers, the American Institute of Electrical Engineers, and the American Institute of Mining and Metallurgical Engineers, and, under Mr. Adams' personal direction, was carried through to completion most effectively in every detail. The carillon of forty-eight bells and the beautiful and symbolic design of the four faces of the great clock constitute a memorial, than which none can be more beautiful or appropriate.

During and notwithstanding his prolonged and intense business activity, Mr. Adams for many years was a patron of Astronomy, particularly interested in total eclipses of the sun. Through his personal co-operation the first paintings of the corona in its true colors were made at Baker, Ore., in 1918, by the distinguished artist, Howard Russell Butler, as a part of the official observations by the Naval Observatory Staff.

The breadth of Mr. Adams' interests in engineering, science, the fine arts, education, and philanthropy, is attested by mention of his active connections with the American Society of Civil Engineers, American Institute of Electrical Engineers, Engineering Foundation, Engineering Societies Library, National Research Council, American Museum of Natural History, New York Botanical Garden, Metropolitan Museum of Art, American Numismatic Society, Institute of Musical Art in the City of New York, Columbia University School of Business, American Scenic and Historical Preservation Society, American Academy in Rome, and American Committee for Devastated France. This list is far from complete; it includes none of the fifty or more organizations of which he was merely a member, nor his twenty-eight clubs. These facts evidence a rare scope of industrial, financial, and scientific achievement, both by personal effort and by support of other workers, coupled with an extraordinary number and diversity of human contacts.

Mr. Adams was an active member of the Protestant Episcopal Church and, for many years, was Senior Warden of the Parish of St. George, Rumson, N. J.

His Alma Mater, Norwich University, conferred upon him the degree of Master of Science (M.Sc.) in 1897, Doctor of Laws (LL.D.), in 1906, Master of Arts (M.A.), in 1908, and Doctor of Science (Sc.D.), in 1927. Honorary degrees of Doctor of Science were conferred upon him in 1928 by the University of Louvain and in 1930 by Columbia University.

During his remarkable career, Mr. Adams was invested by Foreign Governments with the following honors: Royal Order of the Crown of Prussia, Second Class, 1909; Medaille de Reconnaissance (with silver star), of the French Republic, 1921; John Fritz Gold Medal, 1926; Commander of the Order of the Crown (of Leopold) of Belgium, 1928; Chevalier de l'ordre National de la Legion d'honneur (France), 1929.

In 1872, Mr. Adams was married to Frances Amelia Gutterson, of Boston, who died in 1921. He is survived by a daughter, Mrs. Leighton Lobdell, by a granddaughter, Brenda Lobdell, and by four grandsons, Francis Lobdell, Leighton Mansfield Lobdell, Pierpont Adams, and Kempton Adams.

In physique, Mr. Adams was below the average height, slender rather than stout even in his old age. His bearing was always dignified, but his manner was urbane and courteous. His head was large and his features, especially his large and luminous eyes, strikingly evidenced his character and great ability. His mind was highly analytical and methodical. His mental activity was incessant, and although he examined with great care each factor of a problem, however complicated, he visualized with accurate perspective the problem as a whole. In expressing his views orally or in writing, he was deliberate, clear, and precise. It is safe to say that few men, if any, of his generation have excelled him in intellectual grasp of essential premises or in logical ability to draw sound conclusions from those premises.

Mr. Adams was elected a Fellow of the American Society of Civil Engineers on March 31, 1891.

TRANSACTIONS

OF THE

AMERICAN SOCIETY OF CIVIL ENGINEERS

INDEX

VOLUME 96

1932

SUBJECT INDEX, PAGE 1644
AUTHOR INDEX, PAGE 1654

Titles of papers are in quotation marks when given with the author's name.

VOLUME 96

SUBJECT INDEX

ADDRESSES.

Presidential Address at the Annual Convention at Yellowstone National Park, Wyoming, July 6, 1932. Herbert S. Crocker. 1380.

AIR FLOW.

Models of airplane propellers. 301.

AIRPLANE PROPELLERS.

Models of —. 301.

ARCHES.

"Design of a Reinforced Concrete Skew Arch." Bernard L. Weiner. (With Discussion.) 1212.

"The Buttressed Dam of Uniform Strength." Herman Schorer. (With Discussion.) 666.

BEAMS.

"Analysis of Continuous Frames by Distributing Fixed-End Moments." Hardy Cross. (With Discussion.) 1.

BRIDGES.

"Design of a Reinforced Concrete Skew Arch." Bernard L. Weiner. 1212. Discussion: G. D. Houtman, J. Charles Rathbun, and A. A. Eremin. 1321.

BUTTRESSES.

"The Buttressed Dam of Uniform Strength." Herman Schorer. (With Discussion.) 666.

CAMPS.

— and facilities, Cascade Tunnel construction. 962.

CANALS.

"Flow of Water in Tidal —." Earl I. Brown. (With Discussion.) 749.

See also **IRRIGATION CANALS.**

CEMENT.

— tests for use on locks and dams, Ohio River. 1348.

CITY PLANNING.

"Legal Problems Involved in Establishing Set-Back Lines." Clifton Williams. (With Discussion.) 887.

CLAY.

Physical characteristics of — and shales. 629.

COMPASS.

Application of the gyro-compass to dredging work. 648.

CONCRETE.

— lining of Cascade Tunnel. 978.

"Manufacturing — of Uniform Quality." William M. Hall. 1336. Discussion: Henry B. Seaman, Theodore Belzner, John Sanford Peck, J. W. Kelly, Stanton Walker, Edward E. Bauer, O. Bonney and D. T. Mitchell, C. E. Arnold, Morris Mason, John H. Griffith, and W. B. Wendt. 1359.

Pneumatic — placing practise in the Cascade Tunnel. 993.

CONSTRUCTION PLANT.

"Construction of La Ola Pipe Line in Chile." W. B. Saunders. (With Discussion.) 230.

— , Cascade Tunnel, Great Northern Railway. 950.

"Economical Design of Hydraulic Pipe-Line Dredge." John Francis Cushing. (With Discussion.) 625.

CONTINUOUS GIRDERS.

"Analysis of Continuous Frames by Distributing Fixed-End Moments." Hardy Cross. (With Discussion.) 1.

CONTRACTORS.

Relation between engineers and —. 1381.

CONTRACTORS' PLANT.

See **CONSTRUCTION PLANT.**

COSTS OF WORK.

"Economical Design of Hydraulic Pipe-Line Dredge." John Francis Cushing. (With Discussion.) 625.

Pipe line costs. 246, 269.

Wages paid on Cascade Tunnel, Great Northern Railway. 987.

DAMS.

Construction of a model of an arch dam. 318.

"Gravity — Arched Down Stream." B. J. Lambert. 1178. Discussion: Eugene Kalman, William P. Creager, T. L. E. Haug, B. F. Jakobsen, A. Floris, and Floyd A. Nagler. 1184.

"Some Aspects of Water Conservation." R. A. Sutherland. (With Discussion.) 157.

"Stresses in Gravity — by Principle of Least Work." B. F. Jakobsen. 489. Discussion: William Cain, William Gore, L. J. Mensch, Lars R. Jorgensen, C. A. P. Turner, Donald P Barnes, Anthony Hoadley, Fred W. Ely, M. H. Gerry, Jr., Fredrik Vogt, Eugene Kalman, Fernand Campus, A. Floris, Robert E. Glover, and P. Wilhelm Werner. 514.

"The Buttressed Dam of Uniform Strength." Herman Schorer. 666. Discussion: James Girand, Fred A. Noetzli, B. F. Jakobsen, Calvin V. Davis, A. A. Eremin, Eugene Kalman, A. Floris, Fredrik Vogt, A. C. Schwager, Paul Bauman, and I. M. Nelidov. 689.

"The Don Martin Project." Andrew Weiss. (With Discussion.) 835.

See also **STRUCTURES, THEORY OF.**

DREDGES AND DREDGING.

Application of the gyro-compass to dredging work. 648.

"Economical Design of Hydraulic Pipe-Line Dredge." John Francis Cushing. 625. Discussion: R. L. Vaughn, and Arthur L. Collins. 651.

DRILLING.

Drill-steel sharpening practise at west portal, Cascade Tunnel. 989.

ENGINEERING.

"Some Reflections Regarding — Practice": Address at the Annual Convention, Yellowstone National Park, Wyoming, July 6, 1932. Herbert S. Crocker. 1380.

ENGINEERS.

Relation between — and contractors. 1381.

FLUMES.

"Tests of Broad-Crested Weirs." James G. Woodburn. (With Discussion.) 387.

FRAMEWORKS.

"Analysis of Continuous Frames by Distributing Fixed-End Moments." Hardy Cross. (With Discussion.) 1.

GROUND-WATER.

"Flow of — as Applied to Drainage Wells." M. R. Lewis. 1194. Discussion: L. Standish Hall, and O. W. Israelsen. 1207.

HIGHWAYS.

"Highway Location: General Considerations." Gibb Gilchrist. (With Discussion.) 454.

"Highway Location: Practical Considerations." A. R. Losh. (With Discussion.) 468.

HYDRAULIC JUMP.

"Tests of Broad-Crested Weirs." James G. Woodburn. (With Discussion.) 387.

Weirs with Aprons Inclined Up Stream and Down Stream. Alex. R. Webb. 408.

INSPECTION.

"Manufacturing Concrete of Uniform Quality." William M. Hall. (With Discussion.) 1336.

"Some Reflections Regarding Engineering Practice": Address at the Annual Convention, Yellowstone National Park, Wyoming, July 6, 1932. Herbert S. Crocker. 1380.

IRRIGATION.

"Measuring — Deliveries in the Panjab." E. S. Lindley. 1005.

"The Don Martin Project." Andrew Weiss. 835. Discussion: C. H. Howell, and Robert S. Stockton. 855.

IRRIGATION CANALS.

"Measuring Irrigation Deliveries in the Panjab." E. S. Lindley. 1005.

"The Don Martin Project." Andrew Weiss. (With Discussion.) 835.

LAW.

"Legal Problems Involved in Establishing Set-Back Lines." Clifton Williams. (With Discussion.) 887.

MAPPING.

See **MAPS.**

MAPS.

Mapping procedure, Swift River Reservoir. 1027.

MATHEMATICS.

"Theory of Similarity and Models." Benjamin F. Groat. (With Discussion.) 273.

MEMOIRS OF DECEASED MEMBERS.

Adams, Edward Dean. 1636.
Adams, Raymond Edmond. 1388.
Addis, William George. 1389.
Adey, William Henry. 1390.
Alger, Julian Wilmot. 1590.
Allen, Andrews. 1391.
Ash, Louis Russell. 1393.
Asserson, Peter Christian. 1397.
Bailey, Albert Ross. 1592.
Bailey, Frank Harrison. 1403.
Baird, Samuel Pond. 1406.
Behrend, Bernard Arthur. 1407.
Billingsley, James Wartelle. 1411.
Bisbee, Fred Milton. 1413.
Boardman, Howard Edward. 1415.
Boucher, William James. 1417.
Brackenridge, William Algernon. 1419.
Braune, Gustave Maurice. 1422.
Breckinridge, George Castleman. 1594.
Brodie, Hugh. 1426.
Bunn, Paul King. 1428.
Burden, Morton. 1595.
Burroughs, Hector Robins. 1429.
Campbell, George Raymond. 1431.
Carson, Howard Adams. 1386.
Carter, Edward Carlos. 1433.
Chipman, Willis. 1435.
Christensen, John Christian Valdemar. 1438.
Clarke, St. John. 1439.
Comly, James Retzer. 1442.
Coppée, Henry St. Leger. 1444.
Craigmile, Charles James. 1445.
Cummings, William Warren. 1447.
Dahm, Sverre. 1448.
Davidson, Frank Eugene. 1451.
Davis, Joseph Phineas. 1453.
Dawson, John Stanley. 1457.
Dessery, Floyd Gossett. 1458.
Detwiler, Harry Espenship. 1460.
Drake, Robert Morris. 1597.
Dunlap, Frederic Clark. 1461.
Easterbrooks, Preston Burt. 1597.
Edwards, James Harvey. 1463.
Ellis, George Ezra. 1466.
Elwell, Charles Clement. 1467.
Emerson, Frank Collins. 1470.
Emory, Lloyd Tilghman. 1474.
Farnsworth, Howard Richards. 1476.
Foster, Frank. 1478.
Fougner, Hermann. 1480.

Fuertes, James Hillhouse. 1482.
Gagel, Edward. 1486.
Gallivan, James Henry. 1600.
Hallett, James Hendricks. 1601.
Hardaway, Benjamin Hunt. 1489.
Harding, James Clark. 1494.
Hereford, Lewis Stirling. 1603.
Howe, Edward Willard. 1495.
Howe, William Chaffin. 1603.
Hughes, Thomas Rogers. 1625.
Hunt, Andrew Murray. 1497.
Jadwin, Edgar. 1498.
Jensen, Andrew Martin. 1504.
Jones, William Henry. 1505.
Karner, William Josiah. 1630.
Kehoe, Alfred Lawrence. 1506.
Kiel, Frank Burdette. 1604.
Kondo, Sentaro. 1507.
Lawson, Elmer Barthold. 1626.
Lawton, William Henry. 1508.
Leland, George Herbert. 1510.
MacDiarmid, Milo Stuart. 1511.
McCullough, Ernest. 1512.
Mailey, John Bruce. 1607.
Marshall, Horace Miller. 1514.
Meigs, Montgomery, 1516.
Merickel, Bernard Joseph. 1627.
Metcalfe, Joseph Davis. 1518.
Miller, John Harry. 1628.
Mirick, Alfred Stowe. 1608.
Molera, Eusebius Joseph. 1520.
Montfort, Richard. 1521.
Murphy, Joseph Lincoln. 1609.
Noyes, Stephen Henley. 1521.
Odell, Frederick Sylvester. 1524.
Onderdonk, Arthur. 1525.
Pardee, Robert McKnight. 1614.
Phelps, William Collins. 1527.
Porter, Henry Tegmeyer. 1528.
Preston, Harry Longyear. 1531.
Randolph, William Couts, Jr. 1615.
Raymer, Albert Reesor. 1534.
Reid, Homer Austin. 1616.
Reynolds, Abraham Moreau. 1536.
Riser, Knud Sophus. 1537.
Roberts, Earl Ivan. 1539.
Saunders, William Lawrence. 1540.
Schneider, Anton. 1542.
Schuyler, Philip. 1547.
Sengoku, Mitsugu. 1549.
Shailer, Robert Ames. 1553.

Simmons, Edward Alfred. 1632.
Slater, Willis Appleford. 1555.
Sowden, Herbert Jones. 1618.
Spencer, Paul Bertram. 1619.
Stannard, Jay L. 1558.
Stern, Eugene Washington. 1560.
Stevens, Frank Stoddard. 1562.
Thornburg, Lewis. 1564.
Tucker, Lester Waldo. 1566.
Tye, William Francis. 1568.
Volckman, George William. 1570.
Wagner, Fred J. 1572.
Wagner, Samuel Tobias. 1574.
Walker, John Simpson. 1576.
Ware, Samuel Houston. 1621.
Wiley, Andrew Jackson. 1577.
Willacker, Charles Edward. 1622.
Wintermute, Ferd Clark. 1624.
Woods, Henry Dickinson. 1584.
Worthington, Charles. 1586.
Yappen, Adolph. 1588.
Zarbell, Elmer. 1589.

MODELS.

— of gravity dams arched down stream. 1178.

"Theory of Similarity and —." Benjamin F. Groat. 273. Discussion: H. Bateman, W. P. Roop, A. V. Karpov, Jasper O. Draffin, Ed S. Smith, and F. J. Eisner. 304.

MOMENTS.

"Analysis of Continuous Frames by Distributing Fixed-Ends —." Hardy Cross. (With Discussion.) 1.

PIPE.

"Construction of La Ola — Line in Chile." W. B. Saunders. (With Discussion.) 230.

PIPE LINES.

"Construction of La Ola Pipe Line in Chile." W. B. Saunders. 230. Discussion: Ernest B. Nelson, H. L. Thackwell, and William E. Rudolph. 266.

"Economical Design of Hydraulic Pipe-Line Dredge." John Francis Cushing. (With Discussion.) 625.

POWER PLANTS.

Montandon Power Station, La Ola Pipe Line. 263.

RAILROADS.

"The Eight-Mile Cascade Tunnel, Great Northern Railway": A Symposium. D. J. Kerr, Frederick Mears, and J. C. Baxter. (With Discussion.) 915.

RAINFALL.

"Formulas for — Intensities of Long Duration." Merrill M. Bernard. 592. Discussion: W. B. Gregory, W. W. Horner, Adolph F. Meyer, R. W. Powell, C. E. Grunsky, and Charles W. Sherman. 607.

REINFORCED CONCRETE.

"Design of a — Skew Arch." Bernard L. Weiner. (With Discussion.) 1212.

RESERVOIRS.

"Some Aspects of Water Conservation." R. A. Sutherland. (Wth Discussion.) 157.

"Surveys for Swift River Reservoir of the Boston Metropolitan Water Supply." N. LeRoy Hammond. (With Discussion.) 1020.

RIGHT OF WAY.

— features of highway location. 475.

RIVETED JOINTS.

Manufacture of riveted steel pipe in the field. 244.

ROADS.

"Highway Location: General Considerations." Gibb Gilchrist. 454. Discussion: C. S. Mullen, H. L. Thackwell, B. P. Harrison, and E. P. Arneson. 462.

"Highway Location: Practical Considerations." A. R. Losh. 468. Discussion: H. K. Bishop, J. C. Carpenter, H. J. Spelman, H. S. Kerr, and H. W. Giffin. 477.

RUN-OFF.

"— -Rational — Formulas." R. L. Gregory and C. E. Arnold. 1038. Discussion: LeRoy K. Sherman, Francis Bates, John W. Raymond, Jr., Reginald A. Ryves, G. S. Tapley, W. I. Hicks, Carl H. Reeves, John M. Kemmerer, Leonard L. Longacre, G. H. Hickox, Donald M. Baker, Albert R. Arledge, Clarence S. Jarvis, C. E. Grunsky, Ralph W. Powell, Merrill M. Bernard, and Charles H. Lee. 1100.

SET-BACK LINES.

See **STREET LINES.**

SHALES.

Physical characteristics of clay and —. 629.

SIMILARITY, THEORY OF.

"Theory of Similarity and Models." Benjamin F. Groat. (With Discussion.) 273.

SNOW SHEDS.

—, Great Northern Railway. 920.

SPILLWAYS.

Round-head buttress type of spillway. 855.

STREET LINES.

"Legal Problems Involved in Establishing Set-Back Lines." Clifton Williams. 887. Discussion: J. M. Albers, J. E. Willoughby, Horace H. Sears, George H. Herrold, Clifford C. Muhs, H. B. Cooley, Walter C. Sadler, H. J. McFarlan, Donald M. Baker, and E. G. Walker. 896.

STREET WIDENING.
See **STREET LINES.**

STRESS.
"Analysis of Continuous Frames by Distributing Fixed-End Moments." Hardy Cross. (With Discussion.) 1.
"Design of a Reinforced Concrete Skew Arch." Bernard L. Weiner. (With Discussion.) 1212.
"Stresses in Gravity Dams by Principle of Least Work." B. F. Jakobsen. (With Discussion.) 489.
"The Buttressed Dam of Uniform Strength." Herman Schorer. (With Discussion.) 666.

STRUCTURES, THEORY OF.
"Analysis of Continuous Frames by Distributing Fixed-End Moments." Hardy Cross. 1. Discussion: C. P. Vetter, L. E. Grinter, S. S. Gorman, A. A. Eremin, Elmer F. Bruhn, A. H. Finlay, R. F. Lyman, R. A. Caughey, Orrin H. Pilkey, I. Oesterblom, Robert A. Black, H. E. Wessman, Edward J. Bednarski, S. N. Mitra, Jens Egede Nielsen, F. E. Richart, William A. Oliver, R. R. Martel, Clyde T. Morris, Francis P. Witmer, T. F. Hickerson, F. H. Constant, W. N. Downey, E. C. Hartmann, Thomas C. Shedd, David M. Wilson, Marshall G. Findley, George E. Large, Sophus Thompson and R. W. Cutler, Alfred Gordon, A. W. Earl, A. Floris, I. M. Nelidov, E. A. MacLean, George M. Dillingham, Donald E. Larson, and J. A. Van den Broek. 11.
See also **BRIDGES.**

SURVEYING.
"Relocating Land Surveys in Texas." B. F. Williams. 735. Discussion: J. M. Howe, Arthur D. Kidder, and W. J. Powell. 744.
Surveys and sub-divisions on the Don Martin Project. 838.
Surveys, construction methods, and a comparison of routes, Cascade Tunnel. 926.
"Surveys for Swift River Reservoir of the Boston Metropolitan Water Supply." N. LeRoy Hammond. 1020. Discussion: C. B. Breed, C. L. Hall, and Frank C. Sellnow. 1029.

SURVEYING, AERIAL.
Aerial surveys for Swift River Reservoir. 1025.
"Highway Location: General Considerations." Gibb Gilchrist. (With Discussion.) 454.

TIDES.
"Flow of Water in Tidal Canals." Earl I. Brown. (With Discussion.) 749.

TRAFFIC.
"Highway Location: Practical Considerations." A. R. Losh. (With Discussion.) 468.

TRIANGULATION.
— procedure, Swift River Reservoir. 1021.

TUNNELS.
Concrete lining of Cascade Tunnel. 978.
Construction plans and methods, Cascade Tunnel. 926.
"The Eight-Mile Cascade Tunnel, Great Northern Railway": A Symposium. D. J. Kerr, Frederick Mears, and J. C. Baxter. 915. Discussion: R. H. Keays, and J. G. Sullivan. 1001.
Tunneling for La Ola Pipe Line. 256.

VENTILATION.
— of Cascade Tunnel. 958.

WATER, FLOW OF, IN OPEN CHANNELS.
"A Discharge Diagram for Uniform Flow in Open Channels." Murray Blanchard. 865. Discussion: Lynn Perry, J. B. Macphail, G. B. Pillsbury, Lynn Crandall, Benjamin E Jones, A. J. Matheson, L. Standish Hall, and Ray E. Mackenzie. 871.
"Flow of Water in Tidal Canals." Earl I. Brown. 749. Discussion: R. L. Faris, G. T. Rude, Eugene E. Halmos, W. M. Black, H. F. Flynn, and Ralph G. Davis. 814.
"Measuring Irrigation Deliveries in the Panjab." E. S. Lindley. 1005.
"Theory of Similarity and Models." Benjamin F. Groat. (With Discussion.) 273.

WATER, FLOW OF, IN PIPES.
"Economical Design of Hydraulic Pipe-Line Dredge." John Francis Cushing. (With Discussion.) 625.
"Theory of Similarity and Models." Benjamin F. Groat. (With Discussion.) 273.

WATER, FLOW OF, OVER DAMS AND WEIRS.
"Tests of Broad-Crested Weirs." James G. Woodburn. 387. Discussion: H. W. King, Sherman M. Woodward, David L. Yarnell, Boris A. Bakhmeteff, D. D. Curtis, Erik G. W. Lindquist, and Ralph L. Parshall. 417.
Weirs with Aprons Inclined Up Stream and Down Stream. Alex. R. Webb. 408.

WATER STORAGE.
"Some Aspects of Water Conservation." R. A. Sutherland. 157. Discussion: E. C. Eaton, A. W. Simonds, W. H. R. Nimmo, C. R. Olberg, John W. Pritchett, and A. J. Gilardi. 198.

WATER-WORKS.
"Construction of La Ola Pipe Line in Chile." W. B. Saunders. (With Discussion.) 230.

WAVES.
"Flow of Water in Tidal Canals." Earl I. Brown. (With Discussion.) 749.

WEIRS.
See **WATER, FLOW OF, OVER DAMS AND WEIRS.**

WELLS.
"Flow of Ground-Water as Applied to Drainage —." M. R. Lewis. (With Discussion.) 1194.

AUTHOR INDEX

ADAMS, EDWARD DEAN.
Memoir of. 1636.

ADAMS, RAYMOND EDMOND.
Memoir of. 1388.

ADDIS, WILLIAM GEORGE.
Memoir of. 1389.

ADEY, WILLIAM HENRY.
Memoir of. 1390.

ALBERS, J. M.
Legal problems in set-back lines. 896.

ALGER, JULIAN WILMOT.
Memoir of. 1590.

ALLEN ANDREWS.
Memoir of. 1391.

ARLEDGE, ALBERT R.
Run-off — rational run-off formulas. 1139.

ARNESON, E. P.
Highway location: General considerations. 465.

ARNOLD, C. E.
Manufacturing concrete of uniform quality. 1367.
"Run-Off — Rational Run-Off Formulas." 1038.

ASH, LOUIS RUSSELL.
Memoir of. 1393.

ASSERSON, PETER CHRISTIAN.
Memoir of. 1397.

BAILEY, ALBERT ROSS.
Memoir of. 1592.

BAILEY, FRANK HARRISON.
Memoir of. 1403.

BAIRD, SAMUEL POND.
Memoir of. 1406.

BAKER, DONALD M.
Legal problems in set-back lines. 910.
Run-off — rational run-off formulas. 1136.

BAKHMETEFF, BORIS A.
Tests of broad-crested weirs. 423.

BARNES, DONALD P.
Stresses in gravity dams. 536.

BATEMAN, H.
Theory of similarity and models. 304.

BATES, FRANCIS.
Run-off — rational run-off formulas. 1100.

BAUER, EDWARD E.
Manufacturing concrete of uniform quality. 1363.

BAUMAN PAUL.
Buttressed dam of uniform strength. 715.

BAXTER, J. C.
"The Eight-Mile Cascade Tunnel, Great Northern Railway: Construction Plans and Methods." 950.

BEDNARSKI, EDWARD J.
Analysis of continuous frames. 52.

BEHREND, BERNARD ARTHUR.
Memoir of. 1407.

BELZNER, THEODORE.
Manufacturing concrete of uniform quality. 1360.

BERNARD, MERRILL M.
"Formulas for Rainfall Intensities of Long Duration." 592.
Run-off — rational run-off formulas. 1150.

BILLINGSLEY, JAMES WARTELLE.
Memoir of. 1411.

BISBEE, FRED MILTON
Memoir of. 1413.

BISHOP, H. K.
Highway location: Practical considerations. 477.

BLACK, ROBERT A.
Analysis of continuous frames. 35.

BLACK, W. M.
Flow of water in tidal canals. 821.

BLANCHARD, MURRAY.
"A Discharge Diagram for Uniform Flow in Open Channels." 865.

BOARDMAN, HOWARD EDWARD.
Memoir of. 1415.

BONNEY, O.
Manufacturing concrete of uniform quality. 1365.

BOUCHER, WILLIAM JAMES.
Memoir of. 1417.

BRACKENRIDGE, WILLIAM ALGERNON.
Memoir of. 1419.

BRAUNE, GUSTAVE MAURICE.
Memoir of. 1422.

BRECKINRIDGE, GEORGE CASTLEMAN.
Memoir of. 1594.

BREED, C. B.
Surveys for Swift River Reservoir. 1029.

BRODIE, HUGH.
Memoir of. 1426.

BROWN, EARL I.
"Flow of Water in Tidal Canals." 749.

BRUHN, ELMER F.
Analysis of continuous frames. 22.

BUNN, PAUL KING.
Memoir of. 1428.

BURDEN, MORTON.
Memoir of. 1595.

BURROUGHS, HECTOR ROBINS.
Memoir of. 1429.

CAIN, WILLIAM.
Stresses in gravity dams. 514.

CAMPBELL, GEORGE RAYMOND.
Memoir of. 1431.

CAMPUS, FERNAND.
Stresses in gravity dams. 567.

CARPENTER, J. C.
Highway location: Practical considerations. 480.

CARSON, HOWARD ADAMS.
Memoir of. 1386.

CARTER, EDWARD CARLOS.
Memoir of. 1433.

CAUGHEY, R. A.
Analysis of continuous frames. 28.

CHIPMAN, WILLIS.
Memoir of. 1435.

CHRISTENSEN, JOHN CHRISTIAN VALDEMAR.
Memoir of. 1438.

CLARKE, ST. JOHN.
Memoir of. 1439.

COLLINS, ARTHUR L.
Design of hydraulic pipe-line dredge. 658.

COMLY, JAMES RETZER.
Memoir of. 1442.

CONSTANT, F. H.
Analysis of continuous frames. 79.

COOLEY, H. B.
Legal problems in set-back lines. 903.

COPPÉE, HENRY ST. LEGER.
Memoir of. 1444.

CRAIGMILE, CHARLES JAMES.
Memoir of. 1445.

CRANDALL, LYNN.
A discharge diagram for uniform flow in open channels. 875.

CREAGER, WILLIAM P.
Gravity dams arched down stream. 1187.

CROCKER, HERBERT S.
"Some Reflections Regarding Engineering Practice": Address at the Annual Convention, Yellowstone National Park, Wyoming, July 6, 1932. 1380.

CROSS, HARDY.
"Analysis of Continuous Frames by Distributing Fixed-End Moments." 1.

CUMMINGS, WILLIAM WARREN.
Memoir of. 1447.

CURTIS, D. D.
Tests of broad-crested weirs. 434.

CUSHING, JOHN FRANCIS.
"Economical Design of Hydraulic Pipe-Line Dredge." 625.

CUTLER, R. W.
Analysis of continuous frames. 108.

DAHM, SVERRE.
Memoir of. 1448.

DAVIDSON, FRANK EUGENE.
Memoir of. 1451.

DAVIS, CALVIN V.
Buttressed dam of uniform strength. 697.

DAVIS, JOSEPH PHINEAS.
Memoir of. 1453.

DAVIS, RALPH G.
Flow of water in tidal canals. 827.

DAWSON, JOHN STANLEY.
Memoir of. 1457.

DESSERY, FLOYD GOSSETT.
Memoir of. 1458.

DETWILER, HARRY ESPENSHIP.
Memoir of. 1460.

DILLINGHAM, GEORGE M.
Analysis of continuous frames. 125.

DOWNEY, W. N.
Analysis of continuous frames. 80.

DRAFFIN, JASPER O.
Theory of similarity and models. 326.

DRAKE, ROBERT MORRIS.
Memoir of. 1597.

DUNLAP, FREDERIC CLARK.
Memoir of. 1461.

EARL, A. W.
Analysis of continuous frames. 112.

EASTERBROOKS, PRESTON BURT.
Memoir of. 1597.

EATON, E. C.
Some aspects of water conservation. 198.

EDWARDS, JAMES HARVEY.
Memoir of. 1463.

EISNER, F. J.
Theory of similarity and models. 329.

ELLIS, GEORGE EZRA.
Memoir of. 1466.

ELWELL, CHARLES CLEMENT.
Memoir of. 1467.

ELY, FRED W.
Stresses in gravity dams. 542.

EMERSON, FRANK COLLINS.
Memoir of. 1470.

EMORY, LLOYD TILGHMAN.
Memoir of. 1474.

EREMIN, A. A.
Analysis of continuous frames. 21.
Buttressed dam of uniform strength. 701.
Design of a skew arch. 1327.

FARIS, R. L.
Flow of water in tidal canals. 814.

FARNSWORTH, HOWARD RICHARDS.
Memoir of. 1476.

FINDLEY, MARSHALL G.
Analysis of continuous frames. 101.

FINLAY, A. H.
Analysis of continuous frames. 23.

FLORIS, A.
Analysis of continuous frames. 116.
Buttressed dam of uniform strength. 710.
Gravity dams arched down stream. 1190.
Stresses in gravity dams. 571.

FLYNN, H. F.
Flow of water in tidal canals. 822.

FOSTER, FRANK.
Memoir of. 1478.

FOUGNER, HERMANN.
Memoir of. 1480.

FUERTES, JAMES HILLHOUSE.
Memoir of. 1482.

GAGEL, EDWARD.
Memoir of. 1486.

GALLIVAN, JAMES HENRY.
Memoir of. 1600.

GERRY, M. H., Jr.
Stresses in gravity dams. 547.

GIFFIN, H. W.
Highway location: Practical considerations. 485.

GILARDI, A. J.
Some aspects of water conservation. 220.

GILCHRIST, GIBB.
"Highway Location: General Considerations." 454.

GIRAND, JAMES.
Buttressed dam of uniform strength. 689.

GLOVER, ROBERT E.
Stresses in gravity dams. 578.

GORDON, ALFRED.
Analysis of continuous frames. 111.

GORE, WILLIAM.
Stresses in gravity dams. 520.

GORMAN, S. S.
Analysis of continuous frames. 20.

GREGORY, R. L.
"Run-Off — Rational Run-Off Formulas." 1038.

GREGORY, W. B.
Formulas for rainfall intensities. 607.

GRIFFITH, JOHN H.
Manufacturing concrete of uniform quality. 1372.

GRINTER, L. E.
Analysis of continuous frames. 11.

GROAT, BENJAMIN F.
"Theory of Similarity and Models." 273.

GRUNSKY, C. E.
Formulas for rainfall intensities. 612.
Run-off — rational run-off formulas. 1142.

HALL, C. L.
Surveys for Swift River Reservoir. 1033.

HALL, L. STANDISH.
A discharge diagram for uniform flow in open channels. 879.
Ground-water flow to wells. 1207.

HALL, WILLIAM M.
"Manufacturing Concrete of Uniform Quality." 1336.

HALLETT, JAMES HENDRICKS.
Memoir of. 1601.

HALMOS, EUGENE E.
Flow of water in tidal canals. 819.

HAMMOND, N. LEROY.
"Surveys for Swift River Reservoir of the Boston Metropolitan Water Supply." 1020.

HARDWAY, BENJAMIN HURT.
Memoir of. 1489.

HARDING, JAMES CLARK.
Memoir of. 1494.

HARRISON, B. P.
Highway location: General considerations. 464.

HARTMANN, E. C.
Analysis of continuous frames. 93.

HAUG, T. L. E.
Gravity dams arched down stream. 1188.

HEREFORD, LEWIS STIRLING.
Memoir of. 1603.

HERROLD, GEORGE H.
Legal problems in set-back lines. 901.

HICKERSON, T. F.
Analysis of continuous frames. 74.

HICKOX, G. H.
Run-off — rational run-off formulas. 1134.

HICKS, W. I.
Run-off — rational run-off formulas. 1124.

HOADLEY, ANTHONY.
Stresses in gravity dams. 539.

HORNER, W. W.
Formulas for rainfall intensities. 607.

HOUTMAN, G. D.
Design of a skew arch. 1321.

HOWE, EDWARD WILLARD.
Memoir of. 1495.

HOWE, J. M.
Relocating land surveys in Texas. 744.

HOWE, WILLIAM CHAFFIN.
Memoir of. 1603.

HOWELL, C. H.
The Don Martin project. 855.

HUGHES, THOMAS ROGERS.
Memoir of. 1625.

HUNT, ANDREW MURRAY.
Memoir of. 1497.

ISRAELSEN, O. W.
Ground-water flow to wells. 1208.

JADWIN, EDGAR.
Memoir of. 1498.

JAKOBSEN, B. F.
Buttressed dam of uniform strength. 696.
Gravity dams arched down stream. 1189.
"Stresses in Gravity Dams by Principle of Least Work." 489.

JARVIS, CLARENCE S.
Run-off — rational run-off formulas. 1140.

JENSEN, ANDREW MARTIN.
Memoir of. 1504.

JONES, BENJAMIN E.
A discharge diagram for uniform flow in open channels. 876.

JONES, WILLIAM HENRY.
Memoir of. 1505.

JORGENSEN, LARS R.
Stresses in gravity dams. 528.

KALMAN, EUGENE.
Buttressed dam of uniform strength. 703.
Gravity dams arched down stream. 1184.
Stresses in gravity dams. 556.

KARNER, WILLIAM JOSIAH.
Memoir of. 1630.

KARPOV, A. V.
Theory of similarity and models. 308.

KEAYS, R. H.
Cascade Tunnel, Great Northern Railway. 1001.

KEHOE, ALFRED LAWRENCE.
Memoir of. 1506.

KELLY, J. W.
Manufacturing concrete of uniform quality. 1361.

KEMMERER, JOHN M.
Run-off — rational run-off formulas. 1127.

KERR, D. J.
"The Eight-Mile Cascade Tunnel, Great Northern Railway: Preliminary Studies and Results of Improving Cascade Crossing." 917.

KERR, H. S.
Highway location: Practical considerations. 483.

KIDDER, ARTHUR D.
Relocating land surveys in Texas. 745.

KIEL, FRANK BURDETTE.
Memoir of. 1604.

KING, H. J.
Drill-steel sharpening practise at West Portal, New Cascade Tunnel. 989.
Pneumatic concrete placing practise in the New Cascade Tunnel. 993.

KING, H. W.
Tests of broad-crested weirs. 417.

KONDO, SENTARO.
Memoir of. 1507.

LAMBERT, B. J.
"Gravity Dams Arched Down Stream." 1178.

LARGE, GEORGE E.
Analysis of continuous frames. 101.

LARSON, DONALD E.
Analysis of continuous frames. 127.

LAWSON, ELMER BARTHOLD.
Memoir of. 1626.

LAWTON, WILLIAM HENRY.
Memoir of. 1508.

LEE, CHARLES H.
Run-off — rational run-off formulas. 1161.

LELAND, GEORGE HERBERT.
Memoir of. 1510.

LEWIS, M. R.
"Flow of Ground-Water as Applied to Drainage Wells." 1194.

LINDLEY, E. S.
"Measuring Irrigation Deliveries in the Panjab." 1005.

LINDQUIST, ERIK G. W.
Tests of broad-crested weirs. 435.

LITTLE, CLIFFORD F.
Making and seasoning concrete test cylinders in the field. 1353.

LONGACRE, LEONARD L.
Run-off — rational run-off formulas. 1128.

LOSH, A. R.
"Highway Location: Practical Considerations." 468.

LYMAN, R. F.
Analysis of continuous frames. 27.

MacDIARMID, MILO STUART.
Memoir of. 1511.

MACKENZIE, RAY E.
A discharge diagram for uniform flow in open channels. 880.

MacLEAN, E. A.
Analysis of continuous frames. 123.

MACPHAIL, J. B.
A discharge diagram for uniform flow in open channels. 873.

McBETH, R. W.
Inspection of concrete construction. 1354.

McCULLOUGH, ERNEST.
Memoir of. 1512.

McFARLAN, H. J.
Legal problems in set-back lines. 907.

MAILEY, JOHN BRUCE.
Memoir of. 1607.

MARSHALL, HORACE MILLER.
Memoir of. 1514.

MARTEL, R. R.
Analysis of continuous frames. 60.

MASON, MORRIS.
Manufacturing concrete of uniform quality. 1371.

MATHESON, A. J.
A discharge diagram for uniform flow in open channels. 878.

MEARS, FREDERICK.
"The Eight-Mile Cascade Tunnel, Great Northern Railway: Surveys, Construction Methods, and a Comparison of Routes." 926.

MEIGS, MONTGOMERY.
Memoir of. 1516.

MENSCH, L. J.
Stresses in gravity dams. 524.

MERICKEL, BERNARD JOSEPH.
Memoir of. 1627.

METCALFE, JOSEPH DAVIS.
Memoir of. 1518.

MEYER, ADOLPH F.
Formulas for rainfall intensities. 608.

MILLER, JOHN HARRY.
Memoir of. 1628.

MIRICK, ALFRED STOWE.
Memoir of. 1608.

MITCHELL, D. T.
Manufacturing concrete of uniform quality. 1365.

MITRA, S. N.
Analysis of continuous frames. 55.

MOLERA, EUSEBIUS JOSEPH.
Memoir of. 1520.

MONTFORT, RICHARD.
Memoir of. 1521.

MORRIS, CLYDE T.
Analysis of continuous frames. 66.

MUHS, CLIFFORD C.
Legal problems in set-back lines. 902.

MULLEN, C. S.
Highway location: General considerations. 462.

MURPHY, JOSEPH LINCOLN.
Memoir of. 1609.

NAGLER, FLOYD A.
Gravity dams arched down stream. 1191.

NELIDOV, I. M.
Analysis of continuous frames. 117.
Buttressed dam of uniform strength. 716.

NELSON, ERNEST B.
La Ola pipe line, Chile. 266.

NIELSEN, JENS EGEDE.
Analysis of continuous frames. 55.

NIMMO, W. H. R.
Some aspects of water conservation. 207.

NOETZLI, FRED A.
Buttressed dam of uniform strength. 692.

NOYES, STEPHEN HENLEY.
Memoir of. 1521.

ODELL, FREDERICK SYLVESTER.
Memoir of. 1524.

OESTERBLOM, I.
Analysis of continuous frames. 32.

OLBERG, C. R.
Some aspects of water conservation. 212.

OLIVER, WILLIAM A.
Analysis of continuous frames. 60.

ONDERDONK, ARTHUR.
Memoir of. 1525.

PARDEE, ROBERT McKNIGHT.
Memoir of. 1614.

PARSHALL, RALPH L.
Tests of broad-crested weirs. 440.

PECK, JOHN SANFORD.
Manufacturing concrete of uniform quality. 1360.

PERRY, LYNN.
A discharge diagram for uniform flow in open channels. 871

PHELPS, WILLIAM COLLINS.
Memoir of. 1527.

PILKEY, ORRIN H.
Analysis of continuous frames. 29.

PILLSBURY, G. B.
A discharge diagram for uniform flow in open channels. 873.

PORTER, HENRY TEGMEYER.
Memoir of. 1528.

POWELL, R. W.
Formulas for rainfall intensities. 609.
Run-off — rational run-off formulas. 1149.

POWELL, W. J.
Relocating land surveys in Texas. 745.

PRESTON, HARRY LONGYEAR.
Memoir of. 1531.

PRITCHETT, JOHN W.
Some aspects of water conservation. 215.

RANDOLPH, WILLIAM COUTS, Jr.
Memoir of. 1615.

RATHBUN, J. CHARLES.
Design of a skew arch. 1323.

RAYMER, ALBERT REESOR.
Memoir of. 1534.

RAYMOND, JOHN W., Jr.
Run-off — rational run-off formulas. 1115.

REEVES, CARL H.
Run-off — rational run-off formulas. 1126.

REID, HOMER AUSTIN.
Memoir of. 1616.

REYNOLDS, ABRAHAM MOREAU.
Memoir of. 1536.

RICHART, F. E.
Analysis of continuous frames. 56.

RISER, KNUD SOPHUS.
Memoir of. 1537.

ROBERTS, EARL IVAN.
Memoir of. 1539.

ROOP, W. P.
Theory of similarity and models. 305.

RUDE, G. T.
Flow of water in tidal canals. 815.

RUDOLPH, WILLIAM E.
La Ola pipe line, Chile. 268.

RYVES, REGINALD A.
Run-off — rational run-off formulas. 1118.

SADLER, WALTER C.
Legal problems in set-back lines. 905.

SAUNDERS, W. B.
"Construction of La Ola Pipe Line, in Chile." 230.

SAUNDERS, WILLIAM LAWRENCE.
Memoir of. 1540.

SCHNEIDER, ANTON.
Memoir of. 1542.

SCHORER, HERMAN.
"The Buttressed Dam of Uniform Strength." 666.

SCHUYLER, PHILIP.
Memoir of. 1547.

SCHWAGER, A. C.
Buttressed dam of uniform strength. 714.

SEAMAN, HENRY B.
Manufacturing concrete of uniform quality. 1359.

SEARS, HORACE H.
Legal problems in set-back lines. 899.

SELLNOW, FRANK C.
Surveys for Swift River Reservoir. 1033.

SENGOKU, MITSUGU.
Memoir of. 1549.

SHAILER, ROBERT AMES.
Memoir of. 1553.

SHEDD, THOMAS C.
Analysis of continuous frames. 94.

SHERMAN, CHARLES W.
Formulas for rainfall intensities. 616.

SHERMAN, LeROY K.
Run-off — rational run-off formulas. 1100.

SIMMONS, EDWARD ALFRED.
Memoir of. 1632.

SIMONDS, A. W.
Some aspects of water conservation. 206.

SLATER, WILLIS APPLEFORD.
Memoir of. 1555.

SMITH, ED S., Jr.
Theory of similarity and models. 327.

SOWDEN, HERBERT JONES.
Memoir of. 1618.

SPELMAN, H. J.
Highway location: Practical considerations. 482.

SPENCER, PAUL BERTRAM.
Memoir of. 1619.

STANNARD, JAY L.
Memoir of. 1558.

STERN, EUGENE WASHINGTON.
Memoir of. 1560.

STEVENS, FRANK STODDARD.
Memoir of. 1562.

STOCKTON, ROBERT S.
The Don Martin project. 857.

SULLIVAN, J. G.
Cascade Tunnel, Great Northern Railway. 1003.

SUTHERLAND, R. A.
"Some Aspects of Water Conservation." 157.

TAPLEY, G. S.
Run-off — rational run-off formulas. 1122.

THACKWELL, H. L.
Highway location: General considerations. 462.
La Ola pipe line, Chile. 267.

THOMPSON, SOPHUS.
Analysis of continuous frames. 108.

THORNBURG, LEWIS.
Memoir of. 1564.

TUCKER, LESTER WALDO.
Memoir of. 1566.

TURNER, C. A. P.
Stresses in gravity dams. 530.

TYE, WILLIAM FRANCIS.
Memoir of. 1568.

VAN den BROEK, J. A.
Analysis of continuous frames. 133.

VAUGHN, R. L.
Design of hydraulic pipe-line dredge. 651.

VETTER, C. P.
Analysis of continuous frames. 11.

VOGT, FREDRIK.
Buttressed dam of uniform strength. 713.
Stresses in gravity dams. 548.

VOLCKMAN, GEORGE WILLIAM.
Memoir of. 1570.

WAGNER, FRED J.
Memoir of. 1572.

WAGNER, SAMUEL TOBIAS.
Memoir of. 1574.

WALKER, E. G.
Legal problems in set-back lines. 912.

WALKER, JOHN SIMPSON.
Memoir of. 1576.

WALKER, STANTON.
Manufacturing concrete of uniform quality. 1362.

WARE, SAMUEL HOUSTON.
Memoir of. 1621.

WEBB, ALEX. R.
Weirs with Aprons Inclined Up Stream and Down Stream. 408.

WEINER, BERNARD L.
"Design of a Reinforced Concrete Skew Arch." 1212.

WEISS, ANDREW.
"The Don Martin Project." 835.

WENDT, W. B.
Manufacturing concrete of uniform quality. 1374.

WERNER, P. WILHELM.
Stresses in gravity dams. 582.

WESSMAN, H. E.
Analysis of continuous frames. 40.

WILEY, ANDREW JACKSON.
Memoir of. 1577.

WILLACKER, CHARLES EDWARD.
Memoir of. 1622.

WILLIAMS, B. F.
"Relocating Land Surveys in Texas." 735.

WILLIAMS, CLIFTON.
"Legal Problems Involved in Establishing Set-Back Lines." 887.

WILLOUGHBY, J. E.
Legal problems in set-back lines. 898.

WILSON, DAVID M.
Analysis of continuous frames. 95.

WINTERMUTE, FERD CLARK.
Memoir of. 1624.

WITMER, FRANCIS P.
Analysis of continuous frames. 69.

WOODBURN, JAMES G.
"Tests of Broad-Crested Weirs." 387.

WOODS, HENRY DICKINSON.
Memoir of. 1584.

WOODWARD, SHERMAN M.
Tests of broad-crested weirs. 418.

WORTHINGTON, CHARLES.
Memoir of. 1586.

YAPPEN, ADOLPH.
Memoir of. 1588.

YARNELL, DAVID L.
Tests of broad-crested weirs. 419.

ZARBELL, ELMER.
Memoir of. 1589.